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OPTIMIZING HIGHWAY EFFICIENCY IN REAL-TIME FREEWAY DESIGN CAPACITY

Final Report

by

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EXECUTIVE SUMMARY

The estimation of capacity as a parameter to assess traffic flow performance on freeway facilities has received considerable attention in the literature. Despite the general acceptance of the stochastic notion of capacity, limited research has been conducted on how to select a single representative design value from a capacity distribution function. This study reports the results of an empirical comparison between conventional capacity estimates and those obtained by maximizing the Sustained Flow Index (SFI) for 19 U.S. freeway sections. The SFI is defined as the product of the traffic volume and the probability of survival at this volume. The capacity of each cross section was estimated by analyzing the speed-flow relationship and applying methods for stochastic capacity analysis. The results show that the optimum volumes obtained by maximizing the SFI estimated in 5-minute intervals correspond well to the 15 percent probability of breakdown proposed in the HCM 6th edition to estimate the capacity from field data. However, for 15-minute intervals, the optimum volumes obtained in 15-minute intervals correspond to 4 percent probability of breakdown. From these results, it was concluded that maximizing the SFI can be considered a preferred approach to estimate a single, representative value of freeway capacity.

1.0 INTRODUCTION

Capacity is among the most influential parameters used to assess traffic flow performance on freeway facilities. In general, capacity is defined as the maximum volume that can traverse a homogenous freeway section under prevailing traffic and control conditions. In conventional analyses, the capacity of a freeway is treated as a constant value. Thus, operational capacity in this traditional sense can be empirically estimated by identifying the apex volume from a calibrated speed-flow-density model. However, this approach disregards the stochastic nature of capacity. In fact, the apex volume is just one possible estimate for the capacity, because the capacity of a freeway segment can change momentarily as individual driving behaviors also change. If used for capacity analysis, the conventional approach may also be criticized as it does not take into account the breakdown phenomena. Fundamentally, uncongested and congested traffic states represent different capacities due to the capacity drop. Fitting the fundamental diagram through both uncongested and congested observations can bias the capacity estimation results.

In contrast to the conventional perception of capacity, many researchers have reported that freeway facilities reach their capacities at different traffic volumes even under similar prevailing conditions (1, 2, 3, 4, 5). This suggests that capacity is not a fixed value but a random variable from a unique distribution function, subject to the behavioral variability of individual drivers (6). Thus, a capacity distribution function can be estimated by examining traffic flow breakdowns. However, it is important to note that freeways are not designed based on all of the observed capacity values. Rather, a single, representative capacity value is typically used for analysis and freeway design in practice.

Despite the wide acceptance of the stochastic nature of capacity and different methods developed to estimate its distribution function, there has been limited efforts to develop an analytical method to compute a single capacity value from the capacity distribution function. To select a single capacity value, some researchers have superimposed the conventional speed-flow diagrams over the capacity distribution functions. From these plots, average probabilities of breakdown corresponding to apex volumes are commonly selected as benchmarks (7, 8). However, since a capacity estimated using this approach ultimately relies on a calibrated speed-flow model, it is subject to the same limitations of the conventional approach discussed above.

A recent study by Shojaat et al. (9) introduced the Sustained Flow Index (SFI) as a measure of freeway flow performance. This measure is defined as the product of the traffic volume and the probability of “survival”, i.e. the probability that traffic flow does not break down at this volume. A first application based on German freeway data indicated that the optimum volume calculated by maximizing the SFI might be a fair approximation of freeway capacity consistent with the HCM (10). The strength of this approach is that it solely relies on the capacity distribution function representing the risk of a flow breakdown in fluid traffic conditions.

The goal of this paper is to investigate the validity of the optimum volume suggested by Shojaat et al. (9) as reasonable capacity estimate. A thorough comparison between optimum volumes and conventional capacity estimates is conducted based on a large sample of U.S. freeway data. The conventional (deterministic) capacity estimates, capacity distribution functions

and optimum volumes are estimated and compared for 19 U.S. freeway bottleneck sections. In addition, a new procedure is developed to estimate the Wald and Bootstrap confidence intervals for the optimum volumes and determine their stability. The paper also proposes new mathematical derivations which suggests the probability of breakdown at the optimum volume may solely depend on the shape parameter of the capacity distribution function. This finding indicates that probability of breakdown at the capacity may remain constant for roads with different numbers of lanes. A comparison of these methods shows that the optimum volumes obtained by maximizing the SFI are, on average, 1.2 percent different from the conventional capacity estimates. It is also observed that the optimum volumes are stable as their confidence intervals are fairly small. These findings also provide a practical method for estimating the capacity distribution function for sections for which a conventional capacity estimate is already available or for circumstances in which a reliable capacity distribution function cannot be estimated.

The paper starts with a literature review to describe the evolution of capacity estimation methods that have led to this research. Next, a description of the conventional and stochastic methodologies used to estimate capacity values in this research is provided and the procedure to derive the confidence intervals for the optimum volumes is introduced. Then, to demonstrate an application of the experimental method, the proposed methodologies are applied to the selected freeway bottlenecks to investigate similarities between the calculated results. The paper concludes with a summary of the findings and recommendations for future application of this work.

2.0 LITERATURE REVIEW

Capacity is traditionally viewed as the maximum volume that can be sustained by the facility. To estimate the capacity of basic freeway segments, the HCM (10) delivers a set of base capacities dependent upon the free-flow speed. Although these capacity values are found to be fairly representative for US freeways, local conditions may significantly affect the accuracy of the capacity estimates. Thus, many researchers have suggested different mathematical functions to locally determine the relationship between speed, volume, and density. Given the empirical observations of speed, flow, and density, parameters of the pre-determined model are calibrated and the apex volume of the speed-flow diagram is considered as the capacity of the segment. Hence, selecting an appropriate model plays a key role to estimate realistic capacity values.

