Solving Large Optimization Problems in Finance:

How MATLAB Can Help You

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Optimization

Definition of *optimize* in English:

optimize

Oxford Dictionaries
Language matters

1 Make the best or most effective use of (a situation or resource):

Optimum (disambiguation)

From Wikipedia, the free encyclopedia

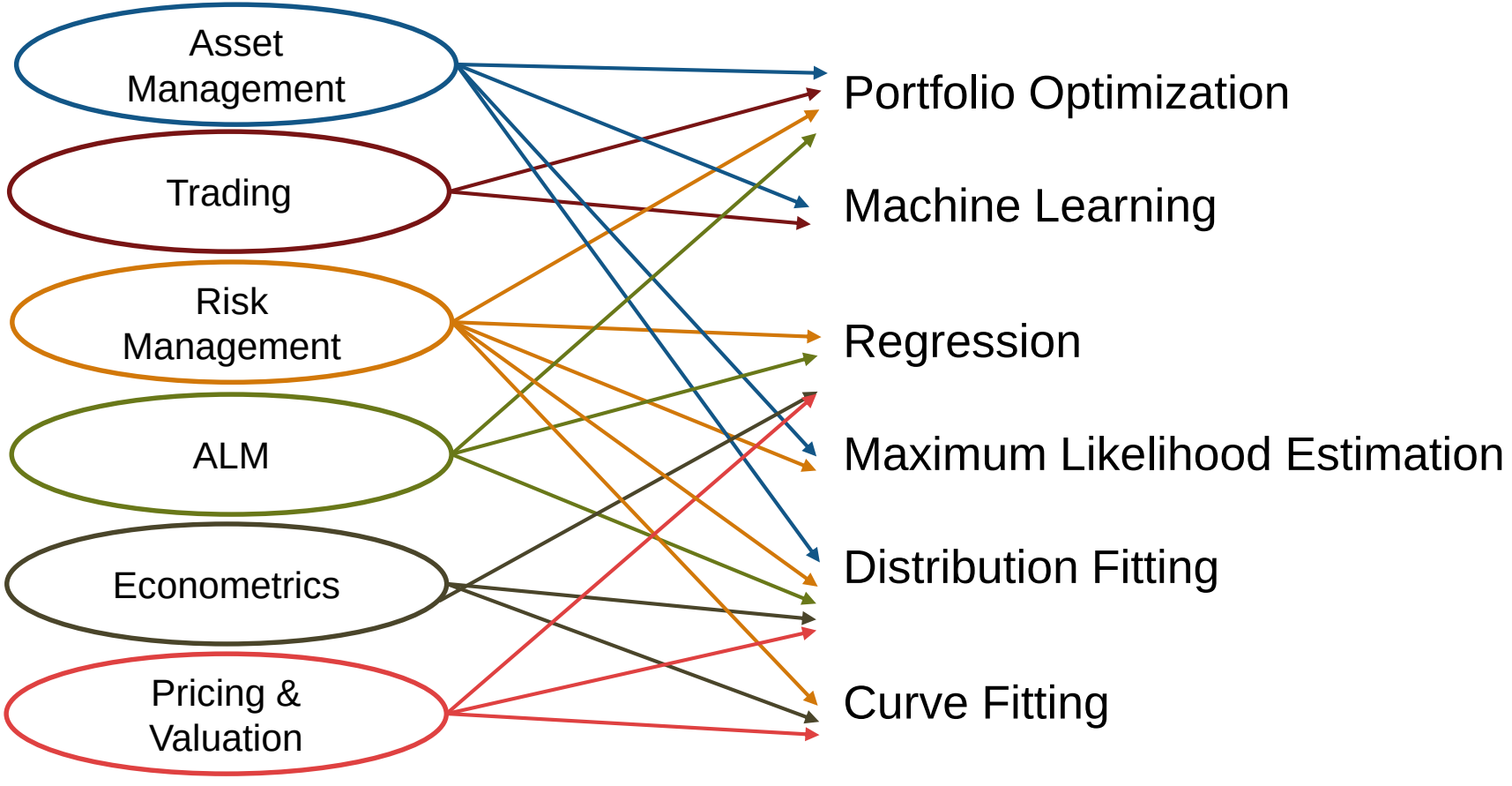
The **optimum** is the best or most favorable condition, or the greatest amount or degree possible under specific sets of comparable circumstances.

