



# Opportunities for Learning Mathematics

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## A Workbook for Mathematics Educators Aligned to the Common Core State Standards

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# Scenarios for Teaching K-12 Mathematics

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The goal of this publication is to engage mathematics educators and researchers in analyzing scenarios of K-5 mathematics classrooms and provide alternative suggestions for furthering student understanding. Each scenario is a description of an episode that occurred in a mathematics lesson. These lessons were fully scripted during the process of observing teachers and students in the classroom as part of the Leadership Institute for Teachers (LIFT), a current NSF-funded project. During these observations, researchers documented situations in which opportunities for clarifying and deepening understanding of key mathematical concepts were missed.

Over 150 hours of observation were completed for a cohort of 31 teachers participating in the first year of LIFT. The grade levels observed ranged from Kindergarten through High School. The teachers were observed routinely, not by appointment, and were accustomed to observers sitting in their classrooms. In addition to the quantitative ratings of observations on two instruments, the scripting of lessons added a valuable component to the research. This qualitative approach served three purposes: 1) to document the scenario by listing the math concept, situation, and what the students did or did not understand; 2) to consider how the missed opportunities could be changed to better support student math learning, and 3) to add how the content of the lesson relates to the new Common Core State Standards for Mathematics (CCSS-M). At the end of three semesters of observations, the research team analyzed all the scenarios and looked for common themes. One of the themes that emerged related to number and operations, and this initial document focuses specifically on building an understanding of number and operations as well as algebraic thinking. These scenarios were taken from classroom practice prior to the implementation of Common Core State Standards. Once CCSS-M was adopted in our state, we analyzed the data through the filter of Common Core.

Mathematicians and educators who served as course developers for LIFT were interested in adding more examples from classroom practice into the LIFT Institute courses. The research team decided to further analyze and group the themes found in the scripted notes and put together a workbook on scenarios in number and operations. The research team wanted to provide actual classroom examples so that the course developers could use them as part of their course on Number to Algebra.

In addition, it is hoped that K-12 mathematics educators, coaches and researchers will use these scenarios as part of professional development with teachers. The workbook provides opportunities for the users to come up with their own enhancement to the lesson and then compare their work with experts. It also allows users to become familiar with the adopted CCSS-M for their grade levels while working on problem solving in classroom practice. This document covers grades K-5 and is part of a series. We are currently beginning work on a part two, which will cover grades 6 -12. Because this document is directed towards K-12, educators, course developers and researchers will be able to see the progression or vertical alignment of Common Core State Standards for Mathematics – Number and Operations in Base Ten and Operations and Algebraic Thinking Domains.

**Potential questions for readers to reflect on while reviewing these scenarios:**

- ❖ How would students respond if the questions were to be asked in a different way than as a yes/no or single answer response?
- ❖ Are there alternative methods that could be used in response to these episodes?
- ❖ Could the students have used representation, graphic organizers or manipulatives to develop more conceptual understanding?
- ❖ What math knowledge does the teacher need to possess in order to use these episodes as opportunities for learning?

**About the Authors:**

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# Table of Contents

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<b>Opportunities for Learning Mathematics</b> .....	0
Scenarios for Teaching K-12 Mathematics .....	i
<b>Table of Contents</b> .....	iii
<b>Getting Started</b> .....	1
<b>Scenarios and Questions</b> .....	3
Scenario A: Adding Doubles (Kindergarten) .....	4
Scenario B: Tens and Ones (Grade 1) .....	6
Scenario C: Number lines in Español (Grade 2) .....	8
Scenario D: Multiplication/Grouping (Grade 3).....	10
Scenario E: Fractions (Grade 4).....	12
Scenario F: Fact Families (Grade 5).....	14
<b>Authors' Analyses</b> .....	16
Scenario A: Adding Doubles (Kindergarten) .....	17
Scenario B: Tens and Ones (Grade 1) .....	21
Scenario C: Number lines in Español (Grade 2) .....	24
Scenario D: Multiplication/Grouping (Grade 3).....	27
Scenario E: Fractions (Grade 4).....	30
Scenario F: Fact Families (Grade 5).....	33

# Getting Started

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The intent of this workbook is to engage teachers and math coaches in discussions about math instruction as it ties to the CCSS. There is no right or wrong answers in this workbook, but rather these scenarios are intended as a resource to facilitate understanding of the CCSS by providing a stage with which to apply the standards to real-life scenarios. Each grade level scenario can be completed individually or as a small group. After reading the prologue, please follow these instructions for better use of this workbook.

1. Have a copy of the CCSS-M document at hand. The entire document is important so that users can become familiar with how to navigate the content standards and standards for mathematical practices. The document also leads to learning about math progressions and changes between grade levels. The document can be obtained from:  
<http://corestandards.org/the-standards/mathematics>.
2. Users should read the context and scenario starting on page 3.
3. Based on the context and scenario, users should independently respond to the following questions:
  - a. What do you think the students have understood or not understood about the lesson?
  - b. How could the content have been taught differently in order to improve the lesson?
  - c. What is the relationship of this lesson to the Common Core Standards (domain, cluster heading, cluster standards)?
  - d. What Standards for Mathematical Practices are evident in the scenario?Please note that there may be more than one response to each of these questions.
4. If working as a group, after completing the above questions independently, users may orally discuss their answers as a group and look for similarities, discrepancies, etc. between users.
5. After completing the questions and discussion, users may refer to the Author's Analyses section of this book to continue discussion and/or compare responses to the authors' analyses, as well as to obtain additional strategies that could have been applied to the case scenarios.

# Scenarios and Questions

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## Scenario A: Adding Doubles (Kindergarten)

### Context:

Kindergarten teachers are now expected to teach their students a higher level of mathematics than they did in the past. For example, according to the Common Core, by the end of kindergarten students are expected to be able to count to 20 by 1s, 5s and 10s. They also need to be able to understand how to add and subtract numbers up to five. The idea is to do lots of computation with small numbers so that students get the basic idea of number operations without having to memorize addition with large numbers. Using small numbers, the students can also learn useful strategies with a quantity they can handle and can use manipulatives to support simple addition and subtraction.

### Scenario:

Ms. Smith has drawn an octopus on the board with 8 tentacles and then draws a second octopus with no tentacles. She then asks the students to draw both octopi and add the right number of tentacles to the second octopus.

The students are drawing their two octopi. Beth says, "18."

The teacher says, "Can you show me how you know it is 18?"

Beth doesn't respond. Ms. Smith says to the class, "Draw it, we are looking at the tentacles."

Beth who had suggested an answer stops working and doesn't draw.

Juan, another student, shows the teacher his picture. "I got 8 and 11 on this one."

Pointing at the second Octopus, Ms. Smith says, "That one has too many, how can you fix it?"

Juan erases all of the tentacles on the second octopus and doesn't draw any tentacles on the second octopus. Ms. Smith tells Juan, "Count on."

Juan doesn't understand and says "Eight." He now has only one drawing and there are only eight tentacles. Ms. Smith then says, "Count with me, 9, 10, 11" using her fingers to show these numbers.

