

# Optimal PMU Placement with Contingency Situations

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**Abstract**— In this paper a new matrix reduction method has been presented to obtain minimum number of phasor measurement unit (PMU) in power system network in case of PMU failure and line outage for full system observability. Method is based on simple algorithm without having any cost function. Different steps have been discussed for different contingency situations in detailed for IEEE-14 bus system while maintaining all bus observable. Results show that proposed matrix reduction based method is compatible with other techniques of PMU placement.

**Index Terms**—PMU, system observability, IEEE bus.

## 1 INTRODUCTION

Phasor Measurement Units (PMUs) are the power system device used in smart grids to provide synchronized measurement data for network automation and information systems at transmission level. A power system is called observable if the state of the system can be uniquely identified. To make system observable PMUs are installed. However it is not necessary to installed PMU at every bus because PMU placed at any bus can gather data of all adjacent buses [1]. [2] Explains how PMU placement on individual bus is impractical either due to communication problem at some stations or the cost factor. Moreover, as a consequence of Ohm's law, when a PMU is placed at a bus, neighboring buses also become observable and hence a system can be made observable with less number of PMUs than the number of buses. To find out minimum number of PMU that should be installed and location where it should be installed to collect all data many techniques have been proposed [3-4]. There are two techniques to determine observability; numerical observability technique and topological observability technique as presented in [5]. The function of optimal PMU is presented in using different constrained [6]. In case of limited resources and when complete observability is not achieved, a method for PMU placement in order to reach maximum coverage is presented in [7]. In [8], a new three stage PMU placement technique is discussed; step I and II are based on process algorithm and third stage is based on pruning operation. Here in this work PMU placement method based on linear programming is presented into [9]; two main criteria are used to assess dynamic error of system response stability. One is based on the real part of dominant Eigen values of the system and the other one is based on MSE (Mean Squared Error) of system response. In [10], reduction of the total number of PMUs required for system observability through judicious placement of the conventional power flow measurements is presented. Few important works on Optimal PMU placement are such as, technique based on Tabu's search is presented in [11], and particle swarm optimization based search method has been discussed in [12]. Genetic algorithm based PMU placement technique is presented in [13]. In [14] bisecting search method, in [15] sorting based genetic algorithm method and in [16] graph theory approach has been presented. State estimator is a tool which provides real time state of system

and it is integral part of energy management system for security analysis [17]. State estimator of power system can only be ensured by available measurement of buses and these measurements should build a spanning tree of full rank of the system [18]. In this paper a new optimal PMU placement method based on priority elimination of buses has been proposed. Proposed methods are divided into four steps such as formation of binary bus matrix, count matrix, priority matrix and finally PMU bus selection step for system with and without considering zero injection bus (ZIB). The algorithm of the proposed method is very simple compared to others and results show that proposed method is compatible to others. The discussion section explains steps in details.

## 2 PROBLEM FORMULATION FOR OPTIMAL PMU

The development of Phase measurement unit (PMU) is inspired by symmetrical component distance relay (SCDR). PMU is installed with GPS system to measure the voltage phasor on a bus and current of all adjacent buses [19]. The phasor quantities can be converted into complex value comprised of magnitude and angle, with time sampling can be send over wireless communication channel. The data of every bus can be monitored and performance of system can be improved such as branch outage and measurement loss monitoring etc [20].