Optimization in Financial Applications with MATLAB

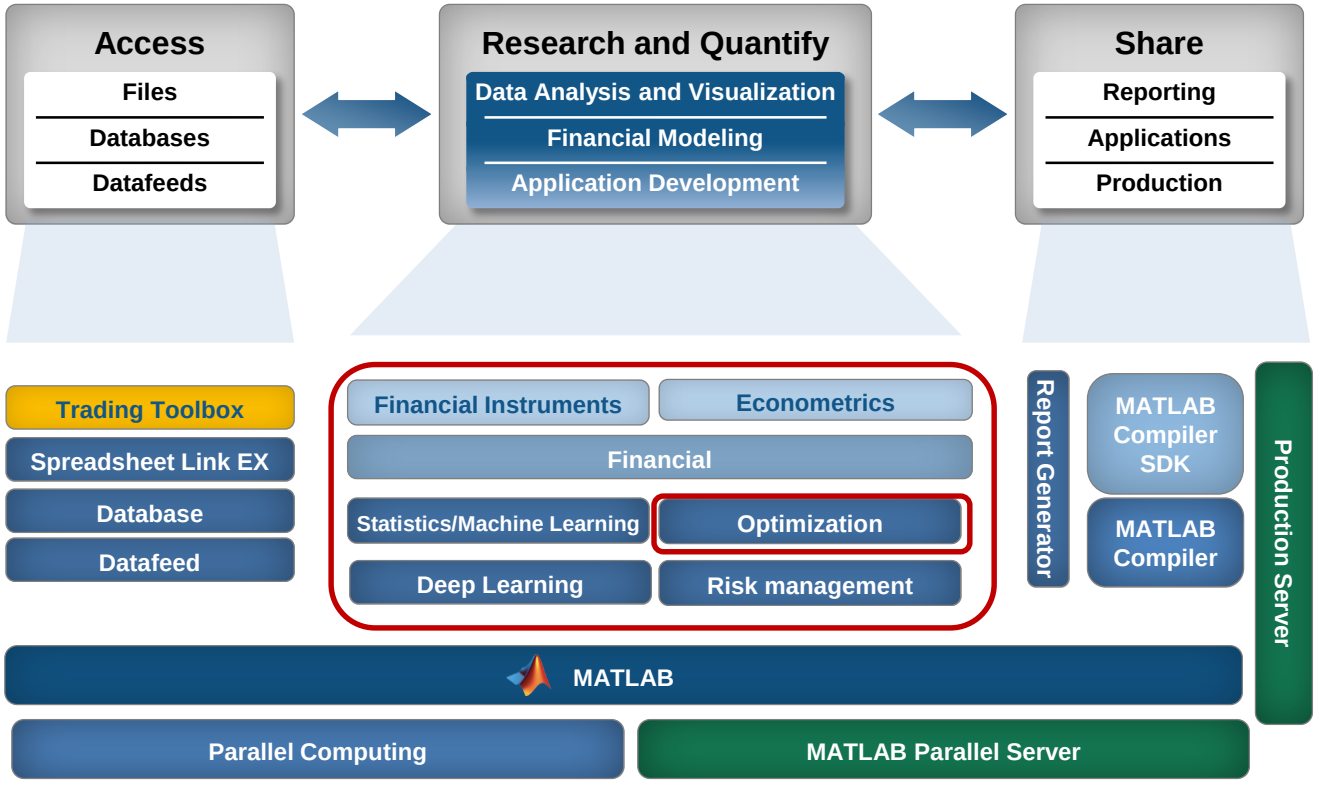
- Financial Optimization
- Optimization Methods
- Customized Optimization Models

