Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Optical Design with Zemax

Lecture 10: Physical Optical Modelling II

2012-11-22

Herbert Gross

10 Physical Optical Modelling II
Preliminary time schedule

1	17.07.	Introduction	Zemax interface, menus, file handling, system description, editors, preferences, updates, system reports, coordinate systems, aperture, field, wavelength, glass catalogs, layouts, raytrace, diameters, stop and pupil
			pick ups, solves, variables, ray fans, quick focus, 3D geometry, ideal lenses, vignetting, footprints, system insertion, scaling, component reversal
2	24.07.	Properties of optical systems	aspheres, gradient media, gratings and diffractive surfaces, special types of surfaces
			telecentricity, ray aiming, afocal systems, Delano diagram, lens catalogs
3	14.08.	Aberrations	representations, spot, Seidel, transverse aberration curves, Zernike wave aberrations
			PSF, MTF, ESF
4	28.08.	Optimization	algorithms, merit function, methodology, correction process
			first examples
5	11.09.	Imaging and illumination	Fourier imaging, geometrical images
			non-sequential option
6	25.09.	Advanced handling	slider, universal plot, I/O of data, material index fit, multi configuration
			macro language, link of DLLs, MDD Matlab coupling
7	09.10.	Correction I	simple systems
			systems with medium complexity
8	23.10.	Correction II	layout and correction of a microscopic objective lens
			design and correction of a zoom system
9	06.11.	Physical optical modelling I	Gaussian beams, POP propagation
			polarization raytrace, transmission, aberrations
10	20.11.	Physical optical modelling II	coatings, representations, transmission and phase effects
			Ghost imaging, general straylight with BRDF

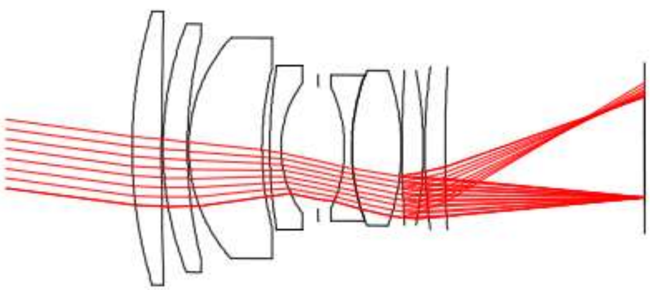
Contents

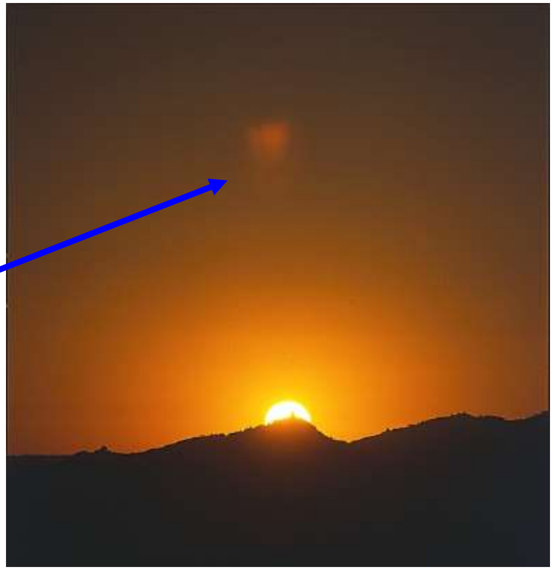
1. Coatings
2. Coating transmission and phase effects
3. Ghost imaging
4. Straylight
5. Scattering in Zemax
6. Fresnel equations
7. Overview
8. Design and calculation of thin films
9. Simple layers
10. Representations
11. Applications and examples
12. Coatings in Zemax

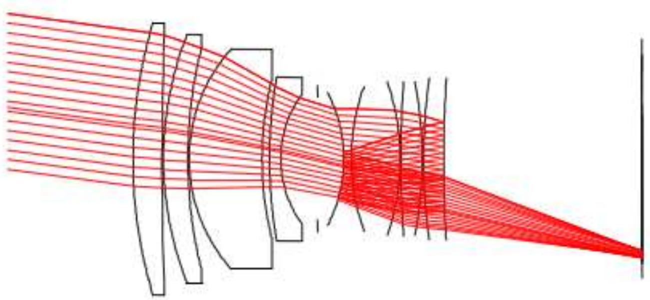
10 Physical Optical Modelling II
Ghost Images

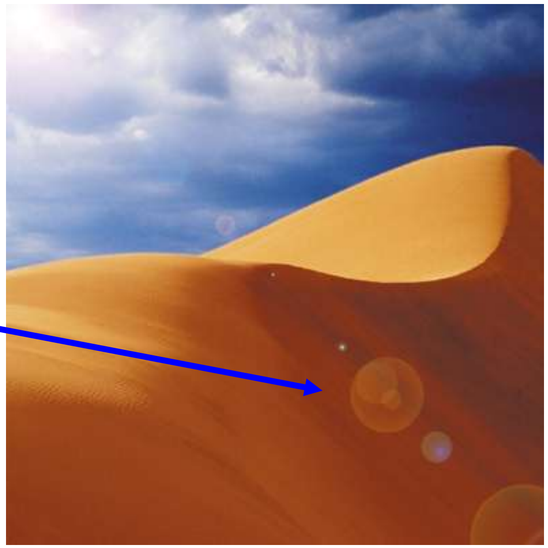
Ghost image in photographic lenses:

Reflex film / surface









Ref: K. Uhlendorf, D. Gängler

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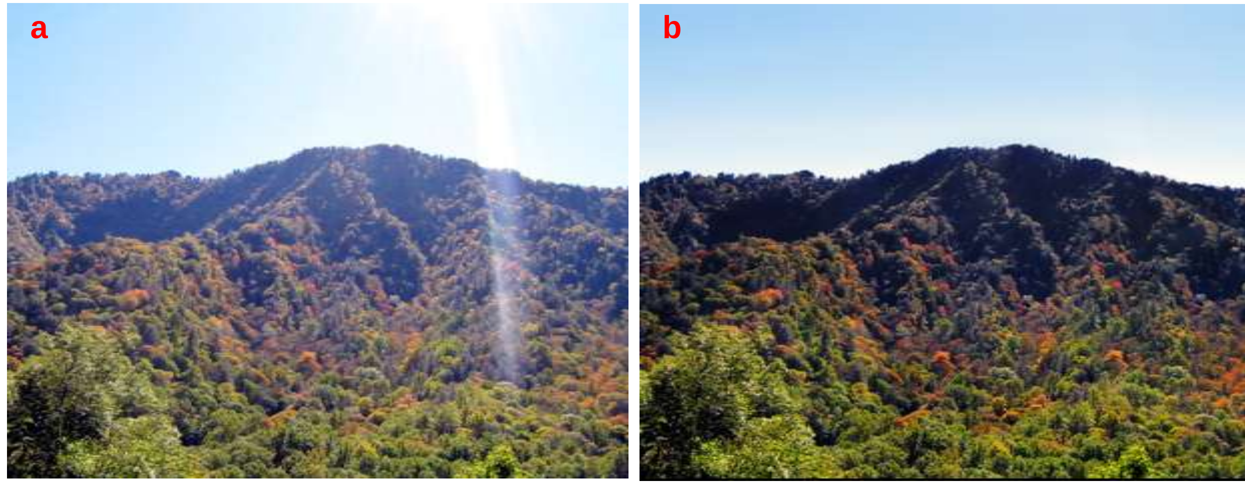
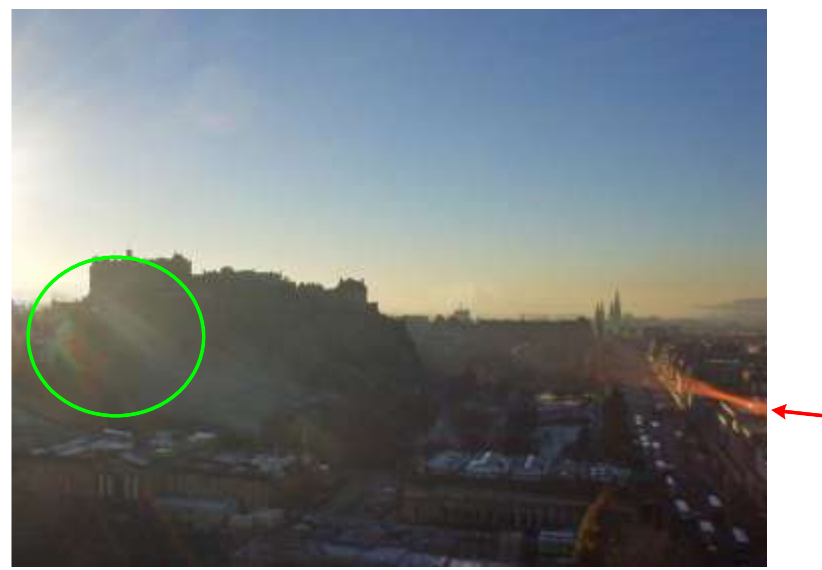
Straylight and Ghost Images

- Different reasons
- Various distributions



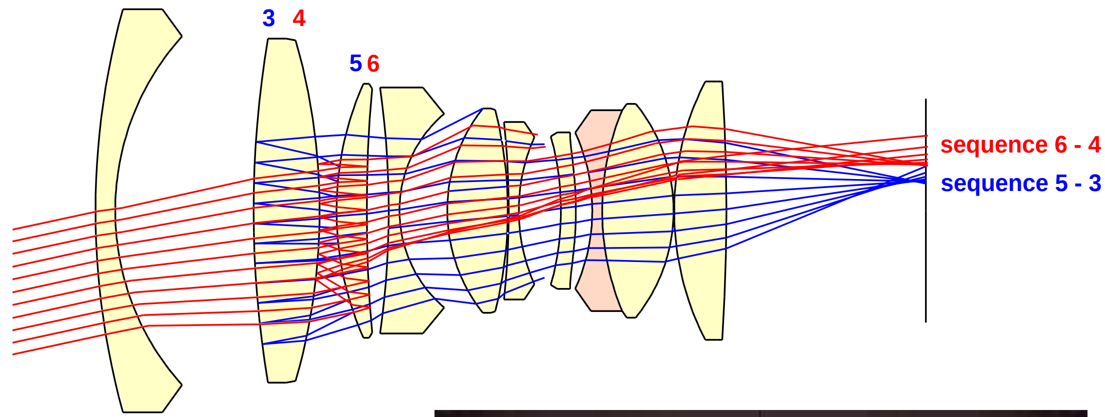


- Scattering of light in diffuse media like frog

Ref: W. Osten

10 Physical Optical Modelling II
Straylight and Ghost Images

- Calculation of reflected light



- Colour effects due to coatings

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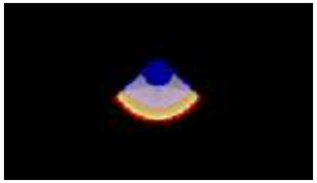

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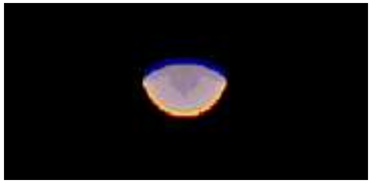

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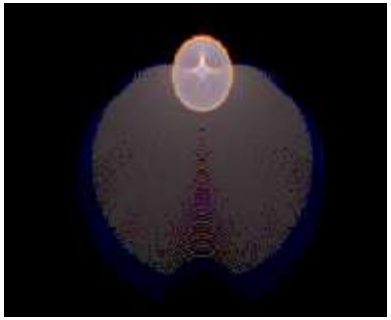

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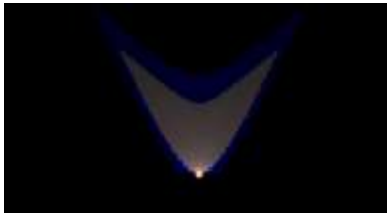

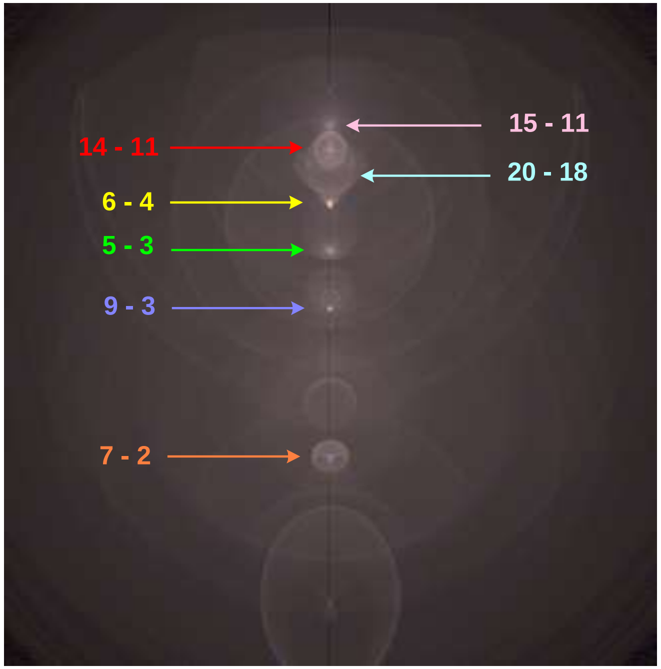

Ref.: M. Peschka

Definition of Scattering

- Physical reasons for scattering:
  - Interaction of light with matter, excitation of atomic vibration level dipols
  - Resonant scattering possible, in case of re-emission λ-shift possible
  - Direction of light is changed in complicated way, polarization-dependent

- Phenomenological description (macroscopic averaged statistics)
  1. Surface scattering:
     - 1.1 Diffraction at regular structures and boundaries: gratings, edges (deterministic: scattering ?)
     - 1.2 Extended area with statistical distributed micro structures
     - 1.3 Single micros structure: contamination, imperfections
  2. Volume scattering:
     - 2.1 Inhomogeneity of refractive index, striae, atmospheric turbulence
     - 2.2 Ensemble of single scattering centers (inclusions, bubble)

Therefore more general definition:

- Interaction of light with small scale structures
- Small scale structures usually statistically distributed (exception: edge, grating)
- No absorption, wavelength preserved
- Propagation of light can not be described by simple means (refraction/reflection)

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Scattering Mechanisms

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1. Surface scattering
   1.1 Edge diffraction
   1.2 Scattering at topological structures of surfaces
   Dimensions: micro roughness – macroscopic ripple due to manufacturing errors
   1.3 Scattering at single localized defects (contamination, micro defects)
   perturbation of phase- and amplitude

2. Particle scattering :
   2.1 Rayleigh regime , $d \ll \lambda$
   2.2 Theory of Rayleigh-Debye , $d < \lambda$
   2.3 Mie-scattering, spherical particles $d > \lambda$

3. Volume scattering
   3.1 Scattering at refractive index inhomogeneities,
   atmospheric turbulence, striae in glass
   3.2 Scattering at crystal boundaries (ceramic)
   3.3 Scattering in statistical dense material/particle ensemble
   biological tissue, multiple scattering events

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Aspects of Scattering

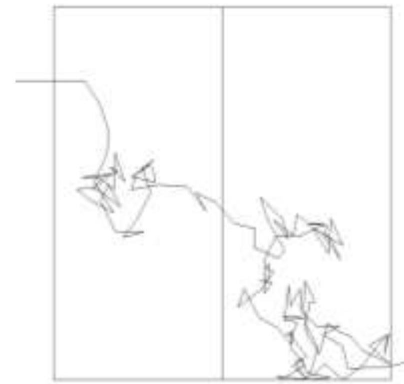
- Geometry regular - statistical distributed
- Single - multi scattering
- Density of scatterers low - high, independence, saturation, change of illumination
- Near - far field
- Scaling, size of scatterers vs. wavelength, micro - macro
- Coherence, scattering vs. re-emission
- Polarization dependence
- Discret scatterers vs. continuous n-variations
- Absorption
- Diffraction vs. geometrical approach
- Steady state vs time dependence
- Wavelength dispersion of material parameters
- Finite volume size - boundary conditions

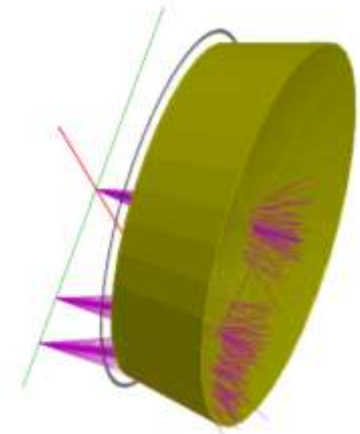
Approximations in Scattering Models

- Geometry simplified
- Boundaries simplified, mostly at infinity
- Isotropic scattering characteristic
- Perfect statistics of distributed particles
- Multiple scattering neglected
- Discretization of volume
- Angle dependence of phase function simplified
- Scattering centers independent
- Scatterers point like objects
- Spatially varying material parameters ignored
- Field assumed to be scalar
- Decoherence effects neglected
- Absorption neglected
- Interaction of scatterers neglected
- λ-dispersion of material data neglected

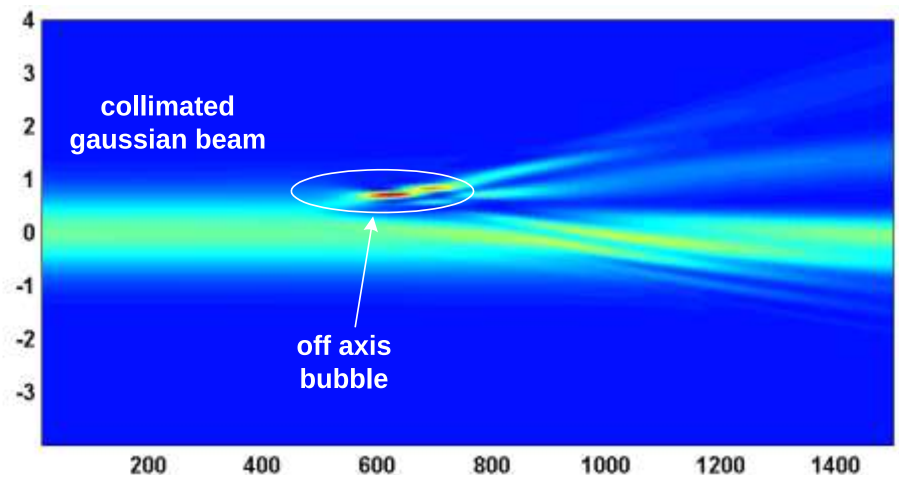
- Calculation:

  1. Geometrical approximation:
     Raytrace with Monte-Carlo-method
     time consuming, non-smooth results





  2. Wave optical with beam propagation
     Scalar approach only for large scales



  3. Diffusion theory:
     In strongly scattering media with isotropic behavior; tissue

- Length scale below 1 cm: Tatarski function due to viscosity
  PSD

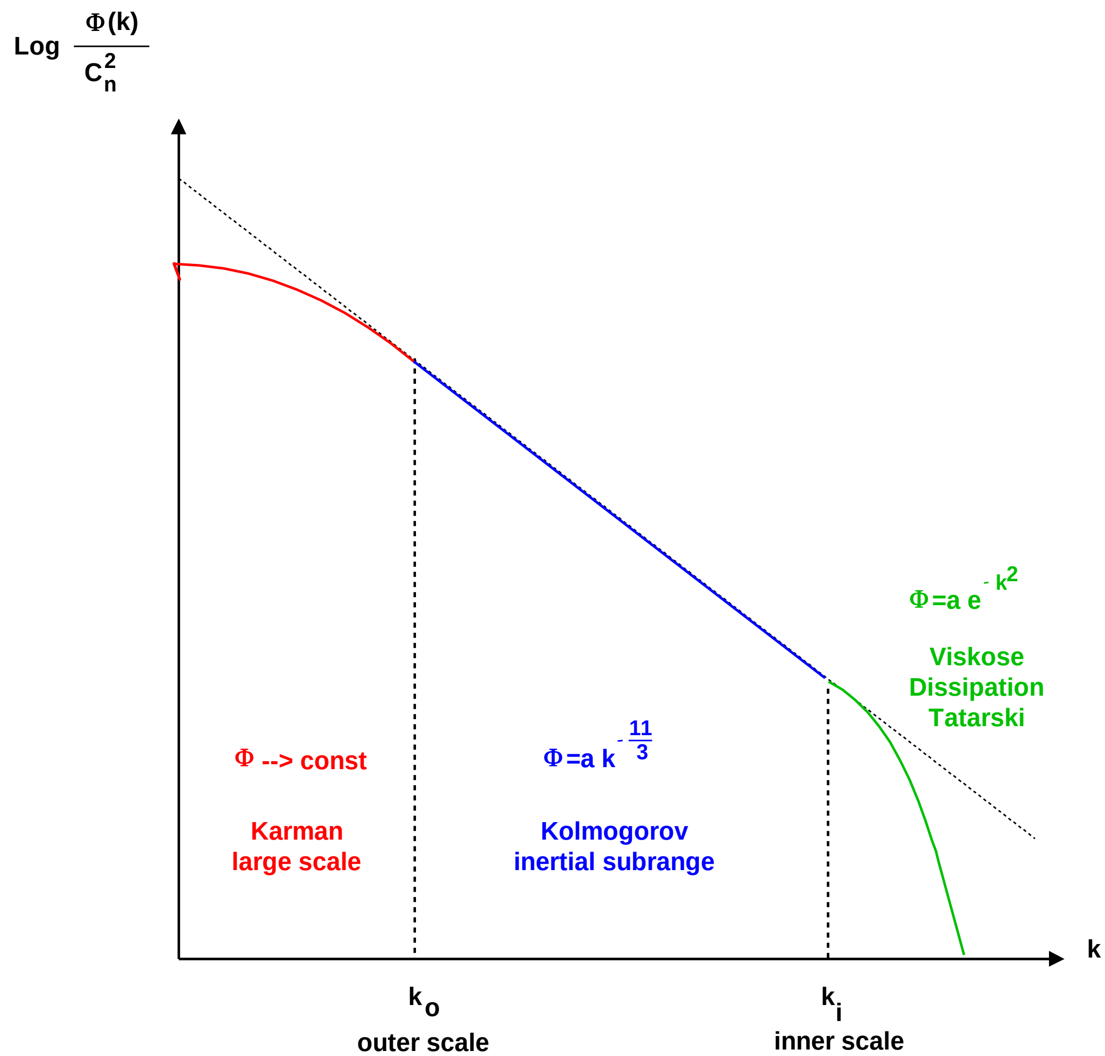
$$\Phi = b_{Ta} \cdot e^{-\left(\frac{k}{k_i}\right)^2}$$

- Lengths scale up to 5 m: Kolmogorov spectrum
  PSD

$$\Phi = b_{Ta} \cdot k^{-\frac{11}{3}}$$

- Length scale above: Karman spectrum
  PSD

$$\Phi = b_{Ka} \cdot \left(k^2 + \frac{4\pi^2}{L_o^2}\right)^{-\frac{11}{6}}$$



Analytical solutions:

- Spherical particles
  1. generalized Lorentz-Mie theory, near and far field
  2. multi sphere configurations
  3. layered structures
- Spheroids
- Cylinders
  1. single cylinders, with oblique incidence, near and far field
  2. stacked cylinders
  3. multi cylinder configurations, perpendicular incidence

Numerical solutions in time domain:

- Arbitrary geometies
- Finite difference time domain method (FDTD), only small volumes ($2\mu m^3$), $\Delta x = \lambda/20$
- Pseudospectral method (PSTD) , $\Delta x = \lambda/4$

Stationary solutions:

- Discrete dipole approximation for arbitrary geomtries
- T-matrix method

Ref: A. Kienle

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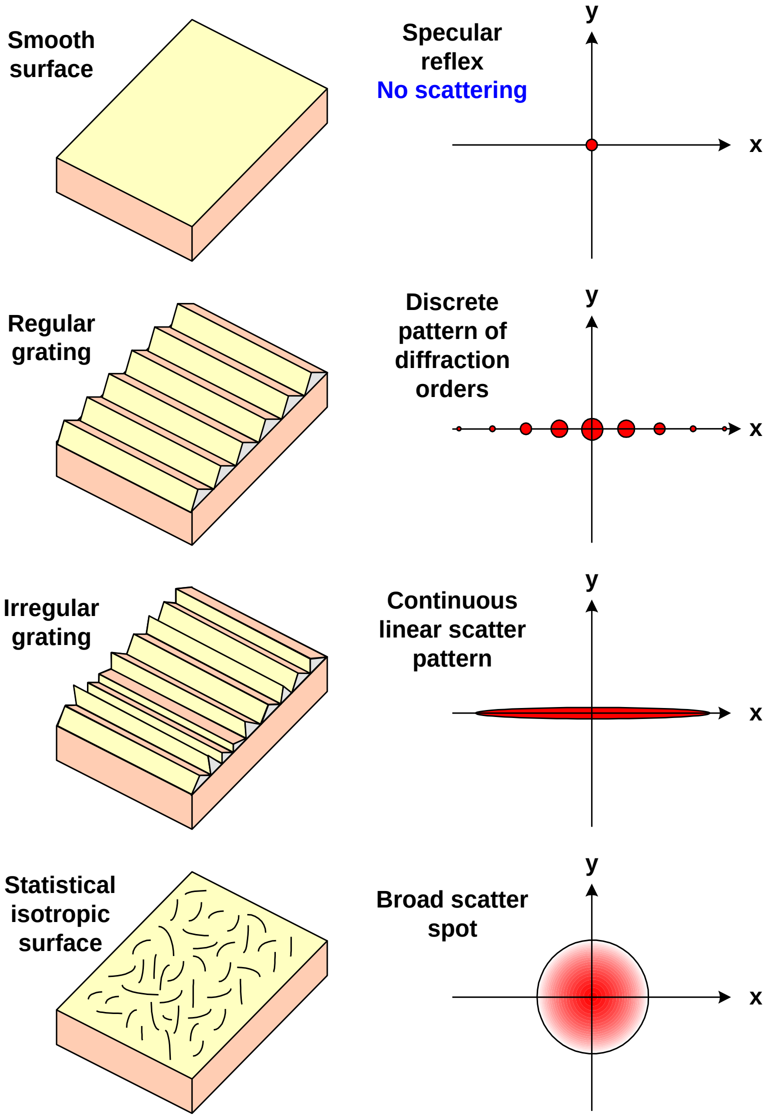
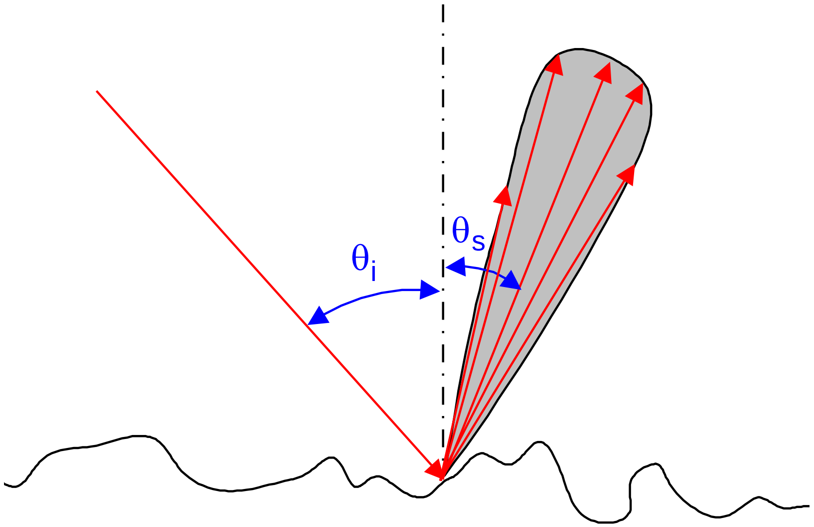
Different surface geometries:

Every micro-structure generates a specific straylight distribution

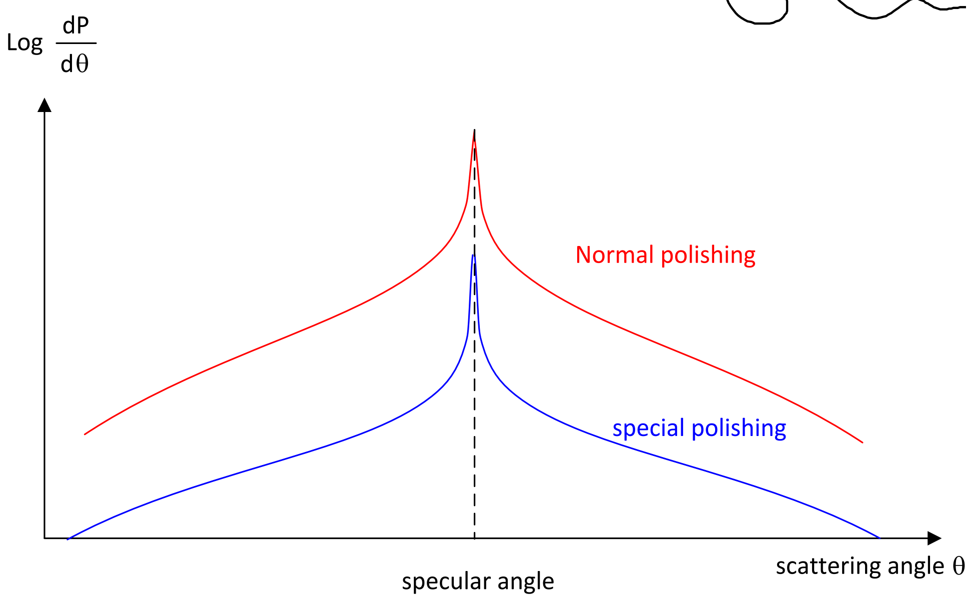


Ref.: J. Stover, p.10

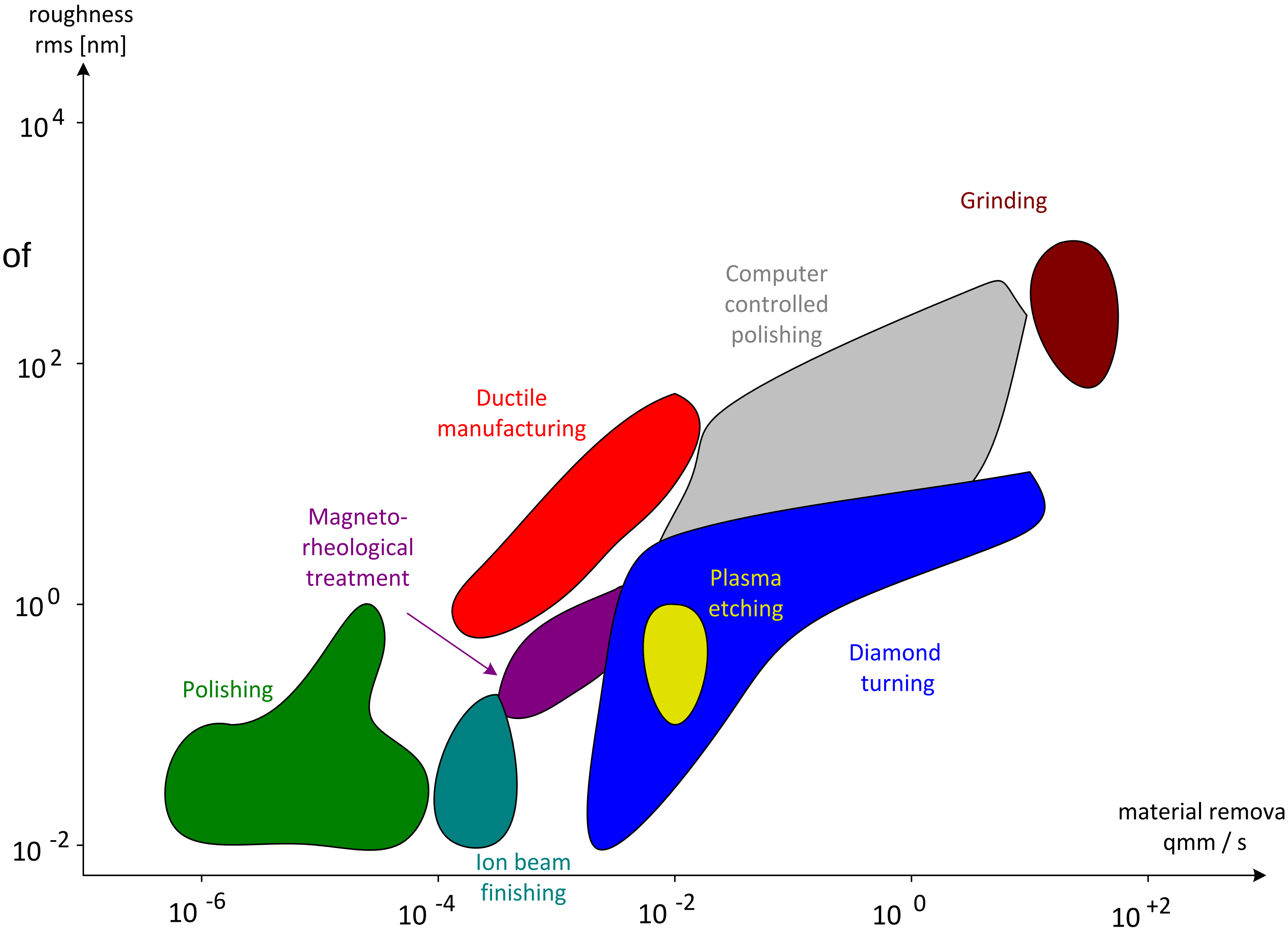
- Scattering at rough surfaces:
  statistical distribution of light scattering in the angle domain

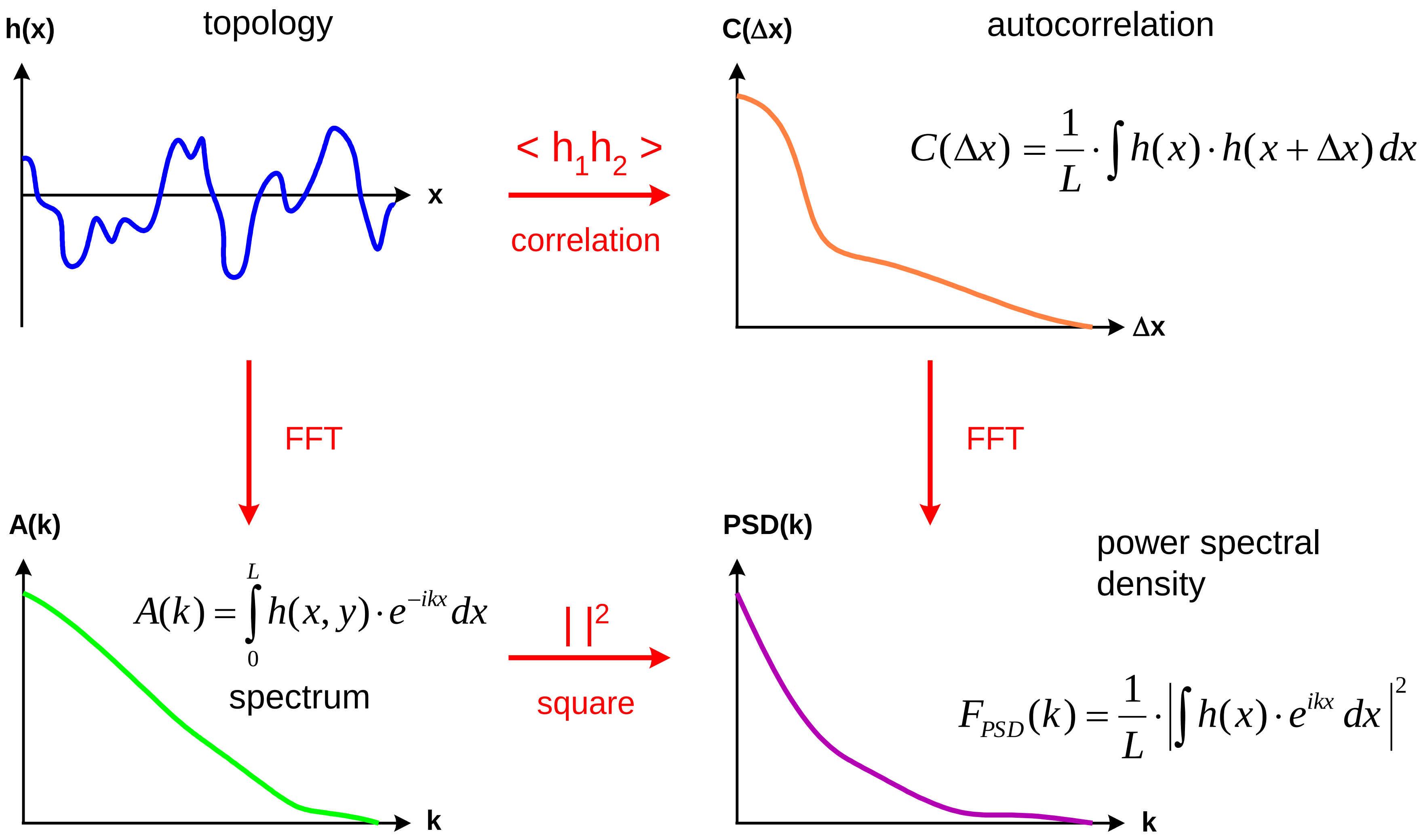


- Angle indicatrix of scattering:
  - peak around the specular angle
  - decay of larger angle distributions depends on surface treatment

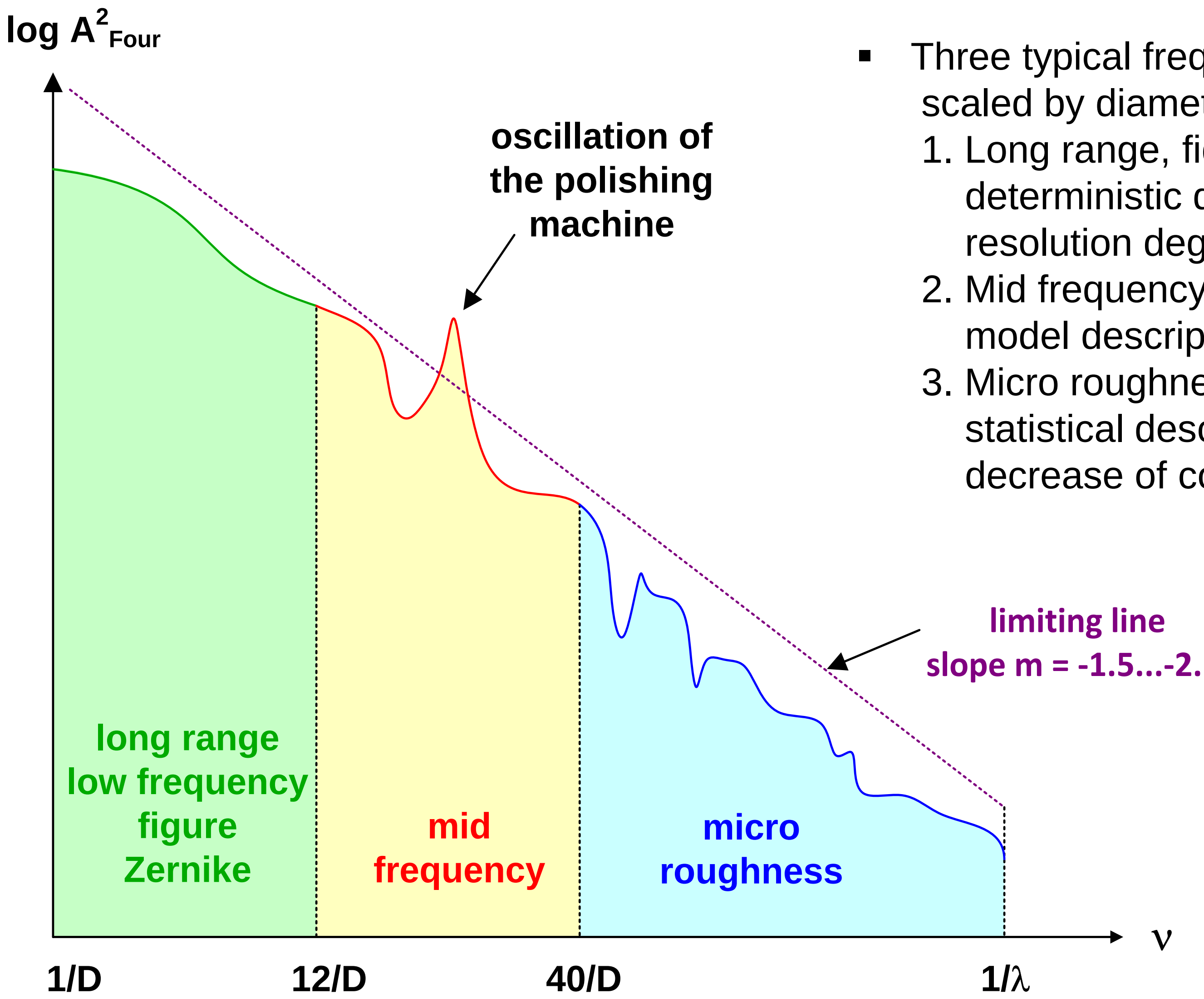


Roughness of optical surfaces, Dependence of treatment technology




h(x)
topology
x
$< h_1 h_2 >$
correlation
C(Δx)
autocorrelation
$C(\Delta x) = \frac{1}{L} \cdot \int h(x) \cdot h(x + \Delta x)\, dx$
Δx
FFT
FFT
A(k)
$A(k) = \int_0^L h(x,y) \cdot e^{-ikx}\, dx$
spectrum
k
$| |^2$
square
PSD(k)
power spectral density
$F_{PSD}(k) = \frac{1}{L} \cdot \left| \int h(x) \cdot e^{ikx}\, dx \right|^2$
k

Spatial Frequency of Surface Perturbations



- Power spectral density of the perturbation
- Three typical frequency ranges, scaled by diameter D
  1. Long range, figure error
     deterministic description
     resolution degradation
  2. Mid frequency, critical
     model description complicated
  3. Micro roughness
     statistical description
     decrease of contrast

- Description of scattering characteristic of a surface: BSDF (bidirectional scattering distribution function)

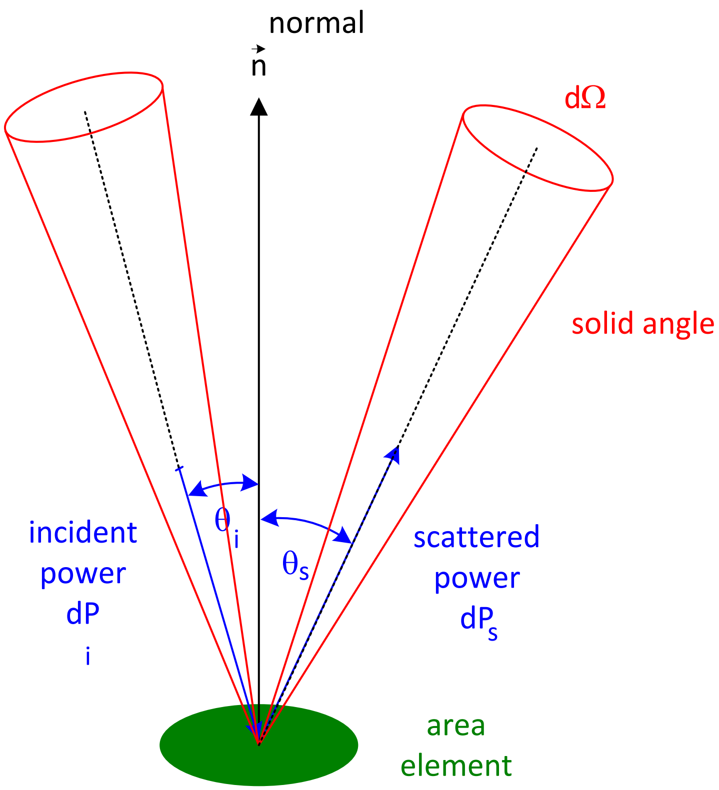
- Straylight power into the solid angle $d\Omega$ from the area element dA relative to the incident power P_i

$$F_{BSDF} = \frac{dL_s}{dP_i} = \frac{dP_s}{\cos\theta \cdot dP_i \cdot d\Omega}$$

- The BSDF works as the angle response function

$$P(\theta,\varphi) = \int F_{BSDF}\left(\theta_i,\varphi_i,\theta,\varphi\right) \cdot P(\theta_i,\varphi_i) \cdot \cos\theta_i d\Omega_i$$

- Special cases: formulation as convolution integral



- Exponential correlation decay
  PSD is Lorentzian function

$$C(x) = \sigma_{rms}^2 \cdot e^{-\frac{x}{\tau_c}}$$

$$F_{PSD}(s) = \frac{1}{\pi} \cdot \frac{\sigma_{rms}^2 \cdot \tau_c}{1 + (s \cdot \tau_c)^2}$$

- Gaussian coerrelation

$$C(x) = \sigma_{rms}^2 \cdot e^{-\frac{1}{2}\left(\frac{x}{\tau_c}\right)^2}$$

$$F_{PSD}(s) = \frac{\tau_c \cdot \sigma_{rms}^2}{\sqrt{4\pi}} \cdot e^{-\left(\frac{s\tau_c}{2}\right)^2}$$

- Fractal surface with Hausdorf parameter D

$$F_{PSD}(s) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{1}{2}\right) \cdot \Gamma\left(\frac{n}{2}\right)} \cdot \frac{K_n}{s^{n+1}}$$

- K correlation model parameter B, s

$$F_{PSD}(s) = \cdot \frac{A}{\left[1 + (s \cdot B)^2\right]^{C/2}}$$

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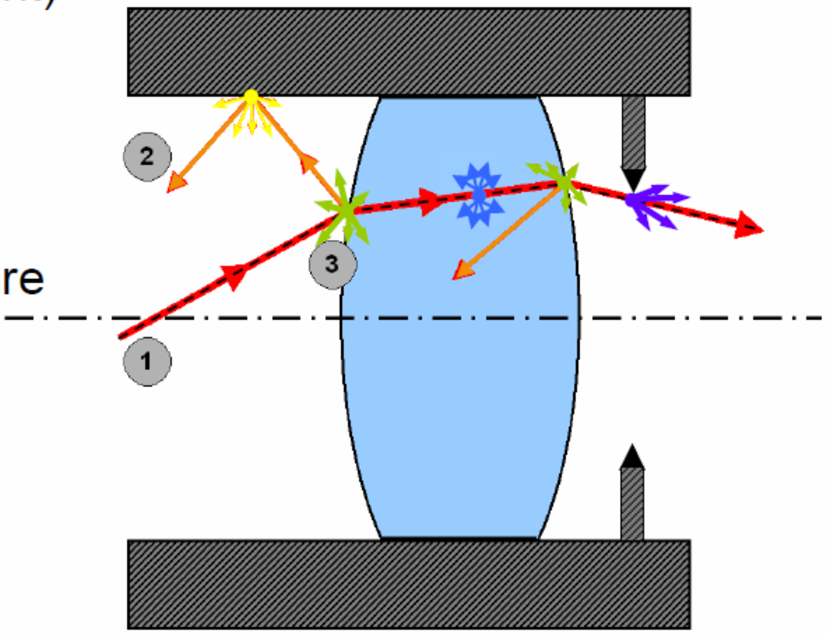
1. Direct imaging ray path
2. Direct reflected ray (zero order false light)
   - Reflected from optical surface
   - Reflected by mechanical parts
3. Scattered light
   - Scattered on surface micro structure
     - Optical surface
     - Mechanical surface
   - Scattered on particles
   - Volume scattering
4. Diffraction
   - Apertures and baffles



Ref: B. Goerz

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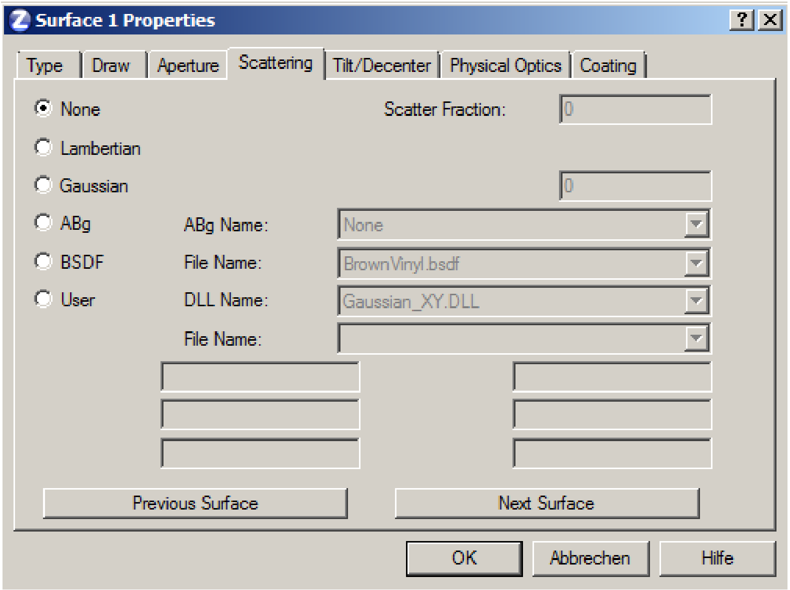
Scattering in Zemax



- Definition of scattering at every surface in the surface properties
- Possible options:
  1. Lambertian scattering indicatrix
  2. Gaussian scattering function
  3. ABg scattering function
  4. BSDF scattering function
  5. User defined
- More complex problems only make sense in the non-sequential mode of Zemax, here also non-optical surfaces (mechanics) can be included
- Surface and volume scattering possible
- Optional ray-splitting possible
- Relative fraction of scattering light can be specified

10 Physical Optical Modelling II
Scattering in Zemax

- Surface scattering:
  Projection of the scattered ray on the surface, difference x to the specular ray: x

- Volumne scattering:
  Angle scattering description by probability P

- Lambertian scattering:
  isotropic

- Gaussian scattering

$$F_{BSDF}(x) = A \cdot e^{-\frac{|x|^2}{\sigma^2}}$$

- ABg model scatter

$$F_{BSDF}(x) = \frac{A}{B + |x|^g}$$

- Henyey-Greenstein volume scattering
  (biological tissue model)

$$P(\theta) = \frac{1 - g^2}{4\pi \cdot \left(1 + g^2 - 2g\cos\theta\right)^{3/2}}$$