Ms. Smith notices Amanda drew 12 tentacles on the second octopus and asks Amanda, "How many are there supposed to be?" Amanda fixes her drawing so there are 8 tentacles on both octopi. Amanda then counts and writes 17 tentacles because she counted 9 tentacles on the first octopus. The teacher does not notice.

**Based on this scenario on adding doubles, what may the students have understood or not understood about the lesson?**

**Understood:** \_\_\_\_\_

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**Not Understood:** \_\_\_\_\_  
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**How could the content have been taught differently in order to improve the lesson?**

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**What is the relationship of this lesson to the Common Core Standards?**

**Domain:** \_\_\_\_\_

**Cluster Heading:** \_\_\_\_\_

**Cluster Standards:** \_\_\_\_\_

**What Standards for Mathematical Practices are evident in this scenario?**

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## Scenario B: Tens and Ones (Grade 1)

### **Context:**

Place value is important for students to understand early in their schooling. Understanding numbers and developing number sense requires that students understand the meaning of the order in which numbers are read and written. This understanding is critical later for addition and subtraction.

### **Scenario:**

Mr. Jones is teaching in a first grade inclusion classroom with one ELL student. He had drawn the following chart on the board as part of their opening calendar activity related to October 13th:

1	3
Tens	Ones

At the end of the calendar activity, Rocio asks: "Why is the number one not with the word one?" The teacher does not respond to Rocio, but asks the students to begin another activity unrelated to the idea of place value.

**Based on this scenario on tens and ones, what may the students have understood or not understood about the lesson?**

**Understood:** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Not Understood:** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**How could the content have been taught differently in order to improve the lesson?**

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**What is the relationship of this lesson to the Common Core Standards?**

Domain: \_\_\_\_\_

Cluster Heading: \_\_\_\_\_

Cluster Standard: \_\_\_\_\_

**What Standards for Mathematical Practices are evident in this scenario?**

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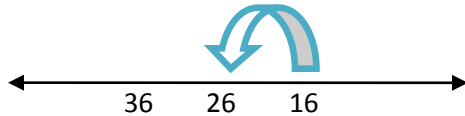
## Scenario C: Number lines in Español (Grade 2)

### Context:

Number lines are used in many successful curricula in places like Singapore, Holland and Finland to help students gain a representational model for number comparisons and operations. Number lines help students learn the directionality of numbers and can be eventually used to introduce negative as well as positive numbers. The number line can be used as an addition or subtraction strategy. This lesson was taught in a second grade dual language Spanish-English classroom on a day in which Spanish was the language of teaching and learning. The teacher-to-student conversation is translated to English in parentheses.

### Scenario:

Ms. Garcia is teaching subtraction using a number line model to her dual language, second grade classroom. Students have to show how to solve  $36 - 20$  using any strategy they had learned. Ms. Garcia offers a possible strategy by saying, "Pudo ser  $36 - 10 - 10$ ." (*Could have been  $36 - 10 - 10$ .*) Pedro was excited by this offering and said, "Yo hice la recta numérica." (I did the number line.) Ms. Garcia asks, "¿Y como quitaste, de 10 en 10?" (*How did you take away, 10 by 10?*) Pedro went to the board and drew this representation but did not explain what he did:



Ms. Garcia provides feedback to Pedro by saying, "Recuerda de escribir los saltos, no se te olvide poner -10 y -10, muy bien." (*Remember to write your jumps, don't forget to put minus 10 and minus 10, very good.*) Pedro goes back to his seat.

### **Based on this scenario on number lines, what may the students have understood or not understood about the lesson?**

**Understood:** \_\_\_\_\_

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**Not Understood:** \_\_\_\_\_

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**How could the content have been taught differently in order to improve the lesson?**

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**What is the relationship of this lesson to the Common Core Standards?**

Domain: \_\_\_\_\_

Cluster Heading: \_\_\_\_\_

Cluster Standard: \_\_\_\_\_

**What Standards for Mathematical Practices are evident in this scenario?**

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## Scenario D: Multiplication/Grouping (Grade 3)

### Context:

Multiplication is usually introduced using equal groups. The most common problems that we use are putting things into equal groups or separating amounts into equal groups. In CCSS this is referred to as:

Group Size Unknown (How many in each group?)	$3 \times ? = 18$
Number of Groups Unknown (How many groups?)	$? \times 6 = 18$

According to the Operations and Algebraic Thinking domain of CCSS for Grade 1, students are expected to “understand and apply properties of operations and the relationships between addition and subtraction: apply properties of operations as strategies to add and subtract. Examples: If  $8 + 3 = 11$  is known, then  $3 + 8 = 11$  is also known (commutative property of addition)” (p.15).

According to the Operations and Algebraic Thinking domain of CCSS for Grade 2, students are expected to “work with equal groups of objects to gain foundations for multiplication: use additions to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.” (p. 19).

According to the Operations and Algebraic Thinking domain of CCSS for Grade 3, students are expected to “understand properties of multiplication and the relationship between multiplication and division: apply properties of operations as strategies to multiple and divide. Examples: if  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known (commutative property of multiplication).” (p. 23).

Ms. Burns is teaching multiplication using a model with a missing factor, where the size of the group is unknown.

### Scenario:

Ms. Burns writes the following equation on the board for  $6 \times \square = 42$  and asks the students to find the missing factor and to tell her how they can group them. Naomi draws 42 tally marks to represent the total and says, “By 6”. The teacher asks, “Why?” and the Naomi answers, “I don’t know.”

Natalie says, “Because  $7 \times 6$  is 42.”

Ms. Burns didn’t hear Natalie and asks the class, “Who wants to group by 6?”

The teacher asks Luis to go to the board to group by 6 (making 7 groups).

Jose then says, “I think by 7 because  $7 \times 6$ .”

(At this point Jose has suggested that they group by 7 but Luis is already at the board grouping by 6)

Ms. Burns does not respond and waits for Luis to finish grouping by 6. She then asks the class, “So what did we find? “

There is no response from the students in the class.

Ms. Burns then asks, "Is it 7?"

The students respond, "Yes."

**Based on this scenario on multiplication/grouping, what may the students have understood or not understood about the lesson?**

**Understood:** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Not Understood:** \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**How could the content have been taught differently in order to improve the lesson?**

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**What is the relationship of this lesson to the Common Core Standards?**

**Domain:** \_\_\_\_\_

**Cluster Heading:** \_\_\_\_\_

**Cluster Standard:** \_\_\_\_\_

**What Standards for Mathematical Practices are evident in this scenario?**

\_\_\_\_\_  
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\_\_\_\_\_  
\_\_\_\_\_



## Scenario E: Fractions (Grade 4)

### Context:

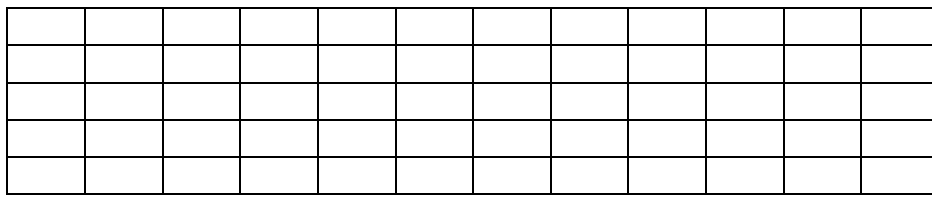
Fractions are one of the hardest areas for students to understand as represented by their standardized tests results and observations of their learning in classrooms. The scenario below shows an example of a misrepresentation of fractions that could haunt students for many years especially as they move into relating fractions and percents.