Financial Optimization

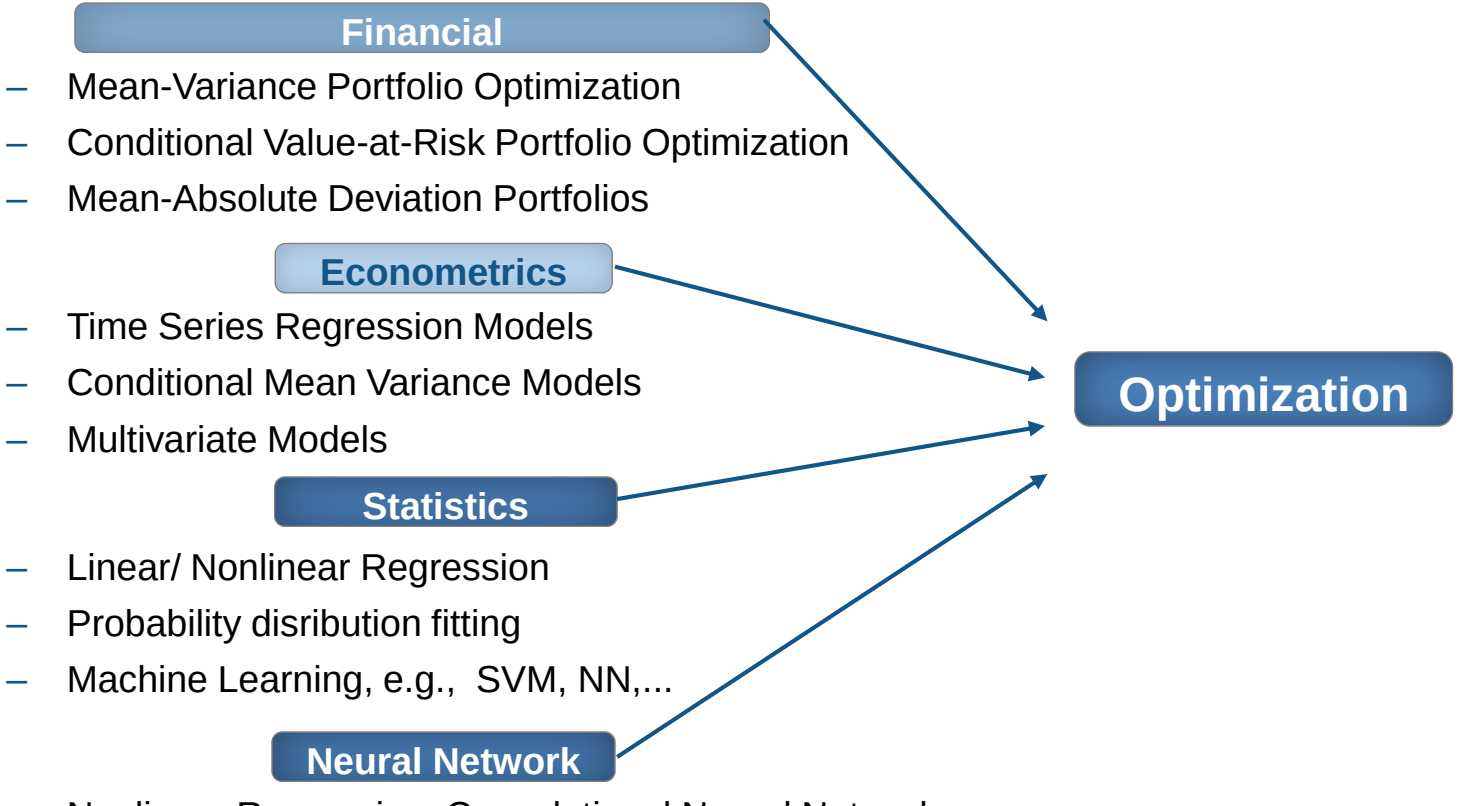
Financial Applications and Optimization



MATLAB – The Financial Development Platform



Financial Optimization in MATLAB



Optimization Methods

Optimization Problem

Objective Function

$$\min_{x} f(x)$$

Typically a linear or nonlinear function

Decision variables (can be discrete or integer)

Subject to Constraints

Linear constraints
- inequalities
- equalities
- bounds

$$Ax \leq b$$

$$A_{eq}x = b_{eq}$$

$$l \leq x \leq u$$

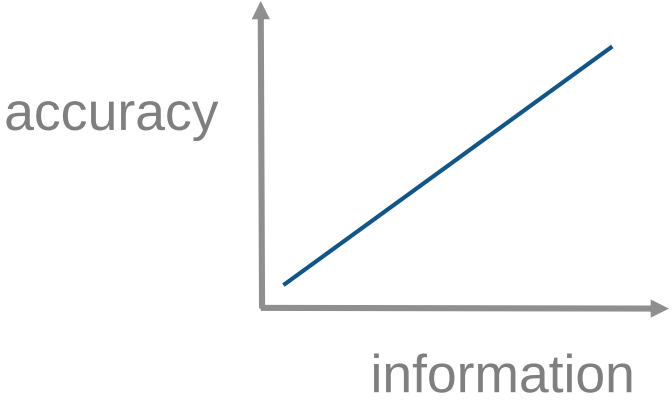
$$c(x) \leq 0$$

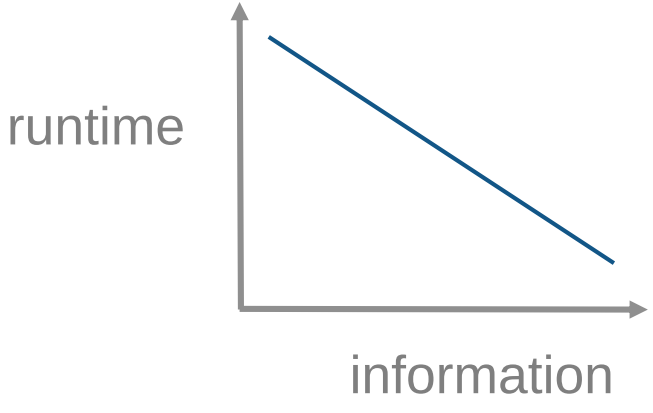
$$c_{eq}(x) = 0$$

Nonlinear constraints
- inequalities
- equalities

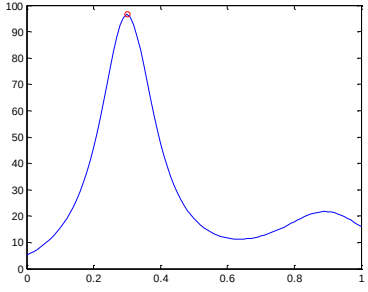
How to solve an optimization problem ?

