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Optimization and Modeling of Antireflective Layers for Silicon Solar Cells: In Search of Optimal Materials

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Abstract

Depositing an antireflection coating on the front surface of solar cells allows a significant reduction in reflection losses. It thus allows an increase in the efficiency of the cells. A modeling of the refractive indices and the thicknesses of an optimal antireflection coating has been proposed. Thus, the average reflective losses can be reduced to less than 8% and less than 2.4% in a large wavelength range respectively for a single-layer and double-layer anti-reflective coating types. However, the difficulty of finding these model materials (materials with the same refractive index) led us to introduce two notions: the refractive index difference and the thickness difference. These two notions allowed us to compare the reflectivity of the antireflection layer in silicon surface. Thus, the lower the refractive index difference is, the more the material resembles to the ideal material (in refractive index), and thus its reflective losses are minimal. SiNx and SiO_2/TiO_2 antireflection layers, in the wavelength range between 400 and 1100 nm, have reduced the average reflectivity losses to less than 9% and 2.3% respectively.

Keywords

Antireflection Layer, Reflectivity, Refractive Index, Thickness

1. Introduction

Antireflection coatings (ARC) are used in processing solar cells to reduce reflection and to offer better passivation properties [1]. This results in a significant increase in the current generation, leading to an improvement in the cell efficiency. Modern ARCs are usually fabricated using single or multilayer thin films.

An ideal antireflection structure should lead to zero reflection loss on solar cell surfaces over an extended solar spectral range for all incident angles. It is well known that normal-incidence reflection at a specific wavelength (usually 600 nm) can be minimized using a single layer coating with quarter wavelength optical thickness. However, the materials used to deposit an antireflection coating must have a refractive index $n = (n\text{Si} \times n_0)1/2$, where n_0 is the refractive index of the surrounding medium. Single-layer thin film antireflective coatings are limited by the availability of materials with required refractive indices [2]. But a single-layer antireflection coating is also known to be unable to cover a broad range of the solar spectrum [3] [4], and using double-layer antireflection coating is considered. The refractive indices and thickness of each of the upper and bottom layers forming the antireflection stack must satisfy a number of conditions.

In this study, a numerical optimization of the antireflection proprieties of simple layer and double layers are performed by modeling the refractive index and thickness of adequate materials. Thus, to better evaluate the reflectivity on the surface of a cell coated with a single or double antireflection layer, two new concepts will be introduced: the refractive index difference (RID) and the thickness difference (TD). RID and TD values will help to predict how the performance of a solar cell panels will be affected from reflectivity losses.

2. Models Description and Optimization Procedure

The matrix method for calculating spectral coefficients of the first layered media was suggested by F. Abeles (1950) and has been ever since widely employed [5]. For each configuration, a modified transfer matrix method [6] [7] [8] is used to calculate the reflectance from the silicon surface. It is used for the reflectance simulation. It is essential to calculate the average reflectivity R_w due to the solar spectrum's long range of wavelength from 400 to 1100 nm.

For a PV cell, it is important to have a minimum of reflection over a whole spectral range. The average residual reflection factor is defined by [9] [10]:

$$R_m = \frac{1}{\lambda_{\max} - \lambda_{\min}} \int_{\lambda_{\min}}^{\lambda_{\max}} R(\lambda)\,\mathrm{d}\lambda \tag{1}$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum values of the wavelength range respectively. $R(\lambda)$ is the reflection factor. In order to compare the effectiveness of an AR coating on a solar cell, it is important to take the AM 1.5 solar spectrum into account. For this investigation, we have chosen the wavelength range from 400 nm to 1100 nm of the solar spectrum. The reason is that for wavelengths shorter than 400 nm, the spectral power density in the AM 1.5 spectrum is almost zero, while photons with wavelengths longer than 1100 nm are hardly absorbed by the Silicon.

The reflection $R(\lambda)$ can be calculated by the transmission matrices of the ARC layers. It depends on both $n(\lambda)$ and $k(\lambda)$. For each configuration, a transfer matrix method [6] [11] [12] [13] is used to calculate the reflectance from the silicon surface. The ARC structures represented in **Figure 1(b)** can be

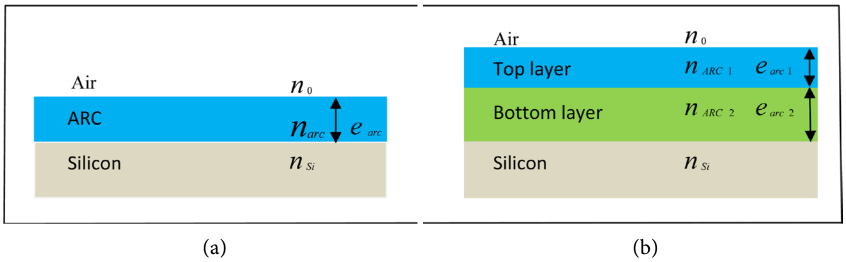

Figure 1. (a) Structure of silicon solar cell with a single layer antireflection coating; (b) Structure of silicon solar cell with a double stack antireflection coatings.

considered as composed of two layers (therefore, three interfaces) on a Si substrate (Figure 1(b)).

First, we describe the characteristic matrix of a single layer. The relationship of matrix defining the problem of single antireflection layer is given by the following relation

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = M \begin{pmatrix} E(Si) \\ H(Si) \end{pmatrix} \tag{2}$$

M is a matrix given by:

$$M = \begin{pmatrix} \cos\varphi & \dfrac{i \cdot \sin\varphi}{n_{arc}} \\ i \cdot n_{arc} \cdot \sin\varphi & \cos\varphi \end{pmatrix} \tag{3}$$

The characteristic matrix of a multilayer is a product of corresponding single layer matrices. For a double stack antireflection coating, relation 2 become:

