Order Reduction of Power System Models using Square-Root Balanced Approach

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***Abstract*—This paper deals with the development of square-root balanced truncation algorithm to reduce the order of power system models along with the preservation of interaction relations between study area and the external area. Further, the balanced approach is extended with the singular perturbation approximation technique. For illustration of developed algorithms, external area model of a four machine system is reduced to 3rd order. The resulting reduced order models are guaranteed to preserve the characteristics of original systems.**

Keywords—model order reduction; square-root balanced truncation; singular perturbation approximation; power system models.

I. INTRODUCTION

Higher order systems are usually described by large number of differential or difference equations that lead to state variable or transfer function models. The dimensionality problem of such models is well known and in many situations it is often desirable and sometimes necessary to replace the higher order system by a lower order system for design and analysis purposes. The reduced order model should retain the important and key qualitative properties such as stability, steady state value etc. of the original higher order system.

Moore [1] introduced the balanced realization theory and suggested that input-output behavior of the system is not changed too much if the least controllable and/or least observable part is deleted. This approach is based on simultaneous diagonalization of controllability and observability Gramians. Further, it is shown that all their reduced order models are stable, their solutions are of closed form and they have a priori frequency response error bounds. Later, Pernebo and Silverman [2] presented the proof for several stages of Moore's work [1] and reiterated that input/output behavior of the system does not change after truncation of least controllable and/or least observable states along with extension of the theory for discrete time systems. Fernando and Nicholson [3]-[4] applied balanced realization along with singular perturbation theory [5] for model order reduction and it is later discussed by Liu and Anderson [6] with several properties in both the time and frequency domains. As far as reduction of power system models are concerned, the coherency equivalent approach [7]-[10] and model equivalent approach [11]-[12] are introduced. Later, Chaniotis and pai [13] applied Krylov subspace method [14]-[15] for order reduction of external area of a power system model. Recently, Ghosh and Senroy [16]-[17] used balanced truncation [1] for simplification of power system models. Several other methods [19]-[24] are developed for simplification/ reduction of power system model by various authors.

In this paper, modeling and order reduction of external area system is discussed through square-root balanced truncation and square-root balanced singular perturbation approximation algorithm as an extension of Hung et al's approach [18]. In section II, an interconnected power system network division as study area and external area is included whereas in section III, square-root balanced truncation and square-root balanced singular perturbation approximation algorithms are discussed. In section IV, a four machine power system is considered as example problem and external area model of this system is reduced to 3rd order. It is observed from the step responses and H_∞ norm computation that reduced order model preserves the characteristics of original higher order model.

II. SYSTEM MODELING [18], [25]

For the interconnected power system network, it is possible to divide power network into two areas: a) Study area, with system variables of interest where detailed modeling and analysis of generator and loads are performed; b) External area, which is only of interest in terms of its effects on study area. With this assumption, the external area is transformed into a standard linear time invariant system by expanding the input variables. Further, order reduction of transformed external area is discussed to obtain an equivalent power system network by preserving the interaction relation between study area and external area. The topology of study and external area [16]-[18], [25] of a power system network is shown in Fig. 1 which is partitioned into study and external area by *k* buses, called as partitioned buses.

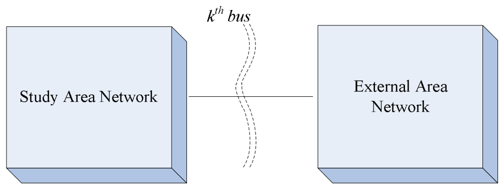

Fig. 1. Division of Power system network

Considering m generators in external area model, each generator may be represented by following set of state equations [18], [25]:

$$\frac{d\delta_{ri}}{dt} = \omega_{ri} - \omega_s \quad (1a)$$

$$\tau'_{dio}\frac{dE'_{qi}}{dt} = \left\{E_{fi} - E'_{qi} - (x_{di} - x'_{di})I_{di}\right\} \quad (1b)$$

$$\tau'_{qio}\frac{dE'_{di}}{dt} = \left\{-E'_{di} + (x_{qi} - x'_{qi})I_{qi}\right\} \quad (1c)$$

$$\frac{\tau_{gi}}{\omega_s}\frac{d\omega_{ri}}{dt} = \left[P_{mgi} - \frac{\xi_{gi}}{\omega_s}(\omega_{ri} - \omega_s) - \{E'_{qi} I_{qi} + E'_{di} I_{di} - (x'_{di} - x'_{qi})I_{di}I_{qi}\}\right] \quad (1d)$$

$$\tau_{Ti}\frac{dP_{mgi}}{dt} = \left(-P_{mgi} + u_{gi}\right) \quad (1e)$$

$$\tau_{Gi}\frac{du_{gi}}{dt} = \left(-u_{gi} + P_{ei}^{ref} - \frac{(\omega_{ri} - \omega_s)}{R_{pi}\omega_s}\right) \quad (1f)$$