What do you know about your optimization problem ?





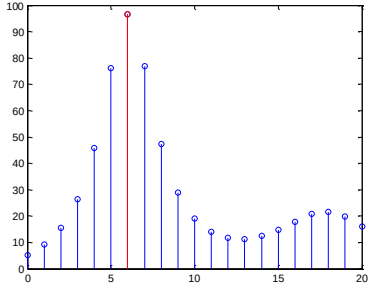
Variables & Constraints



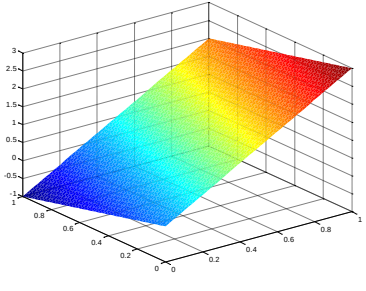
Variables

Continuous

Discrete



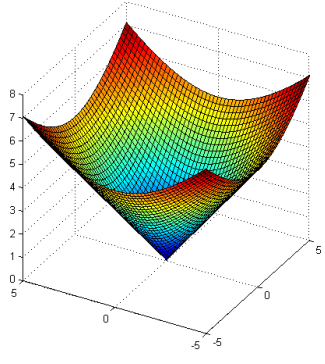
Integer Programming



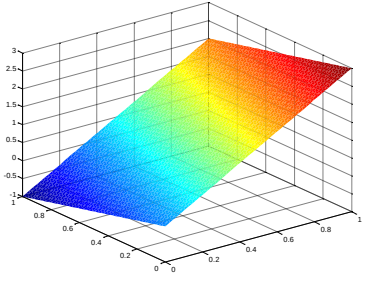
Constraints

Linear

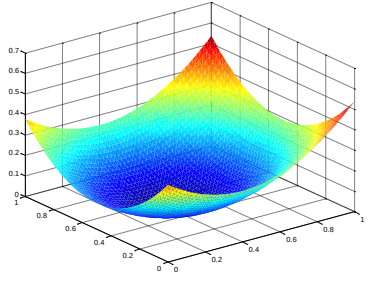
Nonlinear



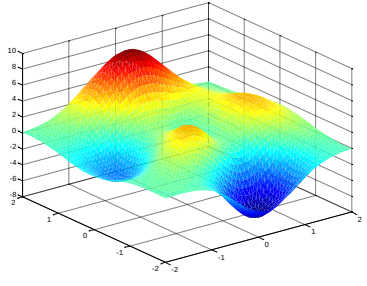
Objective Function



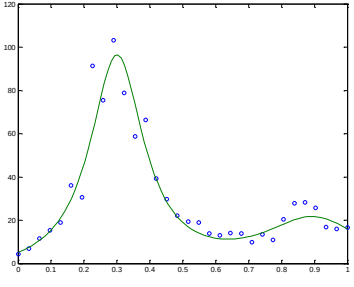
Linear $f(x) = A^{\mathrm{T}}x$



Quadratic $f(x) = x^{\mathrm{T}}Ax$



General $f(x)$

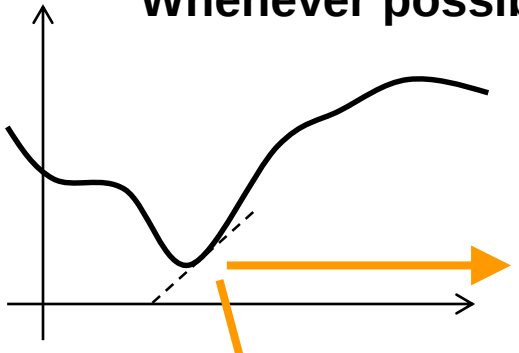


Least-squares/curve fitting

$f(c)=\Sigma[g(x_k;c) - y_k]^2$

Numerical Optimization

Whenever possible, provide gradient/hessian information!