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = M_1 \cdot M_2 \begin{pmatrix} E(Si) \\ H(Si) \end{pmatrix} = M_t \begin{pmatrix} E(Si) \\ H(Si) \end{pmatrix} \tag{4}$$

where

$$M_t = \begin{pmatrix} \cos\varphi & \dfrac{i \cdot \sin\varphi_1}{n_{arc1}} \\ i \cdot n_{arc1} \cdot \sin\varphi & \cos\varphi_1 \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi_2 & \dfrac{i \cdot \sin\varphi_2}{n_{arc2}} \\ i \cdot n_{arc2} \cdot \sin\varphi_2 & \cos\varphi_2 \end{pmatrix} \tag{5}$$

with $i^2 = -1$, n_{arc1}, n_{arc2} represent respectively the refractive index of the top and the bottom antireflection layers. φ_1 and φ_2 are respectively dephasing between the reflected waves of layers k and $k + 1$.

$$\varphi_1 = \frac{2\pi}{\lambda} \cdot n_{arc1} \cdot e_{arc1}, \quad \varphi_2 = \frac{2\pi}{\lambda} \cdot n_{arc2} \cdot e_{arc2} \tag{6}$$

The detailed derivation of amplitude reflection (r) and transmission (t) coefficients is given in [14] [15]. The resulting expressions are shown in Equation (6) and Equation (7).

$$r = \frac{n_0 \cdot M_{11} + n_0 \cdot n_{Si} \cdot M_{12} + M_{21} - n_{Si} \cdot M_{22}}{n_0 \cdot M_{11} + n_0 \cdot n_{Si} \cdot M_{12} + M_{21} + n_{Si} \cdot M_{22}} \tag{7}$$

$$t = \frac{2n_0}{n_0 \cdot M_{11} + n_0 \cdot n_{Si} \cdot M_{12} + M_{21} + n_{Si} \cdot M_{22}} \tag{8}$$

M_{ij} are the elements of the characteristic matrix of the multilayer. The energy coefficients (reflectivity, transmissivity, and absorptance) are given by:

$$R = |r|^2 \tag{9}$$

$$T = \frac{n_{Si}}{n_0}|t|^2 \tag{10}$$

$$A = 1 - R - T \tag{11}$$

with n_{Si} and n_o is refractive index of the silicon and vacuum respectively.

2.1. Simple Layer Antireflection Coating

In order to reduce reflection, a film with intermediate index of refraction can be applied according to Figure 2. If the optical film thickness ($d \times n_{Si}$) is 1/4 of the wavelength (λ), the phase difference becomes π and the two reflected waves cancel out. For complete annihilation, the amplitude of the interfering radiation also has to be identical. Under normal incidence, this requirement is fulfilled when the refractive index of the film is equal to the square root of the refractive index of the substrate [16] [17] [18]. Then, optimal thickness and optimal refractive index of the antireflection layer are given respectively by following relations:

$$e_{car} = \frac{\lambda}{4n_{arc}} \tag{12}$$

$$n_{car} = \sqrt{n_0 \cdot n_{Si}} \tag{13}$$

λ is wavelength (m), n_0 is the air refractive index and it taken equal to 1. n_{car} and n_{Si} represent refractive indices of AR layer and silicon respectively. Equation (12) and Equation (13) permit us to obtain the following figure.

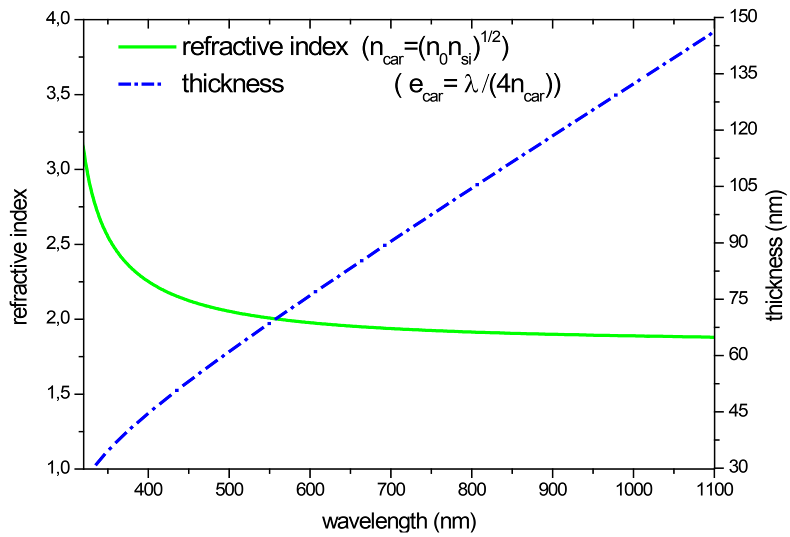

Figure 2. Optimal refractive index (curve green) and optimal thickness (blue) of simple layer antireflective coating for silicon, for annihilation of reflectance at wavelength λ(nm).

Figure 3 shows the result refractive index and thickness of optimal simple ARC as a function of wavelength. According to Figures 2-5, the optimal thickness of an ARC for silicon (blue curve) to cancel the reflection at the wavelength *λ*, can be approximated as the following equation:

$$e(\lambda) = 0.144\lambda - 11.12 \tag{14}$$

By replacing this thickness in Equation (2), the optimal refractive index is then given by:

$$n_{arc} = \frac{\lambda}{4e_{car}} = \frac{\lambda}{0.576\lambda - 44.5}$$

Relation which can be written as follows:

$$n_{arc} = A + \frac{B}{\lambda - C}$$

with $A = 1.736$, $B = 134.12$ and $C = 77.26$

$$n(\lambda) = 1.725 + \frac{134}{\lambda - 77.3} \tag{15}$$

We find the Cornu equation [19] giving the refractive index of a transparent materials.

The refractive index can also be found by extrapolation of the curve 2-5. We obtain the Cauchy equation [20]. These equations are completely empirical and were first proposed by Cauchy (1789-1827).

