

**Optimization of Spectrum Sensing Technique in  
Cognitive Radio**

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### 1 Abstract

One of the main problems in wireless communications is the scarcity of radio resources. To overcome spectrum scarcity problem, new devices use the underutilized spectrum in an opportunistic manner, which is the core idea behind the cognitive radio (CR). Cooperative Spectrum sensing (CSS) models are frequently utilized for looking free channels to be used by CR. As major task in CR networks is spectrum sensing and that makes Secondary Users (SUs) to sense Primary User (PU) actions and come to a decision to utilize free channels.

Because of the capability to enhance the detection accuracy, well-known sensing method Cooperative spectrum sensing, which is based on sharing information about channel activities among SUs in the network is widely used. At Fusion Center, certain Hard Decision Fusion (HDF) techniques like logical AND or logical OR may be applied to ascertain the presence of PU. As in CSS, sharing of local spectrum sensing result between the cognitive and the FC is challenging process for which the performance of cooperative detection is decided. The limitations offered by the conventional HDF techniques can be greatly

overcome by advanced optimization techniques which can help the SU to adapt to the prevailing situations.

In this research work, Energy detection technique is used for spectrum sensing and the sensing error is significantly reduced using the Jaya Algorithm. Comparison with other techniques like Teaching Learning Based Optimization (TLBO) further elaborates the fact that Jaya algorithm can be utilized for optimizing CSS problem with less computational complexity. The best spectrum from various available spectrums is selected using Analytic Hierarchy Process (AHP) as well as using combined Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and AHP methods.

## **2 Brief description on the state of the art of the research topic**

In recent times, it has been a tremendous advancement in Wireless Technology. Wireless networks are in high demand due to increasing number of users day by day. The requirement of large bandwidth for high speed data services has increased the demand for additional radio spectrum for wireless technology. The scarcity of radio spectrum has become a challenge for the conventional fixed spectrum assignment policy assigned by Federal Communication commission (FCC).

Cognitive radio (CR) is a new paradigm in the area of wireless communication system for effective utilization of radio frequency (RF) spectrum. Cognitive Radio (CR) is used to maximize the available spectrum utilization. When the primary user (PU) does not access the channel, at that time cognitive radio (CR) will be allowed to use the spectrum to communicate with other CRs. Spectrum sensing is the principal task of cognitive radio through which it accurately determines the licensed user's existence (signal) and identifies the available vacant spectrum.

Jaya algorithm based cooperative spectrum sensing is proposed to reduce probability of error and improve the detection performance of cognitive radio (CR) user. The Jaya optimization process is implemented at the fusion centre to optimize the weigh vector for minimization of global probability of error.

### **2.1. Introduction to Cognitive Radio**

Due to rise in number of users day by day and rapid development in wireless technology, available spectrum is not enough to meet current requirements. The spectrum is not utilized by the licensed users effectively and some of the holes remain vacant because of

conventional fixed spectrum assignment policy assigned by Federal Communication Commission (FCC) as shown in Fig. 1. So, it is indeed necessary that the spectrum should be utilized effectively to meet growing demands from users. So FCC has published a report by designing new spectrum strategies to solve the problem of overcrowded bands and allow secondary users to use licensed bands accordingly.

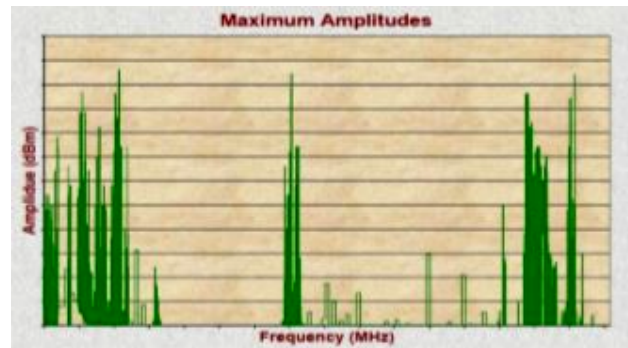


Fig.1 Spectrum Utilization

The usage of spectrum is concentrated on certain portions of spectrum bands whereas considerable portion of spectrum remains unutilized. Hence to improve the effective utilization of spectrum in real time and provide efficient communication the concept of Cognitive Radio technology is introduced. Secondary users use the spectrum when the primary users are not using it in Cognitive Radio (CR). Cognitive radio has the capability of sensing the spectrum in the real time environment.