Van Aerde (11) proposed a mathematical function based on a simple car following model which assumes density to be a function of the current speed and the free-flow speed. Although the suggested model is continuous, the assumption of a linear relationship between speed and density is relaxed and, as a result, it is well capable of describing different traffic states. Van Aerde and Rakha (12) suggested a multivariate calibration of the speed-density scatterplot as an unbiased method when it is not clear which one (of the speed, flow or density) is the dependent and which is the independent variable.

Since the apex volume of the speed-flow diagram is only one possible capacity estimate and freeway facilities may experience saturation at higher or lower volumes, other procedures were developed to estimate the probability of breakdown occurrence as a function of the traffic volume. Van Toorenburg (13) estimated the capacity distribution function based on the analogy

drawn between incomplete lifetime data and freeway capacity. In order to derive the capacity distribution function, observations of both congested and non-congested regimes were included. Brilon et al. (14, 15) used a slightly modified approach and considered only the non-congested observations for the analysis because observations under congested traffic conditions provide no information about the capacity before a breakdown. The researchers also employed different parametric distributions to fit the empirical observations collected from German freeways and found that Weibull distribution provides the best fit to the data.

Geistefeldt (7) applied the method implemented by Brilon et al. to estimate the capacity distribution function and superimposed the Van Aerde speed-flow diagrams over the estimated distribution function for 27 German freeway sections. It was concluded that for 5-minute aggregation intervals, the average probability of breakdown corresponding to the apex volume of the Van Aerde model is nearly 3 percent. Modi et al (8) performed the same analysis and concluded that the peak volume corresponds to nearly 4 percent breakdown probability for U.S freeways. It was also found that for the sections under investigation the capacities provided by the HCM were generally greater than those estimated with other methods.

Shojaat et al. (9) introduced the Sustained Flow Index (SFI) as a stochastic performance measure obtained by multiplying the traffic volume and the probability of survival, which is the complement of the breakdown probability. The volume at which the maximum SFI is reached was referred to as the optimum volume. This optimum volume is the one that provides the best compromise between the probability of breakdown and the unutilized capacity of freeway. As a result, assuming different capacity distribution function types, the breakdown probability corresponding to the optimum volume that maximizes the SFI was introduced as the benchmark to define capacity.

The literature review suggests that in spite of the progress made in the field of capacity analysis, few efforts beyond the research conducted by Shojaat et al. have been made to propose an analytical approach to select a single design value from the capacity distribution function. Thus, this research aims to 1) empirically compare the optimum volumes with capacities obtained in the speed-flow diagram by applying the Van Aerde model to investigate the reasonableness of the estimation results, 2) provide confidence intervals for the parameters of the Weibull distribution as well as the optimum volumes to investigate their variability, and 3) develop a method to transform a conventional capacity estimate into an entire capacity distribution function.

3.0 METHODOLOGY

The research methodology is presented in two parts. The first part discusses the Van Aerde model as a conventional approach to estimate capacity as well as the stochastic approach to estimate a capacity distribution function based on models for censored data. The second part provides a description of the Sustained Flow Index (SFI) as a method to select a single capacity value from the capacity distribution function. This part also provides an explanation of the methods used to determine confidence intervals for the model parameters and the optimum volumes. Subsequently, the results of the application of the presented methodology to data collected from the 19 freeway bottleneck sections are discussed.

3.1 DETERMINISTIC CAPACITY ESTIMATION

To estimate conventional (deterministic) capacities, the Van Aerde model (11), as a function capable of describing the speed-flow-density relationship based on a simple car following equation, was applied in this study. In this model (Equation (1)), the distance headway between consecutive vehicles (h) only depends on the free-flow speed (s_f), the current speed (s), and three parameters (c_1, c_2, c_3). As a continuous traffic flow model, the Van Aerde formula has the capability to accurately estimate the capacity independent of configurations of the congested and non-congested regimes.

$$d = \frac{1}{h} = \frac{1}{c_1 + \frac{c_2}{s_f - s} + c_3 \cdot s} \quad (1)$$

where

- d = density (veh/km)
- h = distance headway between consecutive vehicles (km)
- s_f = free flow speed (km/h)
- c_1 = fixed distance headway parameter (km)
- c_2 = first variable headway parameter (km²/h)
- c_3 = second variable distance headway parameter (h⁻¹)
- s = speed (km/h)

To estimate the model parameters, reasonable starting values for the key traffic flow variables (i.e. capacity, free-flow speed, speed at capacity, and jam density) are assumed and a starting set of parameters (c_1, c_2, c_3, s_f) is calculated. Next, using a non-linear regression in the speed-density-volume plot, an iterative approach is implemented to estimate the model parameters which minimize the sum of squared errors of the model with respect to dependent variable. Thus, the choice of the dependent variable affects the calibration of the parameters and, as a result, the capacity value. As it is not always clear which variable should be chosen as dependent and which as independent, the orthogonal sum of the squared errors can be minimized as an unbiased compromise using multivariate calibration (12). Once the parameters are calibrated, the capacity can be calculated as the apex volume of the speed-flow diagram.

3.2 STOCHASTIC CAPACITY ESTIMATION BASED ON MODELS FOR CENSORED DATA

The estimation of capacity distribution functions is based on the method proposed by Brilon et al. (14, 15). To estimate the capacity distribution function, traffic breakdowns, i.e. the transitions from the uncongested to the congested state, need to be detected. To identify traffic breakdowns, a threshold speed, as the boundary between fluid and congested traffic, is determined by analyzing the speed and flow rate time series. Once the threshold speed is determined, a set of three criteria is applied to detect breakdowns in interval (i) based on 5-minute observations:

- 1- If the average speed in time interval (i) is above the threshold speed, drops below the threshold speed in the next time interval (i+1) and remains below for at least 15 minutes (i.e. three consecutive 5-minute intervals), then interval (i) is considered as uncensored, i.e. the flow rate in interval (i) represents the momentary capacity of the facility.
- 2- If the average speed in time interval (i) is above the threshold speed and remains above the threshold speed in the next time interval (i+1), then interval (i) is considered as censored, i.e. the momentary capacity is greater than the observed flow rate.
- 3- If interval (i) does not satisfy the above conditions, it will not be considered for further analysis.