Following the Cauchy equation, the optimal refractive index of a single ARC is given by the following expression:

$$n(\lambda) = 1.856 + \frac{2.79 \times 10^4}{\lambda^2} + \frac{5.73 \times 10^9}{\lambda^4} \tag{16}$$

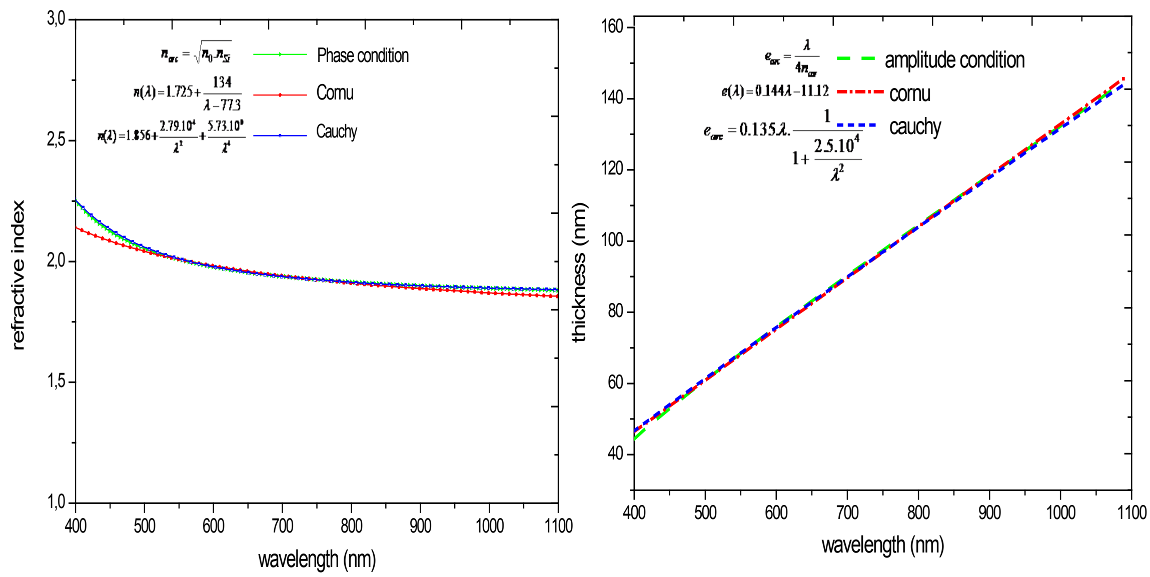

Figure 3. Comparison of different models (Cornu, Cauchy) giving the optimal refractive index (left) and the optimal thickness (right) of single layer ARC with those given by the conditions of phase and amplitude.

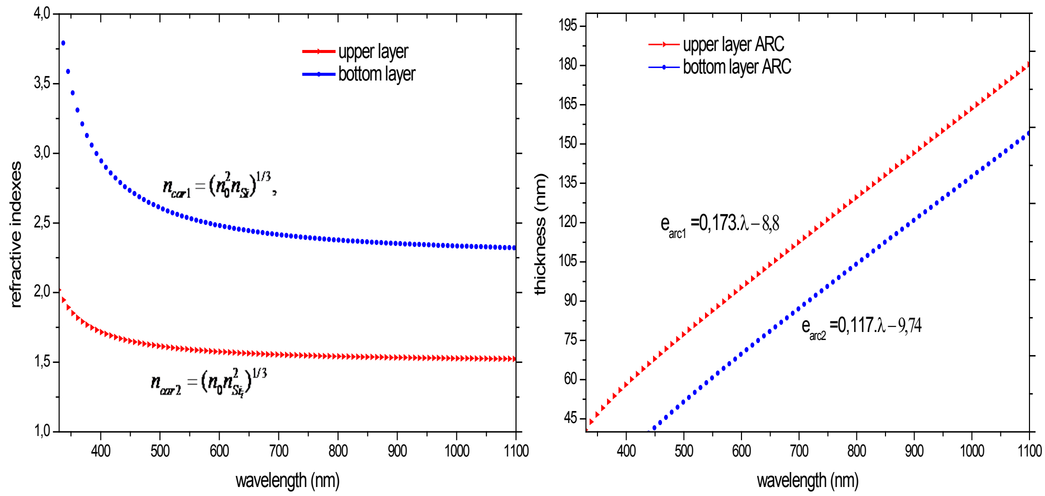

Figure 4. Optimal refractive indices (left) and optimal thicknesses (right) of upper (n_{arc1}, e_{arc1}) and bottom (n_{arc2}, e_{arc2}) antireflection layers for a double stack on the silicon surface.

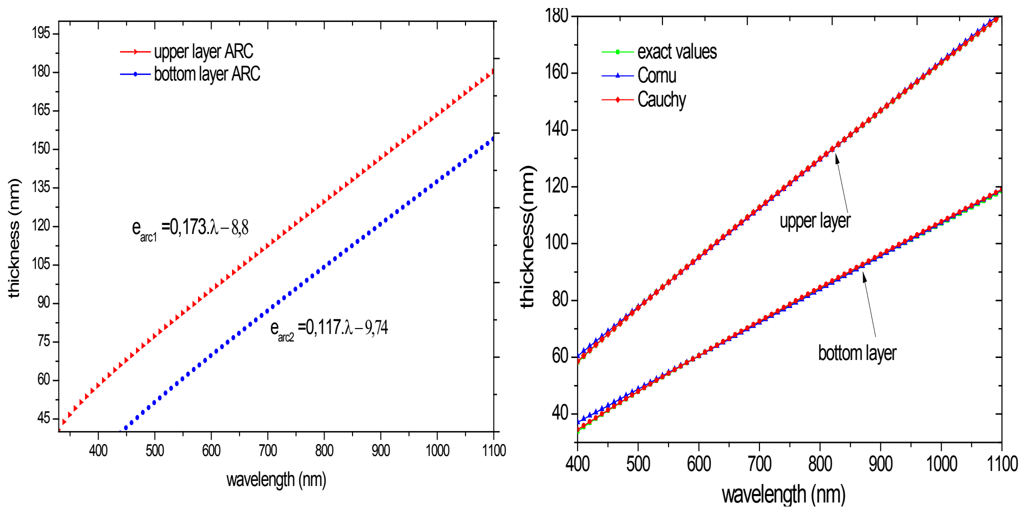

Figure 5. Comparison of different models (Cauchy, Cornu and exact values) giving the optimal refractive indices (left curve) and the optimal thicknesses (right curve) of each of the upper and bottom layers ARC as a function of the wavelength.

These results are summarized in **Table 1**.

