


## MEMPHIS

-     - Optimum Cross Section
o Open-channel systems are usually designed to transport a liquid to a location at a lower elevation at a specified rate under the influence of gravity at the lowest possible cost.
o Since no energy input is required, the cost of an open-channel system consists primarily of the initial construction cost, which is proportional to the physical size of the system.


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o Therefore, for a given channel length, the perimeter of the channel is representative of the system cost, and it should be kept to a minimum in order to minimize the size and thus the cost of the system.
o Resistance to flow is due to wall shear stress $\mathrm{T}_{w}$ and the wall area, which is equivalent to the wetted perimeter per unit channel length.

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- Therefore, for a given flow cross-sectional area $A_{c}$, the smaller the wetted perimeter $P$, the smaller the resistance force, and thus the larger the average velocity and the flow rate.
o A hydraulically optimum cross section in open-channel flow is one that provides maximum conveyance or volume-carrying capacity for a given flow area.


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-     - Optimum Cross Section
o The optimum hydraulic cross section for an open channel is the cross section with the maximum hydraulic radius or equivalently, the one with the minimum wetted perimeter for a specific cross section.
-     - Optimum Cross Section
o If we start with the Manning Expression for average velocity

$$
V=\frac{1}{n} R_{h}^{\frac{2}{3}} S^{\frac{1}{2}}
$$

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o And multiply both sides of the expression to get the volumetric flow rate

$$
v A=\frac{1}{n} R_{h}^{\frac{2}{3}} S^{\frac{1}{2}} A=Q
$$ the ratio of the area to the wetted perimeter

$$
\begin{aligned}
& \frac{1}{n}\left(\frac{A}{P}\right)^{\frac{2}{3}} S^{\frac{1}{2}} A=Q \\
& \frac{1}{n}\left(\frac{1}{P}\right)^{\frac{2}{3}} S^{\frac{1}{2}} A^{\frac{5}{3}}=Q
\end{aligned}
$$

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$\bullet$ - $\quad$ Optimum Cross Section

- If we set the cross section area, the aspect ratio of this area is defined as the ratio of the depth to the channel width

$$
\begin{aligned}
& \frac{1}{n}\left(\frac{A}{P}\right)^{\frac{2}{3}} S^{\frac{1}{2}} A=Q \\
& \frac{1}{n}\left(\frac{1}{P}\right)^{\frac{2}{3}} S^{\frac{1}{2}} A^{\frac{5}{3}}=Q
\end{aligned}
$$

## Optimum Cross Section

o So we can look at how the flow for the cross section changes as the aspect ratio of the channel changes.

$$
\begin{aligned}
& \frac{1}{n}\left(\frac{A}{P}\right)^{\frac{2}{3}} S^{\frac{1}{2}} A=Q \\
& \frac{1}{n}\left(\frac{1}{P}\right)^{\frac{2}{3}} S^{\frac{1}{2}} A^{\frac{5}{3}}=Q \\
& \text { Nonata, Noenemes } 52012
\end{aligned}
$$




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- Optimum Cross Section
o If we write the expression for the wetted perimeter in terms of the depth and channel width

$$
P=2 z+b
$$

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$\bullet \bullet$ Optimum Cross Section
o And we can write b in terms of the Area and the depth

$$
\begin{aligned}
& P=2 z+b \\
& A=b z \\
& b=\frac{A}{z}
\end{aligned}
$$

## MEMPHIS

- $\bullet$ Optimum Cross Section
- Substituting in to the perimeter expression and differentiation with respect to $z$

$$
\begin{aligned}
& P=2 z+\frac{A}{z} \\
& \frac{d P}{d z}=2-\frac{A}{z^{2}}
\end{aligned}
$$

## 

$\bullet$ Optimum Cross Section

- Setting the derivative equal to 0 and replacing the area

$$
\begin{aligned}
& 0=2-\frac{A}{z^{2}} \\
& 0=2-\frac{b z}{z^{2}} \\
& 2=\frac{b}{z}
\end{aligned}
$$

\section*{MEMPHIS <br> - - |  | Optimum Cross Section |
| :--- | :--- |}

o Which is the same value we had from the graph of the aspect ratio.

$$
z=\frac{b}{2}
$$

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$\bullet \bullet$ Optimum Cross Section

- Each cross section shape will have a unique function to describe the optimal channel cross section dimensions

$$
z=\frac{b}{2}
$$

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o The cross sectional area, the downstream slope, and the friction factor are given and the channel geometry are given by this expression for a rectangular channel.