### Scenario:

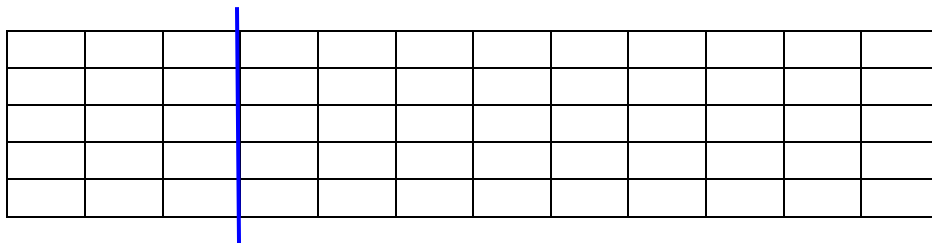
Mr. Sanchez is teaching a fourth grade inclusion classroom that includes special education students.

He shows students a 12 by 5 grid cut into 60 pieces that he has drawn on the board. He says, "In this grid there are 60 of  $\frac{1}{4}$ . Can someone show me  $\frac{1}{4}$  inside of it?"

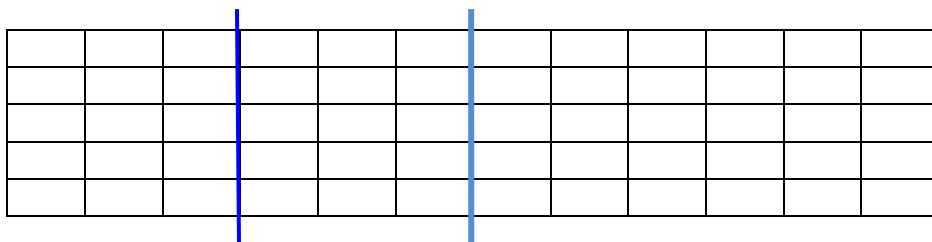
(He is asking the students to show a  $\frac{1}{4}$  of the whole shape, but he isn't asking this question correctly.)



Leah draws a line after the third column of 5 to show a fourth.



Then the teacher draws another line after the 6th column of 5 to show another fourth.



Students are asked to copy this representation onto their papers. There is no discussion as to why this makes a fourth or what these two sections represent.

**Based on this scenario on fractions, what may the students have understood or not understood about the lesson?**

Understood: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Not Understood: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**How could the content have been taught differently in order to improve the lesson?**

\_\_\_\_\_  
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\_\_\_\_\_  
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\_\_\_\_\_  
\_\_\_\_\_

**What is the relationship of this lesson to the Common Core Standards?**

Domain: \_\_\_\_\_

Cluster Heading: \_\_\_\_\_

Cluster Standard: \_\_\_\_\_

**What Standards for Mathematical Practices are evident in this scenario?**

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

## Scenario F: Fact Families (Grade 5)

### Context:

One of the more efficient ways for students to learn their addition and subtraction facts is to learn them through working with Fact Families. Using addition and subtraction fact families helps students understand that subtraction undoes addition and vice versa. Learning the fact families for multiplication and division also supports students in thinking about the relationships between multiplication and division. It also provides a more efficient way to remember multiplication facts. Once you know these facts you can apply the inverse to solve simple division problems. In this example the teacher of a fifth grade class suddenly gets confused about how to present fact families when working with the fact family of 2 and 1.

### Scenario:

Mrs. Winters is teaching a regular fifth grade class and does an introductory lesson using fact families. She asked the students to give her a suggestion for a fact family they would like to show. Jasper suggests the Fact Family for the number 2.

Mrs. Winters writes on the board the following:

$$2 \times 1$$

$$1 \times 2$$

$$2 \div 1$$

Then Mrs. Winters suddenly freezes and is unable to come up with the missing expression. (She may have been thinking that 1 divided by 2 is  $\frac{1}{2}$  and she didn't want to show a fraction. The missing fact is  $2 \div 2 = 1$ ).

The teacher then tells the students that this was a hard one, so let's do a different number. Mrs. Winters writes:

$$22 \times 10 = 220$$

$$10 \times 22 = 220$$

$$220 \div 10 = 22$$

$220 \div 22 = 10$  and completes the Fact Family for 220 instead.

**Based on this scenario on fact families, what may the students have understood or not understood about the lesson?**

**Understood:** \_\_\_\_\_  
\_\_\_\_\_

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**Not Understood:** \_\_\_\_\_  
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**How could the content have been taught differently in order to improve the lesson?**

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**What is the relationship of this lesson to the Common Core Standards?**

**Domain:** \_\_\_\_\_

**Cluster Heading:** \_\_\_\_\_

**Cluster Standard:** \_\_\_\_\_

**What Standards for Mathematical Practices are evident in this scenario?**

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# **Authors' Analyses**

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## Scenario A: Adding Doubles (Kindergarten)

### **Context:**

Kindergarten teachers are now expected to teach their students a higher level of mathematics than they did in the past. For example according to the Common Core, by the end of kindergarten students are expected to be able to count to 20 by 1s, 5s and 10s. They also need to be able to understand how to add and subtract numbers up to five. The idea is to do lots of computation with small numbers so that students get the basic idea of number operations without having to memorize addition with large numbers. Using small numbers, the students can also learn useful strategies with a quantity they can handle and can use manipulatives to support simple addition and subtraction.

### **Scenario:**

Ms. Smith has drawn an octopus on the board with 8 tentacles and then draws a second octopus with no tentacles. She then asks the students to draw both octopi and add the right number of tentacles to the second octopus.

The students are drawing their two octopi. Beth says, "18."

The teacher says, "Can you show me how you know it is 18?"

Beth doesn't respond. Ms. Smith says to the class, "Draw it, we are looking at the tentacles."

Beth who had suggested an answer stops working and doesn't draw.

Juan, another student, shows the teacher his picture. "I got 8 and 11 on this one."

Pointing at the second Octopus, Ms. Smith says, "That one has too many, how can you fix it?"

Juan erases all of the tentacles on the second octopus and doesn't draw any tentacles on the second octopus. Ms. Smith tells Juan, "Count on."

Juan doesn't understand and says "Eight." He now has only one drawing and there are only eight tentacles. Ms. Smith then says, "Count with me, 9, 10, 11" using her fingers to show these numbers.

Ms. Smith notices Amanda drew 12 tentacles on the second octopus and asks Amanda, "How many are there supposed to be?" Amanda fixes her drawing so there are 8 tentacles on both octopi. Amanda then counts and writes 17 tentacles because she counted 9 tentacles on the first octopus. The teacher does not notice.

### **What the students may be understanding, or not understanding:**

Beth attempted to answer the addition question but did not get any feedback and did not know that drawing the picture might help her. She hasn't connected representational objects with numbers, at least in this case.

