Origami-Fun and Mathematics

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ORIGAMI FUN & MATHEMATICS

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PREFACE

Thank you for opening this book.

Many a time people sweat to have sweet fun. This book is an attempt to find an easy way out.

Suppose, I give you a blank sheet of paper. What would you do with it?

You may write something on it. Or else, you may still do something with the paper, A fold here, a crease there you may produce a boat, a box, a flower....

I am sure you will do something. Because Origami, the art of paper folding has already become a part of folklore, in our country.

Nobody teaches a child how to make a boat. She/he learns at school from other children.

To convert a paper into a recognizable shape is a challenge to the intellect. It is more challenging to unfold the same model and reconstruct it.

This challenge has an inherent beauty, of tracing steps backwards, of symmetry, and also mathematics.

We shall visit this challenge in our book.

A fad, a hobby called origami can also condense into a candid lesson in mathematics. This book is really a manual book preparing learning aids in mathematics.

Each page in this book gives an experiment to be conducted in Mathematics Lab.

V.S.S. SASTRY

CONTENTS

1. Fundamentals of Origami

Making a boat, a swan, a box, a tea-coaster is no doubt fun.

It's more fun to discover an angle, discover relationship between numbers, all inside a Origami models made from paper and learn mathematical facts, without a scale, a protractor or a compass or a divider.

Take a piece of paper; make an Origami model with it, then unfold and Lo! There is Maths inside.

This book is an adventure. It is divided into Basics, Beginnings, Adventure, More adventure, etc. This book is for a student of Mathematics, it is also for an Origami enthusiast. But not for Maths enthusiast. Because we do not have any thing here, other than that is available in the curriculum. Central syllabus is followed.

That means all maths we illustrate here is only up to class 10 (SSC) level only. In other words it is a manual for Mathematics Lab.

What is Origami?

Origami is an art of Paper folding. In Japanese language Ori = to fold, and Garni = Paper. There are many shapes that can be folded from a square sheet of paper.

Origami, developed not only in Japan but in China and Spain too. In Spain the Moors (Moslems of Arab Origin) taught geometrical patterns to pupils through paper folding. Folding facilitates symmetrical operations. Now origami is taught in many schools and many people are aware of some or other form of paper-folding.

Which paper you require?

Generally Stationery shops sell Origami papers, which are thin sheets of paper coloured on one side, that are squares of different colours, stacked together in packets.

But for models so can be the described in this book, ordinary paper will suffice. Even computer stationery, printed on one side can be used. Discarded photocopy paper can also be used. Origami models for Maths purpose need not be from a costly paper. A4 size (28x21 cm) paper is preferred to infuse some kind of standard measure in paper-folding.

What happens when you fold?

Take a square paper. Mark ABCD on the edges on either side.

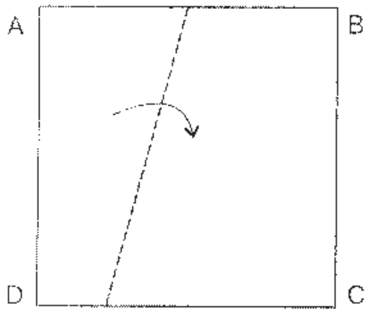


Fig. 1.1

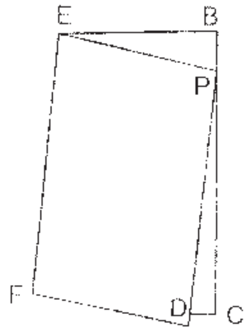


Fig. 1.2

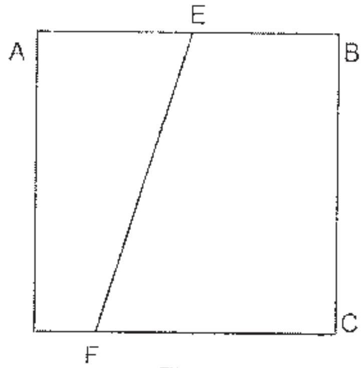


Fig. 1.3

Fold as shown. Crease well. Open it and see. You find a line. This is a straight line, with no cuts, stops or intervals. In Mathematics we call a line a sequence of points. Now make another fold in the same paper ABCD

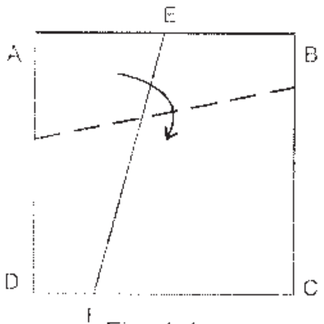


Fig. 1.4

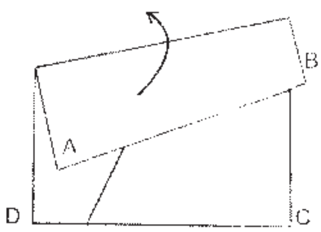


Fig. 1.5

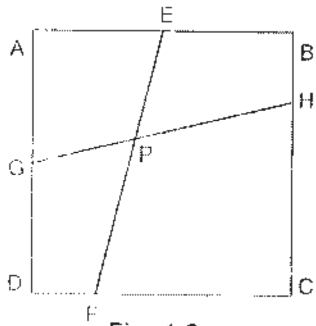


Fig. 1.6

The two lines cut at a point P. Mathematical description for a point P is that it is a circle with no Radius. Again have a look at the same paper. Without any plan or effort on our part we have

$E\hat{P}H = G\hat{P}F$ Vertically opposite angles

$E\hat{P}G = H\hat{P}F$

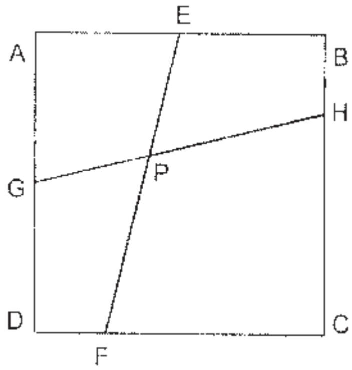


Fig. 1.7

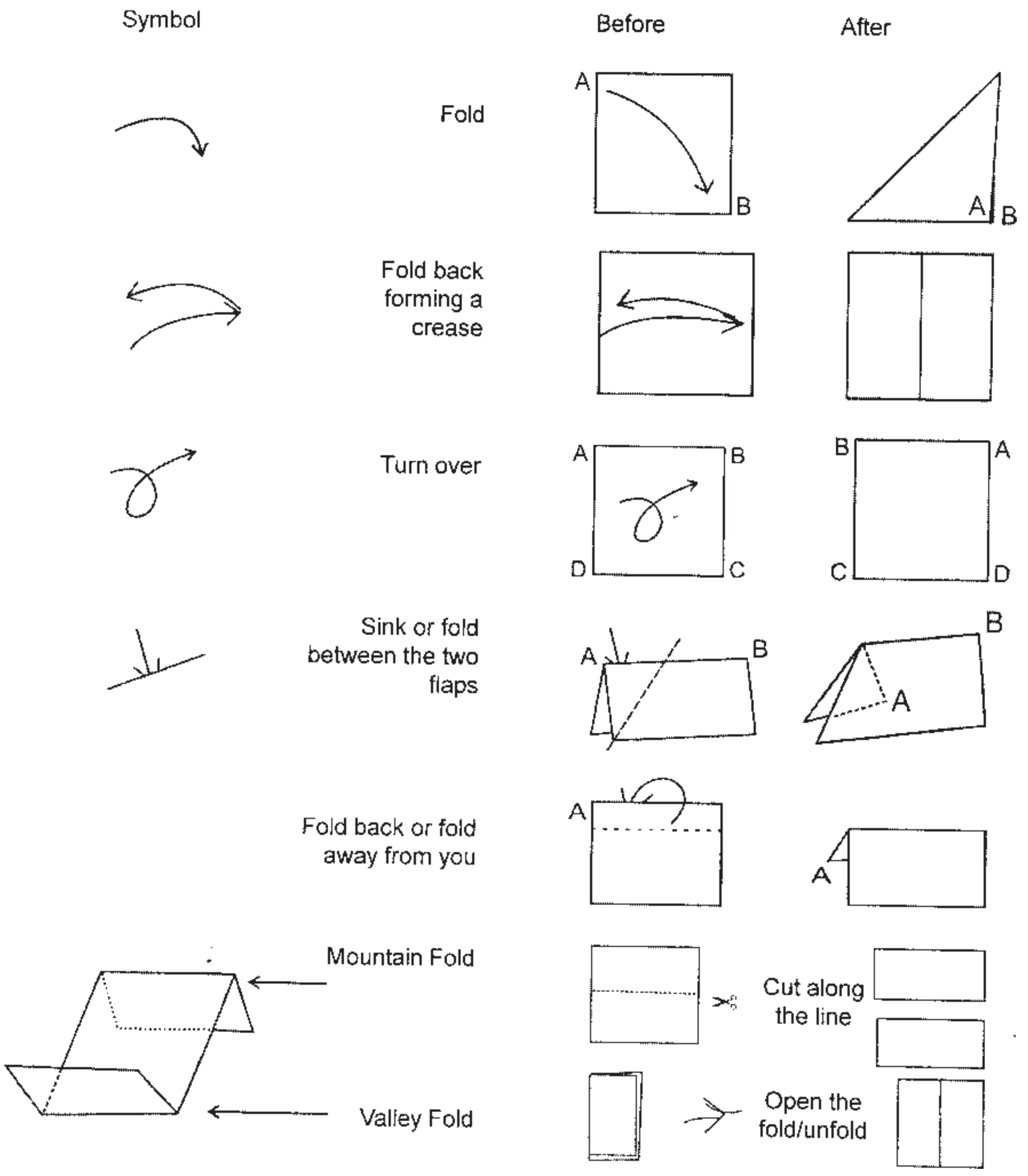
Also

Square ABCD = Quadrilateral AEPG+EBHP+PHCF+GPFD.