$$\tau_{Ei}\frac{dE_{fi}}{dt} = -E_{fi} + K_{ei}\left(V_{ti}^{ref} - \sqrt{V_{di}^2 + V_{qi}^2}\right) \quad (1g)$$

with $i = 1, 2, \dots m$. The (V_x, V_y) and (I_x, I_y) are the voltage and current vectors respectively, the connection relation between network in external area and i^{th} generator is defined by variables $(I_{xi}^E, I_{yi}^E, V_{xi}^E, V_{yi}^E)$ where current vector (I_{xi}^E, I_{yi}^E) describes the effect of power system network on i^{th} generator and voltage vector is used to describe influence on power network from i^{th} generator. Similarly, if the i^{th} bus is partitioned, current vector (I_{xi}^P, I_{yi}^P) describes the effect of study area on external area whereas voltage vector (V_{xi}^P, V_{yi}^P) is used to reflect the influence of external area on the study area.

The stator voltage equations of generator are

$$V_{qi} = E'_{qi} - x'_{di} I_{di} - R_{ai} I_{qi} \quad (2a)$$

$$V_{di} = E'_{di} + x'_{qi} I_{qi} - R_{ai} I_{di} \quad (2b)$$

Further, voltage and current vector are obtained in x-y coordinate axis from the d-q coordinate after transformation as below:

$$\begin{bmatrix} V_{xi} \\ V_{yi} \end{bmatrix} = T\begin{bmatrix} V_{qi} & V_{di} \end{bmatrix}^T \quad (3a)$$

$$\begin{bmatrix} I_{xi} \\ I_{yi} \end{bmatrix} = T\begin{bmatrix} I_{qi} & I_{di} \end{bmatrix}^T \quad (3b)$$

with $T = \begin{bmatrix} \cos\delta_i & \sin\delta_i \\ \sin\delta_i & -\cos\delta_i \end{bmatrix}$.

The network equations for partitioned buses and generator buses of external area [18], [25] are obtained as:

$$\begin{bmatrix} V_{xy} \\ V_{xy}^P \end{bmatrix} = \begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix}\begin{bmatrix} I_{xy} \\ I_{xy}^P \end{bmatrix} \quad (4)$$

where $V_{xy} = (V_{x1}, V_{y1}, \cdots, V_{xm}, V_{ym})^T$, $V_{xy}^P = (V_{x1}^P, V_{y1}^P, \cdots, V_{xm}^P, V_{ym}^P)^T$, $I_{xy} = (I_{x1}, I_{y1}, \cdots, I_{xm}, I_{ym})$, $I_{xy}^P = (I_{x1}^P, I_{y1}^P, \cdots, I_{xm}^P, I_{ym}^P)$ and Z_1, Z_2 Z_3 & Z_4 are block impedance matrices with appropriate dimension.

Further, the output equations of external area are considered as

$$y_{xi} = V_{xi}^P \quad (5a)$$

$$y_{yi} = V_{yi}^P \quad (5b)$$

with $i = 1, 2, \dots k$. After the linearization of Eqs. (1)-(5) around the operating point, the linearized model of external area is obtained such that effects of study area on external area are given by Δw and influences of external area on the study area are included through Δy as below

$$\Delta\dot{x} = A\Delta x + B_u\Delta u^{ref} + F_w\Delta w \quad (6a)$$

$$\Delta y = C\Delta x + D_u\Delta u^{ref} + H_w\Delta w \quad (6b)$$

where $\Delta x = (\Delta\delta_1, \Delta\omega_{g1}, \Delta E'_{q1}, \Delta E'_{d1}, \Delta P_{mg1}, \Delta u_{g1}, \Delta E_{f1}, \cdots, \Delta\delta_m, \Delta\omega_{gm}, \Delta E'_{qm}, \Delta E'_{dm}, \Delta P_{mgm}, \Delta u_{gm}, \Delta E_{fm})$, $\Delta u^{ref} = (\Delta P_{e1}^{ref}, \Delta V_{t1}^{ref}, \cdots \Delta P_{em}^{ref}, \Delta V_{tm}^{ref})$, $\Delta y = (\Delta V_{x1}^P, \Delta V_{y1}^P, \cdots, \Delta V_{xk}^P, \Delta V_{yk}^P)$, $\Delta w = (\Delta I_{x1}^P, I_{y1}^P, \cdots, I_{xk}^P, I_{yk}^P)$ and A, B_u, F_w, C, D_u, H_w are constant matrices.

Transforming the model obtained in Eq. (6) by expanding the input variables as

$$\Delta u = (\Delta u^{ref}, \Delta w) \quad (7)$$

along with $B = (B_u, F_w)$ and $D = (D_u, H_w)$, the linearized model of external area in standard linear time invariant state space model may be expressed as

$$\Delta\dot{x} = A\Delta x + B\Delta u \quad (8a)$$

$$\Delta y = C\Delta x + D\Delta u \quad (8b)$$

which may be reduced to appropriate order using the algorithm discussed in next section.