Juan has drawn two octopi but has the wrong number of tentacles on the second one. Instead of counting the tentacles and erasing only three of them he erases all the tentacles. He also doesn't know what the teacher means by "counting on." The students probably had few or little strategies for counting on by themselves. They may have just been counting with the teacher without knowing they were using the "counting on" strategy.

Amanda after her first try was then able to draw the correct representation of the tentacles. However, she didn't understand the correspondence between numbers and pictures and wasn't careful with her counting.

### **Suggested Alternative Strategies:**

**Using different examples:** it might be inappropriate to start with an octopus, especially for students who live in the desert. Ms. Smith could have modeled this addition problem using a different animal like a roadrunner which has only two legs, a dog with four legs, or a tarantula which has eight legs. *(MP4: Model with mathematics)*

**Using different numbers:** CCSS suggests to decompose numbers less than or equal to 10 into pairs in more than one way. Ms. Smith might have chosen to use smaller numbers to make this task more doable and aligned to the CCSS. For example, Ms. Smith might have chosen to draw two dogs since most students are already familiar with the idea of four-legged dogs. *(MP2: Reason abstractly and quantitatively)*

**Using the "counting on" strategy:** Ms. Smith could have modeled the first complete drawing, then drawn the second two legs with the students, and then have them count on with her to model the strategy of "counting on." Then she could have asked the students to do the task with perhaps two dogs who only have four legs each. *(MP3: Construct viable argument and critique the reasoning of them; MP7: Look for and make use of structure)*

**Using manipulatives:** Ms. Smith, as a kindergarten teacher should have introduced simple manipulatives like counters, beans, or unifix cubes from the beginning of the year so that students could use objects to help them bridge from concrete to representational to abstract thinking. Students at this grade level use unifix cubes to show towers of 10's, thus students could have created a tower of  $8 + 2$  to show a tower of 10. *(MP5: Use appropriate tools strategically)*

**Counting and cardinality:** Ms. Smith could also have taken advantage of this lesson to connect counting to cardinality to the CCSS domain. She could have talked about the first leg, the second leg, etc. *(MP6: Attend to precision)*

### **Relation of Scenario to Common Core Standards:**

**Domain:** Counting and Cardinality

**Cluster Heading:** Count to tell the number of objects.

**Cluster Standard:**

4. Understand the relationship between numbers and quantities: connect counting to cardinality.
  - a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
  - b. Understand that the last number name said tell the number of objects counted.

**Domain:** Operations and Algebraic Thinking

**Cluster Heading:** Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

**Cluster Standards:**

1. Represent addition ... with objects, fingers, mental images, drawing, sounds, acting out situations, verbal explanations, expression or equation.
2. Solve addition (and add and subtract within 10) by using objects or drawings to represent the problem.
5. Fluently add and subtract within 5.

**Standards for Mathematical Practices connected to this Kindergarten case scenario:**

**1. Make sense of problems and persevere in solving them.**

Students in Grade K should find an entry point to the problem. Emphasizing on the root-word “octo” meaning (8), could have led to an entry point for students.

**2. Reason abstractly and quantitatively.**

Students in Grade K decontextualize task into numbers and symbols, number + number = \_\_\_\_, and then solve the task. Students also contextualize the situation by knowing that they need to add the quantity of second object to quantity of the given object.

**3. Construct viable arguments and critique the reasoning of them.**

Students in Grade K accurately use mathematical terms to construct arguments and engage in discussions about problem solving strategies. A student may argue that the reason for this being an addition problem being that the animals joined together rather than going away, which would have indicated a subtraction equation.

**4. Model with mathematics.**

Students in Grade K model real-life situations with a number sentence or an equation, and check to make sure that the equation accurately matches the problem context.

**5. Use appropriate tools strategically.**

During Grade K classroom instruction, students should have access to various mathematical tools as well as to paper, and explain why they use these specific tools. The SMP suggests that in Grade K, students work focus on concrete manipulatives first, then pictorial representations. Tools for this scenario may include counters or hundreds-charts.

**6. Attend to precision.**

Students in Grade K are expected to use grade level appropriate vocabulary accurately, as well as give precise explanations and reasoning regarding their process of finding solutions. In this scenario, Juan and Amanda did not have 8 tentacles for the second octopus. Amanda



represented 8 tentacles on the first octopus but made an error and counted 9 instead. Students are expected to check their work to ensure accuracy and reasonableness.

**7. Look for and make use of structure.**

Students in Grade K carefully look for patterns and structures in the number system. They begin to decompose numbers less than or equal to 10.

**8. Look for and express regularity in repeated reasoning.**

Students in Grade K begin to look for regularity in problem structures. They begin composing and decomposing numbers in different ways, such as:  $4 = 0 + 4$ ;  $4 = 1 + 3$ ;  $4 = 2 + 2$ ;  
 $4 = 3 + 1$ ;  $4 = 4 + 0$ .

## Scenario B: Tens and Ones (Grade 1)

### **Context:**

Place value is important for students to understand early in their schooling. Understanding numbers and developing number sense requires that students understand the meaning of the order in which numbers are read and written. This understanding is critical later for addition and subtraction.

### **Scenario:**

Mr. Jones is teaching in a first grade inclusion classroom with one ELL student. He had drawn the following chart on the board as part of their opening calendar activity related to October 13th:

1	3
Tens	Ones

At the end of the calendar activity, Rocio asks: “Why is the number one not with the word one?” The teacher does not respond to Rocio, but asks the students to begin another activity unrelated to the idea of place value.

### **What the students may be understanding, or not understanding:**

Rocio is able to read words and numbers and wonders why the word Ones is not below the number 1. As a result of a student’s question the teacher missed the opportunity to talk with the class about the meaning of place value.

### **Suggested Alternative Strategies:**

**Probing the student:** Mr. Jones could have asked Rocio to explain what she was asking about. Rocio probably would have wanted the 3 to be above the word tens and the one to be above the word ones. *(MP1: Make sense of problems and persevere in solving them; MP3: Construct viable arguments and critique the reasoning of them; MP6: Attend to precision)*

**Introducing Place Value:** According to the Number and Operations in Base Ten domain of CCSS:

“Use place value understanding and properties of operations to add and subtract: add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models of drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.” (p. 16).

Mr. Jones could also have introduced the idea of singles (ones) and a bundle of singles called ten. Students could have used straws to represent the number of days in the calendar, where each ten days would be bunched together to create a tens unit. *(MP4: Model with mathematics; MP5: Use appropriate tools strategically; MP7: Look for and make use of structure)*

**Decomposing numbers:** The students could have explored the differences between 12 and 21, to show how the numbers can decompose. The table below shows an example of how this could be done:

Tens	Ones	Student Equation
1	2	$10 + 2 = 12$ days
2	1	$20 + 1 = 21$ days

(MP2: Reason abstractly and quantitatively; MP4: Model with mathematics; MP7: Look for and make use of structure)

**Looking for patterns:** In this scenario, students could have looked for patterns in the days of the month that begin with a 1 and the days of the month that end in 1. Notice that 11 will appear in both patterns. (MP8: Look for and express regularity of repeated reasoning)

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Tens Pattern	Ones Pattern
$10 = 10 + 0$	$1 = 0 + 1$
$11 = 10 + 1$	$11 = 10 + 1$
$12 = 10 + 2$	$21 = 20 + 1$
$13 = 10 + 3$	$31 = 30 + 1$
$14 = 10 + 4$	
$15 = 10 + 5$	
$16 = 10 + 6$	
$17 = 10 + 7$	
$18 = 10 + 8$	
$19 = 10 + 9$	

**Relation of Scenario to the Common Core Standards:**

**Domain:** Number and Operations in Base Ten

**Cluster Heading:** Understanding Place Value

**Cluster Standard:**

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
  - a. 10 can be thought of as a bundle of ten ones - called a "Ten".
  - b. The numbers from 11-19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones.

