

Optical Waveguide Theory (D)



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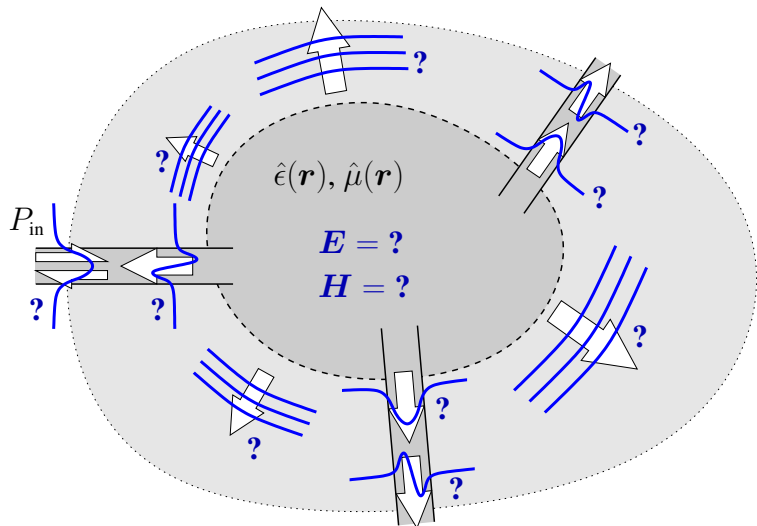
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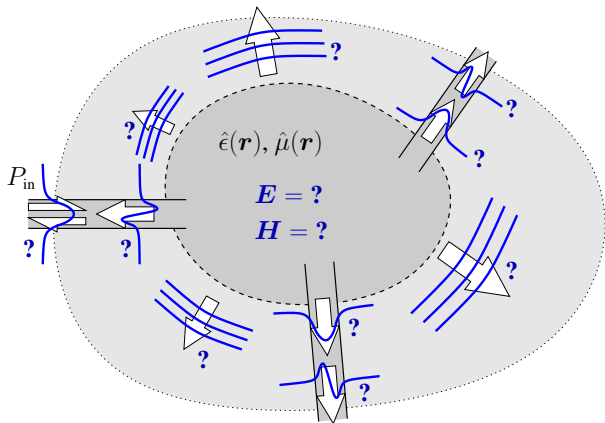
Optical waveguide theory

- A Photonics / integrated optics; theory, motto; phenomena, introductory examples.
- B Brush up on mathematical tools.
- C Maxwell equations, different formulations, interfaces, energy and power flow.
- D Classes of simulation tasks: scattering problems, mode analysis, resonance problems.
- E Normal modes of dielectric optical waveguides, mode interference.
- F Examples for dielectric optical waveguides.
- G Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.
- H Bent optical waveguides; whispering gallery resonances; circular microresonators.
- I Coupled mode theory, perturbation theory.
 - Hybrid analytical / numerical coupled mode theory.
- J A touch of photonic crystals; a touch of plasmonics.
 - Oblique semi-guided waves: 2-D integrated optics.
 - Summary, concluding remarks.

Guided wave scattering problems, schematically



Guided wave scattering problems, schematically



Given $\hat{\epsilon}(\mathbf{r})$, $\hat{\mu}(\mathbf{r})$ & external excitation (incoming guided mode),
determine \mathbf{E} , \mathbf{H} within the computational domain
& determine the optical power carried by outgoing waves.