By changing its various parameters, CR can acquire information from the environment and gets adapted to the environment accordingly. Thus a cognitive radio can sense the spectrum in a better way. Main objective of the cognitive radio is to sense the spectrum, learn from the environment and adapt to the environment. Primary users, has the highest priority for the spectrum usage. Secondary users have to vacate the spectrum as soon as primary users appear. Secondary users can not interfere the operation of the primary users.

The major functions of cognitive radio can then be categorized as [4]:

1. Radio scene analysis: In this function, the unused frequency band is detected.
2. Channel state estimation: The task is concentrated on finding the channel.
3. Spectrum management: The principal aim of this task is effective spectrum sharing of the free channels detected in the spectrum sensing stage.

The main and most key task of the cognitive radio is the procedure of searching used spectrum of primary user (spectrum sensing). Once the white spaces are identified, the cognitive user must select the best available channel that meets Quality-Of-Service (QoS) requirements and its communication (spectrum management). During the occupation of the channel by the CR user if licensed user (i.e. Primary User) want to use this channel, then CR user immediately terminate their transmission and slightly migrate to another unused channel due to a lower priority than the primary user (spectrum mobility). Also, In a CR network, there is some scheduling mechanism to ensure that all CR user get equal opportunities on accessing the spectrum (spectrum sharing).

## 2.2 Basics of Spectrum sensing and Optimization

By the spectrum sensing, the CR user is able to find temporally idle spectrum, which is known as spectrum hole or white space. If the licensed user comes active for communication then CR user has to use another spectrum holes or change its transmission parameter to avoid interference. The spectrum holes [1], [2], [3], [4] theory is illustrated with the help of Fig.2.

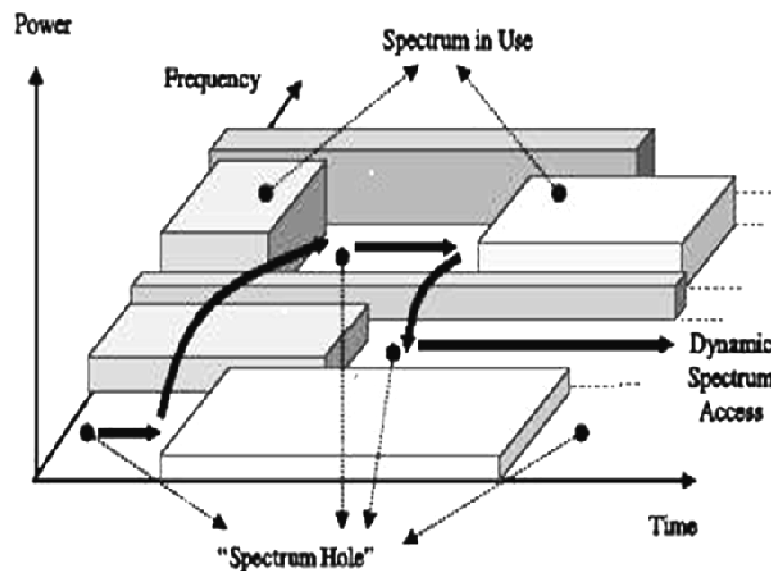


Fig. 2 Illustration of spectrum holes

A basic comparison of the sensing methods is given in Fig. 3 [1]. Waveform-based sensing is more robust than energy detector and cyclostationarity based methods because of the coherent processing that comes from using deterministic signal component. However,

there should be a priori information about the primary user's characteristics and primary users should transmit known patterns or pilots.

The performance of energy detector based sensing is limited when two common assumptions do not hold. The noise may not be stationary and its variance may not be known. Other problems with the energy detector include baseband filter effects and spurious tones. It is stated in literature that cyclostationary-based methods perform worse than energy detector based sensing methods when the noise is stationary. However, in the presence of co-channel or adjacent channel interferers, noise becomes non-stationary. Hence, energy detector based schemes fail while cyclostationarity-based algorithms are not affected. On the other hand, cyclostationary features may be completely lost due to channel fading.