According to recent applications of the stochastic capacity estimation technique, the capacity distribution function is estimated in 5-minute intervals. In addition, since the HCM (10) defines the pre-breakdown flow rate as “the 15-minute average flow rate immediately prior to the breakdown event”, the capacity distribution is also estimated in 15-minute intervals to receive results comparable with the HCM. However, since only short time intervals are appropriate to detect the speed drops, breakdowns are still detected based on 5-minute intervals (as above), but the average of the three consecutive 5-minute flow rates before a breakdown is considered as the uncensored (pre-breakdown) observation. In the same way, the average flow rates of every three 5-minute intervals between a recovery and the following breakdown are considered as censored observations. If the number of 5-minute intervals between the recovery and the breakdown is not an integer multiplier of three, the first one or two 5-minute intervals following the recovery are disregarded.

Once the pre-breakdown (uncensored) and the other non-congested (censored) observations are determined, both non-parametric and parametric approaches can be used to estimate the capacity distribution functions. Equation (2) shows the transformed form of the Product Limit Method (PLM) that is applied to estimate a non-parametric capacity distribution (14, 15).

$$F_c(q) = 1 - S_c(q) = 1 - \prod_{i:q_i < q} \left(\frac{k_i - d_i}{k_i} \right) \quad (2)$$

where

$F_c(q)$ = capacity distribution function

$S_c(q)$ = capacity survival function

q = traffic volume (veh/h)

q_i = traffic volume in interval i (veh/h)

k_i = number of intervals with traffic volume $q_i \leq q$

d_i = number of breakdowns at volume q_i

The Maximum Likelihood Estimation (MLE) technique is employed to estimate the parametric capacity distribution. Here, an a priori assumption about the capacity distribution type is made and the parameters that maximize the log-likelihood value are selected as the calibrated parameters. Equation (3) shows the log-likelihood function that is applied for capacity analysis (14, 15):

$$\ln(L) = \sum_{i=1}^n \{\delta_i \cdot \ln[f_c(q_i)] + (1 - \delta_i) \cdot \ln[1 - F_c(q_i)]\} \quad (3)$$

where

$f_c(q_i)$ = density function of capacity

$F_c(q_i)$ = cumulative distribution function of capacity

n = number of intervals

δ_i = 1, if the interval i is uncensored

δ_i = 0, if the interval i is censored

The Weibull distribution, as shown in equation (4), was suggested by previous research (8, 14, 15) as the function type that best represents the capacity distribution for freeways:

$$F_c(q) = 1 - e^{-\left(\frac{q}{\beta}\right)^\alpha} \quad (4)$$

where

$F_c(q)$ = capacity distribution function

q = traffic volume (veh/h)

α = shape parameter

β = scale parameter (veh/h)

3.3 SUSTAINED FLOW INDEX

Once the capacity distribution function is estimated, the SFI can be calculated as the product of the traffic volume (q_i) and the probability of survival at this volume ($S_c(q_i)$). The SFI, which represents the “theoretical average volume that is sustained without a traffic breakdown” (10), is given in Equation (5).

$$SFI = q_i \cdot S_c(q_i) = q_i \cdot (1 - F_c(q_i)) \quad (5)$$

where

SFI = sustained flow index (veh/h)

$S_c(q_i)$ = probability of survival at volume q_i

$F_c(q_i)$ = probability of breakdown at volume q_i

q_i = traffic volume in interval i (veh/h)

It is desirable to increase both the probability of survival and the traffic volume for a given freeway section. But, since any increase of volume necessarily leads to a decrease of the survival probability and vice versa, the SFI (as the product of the two) provides a joint performance measure. Thus, the volume that leads to the maximum SFI can be regarded as the best compromise between maximizing the throughput and minimizing the risk of a traffic

breakdown. Assuming a Weibull-type capacity distribution, this optimum volume (q_{opt}) is defined in Equation (6).

$$q_{opt} = \beta \left(\frac{1}{\alpha} \right)^{\frac{1}{\alpha}} \quad (6)$$

The SFI and the optimum volume are estimated in both 5- and 15-minute intervals according to the underlying capacity distribution function $F_c(q_i)$.

According to the statistics theory, if a variable is Weibull distributed, its natural logarithm is Smallest Extreme Value (SEV) distributed. Therefore, the natural logarithm of the p-quantile of the Weibull distribution can be written as (16):

$$\text{Ln}(t_p) = \mu + \sigma(\Phi_{SEV}^{-1}(p)) \quad (7)$$

where

- t_p = p-quantile
- = time at which the proportion p of the population fails
- μ = location parameter = $\text{Ln}(\beta)$
- σ = scale parameter = $1/\alpha$
- p = probability of failure
- $\Phi_{SEV}(z) = 1 - e^{(-e^z)}$ = smallest extreme value distribution

Replacing the p-quantile (t_p) in Equation (7) with the optimum volume (q_{opt}) from Equation (6), the probability of breakdown at the optimum volume (P_{opt}) can be calculated as:

$$P_{opt} = 1 - e^{-\frac{1}{\alpha}} \quad (8)$$

As can be seen in Equation (8), the breakdown probability at the optimum volume depends only on the value of the shape parameter of the Weibull distribution. As the shape parameter increases, the probability of breakdown at the optimum volume decreases. Hence, control strategies such as variable speed limits that increase the shape parameter (i.e. reduce variance of the distribution function, cf. (17)) decrease the probability of breakdown.

3.4 CONFIDENCE INTERVALS

Estimated values for the parameters of the capacity distribution function and the optimum volume strongly depend on the collected data sample in the sense that another set of parameters will be estimated if another sample is collected. Thus, it is important to estimate the confidence intervals for these parameters to address sample to sample variations.