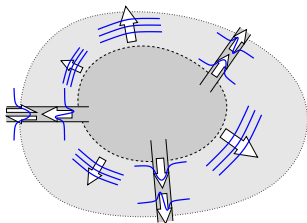
Scattering problems, time domain

(TD)

$$\mathbf{E}(\mathbf{r}, t), \mathbf{H}(\mathbf{r}, t),$$

$$\nabla \times \mathbf{E} = -\mu_0 \hat{\mu} \dot{\mathbf{H}},$$

$$\nabla \times \mathbf{H} = \epsilon_0 \hat{\epsilon} \dot{\mathbf{E}}.$$



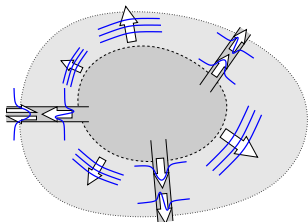
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- $\begin{pmatrix} 3\text{-D} \\ 2\text{-D} \\ 1\text{-D} \end{pmatrix}$ computational domain \times time interval.
- Initial & boundary conditions \longleftrightarrow incident waves.
- “Local” time-explicit iterative schemes possible (e.g. FDTD).
- Time evolution available; direct modeling of pulse propagation.
- Dispersion (... ?).
- Guided wave excitation (... ?).
- Fourier transform \longrightarrow spectral information.

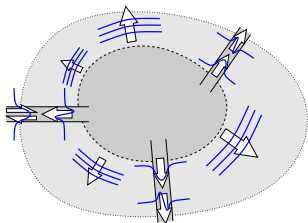
Scattering problems, frequency domain

(FD)

$$\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}), \sim \exp(i\omega t),$$

$$\nabla \times \mathbf{E} = -i\omega\mu_0\hat{\mu}\mathbf{H},$$

$$\nabla \times \mathbf{H} = i\omega\epsilon_0\hat{\epsilon}\mathbf{E}.$$



Scattering problems, frequency domain

(FD)

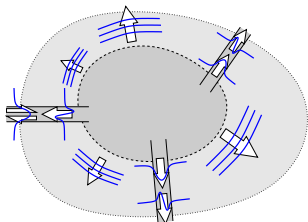
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- $\begin{pmatrix} 3\text{-D} \\ 2\text{-D} \\ 1\text{-D} \end{pmatrix}$ computational domain.

- “ $\mathbf{M}(\text{field}) = \overrightarrow{(\text{excitation})}$ ”;
matrix needs to be determined, stored; system needs to be solved.
- Spectral information directly available.
- Dispersion — straightforward.
- Guided wave excitation — straightforward.
- Fourier transform \longrightarrow time evolution / pulse propagation.



Open problems

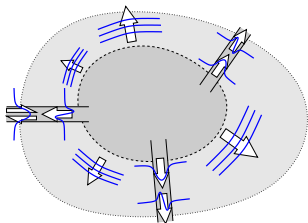
(TD & FD)

“Open” spatial computational domain

~> boundary conditions need to

- permit outgoing radiated fields
& outgoing (reflected) guided modes to exit the domain,
- launch the incoming external excitation.

~<> simulate a nonexistent boundary, an unlimited domain.



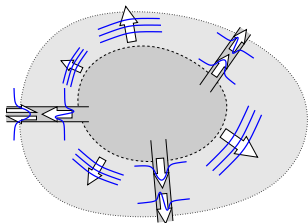
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- ~<> simulate a nonexistent boundary, an unlimited domain.



- Keywords:
- transparent-influx boundary conditions,
 - absorbing boundary conditions,
 - perfectly matched layers (PMLs).



2-D problems

$$\hat{\epsilon} = \epsilon \hat{1}, \quad \hat{\mu} = \mu \hat{1}, \quad \sim \exp(i\omega t) \quad (\text{FD})$$

$$\begin{pmatrix} \partial_y E_z - \partial_z E_y \\ \partial_z E_x - \partial_x E_z \\ \partial_x E_y - \partial_y E_x \end{pmatrix} = -i\omega\mu_0\mu \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad \begin{pmatrix} \partial_y H_z - \partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y - \partial_y H_x \end{pmatrix} = i\omega\epsilon_0\epsilon \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$