It can be seen from **Figure 3** that there is no general trend in the variation of reflectance with the models. However, when the wavelength is inferior to 480 nm, the curve corresponding to Cornu model hasn't the same allure with other. **Table 2** shows the influence of the thickness and the refractive index of an antireflection layer for the annihilation of reflectance at a wavelength λ. As can be seen, the reflectivity is almost zero for each of wavelengths. The choice of an antireflection layer material is accordingly guided by its refractive index (we consider that its absorbance is negligible); then we look for the optimal thickness.

2.2. Double Layer Antireflection Coating

For a double layer coating for a zero reflection minimum in a large wavelength

Table 1. models of optimum thickness and refractive index of a single layer ARC for silicon.

Simple layer ARC	Cornu	Cauchy
Ref. ind. $n_{arc} = \sqrt{n_0 \cdot n_{Si}}$	$n(\lambda) = 1.725 + \frac{134}{\lambda - 77.3}$	$n(\lambda) = 1.856 + \frac{2.79 \times 10^4}{\lambda^2} + \frac{5.73 \times 10^9}{\lambda^4}$
thickness $e_{arc} = \frac{\lambda}{4n_{car}}$	$e(\lambda) = 0.144\lambda - 11.12$	$e_{arc} = 0.135 \cdot \lambda \cdot \frac{1}{1 + \frac{2.5 \times 10^4}{\lambda^2}}$

Table 2. Characteristics of an optimal antireflection layer (for silicon solar cells) to eliminate reflection at the wavelength λ.

wavelength	$\lambda = 500$	$\lambda = 600$	$\lambda = 700$	$\lambda = 800$	$\lambda = 900$	$\lambda = 1000$
thickness	61	76	90	104	118	132
Refractive index	2.05	1.98	1.94	1.91	1.90	1.89
Reflectivity	0.0002%	0.0016%	0.0008%	0.006%	0.0015%	0.0005%

range, the refractive index of the two layers have to fulfill these following relations [21]:

$$n_{arc1} = \left(n_0^2 n_{Si}\right)^{1/3}, \quad n_{arc2} = \left(n_0 n_{Si_i}^2\right)^{1/3} \tag{17}$$

And the optimal thicknesses of the top and bottom layers are respectively equal to:

$$e_{top} = \frac{\lambda_{ref}}{4 \cdot n_{top}}, \quad e_{bot} = \frac{\lambda_{ref}}{4 \cdot n_{bot}} \tag{18}$$

According to Equation (5) and Equation (6), the refractive index and thicknesses of an optimal double antireflection layer are given respectively by **Figure 5** and **Figure 6**. These figures show the optimal values of the refractive index (left) and the thicknesses (right) of each of the upper and bottom layers forming the stack.

According to the curves (3) on the right, these thicknesses are governed by the following equations:

$$e_{top} = 0.173 \cdot \lambda - 8.8 \tag{19}$$

$$e_{bot} = 0.117 \cdot \lambda - 9.74 \tag{20}$$

The thicknesses and wavelength are in nanometer (nm). From relation (6), the optimal refractive index is given by the following expression:

$$n_{top} = \frac{\lambda}{4(0.173 \cdot \lambda - 8.8)} = \frac{\lambda}{0.692 \cdot \lambda - 35.2} \tag{21}$$

$$n_{bot} = \frac{\lambda}{4(0.117 \cdot \lambda - 9.74)} = \frac{\lambda}{0.468 \cdot \lambda - 38.96} \tag{22}$$

These relations can be written in the form of the dispersion equation giving the Cornu refractive index:

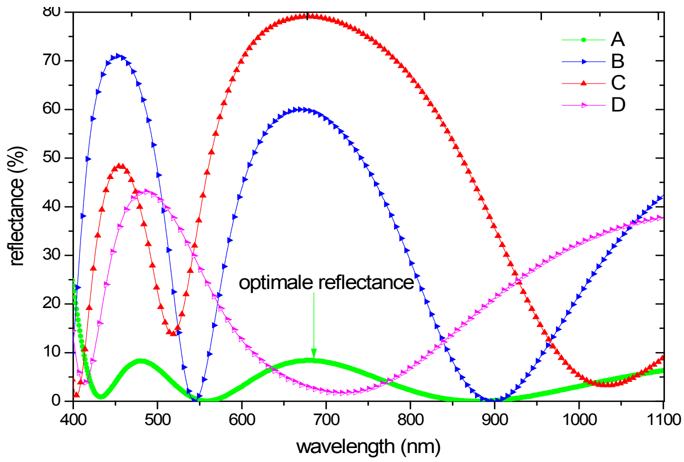

Figure 6. Reflectance spectra for a stack of four-layer ARC with the order of their deposition on silicon (A, B, C and D) under normal incidence.

$$n_{bot} = 2.137 + \frac{177.9}{\lambda - 83.25} \tag{23}$$

Besides by extrapolation of the left **Figure 3**, we obtain the refractive index according to the Cauchy equation. We can then deduce the optimal thicknesses.

These results are summarized in **Table 3**.

Figure 4 on the left shows the comparison of different models giving the refractive index of the optimal antireflection layers (upper and bottom). As can be seen, curves of refractive index of the inner AR layer are same allure. This means that the models used reflect the behavior (refractive index) of the optimal antireflection layer. The same observation is noted for the upper layer. But the Cauchy model better reflects the behavior of the optimal refractive index. Indeed for wavelengths less than 480 nm, the curve giving the refractive index obtained with the model of Cornu is detached from those of Cauchy and exact values. Curve 6 on the right compares the models giving the optimal thicknesses of the upper and inner antireflection layers for a minimization of the average reflectance. As can be seen, these curves have same allure.

3. Refractive Index Difference (RID)

In order to better search for the ideal materials for an antireflection layer on the silicon surface, we introduced the notions of refractive index difference (RID) and thickness difference (TD). Thus, the refractive index difference of a material at the wavelength λ is called the spectral refractive index difference (SRID). Over the whole range of the spectrum considered, we speak of difference of total refractive index difference, which is the sum of all the difference of spectral refractive index on the spectrum [400 - 1100 nm].

3.1. Refractive Index Difference for Simple Layer Antireflection Coating

It is defined as the difference between the refractive index of the antireflection

Table 3. Modeling of refractive index and optimal thicknesses for double layer antireflection coatings ARC_1/ARC_2 with reflective indices n_{arc1}/n_{arc2} and thicknesses e_{arc1}/e_{arc2}.