- Rayleigh scattering

$$P(\theta) = \frac{3}{8\lambda^4} \cdot \left(1 + \cos^2\theta\right)$$

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Scattering in Zemax

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- Data file with scattering functions: ABg-data.dat
- File can be edited

```
NM ABG-EXAMPLE
WV 0.000000000E+000
DT 0 0.000000000E+000 1.000000000E-002 1.500000000E-002 2.000000000E+000
DT 1 4.500000000E+001 1.000000000E-002 5.520000000E-003 2.000000000E+000
DT 2 7.000000000E+001 1.000000000E-002 1.000000000E-003 2.000000000E+000
NM TP_TEST
WV 0.000000000E+000
DT 0 0.000000000E+000 1.000000000E-003 1.500000000E-002 2.000000000E+000
DT 1 4.500000000E+001 1.000000000E-003 5.520000000E-003 2.000000000E+000
DT 2 7.000000000E+001 1.000000000E-003 1.000000000E-003 2.000000000E+000
NM LAMB-SPEC
WV 0.000000000E+000
DT 0 0.000000000E+000 2.000000000E-007 1.000000000E-008 8.000000000E+000
DT 1 8.900000000E+001 1.000000000E-001 4.000000000E-001 0.000000000E+000
NM SPECULAR
WV 0.000000000E+000
DT 0 0.000000000E+000 1.000000000E-007 1.000000000E-008 8.000000000E+000
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DT 2 8.500000000E+001 2.000000000E-003 4.500000000E-002 2.000000000E+000
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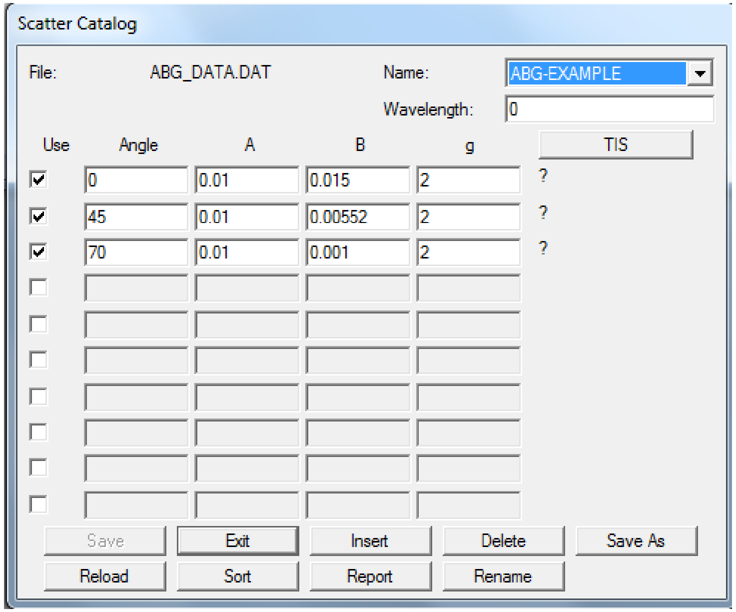
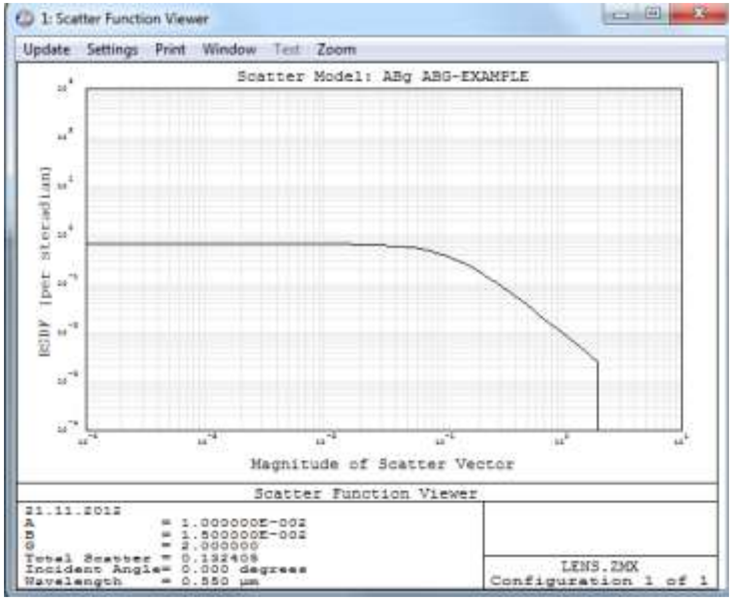
10 Physical Optical Modelling II
Scattering in Zemax

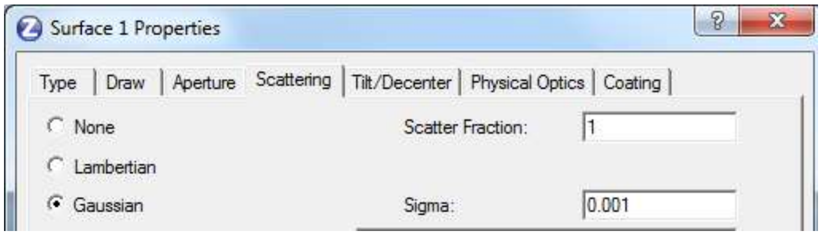
- Tools / Catalogs / ABg Scatter Data Catalogs
- Specification and definition of scattering parameters:
  wavelength, angle, A, B, g

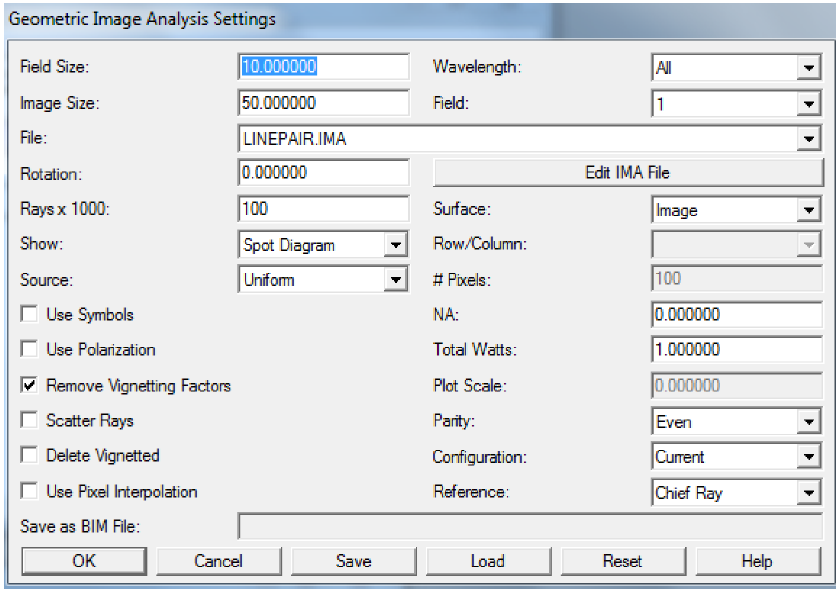


- Tools / Catalogs / SCatter Function Viewer
  Graphical representation of the scattering function

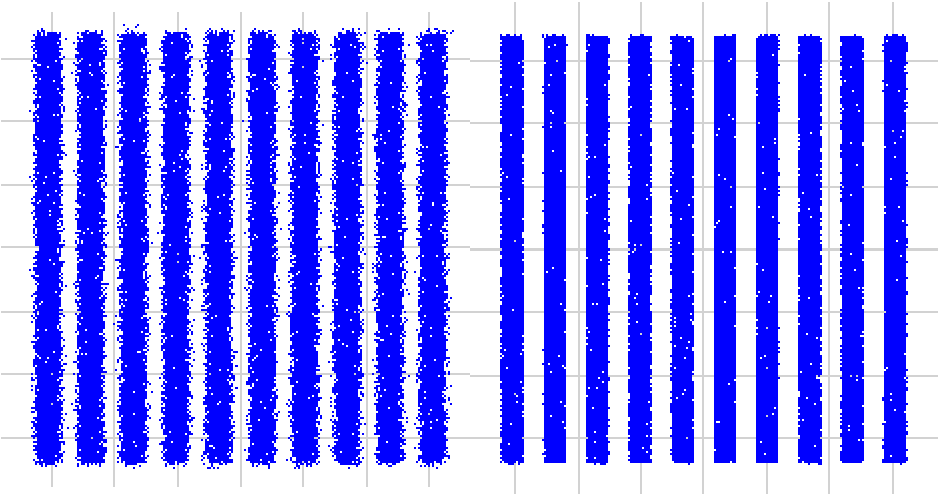


- Simple example: single focussing lens
- Gaussian scattering characteristic at one surface



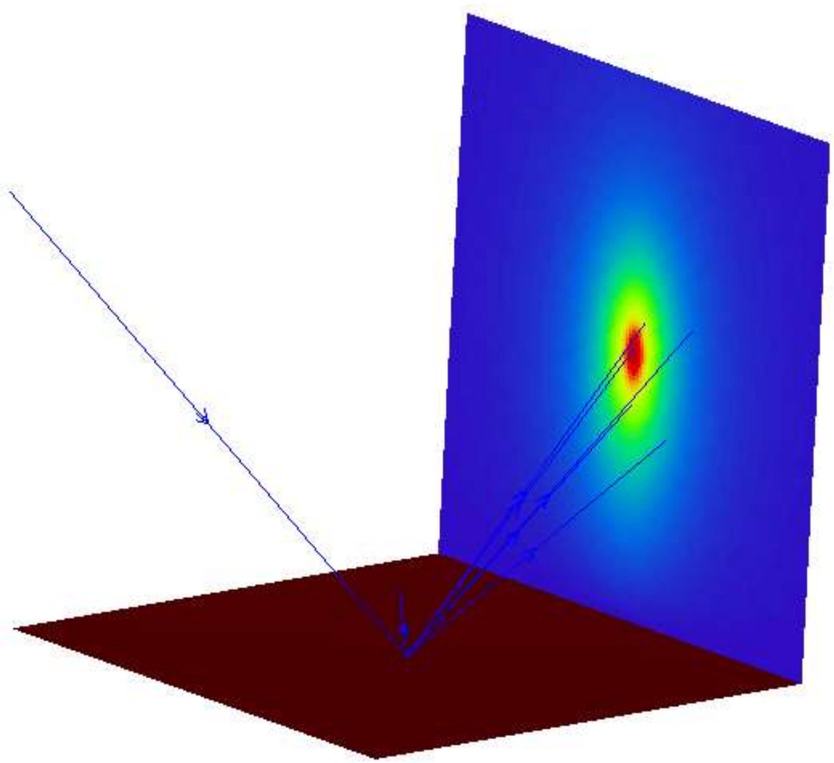


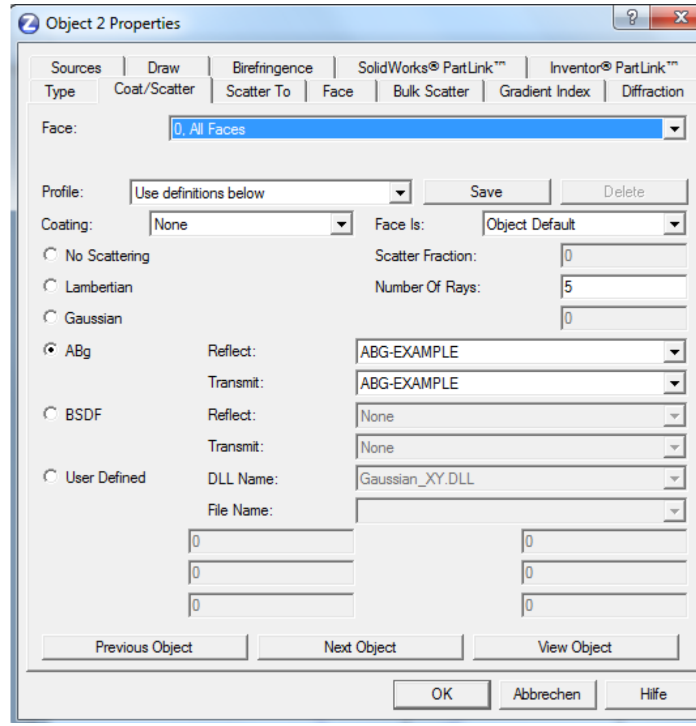
- Geometrical imaging of a bar pattern
- Image with / without Scattering
Scattering must be activated in settings

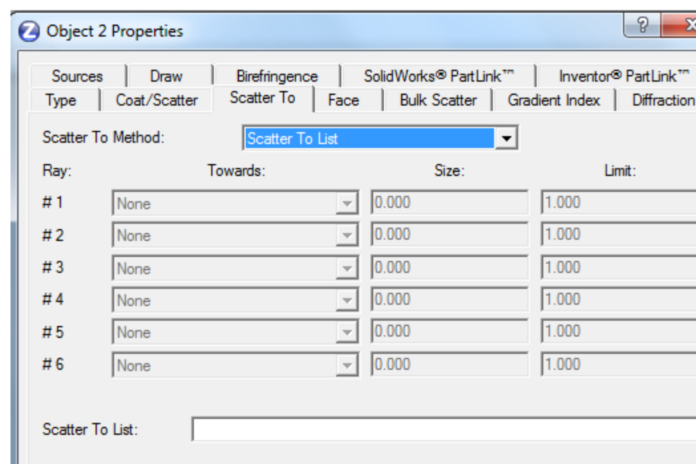


- Blurring increases with growing σ-value

- Example from samples with non-sequential mode
- Important sampling accelerates the calculation







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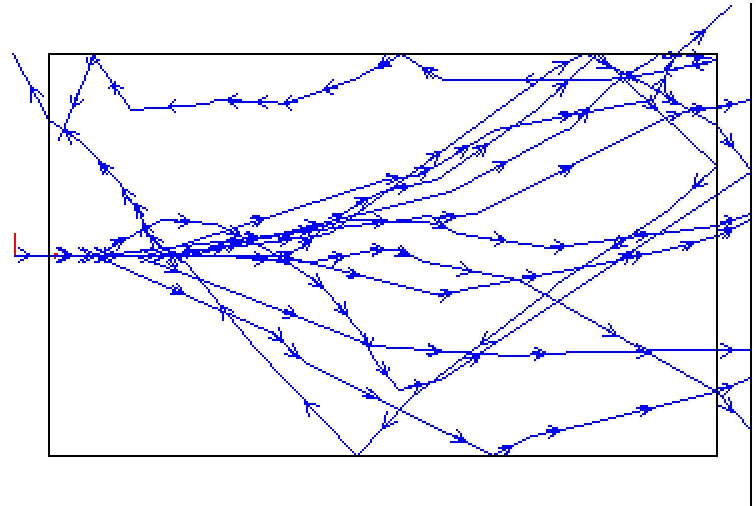
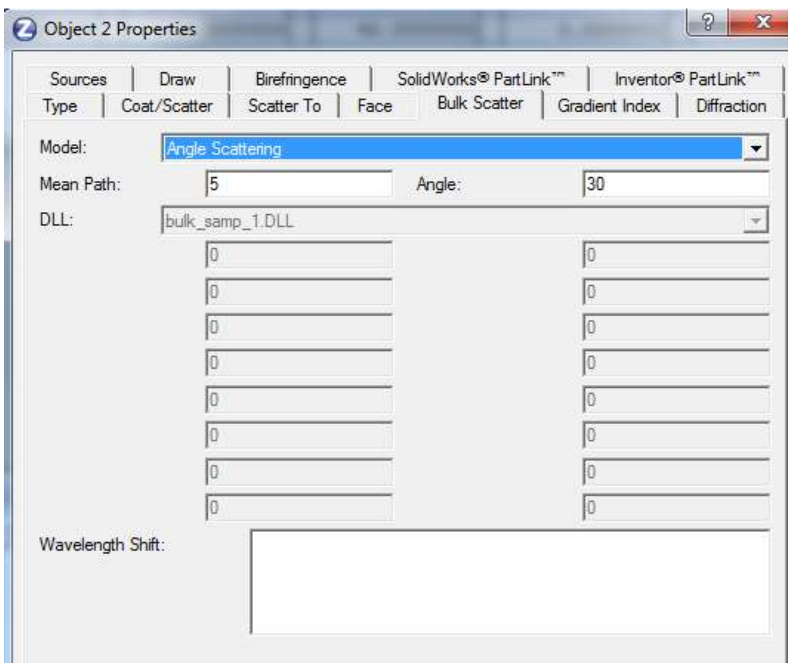
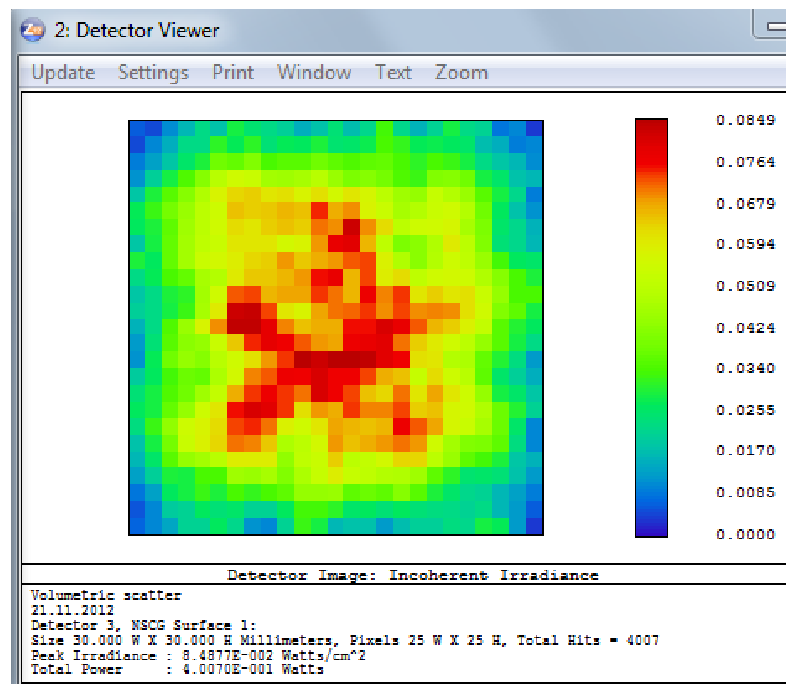
Scattering in Zemax

- Volume scattering example
- Stokes shift is possible for fluorescence

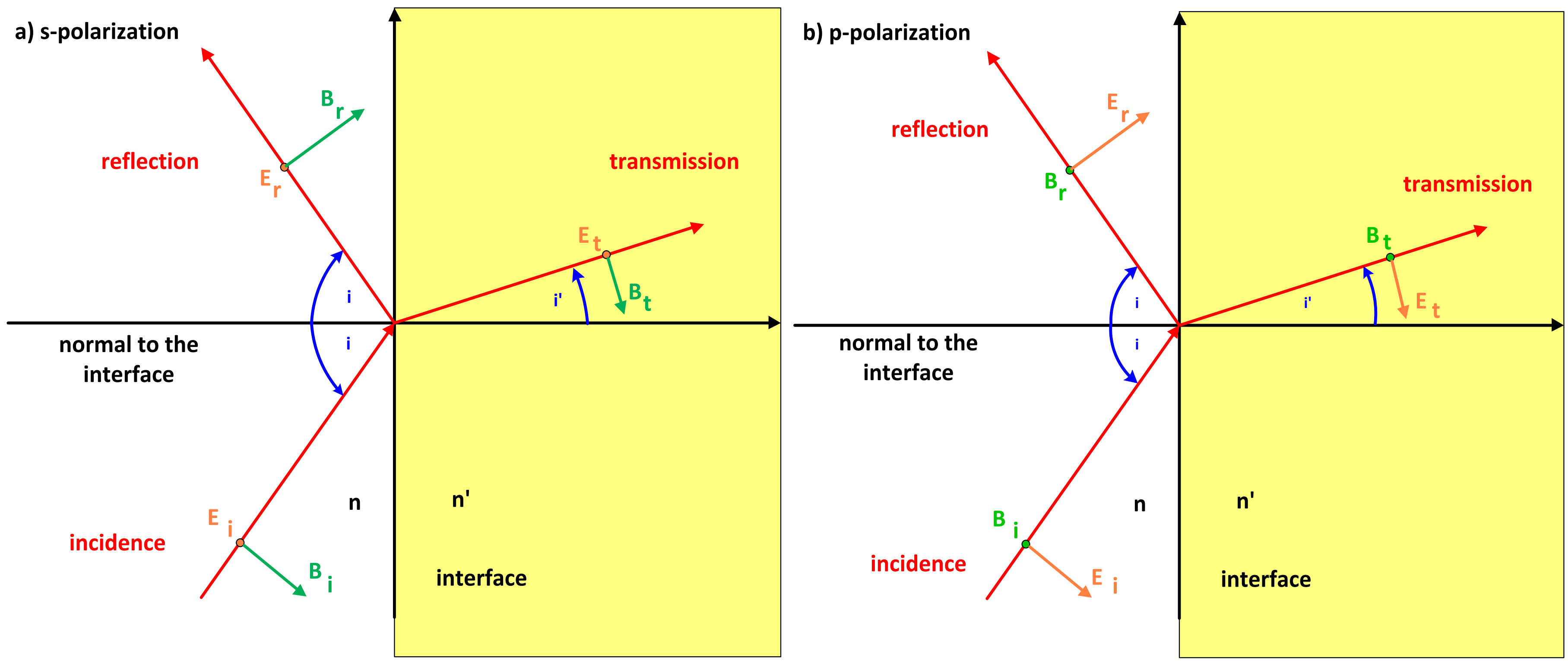






Fresnel Formulas

- Schematical illustration of the ray refraction (reflection at an interface
- The cases of s- and p-polarization must be distinguished



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Fresnel Formulas

- Electrical transverse polarization
  TE, s- or σ-polarization, E perpendicular to incidence plane $\perp$

- Magnetical transverse polarization
  TM, p- or p-polarization, E in incidence plane $\parallel$

- Boundary condition of Maxwell equations
  at a dielectric interface:
  continuous tangential component of E-field

$$\varepsilon_1 \cdot E_{1n} = \varepsilon_2 \cdot E_{2n}$$

$$E_{1t} = E_{2t}$$

- Amplitude coefficients for
  reflected field

$$r_{TE} = \left.\frac{E_r}{E_e}\right|_{TE} \qquad r_{TM} = \left.\frac{E_r}{E_e}\right|_{TM}$$

  transmitted field

$$t_{TE} = \left.\frac{E_t}{E_e}\right|_{TE} = r_{TE} + 1 \qquad t_{TM} = \frac{n}{n'} \cdot (r_{TM} + 1)$$

- Reflectivity and transmission
  of light power

$$R = \frac{P_r}{P_e} = \left| r^2 \right| \qquad T = \frac{P_t}{P_e} = \frac{n' \cdot \cos i'}{n \cdot \cos i} \cdot \left| t^2 \right|$$

Fresnel Formulas: Stokes Relations

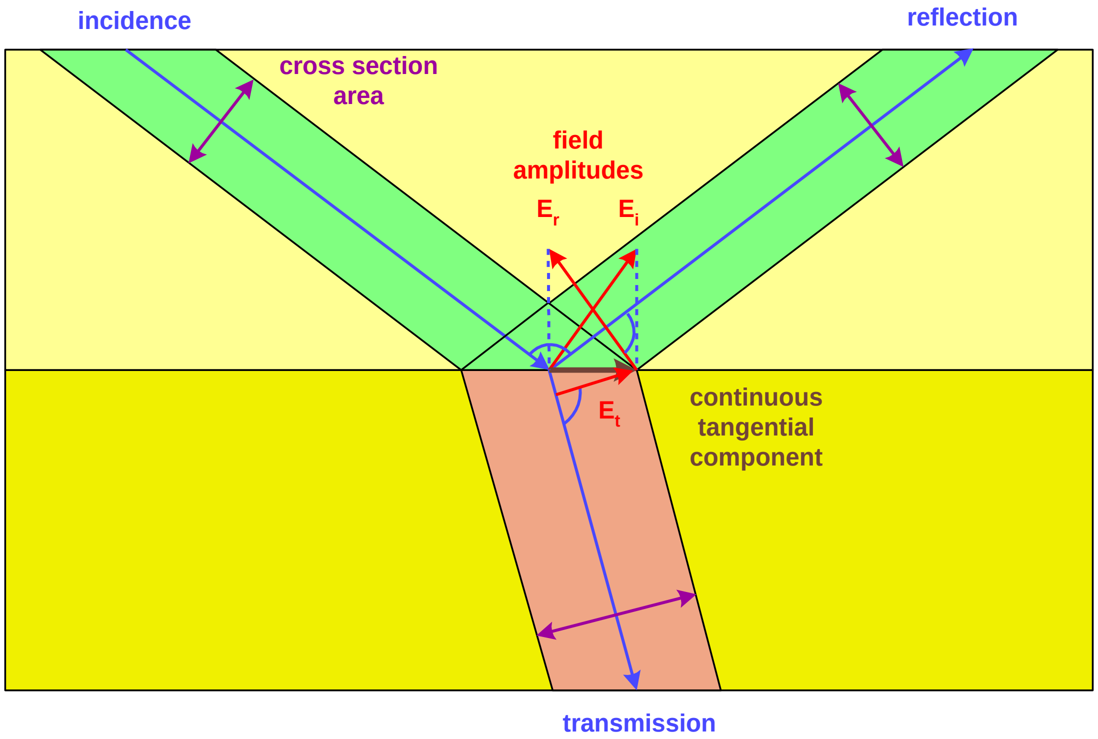
- Relation between the amplitude coefficients for reflection/transmission:

1. s-components:
   field components additive
   minus sign due to phase jump

$$t_{\perp} - r_{\perp} = 1$$

2. p-components:
   energy preservation but change of area size due to projection, correction factor, no additivity of intensities

$$t_{\parallel} \cdot \frac{\cos i'}{\cos i} + r_{\parallel} = 1$$



Fresnel Formulas

- Coefficients of amplitude for reflected rays, s and p