2-D problems

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Assume $\partial_y \epsilon = 0$, $\partial_y \mu = 0$; consider solutions $\partial_y \mathbf{E} = 0$, $\partial_y \mathbf{H} = 0$:

$$\begin{pmatrix} -\partial_z E_y \\ \partial_z E_x - \partial_x E_z \\ \partial_x E_y \end{pmatrix} = -i\omega\mu_0\mu \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix}, \quad \begin{pmatrix} -\partial_z H_y \\ \partial_z H_x - \partial_x H_z \\ \partial_x H_y \end{pmatrix} = i\omega\epsilon_0\epsilon \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}.$$

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↪ Two decoupled sets of equations:

- $\{E_y, H_x, H_z\}$: **transverse electric (TE)** fields, $\mathbf{E} \perp x\text{-}z\text{-plane}$.
- $\{H_y, E_x, E_z\}$: **transverse magnetic (TM)** fields, $\mathbf{H} \perp x\text{-}z\text{-plane}$.

(Different conventions on the use of TE, TM.)

(Applies also to the TD.)

2-D TE waves

$$k^2 = \omega^2/c^2 = \omega^2\epsilon_0\mu_0 \quad (\text{FD})$$

- Principal component E_y ,

$$H_x = \frac{-i}{\omega\mu_0\mu} \partial_z E_y, \quad H_z = \frac{i}{\omega\mu_0\mu} \partial_x E_y, \quad i\omega\epsilon_0\epsilon E_y = \partial_z H_x - \partial_x H_z$$

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- If $\mu = 1$: $\epsilon(x, z)$ (!)

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scalar 2-D (TE) Helmholtz equation (E_y , $\partial_n E_y$ continuous).

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(Reflection / transmission problems: s-polarized waves satisfy (*), (**).)

2-D TM waves

$$k^2 = \omega^2/c^2 = \omega^2\epsilon_0\mu_0 \quad (\text{FD})$$

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$$E_x = \frac{i}{\omega\epsilon_0\epsilon} \partial_z H_y, \quad E_z = \frac{-i}{\omega\epsilon_0\epsilon} \partial_x H_y, \quad -i\omega\mu_0\mu H_y = \partial_z E_x - \partial_x E_z$$

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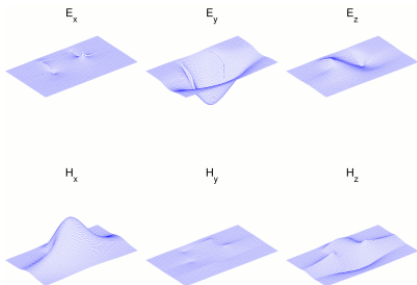
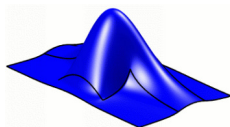
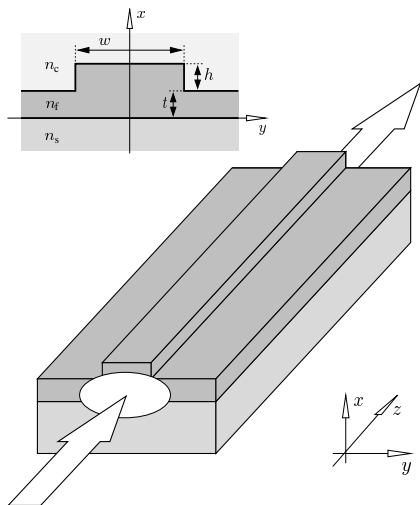
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(Reflection / transmission problems: p-polarized waves satisfy (*), (**).)

Rib waveguide

... variant of an integrated optical waveguide with 2-D confinement



Waveguides: Mode problems

$$\nabla \times \mathbf{E} = -i\omega\mu_0\mu\mathbf{H}, \quad \nabla \times \mathbf{H} = i\omega\epsilon_0\epsilon\mathbf{E}. \quad \sim \exp(i\omega t) \quad (\text{FD})$$

- **Waveguide:** a system that is homogeneous along its axis z ,
 $\partial_z\epsilon = 0, \partial_z\mu = 0, \partial_z n = 0$.