ARCs	Exact valeurs	Cornu model	Cauchy model
Top ARC	$n_{top}=\left(n_0^2 n_{Si}\right)^{1/3}$	$n_{top}=1.445+\frac{73.5}{\lambda-50.87}$	$n_{top}=1.507+\frac{1.86\times10^4}{\lambda^2}+\frac{2.2\times10^9}{\lambda^4}$
Bottom ARC	$n_{bot}=\left(n_0 n_{Si}^2\right)^{1/3}$	$n_{bot}=2.137+\frac{177.9}{\lambda-83.25}$	$n_{bot}=2.259+\frac{6.345\times10^4}{\lambda^2}+\frac{6.49\times10^9}{\lambda^4}$
Thickness	$e_{top}=\frac{\lambda_{ref}}{4\cdot n_{sup}}$ $e_{bot}=\frac{\lambda_{ref}}{4\cdot n_{sup}}$	$e_{top}=0.173\cdot\lambda-8.8$ $e_{bot}=0.117\cdot\lambda-9.74$	$e_{top}=0.166\cdot\lambda\frac{1}{1+\frac{1.23\times10^4}{\lambda^2}+\frac{1.46\times10^9}{\lambda^4}}$ $e_{bot}=0.111\cdot\lambda\frac{1}{1+\frac{2.8\times10^4}{\lambda^2}+\frac{2.87\times10^9}{\lambda^4}}$

layer $n(\lambda)$ and the optimal one, given by the phase condition at the wavelength λ.

$$\Delta n(\lambda)=\left|\left(n_0 n_{Si}\right)^{1/2}-n_{ARC}(\lambda)\right| \tag{25}$$

with $\left(n_0 n_{Si}\right)^{1/2}=n(\lambda)=1.856+\frac{2.79\times10^4}{\lambda^2}+\frac{5.73\times10^9}{\lambda^4}=1.725+\frac{134}{\lambda-77.3}$

$\Delta n(\lambda)$ can be given by the following relation:

$$\Delta n(\lambda)=\left|1.856+\frac{2.79\times10^4}{\lambda^2}+\frac{5.73\times10^9}{\lambda^4}-n_{ARC}(\lambda)\right| \tag{26}$$

Relation (15) is the spectral refractive index difference (SRID).

To obtain the total refractive index difference, we sum on all the wavelengths of the spectrum considered. The expression of the total refractive index difference (or refractive index difference) is then given by the following relation:

$$\Delta n=\left|\left(\int_{400}^{1100}1.856+\frac{2.79\times10^4}{\lambda^2}+\frac{5.73\times10^9}{\lambda^4}-n_{ARC}(\lambda)\right)\mathrm{d}\lambda\right| \tag{27}$$

3.2. Refractive Index Difference for Double Layer Antireflection Coatings

Let $\Delta n_1(\lambda)$ be the refractive index difference between the upper layer of refractive index $n_1(\lambda)$ and that of optimal refractive index for a layer of the same position and $\Delta n_2(\lambda)$ the difference between the refractive index of the bottom layer ARC and that optimal for a layer of the same position (described by relation 13). Then, the difference in refractive index for a double AR layer at a wavelength is given by the average of Δn_1 and Δn_2.

$$\Delta n_1(\lambda)=\left|n_{top}(\lambda)-n_{ARC1}(\lambda)\right| \tag{28}$$

$$\Delta n_2(\lambda)=\left|n_{bot}(\lambda)-n_{ARC2}(\lambda)\right| \tag{29}$$

$$\Delta n(\lambda)=\frac{\Delta n_1(\lambda)+\Delta n_2(\lambda)}{2} \tag{30}$$

$n_{top}(\lambda)$ and $n_{bot}(\lambda)$ represent respectively the optimal refractive indices of the top and the bottom antireflective layers. And $n_1(\lambda), n_2(\lambda)$ represent respectively refractive indices of the chosen antireflective coatings of top and bottom layers

Taking into account relations 20 and 21, it comes:

$$\Delta n_1(\lambda) = \left|1.507 + \frac{1.86\times10^4}{\lambda^2} + \frac{2.2\times10^9}{\lambda^4} - n_{ARC1}(\lambda)\right| \tag{31}$$

$$\Delta n_2(\lambda) = \left|2.259 + \frac{6.345\times10^4}{\lambda^2} + \frac{6.49\times10^9}{\lambda^4} - n_{ARC2}(\lambda)\right| \tag{32}$$

Relation (19) is the spectral refractive index difference, which depending only on the wavelength and the refractive index of the bottom $(n_2(\lambda))$ and top $(n_1(\lambda))$ layers. Over the entire solar spectrum, the refractive index difference (RID) is defined by the sum of all differences of spectral refractive index.

$$\Delta n_1 = \left|\int_{400}^{1100}\left(1.507 + \frac{1.86\times10^4}{\lambda^2} + \frac{2.2\times10^9}{\lambda^4} - n_{ARC1}(\lambda)\right)\mathrm{d}\lambda\right| \tag{33}$$

$$\Delta n_2 = \left|\int_{400}^{1100}\left(2.259 + \frac{6.345\times10^4}{\lambda^2} + \frac{6.49\times10^9}{\lambda^4} - n_{ARC2}(\lambda)\right)\mathrm{d}\lambda\right| \tag{34}$$

$$\Delta n = \frac{\Delta n_1 + \Delta n_2}{2} \tag{35}$$

3.3. Case of Antireflective Multi Layers

If the number of layers is greater than 3, the refractive index difference will be defined in another way. Consider a stack of N layers deposited follow that order $N,(N-1),\cdots,2,1$ with refractive indices $n_N, n_{N-1},\cdots,N_1$, respectively. The spectral refractive index difference for a multilayer coating is defined by the following equation:

$$\begin{aligned}\Delta n &= \left|n_1(\lambda) - n_0\right| + \left|n_2 - n_1(\lambda)\right| + \cdots + \left|n_{Si}(\lambda) - n_N(\lambda)\right| \\ &= \Delta_1 + \Delta_2 + \cdots + \Delta_n\end{aligned} \tag{36}$$