$$r_{E\perp} = -\frac{\sin(i-i')}{\sin(i+i')} = \frac{n\cdot\cos i - \sqrt{n'^2 - n^2\cdot\sin^2 i}}{n\cdot\cos i + \sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{n\cdot\cos i - n'\cdot\cos i'}{n\cdot\cos i + n'\cdot\cos i'} = \frac{k_{ez} - k_{tz}}{k_{ez} + k_{tz}}$$

$$r_{E\parallel} = \frac{\tan(i-i')}{\tan(i+i')} = \frac{n'^2\cdot\cos i - n\cdot\sqrt{n'^2 - n^2\cdot\sin^2 i}}{n'^2\cdot\cos i + n\cdot\sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{n'\cdot\cos i - n\cdot\cos i'}{n'\cdot\cos i + n\cdot\cos i'} = \frac{n'^2\cdot k_{ez} - n^2\cdot k_{tz}}{n'^2\cdot k_{ez} + n^2\cdot k_{tz}}$$

- Coefficients of amplitude for transmitted rays, s and p

$$t_{E\perp} = \frac{2n\cdot\cos i}{n\cdot\cos i + n'\cdot\cos i'} = \frac{2n\cdot\cos i}{n\cdot\cos i + \sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{2n\cdot\cos i}{n\cdot\cos i + n'\cdot\cos i'} = \frac{2k_{ez}}{k_{ez} + k_{tz}}$$

$$t_{E\parallel} = \frac{2n\cdot\cos i}{n'\cdot\cos i + n\cdot\cos i'} = \frac{2n'\cdot n\cdot\cos i}{n'^2\cdot\cos i + n\cdot\sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{2n\cdot\cos i}{n'\cdot\cos i + n\cdot\cos i'} = \frac{2n'^2\cdot k_{ez}}{n'^2\cdot k_{ez} + n^2\cdot k_{tz}}$$

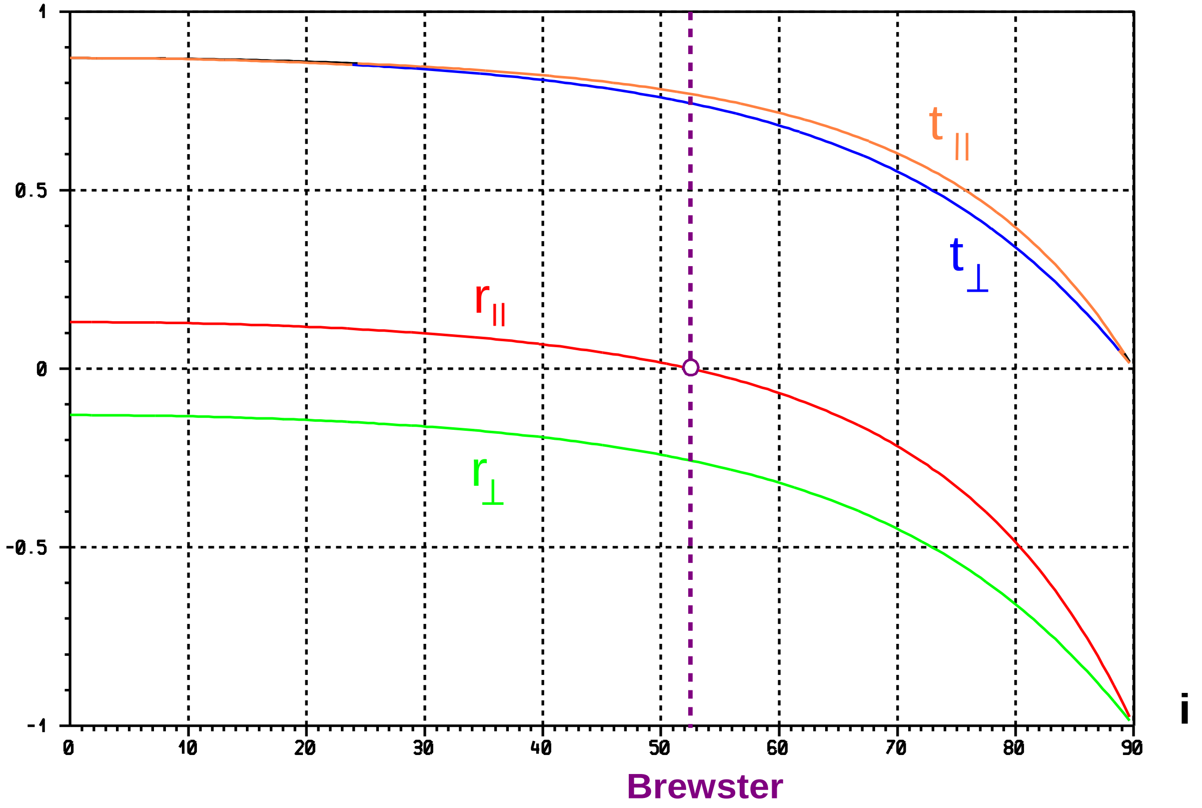
Fresnel Formulas

- Typical behavior of the Fresnel amplitude coefficients as a function of the incidence angle for a fixed combination of refractive indices

- i = 0
  Transmission independent
  on polarization
  Reflected p-rays without
  phase jump
  Reflected s-rays with
  phase jump of π
  (corresponds to r<0)

- i = 90°
  No transmission possible
  Reflected light independent
  on polarization

- Brewster angle:
  completely s-polarized
  reflected light



Fresnel Formulas: Energy vs. Intensity

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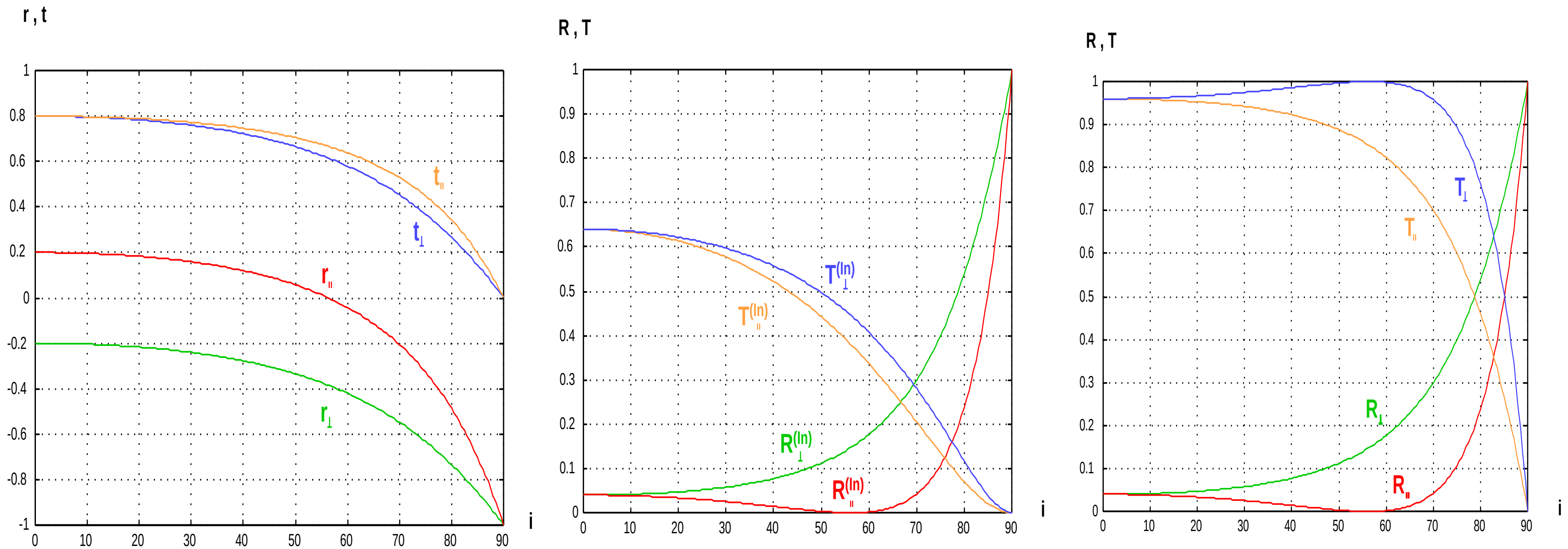
Fresnel formulas, different representations:

1. Amplitude coefficients, with sign

2. Intensity cefficients: no additivity due to area projection

3. Power coefficients: additivity due to energy preservation



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Fresnel Formulas

- Reflectivity and transmittivity of power

$$R_\perp = \frac{\sin^2(i-i')}{\sin^2(i+i')} \qquad R_\parallel = \frac{\tan^2(i-i')}{\tan^2(i+i')} \qquad T_\perp = \frac{\sin 2i' \cdot \cos 2i}{\sin^2(i+i')} \qquad T_\parallel = \frac{\cos 2i \cdot \sin 2i'}{\sin^2(i+i')\cdot\cos^2(i-i')}$$

- Arbitrary azimuthal angle τ of polarization: decompositioin of components

$$R = R_\parallel \cdot \cos^2\tau_e + R_\perp \cdot \sin^2\tau_e \qquad T = T_\parallel \cdot \cos^2\tau_e + T_\perp \cdot \sin^2\tau_e$$

- In case of vanishing absorption:
  Energy preservation

$$R + T = 1$$

- Special case of normal incidence

$$R_\perp = \left(\frac{n-n'}{n+n'}\right)^2 \qquad T_\perp = \frac{4n\cdot n'}{(n+n')^2}$$

- Typical values for some glasses and optical materials in air

n	R
1.4	2.778 %
1.5	4.0 %
1.8	8.16 %
2.4	16.96 %

Introduction and Overview

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- Optical multi layer systems / thin layers:
  Change of reflection, transpossion, polarization

- Application in optical systems:
  1. improved transmissioin
  2. avoiding false light and ghosts

- Thin layer stacks:
  1. interference at many interface planes
  2. the layer thickness is in the range of the wavelength
  3. calculation is quite complex

- Types of coatings:
  1. AR coatings
  2. HR coatings (mirrors)
  3. spectral edge filter
  4. spectral band filter
  5. beam splitter
  6. polarizing coatings
  7. color filtering

Types of Coatings

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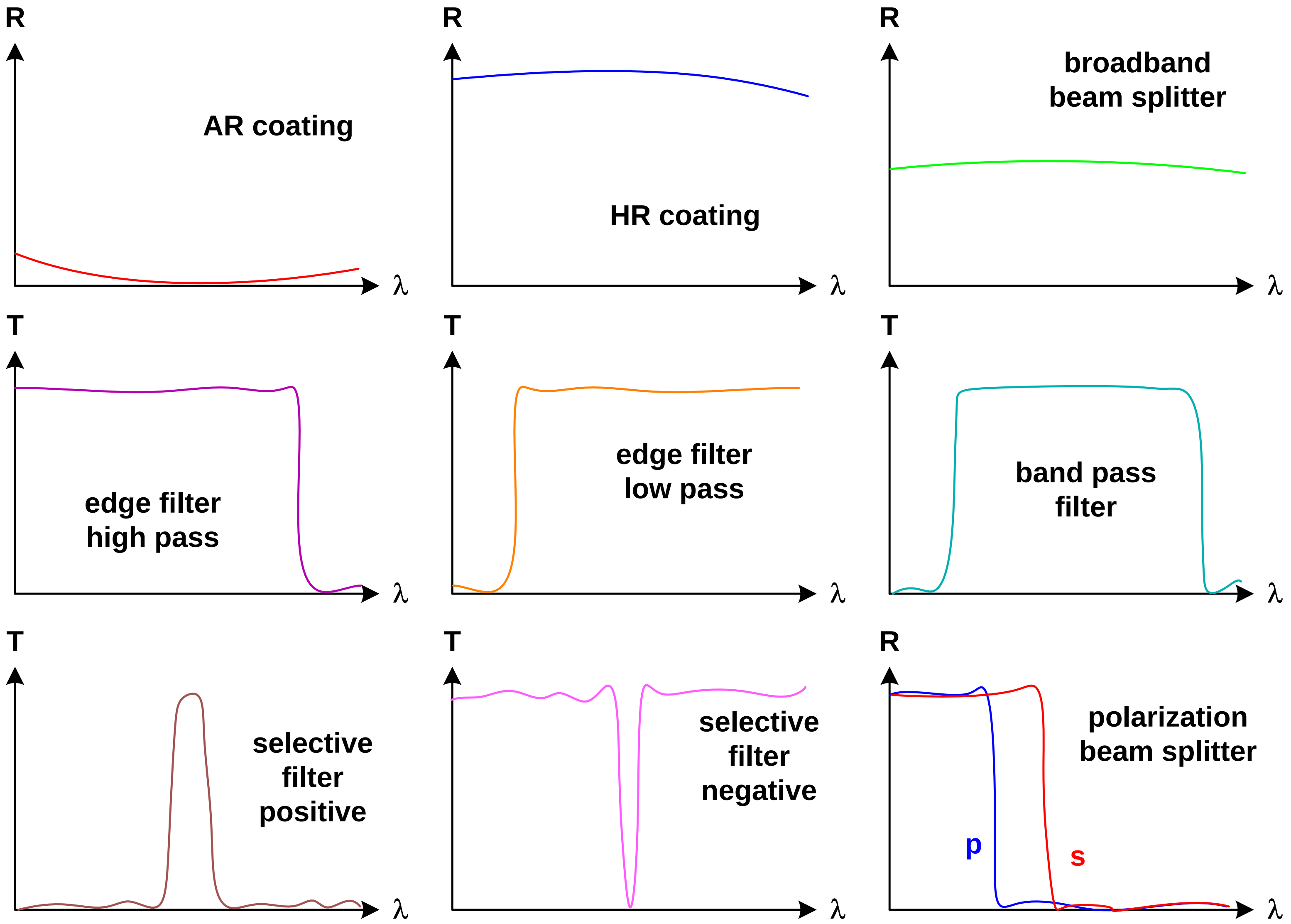
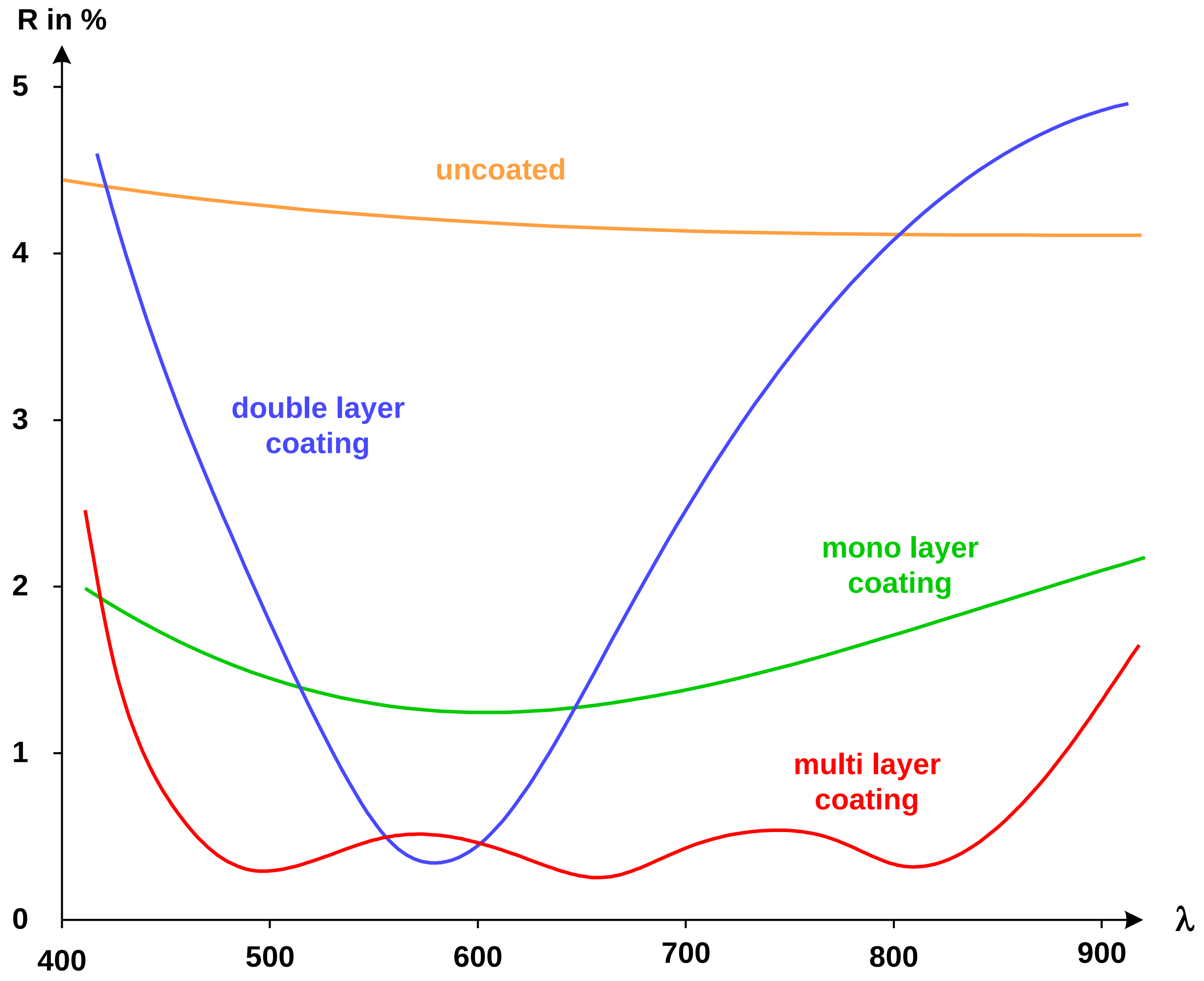


Comparison of Coatings

- Comparison of coatings with different number of layers:
  1. without coating
  2. single layer
  3. double layer
  4. multi-layer

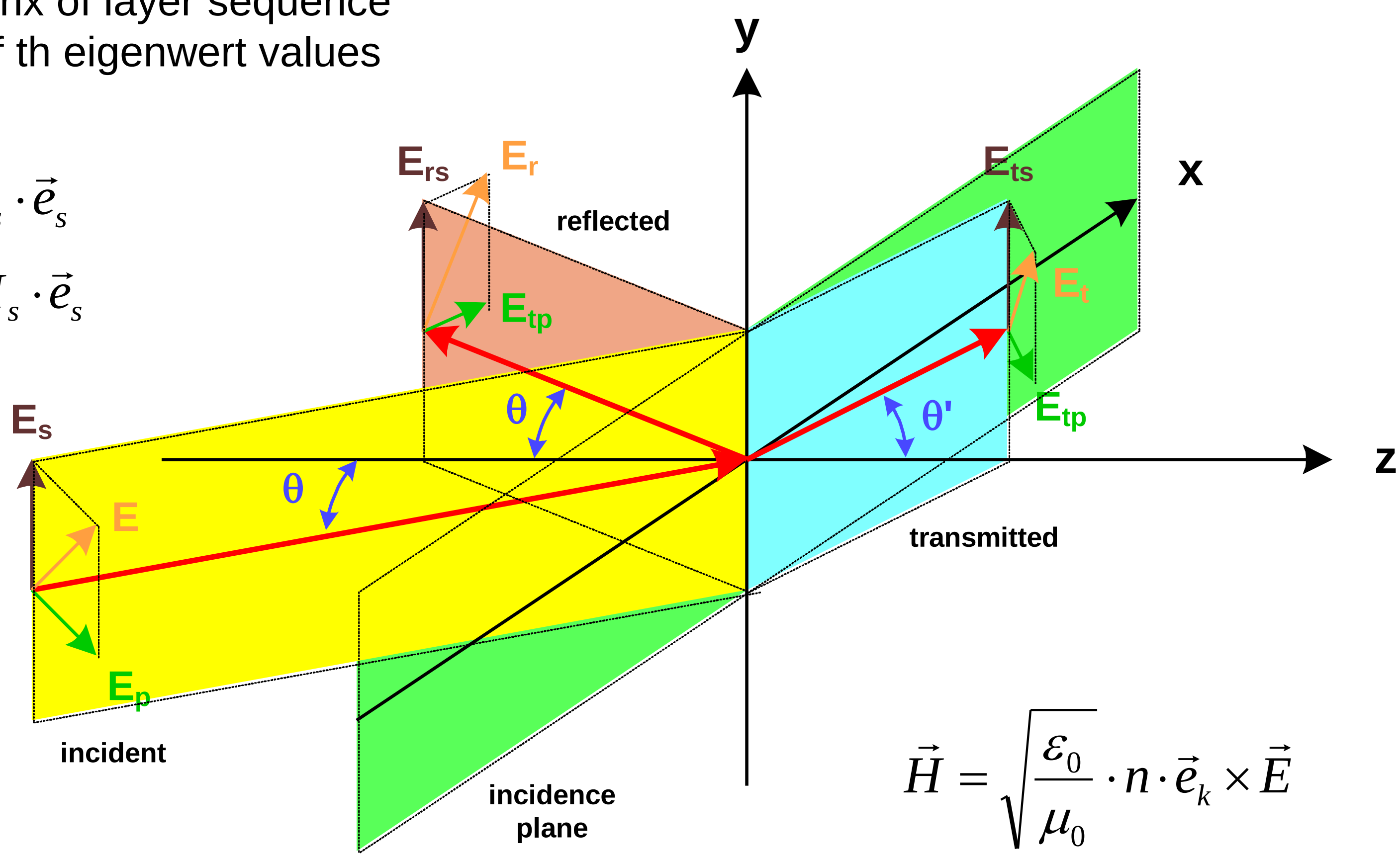


Field Components in Layers

- Principle of calculation of a thin layer system:
  1. decomposition of field components at the plane interfaces
  2. continuity condition of Fresnel equations
  3. resulting matrix of layer sequence
  4. calculation of th eigenwert values