III. Main Results

In this section, Square-root balanced truncation [26] and square-root balanced singular perturbation approximation algorithms are discussed for order reduction of external area model along with Gramians and balanced realization theory.

A. Preliminaries

Let $P > 0$ and $Q > 0$ be the controllability and observability Gramians [27] for the linearized system given in Eq. (8) is defined respectively as

$$P = \int_0^\infty e^{\tau A} BB^* e^{\tau A^*} d\tau \quad (9a)$$

$$Q = \int_0^\infty e^{\tau A^*} C^* C e^{\tau A} d\tau \qquad (9b)$$

which may be obtained by solving following set of Lyapunov equations

$$AP + PA^* + BB^* = 0 \qquad (10a)$$

$$A^* Q + QA + C^* C = 0 \qquad (10b)$$

A balancing transform is used to convert the original system $\{A, B, C, D\}$ into an equivalent "*internally balanced*" system $\{A_{bal}, B_{bal}, C_{bal}, D_{bal}\}$. A lower order truncated model is then obtained by eliminating the least controllable and/or least observable part of the transformed system.

B. Square-Root Balanced Truncation

Tombs and Postlethwaite [26] presented truncated balanced realization algorithm to compute the reduced order model of a stable state-space system that may be arbitrarily close to being unobservable and/or uncontrollable. Eq. (10a) can be solved for *S*, as a factor of *P* such that

$$P = S^T S \qquad (11a)$$

Similarly, *R* matrix may be obtained such that

$$Q = R^T R \qquad (11b)$$

Now, singular value decomposition of SR^T is obtained as below

$$SR^T = U \Sigma V^T \qquad (12)$$

where Σ is the diagonal matrix of the Hankel singular values of the system obtained in Eq. (8) which have been arranged in order of decreasing magnitude diagonally. Then the square-root balanced realized model is given by

$$G_{sr\_bal} = \left[\begin{array}{c|c} A_{sr\_bal} & B_{sr\_bal} \\ \hline C_{sr\_bal} & D \end{array}\right] = \left[\begin{array}{c|c} TAT^+ & TB \\ \hline CT^+ & D \end{array}\right] \qquad (13)$$

with $T = \Sigma_1^{-1/2} V_1^T R$ and $T^+ = S^T U_1 \Sigma_1^{-1/2}$. The r^{th} order reduced model by square-root balanced truncation is obtained by truncating the weak subsystem from the balanced realized model obtained in Eq. (13) as

$$G_{sr\_bt} = \left[\begin{array}{c|c} A_{sr\_bt} & B_{sr\_bt} \\ \hline C_{sr\_bt} & D \end{array}\right] \qquad (14)$$

where $A_{sr\_bt} \in \Re^{r \times r}$.

C. Square-root Singular Perturbation Approximation

The balanced realized model obtained in Eq. (13) may be represented as

$$G_{sr\_bal}(s) = [C_1 \quad C_2] S_A \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + D \qquad (15)$$

where $S_A = \begin{bmatrix} sI_r - A_{11} & -A_{12} \\ -A_{21} & sI_{n-r} - A_{22} \end{bmatrix}^{-1}$

Now, $G_{sr\_bal}(s)$ additively decomposed as

$$G_{sr\_bal}(s) = G_1(s) + G_2(s) \qquad (16)$$

where

$$G_1(s) = C_{bspa}(s) S_1 B_{bspa}(s) + D_{bspa} \qquad (17)$$

and

$$S_1 = (sI_r - A_{bspa}(s))^{-1} \qquad (18)$$

$$A_{bspa}(s) = A_{11} - A_{12}(sI_{n-r} - A_{22})^{-1} A_{21} \qquad (19a)$$

$$B_{bspa}(s) = B_1 + A_{12}(sI_{n-r} - A_{22})^{-1} B_2 \qquad (19b)$$

$$C_{bspa}(s) = C_1 + C_2(sI_{n-r} - A_{22})^{-1} A_{21} \qquad (19c)$$

$$G_2(s) = C_2(sI_{n-r} - A_{22})^{-1} B_2 \qquad (20)$$

If the subsystem $G_2(s)$ is stable and fast (i.e. it's states have very fast transient dynamics and decay rapidly to certain steady state values) in the neighborhood of a given frequency $s = \sigma_0$ then by ignoring the dynamics of the fast subsystem, the system with transfer function $G_{sr\_bal}(s)$ can be written as

$$G_{bspa}(s) = C_{bspa}(\sigma_0) S_{A\sigma} B_{bspa}(\sigma_0) + D_{bspa}(\sigma_0) \qquad (21)$$

where

$$S_{A\sigma} = [sI_r - A_{bspa}(\sigma_0)]^{-1} \qquad (22)$$

and

$$A_{bspa}(\sigma_0) = A_{11} - A_{12}(\sigma_0 I_{n-r} - A_{22})^{-1} A_{21} \qquad (23a)$$

$$B_{bspa}(\sigma_0) = B_1 + A_{12}(\sigma_0 I_{n-r} - A_{22})^{-1} B_2 \qquad (23b)$$

$$C_{bspa}(\sigma_0) = C_1 + C_2(\sigma_0 I_{n-r} - A_{22})^{-1} A_{21} \qquad (23c)$$

$$D_{bspa}(\sigma_0) = D + C_2(\sigma_0 I_{n-r} - A_{22})^{-1} B_2 \qquad (23d)$$