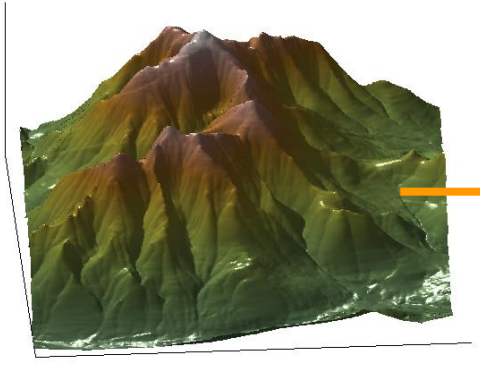


$$f'(x) \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

```
function [f,df] = objective(x)
f = ...    % function value
df = ...   % gradient vector
```

- ✓ **Fewer function evaluations**
- ✓ **More accurate**

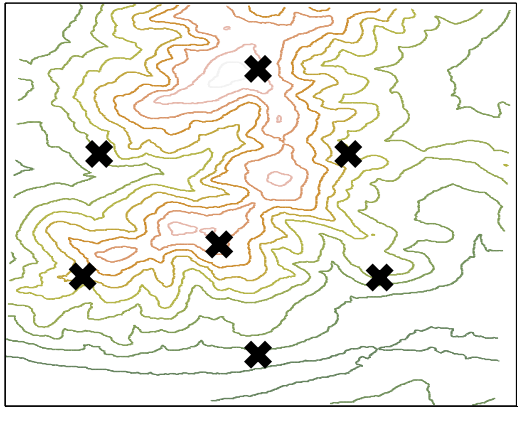
Derivative-Free Optimization



$\nabla f = ?$ → ~~fmincon~~

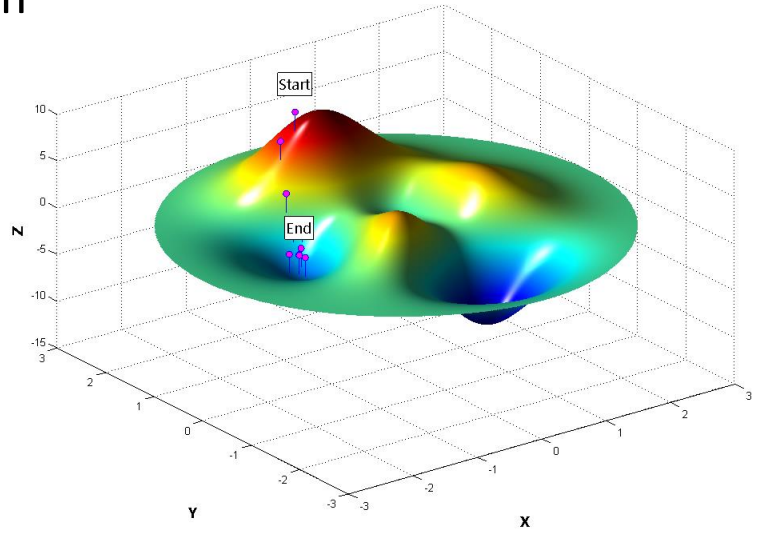
Repeatedly sample several points

Direct Search
Genetic Algorithm



Approaches in MATLAB

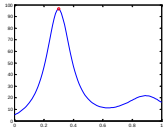
- Local Optimization
  - Finds local minima/maxima
  - Uses supplied gradients or estimates them
  - Applicable for large scale problems with smooth objective function
  - Faster/fewer function evaluations



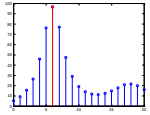
- Global Optimization
  - No gradient information required
  - Solve problems with non-smooth, discontinuous objective function

Solvers

Variables

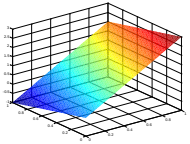


Continuous

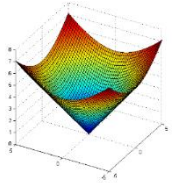


Discrete

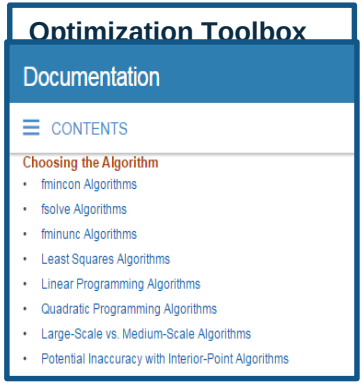
Constraints



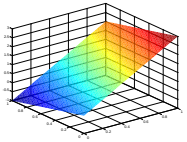
Linear



Nonlinear



Objective function



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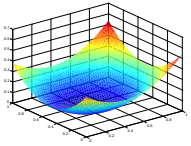
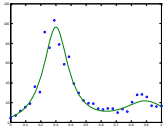
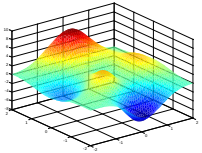