$$\vec{E} = E_p \cdot \vec{e}_p + E_s \cdot \vec{e}_s$$

$$\vec{H} = H_p \cdot \vec{e}_p + H_s \cdot \vec{e}_s$$



$$\vec{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot n \cdot \vec{e}_k \times \vec{E}$$

Calculation of Thin Layer Stacks

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- Optical impedance in vacuum

$$Z_0 = \sqrt{\frac{\varepsilon_o}{\mu_0}} = \frac{H}{E}$$

- Impedance of a system stack for both polarizations

$$Z_p = \frac{H}{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot \frac{n}{\cos\theta} = Z_o \cdot \frac{n}{\cos\theta}$$

$$Z_s = \frac{H}{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot n \cdot \cos\theta = Z_o \cdot n \cdot \cos\theta$$

- Equivalent phase delay of a layer for incidence angle θ

$$\delta = \frac{2\pi \cdot n \cdot d \cdot \cos\theta}{\lambda_o}$$

- Reflectivity of amplitude

$$r_s = \frac{E_{os}^{ref}}{E_{os}^{inc}} \qquad R_s = \left| \frac{E_{os}^{ref}}{E_{os}^{inc}} \right|^2 = |r_s|^2$$

- Transmission of amplitude

$$t_s = \sqrt{\frac{n_g}{n_o}} \cdot \frac{E_{gs}^{trans}}{E_{os}^{inc}} \qquad T_s = \frac{n_g}{n_o} \left| \frac{E_{gs}^{trans}}{E_{os}^{inc}} \right|^2 = |t_s|^2$$

Matrix Model of Stack Calculation

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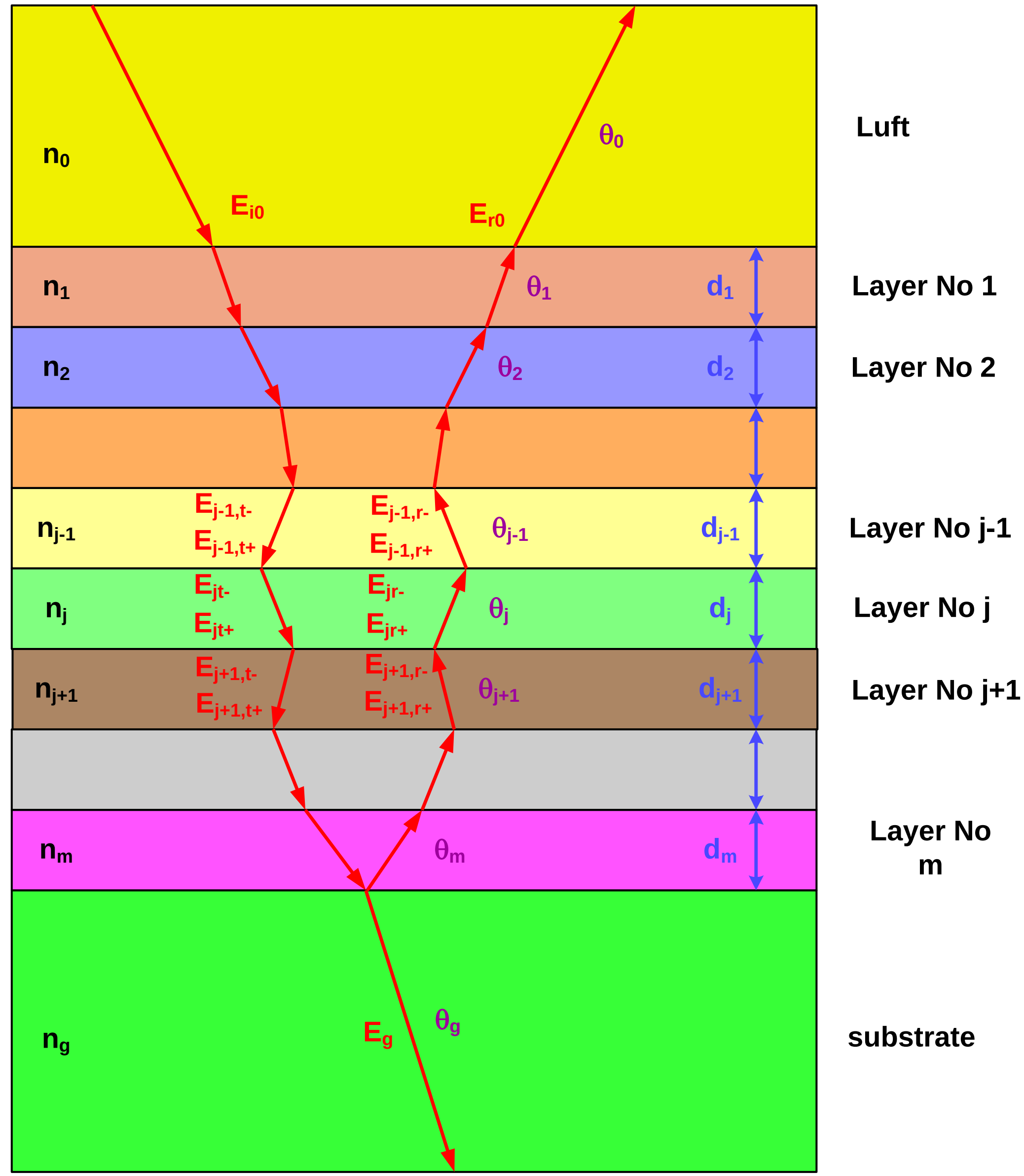
- Field components:
  1. entrance and exit of a layer
  2. E and H-field
  3. forward and backward propagating wave
  4. both polarization components

- Continuity conditions at the interface planes

- Solution of linear system equations

- Also valid for absorbing media



Transfer Matrices

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- Transfer matrices
  - for one layer
  - for both polarizations p and s

$$\begin{pmatrix} E_{j+1,s} \\ H_{j+1,s} \end{pmatrix} = \begin{pmatrix} \cos\delta_j & \dfrac{i}{n_j \cdot \cos\theta_j} \cdot \sin\delta_j \\ i \cdot n_j \cdot \cos\theta_j \cdot \sin\delta_j & \cos\delta_j \end{pmatrix} \cdot \begin{pmatrix} E_{j,s} \\ H_{j,s} \end{pmatrix}$$

$$\begin{pmatrix} E_{j+1,p} \\ H_{j+1,p} \end{pmatrix} = \begin{pmatrix} \cos\delta_j & \dfrac{i \cdot \cos\theta}{n_{j\,j}} \cdot \sin\delta_j \\ i \cdot \dfrac{n_j}{\cos\theta_j} \cdot \sin\delta_j & \cos\delta_j \end{pmatrix} \cdot \begin{pmatrix} E_{j,p} \\ H_{j,p} \end{pmatrix}$$

- Matrix of complete stack (representation with B)

$$\begin{pmatrix} E_j \\ B_j \end{pmatrix} = \underline{M}_{jm} \cdot \begin{pmatrix} E_m \\ B_m \end{pmatrix}$$

$$\underline{M}_{1,m} = \prod \underline{M}_j = \underline{M}_m \cdot \underline{M}_{m-1} \cdot \ldots \cdot \underline{M}_3 \cdot \underline{M}_2 \cdot \underline{M}_1$$

- Impedance of air and substrate must be taken into account

$$Z_{gs} = \sqrt{\frac{\varepsilon_o}{\mu_0}} \cdot n_g \cdot \cos\theta_g \qquad Z_{s0} = \sqrt{\frac{\varepsilon_o}{\mu_0}} \cdot n_0 \cdot \cos\theta$$

Final Calculations Step

- Single layer matrix

$$\underline{M}_j = \begin{pmatrix} \cos\delta_j & \dfrac{i\cdot\sin\delta_j}{Z_j} \\ i\cdot Z_j\cdot\sin\delta_j & \cos\delta_j \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

- Reflectivity of complete system
  for s and p polarization

$$r_s = \frac{Z_{so}M_{11} + Z_{so}Z_{sg}M_{12} - M_{21} - Z_{sg}M_{22}}{Z_{so}M_{11} + Z_{so}Z_{sg}M_{12} + M_{21} + Z_{sg}M_{22}}$$

- Transmission of complete system
  for s and p polarization

$$t_s = \frac{2Z_{so}}{Z_{so}M_{11} + Z_{so}Z_{sg}M_{12} + M_{21} + Z_{sg}M_{22}}$$

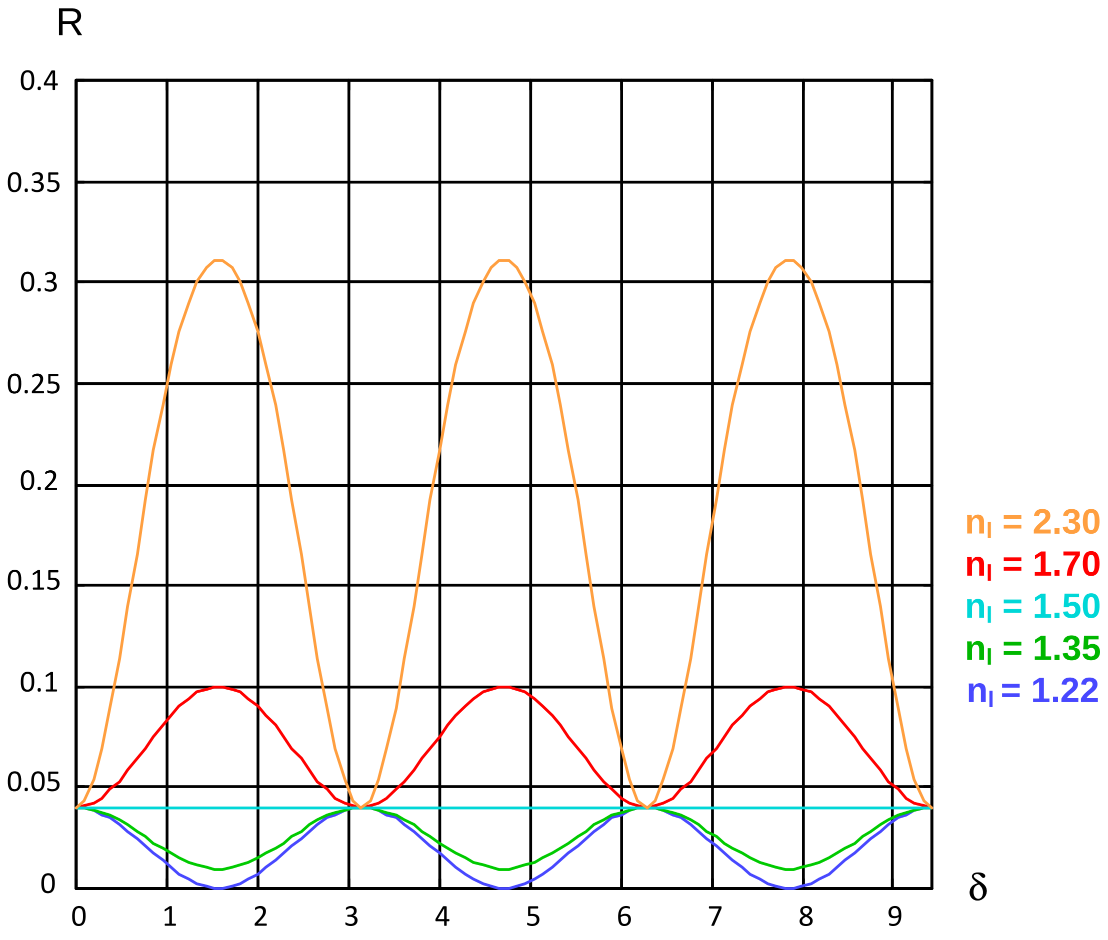
- Matrix approach:
  - analysis method
  - optimization by NLSQ-algorithms
  - problem: periodicity of phase

- In principle solution only for
  1. one wavelength
  2. one incidence angle

- Typically the spectral performance is shown as a function of λ_o / λ

Influence of the Layer Thickness

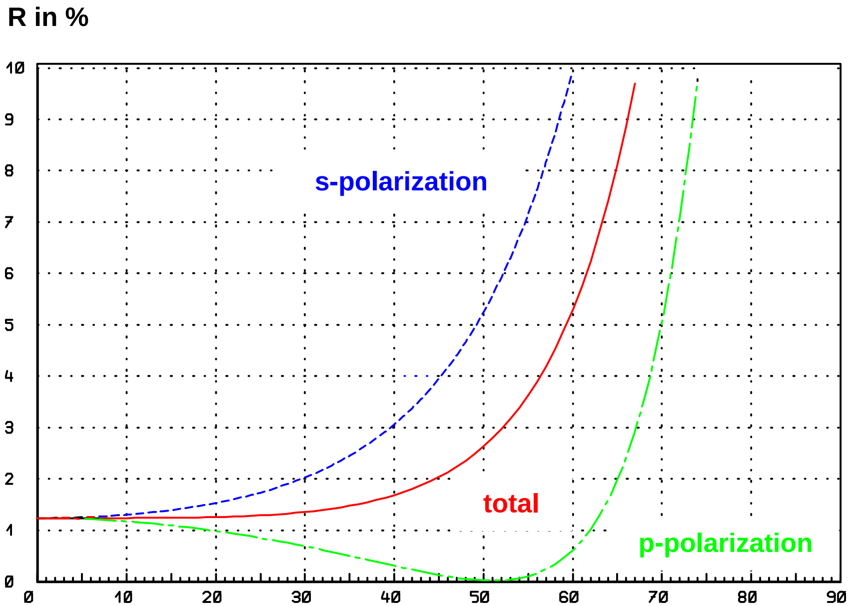
- Periodicity of phase:
  dependence of performance on thickness
- Period is the wavelength
- Special thicknesses:
  waves in phase,
  no effect of the thin layer stack

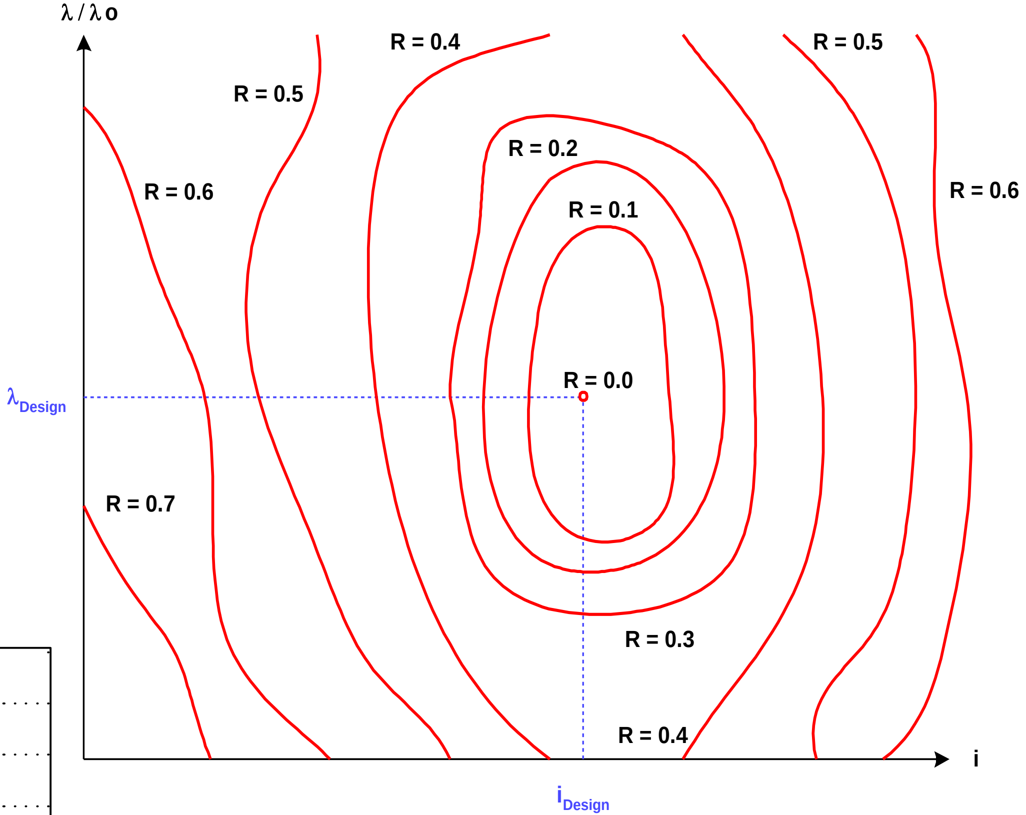


Dependence on Incidence Angle and Polarization

- Fresnel formulas:
  reflectivity in general depends on
  1. incidence angle
  2. polarization

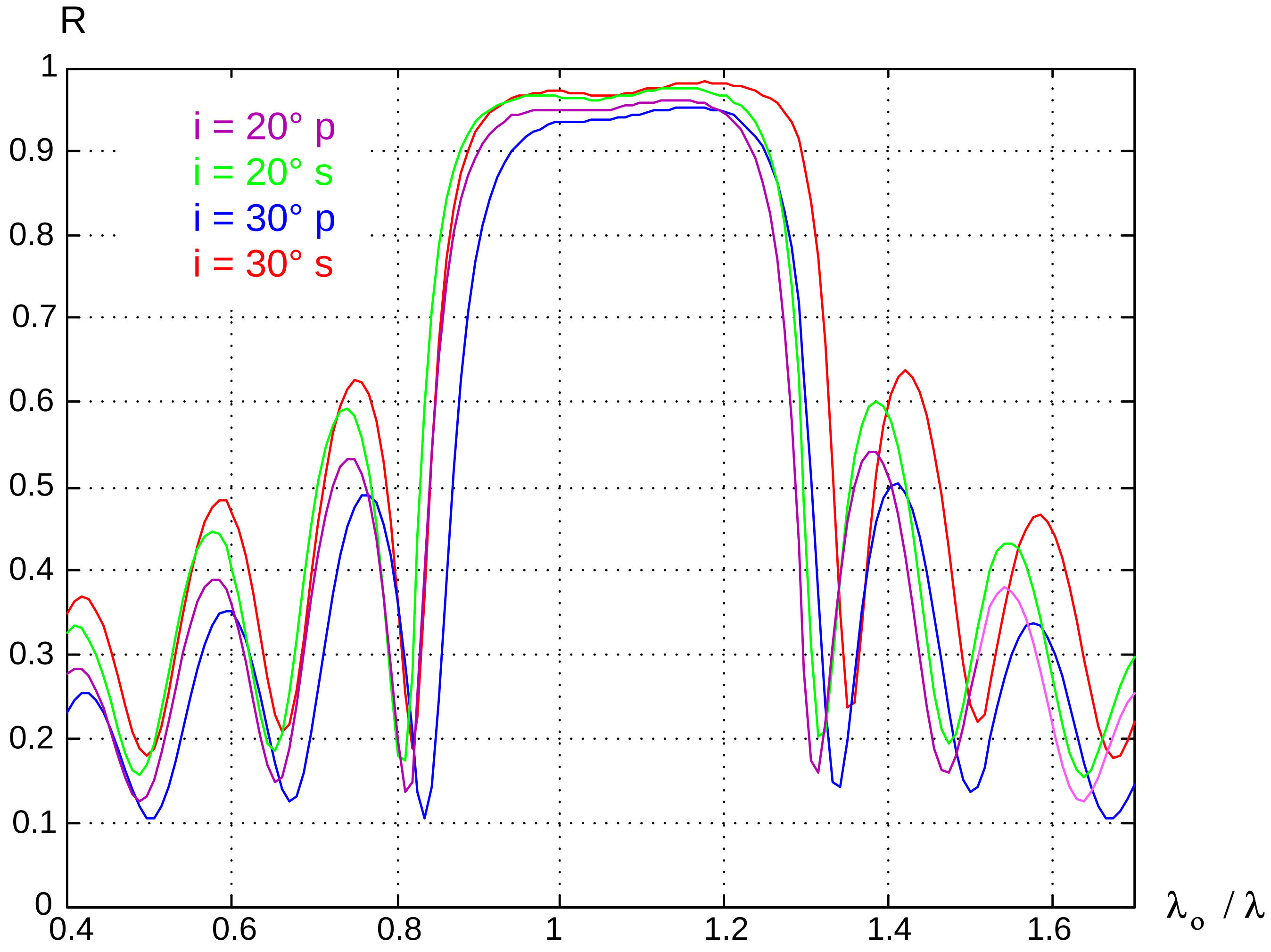