The required reduced order model by proposed algorithm is obtained by putting $\sigma_0 = 0$,

$$G_{bspa}(s) = \begin{bmatrix} A_{11} - A_{12}A_{22}^{-1}A_{21} & B_1 - A_{12}A_{22}^{-1}B_2 \\ C_1 - C_2A_{22}^{-1}A_{21} & D - C_2A_{22}^{-1}B_2 \end{bmatrix} \qquad (24)$$

Remark 2: At $\sigma_0 = \infty$, the balanced singular perturbation approximation technique corresponds to balanced truncation technique as $A_{bspa}(\sigma_0) \to A_{11}$, $B_{bspa}(\sigma_0) \to B_1$, $C_{bspa}(\sigma_0) \to C_1$ and $D_{bspa}(\sigma_0) \to D$.

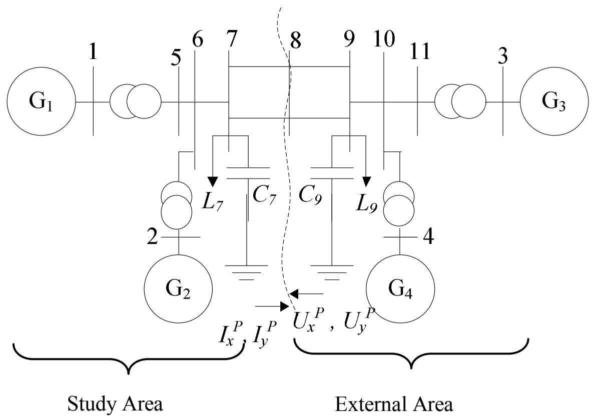

Fig. 2. Two area, Four machine Test system

IV. Simulation Results

In this section, a four machine system is considered as an example problem in which study area and external area are divided by bus 8, shown in Fig. 2. Table I includes the operating points of the system under consideration and system parameter may be referred from [28]. The state space model for external area is obtained as

$$\Delta\dot{x} = A\Delta x + [B_u \quad F_w][\Delta u^{ref} \quad \Delta w]^T \tag{25a}$$

$$\Delta y = C\Delta x + [D_u \quad H_w][\Delta u^{ref} \quad \Delta w]^T \tag{25b}$$

where $\Delta x = \left(\Delta\delta_{r3}, \Delta\omega_{r3}, \Delta E'_{q3}, \Delta E'_{d3}, \Delta P_{mg3}, \Delta u_{g3}, \Delta E_{f3}, \Delta\delta_{r4}, \Delta\omega_{r4}, \Delta E'_{q4}, \Delta E'_{d4}, \Delta P_{mg4}, \Delta u_{g4}, \Delta E_{f4}\right)^T$, $\Delta u^{ref} = \left(\Delta P_{e3}^{ref}, \Delta V_{t3}^{ref}, \Delta P_{e4}^{ref}, \Delta V_{t4}^{ref}\right)$, $\Delta y = \left(\Delta V_x^P, \Delta V_{y1}^P\right)$, $\Delta w = \left(\Delta I_x^P, \Delta I_y^P\right)$ and A, B_u, C, D_u, F_w, H_w matrices are as below

$$A = \begin{bmatrix}
0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
-23.61 & -0.16 & -40.04 & -0.16 & 30.53 & 0.00 & 0.00 & 19.29 & 0.00 & -5.06 & -20.16 & 0.00 & 0.00 & 0.00\\
-0.14 & 0.00 & -0.32 & -0.05 & 0.00 & 0.00 & 0.13 & 0.14 & 0.00 & 0.10 & -0.07 & 0.00 & 0.00 & 0.00\\
0.85 & 0.00 & 0.68 & -4.65 & 0.00 & 0.00 & 0.00 & -0.50 & 0.00 & 1.25 & 1.11 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & -3.33 & 3.33 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & -0.66 & 0.00 & 0.00 & 0.00 & -5.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
932.59 & 0.00 & -11594.85 & -7084.84 & 0.00 & 0.00 & -100.00 & -1826.61 & 0.00 & -5623.95 & -1287.17 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
25.41 & 0.00 & 5.81 & -19.87 & 0.00 & 0.00 & 0.00 & -30.39 & -0.16 & -51.91 & 0.41 & 30.53 & 0.00 & 0.00\\
0.13 & 0.00 & 0.13 & -0.04 & 0.00 & 0.00 & 0.00 & -0.12 & 0.00 & -0.37 & -0.09 & 0.00 & 0.00 & 0.13\\
-1.14 & 0.00 & 0.57 & 1.34 & 0.00 & 0.00 & 0.00 & 1.60 & 0.00 & 1.41 & -5.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -3.33 & 3.33 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -0.66 & 0.00 & 0.00 & 0.00 & -5.00 & 0.00\\
93.25 & 0.00 & -4553.03 & -2531.58 & 0.00 & 0.00 & 0.00 & -1425.21 & 0.00 & -12553.75 & -5288.23 & 0.00 & 0.00 & -100.00
\end{bmatrix}$$