This is what happens when we fold a paper. Multiple folds give multiple lines, much more partitions of Area.

2. Origami Symbols and Signs

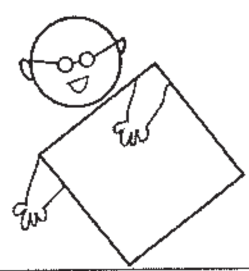
There is a set of symbols to denote folds of origami. The following illustrations are adequate for the models described in this book.



3. Different Geometrical Shapes from Paper

- ☺ Square
- ☺ Right angle Triangle
- ☺ Equilateral Triangle
- ☺ Isosceles Triangle
- ☺ Rhombus
- ☺ Parallelogram
- ☺ Trapezium
- ☺ Circle

ORIGAMIAN SAYS



Origami concretises abstract Concepts

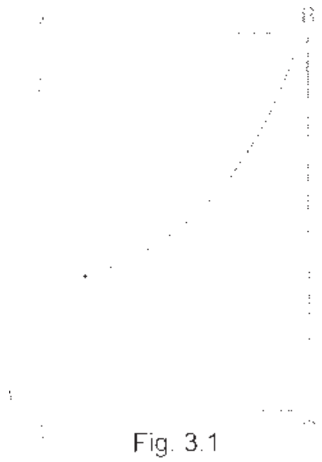


Fig. 3.1
Fold AB to AD

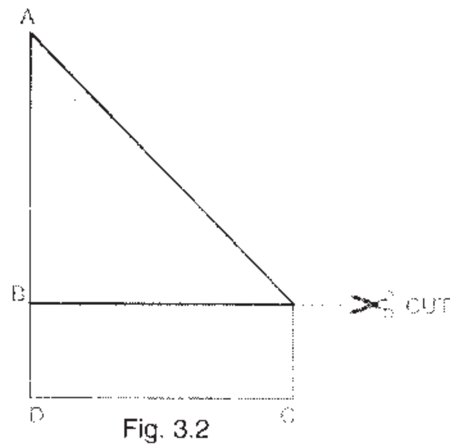


Fig. 3.2
Cut along the crease

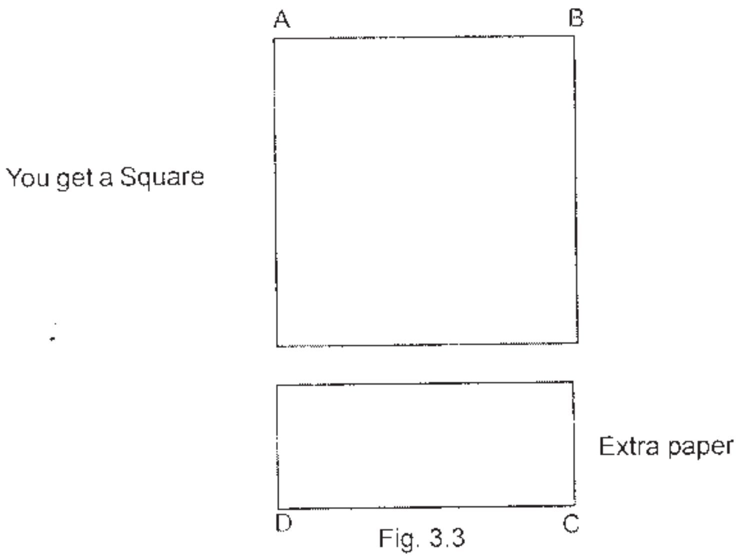


Fig. 3.3

To get Right Angle Triangle from a Square

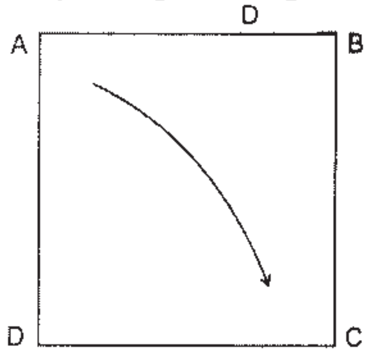


Fig. 3.4

Start from a Square.
Fold A to C

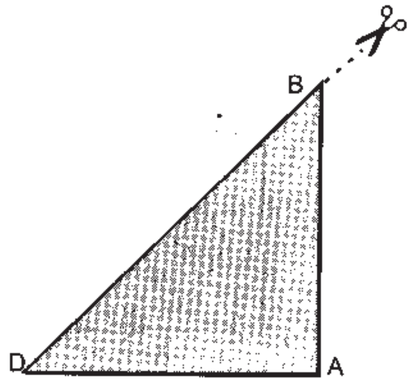


Fig. 3.5

Cut along BD

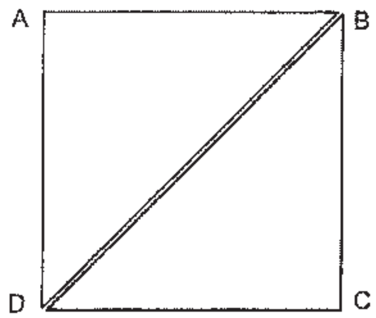


Fig. 3.6

Two Right Angle Triangles

To get Right Angle Triangle from a Rectangle

1.

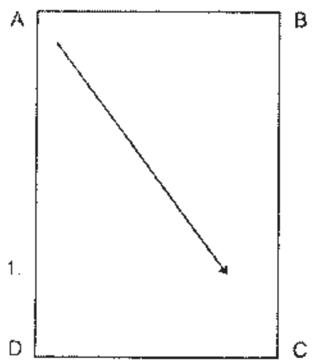


Fig. 3.7

Fold A to BC

2.

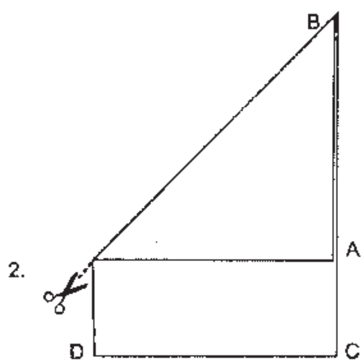


Fig. 3.8

Cut along the crease

3.

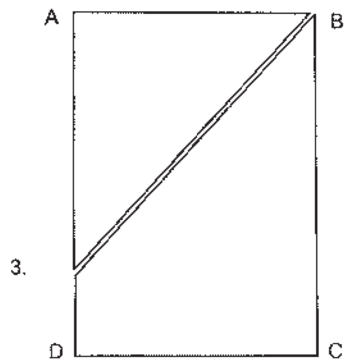


Fig. 3.9

One Right Angled Triangle

To get Equilateral Triangle from a Square

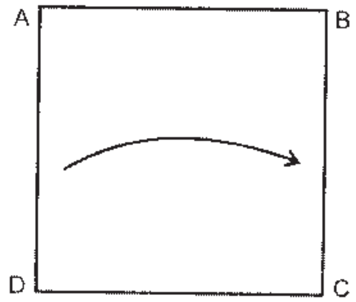


Fig. 3.10
Start from a square

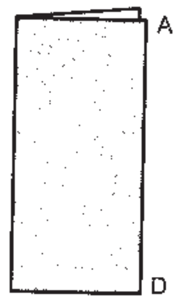


Fig. 3.11
Fold AD upon BC & unfold

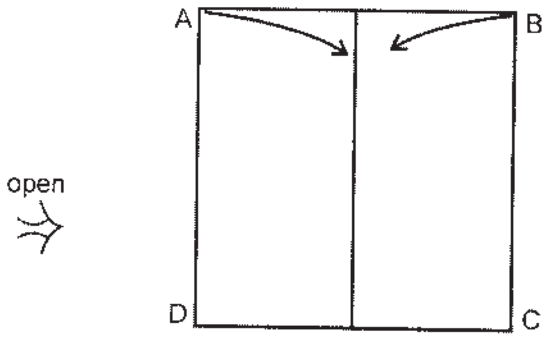


Fig. 3.12
Fold corners A & B to middle line.