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- Look for solutions (**modes**) that vary harmonically with z :

$$\mathbf{E}(x, y, z) = \bar{\mathbf{E}}(x, y) e^{-i\beta z}, \quad \mathbf{H}(x, y, z) = \bar{\mathbf{H}}(x, y) e^{-i\beta z},$$

mode profile $\bar{\mathbf{E}}, \bar{\mathbf{H}}$, propagation constant β .

(drop $\bar{}$)

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vectorial mode equations, variants.

(...)

Waveguides: Mode equations

- Where $\epsilon(\mathbf{r})$, $\mu(\mathbf{r})$: $\sim \exp(i\omega t)$ (FD)

$$\Delta\tilde{\mathbf{E}} + k^2\epsilon\mu\tilde{\mathbf{E}} = 0, \quad \Delta\tilde{\mathbf{H}} + k^2\epsilon\mu\tilde{\mathbf{H}} = 0$$

↪

$$\begin{aligned}\partial_x^2 \mathbf{E} + \partial_y^2 \mathbf{E} + (k^2\epsilon\mu - \beta^2)\mathbf{E} &= 0, \\ \partial_x^2 \mathbf{H} + \partial_y^2 \mathbf{H} + (k^2\epsilon\mu - \beta^2)\mathbf{H} &= 0,\end{aligned}$$

scalar **mode equation**, valid for all components of \mathbf{E} , \mathbf{H} ,
to be supplemented by suitable **boundary** and **interface conditions**.

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“ $\mathbf{M}(\beta)$ (profile) = 0”.

Waveguides: Mode equations

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- **Guided modes**: discrete $\beta \in \mathbb{R}$, $\iint S_z dx dz < \infty$. $(\epsilon, \mu \in \mathbb{R})$

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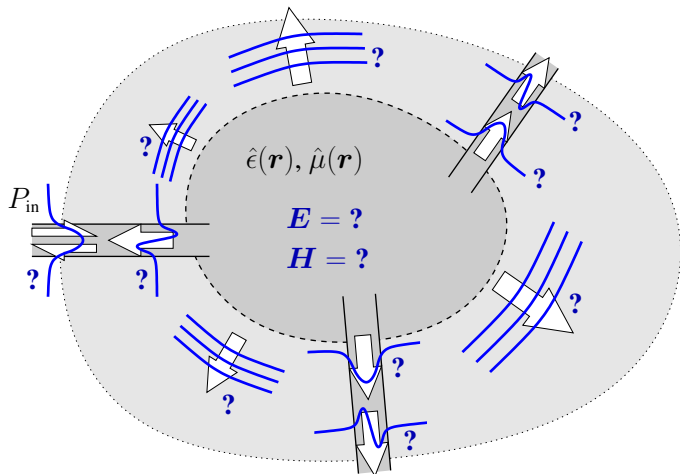
(Radiation modes: continuum of $\beta^2 \in \mathbb{R}$, oscillating external fields.)

(Leaky modes: discrete $\beta \in \mathbb{C}$, outgoing wave boundary conditions.)

(...)

Guided wave scattering problems

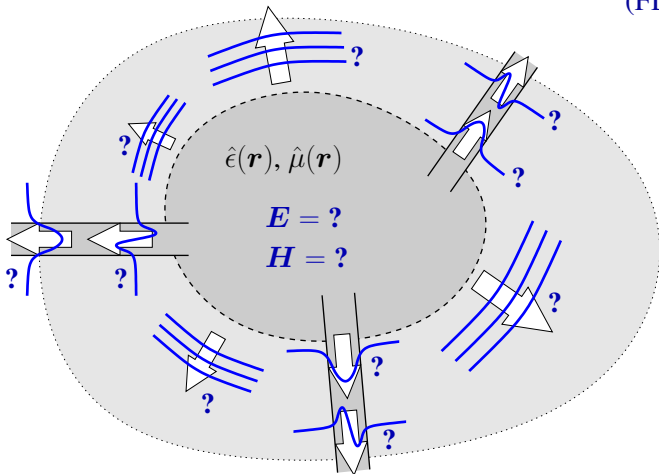
(FD)



Given external excitation $\sim \exp(i\omega t)$, $\omega \in \mathbb{R}$.