While selecting a sensing method, some tradeoffs should be considered. The characteristics of primary users are the main factor in selecting a method. Cyclostationary features contained in the waveform, existence of regularly transmitted pilots, and timing/frequency characteristics are all important. Other factors include required accuracy, sensing duration requirements, computational complexity, and network requirements.

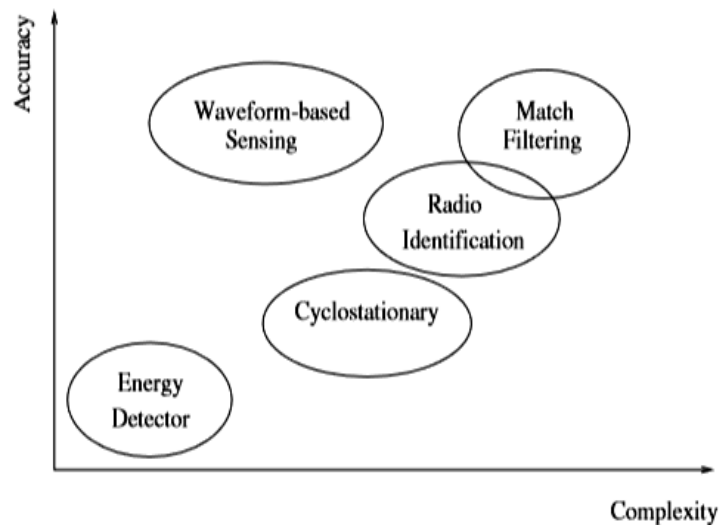


Fig. 3 Main sensing methods in terms of their sensing accuracies and complexities

Many optimization techniques are used to find the optimal solution for performance improvement of spectrum sensing. The optimization techniques optimize the parameters and make them possible as per the required maximum and minimum criterion.

### **3 Definition of the problem**

This research work is carried out to apply the energy detection spectrum sensing technique to find the free spectrum for secondary users. Primary Users have the first priority to use their spectrum and when that spectrum is not in use by Primary Users, Secondary users can utilize it.

But when PU wants that spectrum again, SU has to make that spectrum vacant and it has to switch over to other free spectrum.

So, this research work is carried out to find free spectrum, apply optimization algorithm to find optimal solution for minimum probability of error and to select the best spectrum among available free spectrums by applying optimization algorithm.

### **4 Objective and scope of work**

- To Minimize Probability of Error in spectrum sensing.
- To find lowest probability of error in minimum number of iterations.
- To select the best spectrum among various available spectrums.

### **5 Original contributions by the thesis**

In this thesis, energy detection technique is used for spectrum sensing in cooperative manner. The use of Jaya algorithm as an optimization method is proposed to evaluate optimal weighting coefficient vector of sensing information. Also the best spectrum is selected using Analytic Hierarchy Process (AHP) as well as combined AHP and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) optimization techniques.

The main contributions of this thesis are summarized by following:

1. Energy detection spectrum sensing is used to find free spectrum.
2. Jaya based cooperative spectrum sensing frame work is proposed which optimize thresholds and the weighting coefficients vector of energy level of sensing information so that the total probability of error is minimized.
3. The performance of Jaya based cooperative spectrum sensing is compared with other conventional soft decision fusion schemes like Equal Gain Combining (EGC) as well as hard decision fusion like OR rule.
4. The performance of Jaya algorithm is compared with other advanced optimization technique like Teaching Learning Based Optimization (TLBO) for validation.

5. The selection of the best spectrum is carried out using Analytic Hierarchy Process (AHP) as well as combined AHP and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) optimization techniques.

## 6 Methodology of Research, Results / Comparisons

### 6.1 Energy Detection Technique for Spectrum Sensing

Energy detector based approach, also known as radiometry or periodogram, is the most common way of spectrum sensing because of its low computational and implementation complexities.