- Degradation for broad band
  and incidence angle interval





Example Coating

- Special example:
  13 layer with
  L : n = 1.45
  H : n = 2.35

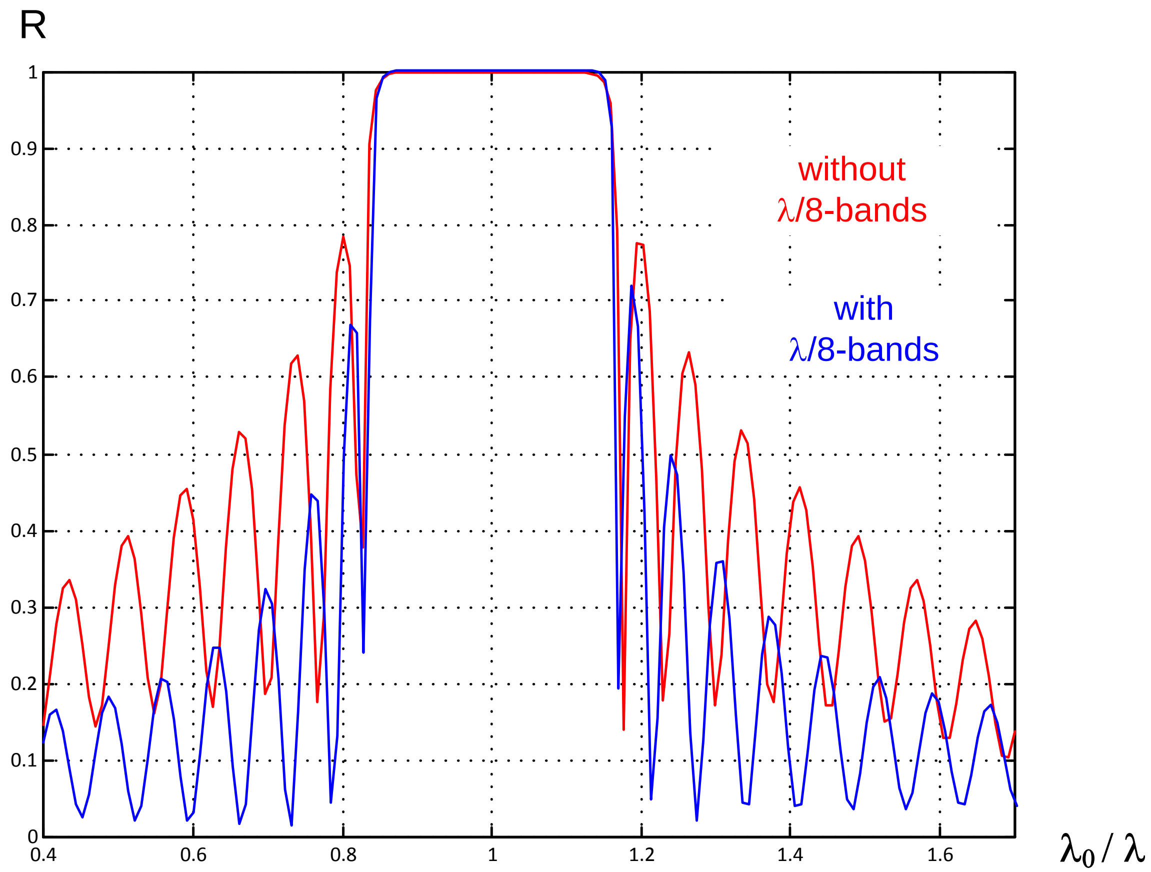


Sidelobs

- Typical coating:
  stack of low-high-materials (LH) with thickness λ / 4

- Remaining sidelobs outside of a centrals spectral interval

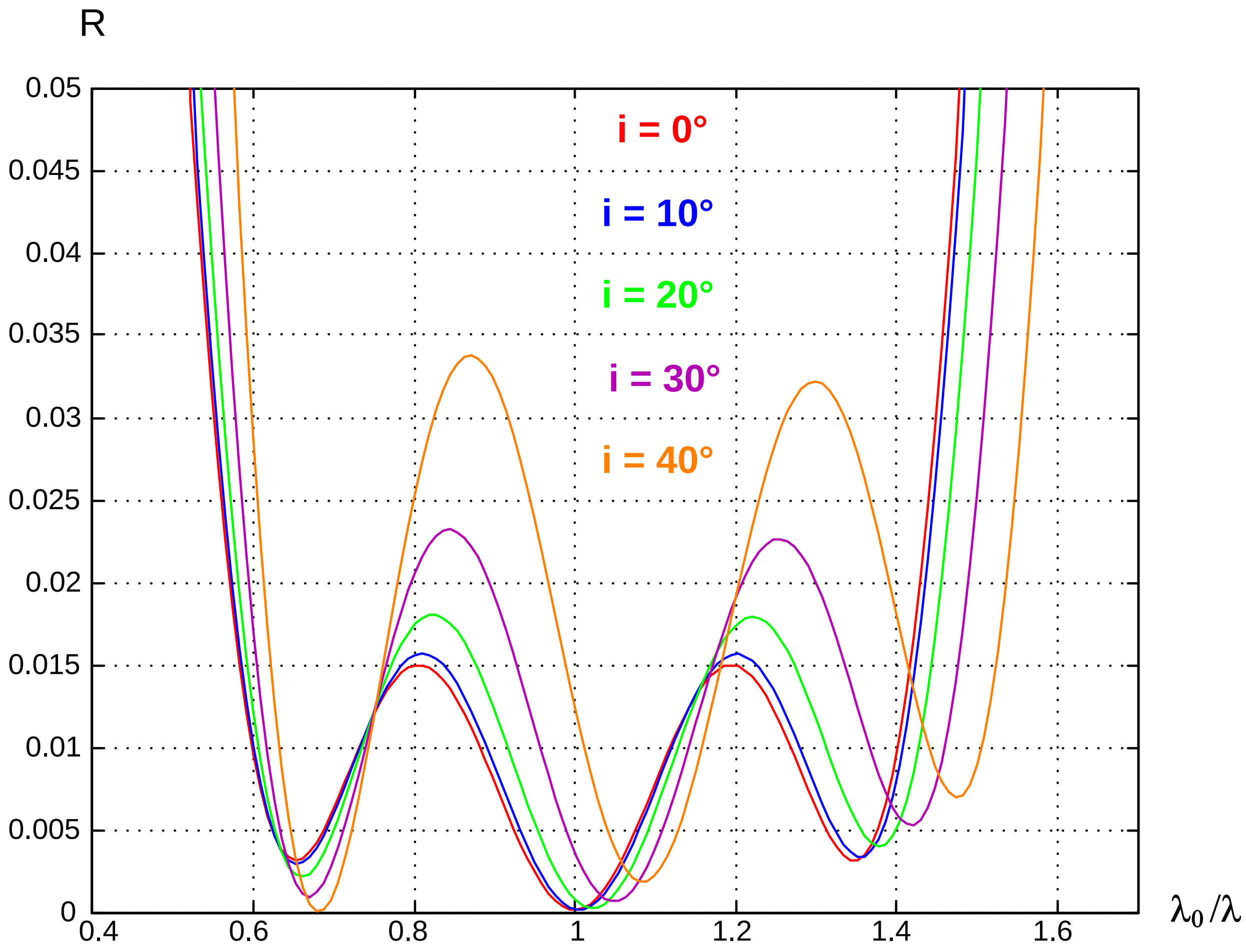
- Suppression of sidelobs:
  additional initial and final layer with λ/8



Example Coating

- Special example:
  4 layer with thickness $\lambda/4$
  L : n = 1.45
  H : n = 2.35

- Reflectivity for different incidence angles

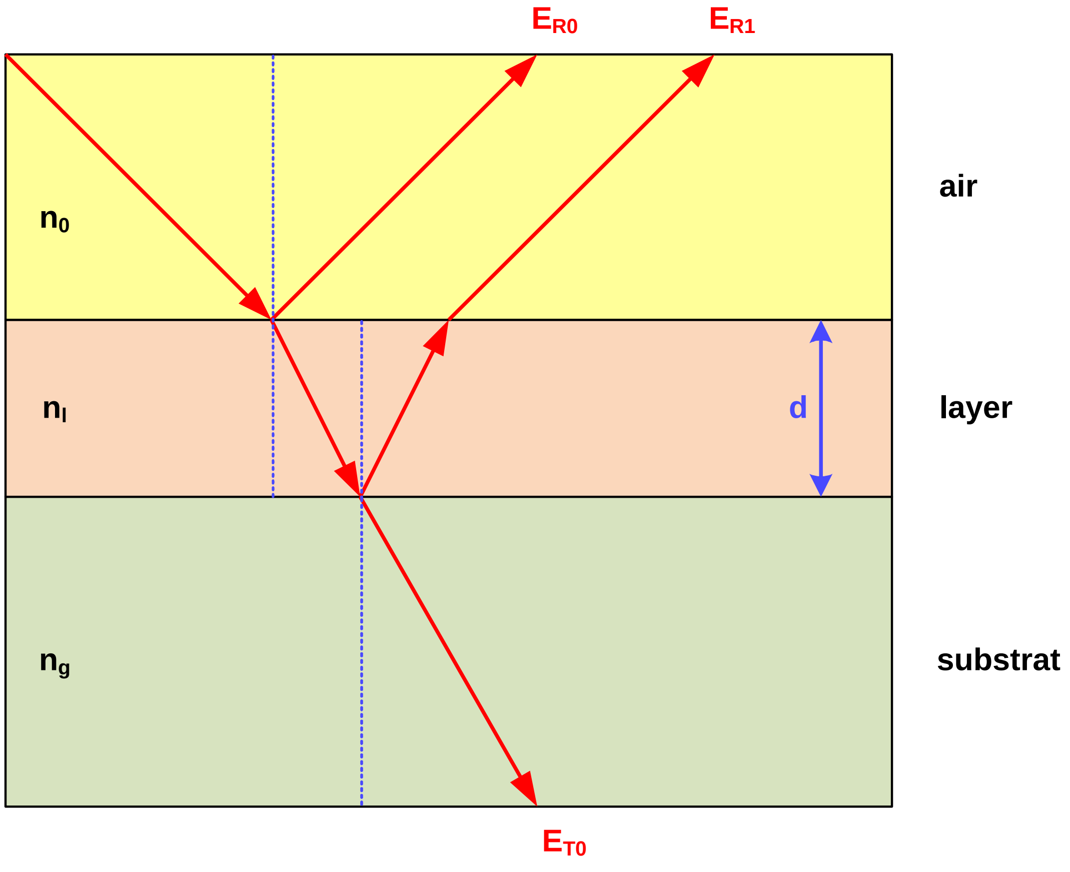


Single λ/4 Layer

- Single layer with thickness λ/4
- Matrix

$$\underline{M}_{\lambda/4} = \begin{pmatrix} 0 & \dfrac{i}{Z_0 n_l} \\ iZ_o n_l & 0 \end{pmatrix}$$

- Destructive interference of the two backwards propagating waves: no reflected light



Single λ/4 Layer

- Reflectivity of amplitude

$$r = \frac{n_l \cdot (n_o - n_g) \cdot \cos\delta + (n_o n_g - n_l^2) \cdot i \cdot \sin\delta}{n_l \cdot (n_o + n_g) \cdot \cos\delta + (n_o n_g + n_l^2) \cdot i \cdot \sin\delta}$$

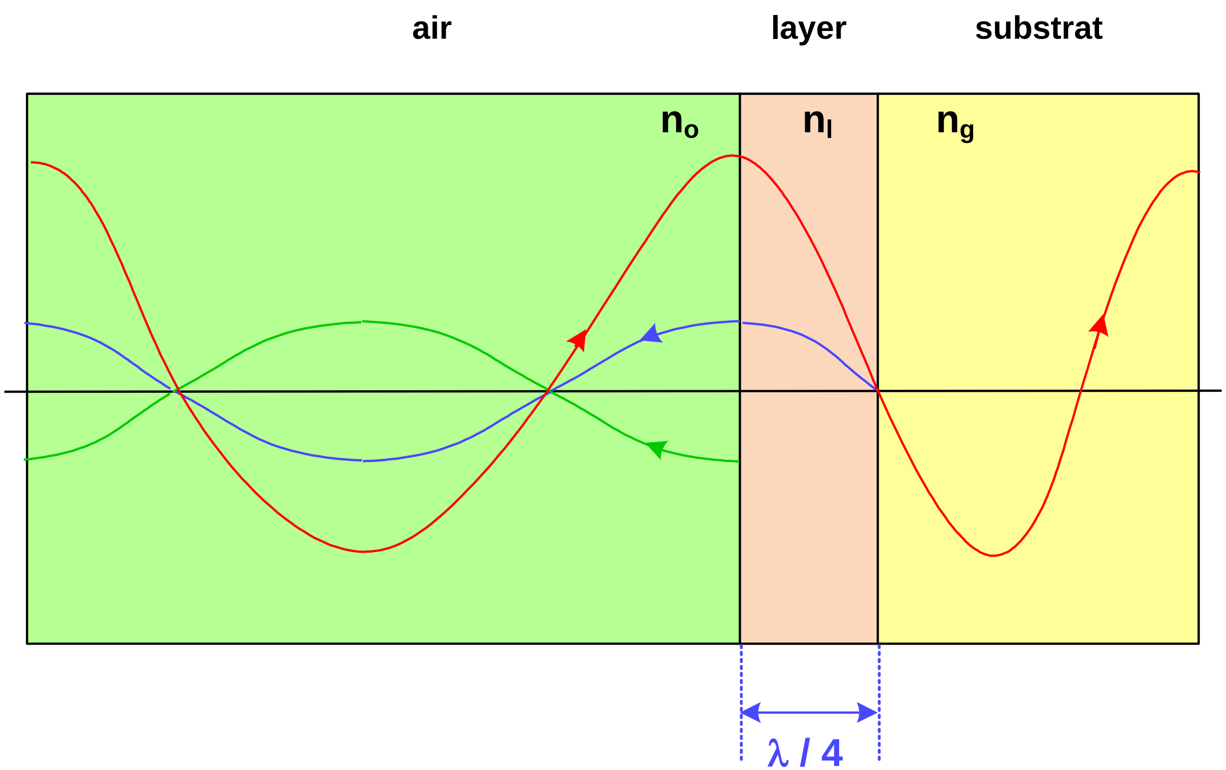
- Phase condition corresponds to λ/4

$$d = \frac{\lambda}{4 \cdot n_l}$$

- Amplitude condition: identical amplitude

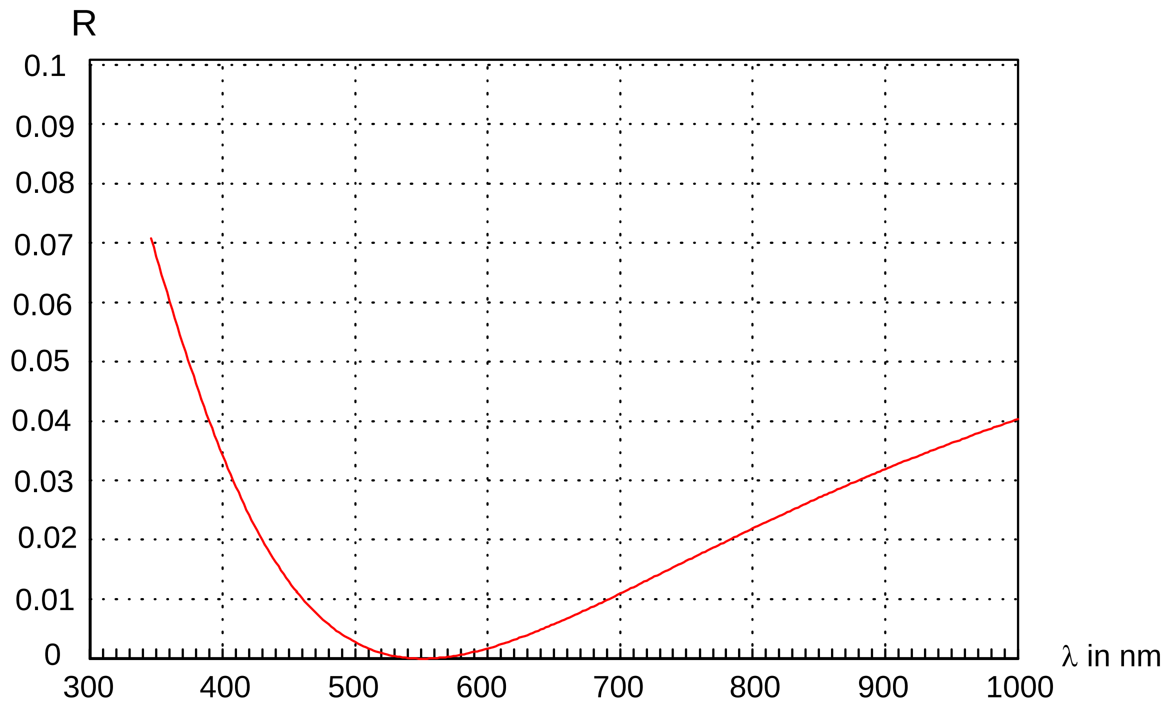
$$n_l = \sqrt{n_o \cdot n_g}$$

- Problem:
  on a glass with n = 1.5
  a layermaterial of
  n=1.23 is needed



Single λ/4 Layer

- Spectral behavior:
  single zero of reflectivity at the desired design wavelength



- Residual reflectivity for violation of the amplitude condition
  (non-ideal refractive index)

$$R_{single} = \left(\frac{n_o \cdot n_s - n_l^2}{n_o \cdot n_s + n_l^2} \right)^2$$

Double Layer

- Double layer: several solutions possible

- One simple solution: thickness

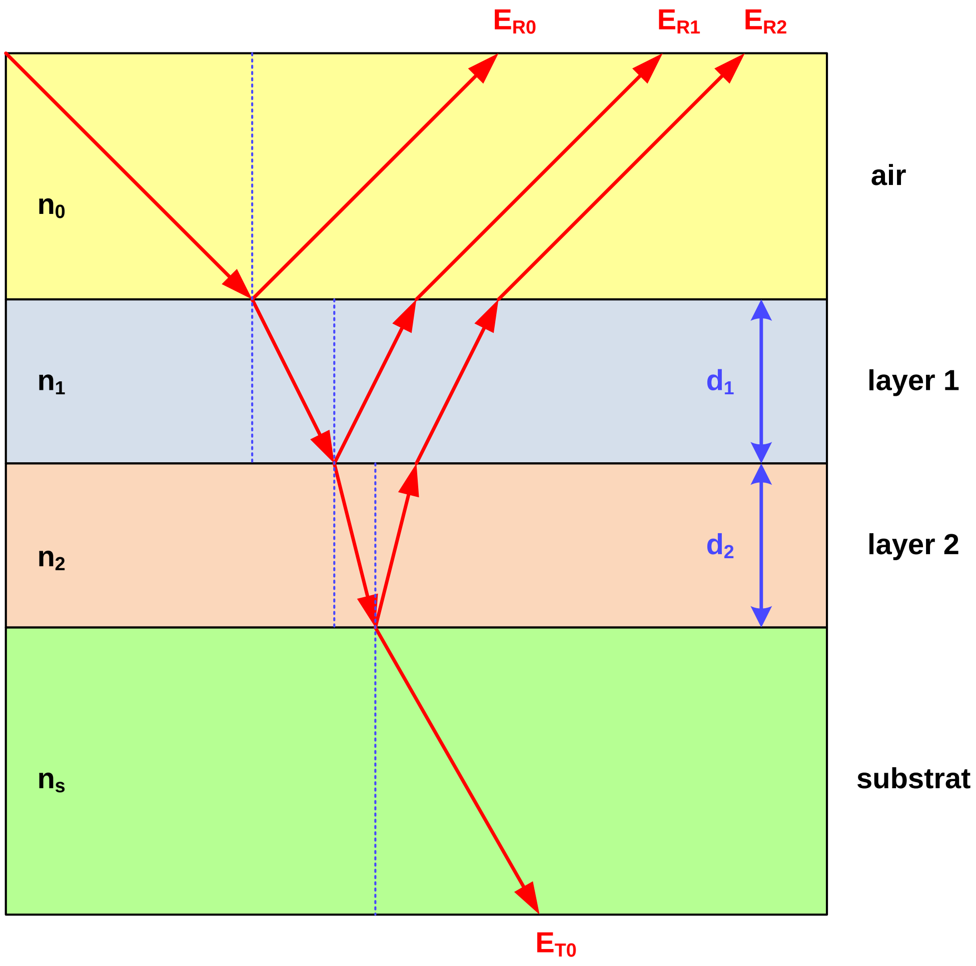
$$n_1 d_1 = n_2 d_2 = \lambda / 4$$

- Corresponding amplitude condition

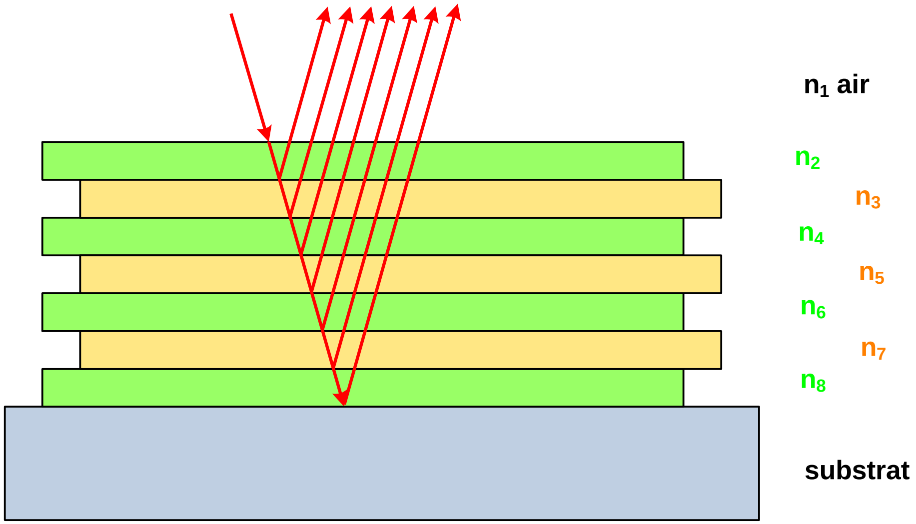
$$\frac{n_1}{n_2} = \sqrt{\frac{n_o}{n_g}}$$

- Residual reflectivity

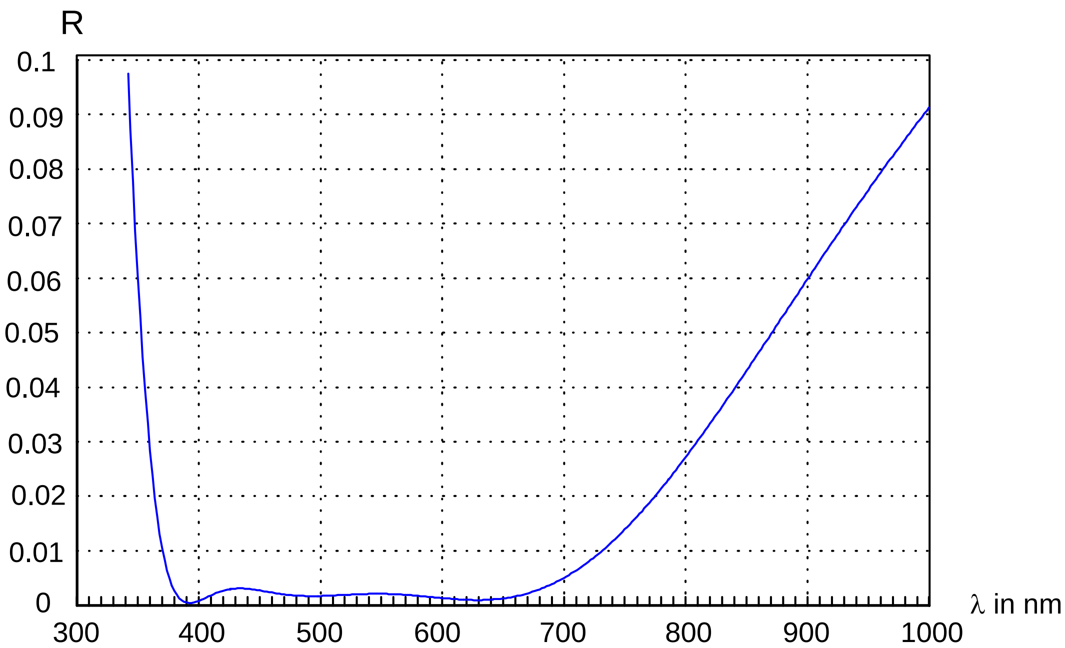
$$R_{double} = \left(\frac{n_0 \cdot n_2^2 - n_g \cdot n_1^2}{n_0 \cdot n_2^2 + n_g \cdot n_1^2} \right)^2$$



- Scheme:



- Typical broad spectral performance for reflectivity

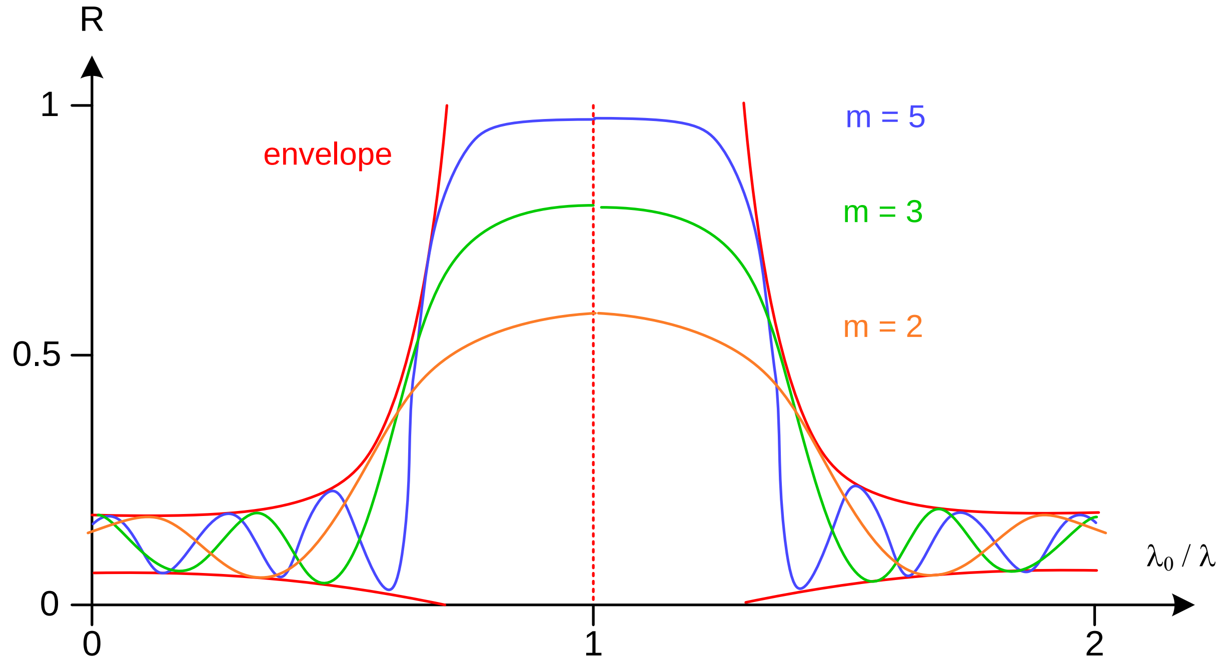


Periodical Layer of 2 Materials

- Especially simple layout: periodical stack with pair of materials

  a(LH)^{m}g

  a : air
  L : low-index material
  H : high-index material
  g : glass substrate
  m : number of pairs



- Approximation of residual reflectivity

$$R \approx 1 - 4 \cdot \frac{n_g}{n_H^2} \cdot \left(\frac{n_L}{n_H} \right)^m$$

- Advantageous:
  1. large ratio of indices
  2. high number of layer pairs

Periodical Layer of 2 Materials

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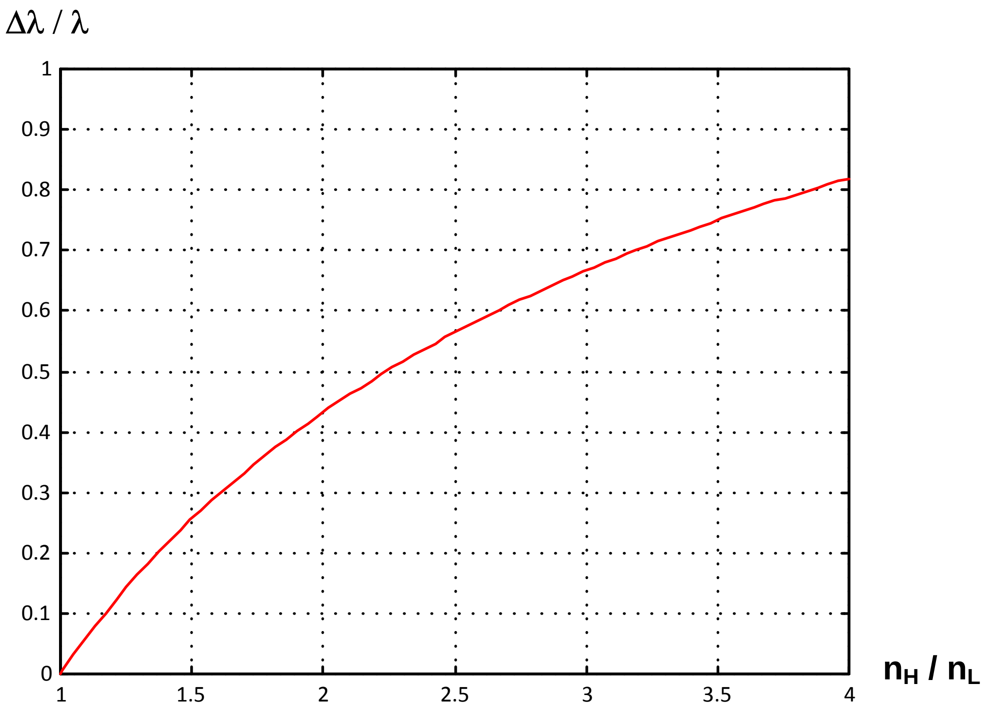
- Comfortable choice:
  layer thickness λ/4

  Peak reflectivity

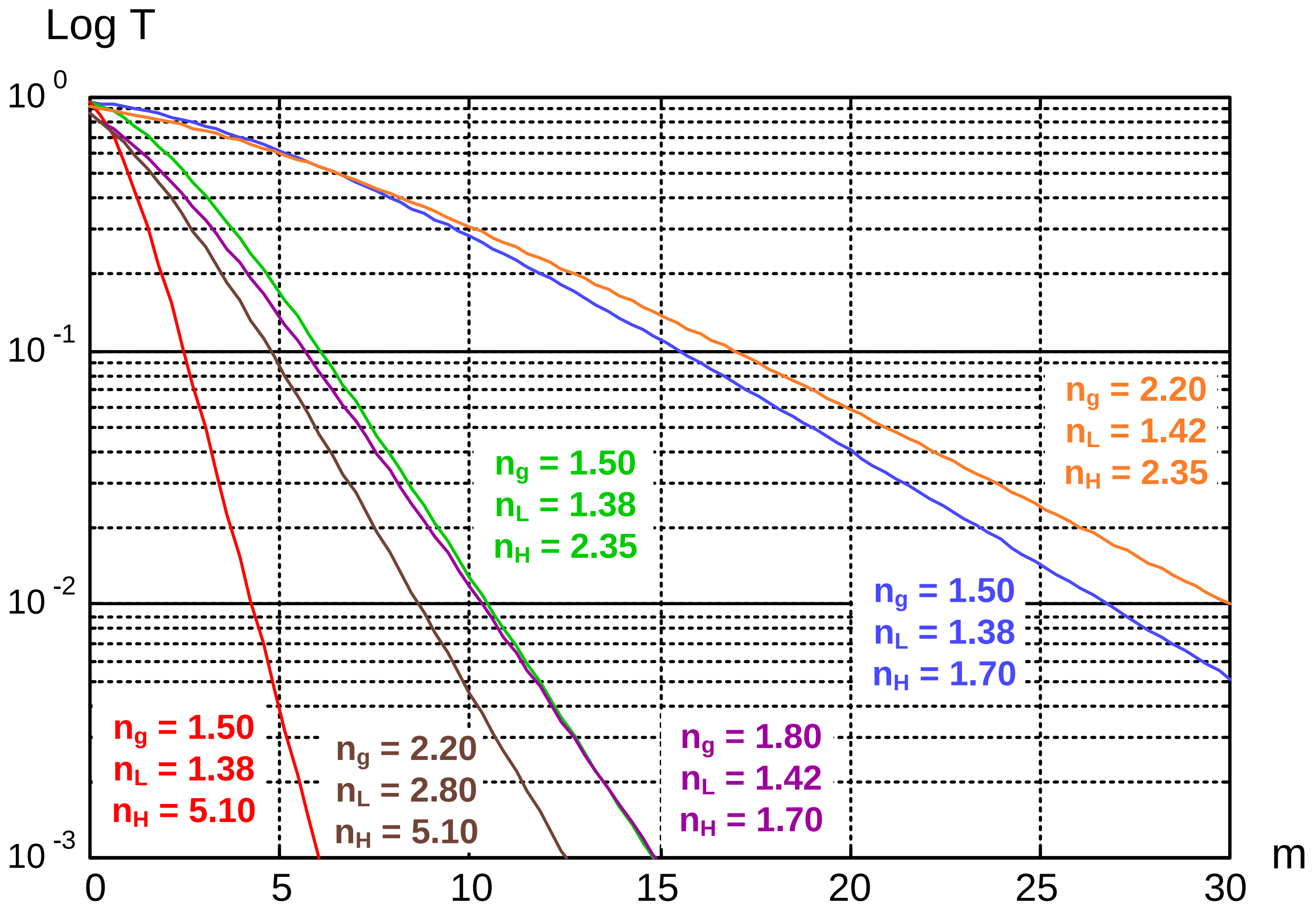
$$R_{\max} = \left[\frac{\frac{n_o}{n_g} - \left(\frac{n_L}{n_H}\right)^{2m}}{\frac{n_o}{n_g} + \left(\frac{n_L}{n_H}\right)^{2m}} \right]^2$$

- Spectral width of the reflectivity interval:
  growing with increased n-ratio



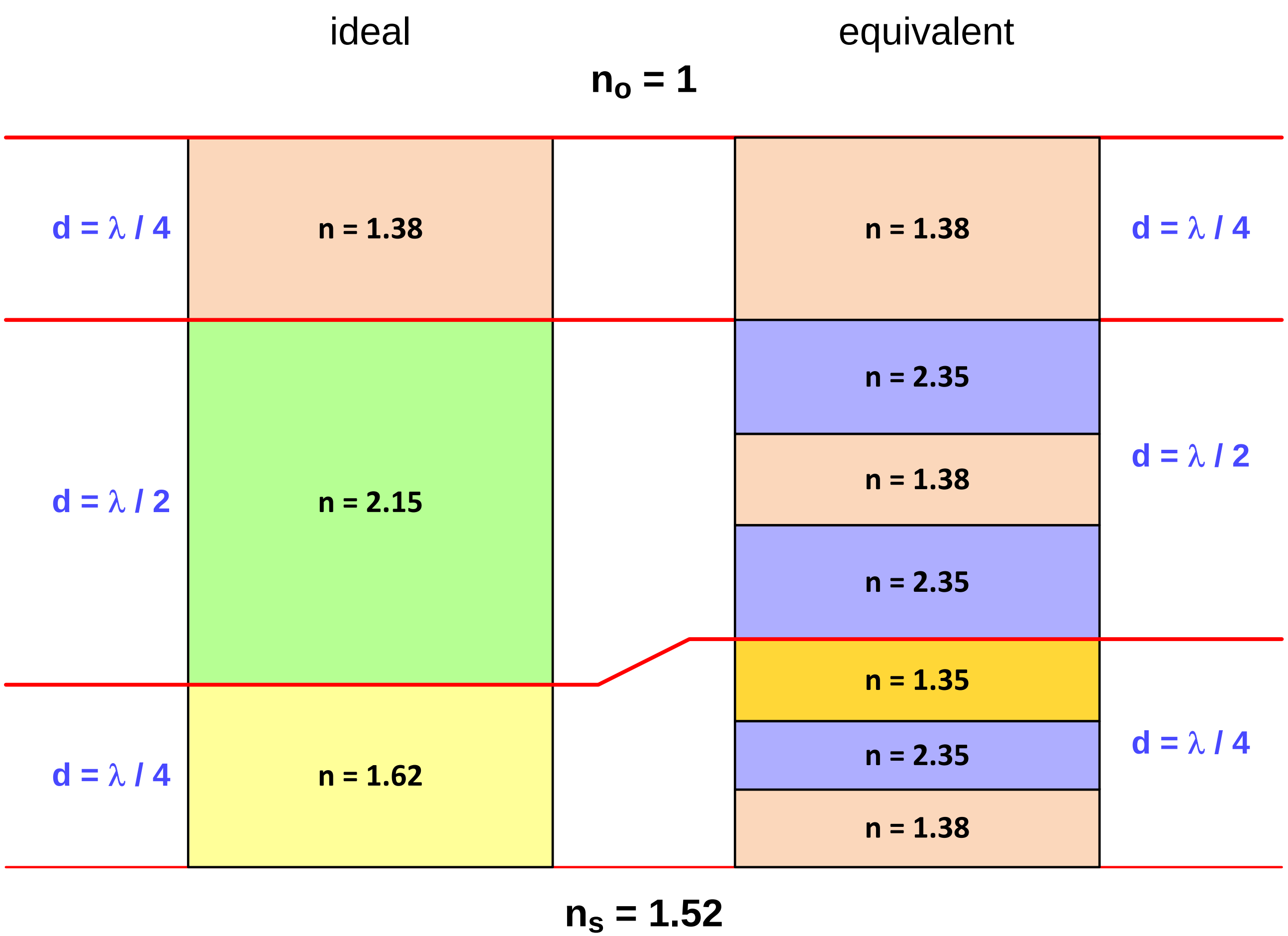
Periodical Layer of 2 Materials

- Performance dependence for different material combinations



Equivalent Layers

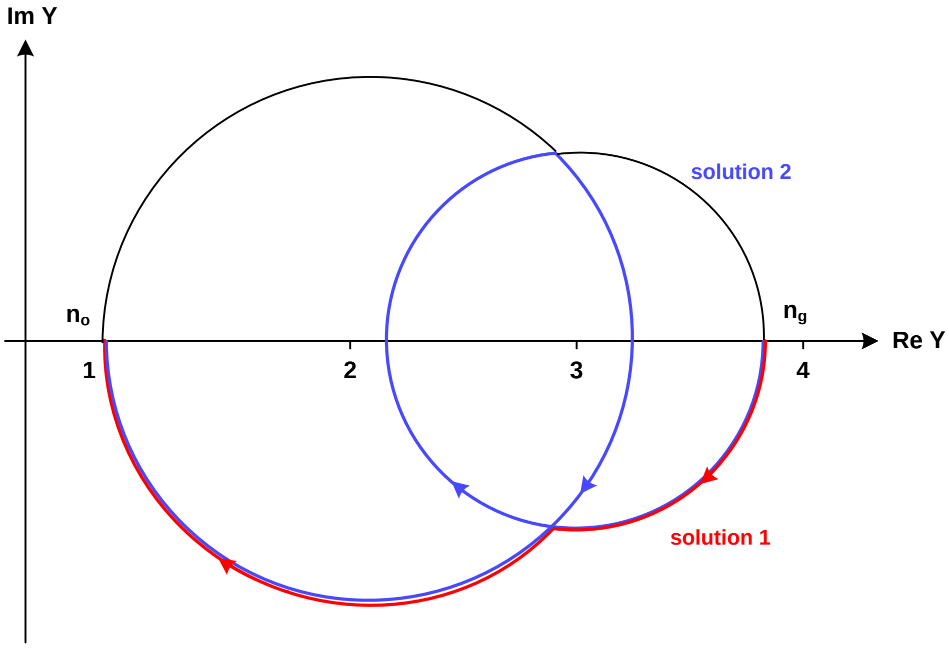
- Combination of layers with real refractive indices can be used to generate arteficial indices

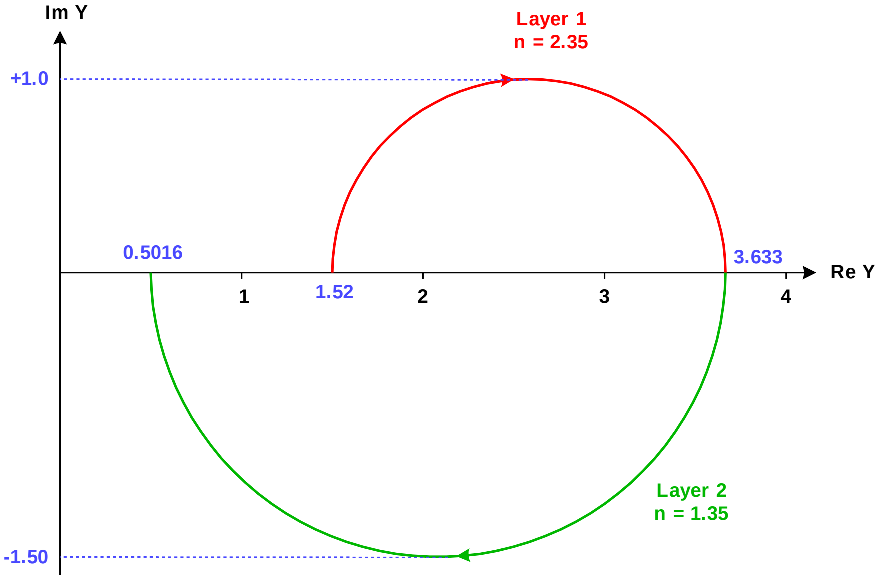


Impedance Diagram

- Graphical visualization of thin layer effects in the complex impedance plane
- Every layer is represented by an arc
- The arc length corresponds to the phase
- Absorbing materilals are represented by spiral curves
- Initial and final point must be on the real axis

  Exception:metals with complex substrate index

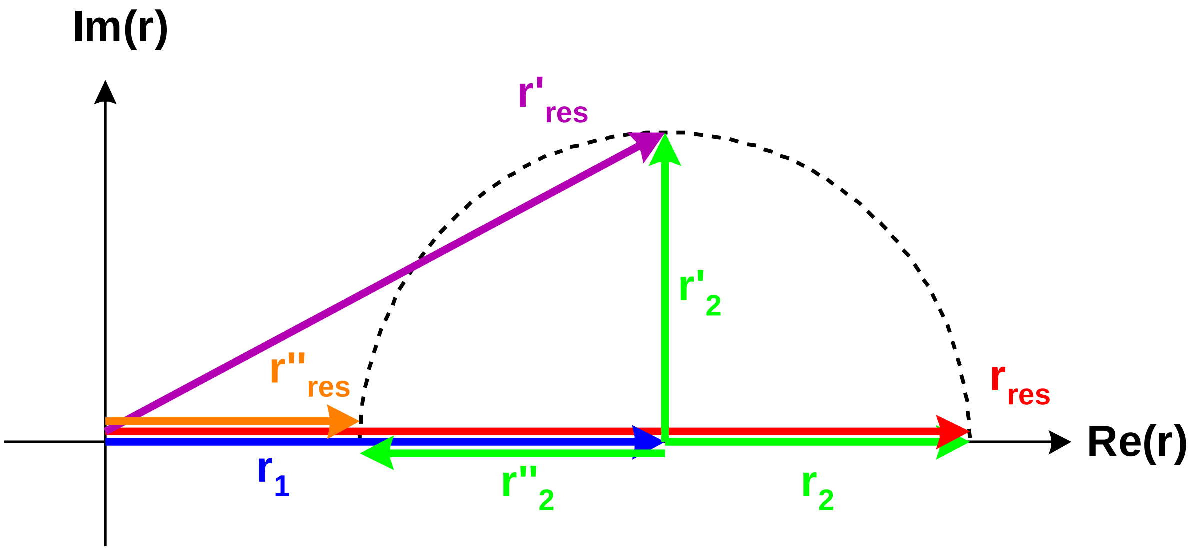




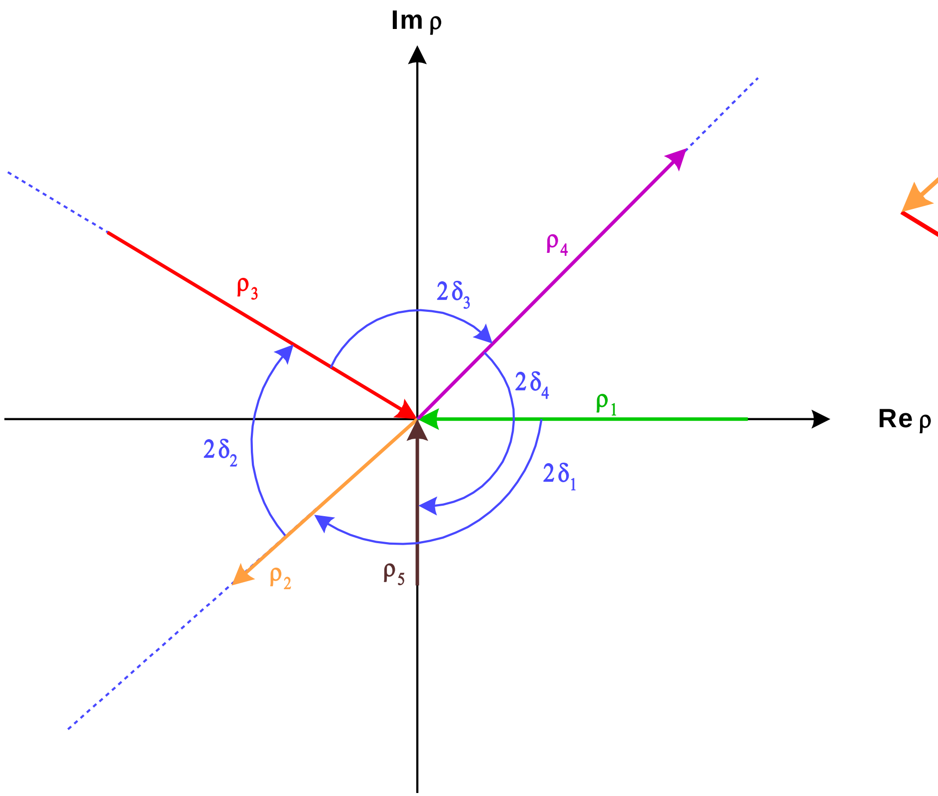
Reflectivity Diagram

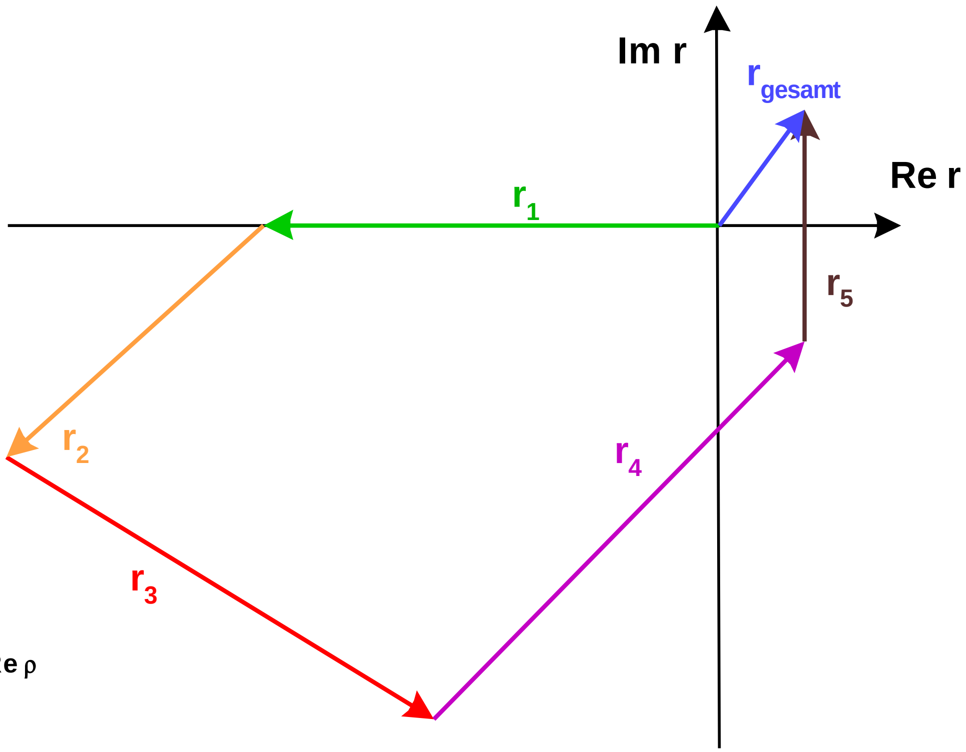
- Graphical illustration of layer stacks in the complex plane of the reflectivities
- Every layer is represented by an arrow with

1. length, corresponds to amplitude