Quadratic $f(x) = x^{\mathrm{T}}Ax$



Least-squares

$f(c)=\Sigma[g(x_k;c) - y_k]^2$



General $f(x)$

Solver

`linprog`

`intlinprog`

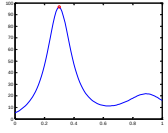
`quadprog`

`lsqlin`
`lsqnonlin`

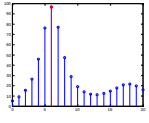
`fminunc`
`fmincon`

Solvers

Variables

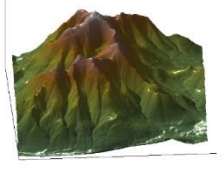


Continuous



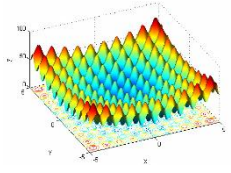
Discrete

Constraints



Non-smooth

Objective function



Nonlinear $f(x)$

Global Optimization Toolbox

Documentation

CONTENTS

Choosing a Solver

Table for Choosing a Solver

There are seven Global Optimization Toolbox solvers:

- `ga` (genetic algorithm)
- `GlobalSearch`
- `MultiStart`
- `patternsearch`, also called direct search
- `particleswarm`
- `simulannealbnd` (simulated annealing)
- `gamultiobj`, which is not a minimizer; see Multiobjective Optimization

Solver

`GlobalSearch`,
`MultiStart`

`patternsearch`,
`ga`,
`simulannealbnd`

`ga`

How to Improve Performance?

- Derivatives calculations
  - Gradients
  - Hessian matrix
- Default approach
  - Approximation: Finite differences perturbations (**VERY EXPENSIVE!**)
- Alternative approaches
  - Cheaper approximations
    - LBFGS
    - Supply your own approximation
    - Analytical calculation

Hessian Analytical Calculation

- Not easy, in general, to calculate analytical Hessian
- Can MATLAB help?
  - YES! **Symbolic MathToolbox**

Sample Problem

- Problem: Volatility surface estimation for the pricing of call options. Calibration using market options.

- Objective: $g(x) = \|f(x)\|^2$

  - $f_j(x) = C(\mathbf{x_j}, K_j, T_j) - C_j$

    - *K strikes*
    - *T maturities*
    - *C closing prices*
    - *BSM model (Black Scholes and Merton)*

Numerical results (problem size 209)

Gradients	Function evaluations	Time (secs)
Numerical	100,056	1351
Numerical (using parallel)	100,056	642
Analytical	52	3.41

MATLAB Code

```
%Define symbolic residuals
x = evalin(symengine,['n:=',num2str(n),';[x[j] $ j = 1..n]']);
f = evalin(symengine,['[f[j] $ j = 1..n]']);
for jj=1:n
    d1 = (log(S(jj)/K(jj)) +
(r+x(jj)^2/2)*T(jj))/(x(jj)*sqrt(T(jj)));
    d2 = d1 - x(jj)*sqrt(T(jj));
    val = S(jj)*0.5*(1+erf(d1/sqrt(2))) - ...
        K(jj)*exp(-r*T(jj)) * 0.5*(1+erf(d2/sqrt(2)));
    f(jj) = (val-CP(jj));
end
obj = f*f.';
```

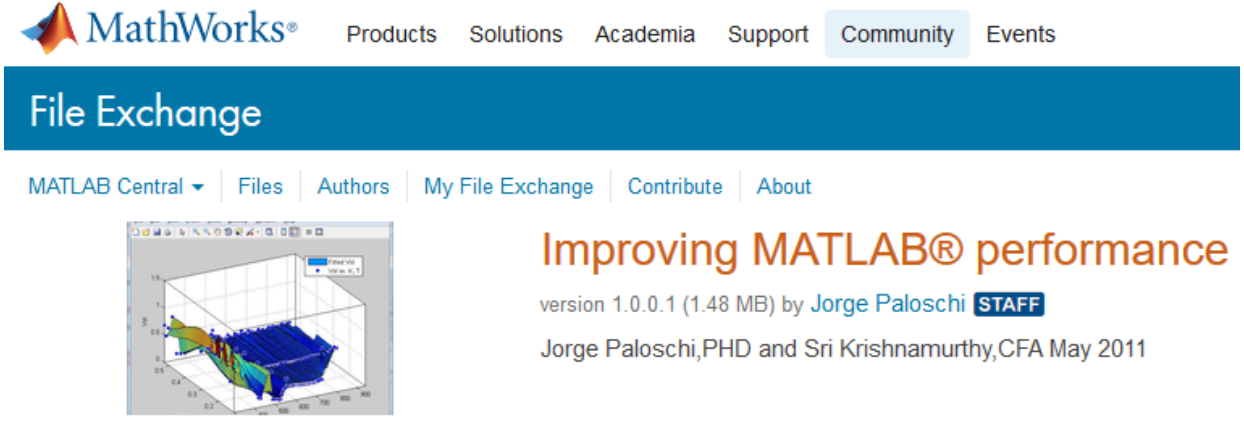
MATLAB Code

```
%Define objective gradient and hessian
grad = diff(obj,x);
hess = jacobian(grad,x);

%Transform symbolics into MATLAB function handles
symGrad = matlabFunction(grad);
symHess = matlabFunction(hess);
```