### **Standards for Mathematical Practices connected to this Grade 1 case scenario:**

**1. Make sense of problems and persevere in solving them.**

In Grade 1, students work builds from Grade K and still relies on concrete manipulatives and pictorial representations. They examine the problem and find an entry point. This scenario shows that the student was making a word/number association but noticed that something did not make sense. At this grade level, if students reach a point at which they are stuck, they can reexamine the task in a different way and continue to solve the task.

**2. Reason abstractly and quantitatively.**

In Grade 1, students represent situations by decontextualizing into numbers and symbols. For example,  $13 = \_ + \_$  and then solve. Grade 1 students also contextualize the task. In this scenario they needed to show that the number 1 in the tens column was equivalent to 10 days, so  $13 = 10 + 3$ .

**3. Construct viable arguments and critique the reasoning of them.**

Students in Grade 1 accurately use definitions and previously established answers to make arguments. They use a variety of strategies to solve tasks. They are expected to share strategies and discuss their classmates' strategies. For example, the number 1 written under the ones heading would represent one day, versus the number 1 written under the tens heading would represent 10 days.

**4. Model with mathematics.**

Grade 1 students use concrete manipulatives or pictorial representations and are expected to write an equation to model the problem.

**5. Use appropriate tools strategically.**

Grade 1 students have access to and use tools appropriately. They should be able to explain that the 1 in the tens place refers to one set of 10 units. In this case the use of straws would have been appropriate.

**6. Attend to precision.**

Mathematically proficient students in Grade 1 are precise in their communication, calculations and measurements. In this scenario, the 1 in the tens column has more quantity than if 1 had been in the ones column. Students could check to ensure the accuracy for this solution.

**7. Look for and make use of structure.**

Students in Grade 1 realize that any two-digit number can be broken into tens and ones.

**8. Look for and express regularity in repeated reasoning.**

Students in Grade 1 begin to look for regularity, such as patterns, in problem structures.

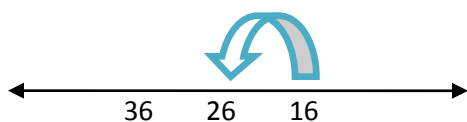
## Scenario C: Number lines in Español (Grade 2)

### **Context:**

Number lines are used in many successful curricula in places like Singapore, Holland and Finland to help students gain a representational model for number comparisons and operations. Number lines help students learn the directionality of numbers and can be eventually used to introduce negative as well as positive numbers. The number line can be used as an addition or subtraction strategy. This lesson was taught in a second grade dual language Spanish-English classroom on a day in which Spanish was the language of teaching and learning. The teacher-to-student conversation is translated to English in parentheses.

### **Scenario:**

Ms. Garcia is teaching subtraction using a number line model to her dual language, second grade classroom. Students have to show how to solve  $36 - 20$  using any strategy they had learned. Ms. Garcia offers a possible strategy by saying, “Pudo ser  $36 - 10 - 10$ .” (*Could have been  $36 - 10 - 10$ .*) Pedro was excited by this offering and said, “Yo hice la recta numérica.” (I did the number line.) Ms. Garcia asks, “¿Y como quitaste, de 10 en 10?” (*How did you take away, 10 by 10?*) Pedro went to the board and drew this representation but did not explain what he did:



Ms. Garcia provides feedback to Pedro by saying, “Recuerda de escribir los saltos, no se te olvide poner  $-10$  y  $-10$ , muy bien.” (*Remember to write your jumps, don’t forget to put minus 10 and minus 10, very good.*) Pedro goes back to his seat.

### **What the students may be understanding, or not understanding:**

Pedro may understand that a subtraction problem in a number line goes from the bigger number to the smaller number and therefore drew the direction of the numbers in the number line as such. Pedro did not understand that moving to the left on a number line represents subtraction, and moving to the right represents addition.

The student only represents one of the tens that need to be subtracted from the total of 36.

### **Suggested Alternative Strategies:**

**Direction of a number line:** An accurate representation of the number line showing direction should have been the basis for this lesson. Once students know the number line (0-100) and that they can subtract by moving backwards and add by moving forwards, they will have a stronger mental representation of these operations. Once students are familiar with the number line extending from 0 to 100, they may alter how they represent it to use it for operations. *(MP1: Makes sense of problems and persevere in solving them; MP4: Model with mathematics)*

**Discussing the number line:** Pedro could have talked about why he jumped backwards by ten twice on the number line. *(MP3: Construct viable arguments and critique the reasoning of them; MP6: Attend to precision; MP8: Look for and express regularity in repeated reasoning)*

**Comparing student strategies:** If another student solved the problem also using a number line, the two number lines could have been compared. *(MP2: Reason abstractly and quantitatively; MP3: Construct viable arguments and critique the reasoning of them; MP4: Model with mathematics; MP7: Look for and make use of structure; MP8: Look for and express regularity in repeated reasoning)*

**Labeling the jumps:** The teacher's focus was on labeling the jumps, not the direction of the number line. The teacher did not model labeling the jumps for the students. *(MP4: Model with mathematics; MP5: Use appropriate tools strategically)*

**Using a hundreds-chart:** This problem could have also been modeled on a hundreds-chart. *(MP4: Model with mathematics; MP5: Use appropriate tools strategically)*

### **Relation of Scenario to the Common Core Standards:**

**Domain:** Operations and Algebraic Thinking

**Cluster Heading:** Represent and solve problems involving addition and subtraction.

**Cluster Standard:**

1. Using addition and subtraction within 100 drawing to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.

### **Standards for Mathematical Practices connected to this Grade 2 case scenario:**

#### **1. Make sense of problems and persevere in solving them.**

Grade 2 students find an entry point and develop a foundation for problem solving strategies. In this scenario, Pedro used a number line as an entry point; however, after completing the number line he failed to reexamine that the numbers 36, 26, and 16 were not in the appropriate order. Pedro should have placed the numbers on the line as 16, 26, and 36.

#### **2. Reason abstractly and quantitatively.**

In grade 2, students represent situations by decontextualizing tasks into numbers and symbols and write equations. In this scenario, the problem was  $36 - 20$ , in which the number 20 was

decomposed - 10 - 10. Students also contextualize situations. Students can refer to the context that  $10 + 10 = 20$ , which was the total amount they had to subtract.