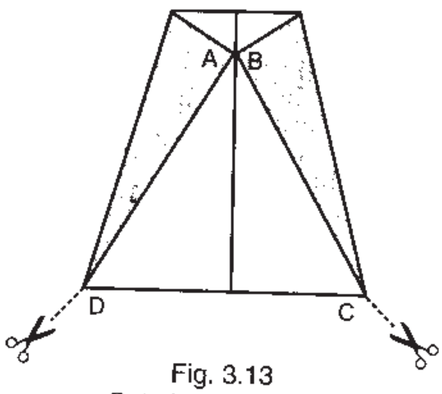


Fig. 3.13
Cut along DA and BC

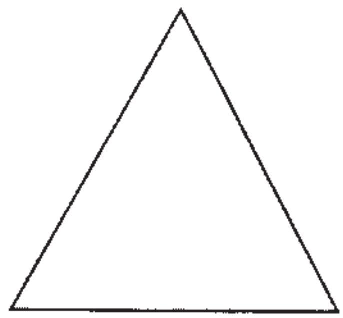


Fig. 3.14
Equilateral Triangle

To get an Equilateral Triangle from a Rectangle

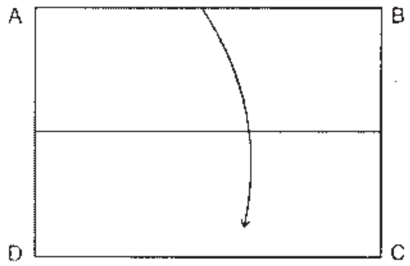


Fig. 3.15
Fold AB to DC

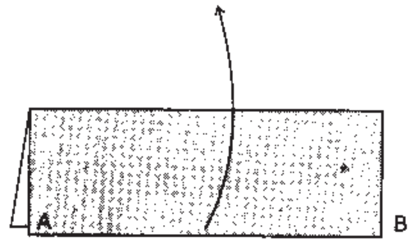


Fig. 3.16
Fold back,observe a line

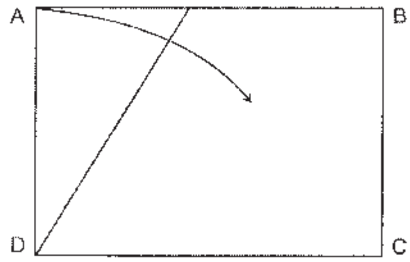


Fig. 3.17
Fold A to middle-line.

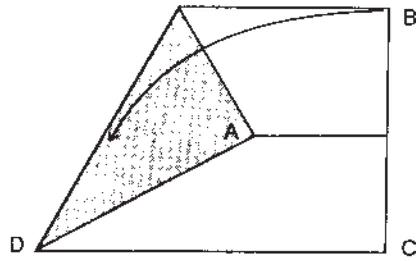


Fig. 3.18
Wrap B upon creased line

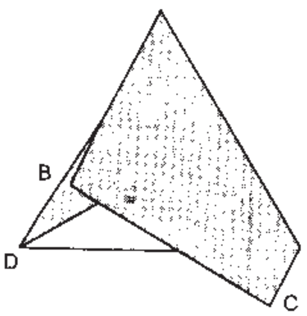


Fig. 3.19
It looks like this. Open it.

open

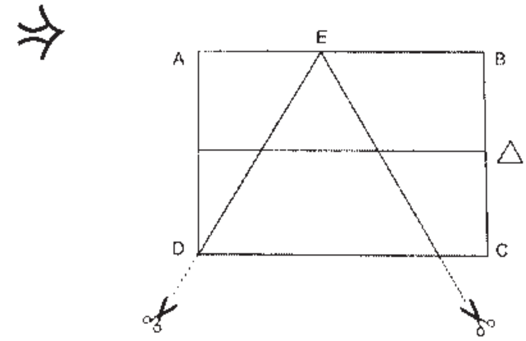


Fig. 3.20
Cut along creased lines.
You get Equilateral Triangle

To get an Isosceles Triangle from a Square

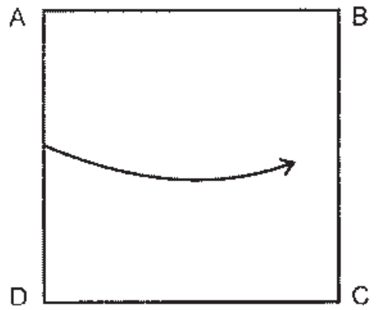


Fig. 3.21
Start From A Square
Fold AD upon BC

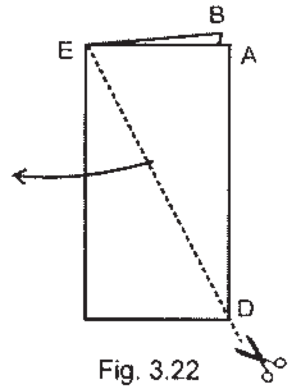


Fig. 3.22
Cut along ED & open

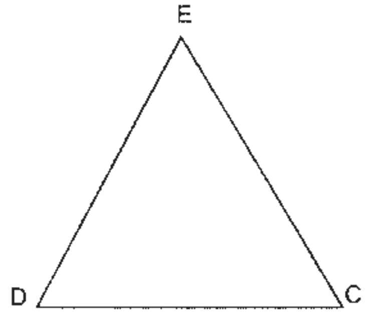


Fig. 3.23
Isosceles Trangle

From a rectangle

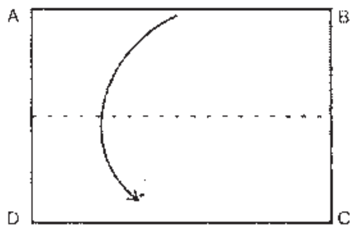


Fig. 3.24
Fold AB on DC

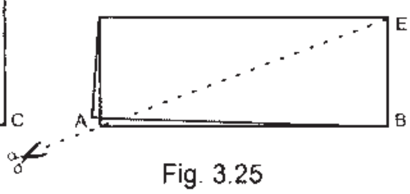


Fig. 3.25
Select any point E on the Midline. Depending on E the angle of the Isosceles triangles are formed. Cut along AE.

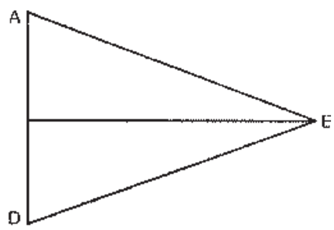


Fig. 3.26
Isosceles Triangle

To get a Rhombus from a Square

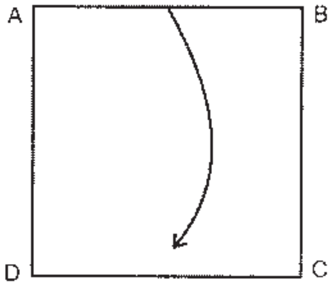


Fig. 3.27
Start From A Square
Fold AB upon DC

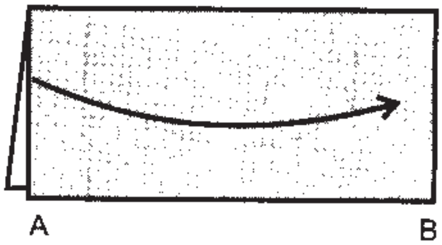


Fig. 3.28
Fold A to B

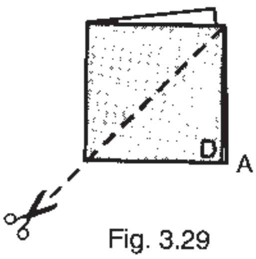


Fig. 3.29
Cut along
diagonal

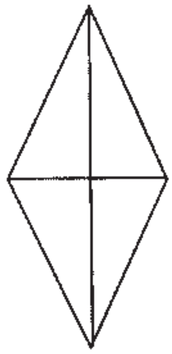


Fig. 3.30
Rhombus

From a Rectangle

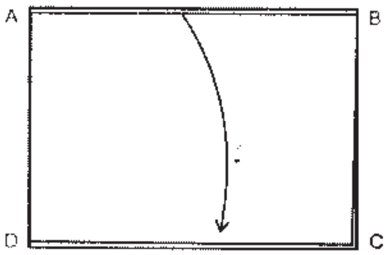


Fig. 3.31

Fold AB upon DC

Fig. 3.32

Fold A to B

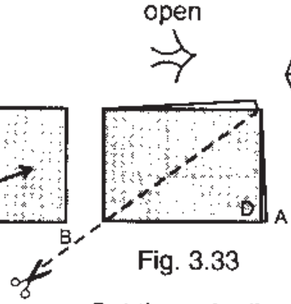


Fig. 3.33

Cut through all
Layers along diagonal

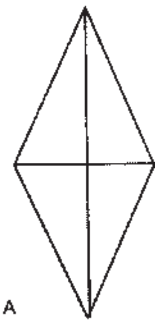


Fig. 3.34

Rhombus

To get a Parallelogram from a Square

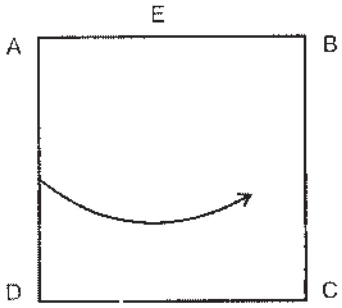


Fig. 3.36

Start From A Square
Fold AD on BC

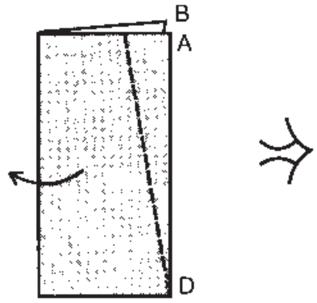


Fig. 3.37

Choose a Point E on AB,cut along DE

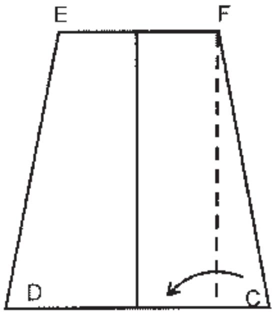


Fig. 3.38

Fold perpendicular from F upon DC.