Example of Use

- References
  - Paloschi J and Krishnamurthy S – Improving MATLAB performance when solving financial optimization problems – Wilmott Magazine – May 2011
  - https://uk.mathworks.com/matlabcentral/fileexchange/33597-improving-matlab-performance-when-solving-financial-optimization-problems



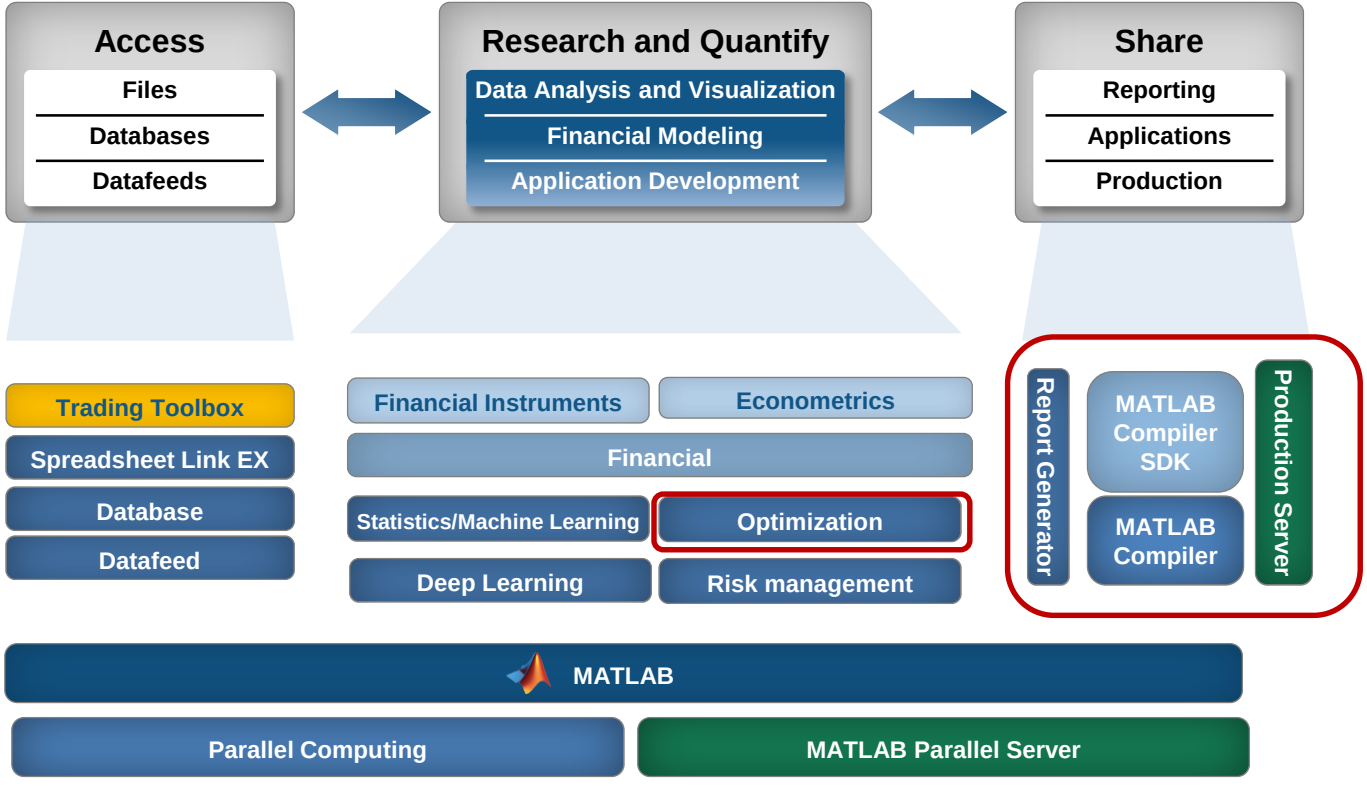
Customized Optimization Models

Supported Portfolio Optimization Models

Financial Toolbox

- **Mean-Variance Portfolio Optimization**
- **Conditional Value-at-Risk Portfolio Optimization**
- **Mean-Absolute Deviation Portfolio Optimization**

MATLAB – The Financial Development Platform

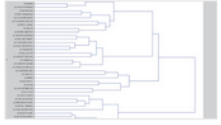
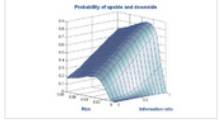
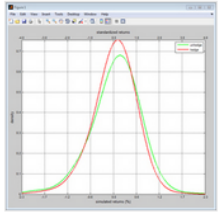
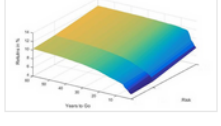