**3. Construct viable arguments and critique the reasoning of them.**

Mathematically proficient students in Grade 2 accurately use definitions and previously established solutions to construct viable arguments about mathematics; they also critique strategies and solutions of their classmates. In this case, the answer to the problem was correct, but the solution was not because the direction of the numbers on the number line was reversed.

**4. Model with mathematics.**

Mathematically proficient students in Grade 2 model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. In this scenario, the answer to  $36 - 10 - 10 = 16$  was correct, but the pictorial representation of the number line was incorrect.

**5. Use appropriate tools strategically.**

Mathematically proficient students in Grade 2 have access to and use tools appropriately. The use of the number line was one tool to support learning. Other tools could have been hundreds-charts and unifix cubes to show units of tens and ones.

**6. Attend to precision.**

Students in Grade 2 are expected to communicate and calculate accurately and showing reasonableness in their solutions. In this scenario, directionality of the number line should have been discussed.

**7. Look for and make use of structure.**

Students in Grade 2 carefully look for patterns and structures. Students can skip count by 10's to show addition or subtraction. Students in grade 2 also make use of structure to show that  $36 - 20 = \underline{\quad}$  can also be written as  $20 + \underline{\quad} = 36$ .

**8. Look for and express regularity in repeated reasoning.**

Students in grade 2 begin to look for regularity in problem structures. After solving  $36 - 20$ , by decomposing the second number to  $36 - 10 - 10$ , students may begin to generalize and apply that strategy to the future. For example, the students could apply the same strategies to the equation  $58 - 30 = \underline{\quad}$ , written as  $58 - 10 - 10 - 10 = \underline{\quad}$ .

## Scenario D: Multiplication/Grouping (Grade 3)

### Context:

Multiplication is usually introduced using equal groups. The most common problems that we use are putting things into equal groups or separating amounts into equal groups. In CCSS this is referred to as:

Group Size Unknown (How many in each group?)	$3 \times ? = 18$
Number of Groups Unknown (How many groups?)	$? \times 6 = 18$

According to the Operations and Algebraic Thinking domain of CCSS for Grade 1, students are expected to “understand and apply properties of operations and the relationships between addition and subtraction: apply properties of operations as strategies to add and subtract. Examples: If  $8 + 3 = 11$  is known, then  $3 + 8 = 11$  is also known (commutative property of addition)” (p.15).

According to the Operations and Algebraic Thinking domain of CCSS for Grade 2, students are expected to “work with equal groups of objects to gain foundations for multiplication: use additions to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.” (p. 19).

According to the Operations and Algebraic Thinking domain of CCSS for Grade 3, students are expected to “understand properties of multiplication and the relationship between multiplication and division: apply properties of operations as strategies to multiple and divide. Examples: if  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known (commutative property of multiplication).” (p. 23).

Ms. Burns is teaching multiplication using a model with a missing factor, where the size of the group is unknown.

### Scenario:

Ms. Burns writes the following equation on the board for  $6 \times \square = 42$  and asks the students to find the missing factor and to tell her how they can group them. Naomi draws 42 tally marks to represent the total and says, “By 6”. The teacher asks, “Why?” and the Naomi answers, “I don’t know.”

Natalie says, “Because  $7 \times 6$  is 42.”

Ms. Burns didn’t hear Natalie and asks the class, “Who wants to group by 6?”

The teacher asks Luis to go to the board to group by 6 (making 7 groups).

Jose then says, “I think by 7 because  $7 \times 6$ .”

(At this point Jose has suggested that they group by 7 but Luis is already at the board grouping by 6)

Ms. Burns does not respond and waits for Luis to finish grouping by 6. She then asks the class, “So what did we find? “



There is no response from the students in the class.

Ms. Burns then asks, "Is it 7?"

The students respond, "Yes."

**What the students may be understanding, or not understanding:**

Naomi understands that the whole set of numbers is 42 and then says "by 6" but doesn't know why. Natalie says why Naomi's answer is 6, by giving the correct equation and saying, "7 X 6 is 42, so 6." Jose also tries to give the correct answer. Both students are solving for the missing number based on memorization of multiplication facts, and probably not understanding the meaning of the missing box. None of the students in the classroom had the opportunity to explain why the answer is 7 using the representational objects on the board.

**Suggested Alternative Strategies:**

**Clarifying the Question:** Ms. Burns' request, "Find the missing factor and share how you can group them" could have been asked in a more concise manner, such as: "The problem reads  $6 \times \underline{\quad} = 42$ . So, if 42 apples are shared equally into 6 baskets, then how many apples will be in each basket?"

**Dissecting what the problem is asking for:** Having an emphasis on grouping (size of the set and the number of sets) is important for building an understanding of the multiplication process. There is a total of 42 and there are 6 items (size) in each group (set); the number of groups (sets) is unknown. *(MP1: Making sense and persevere in solving them; MP2: Reason abstractly and quantitatively, MP6: Attend to precision)*

**Comparing Student Responses/Strategies:** The four students in this scenario could have presented their strategies, and the class could have collectively come up with the strategy that made the most sense. Students in this scenario could have explored the strategies offered by the other students to solve for the missing factor. *(MP3: Construct viable arguments and critique the reasoning of others; MP4: Model with mathematics; MP6: Attend to precision; MP7: Look for and make use of structure)*

**Relation of Scenario to the Common Core Standards:**

**Domain:** Operations and Algebraic Thinking

**Cluster Heading:** Represent and solve problems involving multiplication and division.

**Cluster Standard:**

3. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example: Determine the unknown number that makes the equation true:  $8 \times ? = 48$ .

**Standards for Mathematical Practices connected to this Grade 3 case scenario:**

1. **Make sense of problems and persevere in solving them.**

In Grade 3, students discuss how they solve problems. They are expected to explain to

themselves the meaning of a problem, use concrete objects or pictures, and try different approaches when finding a solution and use another method to check their answers.

**2. Reason abstractly and quantitatively.**

Students in Grade 3 should recognize that a number represents a specific quantity and they connect the quantity to a written symbol to create a logical representation, considering units involved and the meaning of quantities. For example, 6 tallies in X amount of groups is equal to 42 tallies.

**3. Construct viable arguments and critique the reasoning of them.**

Students in Grade 3 communicate their mathematical thinking by explaining their own thinking and responding to others' thinking.

**4. Model with mathematics.**

Students in Grade 3 experiment with representing situations using pictures, objects, charts, lists, etc. For example, the students' initial process was to count 6 tallies and group it, and so on, until a total of 7 groups. Another student had suggested 7 tallies inside each group, for a total of 6 groups. Students in this scenario should have evaluated their results of these representations to determine which of the two made sense.

**5. Use appropriate tools strategically.**

Grade 3 students consider available tools (including estimation) to solve math problems and decide when tools might be helpful.

**6. Attend to precision.**

Grade 3 students develop their math communication skills, attempt to use clear and precise language in their discussions with others, and in their own reasoning. They specify units of measure and say the meaning of the symbols they choose.

**7. Look for and make use of structure.**

Students in Grade 3 discover patterns and structures. For example, the commutative property could have been fleshed out through a discussion of the difference between 6 groups of 7 units, and 7 groups of 6 units.