2. direction, corresponds to phase



- AR coating:
  vectorial sum of vectors must vanish





Coating Materials

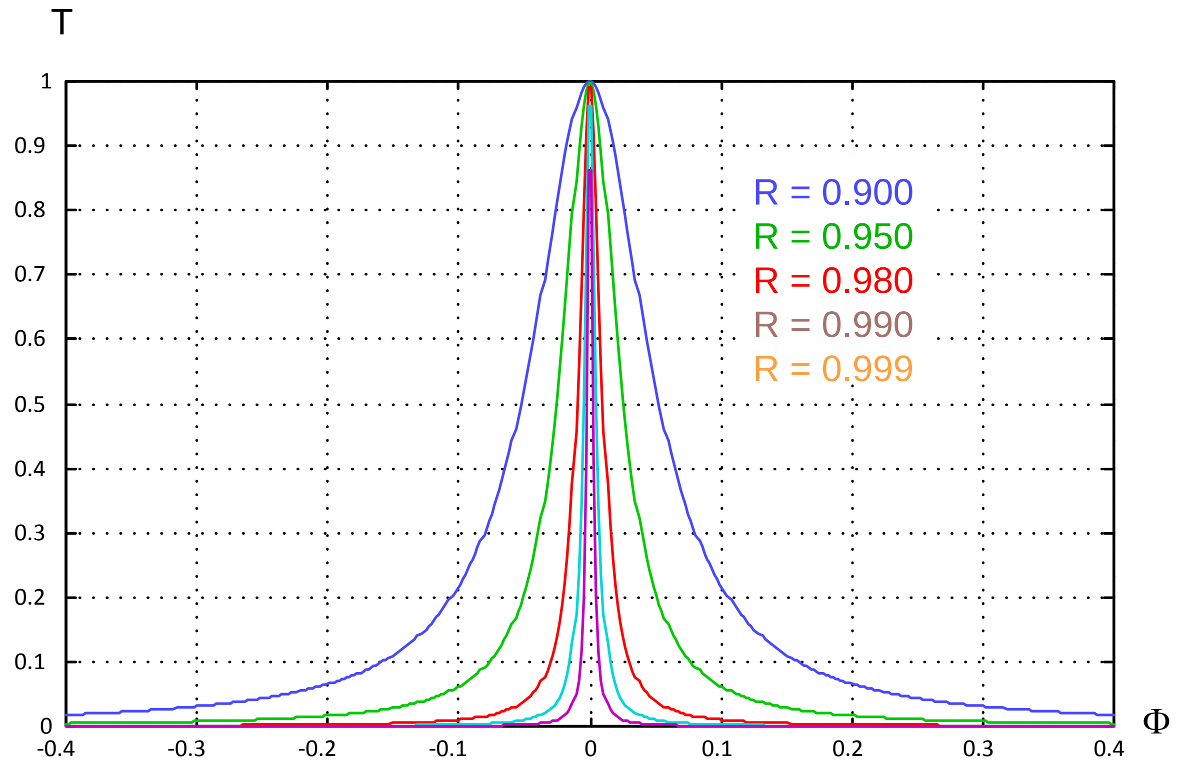
Material	Typ	Refractive index n	Spectral range
Magnesiumfluorid	L	1.38	UV - nIR
Cerfluorid	L	1.62	
Aluminiumoxid		1.62	UV - nIR
Cryolite		1.35	UV - nIR
Siliziumdioxid		1.48	
Magnesiumoxid	L	1.72	
Zirkonium Dioxid	H	2.00	UV - nIR
Hafniumdioxid	H	1.98	
Titandioxid	H	2.45	vis - nIR
Zinksulfid	H	2.30	vis - nIR
Cerdioxid		2.20	vis - nIR
Thoriumfluorid		1.52	UV - nIR
Silizium Monoxid		1.95	vis - nIR
Silizium		3.50	nIR - IR
Germanium		4.20	nIR - IR
Zinkselenid		2.44	nIR - IR
Cadmiumtellurid		2.69	nIR - IR
Bleitellurid		5.5	IR
Tellur	H	4.80	
Lithiumfluorid	L	1.37	

Interference Coating

- Multi-beam interference analogous to the Fabry-Perot
- Extrem narrow spectral transmission
- Calculation of transmission:

Airy formula

$$I_T(\varphi) = \frac{(1-\sqrt{R})^2}{1+R^2-2R\cos\varphi}$$



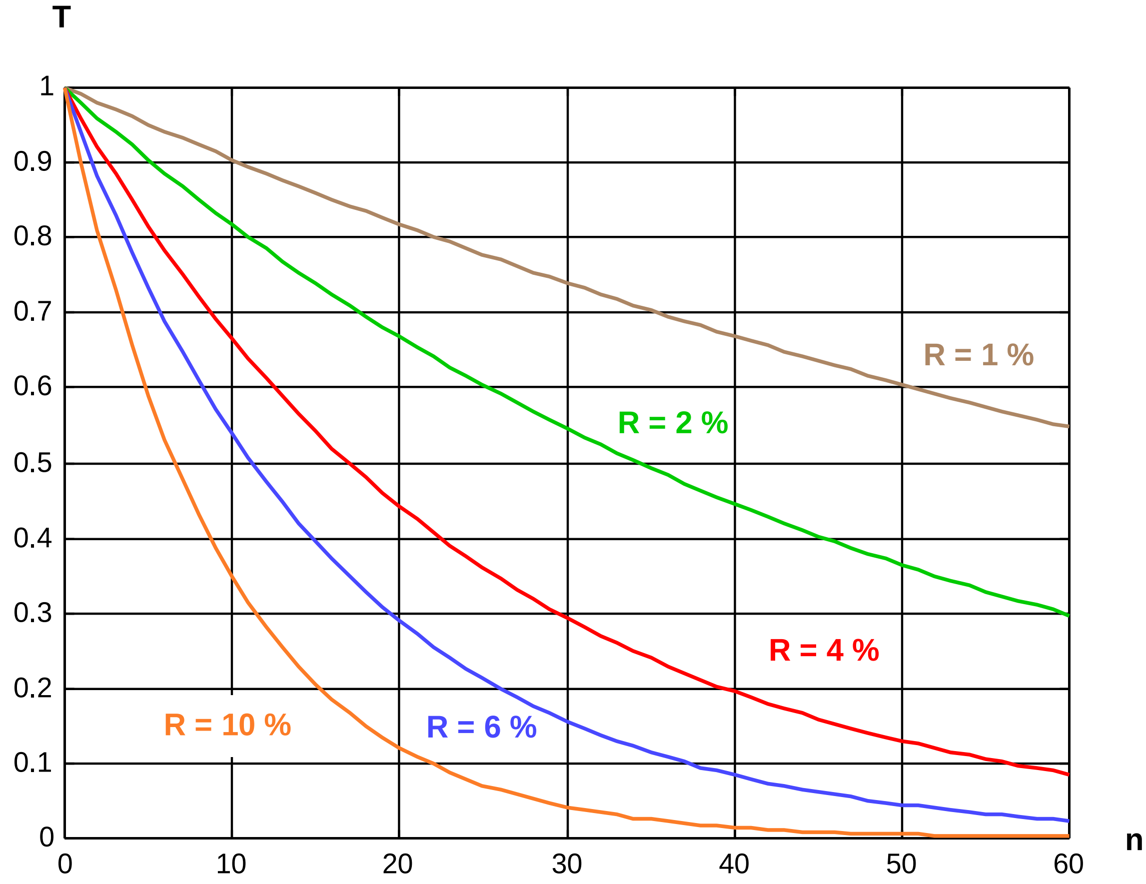
Transmission in Optical Systems

- Residual reflectivity of the (identical) surfaces in an optical system with n surfaces:
  Overall transmission of energy:

$$T_{ges} = (1 - R)^n$$

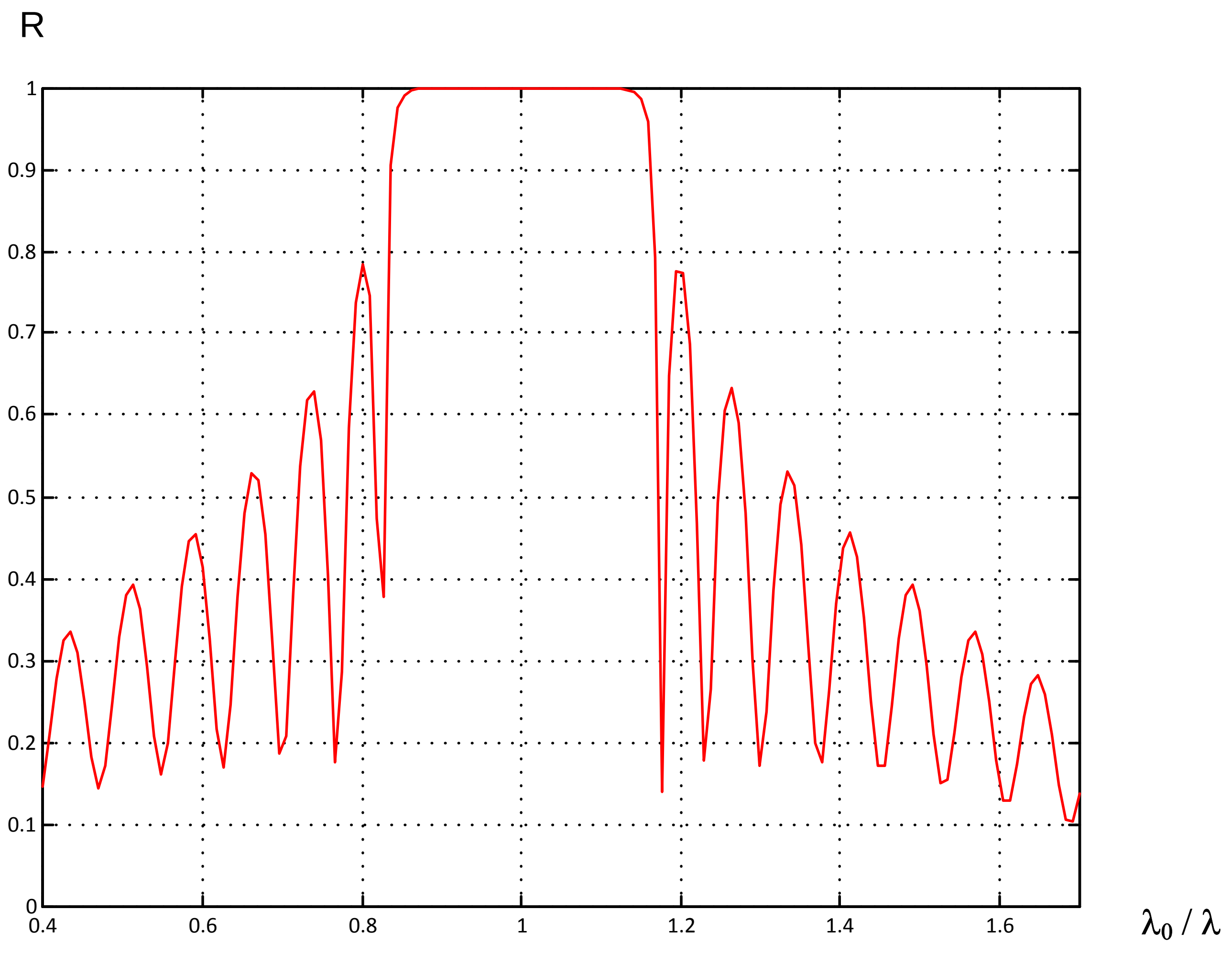
- Transmission decreases nonlinear

- Practical consequences:
  1. loss of signal energy
  2. contrast reduction in case of imaging
  3. occurence of ghost images



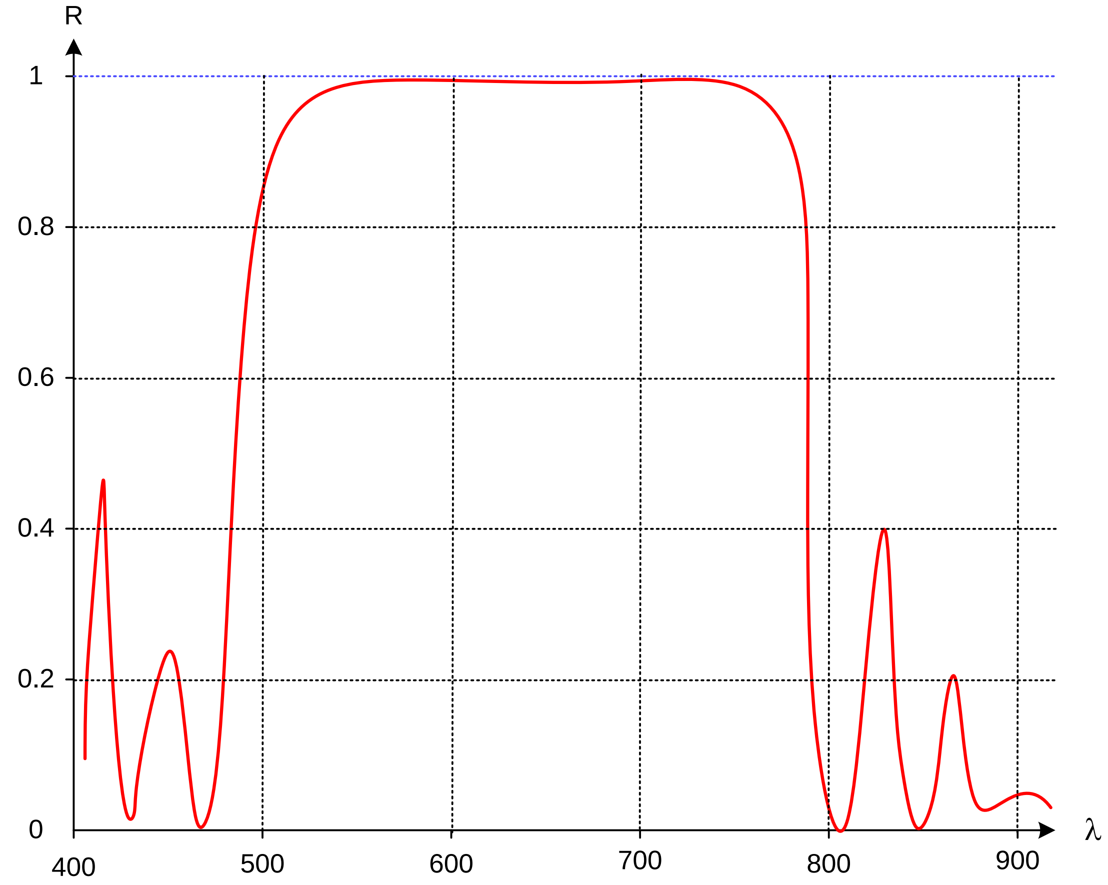
Example Coating

- Example of a concrete multi-layer stack



Example: HR Coating

- Dielectric HR coating
- Problems:
  1. absorbing substrates (metal mirrors)
  2. Polarization splitting for oblique incidence



Thin Layer as Edge Filter

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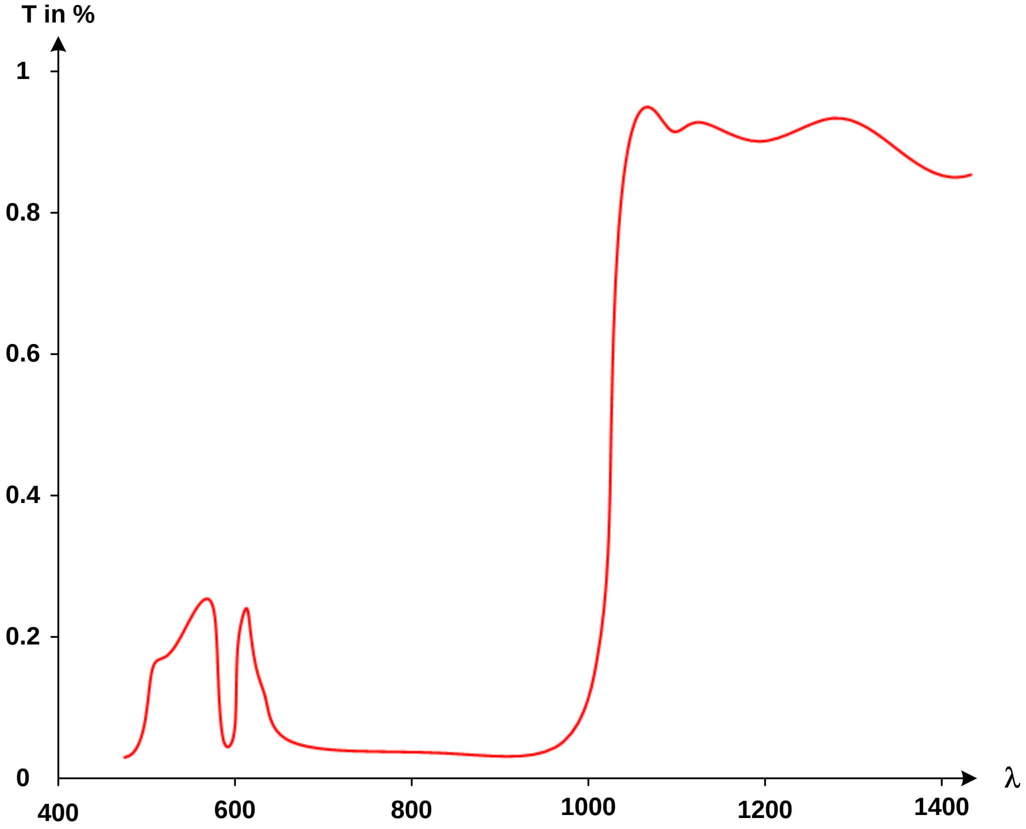
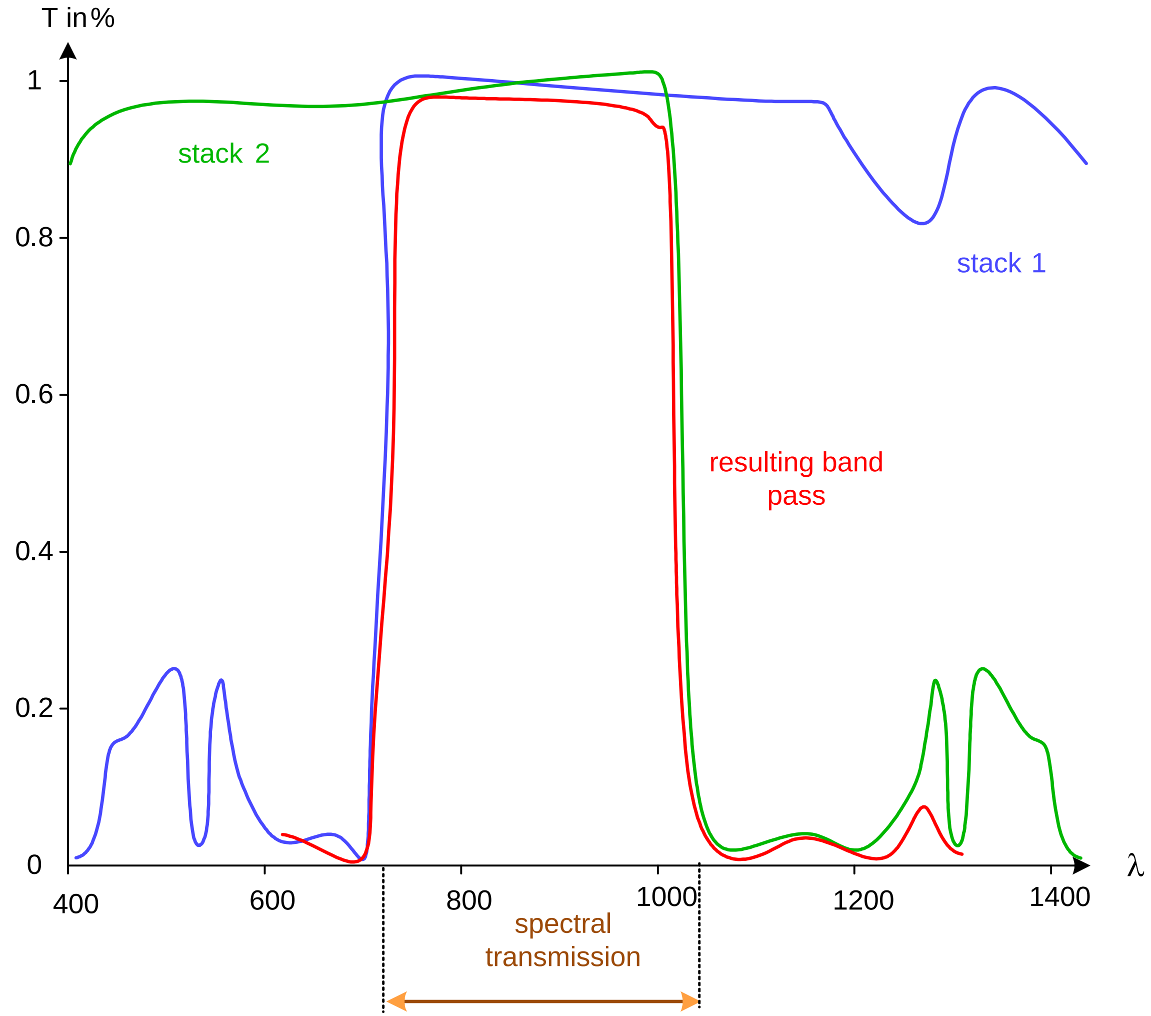
- Requirements:
  1. steep edge
  2. high reflectivity R on one side
  3. high transmission T on the other side
  4. no oscillations

- Application:
  cold mirror, blocking of infrared light



Thin Film Layers for Bandpass Filtering

- Superposition of two edge filters: Band pass
- Transmission of a selcted spectral range



Color Filtering

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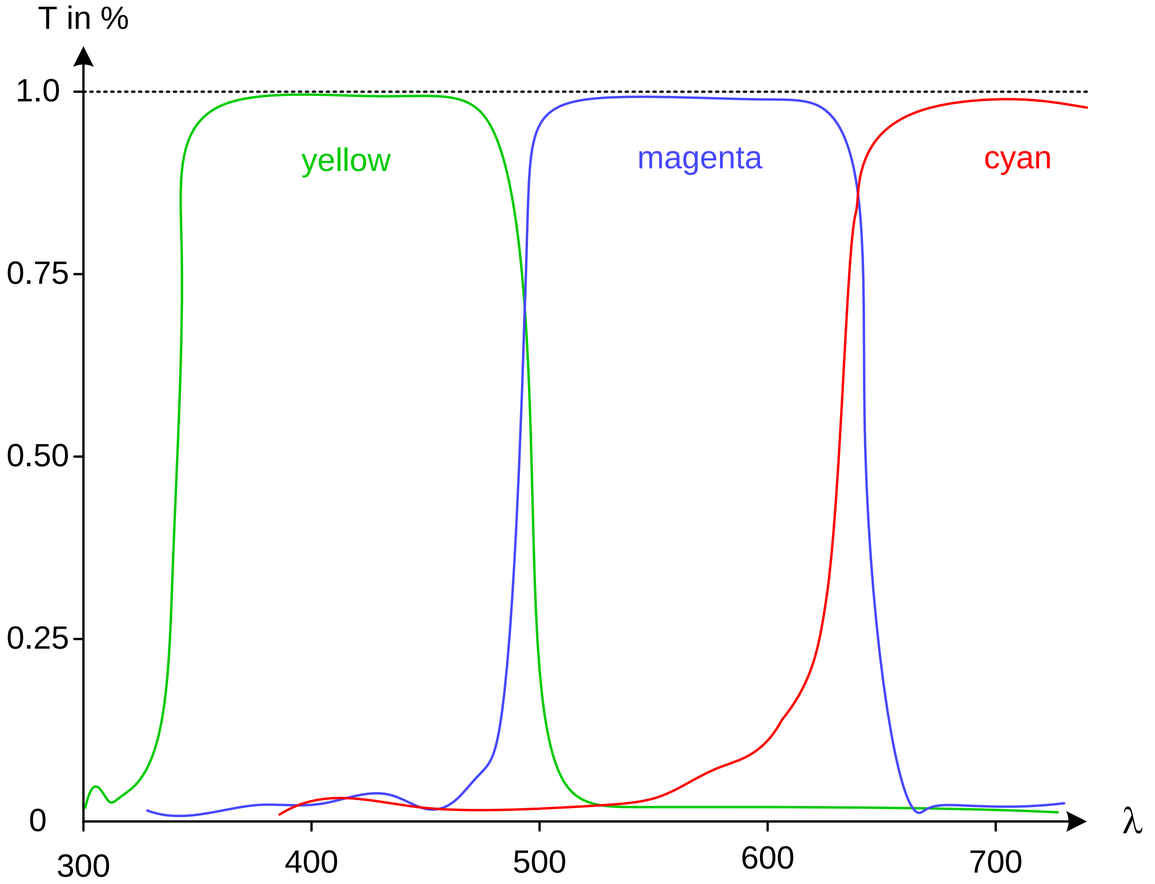
- Special coloring properties by spectral filtering

- Example application:

  selective spectral transmission

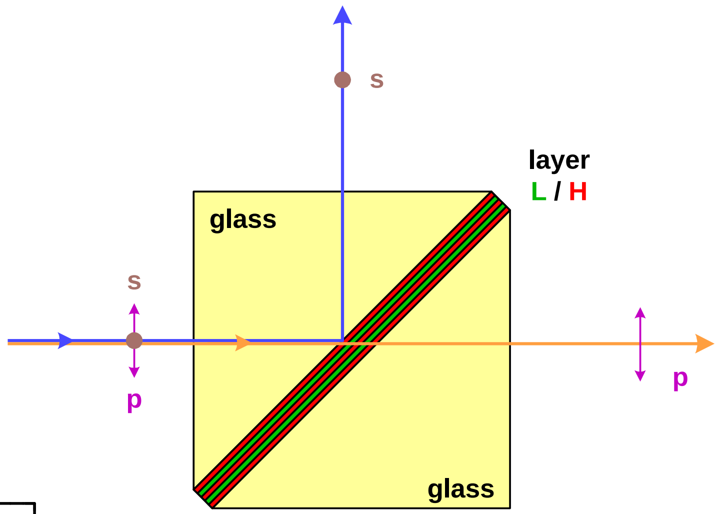
  for subtractive color printing

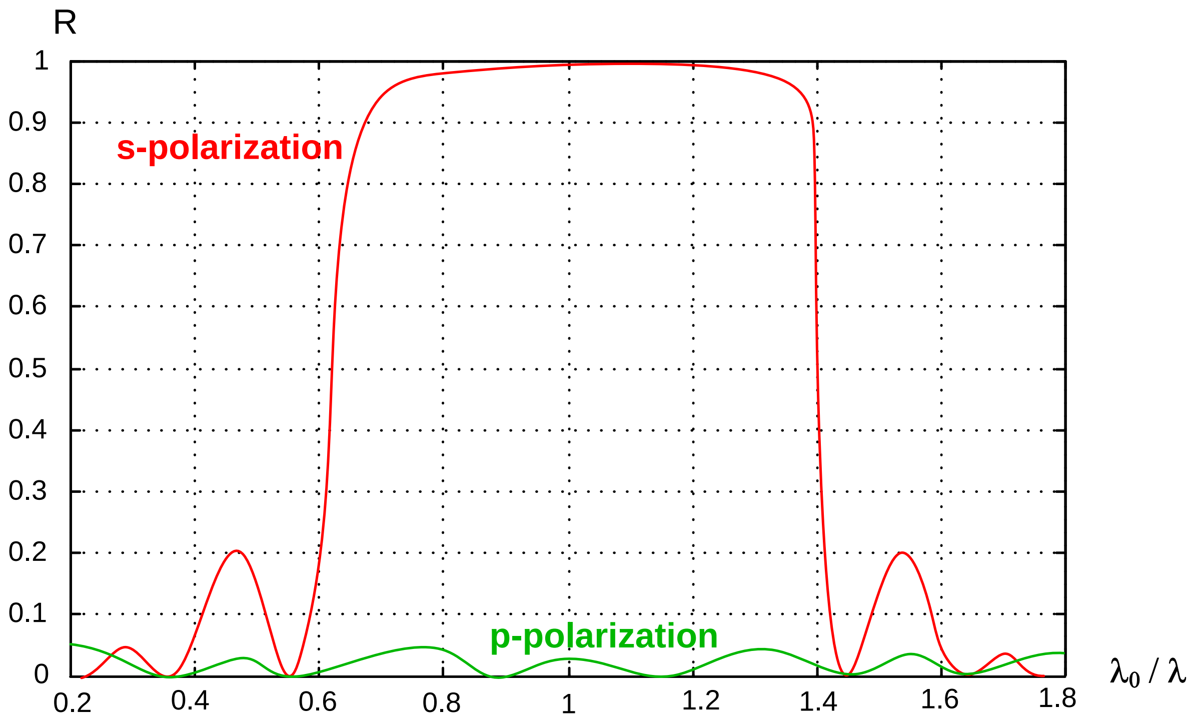


Polarizing Beam Splitting Coating

- Special coating for polarization splitting into s and p

- Application: cubic beam splitter





10 Physical Optical Modelling II Coatings in Zemax

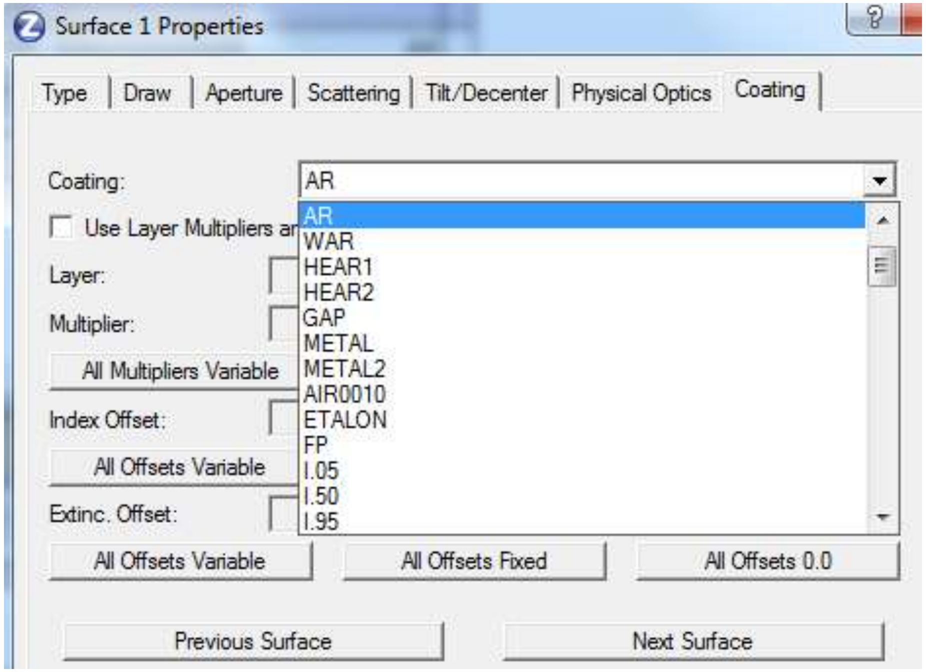
- Last column in the lens data manager:
  coating on the surface

Lens Data Editor

Edit Solves View Help

	Surf:Type	Radius	Thickness	Glass	Semi-Diameter	Coating
OBJ	Standard	Infinity	Infinity		Infinity	
1	Standard	54.1532462 V	8.7466579	SK2	29.2252978	AR
2	Standard	152.521921 V	0.5000000		28.1409540	AR
3	Standard	35.9506245 V	14.0000000	SK16	24.2958124	AR
4	Standard	Infinity	3.7769659	F5	21.2971909	
5	Standard	22.2699246 V	14.2530593		14.9193526	AR
STO	Standard	Infinity	12.4281291		10.2288352	
7	Standard	-25.685033 V	3.7769659	F5	13.1877585	AR
8	Standard	Infinity	10.8339285	SK16	16.4681224	
9	Standard	-36.980221 V	0.5000000		18.9295675	AR
10	Standard	196.417334 V	6.8581749	SK16	21.3107647	AR
11	Standard	-67.147550 V	57.3145379 V		21.6462584	AR
IMA	Standard	Infinity	-		24.5705331	

- One-Step-Coating of the complete system in:
  Tools/Catalogs/
  Add Coationgs to all Surfaces



- On every surface:
  specification of coating data form data file
  coating.dat

- File can be edited in:
  Tools/Catalogs/Edit Coating File

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- Syntax of the coating file, Tools/Catalogs/Coating Listing

1. Material specifications:
   different indices of the substrate
2. coatings:
   all layers: material, thickness in waves
   material must be given
3. ideal coatings:
   described by transmission value
4. coatings specified by table
   explicite values of Rs, Rp, Ts, Tp for each wavelength and incidence angle
5. encripted coatings
   (binary format)