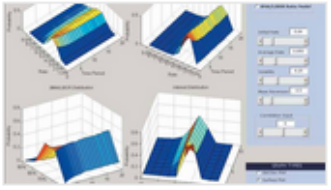
Customized Portfolio Optimization - Deployment

- Compile your MATLAB optimization model for your dedicated platform
- Make it available for your enterprise environment

Customer User Stories Involving Optimization in Finance

- https://uk.mathworks.com/company/user_stories/search.html?q=&fq=product:OP%20marketing-industry:financial-services

	**A2A Develops Comprehensive Risk Management Solution for Energy Markets** A2A mitigated risk by using MATLAB and companion toolboxes to process data, develop risk and pricing models, and deploy an interactive dashboard.	A2A SpA
	**Banque Cantonale Vaudoise Speeds Financial Analysis Tasks** Banque Cantonale Vaudoise modeled the clustering of equity industrial indices using MATLAB and related toolboxes.	Banque Cantonale Vaudoise
	**CAMRADATA Models Dependencies for Quantitative Risk Assessment** CAMRADATA used MATLAB to develop quantitative tools for factor analysis, risk analysis, and defensive asset allocation.	CAMRADATA
	**Fulcrum Asset Management Develops Custom Quantitative Risk Management System** Fulcrum imported financial data from multiple sources, developed sophisticated risk models, and ran optimizations and scenarios analysis.	Fulcrum Asset Management
	**Gas Natural Fenosa Predicts Energy Supply and Demand** Gas Natural Fenosa built models that incorporate historical usage patterns, weather forecasts, production costs, and regulatory rules.	Gas Natural Fenosa
	**ICICI Securities Develops Online Financial Planning and Advisory Platform** ICICI Securities deployed algorithms for determining market correlations, running Monte Carlo simulations, and matching liabilities with assets.	ICICI Securities



Intuitive Analytics Builds Quantitative Tools to Help Bond Issuers Manage Risk
Intuitive Analytics used MATLAB to develop algorithms, visualize results, and simplify deployment of an advanced analytical tool.

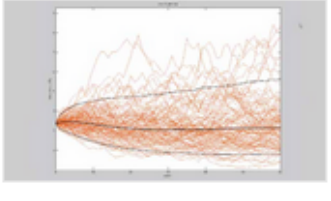
Intuitive Analytics

MATLAB Used to Predict Financial Crises in Emerging Markets
Georgetown University developed a model that applies linear methods and neural networks to analyze trends in currency demand over a selected period.

Georgetown University

Risk Management with MATLAB: Q&A with Intuitive Analytics
Intuitive Analytics used MATLAB build a quantitative tool for reducing expected cost and risk for municipal bond issuers.

Intuitive Analytics



Robeco Develops Quantitative Stock Selection and Portfolio Optimization Models
Robeco used MATLAB and MATLAB Compiler SDK to develop algorithms, build quantitative models, and deploy portfolio construction and management solutions.

Robeco

Sanlam Multi-Manager International Develops Dashboard for Quantitative Risk Analysis

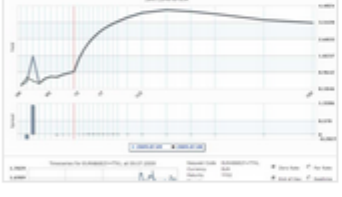
SMMI used MATLAB to model risk inputs, generate optimized portfolios, and develop a dashboard for visualizing results.

Sanlam Multi-Manager International

Trient Develops Financial Analytics Platform to Support Its Investment Team

Trient retrieved and cleaned market data, calculated prices, optimized factor weights, ran Monte Carlo simulations, and developed screening models.

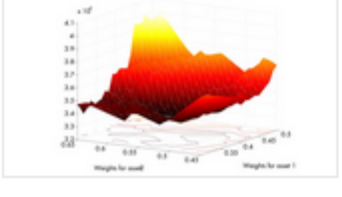
Trient Asset Management



UniCredit Bank Austria Develops and Rapidly Deploys a Consistent, Enterprise-Wide Market Data Engine

UniCredit Bank Austria built and rapidly deployed an enterprise-wide data warehouse to improve risk management operations.

UniCredit Bank Austria



University of Geneva Develops Advanced Portfolio Optimization Techniques

The University of Geneva developed portfolio optimization algorithms, visualized results, and rapidly computed solutions.

University of Geneva

Summary

- Optimization for financial applications is built within MATLAB toolboxes covering many standard applications
- A large variety of optimization algorithms available in MATLAB® Optimization Toolbox™ and Global Optimization Toolbox™
- Customized optimization models made easy by
  - quick modeling (Math to MATLAB)
  - advanced optimization process diagnostics
  - rapid deployment

Thank you !