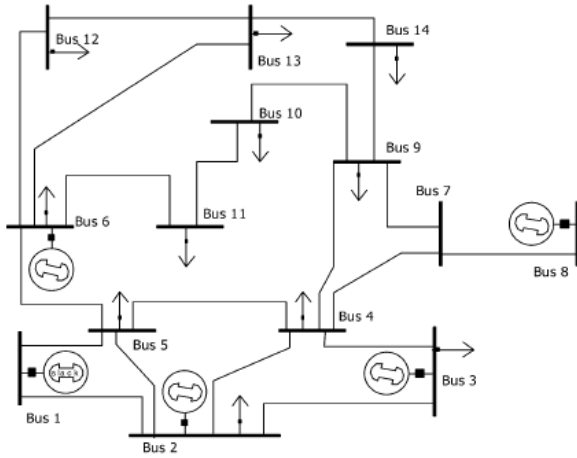


Figure 1 - Standard IEEE-14 Bus System [3]

For N bus system the optimal PMU problem can be formulated as equation (1-3);

$$\text{s.t. } f(b) \geq c \tag{1}$$

Where N is length,  $A_i$  is binary bus matrix,  $b_i$  is binary decision variable and c is unit vector of length N.  $c=[1,1,1,1,\dots]^T$ , element of binary bus matrix  $A_i$  can be defined as;

$$A_{mn} = \begin{cases} 1 & \text{if } m = n \\ 1 & \text{if } m \text{ connected to } n \\ 0 & \text{else} \end{cases} \tag{2}$$

$$b_i = \begin{cases} 1 & \text{if PMU is at this bus} \\ 0 & \text{else} \end{cases} \tag{3}$$

PMU are used for complete system observability and it should be installed in such manner that cost is minimum. PMUs selection based on reliability of complete system has been discussed in [21]. Figs. 1 shows standard IEEE-14 bus system used for explanation of proposed method.

### 3 PBET METHOD FOR OPTIMAL PMU

PBET method do not involves any cost function but based on four steps optimal PMU location is obtained. The first step is governed by type of system selected. The four steps are explained as follows [24]:

### 3.1 Binary BUS matrix

Construct binary bus matrix which depends on type of bus system. If two buses are connected then  $A_{mn}=1$  and if not then  $A_{mn}=0$ . For IEEE-14 bus system as mentioned in Fig.1, matrix is shown in Table 1.

TABLE - 1  
 Binary Bus Matrix for IEEE-14 Bus System

Bus Number	Bus Branch													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	1	0	0	1	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	0	0	0	0	0	0	0	0	0
3	0	1	1	1	0	0	0	0	0	0	0	0	0	0
4	0	1	1	1	1	0	1	0	1	0	0	0	0	0
5	1	1	0	1	1	1	0	0	0	0	0	0	0	0
6	0	0	0	0	1	1	0	0	0	0	1	1	1	0
7	0	0	0	1	0	0	1	1	1	0	0	0	0	0
8	0	0	0	0	0	0	1	1	0	0	0	0	0	0
9	0	0	0	1	0	0	1	0	1	1	0	0	0	1
10	0	0	0	0	0	0	0	0	1	1	1	0	0	0
11	0	0	0	0	0	1	0	0	0	1	1	0	0	0
12	0	0	0	0	0	1	0	0	0	0	0	1	1	0
13	0	0	0	0	0	1	0	0	0	0	0	1	1	1
14	0	0	0	0	0	0	0	0	1	0	0	0	1	1

### 3.2 Count BUS matrix

Count number of 1's in each row and create new matrix. For example in IEEE-14 bus system bus number 1 is connected to three branches, bus number 2 connected with 5 branches and so on. Table 2 is new matrix for IEEE-14 bus system. Observability index of individual bus with their links are also mentioned in Table 2. Observability index helps to determine optimal PMU location in contingency situations. Observability index and link are additional to PBET method and count matrix if modified as discussed in [24].