In the case where the study is done on all the wavelengths, refractive index difference (RID) is defined by the following relation:

$$\Delta n = \int_{400}^{1100} \Delta_m(\lambda)\,\mathrm{d}\lambda \tag{37}$$

$$\begin{aligned}\Delta n &= \int_{400}^{1100}\left(\Delta n_{1,0} + \Delta n_{2,1} + \cdots + \Delta n_{Si,N}\right)\mathrm{d}\lambda \\ &= \Delta n_{1,0} + \Delta n_{2,1} + \cdots + \Delta n_{Si,N}\end{aligned}$$

4. Thickness Difference (TD)

4.1. Thickness Difference for Single AR Layer

It is defined as the difference between the optimal thickness (e_{arc}) of an antireflection layer for silicon and that given by the amplitude condition.

$$\Delta e(\lambda) = e_{arc} - \frac{\lambda}{4 \cdot n_{arc}(\lambda)} \tag{38}$$

Following relations (14) and (38), thickness difference (TD), is related to the wavelength λ by the following expression:

$$\Delta e(\lambda) = -0.144 \cdot \lambda + 11.12 + e_{arc} \tag{39}$$

4.2. Case of Double AR Layers

Let e_1 and e_2 be the thicknesses of the respective lower and upper layers forming the AR double layer stack. Noting e_{arc1} and e_{arc2} those optimal for a double-layer antireflection coating, the thickness difference (TD) of a double antireflection layer is defined by the following relations:

$$\Delta e_1 = e_{arc1} - \frac{\lambda_{ref}}{4n_{arc1}} \tag{40}$$

$$\Delta e_2 = e_{arc2} - \frac{\lambda_{ref}}{4n_{arc2}} \tag{41}$$

n_{car1} and n_{car2} respectively represent the optimal refractive index of the inner and the upper antireflection layers. According to relations (19) and (20), these expressions can be written as following relations:

$$\Delta e_1 = e_{arc1} - 0.173 \cdot \lambda - 8.8 \tag{42}$$

$$\Delta e_2 = e_{arc2} - 0.117 \cdot \lambda - 9.74 \tag{43}$$

Thickness difference (TD) of a double layers is presented as follows: $\Delta e = \Delta e_1 / \Delta e_2$, λ = 600nm is chosen as wavelength of reference in this paper.

5. Results and Discussion

Average Reflectance (R_{av}) Depending on Refractive Index Difference (RID)

1) Case of Simple Layer Antireflection Coating.

Table 4 shows the average reflectance values and the corresponding refractive indices difference for each of the AR layers. As can be seen, there is a dependency relationship between the refractive index difference and the average reflectance. Reflectivity is as lower than this difference is low. For example, a low reflectivity of 9.1% is obtained with the SiN_x corresponding to a low refractive index difference of 61; and a refractive index difference of 445 corresponds to a large reflectivity of 15%. These results are explained by the fact that this refractive index difference is even bottom; the refractive index of the AR layer is close to that of the optimal antireflection layer (given by the phase condition). Using Table 4, the variation of the average reflectance R_{av} on the silicon coated with a single antireflection layer can be modeled as a function of the total refractive index difference (Δn) by the following relation:

$$R_{av} = -5.133 \times 10^{-5} \cdot \Delta n^2 + 0.049 \cdot \Delta n + 3.36 \tag{43}$$

Table 4. Reflectance on silicon surface coated a single antireflection layer, depending of refractive index difference. Optimal thickness (Δe = 0) were taken for each of the AR layer.

Antireflection layer	Refractive index at λ = 600 nm	RID (Δn_t)	Average reflectance R (%)
SiN_x	1.86	67	9.1
Al_2O_3	1.77	127	9.2
ZrO_2	2.16	142	9.4
SiN_x	2.2	161	10.8
SiN_x	2.4	279	13.1
SiO_2	1.46	344	14.3
TiO_2	2.60	445	15

2) Case of Double Layer Antireflection Coatings.

Reflectance losses values and refractive index difference (RID) corresponding for each of double stack ARCs are presented in Table 5.

Table 5 shows the dependence between refractive index difference and average reflectance. As can be seen, the reflection is weaker as this difference is small. The average reflectance of the SiO_2/TiO_2 AR double layer is lower, corresponding to an index difference of 84. This reflection is greater for the double SiO_2 /Al_2O_3 antireflection layer (10.8%) corresponding to a refractive index difference of 278. However, the refraction index difference for a single antireflection layer is not comparable to that coated with a double antireflection layer. In other words, the reflection on a cell coated with a double AR layer may be greater than that coated with a single AR layer while the latter has a difference in refractive index greater than the first. The average reflectance (R_{av}) on the silicon surface coated with a double antireflection layer is a function of the refractive index difference (Δn) and can be modeled as follows:

$$R_{av} = 1.82 \times 10^{-4} \cdot \Delta n^2 - 0.022 \cdot \Delta n + 2.86 \tag{44}$$

3) Case of multilayer antireflection coatings

Consider these different configurations of deposition of the AR layers on silicon with the same materials:

- A. $TiO_2/Al_2O_3/ZrO_2/SiO_2$ ARC on silicon (degrowth of refractive indices).
- B. SiO_2/Al2$O_3/ZrO_2/TiO_2$ ARC on silicon (growth of the refractive indices).
- C. $ZrO_2/Al_2O_3/TiO_2/SiO_2$ ARC on silicon (refractive indices neither growth nor degrowth).
- D. $Al_2O_3/ZrO_2/SiO_2/TiO_2$ ARC on silicon (refractive indices neither growth nor degrowth).

Table 6 shows the reflectivity of a stack multilayer depends greatly on deposition order of these layers on silicon surface. In fact that deposition order governs the refractive indices difference values those impacts directly on solar cell reflectivity. Figure 6 confirms theses results.

Table 5. Dependences of the average reflectance on the refractive index difference (RID) for a silicon surface coated double layer antireflection coatings.