```
ENCRYPTED ZEC_HEA673
ENCRYPTED VISNIR
ENCRYPTED ZEC_UVVIS
ENCRYPTED ZEC_HEA613
```

```
MATE BK7
0.4  1.5308485 0
0.46 1.5244335 0
0.5  1.5214145 0
0.7  1.5130640 0
0.8  1.5107762 0
1.0  1.5075022 0
2.0  1.4945016 0
```

```
COAT AR
MGF2 .25

COAT HEAR1
MGF2 .25
ZRO2 .50
CEF3 .25
```

```
COAT      CZ2301,45
MGF2_G    0.08450000  1 0
XIV       0.02640000  1 0
MGF2_G    0.01280000  1 0
XIV       0.07600000  1 0
MGF2_G    0.01380000  1 0
XIV       0.02930000  1 0
MGF2_G    0.03260000  1 0
XIV       0.01070000  1 0
```

```
COAT I.05
COAT I.50
COAT I.95
```

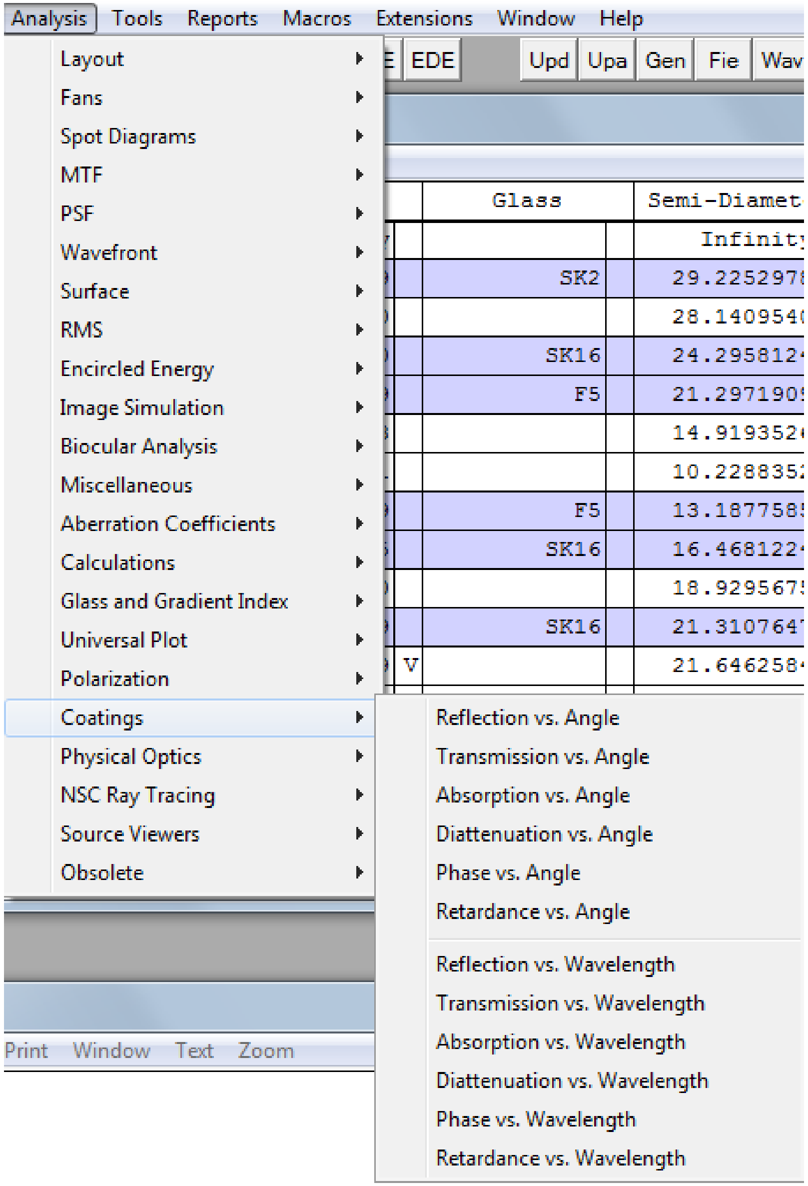
```
TABLE PASS45
ANGL 0.0
WAVE 0.55 1.0 1.0 0.0 0.0 0.0 0.0 0.0 0.0
ANGL 45.0
WAVE 0.55 0.0 0.0 1.0 1.0 0.0 0.0 0.0 0.0
ANGL 90.0
WAVE 0.55 1.0 1.0 0.0 0.0 0.0 0.0 0.0 0.0
```

- Selection of coating/polarization analysis

- Tapering of coatings is possible:
radial polynomial of cosine-shaped
spatial thickness variation

- Import of professional coating software
data is possible

- Given coatings can be incorporated into the
optimization to a certain flexibility
- thickness scaling
- offset of refractive index
- offset of extinction coefficient



9 Physical Optical Modelling I
Coating Analysis in Zemax

- Detailed polarization analyses are possible at the individual surfaces by using the coating menue options