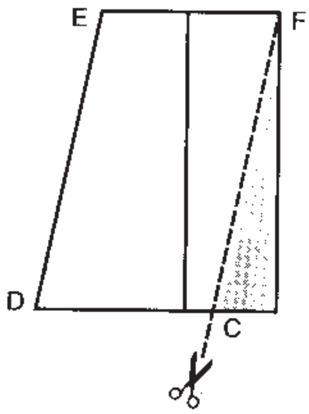


Fig. 3.39

Cut along FC.

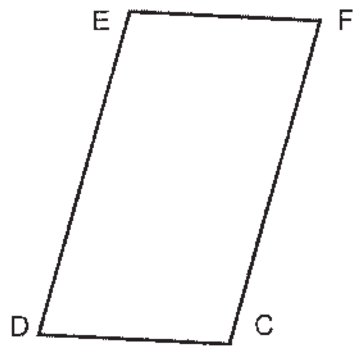


Fig. 3.40

Parallelogram

To get a Trapezium from a Square

First Method

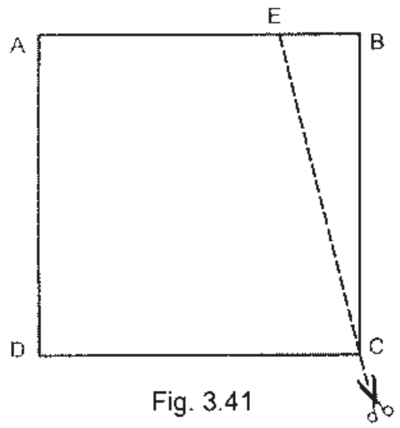


Fig. 3.41

Start From A Square
Choose a point E on AB
and cut CE.

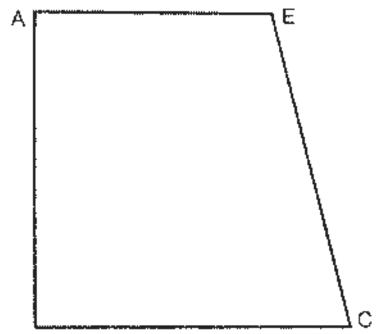


Fig. 3.42

Trapezium

Second Method

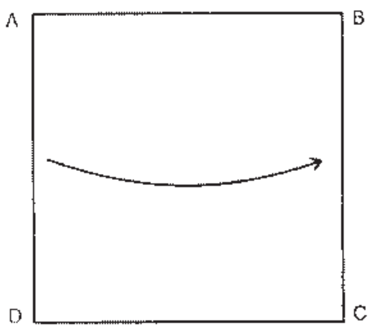


Fig. 3.43

Fold AD on BC

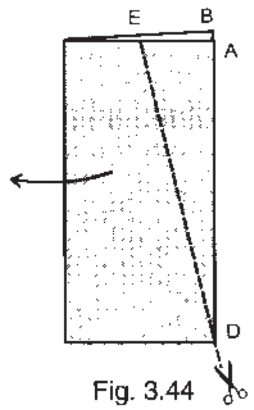


Fig. 3.44

Choose E on AB and cut.
through both Layers

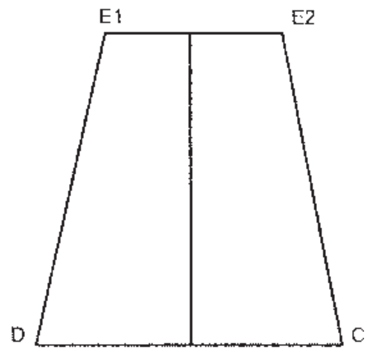


Fig. 3.45

Trapezium

To cut a Circle from a Square (or Rectangle)

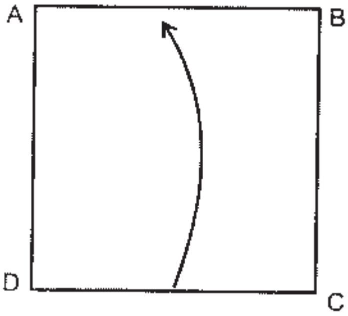


Fig. 3.46
Start From A Square
Fold DC to AB

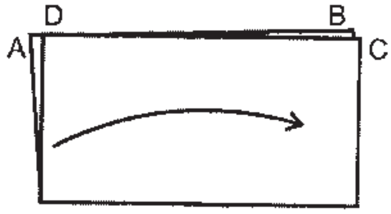


Fig. 3.47
Fold D to C

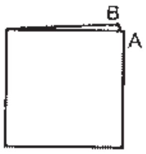


Fig. 3.48
Turn over

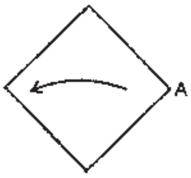


Fig. 3.49
Fold A to a side.

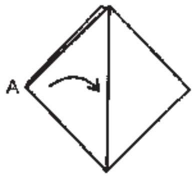


Fig. 3.50
Fold back A to midline and crease.

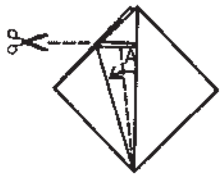


Fig. 3.51
Fold on the crease

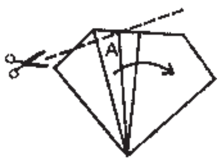


Fig. 3.52
Cut along Sharp edges.

Take a Square Paper 20cmx20cm. Follow the step correctly. Cut along the contour.

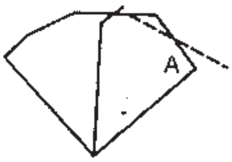


Fig. 3.53

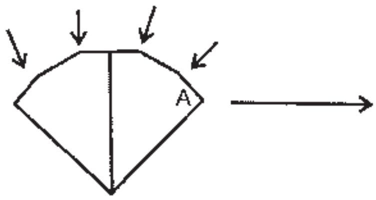


Fig. 3.54
Trim the edges

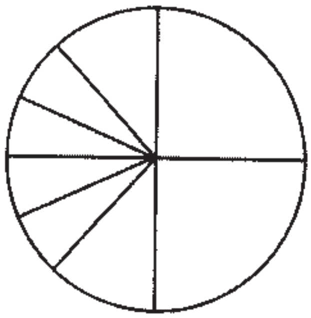
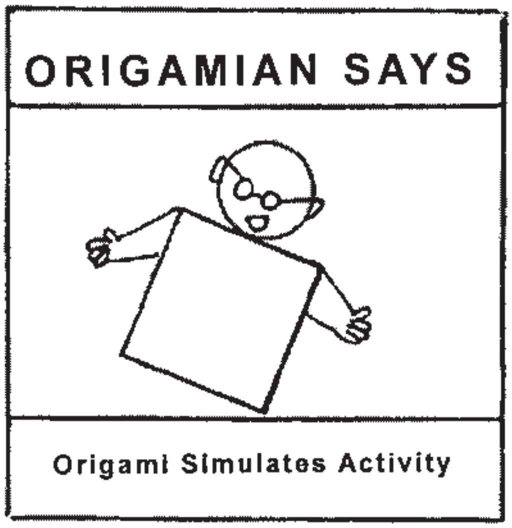


Fig. 3.55
Circle

4. Fun and Facts of Number Theory

- ☺ What is a Number?
- ☺ Representing a Number using Paper
- ☺ Triangular Numbers
- ☺ Square Numbers
- ☺ Square Number is sum of the consecutive Triangular numbers
- ☺ Dividing and counting Triangles and Squares



Fun and Facts of Number Theory

We showed in earlier chapters how we deal about with points, lines, angles, areas.

In mathematics we deal with numbers also.

What is a number? Can somebody show No. 5 or No. 7

But we can always get 5 apples or 7 objects

What we mean to say is that a number is an abstraction?

A number has no dimensions.

A number is used to denote, a quantity, a volume, or an area which are of different dimensions.

A number is used to express, an order of things, value of things and as a symbol too.

Surely all this is confusing.
Let me illustrate with examples

- When we say, "These apples weigh 5 kgs"
  - the number is being used to denote quantity.
- When we say, "The 5th apple is Rotten"
  - the number is being used to denote order.
- When we say, "soldier 5 is on sentry duty"
  - the number is being used as a symbol.

....these numbers can also be represented through paper-folding. Look at this.