$$
z=\frac{b}{2}
$$

## Optimum Cross Section

- Considering a trapezoidal channel with a base width $b$ and a side slope of 1:m


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$\bullet \bullet$ Optimum Cross Section
o Rather than use the $m$ in the expression, we can consider the angle $\theta$ that the sides make with the horizontal

$$
\theta=\arctan \left(\frac{1}{m}\right)
$$

$\bullet$ - Optimum Cross Section
o Rewriting the expression for the wetted perimeter in terms of $\theta$

$$
P=b+\frac{2 z}{\sin (\theta)}
$$

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$\bullet \bullet$ Optimum Cross Section

- Since we are dealing with a set Cross Section area, the value for $b$ can be developed in terms of the area and $z$
$A=\frac{\left(2 b+2 \frac{z}{\tan (\theta)}\right)}{2} z \quad P=b+\frac{2 z}{\sin (\theta)}$


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- Solving the area expression for $b$

$$
A=\left(b+\frac{z}{\tan (\theta)}\right) z
$$

$\frac{A}{z}=b+\frac{z}{\tan (\theta)}$
$P=b+\frac{2 z}{\sin (\theta)}$
$b=\frac{A}{z}-\frac{z}{\tan (\theta)}$

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$\bullet \bullet$ Optimum Cross Section
o Substituting for $b$ in the wetted perimeter expression

$$
P=\frac{A}{z}-\frac{z}{\tan (\theta)}+\frac{2 z}{\sin (\theta)}
$$

$\bullet$ - Optimum Cross Section
o We want $P$ to be a minimum so we differentiate with respect to $z$ and set the derivative equal to 0

$$
\frac{d P}{d z}=-\frac{A}{z^{2}}-\frac{1}{\tan (\theta)}+\frac{2}{\sin (\theta)}=0
$$

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- We can substitute back the expression for $A$
$\frac{d P}{d z}=-\frac{\left(b+\frac{z}{\tan (\theta)}\right) z}{z^{2}}-\frac{1}{\tan (\theta)}+\frac{2}{\sin (\theta)}=0$


## 

$\bullet$ - Optimum Cross Section

- Collecting terms

$$
\begin{aligned}
& -\frac{\left(b+\frac{z}{\tan (\theta)}\right) z}{z^{2}}-\frac{1}{\tan (\theta)}+\frac{2}{\sin (\theta)}=0 \\
& -\frac{b}{z}-\frac{1}{\tan (\theta)}-\frac{1}{\tan (\theta)}+\frac{2}{\sin (\theta)}=0 \\
& -\frac{b}{z}-\frac{2}{\tan (\theta)}+\frac{2}{\sin (\theta)}=0
\end{aligned}
$$

## MEMPHIS

## Optimum Cross Section

o Collecting terms

$$
\begin{aligned}
& -\frac{2}{\tan (\theta)}+\frac{2}{\sin (\theta)}=\frac{b}{z} \\
& -\frac{2 \cos (\theta)}{\sin (\theta)}+\frac{2}{\sin (\theta)}=\frac{b}{z} \\
& \frac{2(1-\cos (\theta))}{\sin (\theta)}=\frac{b}{z} \\
& z=\frac{b \sin (\theta)}{2(1-\cos (\theta))}
\end{aligned}
$$

o Water is to be transported at a rate of $2 \mathrm{~m}^{3} / \mathrm{s}$ in uniform flow in an open channel whose surfaces are asphalt lined.
o The bottom slope is 0.001 .
o Determine the dimensions of the best cross section if the shape of the channel is (a) rectangular and (b) trapezoidal with a 60 degree slope

- • $\quad$ Homework 25-1
- Using the same logic as was developed in these slides, derive what would be the optimal value for $\theta$ in a trapezoidal channel.
o Water is flowing uniformly in a finishedconcrete channel of trapezoidal cross section with a bottom width of 0.6 m , trapezoid angle of $50^{\circ}$, and a slope angle of $0.4^{\circ}$. If the flow depth is measured to be 0.45 m , determine the flow rate of water through the channel.

-     - Homework 25-3
o Consider water flow through two identical channels with square flow sections of 3 m by 3 m . Now the two channels are combined, forming a $6-\mathrm{m}$-wide channel. The flow rate is adjusted so that the flow depth remains constant at 3 m . Determine the percent increase in flow rate as a result of combining the channels.