TABLE I. Operating Points of Test System at Base 900 MW [18]

Unit	δ_0	E'_{q0}	E'_{d0}	V_{q0}	V_{d0}	I_{q0}	I_{d0}	P_{m0}
G1	1.1062	0.94781	0.47927	0.7492	0.7069	0.4167	0.6586	0.77778
G2	0.9231	0.95439	0.46357	0.7436	0.6835	0.4031	0.6994	0.77778
G3	0.6576	0.93787	0.48931	0.7349	0.7216	0.4255	0.673	0.79832
G4	709	0.93255	0.47527	0.7272	0.7009	0.4132	0.6809	0.77778

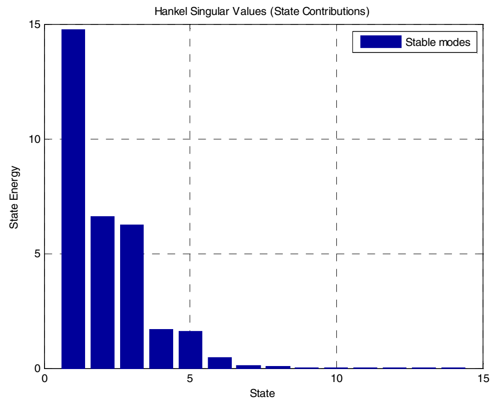

Fig. 3. Bar Chart of Hankel Singular values for external area model

$$B_u^T = \begin{bmatrix}
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 5.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 20000.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 5.00 & 0.00\\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 20000.00
\end{bmatrix}$$

$$F_w^T = \begin{bmatrix}
0.00 & 9.90 & 0.07 & -0.28 & 0.00 & 0.00 & -823.41 & 0.00 & 15.04 & 0.10 & -0.52 & 0.00 & 0.00 & -834.52\\
0.00 & 10.43 & 0.00 & -0.92 & 0.00 & 0.00 & 2573.40 & 0.00 & 11.50 & -0.03 & -1.19 & 0.00 & 0.00 & 3778.46
\end{bmatrix}$$

$$C = \begin{bmatrix}
0.22 & 0.00 & 0.35 & -0.01 & 0.00 & 0.00 & 0.00 & 0.38 & 0.00 & 0.50 & -0.09 & 0.00 & 0.00 & 0.00\\
0.36 & 0.00 & -0.04 & -0.35 & 0.00 & 0.00 & 0.00 & 0.36 & 0.00 & -0.21 & -0.44 & 0.00 & 0.00 & 0.00
\end{bmatrix}$$

$$H_w = \begin{bmatrix} 0.26 & -0.73\\ 0.78 & 0.34 \end{bmatrix} \text{ and } D_u = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00\\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}$$

The Hankel singular values (HSVs) for the above model are obtained as

$$HSV = [14.7441 \;\; 6.5870 \;\; 6.2303 \;\; 1.6806 \;\; 1.5866 \;\; 0.4433 \;\; 0.1237 \;\; 0.0769 \;\; 0.0381 \;\; 0.0116 \;\; 0.0083 \;\; 0.0039 \;\; 0.0022 \;\; 0.0002]$$

Further, the bar chart of HSVs is plotted in Fig. 2 from which it is clear that only first few HSVs are significant and these should be preserved in the reduced order model. For the purpose of comparison with [18], the above model is reduced to 3rd order. The transformation matrices are obtained as

$$T^T = \begin{bmatrix}
0.1235 & 0.0369 & -0.0442 & -0.0145 & 0.2680 & 0.1539 & -0.0001 & 0.1274 & 0.0344 & -0.0966 & -0.0201 & 0.2501 & 0.1444 & -0.0001\\
-0.0048 & -0.0159 & 0.0210 & -0.0455 & -0.3189 & -0.2269 & 0.0000 & -0.0157 & -0.0178 & 0.0630 & -0.0543 & -0.3063 & -0.2113 & 0.0001\\
-0.0417 & 0.0291 & -0.0449 & 0.0037 & 0.0682 & -0.0543 & -0.0001 & -0.0286 & 0.0255 & -0.0700 & -0.0188 & 0.0412 & -0.0638 & -0.0001
\end{bmatrix}$$