Couches AR	Δn (600 nm)	Δn (800 nm)	Δn_1	Δn_2	Δn	R_{av} (%)
SiO_2/TiO_2	0.12	0.12	76	92	84	2.3
SiO_2/SiN_x (n = 2.4)	0.09	0.10	76	73	75	2.4
SiO_2/SiN_x (n = 2.2)	0.19	0.17	76	192	134	3.3
SiO_2/ZrO_2	0.22	0.17	76	211	144	3.6
SiO_2/SiN_x (n = 2)	0.28	0.23	76	302	189	5.2
SiO_2/SiN_x (n = 1.8)	0.37	0.31	76	420	248	8.4
SiO_2/Al_2O_3	0.42	0.35	76	480	278	10.8

Table 6. Importance of deposition configuration of the multi stack antireflection coating on silicon surface for minimizing reflectance losses.

multi stack	$\Delta n_{1,0}$	$\Delta n_{2,1}$	$\Delta n_{3,2}$	$\Delta n_{4,3}$	$\Delta n_{Si,4}$	Δn	R_{av} (%)
A	319	216	269	304	884	398	3.97
D	535	269	485	789	884	592	20.11
B	1109	304	269	216	1673	714	33.5
C	805	269	573	789	1673	822	42.7

With $\Delta n_{j,k}$ Refractive index difference (RID) between layer number j and layer number k: then $\Delta n_{1,0}$ represents RID between the n_0 (air refractive index) and n_1 (refractive index of upper layer).

Figure 6 shows the variation of the reflectivity of a four-layer AR stack according to their deposition orders (A, B, C and D) on the silicon surface. Among these different curves, that A representing the order of deposition where the refractive index increase from nair to that of silicon (n_{Si}) leads to a lower reflectance (less than 4%) over the entire spectrum wavelength range of solar. These results were expected. Indeed, the high reflectivity on the surface of silicon solar cells is due to the large discontinuity between the refractive index that exists at the interface (air-cell). The deposition of a stack where the refractive indices grow, makes a sudden change in this index is replaced by a continuous transition from a low refractive index material to a high refractive index material. Thus, for a minimization of the reflectivity with a multilayer stack, the refractive index difference must be minimal. Consequently, the refractive indices of the different layers composing this AR stack must grow from the refractive index of air to that of silicon.

6. Average Reflectance Depending on Thickness Difference (TD)

6.1. Case of Simple Layer Antireflection Coating

As shown in **Figure 7**, the average reflectance increases rapidly with this thickness difference. In fact, for antireflection layers of thicknesses less than or greater

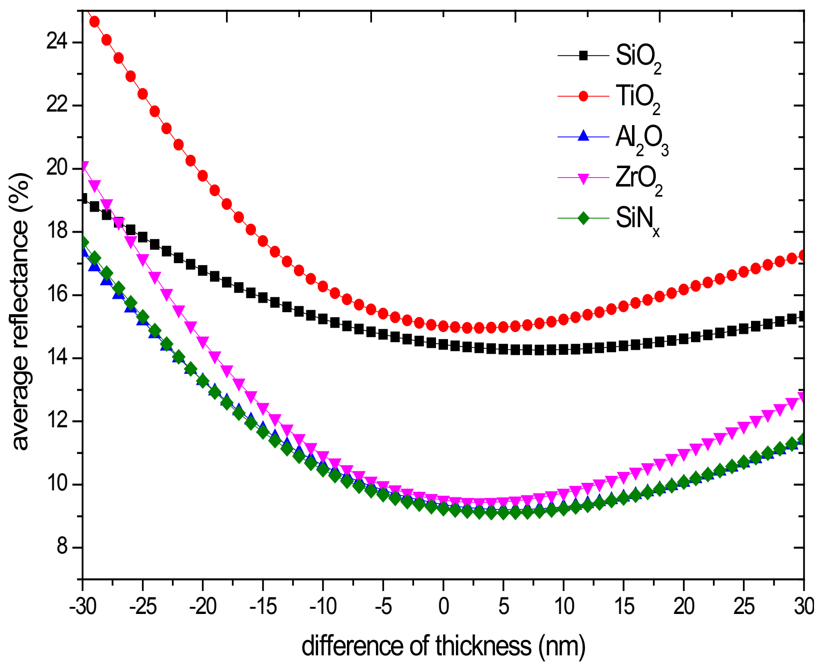

Figure 7. Thickness difference (TD) dependence of the reflectance spectra of different single antireflection layer.

than optimal, in other words, as one moves away from the zero thickness difference ($\Delta e = 0$), the reflectivity grow rapidly. ZrO_2 ARC is an example where a zero refractive index difference ($\Delta e = 0$ then $e = 72$ nm) corresponds to an average reflectance of 9.4% while thickness difference of $\Delta e = 10$ ($e = 82$ nm) and $\Delta e = -10$ ($e = 62$) correspond to mean reflectivity of 10% and 10.4%, respectively. That is an increase in reflection of more than 6% in each case.

6.2. Case of Double Layers Anti-Reflection Coatings

At the reference wavelength $\lambda_{ref} = 600$ nm, the thickness value of each layer given by the phase condition, (relation 8) is compared with that optimal (making the minimum reflectance) for different types of antireflection layer. The thickness difference (TD) and average reflectance values then obtained with each of antireflection layer) are summarized in Table 7.

In Table 7, for each antireflection layer, optimal thickness is used for calculating average reflectance. As can we see, there is little difference between optimal thickness and that given by phase condition. Consequently, the thickness difference (TD) values are low.

Figures 8(a)-(f) show variation of reflectance losses with thickness difference (TD) for each of stack double layer ARC on silicon.

As can be seen, the reflectivity on a double layer AR strongly depending on the respective thicknesses of the different stacks constituting it. For each type of AR layer, there is a couple of optimal thickness minimizing the average reflectance of the light rays arriving on the surface. As can be seen, this pair of thickness differs little from that given by the relationships (25) which would lead to a