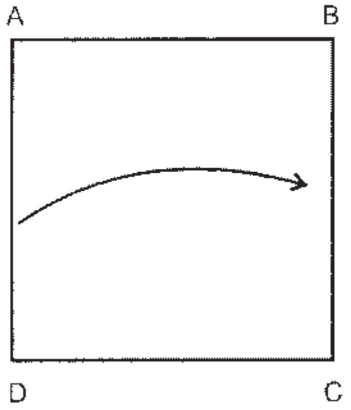


Fig. 4.1

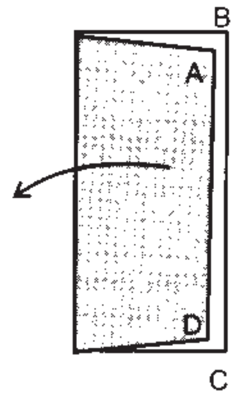


Fig. 4.2

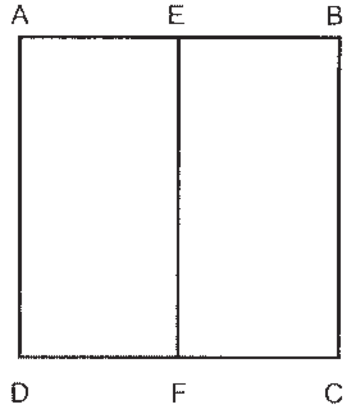


Fig. 4.3

ABCD is folded in half. So that it gets equally divided into two parts. Now do the same thrice. Through these foldings ABCD is divided in eight equal rectangles. Repeat these foldings horizontally

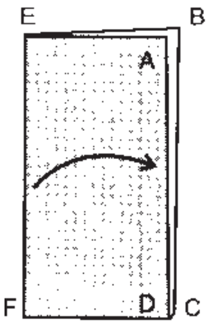


Fig. 4.4

Fig. 4.5

Fig. 4.6

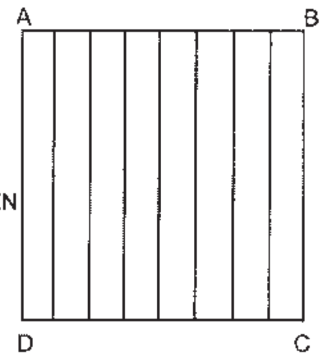


Fig. 4.7

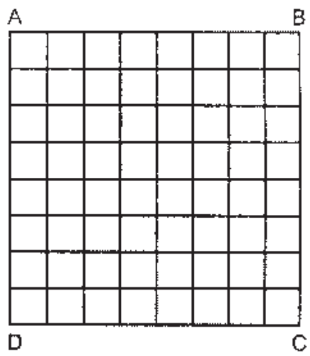


Fig. 4.8

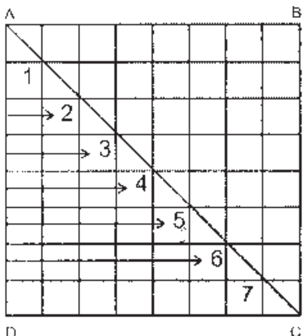


Fig. 4.9

Observe that below A there is a right angled triangle

Below that there is one full square

Below that there are two squares

Below that three squares

then four squares etc.

So here we have a sequence of natural numbers 1, 2, 3, ...7. If the square is larger sequence is also longer. This is how we represent numbers, by equal sized squares or equal sized geometrical figures of any length or breadth or area, folded inside a regular geometrical figure.

Generating the Sequence of Triangular Numbers

We saw that the natural numbers can be represented on a square paper throug' small squares tessellated into it.

Two types of numbers are very famous. They are triangular number and square numbers.

What are they?

Some numbers can be arranged in a triangle let us say 3. The natural numbers upto 3 are 1, 2, 3. All the numbers can be arranged to form a Triangle.

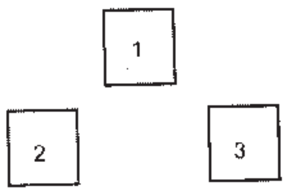


Fig. 4.9

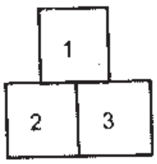


Fig. 4.10

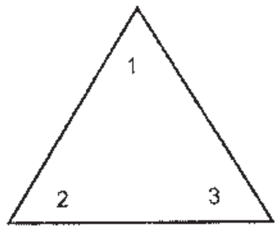


Fig. 4.11

If we want any other triangular number we have to necessarily arrange them in a Triangle. There is no other way. But there is a paper folding method which generates Triangular numbers, as many as you want.

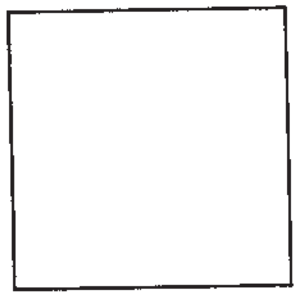


Fig. 4.12
Start with 30 cm x 30 cms paper. Tessellate with 64 Squares

Take a square paper.
Fold 64 squares, Fold diagonally. Write natural number 1,2,3,4,5,.. as shown in ABD.

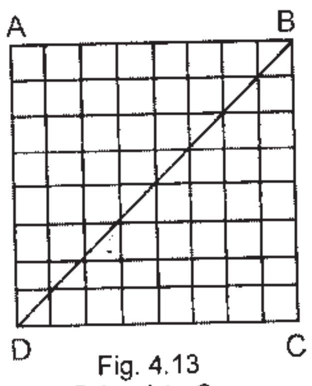


Fig. 4.13
Bring A to C

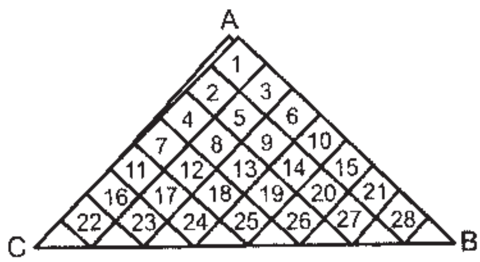


Fig. 4.14
Write numbers 1 to 28

The number on RHS squares on AB 1, 3, 6, 10, 15, 21,28.....is a sequence of triangular numbers. Observe how each of these numbers associates with a triangle.

Generating the Sequence of Square Numbers

Similar to triangular numbers square numbers can be visualised. For example take a No. 4. The natural numbers upto 4 are 1, 2, 3, 4, These form a square

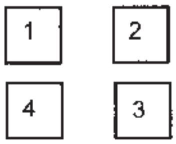


Fig. 4.15

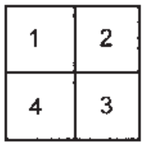


Fig. 4.16

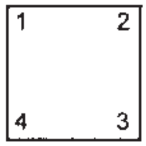


Fig. 4.17

We have got a sequence of triangular numbers in a 64 square grid. The lower portion of the square i.e. ΔBDC is not filled up. We shall use this ΔBCD

Bring C to the mid point of BD and fold twice on itself each time (in half). Then open. You will see that ΔBCD is now tessellated into Right angled Triangles.

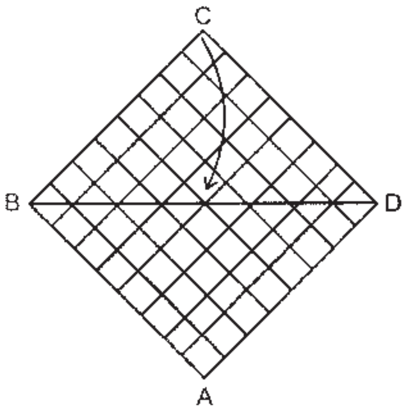


Fig. 4.18

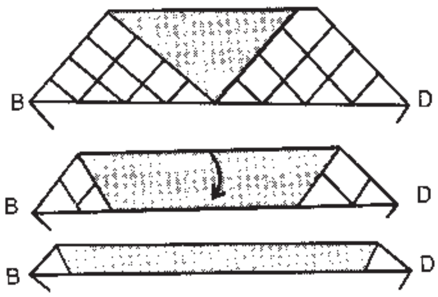


Fig. 4.19

Start filling up these triangles from C as shown.

You can observe the CD contains a sequence of square numbers 1, 4, 9, 16, 25, 36, 49 & 64.

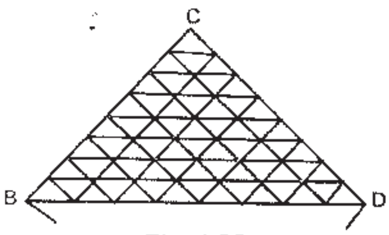


Fig. 4.20

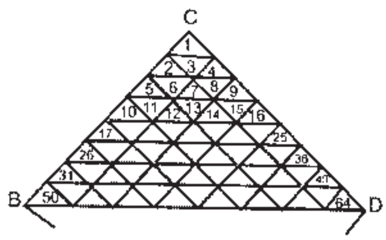


Fig. 4.21

An intersting way to demonstrate relationship between Triangular & Square numbers

We have just seen sequences of Triangular numbers and square numbers. Let us prepare a square ABCD in which ABC is filled with Triangular No's and ΔADC is filled with square numbers.

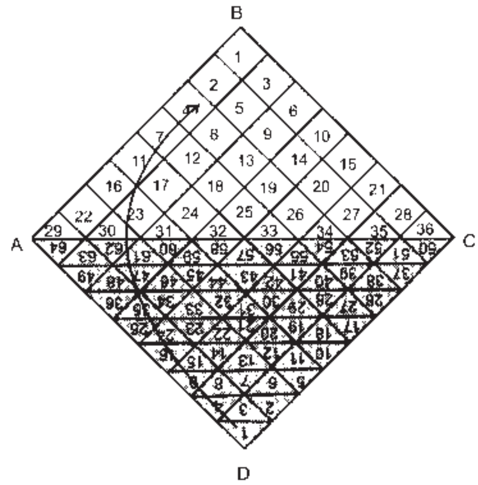


Fig. 4.22

Fold D to B. Now you can't see No's.