$$T^{+T} = \begin{bmatrix} 3.2839 & 3.0731 & -0.1490 & 0.0052 & -0.1542 & -0.3106 & -0.9753 \\ 2.9153 & 3.3801 & -0.0811 & 0.0719 & -1.0741 & -1.2572 & 0.5693 \\ -2.8341 & 13.6049 & 0.1177 & -0.3133 & 0.1469 & -0.8511 & -4.9387 \end{bmatrix}$$

$$\begin{bmatrix} 3.7411 & 3.3165 & -0.6753 & 0.1946 & -0.1710 & -0.3173 & -3.0044 \\ 2.4021 & 2.9466 & 0.1074 & 0.0111 & -0.9932 & -1.0594 & 9.7156 \\ -2.6598 & 11.6521 & 0.0350 & -0.3320 & 0.0147 & -0.9026 & -7.1332 \end{bmatrix}$$

Finally, the 3rd order model by Square-root balanced truncation algorithm as discussed in section III, is obtained as

$$G_{srbt} = \left[\begin{array}{c|cc} \mathrm{A_{srbt}} & \mathrm{B_{u\_srbt}} & \mathrm{F_{w\_srbt}} \\ \hline \mathrm{C_{srbt}} & \mathrm{D_{u\_srbt}} & \mathrm{H_{w\_srbt}} \end{array}\right]$$

where

$$A_{srbt} = \begin{bmatrix} -0.3297 & -0.9088 & 0.8081 \\ 0.8851 & -0.4530 & 3.4050 \\ -0.7617 & -2.6903 & -0.4323 \end{bmatrix}$$

$$B_{u\_srbt} = \begin{bmatrix} 0.7695 & -1.1382 & 0.7222 & -2.4896 \\ -1.1345 & 0.5941 & -1.0563 & 1.7232 \\ -0.2715 & -1.1614 & -0.3188 & -1.7937 \end{bmatrix}, F_{w\_srbt} = \begin{bmatrix} 1.0353 & 0.2040 \\ -0.4730 & 0.1359 \\ 0.7926 & 0.1294 \end{bmatrix},$$

$$C_{srbt} = \begin{bmatrix} 1.7367 & 1.5778 & -1.5425 \\ 2.5893 & 1.8649 & -1.7342 \end{bmatrix}, \; D_{u\_srbt} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H_{w\_srbt} = \begin{bmatrix} 0.2600 & -0.7300 \\ 0.7800 & 0.3400 \end{bmatrix},$$

and the 3rd order reduced model by square-root balanced singular perturbation approximation algorithm is given by

$$G_{srbt} = \left[\begin{array}{c|cc} \mathrm{A_{srbt}} & \mathrm{B_{u\_srbt}} & \mathrm{F_{w\_srbt}} \\ \hline \mathrm{C_{srbt}} & \mathrm{D_{u\_srbt}} & \mathrm{H_{w\_srbt}} \end{array}\right]$$

where

$$A_{srbspa} = \begin{bmatrix} -0.331 & -0.9077 & 0.8058 \\ 0.8807 & -0.447 & 3.396 \\ -0.7566 & -2.697 & -0.4224 \end{bmatrix}$$

$$B_{u\_srbspa} = \begin{bmatrix} 0.7746 & -1.205 & 0.7341 & -2.463 \\ -1.126 & 0.4727 & -1.037 & 1.758 \\ -0.28 & -1.038 & -0.3385 & -1.826 \end{bmatrix}, F_{w\_srbspa} = \begin{bmatrix} 1.034 & 0.1815 \\ -0.4743 & 0.09396 \\ 0.7938 & 0.172 \end{bmatrix},$$

$$C_{sr\_bspa} = \begin{bmatrix} 1.842 & 1.425 & -1.351 \\ 2.523 & 1.965 & -1.854 \end{bmatrix}, H_{w\_srbspa} = \begin{bmatrix} 0.2502 & -0.5303 \\ 0.7921 & 0.3294 \end{bmatrix},$$

$$D_{u\_srbspa} = \begin{bmatrix} -0.008125 & 0.4013 & -0.0117 & 0.4398 \\ -0.02364 & 0.09802 & -0.05993 & -0.4569 \end{bmatrix},$$

Step responses for original and reduced order models by square-root balanced truncation (SR-BT) and square-root balanced singular perturbation approximation (SR-BSPA) are plotted in Fig. 3 from which it is clear that both the developed algorithms preserve the characteristics of original system. Frequency domain computations for the reduced order models are listed in Table II. It is observed that relative error bound for the SR-BT and SR-BSPA is less than the Hung et al's approach [18].