It is more generic compared to other techniques as receivers do not need any knowledge of the primary user's signal. The signal is detected by comparing the output of the energy detector with a threshold which depends on the noise floor.

Let us assume that the received signal has the following simple form

$$y(n) = s(n) + w(n)$$

where,  $s(n)$  is the signal to be detected,  $w(n)$  is the additive white Gaussian noise (AWGN) sample, and  $n$  is the sample index.

Note that  $s(n)=0$  when there is no transmission by primary user. The decision metric for the energy detector can be written as

$$M = \sum_{n=0}^N |y(n)|^2$$

where  $N$  is the size of the observation vector. The decision on the occupancy of a band can be obtained by comparing the decision metric  $M$  against a fixed threshold  $\lambda_E$ . This is equivalent to distinguishing between the following two hypotheses:

$$H_0: y(n) = w(n),$$

$$H_1: y(n) = s(n) + w(n)$$

The performance of the detection algorithm can be summarized with two probabilities:

1. Probability of Detection  $P_D$
2. Probability of False Alarm  $P_F$

$$P_D = P_r ( M > \lambda_E \mid H_1 )$$

$$P_F = P_r ( M > \lambda_E \mid H_0 )$$



Fig.4 below shows Receiver Operating Characteristics (ROC) curves for energy detector under different SNR values.

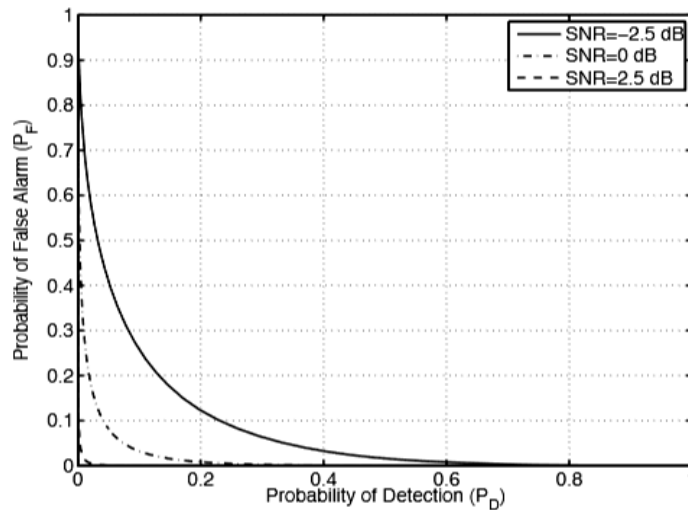


Fig. 4 ROC curves for energy detector based spectrum sensing under different SNR values

## 6.2 Jaya Optimization Algorithm

A simple yet powerful optimization algorithm is proposed in this research work for solving the constrained and unconstrained optimization problems. This algorithm is based on the concept

that the solution obtained for a given problem should move towards the best solution and should avoid the worst solution. This algorithm requires only the common control parameters and does not require any algorithm-specific control parameters.

The Jaya algorithm is found to secure first rank for the ‘best’ and ‘mean’ solutions in the Friedman’s rank test for all the 24 constrained benchmark problems. In addition to solving the constrained benchmark problems, the algorithm is also investigated on 30 unconstrained benchmark problems taken from the literature and the performance of the algorithm is found better [21].

Keeping in view of the success of the Teaching Learning Based Optimization (TLBO) algorithm, this another algorithm-specific parameter-less algorithm is proposed by Dr. Rao [19][20]. However, unlike two phases (i.e. teacher phase and the learner phase) of the TLBO algorithm, the proposed algorithm has only one phase and it is comparatively simpler to

apply. The working of the proposed algorithm is much different from that of the TLBO algorithm.