TABLE - 2  
Count Bus Matrix and Observability Index Matrix

Bus Number	IEEE-14	P	Links
1	3	1	1-5, 1-2
2	5	1	2-1, 2-5, 2-4, 2-3
3	3	1	3-4, 3-2
4	6	3	4-3, 4-7, 4-9, 4-5, 4-2
5	5	2	5-4, 5-2, 5-1, 5-6
6	5	1	6-5, 6-11, 6-13, 6-12
7	4	2	7-8, 7-4, 7-9
8	2	1	8-7
9	5	2	9-7, 9-4, 9-14, 9-10
10	3	1	10-9, 10-11
11	3	1	11-10, 11-6
12	3	1	12-13, 12-6
13	4	1	13-6, 13-12, 13-14
14	3	1	14-9, 14-13

### 3.3 Priority BUS matrix

PMUs should be installed in such a way that it should measure maximum bus data to ensure system observability. Hence bus with lower number of branches will be assign to lower priority of having PMU. Based on this assumption neglecting all buses with lower number of branches and this will reduce matrix size that should have PMUs. Thought it's not necessary that bus with higher number of branches will have PMU but it has higher probability. Table 3 shows the priority buses derived from count bus matrix for different type of bus system. For system without zero injection bus (ZIB) eliminate least two priority bus from Table -3, and with ZIB eliminate last priority bus and then make list of selected bus associate with bus branches.

TABLE - 3  
Priority Bus Matrix without ZIB

No. of branches	8	7	6	5	4	3	2
IEEE-14 Bus	-	-	4	2, 5, 6, 9	7, 13	1, 3, 10, 11, 12, 14	8

### 3.4 PMU selection criteria

**3.4.1**Form a list that has buses and associated bus numbers. Select all those buses which has branch that does not associated with any other bus. For example in IEEE-14 bus system branch number 11 is connected to bus number 6 only shown in Table - 4 hence select bus 6 as PMU location bus.

**3.4.2**If all of the branches of any bus M are associated with other bus N and bus N has more branches connected to it then eliminate bus M for PMU location bus. For example in IEEE-14 bus system branch 1, 2 are associated with bus number 5 and 2 but bus number 2 has one more branch number 3, hence eliminate branch bus number 5 for PMU placement.

**3.4.3**If any branch is left and does not include with any bus number then select that branch as PMU location bus and then apply pruning process.

TABLE - 4  
Buses Selected From Priority Matrix with Associated Buses (Without ZIB)

Bus no.	Associate bus
6	5, 6, 11, 12, 13
13	6, 12, 13, 14
2	1, 2, 3, 4, 5
9	4, 7, 9, 10, 14
5	1, 2, 4, 5, 6
7	4, 7, 8, 9
4	2, 3, 4, 5, 7, 9

For zero injection bus eliminate all those bus which is selected as ZIB bus and if any bus connected to only ZIB bus eliminate that also for PMU then repeat all steps as mentioned above.

TABLE - 5  
Buses Selected From Priority Matrix with Associated Buses (With ZIB)

IEEE- 14 Bus	
Bus no.	Associate bus
6	6, 11, 12, 13
13	6, 12, 13, 14
2	1, 2, 3, 4, 5
9	4, 7, 9, 10, 14
5	1, 2, 4, 5
4	2, 3, 4, 5, 7, 9

## 4 PROPOSED MATRIX REDUCTION METHOD FOR CONTINGENCY

### CASE - A Loss of PMU measurement

Under the loss of measurement the overall observability of system is compromised and new PMU must be added to ensure the complete Observability of power system network. In proposed method, we first count the observability index of individual bus and apply matrix reduction based algorithm to find new PMU locations. The observability index for IEEE 14 bus system has been show in Table 2. The proposed algorithm is as follows:

**A.1** Start from BUS number  $n(=1)$  and so on, if count value is two then put PMU on both the associated buses as mentioned in Table 2 and update the observability indices P.

**A.2** If observability index  $P \geq 2$  then eliminate all those buses from the matrix (Table 2).

**A.3:** if  $P=1$  then start from bus number  $n(=1)$   
 Read count value for bus 'n'  
 Read links of bus 'n'  
 eliminates if old PMU location is present  
 Read count value for buses remaining from links  
 New PMU location = Bus with high count value  
 Update P matrix

**CASE - B Single Line Outage in Network**

**B.1** Start from BUS number 1, if count value =2 then put PMU on both the buses.

**B.2** If  $P \geq 2$  then eliminate those buses  
 else step B.3

**B.3** Start from BUS number 'n' (=1)  
 Read count value for bus 'n'  
 Read links for bus 'n'  
 Eliminate old PMU location from links  
 New PMU = BUS which is common in both the links  
 Update observability index P.