Resonance problems

(FD ...)



Omit excitation, look for nonzero solutions that decay in time.

Resonance problems

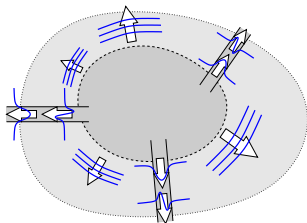
(FD ...)

$$\mathbf{E}(\mathbf{r}), \mathbf{H}(\mathbf{r}), \sim \exp(i\omega t), \omega = ?$$

$$\nabla \times \mathbf{E} = -i\omega\mu_0\hat{\mu}\mathbf{H},$$

$$\nabla \times \mathbf{H} = i\omega\epsilon_0\hat{\epsilon}\mathbf{E},$$

& outgoing wave boundary conditions.



Resonance problems

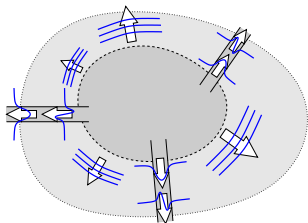
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$$\nabla \times \mathbf{H} = i\omega\epsilon_0\hat{\epsilon}\mathbf{E},$$

& outgoing wave boundary conditions.



- Look for nonzero solutions with $\omega \in \mathbb{C}$ that oscillate and decay (slowly ...) in time.
- “ $\mathbf{M}(\omega) (\overrightarrow{\text{field}}) = 0$ ”, eigenvalue problem.
- Solutions: discrete eigenfrequencies ω , resonant mode profiles.

Keyword: “Quasi-Normal-Modes”, QNMs.

Scalar approximation

Linear, isotropic, nonmagnetic media, $\epsilon = n^2$;
a structure with “small” variations in ϵ :

A **scalar approximation** may be adequate,

$$\nabla \cdot (\epsilon \mathbf{E}) \approx \epsilon \nabla \cdot \mathbf{E}$$

$$\curvearrowright \Delta \psi - \frac{1}{c^2} \epsilon \ddot{\psi} = 0, \quad \text{(TD)}$$

$$\Delta \psi + k^2 \epsilon \psi = 0, \quad \text{(FD)}$$

satisfied by all components ψ of \mathbf{E} , \mathbf{H} .

(Applicable to basically all types of problems.)

Beam propagation method

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“small” changes in $\epsilon = n^2$ along a propagation coordinate z .

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- Ansatz: $\psi(x, y, z) = \psi_0(x, y, z) e^{-ikn_r z}$,
reference effective index n_r ,

assume that ψ_0 varies “slowly” along z \longleftrightarrow neglect $\partial_z^2\psi_0$.

$\hookrightarrow -i2kn_r\partial_z\psi_0 + (\partial_x^2 + \partial_y^2)\psi_0 + k^2(\epsilon - n_r^2)\psi_0 = 0,$

PDE of first order in z , solved as an initial value problem.

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- Restriction to unidirectional propagation, reflections are neglected.
- Paraxial propagation, errors for waves with effective indices $\neq n_r$.

(Many variants (vectorial, wide-angle, bi-directional, ...) have been proposed.)

(Other ways of motivating the approximation exist.)

(Term “BPM” in use also for other types of methods.)

- Keywords: Paraxial approximation,
Slowly-varying-envelope approximation (SVEA),
Beam propagation method (BPM).

Upcoming

Next lectures:

- Normal modes of dielectric optical waveguides, mode interference.
- Examples for dielectric optical waveguides.
- Waveguide discontinuities & circuits, scattering matrices, reciprocal circuits.