Assuming a Weibull distribution for the capacity of a freeway, Wald confidence intervals can be estimated for both the parameters of the capacity distribution function and the optimum volume. However, if another assumption is made for the capacity distribution function (e.g. normal distribution), the optimum volume may only be derived with numeric calculations and the Wald confidence intervals cannot be easily computed. Thus, this research provides Bootstrap confidence intervals, which are computed numerically and widely used in many statistical analyses, to compare the proximity of the estimation results for future reference in case only the Bootstrap confidence intervals are applicable.

Wald Confidence Intervals

Once the scale parameter of the Weibull-type capacity distribution function is calibrated, its standard error can be easily estimated with the Taylor series approximation and the Wald confidence interval is calculated according to Equation (9). The same method can be applied to estimate the Wald confidence interval for the shape parameter:

$$\hat{\beta} \pm z_{(1-\alpha/2)} \hat{s}\hat{e}_{\hat{\beta}} \quad (9)$$

where

$\hat{\beta}$ = scale parameter (veh/h)

$\hat{s}\hat{e}_{\hat{\beta}}$ = standard error the scale parameter

$z_{(1-\alpha/2)}$ = standard score at $\alpha\%$ significance level

To estimate the Wald confidence interval for the optimum volume, a hand calculation is needed to estimate its standard error. Thus, if the p-quantile (t_p) in Equation (7) is replaced with the optimum volume (q_{opt}) from Equation (6), its variance can be estimated with Equation (10) using the Taylor series approximation.

$$\widehat{Var}_{\widehat{Ln}(q_{opt})} = \left[\frac{\partial \widehat{Ln}(q_{opt})}{\partial \hat{\mu}} \right]^2 \cdot \widehat{Var}(\hat{\mu}) + \left[\frac{\partial \widehat{Ln}(q_{opt})}{\partial \hat{\sigma}} \right]^2 \cdot \widehat{Var}(\hat{\sigma}) + 2 \cdot \frac{\partial \widehat{Ln}(q_{opt})}{\partial \hat{\mu}} \cdot \frac{\partial \widehat{Ln}(q_{opt})}{\partial \hat{\sigma}} \cdot \widehat{Cov}(\hat{\mu}, \hat{\sigma}) \quad (10)$$

Since $\frac{\partial \widehat{Ln}(q_{opt})}{\partial \hat{\mu}} = 1$, $\frac{\partial \widehat{Ln}(q_{opt})}{\partial \hat{\sigma}} = -\widehat{Ln}(\alpha)$, $\widehat{VAR}(\hat{\mu})$, $\widehat{VAR}(\hat{\sigma})$, and $\widehat{COV}(\hat{\mu}, \hat{\sigma})$ are provided by the software, the variance and, as a result, the standard error of the natural logarithm of the optimum volume ($\widehat{Ln}(q_{opt})$) can be calculated. Thus, the confidence interval for the natural logarithm of the optimum volume can be estimated with Equation (11).

$$\widehat{Ln}(q_{opt}) \pm z_{(1-\alpha/2)} \hat{s}\hat{e}_{\widehat{Ln}(q_{opt})} \quad (11)$$

Next, an antilog can be taken from the upper and lower bounds of Equation (11) to estimate the Wald confidence interval for the optimum volume (q_{opt}).

Bootstrap Confidence Intervals

Bootstrap confidence intervals are approximate confidence intervals that are built based on sampling with replacement. In this method, once the sample data (of size n) is collected, B Bootstrap samples of the same size (also called resamples) are taken from the initial sample with replacement. This procedure suggests that in a single Bootstrap sample some observations may be repeated whereas some others may not be selected even once. Thus, new parameters are estimated for each of the Bootstrap samples. If a sufficient number (usually between 2,000 and 5,000) of Bootstrap samples is selected, the confidence intervals delivered for the parameters by this method are quite similar to those delivered by exact methods. Also, Bootstrap confidence intervals have the advantage of relaxing the assumption of the underlying distribution of the data that is used to build the conventional confidence intervals and, as a result, are used in many statistical analyses (16).

4.0 EMPIRICAL RESULTS

To compare the capacity values delivered with the conventional and stochastic methods discussed above, an empirical analysis was performed based on traffic flow data in 5-minute intervals collected from 19 urban freeway bottleneck sections located in California. All sections under study were merge bottlenecks with a considerable share of commuter traffic. To avoid that spillbacks from downstream affect the capacity estimation, only distinct bottlenecks were selected by analyzing long-time speed contour plots. A same sample size of one year was used to estimate both deterministic and stochastic capacities for every section under investigation. With this large sample size, a sufficient number of traffic breakdowns could be observed at each bottleneck section in order to estimate a reliable capacity distribution function. To consider the influence of different prevailing trip purposes on the capacity, only workdays were considered for analysis. Also, to assure that non-recurrent congestions (such as those caused by accidents or incidents) did not affect the estimated capacity values, traffic breakdowns at volumes less than 1,200 veh/h/lane were not considered for analysis (18).

In order to calculate the conventional capacity values, parameters of the Van Aerde model were estimated by applying multivariate calibration. Thus, the SPD_CAL software was used to minimize the orthogonal sum of squared errors of speed, volume, and density using nonlinear regression (19,20). The capacity obtained as the volume at the apex of the Van Aerde model is referred to as c_{VAM} in the following.

To calculate the optimum volumes by maximizing the SFI, both non-parametric and parametric capacity distribution functions were estimated. To calibrate the parametric distribution function, the Weibull distribution was assumed to represent the capacity. Once the parameters were calibrated, optimum volumes and their corresponding breakdown probabilities were calculated according to Equations (6) and (8), respectively. Figure 1 shows the capacity distribution functions and the SFI curves estimated based on 5-minute and 15-minute intervals for a bottleneck section located at Interstate 5. As can be seen, for 5-minute intervals, the freeway section reaches its maximum SFI at a volume of 7,630 veh/h. This optimum volume corresponds to a 4.6 and 15.6 percent probability of breakdown for the distribution functions estimated in 5- and 15-minute intervals, respectively. Based on 15-minute data, an optimum volume of 7,185 veh/h was estimated, which corresponds to a 4.5 percent probability of breakdown for the capacity distribution function estimated in 15-minute intervals.