Let  $f(x)$  is the objective function to be minimized (or maximized). At any iteration  $i$ , assume that there are ‘ $m$ ’ number of design variables (i.e.  $j=1,2,\dots,m$ ), ‘ $n$ ’ number of candidate solutions (i.e. population size,  $k=1,2,\dots,n$ ). Let the best candidate best obtains the best value of  $f(x)$  (i.e.  $f(x)_{\text{best}}$ ) in the entire candidate solutions and the worst candidate worst obtains the worst value of  $f(x)$  (i.e.  $f(x)_{\text{worst}}$ ) in the entire candidate solutions. If  $X_{j,k,i}$  is the value of the  $j^{\text{th}}$  variable for the  $k^{\text{th}}$  candidate during the  $i^{\text{th}}$  iteration, then this value is modified as per the following equation:

$$X'_{j,k,i} = X_{j,k,i} + r_{1,j,i} (X_{j,\text{best},i} - |X_{j,k,i}|) - r_{2,j,i} (X_{j,\text{worst},i} - |X_{j,k,i}|)$$

Where,  $X_{j,\text{best},i}$  is the value of the variable  $j$  for the best candidate and  $X_{j,\text{worst},i}$  is the value of the variable  $j$  for the worst candidate.  $X'_{j,k,i}$  is the updated value of  $X_{j,k,i}$  and  $r_{1,j,i}$  and  $r_{2,j,i}$  are the two random numbers for the  $j$ th variable during the  $i$ th iteration in the range  $[0, 1]$ . The term “ $r_{1,j,i} (X_{j,\text{best},i} - |X_{j,k,i}|)$ ” indicates the tendency of the solution to move closer to the best solution and the term “ $-r_{2,j,i} (X_{j,\text{worst},i} - |X_{j,k,i}|)$ ” indicates the tendency of the solution to avoid the worst solution.  $X'_{j,k,i}$  is accepted if it gives better function value. All the accepted function values at the end of iteration are maintained and these values become the input to the next iteration.

Fig.5 shows the flowchart of the Jaya algorithm [21]. The algorithm always tries to get closer to success (i.e. reaching the best solution) and tries to avoid failure (i.e. moving away from the worst solution). The algorithm strives to become victorious by reaching the best solution and hence it is named as Jaya (a Sanskrit word meaning victory).

### 6.3 Results

The performance of proposed JAYA algorithm is checked by the simulation. Comparison of Probability of Error ( $P_e$ ) versus  $\lambda$  is shown in figure 6. It is compared with the conventional SDF technique EGC and convention HDF technique OR rule. It can be clearly observed that the JAYA algorithm generates the best weighting coefficients vector leading to minimized probability of error for CSS compared to other schemes.

The convergence performance of JAYA algorithm is shown in figure 7 and also compared with the convergence of TLBO Based algorithm. JAYA algorithm perform better

than TLBO and it is so fast for convergence that can ensure to meet real time requirements of cooperative spectrum sensing in cognitive radio.