V. CONCLUSIONS

In this paper order reduction algorithm based on square-root balanced truncation and square-root balanced singular perturbation approximation are discussed. For the verification of the algorithms, a four machine power system network is considered with the division in study area and external area at 8th bus by maintaining the interaction relations between study area and external area. The external area of this model is reduced to 3rd order and simulation results indicate that reduced order model can approximate the original system in well manner.

REFERENCES

[1] B.C. Moore, "Principal component analysis in linear systems: controllability, observability and model reduction", *IEEE Trans. Automat. Contr.*, vol. 26, pp. 17–32, 1981.

[2] L. Pernebo and L.M. Silverman, "Model reduction via balanced state space representations", *IEEE Trans Automat. Contr.*, vol. 27, no. 2, pp. 382–387, 1982.

[3] K.V. Fernando, H. Nicholson, "Singular perturbational model reduction of balanced systems", *IEEE Trans Automat. Contr*, vol. 27, pp. 466–468, 1982.

[4] K.V. Fernando, H. Nicholson, "Singular perturbation approximation for discrete-time balanced systems", *IEEE Trans Automat. Contr*, vol. 28, pp. 240–242, 1983.

[5] P.V. Kokotovic, R.E. O'Malley and P. Sannuti, "Singular perturbations and order reduction in control theory-an overview, *Automatica,* vol. 12, pp. 123-132, 1976

[6] Y. Liu and B.D.O. Anderson, "Singular perturbation approximation of balanced systems", *Int. J. Contr.*, vol. 50, no. 4, pp. 1339-1405, 1989.

[7] R. Podmore, "Identification of coherent generators for dynamic equivalents," *IEEE Trans. on Power Systems*, vol. PAS-97, no. 4, pp. 1344-1354, 1978.

[8] S. B. Yusof, G. J. Rogers, and R. T. H. Alden, "Slow coherency based network partitioning including load buses," *IEEE Trans. on Power Systems*, vol. 8, no. 3, pp. 1375−1381, 1993.

[9] M. L. Ourari, L. A. Dessaint, and V. Q. Do, "Dynamic equivalent modeling of large power systems using structure preservation technique," *IEEE Trans. on Power Systems*, vol. 21, no. 3, pp. 1284-1295, 2006.

[10] L. Mariotto, H. Pinheiro, G. Cardoso Jr, A. P. Morais, and M. R. Muraro, "Power systems transient stability indices: an algorithm based on equivalent clusters of coherent generators," *IET Generation, Transmission & Distribution*, vol. 5, no. 11, pp. 1223-1235, 2010.

[11] L.Wang, M. Klein, S. Yirga, and P. Kundur, "Dynamic reduction of large power systems for stability studies," *IEEE Trans. on Power Systems*, vol. 12, no. 2, pp. 889-895, 1997.

[12] H. Kim, G. Jang, and K. Song, "Dynamic reduction of the large-scale power systems using relation factor," *IEEE Trans. on Power Systems*, vol. 19, no. 3, pp. 1696-1699, 2004.

[13] D. Chaniotis and M. A. Pai, "Model reduction in power systems using Krylov subspace methods," *IEEE Trans. on Power Systems*, vol. 20, no. 2, pp. 888-894, 2005.

[14] R.W. Freund, "Model reduction methods based on Krylov subspaces", *Acta Numerica*, vol. 12, pp.267-319, 2003

[15] E.J. Grimme, "Krylov projection methods for model reduction", *Ph.D. Thesis*, Dept. of Electrical Engg., University of Illinois at Urbana Champaign, 1997

[16] S. Ghosh and N. Senroy, "Balanced truncation approach to power system model reduction", *Electric Power Components and Systems*, vol. 41, no. 8, pp. 747-764, 2013

[17] S. Ghosh and N. Senroy, "A comparative study of two model order reduction approaches for application in power systems," *IEEE Power and Energy Society General Meeting*, pp.1-8, 2012

[18] C. Huang, K. Zhang, X. Dai and W. Wang, "Model reduction of power systems based on the balanced residualization method," *IEEE Power and Energy Society General Meeting*, pp.1-7, 2012

[19] N. Nihei and T. Oyama "A study on decomposition and model reduction for wide area power system stability assessment" *IEEE Power Engineering Society*, pp. 651-654,1999.

[20] S. K. Joo, C. C. Liu, L. E. Jones, and J. W. Choe "Coherency and aggregation techniques incorporating rotor and voltage dynamics" *IEEE Trans. on Power Systems.*, vol. 19, no. 2, pp.1068-1075, 2004

[21] P. A. Parrilo et al, "Model reduction for analysis of cascading failures in power systems" in *Proc. American Control Conference*,vol. 6, pp. 4208-4212, 1999.

[22] J. J. Sanchez-Gasca, J. H. Chow "Power system reduction to simplify the design of damping controllers for interarea oscillations" *IEEE Trans. on Power Systems.*, vol. 11, no. 3, pp. 1342-1349, Aug. 1996.