For better illustration, Figure 2 shows the SFI superimposed over the Van Aerde model for the same four-lane freeway section. Here, the conventional capacity estimated in 5-minute intervals by multivariate calibration of the Van Aerde model is 7,472 veh/h. Once both the conventional capacity values and the optimum volumes are estimated, they can be compared based on the percentage difference between the results delivered by the two methods. For the section shown in Figure 2, the optimum volume is 2 percent greater than the conventional capacity estimate.

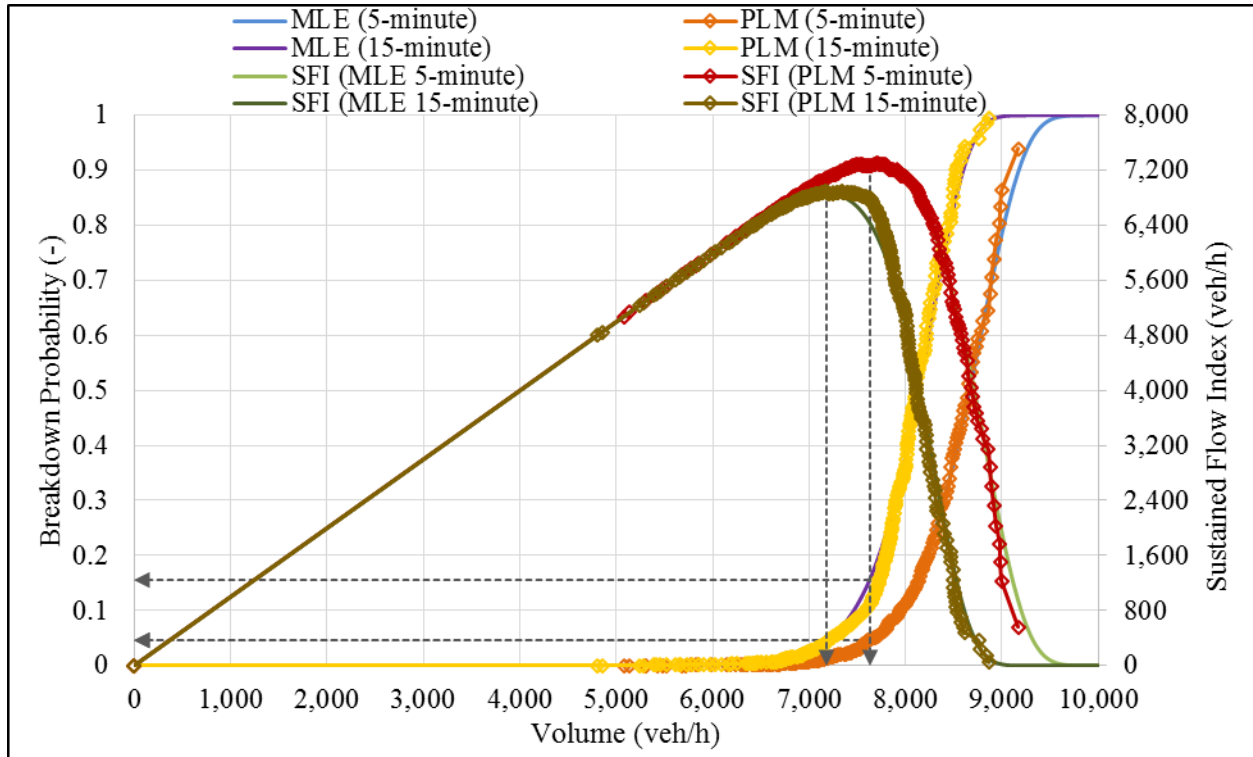


Figure 1: Estimated capacity distribution functions obtained with the Product Limit Method (PLM) and the Maximum Likelihood Estimation technique (MLE) and corresponding Sustained Flow Indexes (SFI) based on 5- and 15-minute intervals for a 4-lane freeway cross section.