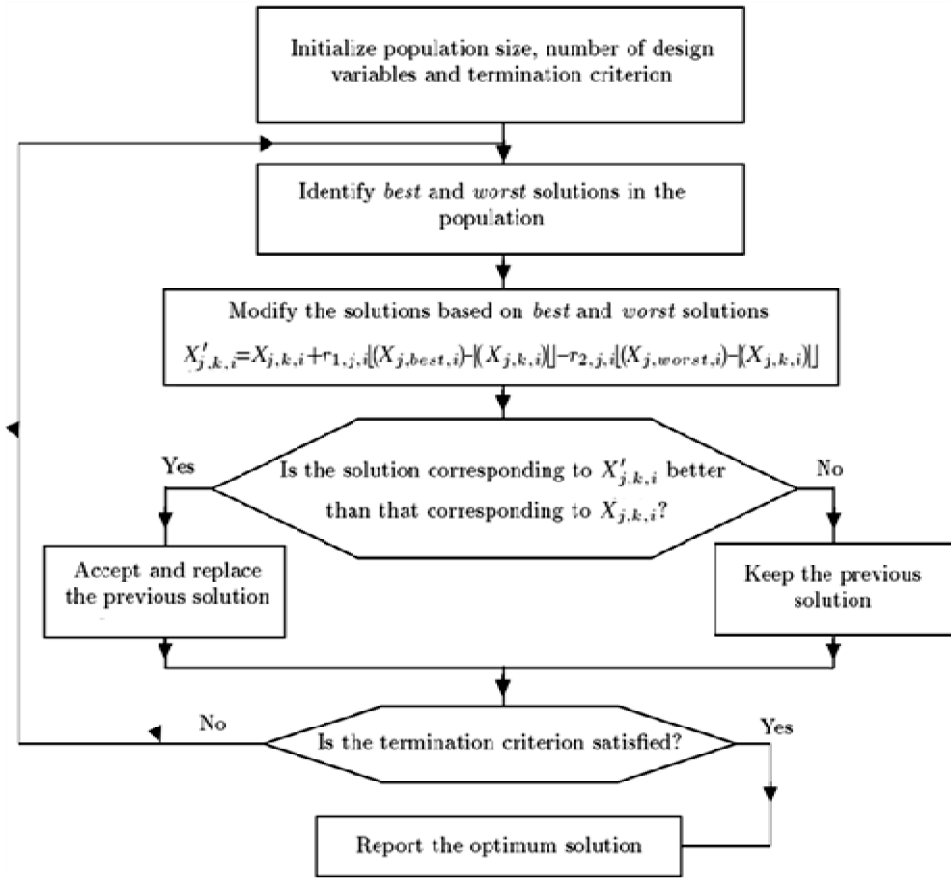


Fig. 5 Flow Chart of JAYA Algorithm

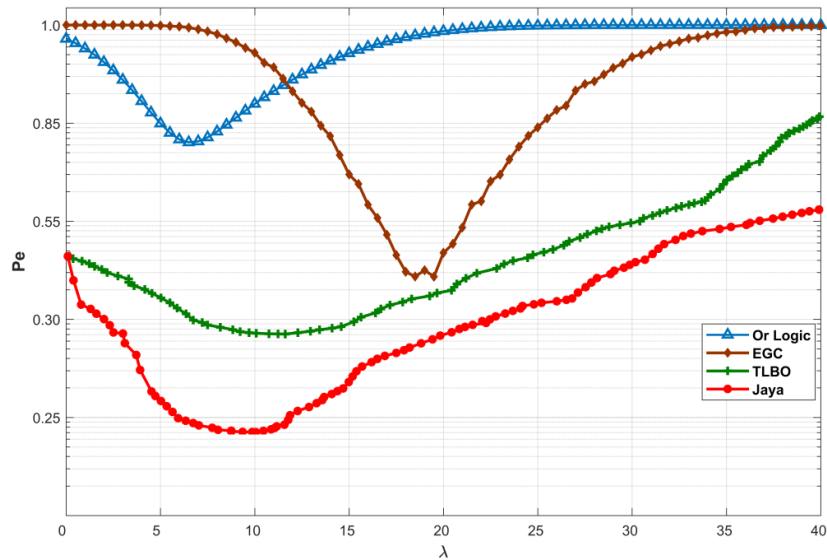


Fig.6 Probability of Error (Pe) versus lambda

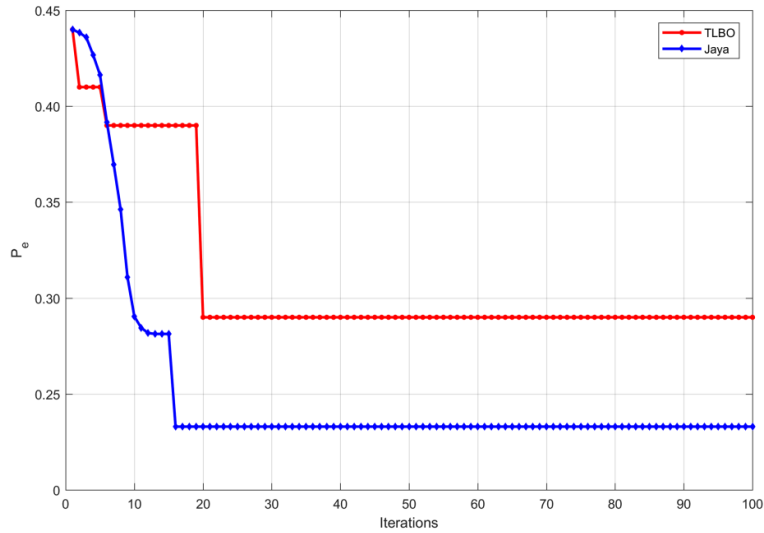


Fig.7 Probability of Error ( $P_e$ ) versus Iterations

A Cognitive Radio Network with a maximum of 8 spectrum holes is assumed that can be opportunistically detected at a specific period of time by the secondary user. The Spectrum Management Center (SMC) is able to communicate with secondary user and exchange the characteristics of the available spectrum for an efficient selection. The spectrum sensing gives the following spectrum binarization result: [0 1 1 0 0 1 1 0].