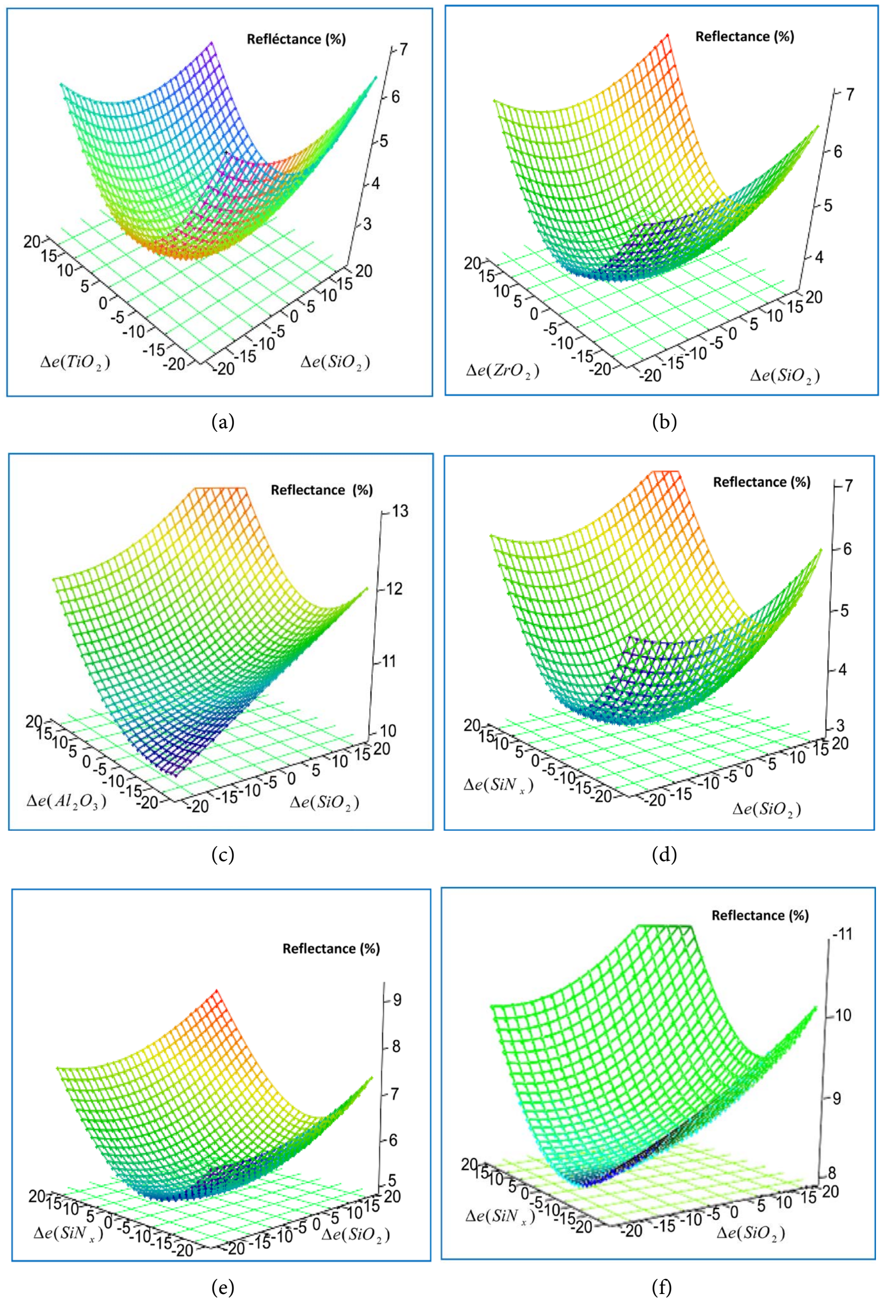

Figure 8. (a) Reflectance according to the thickness difference of the SiO_2/TiO_2 double layer antireflection coatings; (b) Reflectance according to the thickness difference of the SiO_2/ZrO_2 double layer antireflection coatings; (c) Reflectance according to the thickness difference of the SiO_2/Al_2O_3 double layer antireflection coatings; (d) Reflectance according to the thickness difference of the SiO_2/SiN_x (n = 2.2) double layer antireflection coatings; (e) Reflectance according to the thickness difference of the SiO_2/SiN_X (n = 2) double layer antireflection coatings; (f) Reflectance according to the thickness difference of the SiO_2/SiN_X (n = 1.8) double layer antireflection coatings.

zero thickness difference ($\Delta e_1 = \Delta e_2 = 0$) of the curves 8, only the one representing the AR double-layer stack SiO_2/SiN_x (n = 1.8) leads to a minimal reflectance outside the zone where the thickness differences are zero. Indeed, in

Table 7. Influence of the thickness difference of each antireflection layer on average reflectance.

Antireflection coating on Si	$e = \frac{\lambda_{ref}}{4 \cdot n}$	Optimal thickness (nm)	Thickness Difference (TD)	Average Reflectance (%)
SiO_2	103	106	3	14.3
ZrO_2	69	72	3	9.4
TiO_2	58	58	0	15
Al_2O_3	85	88	3	9.2
SiN_x (n = 1.8)	81	85	4	9.1
SiN_x (n = 2.2)	68	72	4	10.8
SiO_2/TiO_2	103/58	103/58	0/0	2.3
SiO_2/ZrO_2	103/69	103/69	0/0	3.6
SiO_2/Al_2O_3	103/85	95/78	−8/−7	9.0
SiO_2/SiN_x (n = 1.8)	103/81	110/89	7/8	2.0
SiO_2/SiN_x (n = 2)	103/74	105/77	2/3	2.0
SiO_2/SiN_x (n = 2.2)	103/68	100/66	−3/−2	2.2

this case the reflectivity is minimal (2% on average) if Δe (SiN_x) is between −15 and 0; and Δe (SiO_2) is between −20 and −10.

7. Conclusion

We have modeled the optimal refractive index for single layer and multilayer antireflection coatings for minimization of reflectivity losses at the silicon surface. Silicon nitride (SiN_x, n = 1.8) single layer ARC reduces consequently reflectance but It was found that the antireflection effect of SiO_2/ARC2 double-layer ARC is better than that of single layer. For SiO_2/SiN_x double-layer ARC, the optimal antireflection effect is obtained with refractive indices of 1.46 and 2 for the top and the bottom layer, respectively. Refraction Index Difference (RID) and thickness Difference (TD) have allowed us to better understand the mechanisms of photon losses at the surface of silicon solar cells coated antireflection layer: Reflectivity is even lower than these refractive index difference and thickness difference are low. The results reported in this study can be used as a significant tool for efficiency improvement in thin film silicon solar cells. However, for an ideal antireflection layer, the absorption loss especially in the wavelength range [400 - 1100 nm], must be low.

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