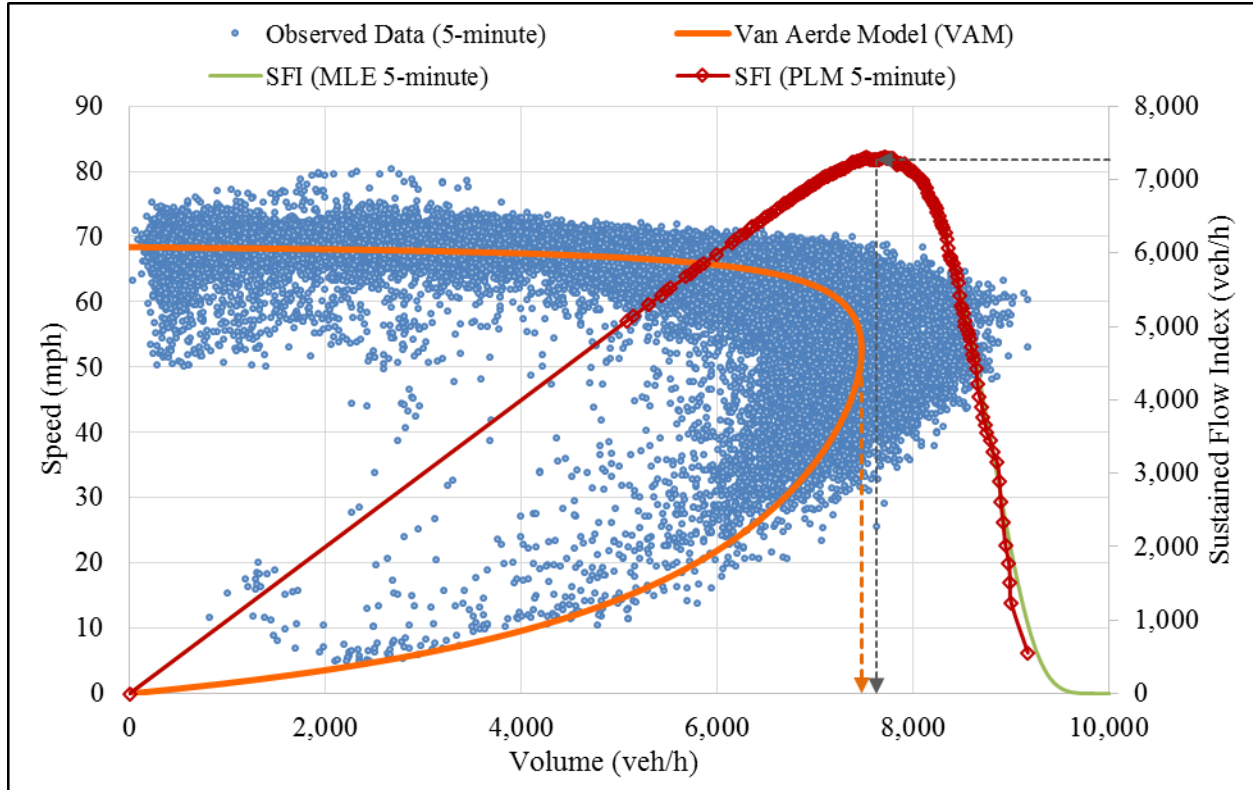


Figure 2: Conventional capacity vs. optimum volume estimated in 5-minute intervals for a 4-lane freeway cross section.

Table 1 shows the results of applying the above procedure for all sections under investigation. As can be seen, the average probability of breakdown corresponding to the 5-minute optimum volume is 4.5 and 16.4 percent for the 5- and 15-minute capacity distribution functions, respectively. The 16.4 percent probability of breakdown found in this research is roughly consistent with the 15 percent probability of breakdown recommended by the HCM 6th edition (10) to select a single value from the 15-minute capacity distribution function. If the optimum volume is estimated based on 15-minute intervals, the average probability of breakdown in 15-minute intervals corresponding to this 15-minute optimum volume is 3.9 percent.

The average probability of breakdown corresponding to the conventional capacity values c_{VAM} estimated with the Van Aerde model is 3.7 percent for the 5-minute capacity distribution function. This percentage is similar to the 4 percent probability of breakdown obtained by Modi et al (8) for U.S. freeways. The results also reveal that the standard deviation of the probability of breakdown at the optimum volume is less than that of the conventional capacity: $stdev(F_{c,5}(q_{opt})) = 0.42 < stdev(F_{c,5}(c_{VAM})) = 1.22$. This indicates that the probability of breakdown at the optimum volume varies less than the probability of breakdown corresponding to the volume at the apex of the Van Aerde curve.

The average percentage difference between the optimum volumes and the capacities estimated with the Van Aerde model is 1.2 percent and the correlation between them is more than 99 percent. These findings suggest that the optimum volumes obtained by maximizing the

SFI are on average within a reasonable range of the conventional capacity estimates and can therefore be considered as suitable capacity estimates.

Table 1: Estimated conventional capacity values and optimum volumes based on 5- and 15-minute intervals as well as their corresponding probabilities of breakdown for the freeway sections under investigation.

Section	Lanes	Capacity		Optimum			Optimum	
		$cv_{AM,5}$ (veh/h)	$F_{c,5}(cv_{AM,5})$	Volume $q_{opt,5}$ (veh/h)	$F_{c,5}(q_{opt,5})$	$F_{c,15}(q_{opt,5})$	Volume $q_{opt,15}$ (veh/h)	$F_{c,15}(q_{opt,15})$
1	2	3,516	2.9	3,611	4.8	20.5	3,363	4.1
2	2	3,942	5.8	3,899	4.7	18.5	3,653	3.9
3	2	3,762	2.3	3,880	4.4	15.3	3,670	3.6
4	2	3,735	2.7	3,815	4.4	15.3	3,611	3.5
5	3	5,574	3.6	5,639	4.6	20	5,253	4.2
6	3	5,363	6.6	5,286	5.1	16.6	4,977	3.9
7	3	5,696	3.4	5,800	4.9	16.8	5,450	4.2
8	3	5,994	4.4	5,990	4.3	14.5	5,678	3.7
9	4	8,306	4.1	8,254	3.4	11.6	7,921	2.7
10	4	8,378	4.2	8,393	4.4	14.4	7,960	3.6
11	4	7,486	4.3	7,494	4.4	12.6	7,138	4.1
12	4	8,416	2.9	8,635	4.8	17.5	8,085	4.5
13	4	7,472	3	7,630	4.6	15.6	7,185	4.5
14	4	7,684	2.5	7,841	4.1	15.8	7,596	3.6
15	4	7,154	2.6	7,273	3.9	14.6	6,897	3.5
16	4	7,826	2.7	8,098	5.1	19.2	7,543	4.6
17	5	9,249	5.5	9,151	4.3	18.4	8,550	4.4
18	5	8,001	2.8	8,155	4.3	21.3	7,591	3.9
19	5	9,995	4.1	10,004	4.2	13.3	9,509	3.9
Average Prob. of Breakdown			3.7		4.5	16.4		3.9

As can be seen in Table 1, the estimation results suggest that there is no evident empirical relationship between the number of lanes and the probability of breakdown at the optimum volume for the distribution functions estimated based on 5- and 15-minute intervals. This seems reasonable since, according to Equation (8), the probability of breakdown at the optimum volume only depends on the shape parameter of the Weibull distribution and previous research did not find a meaningful relationship between the number of lanes and the shape parameter either. Therefore, it can be concluded that the probability of breakdown at the optimum volume for freeways with different number of lanes is nearly constant.