Table 1 below gives four spectrum characteristics for simulation purpose in terms of available Bandwidth (BW) in MHz, Signal to noise ratio (SNR) in dB, transmission power ( $P_w$ ) in dBm and interference (INT) in dB.

Table I. Spectrum Characteristics Matrix [13]

Spectrum	Spectrum BW (MHz)	SNR (dB)	$P_w$ (dBm)	INT (dB)
1	20	8	38	2.76
2	25	14	33	2.15
3	15	14	32	3.92
4	25	10	38	3.75
5	20	13	32	5.58
6	20	12	29	2.42
7	20	13	28	4.79
8	25	11	35	4.77

With AHP method, priority values are 0.818156, 1.089836, 0.819609, 0.936111, 0.885817, 0.950279, 0.916672 and 0.950112 for 1 to 8 spectrums respectively. From these priority values, ranking of spectrums for secondary users is 2-6-8-4-7-5-3-1.

With combined TOPSIS and AHP method, relative closeness values are 0.4508, 0.9196, 0.3662, 0.6893, 0.4953, 0.5929, 0.5337 and 0.6885 for 1 to 8 spectrums respectively. From these priority values, ranking of spectrums for secondary users is 2-4-8-6-7-5-1-3.

From the above results, it can be seen that the secondary user can select spectrum 2 and 4 for the transmission with spectrum 2 having the most desirable transmission quality with a very good available bandwidth and signal to noise ratio.

## **7 Achievements with respect to objectives**

By applying Energy detection technique for spectrum sensing and Jaya algorithm for optimization, value of probability of error is 0.23 at threshold value of 8 in 15 iterations which is better and fast than other optimization techniques.

## **8 Conclusions**

To meet high demand of spectrum in current era, cognitive Radio plays an important role. Spectrum sensing is the main task of cognitive radio to find the free spectrum for Secondary users. In this research work, energy detection technique is used to find the free spectrum. After finding free spectrum, this research work also work on selecting the best spectrum from available spectrums.

From the simulation results, it is concluded that the proposed method is efficient and stable and it outperforms other algorithm-specific parameter-less algorithm to obtain minimum probability of error with less computational complexity.

## **9 Publications**

1. A. A. Vithalani, Dr. C. H. Vithalani, " Application of combined TOPSIS and AHP method for Spectrum Selection in Cognitive Radio by Channel Characteristic Evaluation" International Journal of Electronics & Communication Engineering (IJECE)" ISSN 0974-2166, Volume 10, Number 2,2017, pp. 71-79.

2. A. A. Vithalani, Dr. C. H. Vithalani, "Optimized Spectrum Selection in Cognitive Radio by Channel Characteristic Evaluation using AHP method" Electronics, Communication and Aerospace Technology (ICECA), 2018 International conference IEEE held at Coimbatore on 29th and 30th March, 2018.
3. A. A. Vithalani, Dr. C. H. Vithalani, "A Survey on Optimization Techniques for Spectrum Sensing in Cognitive Radio" International Journal of Research in Electronics and Computer Engineering (IJRECE) ISSN: 2348-2281, Vol. 7, Issue 1 (January-March 2019), pp. 16-19. (UGC Approved Journal)
4. A. A. Vithalani, Dr. C. H. Vithalani, "Optimization in Cooperative Spectrum Sensing in Cognitive Radio using JAYA Algorithm" Journal of Advanced Research in Dynamical and Control Systems (JARDCS) ISSN: 1943-023X, Vol. 11, 07-Special Issue, 2019, pp. 750-756. (SCOPUS Indexed Journal)

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