[23] J. H. Chow, R. Galarza, P. Accari and W. W. Price, "Inertial and slow coherency aggregation algorithms for power system dynamic model Reduction," *IEEE Trans. on Power Systems.*, vol. 10, no. 2, pp. 680-685, 1995

[24] J. H. Chow "A nonlinear model reduction formulation for power system slow coherency and aggregation," in *Proc. Workshop on Advances in Control and its Applications*, vol. 208, pp. 282-298, 1996

[25] G. H. Shakouri and R. H. Reza, "Identification of a continuous time nonlinear state space model for the external power system dynamic equivalent by neural networks," *Int. Journal of Electrical Power & Energy Systems*, vol. 31, no. 7-8, pp.334-344, 2009

[26] M.S. tombs, I. Postlethwaite, Truncated balanced realization of a stable non-minimal state space system, *Int. J. Contr.,* vol. 46, pp. 1319-1330, 1987.

[27] K. Glover, "All optimal Hankel norm approximation of linear multivariable systems and their L_∞ error bounds", *Int. J. Contr.,* vol. 39, pp. 1115–1193, 1984

[28] P. Kundur, "Power system stability and control," New York: *McGraw-Hill Inc.*, pp. 813-814, 1994.

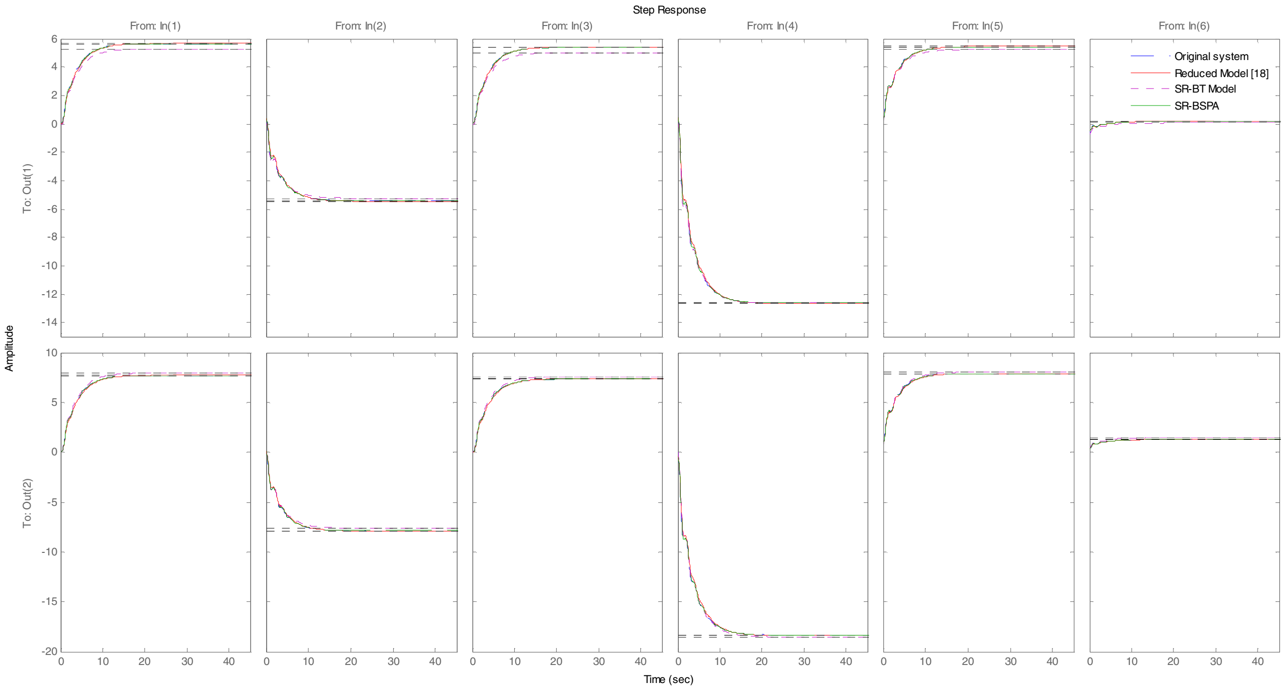

Fig. 4. Step responses for original external area model and reduced order models

TABLE II. FREQUNECY DOMAIN COMPUTATIONS

S.N.	**Frequency domain computations** $\|G(z)\|_\infty = 29.2331$	**Algorithm**	**Values**
1.	H_∞ norm	1. SR-BT 2. SR-BSPA 3.Reduced Model [18]	29.2654 29.2331 29.3313
2.	Actual H_∞ norm error bound in modeling $= \|G_r(z) - G(z)\|_\infty$	1. SR-BT 2. SR-BSPA 3.Reduced Model [18]	3.3220 3.2630 3.2472
3.	Actual relative error bound $= \frac{Actual\ error\ norm}{\|G(z)\|_\infty}$	1. SR-BT 2. SR-BSPA 3.Reduced Model [18]	0.1136 0.1116 0.1111