**8. Look for and express regularity in repeated reasoning.**

Grade 3 students should notice repetitive actions in computation and look for shortcut methods.

## Scenario E: Fractions (Grade 4)

### Context:

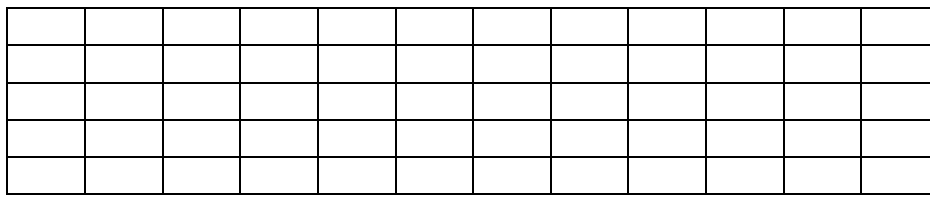
Fractions are one of the hardest areas for students to understand as represented by their standardized tests results and observations of their learning in classrooms. The scenario below shows an example of a misrepresentation of fractions that could haunt students for many years especially as they move into relating fractions and percents.

### Scenario:

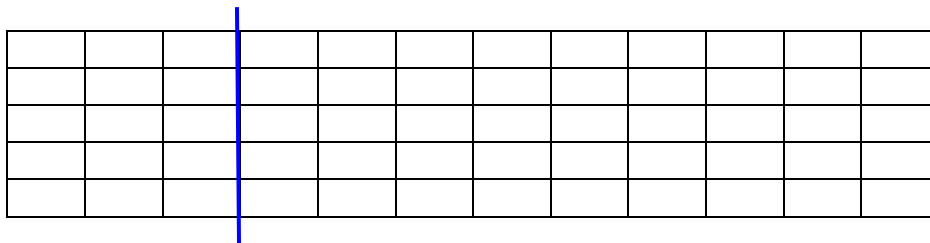
Mr. Sanchez is teaching a fourth grade inclusion classroom that includes special education students.

He shows students a 12 by 5 grid cut into 60 pieces that he has drawn on the board. He says, "In this grid there are 60 of  $\frac{1}{4}$ . Can someone show me  $\frac{1}{4}$  inside of it?"

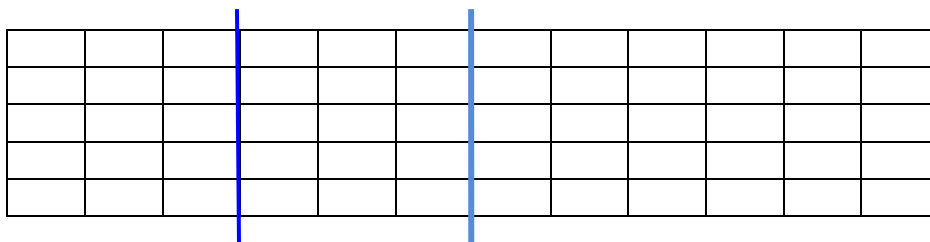
(He is asking the students to show a  $\frac{1}{4}$  of the whole shape, but he isn't asking this question correctly.)



Leah draws a line after the third column of 5 to show a fourth.



Then the teacher draws another line after the 6th column of 5 to show another fourth.



Students are asked to copy this representation onto their papers. There is no discussion as to why this makes a fourth or what these two sections represent.

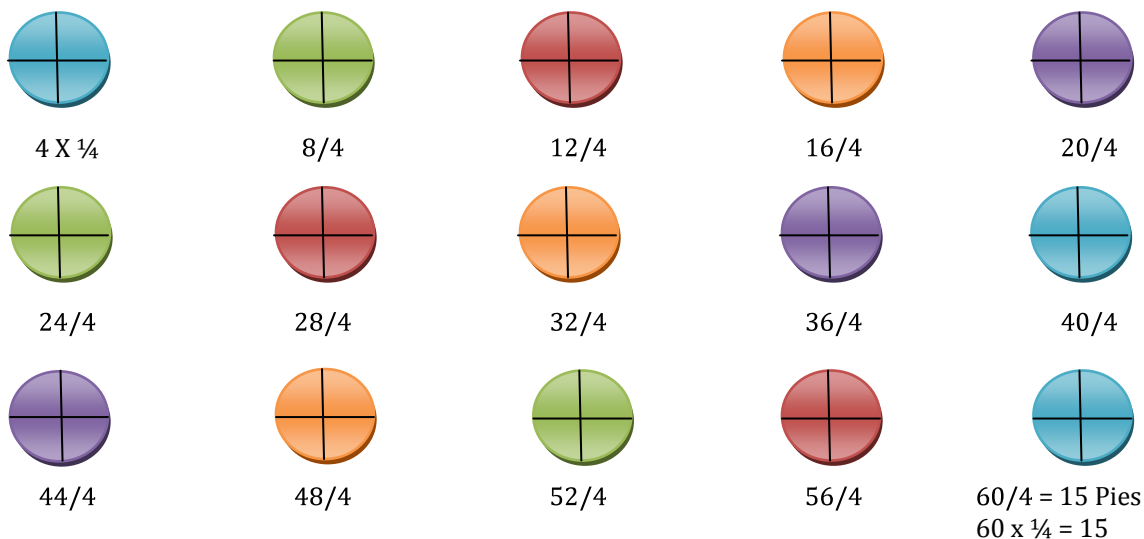
**What the students may be understanding, or not understanding:**

The students were not given an opportunity to talk about what they were learning. We really don't know what they understood or did not understand. Leah did put the line in the right place so she understood that the teacher meant  $\frac{1}{4}$  of 60. However, most students just did what the teacher said which is to copy his picture.

**Suggested Alternative Strategies:**

**Clarifying Question:** Mr. Sanchez said, "In this grid there are 60 of  $\frac{1}{4}$ . Can someone show me  $\frac{1}{4}$  inside of it?" but was really asking the students to show  $\frac{1}{4}$  of 60. The drawing he showed was correct, however the language was not. Fractions are difficult to teach and require that the teacher's language match the representation provided to the students. The two representations of fractions and multiplication are quite different.

**Different Representation:** What Mr. Sanchez asked was 60 of  $\frac{1}{4}$  is not the same as  $\frac{1}{4}$  of 60. What he asked the students to think about was 60  $\frac{1}{4}$ s. This would be like asking the students to look at multiple pies divided into fourths and then adding up the fourths to reach 60 pieces.



**Relation of Scenario to the Common Core Standards:**

There was no standard in the Grade 4 CCSS that corresponds to this scenario. However, we were able to match this scenario to the Grade 3 standard listed below. The problem was more complex since the students are asked to find  $\frac{1}{4}$  of many objects rather than one whole.

**Domain:** Number and Operations – Fractions (*Grade 3*)

**Cluster Heading:** Develop understanding of fractions as numbers

**Cluster Standard:**

1. Understand a fraction  $\frac{1}{b}$  as the quantity formed by one part when a whole is partitioned into  $b$  equal parts. Understand a fraction  $\frac{a}{b}$  as the quantity formed by a part of size of  $\frac{1}{b}$ .