**B.4 Check the Redundancy**

First of all check the observability (P) for each bus, if  $P > 2$  then check if that particular bus is already observable by two other buses, then remove PMU from that particular bus which does not include the bus having count value of 2. After checking the redundancy PMU count in IEEE- 14 bus bar system is found to be eight.

**5 RESULT AND DISCUSSION**

Optimal PMU location for normal operation without zero-injection and with zero injection bus system has been evaluated as discussed in [24] using PBET method. As the optimal PMU location has been obtained, the proposed matrix reduction based method is incorporated to obtain optimal PMU location under contingency situations as discussed in Case A and Case B in previous section. The results shown in Table 6, 7 are without ZIB for IEEE- 14 bus system with normal working conditions. Table IX shows optimal PMU location obtain from PBET method with observable buses. When buses with zero injection bus (ZIB) characteristics are introduced, eliminate all that bus and select one lower priority branch buses from Table III and apply same process as in without ZIB. Table X shows the comparison with other published work under normal and with zero injection bus consideration.

**TABLE - 6**  
**PMUs Location after Step 3.4.1**

IEEE- 14 Bus	
Bus no.	Associate bus
6	PMU
13	--
2	1, 2, 3
9	PMU
5	1, 2
7	PMU
4	2, 3

Table 11 shows the comparison of different technique for optimal PMU placement under different contingency

situations.

TABLE - 7  
PMUs Location after Step 3.4.2

IEEE- 14 Bus	
Bus no.	Associate bus
6	PMU
13	--
2	PMU
9	PMU
5	--
7	PMU
4	--

TABLE - 8  
PMU Location after Step 3.4.3

IEEE- 14 Bus	
Bus no.	Associate bus
6	PMU
13	--
2	PMU
9	PMU
5	--
7	PMU
4	--

TABLE - 9  
Optimal PMUs for Different Type of System (Without and with ZIB)

	PMU Bus	Branch
IEEE-14	2, 6, 7, 9	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
IEEE-14	2, 6, 9	1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14 Bus 7 as ZIB

TABLE - 10  
Comparison of Different Technique (Without and with ZIB)

Methods	IEEE-14 Bus Without ZIB	IEEE-14 Bus With ZIB at Bus 7
Proposed	4	3
B.K. Saha Roy, A.K. Sinha and A.K. Pradhan [10]	4	3
Xu and Abur [4]	4	3
Chakrabarti and Kyriakides [3]	4	3
Hurtgen and Maun [22]	4	3

TABLE - 11  
Comparison of Different Technique (Without ZIB) in case of PMU failure

Methods	Loss of Measurement W_ZIB/ZIB	Line Outage
Proposed	9/7	8/7
B.K. Saha Roy, A.K. Sinha and A.K. Pradhan [10]	NA/7	NA/NA
Chakrabarti and Kyriakides [3]	NA/7	NA/7

## 6 CONCLUSION

A new matrix reduction based technique for optimal PMU placement in contingency situations such as failure of measurement and Line Outage have been presented in this paper. The basic algorithm for normal condition is based on PBET methods, and modified for determination of optimal PMU under contingencies. Reduction based matrix method uses observability index and links to determine new locations for PMU placement. Section IV, discusses the detailed algorithm. In case of IEEE-14, bus system 9 numbers of PMU in loss of measurement case, 8 number of PMU in Line Outage case are sufficient for full system observability for without ZIB. With ZIB numbers are 7 for both types of contingencies. Results obtained from proposed method shows competition with other methods.

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