Arbitrarily bring down D towards AC

Then you see a square with top half triangular numbers and bottom half square numbers.

Both together form a square shape.

How many small squares are there inside this square? Count them. In this example they are 4x4=16

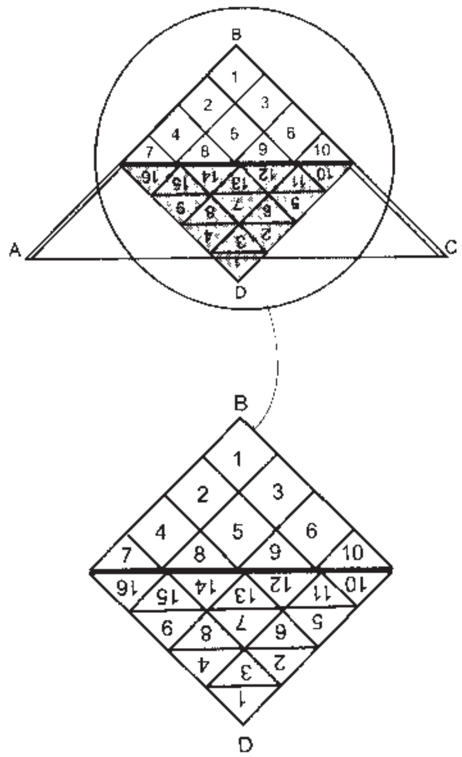


Fig. 4.23

Spot that numbers 16 on left hand corner. This explains why 16 is a square number.

This square has two Right angled Δs (one of the two Δs No. 16 appears.) Therefore square number must contain two triangular no's. Look to RHS. You see no. 10 and 6. 10+6=16.

Repeat this folding and compare the numbers. For all Square Numbers this is true. Therefore we have the relationship:

Any square no= sum of consecutive triangular numbers.

5. Origami Models from Different Geometric Shapes

Usually Origami models are folded from a square shaped paper.

But this tradition is not observed nowadays.

The aim of this book is to introduce mathematical concepts, elements and ideas through origami.

Hence we have collected here, origami models which are folded from a given Geometric shape.

Note that it covers all the shapes found in your school curriculum.

Hence these could easily be used as learning aids by creative teachers to illustrate different mathematical ideas. By practising these models the child gets accustomed to precise geometrical shapes.

From

☺ **Isosceles Triangle**

☺ **Square**

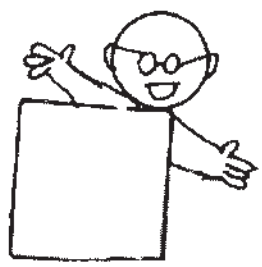
☺ **Equilateral Triangle**

☺ **Right Angled Triangle**

☺ **Trapezium**

☺ **Circle**

ORIGAMIAN SAYS



Origami Provides Variety of Methods

Elephant from an Isosceles Triangle

Fig. 5.1

Start with a rectangle folded in half longitudinally

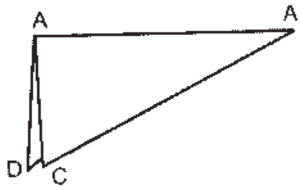


Fig. 5.2

Cut along diagonals

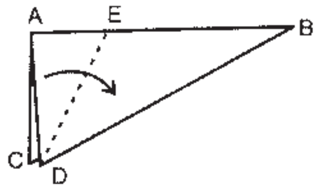


Fig. 5.3

Fold a Right angle △ to side

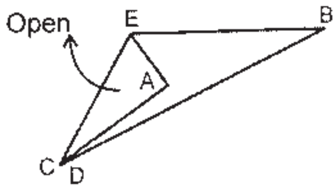


Fig. 5.4

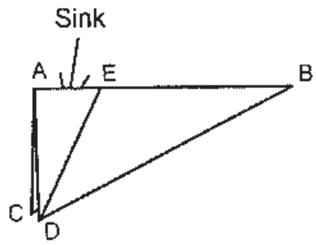


Fig. 5.5

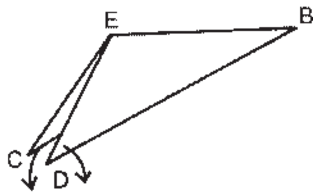


Fig. 5.6
Fold C, D Down

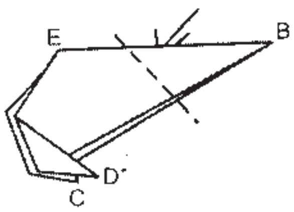


Fig. 5.7
Sink

Fig. 5.8
Elephant Head

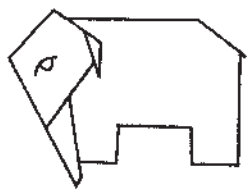


Fig. 5.9
Full Elephant

Airplane from a Square

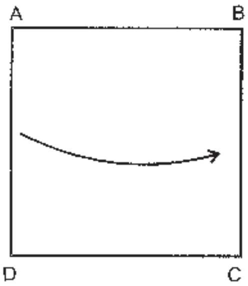


Fig. 5.10
Take a Square

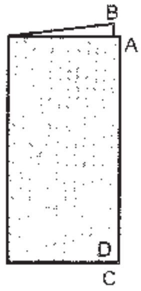


Fig. 5.11
Fold midline

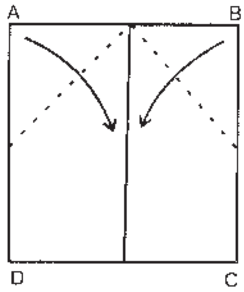


Fig. 5.12
Bring A&B to midline

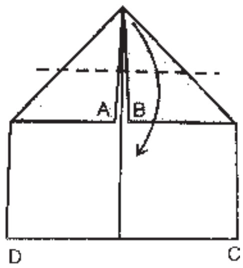


Fig. 5.13
Bring the Tip down

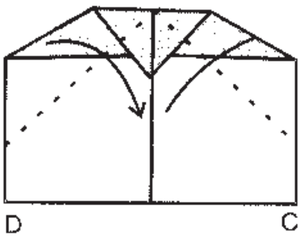


Fig. 5.14
Fold sides to centre

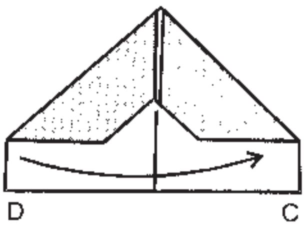


Fig. 5.15
Fold D to C

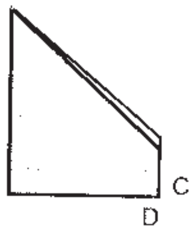


Fig. 5.16
Turn over

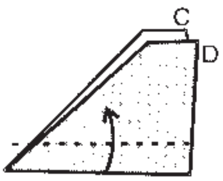


Fig. 5.17
Fold about an inch up

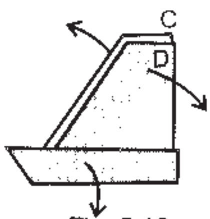


Fig. 5.18

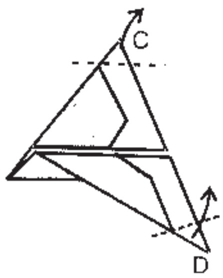
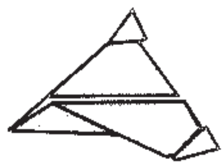


Fig. 5.19
Lift edges up

This plane flies beautifully. Hold the head in two fingers, throw forward, Plane flies straight.



Airplane

Fig. 5.20

Star of David from an Equilateral Triangle

Start from an Equilateral Triangle (see page 14)