### **Standards for Mathematical Practices connected to this Grade 4 case scenario:**

**1. Make sense of problems and persevere in solving them.**

In Grade 4, students explain to themselves the meaning to a problem and plan a solution pathway rather than simply jumping into a solution. They are expected to draw diagrams of important features and relationships, graphs, and search for regularity or trends and ask themselves, “does this make sense”, and understand the approaches of others to solving complex problems.

**2. Reason abstractly and quantitatively.**

Students in Grade 4 should use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities, not just how to compute them.

**3. Construct viable arguments and critique the reasoning of them.**

Students in Grade 4 make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are expected to analyze situations by breaking them into cases and justify their conclusions, communicate them to others, and respond to the arguments of others. Grade 4 students also construct arguments using concrete referents such as objects, drawings, diagrams and actions.

**4. Model with mathematics.**

Students in Grade 4 apply the mathematics they know to solve problems arising in everyday life. In this scenario, students could have talked about what they know about a fourth of one before figuring out a fourth of 60. Students in Grade 4 also reflect whether the results make sense, possibly improving the model if it has not served its purpose.

**5. Use appropriate tools strategically.**

Grade 4 students consider available tools to solve math problems and decide when tools might be helpful. Tools might include pencil and paper, concrete models, etc. Students in Grade 4 are expected to identify relevant mathematical resources and use them to pose or solve problems.

**6. Attend to precision.**

Grade 4 students try to communicate precisely to others and specify units of measure to clarify the correspondence with quantities in a problem, and also give carefully formulated explanations to each other.

**7. Look for and make use of structure.**

Students in Grade 4 closely look to discern a pattern or structure and see complicated things as single objects or composed of several objects.

**8. Look for and express regularity in repeated reasoning.**

Grade 4 students look for general methods and for shortcuts. In this scenario, students might notice that a 60 pieces can also be represented as 15 pies cut into fourths.

## Scenario F: Fact Families (Grade 5)

### **Context:**

One of the CCSS Grade 5 Critical Areas is developing fluency with whole number and decimal operations. Students develop understanding of why division procedures work; analyze fluency with multi-digit addition, subtraction, multiplication and division; and apply their understanding to decimals and fractions. In this example the teacher of a fifth grade class suddenly gets confused about how to present fact families when working with the fact family of 2 and 1. Although this scenario is based on a Grade 5 classroom, the concept being taught is found in the Grade 3 standards.

Many students learn their addition and subtraction facts through working with Fact Families. Using addition and subtraction for fact families helps students understand that subtraction undoes addition and vice versa. Learning the fact families for multiplication and division also supports students in thinking about the relationships between multiplication and division. It also provides a more efficient way to remember multiplication facts and apply the inverse to solve simple division problems.

### **Scenario:**

Mrs. Winters is teaching a regular fifth grade class and does an introductory lesson using fact families. She asked the students to give her a suggestion for a fact family they would like to show. Jasper suggests the Fact Family for the number 2.

Mrs. Winters writes on the board the following:

$$2 \times 1$$

$$1 \times 2$$

$$2 \div 1$$

Then Mrs. Winters suddenly freezes and is unable to come up with the missing expression. (She may have been thinking that 1 divided by 2 is  $\frac{1}{2}$  and she didn't want to show a fraction. The missing fact is  $2 \div 2 = 1$ ).

The teacher then tells the students that this was a hard one, so let's do a different number. Mrs. Winters writes:

$$22 \times 10 = 220$$

$$10 \times 22 = 220$$

$$220 \div 10 = 22$$

$220 \div 22 = 10$  and completes the Fact Family for 220 instead.

**What the students may be understanding, or not understanding:**

Students are used to having three numbers in their fact family. In the first example of 2, only two numbers are involved which is unusual. Most fact families have three numbers and students and the teacher became confused when there were only two numbers involved.

**Suggested Alternative Strategies:**

**Writing a Complete Equation:** In the first example, Mrs. Winters only wrote numerical expressions for the fact families. Writing out the full equation, including products, quotients, and equal signs, would have allowed the teacher and students to see the missing fact.

**Doubles:** When working with any doubles there will only be two equations in the fact family, for example,  $4 \times 4 = 16$  and  $16 \div 4 = 4$ .

**Division by Zero:** Zero is the mathematical representation of the concept of “nothing”. This might cause some misconceptions in division (or sharing). If you share “nothing” among any number of people, each person gets “nothing” or zero. This is represented as  $0 \div \text{each person involved} = 0$ .

A fact family with zero might look like this:

$$4 \times 0 = 0$$

$$0 \times 4 = 0$$

$$0 \div 4 = 0 \quad (\text{dividing nothing among four people, the four people get nothing})$$

$$4 \div 0 = \text{undefined} \quad (\text{four apples divided by nothing, the four apples are still there})$$

To further see this concept, we can think about multiplication undoing division:

$$\text{If } 3 \times 4 = 12, \text{ then } 12 \div 4 = 3. \quad \text{If } 0 \times 4 = 0, \text{ then } 0 \div 4 = 0.$$

$$\text{If } 4 \times 3 = 12, \text{ then } 12 \div 3 = 4. \quad \text{If } 4 \times 0 = 0, \text{ then } 0 \div 0 \text{ is NOT equal to } 4, \text{ and is undefined.}$$

**Relation of Scenario to the Common Core Standards:**

**Domain:** Operations and Algebraic Thinking (Third Grade)

**Cluster Heading:** Multiply and Divide with in 100.

**Cluster Standard:**

7. Fluently multiply and divide within 100 using strategies such as the relationship between multiplication and division (example: knowing that  $8 \times 5 = 40$ ,  $40 \div 5 = 8$ ) or properties of operations. By the end of third grade, know from memory all products of two one-digit numbers.

**Standards for Mathematical Practices connected to this Grade 5 case scenario:**

1. **Make sense of problems and persevere in solving them.**

Students in Grade 5 should solve problems by applying their understanding of operations with

whole numbers, decimals, and fractions, including mixed numbers. They ask themselves whether they can solve problems in different ways.

**2. Reason abstractly and quantitatively.**

Grade 5 students should recognize that a number represents a specific quantity. They connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals.

**3. Construct viable arguments and critique the reasoning of them.**

Students in Grade 5 may construct arguments using objects, pictures, and drawings. They explain calculations based models, properties of operations, and rules that generate patterns. Students communicate with each other by asking questions like: “how did you get that?” and “why is that true?”.

**4. Model with mathematics.**

Students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, and making charts, lists, graphs, or creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

**5. Use appropriate tools strategically.**

Grade 5 students who are mathematically proficient consider the available tools (including estimation) when solving a problem and decide when certain tools might be helpful. They may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions, graph paper to accurately create graphs and solve problems, or make predictions from real world data.

**6. Attend to precision.**

Grade 5 students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and when expressing their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids.

**7. Look for and make use of structure.**

Students in Grade 5 closely look to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals.

**8. Look for and express regularity in repeated reasoning.**

Students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. They connect place value and their prior work with operations to understand algorithms and to fluently multiply multi-digit numbers. Students in this grade level explore operations with fractions with visual models and begin to formulate generalizations.