Based on the calibrated parameters of the Weibull distribution for 5-minute intervals, Bootstrap confidence intervals were calculated. For this, 10,000 Bootstrap samples were drawn (with replacement) from the initial sample used to estimate parameters of the distribution function. For each of the Bootstrap samples, a unique set of parameters and, as a result, the optimum volumes were estimated.

After estimating the parameters and the optimum volumes, their confidence intervals at a significance level of 5% were calculated. To estimate the Wald confidence intervals for the estimated parameters and optimum volumes, Equations (9) and (11) were applied respectively. Both Bootstrap-based and Wald confidence intervals estimated at a significance level of 5 percent are shown in Table 2. As can be seen, the results delivered by both methods are very

similar, which means that applying the Bootstrapping technique delivers a suitable approximation of confidence intervals for the optimum volume in case that another function type is assumed for the capacity distribution (whose optimum volume can only be estimated numerically). Also, since the estimated 95% confidence intervals are relatively small, it can be inferred that the optimum volume is a fairly stable indicator of freeway capacity.

Table 2: Estimated Wald and Bootstrap confidence intervals for the parameters and optimum volumes.

Section Number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Shape Parameter		20.2	20.7	22.0	22.5	21.3	19.2	20.1	22.8	28.6	22.2	22.4	20.1	21.1	23.9	25.1	19.0	22.6	22.7	23.1
Wald CI	Lower Bound	19.4	19.6	20.9	21.1	19.2	17.9	18.7	21.4	26.9	20.5	21.5	19.4	20.0	22.2	23.4	18.0	21.9	21.1	21.7
	Upper Bound	21.0	21.8	23.2	24.0	23.5	20.6	21.6	24.4	30.6	24.4	23.4	21.0	22.2	26.0	27.2	20.1	23.4	24.7	24.7
Bootstrap CI	Lower Bound	19.2	19.7	20.5	21.0	19.2	17.9	18.5	21.0	26.0	19.9	21.4	19.1	19.9	22.0	22.8	17.4	21.4	21.0	20.9
	Upper Bound	21.2	21.7	23.7	24.1	23.8	20.8	21.8	24.8	32.3	24.8	23.5	21.2	22.3	26.0	27.9	21.1	23.9	24.8	25.5
Scale Parameter		4,190	4,515	4,465	4,382	6,509	6,164	6,735	6,871	9,281	9,649	8,609	10,023	8,818	8,954	8,269	9,457	10,504	9,357	11,460
Wald CI	Lower Bound	4,165	4,481	4,435	4,349	6,419	6,110	6,661	6,809	9,226	9,548	8,567	9,961	8,753	8,882	8,211	9,361	10,463	9,288	11,381
	Upper Bound	4,216	4,549	4,496	4,415	6,621	6,227	6,811	6,934	9,336	9,751	8,652	10,087	8,883	9,026	8,327	9,554	10,545	9,426	11,539
Bootstrap CI	Lower Bound	4,162	4,482	4,427	4,346	6,414	6,104	6,655	6,796	9,187	9,543	8,564	9,947	8,756	8,881	8,205	9,279	10,447	9,289	11,364
	Upper Bound	4,221	4,550	4,508	4,421	6,626	6,232	6,827	6,953	9,373	9,780	8,657	10,108	8,888	9,039	8,343	9,623	10,573	9,429	11,578
Optimum Volume		3,611	3,899	3,880	3,815	5,639	5,286	5,800	5,990	8,254	8,393	7,494	8,635	7,630	7,841	7,273	8,098	9,151	8,155	10,004
Wald CI	Lower Bound	3,598	3,880	3,861	3,793	5,593	5,252	5,760	5,959	8,211	8,330	7,463	8,598	7,589	7,791	7,223	8,056	9,118	8,082	9,950
	Upper Bound	3,624	3,918	3,898	3,837	5,686	5,321	5,841	6,021	8,298	8,457	7,526	8,673	7,671	7,891	7,324	8,140	9,184	8,228	10,059
Bootstrap CI	Lower Bound	3,598	3,881	3,863	3,797	5,594	5,257	5,761	5,958	8,221	8,338	7,468	8,596	7,594	7,801	7,235	8,024	9,123	8,102	9,962
	Upper Bound	3,625	3,919	3,898	3,835	5,696	5,319	5,846	6,027	8,293	8,448	7,521	8,676	7,669	7,881	7,316	8,175	9,179	8,213	10,050

The formulation of the optimum volume and its correspondence with conventional capacity (i.e. $q_{opt} = \beta(1/\alpha)^{1/\alpha}$ and $c \approx q_{opt}$ thus $c \approx \beta(1/\alpha)^{1/\alpha}$) implies that a relationship exists between the shape and scale parameters of the capacity distribution function for any capacity value. Thus, as shown in Figure 3, different combinations of the shape and the scale parameters may lead to the same capacity value. Since the relationship between the shape and scale parameter is true for any capacity value, a set of indifference curves (one for each capacity value) was created.