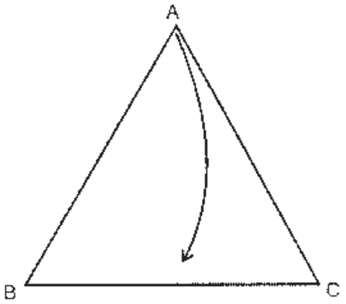


Fig. 5.21
Mark A, B, C, Fold A down

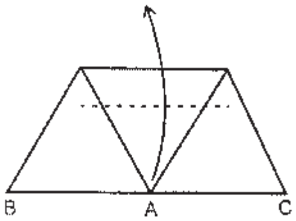


Fig. 5.22
Lift A

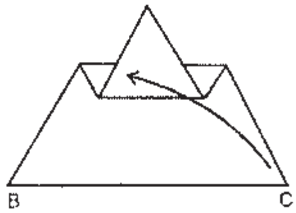


Fig. 5.23
Fold C on to A

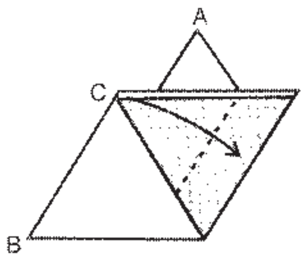


Fig. 5.24
Fold C back

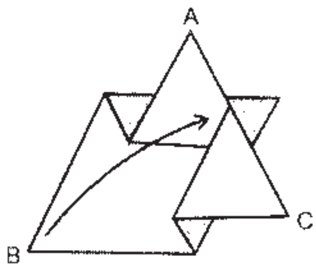


Fig. 5.25
Fold B onto A & C

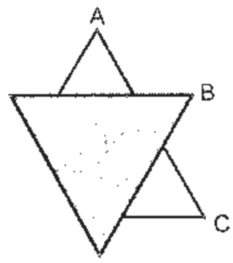


Fig. 5.26
Fold B back

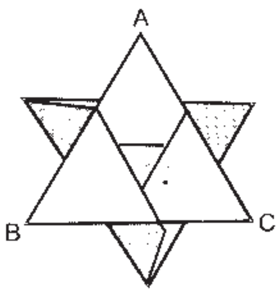


Fig. 5.27
Tuck B under C

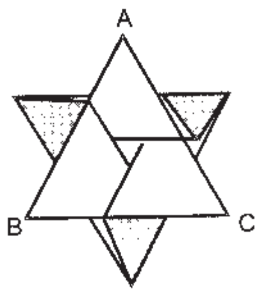


Fig. 5.28
Star of David

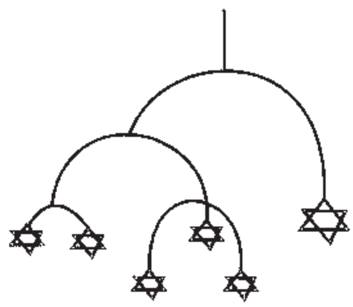


Fig. 5.29
Mobile sculpture with
Stars of David.

King Fisher from a Rightangled Triangle

Start with a Rightangle Triangle cut from a Square (see page 7)

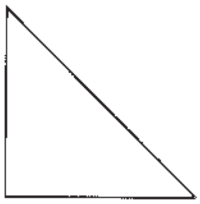


Fig. 5.30

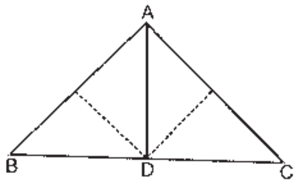


Fig. 5.31

Fold B & C to A

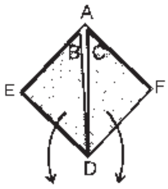


Fig. 5.32

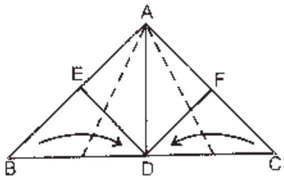


Fig. 5.33

Fold AC and AB on AD
Bisect A on both
sides and turn over

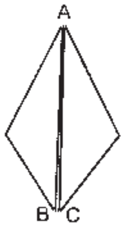


Fig. 5.34

Bring F, G H
together and
crease CI

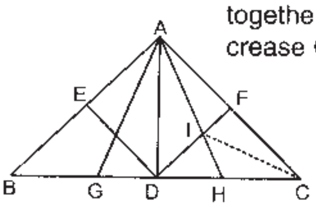


Fig. 5.35

Fold C to I

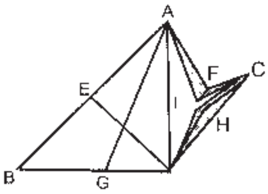


Fig. 5.36

FCH props up

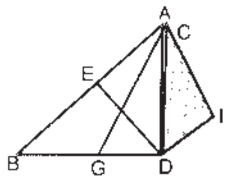


Fig. 5.37

Do the same on
other(ABD) side also

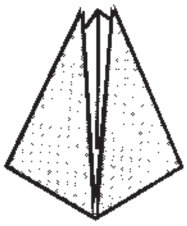


Fig. 5.38

Turn over

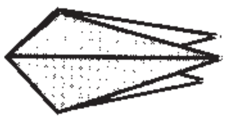


Fig. 5.39

Fold in the middle

Fig. 5.40

Fig. 5.41

Fold the wings of B

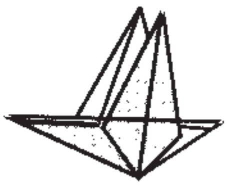


Fig. 5.42

Fig. 5.43

The Bird

Another Plane from a Trapezium

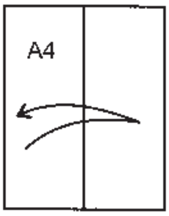


Fig. 5.44
Start with a Square
Make Central line

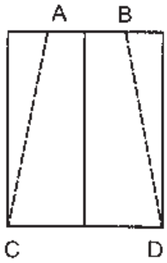


Fig. 5.45
Measure 2 cms from edges & mark A, B

ABCD is a Trapezium

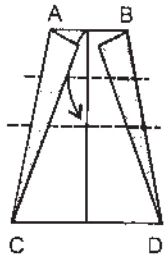


Fig. 5.46
Fold along AC & BD

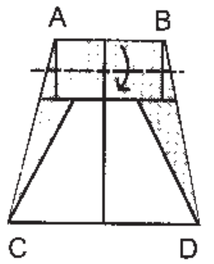


Fig. 5.47

Fold AB to the middle

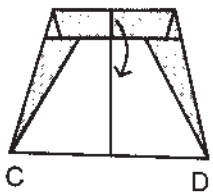


Fig. 5.48

Fold further

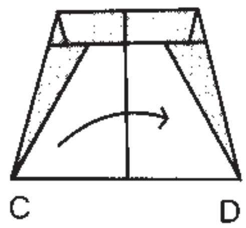


Fig. 5.49

Fold down C to D

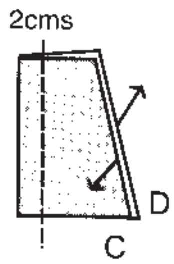


Fig. 5.50
On the line shown at 2cms
Crease 2 cms
Pull out C & D

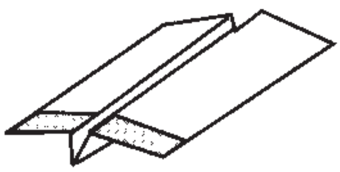


Fig. 5.51

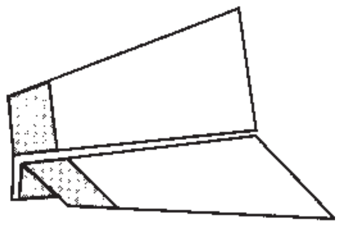


Fig. 5.52
This plane has wonderful Aerofoil properties.

Hexagonal from a Circle

A Hexagonal Bowl.

Usually circles are not chosen to fold origami models. We have given here two models incorporating mathematical ideas. Start with a circle with Radius 15 cms. For this choose a square 15 cm side and cut as shown in page 17

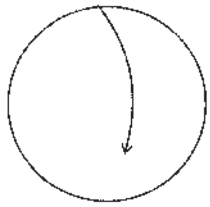


Fig. 5.53

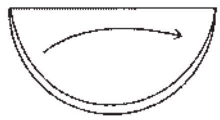


Fig. 5.54

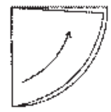


Fig. 5.55

Follow the sequence strictly while Folding.

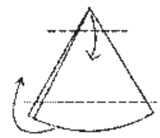


Fig. 5.56

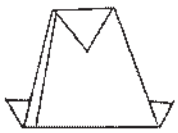


Fig. 5.57

Open

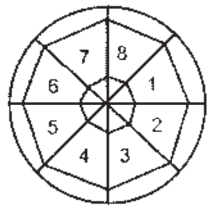


Fig. 5.58

Mark 1-8 in inner and outer octagon

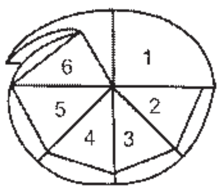


Fig. 5.59

Fold 7 & 8 face to face tuck it under 6. It forms a cup.

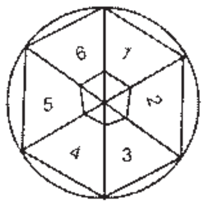


Fig. 5.60

Fold edges down.

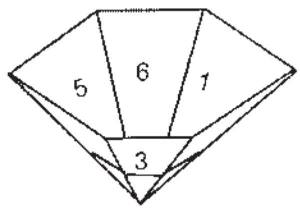


Fig. 5.61

Cup actually looks like this. Push the lower octagon up.

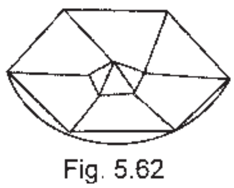


Fig. 5.62

You get a Hexagonal Bowl

Lamp from a Circle

Lamp from a circle

Take a paper circle, preferably Yellow on one side and a dark colour (Black/blue) on the back.