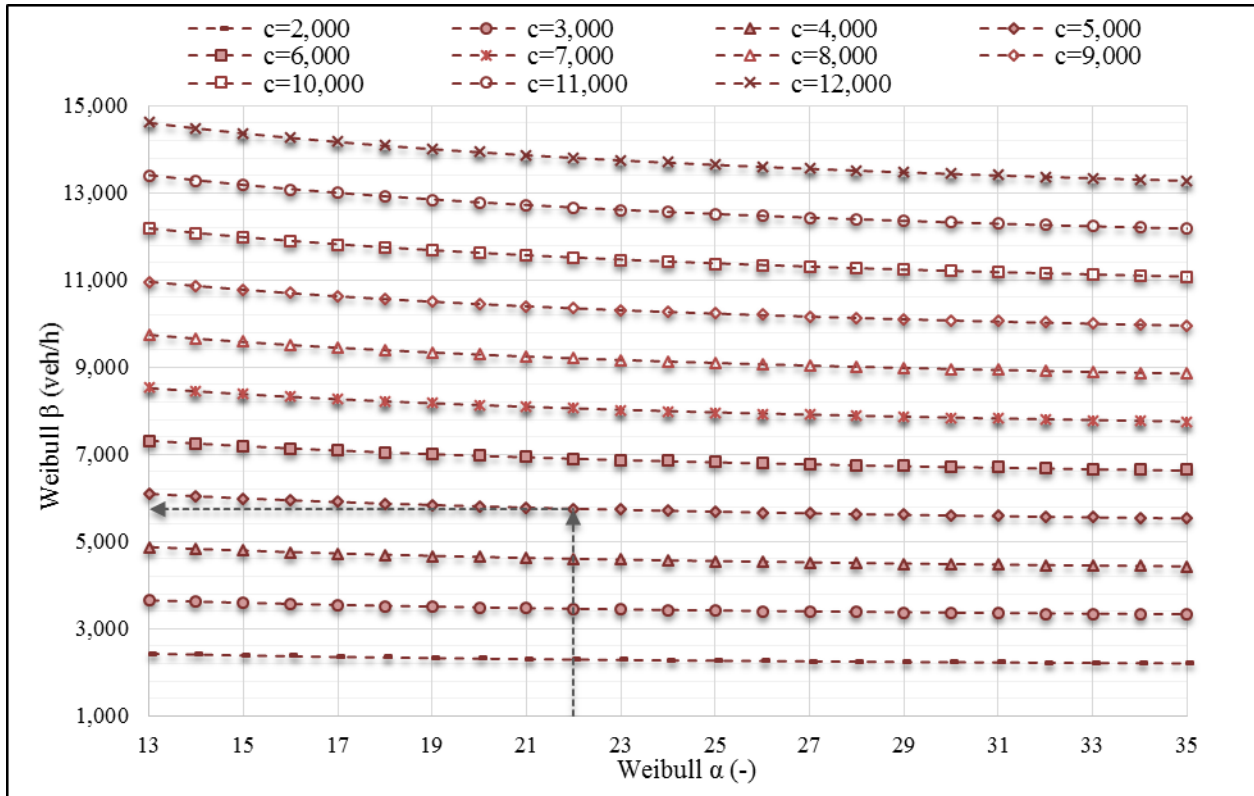


Figure 3 Indifference curves for different capacity values.

Given a conventional capacity value, e.g. the design capacity obtained from the HCM (10), and a reasonable assumption for the shape parameter, the scale parameter can be calculated and the capacity distribution function can be estimated. For standard conditions, practitioners may assume a shape parameter of $\alpha = 22$ (according to the average value given in Table 2). However, given the fact that the shape parameter is proportional to the variance of the capacity distribution function, in case it is believed that the capacity variance is lower or higher than for standard conditions, a smaller or greater shape parameter may be selected. For example, it seems reasonable to assume a greater shape parameter if control strategies such as variable speed limits that reduce the capacity variance (cf. (17)) are implemented in the segment under investigation. This method allows practitioners to estimate a capacity distribution function for segments whose conventional capacity has already been estimated or can be taken from the guidelines in case that an empirical estimation of the capacity distribution function is not feasible.

5.0 CONCLUSIONS

The stochastic variability of freeway capacity has evolved to become a widely accepted concept in traffic engineering. Different methods have been proposed and applied to estimate the capacity distribution function. Despite traffic flow assessment procedures given in guidelines like the HCM (10) are still mostly based on conventional (deterministic) design capacities, only limited research has been carried out on how to select a single, representative, capacity value from the estimated capacity distribution function. A recent study by Shojaat et al. (9) introduced the Sustained Flow Index (SFI) as a new performance measure based on the stochastic notion of the capacity and suggested selecting the volume that maximizes the SFI (referred to as optimum volume) as the capacity of the freeway section. However, so far these values were not compared with conventional capacity estimates to assess their validity. To verify the reasonableness of the results obtained by maximizing the SFI, in this study, optimum volumes were compared with the conventional capacities based on data collected for 19 U.S. freeway bottleneck sections. Despite the application of two completely different methodologies, the obtained average differences between the estimated capacity values are small (nearly 1.2 percent), and a very strong correlation between the capacity estimates exists.

The empirical results revealed that the optimum volume estimated in 5-minute intervals is roughly equivalent to a 5 percent probability of breakdown in 5-minute intervals and a 15 percent probability of breakdown in 15-minute intervals respectively. Hence, the optimum volume obtained by maximizing the SFI in 5-minute intervals well corresponds with the capacity according to the HCM (10), which proposes a 15 percent probability of breakdown for the capacity estimation from field data. Mathematical derivations also yielded that the probability of breakdown at the optimum volume solely depends on the shape parameter of the Weibull-type capacity distribution and hence can be expected to be relatively constant for freeways with different numbers of lanes.

To investigate the stability of the estimated optimum volumes, a new procedure was developed to calculate the 95% confidence intervals with the Wald and Bootstrap methods. The results obtained by the two methods are very similar and suggest that the confidence intervals of the optimum volumes are relatively small. Hence, it can be concluded that the optimum volume is a fairly stable indicator of freeway capacity. Furthermore, based on the correspondence between the optimum volume and the conventional capacity, a practical method to estimate the entire capacity distribution function given a conventional capacity estimate was developed.

Based on the presented findings, it is suggested that the optimum volume obtained by maximizing the SFI may be considered as a preferred approach to select a single capacity value from the capacity distribution function. Compared with the conventional approach of estimating the capacity in the speed-flow diagram, the optimum volume solely relies on uncongested flow observations and therefore best represents the pre-breakdown capacity. The application of the proposed approach is simple as the optimum volume can be calculated from the parameters of the capacity distribution function by a single equation.

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