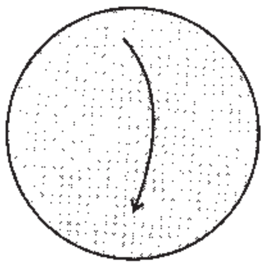


Fig. 5.63

Fold Half

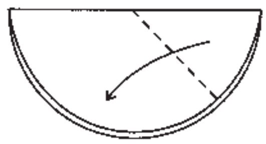


Fig. 5.64

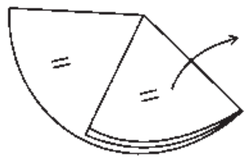


Fig. 5.65

Fold so that the two portions are equal

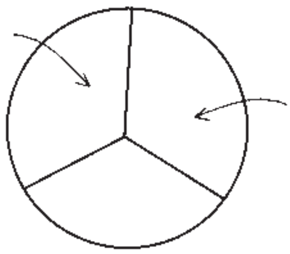


Fig. 5.66

Open the Folds. The circle is divided into three equal segments.

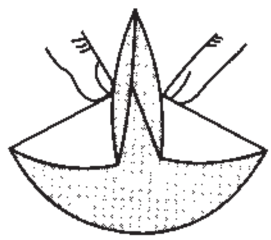


Fig. 5.67

Press away from the centre towards the circumference.

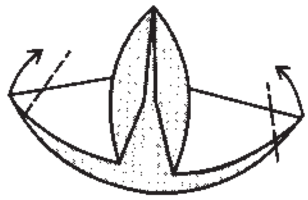
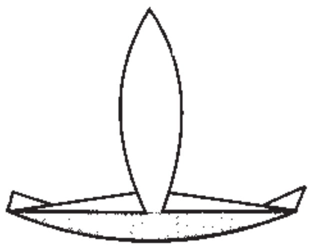


Fig. 5.68

Lift the edges



The lamp is ready

Fig. 5.69

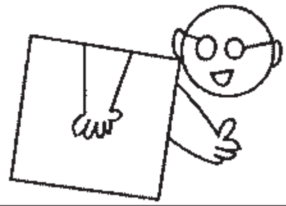
6. Fun in Every Corner

☺ **Angles in Geometric Shapes**

☺ **Bisecting an Angles- Paper Folding way**

☺ **Trisecting an Angle - Paper Folding way**

ORIGAMIAN SAYS



Origami Improves Logical Analytical Ability

Geometrical figures and their corners

Do you know that ancient Egyptians who built magnificent Pyramids, did not have a protractor.

Protractor is the ordinary instrument today carried routinely in an Instrument box. Egyptians were not aware of different angles.

It was in Babylonia that a circle was divided into 360 parts and the Angle measure as we know today came into Mathematics. Seldom we notice the Angles in a given Geometric figure.

It will be funny to know how they arrived at a Right-angle (as we call it today). They took a long rope and made dozen knots at equal distances.

Then they held them to form a triangle, with 3 and 4 spaces between the knots for the sides. Then automatically the third side had 5 spaces between knots. And angle between sides with 3 and 5 knots was 90^0= Right Angle.

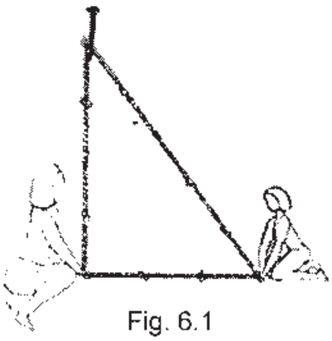


Fig. 6.1

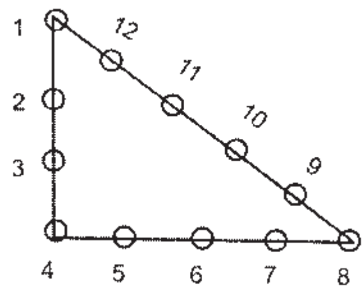


Fig. 6.2

The following figures will familiarise you with Angle measure, equal angles, etc., in the Geometric figures dealt in our class rooms. The introduction of the concept of angle in origami is itself a learning experience. Because just by folding not only you can compare different angles, but also you can divide given angles.

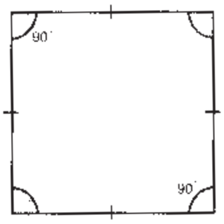


Fig. 6.3

Square

All angles are 90^0 each
All sides are equal

Fig. 6.4

Rectangle

Opposite sides are Equal .All angles are 90^0 each

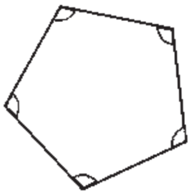


Fig. 6.5

Pentagon

All angles are equal in Regular Pentagon. Each Angle = 108^0

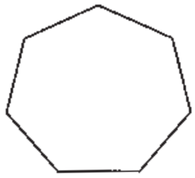


Fig. 6.6

Heptagon

All sides are equal.
Each internal angle is 128^0 57'

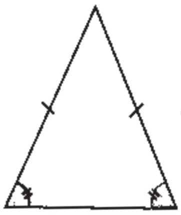


Fig. 6.7

Isosceles Triangle

Two sides are equal
Angles opposite to the equal sides are equal

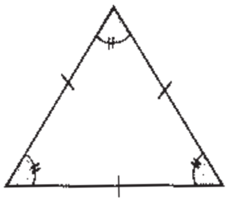


Fig. 6.8

Equilateral Triangle

All angles are 60^0 each
All sides are equal

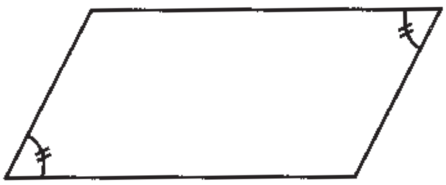


Fig. 6.9

Parallelogram

Opposite sides are equal and parallel
opposite angles are equal

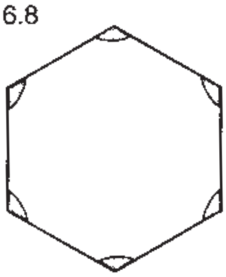


Fig. 6.10

Hexagon

In regular Hexagon all angles are equal
Each angle = 120^0

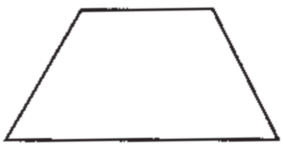


Fig. 6.11

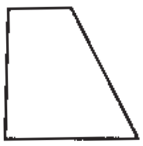


Fig. 6.12

Trapezium

Two sides are parallel
None of the angles are equal

Dividing Angles into Equal parts

The funniest advantages in Paper folding is that you can bisect or trisect any angle in a geometric shaped paper by folding paper in a particular way.

You do not need the geometrical exercise like using a protractor, cutting arc lengths etc.

We have chosen here two squares. They are required in further adventures in paper foldings.

Bisecting an Angle in a Square

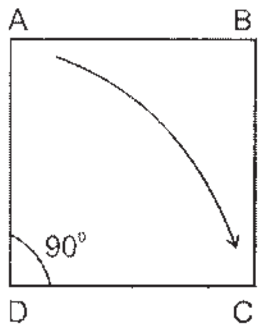


Fig. 6.13

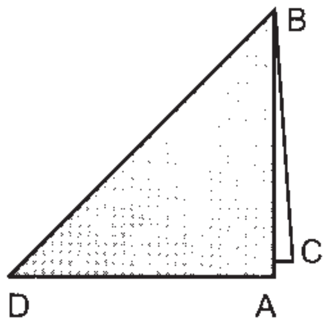


Fig. 6.14

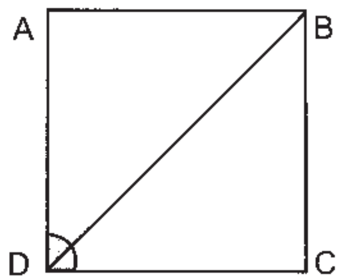


Fig. 6.15

$A\hat{D}B = B\hat{D}C$

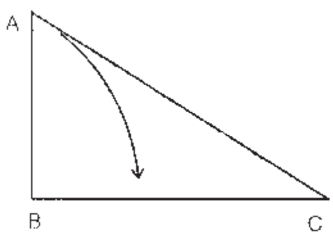


Fig. 6.16

Take Right Angle Triangle in paper

Fold A upon BC

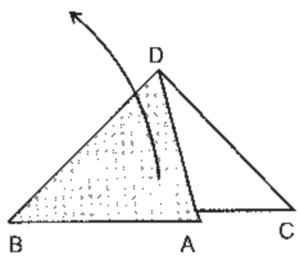


Fig. 6.17

Unfold

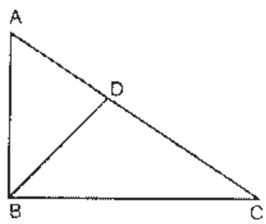


Fig. 6.18

Here $A\hat{B}D = D\hat{B}C$

Trisecting an Angle in a Square

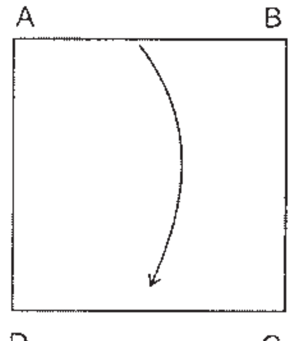


Fig. 6.19
Take a Square

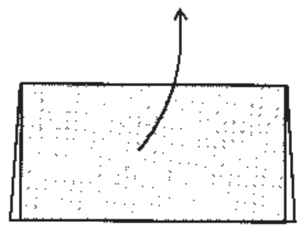


Fig. 6.20
Bring AB upon DC

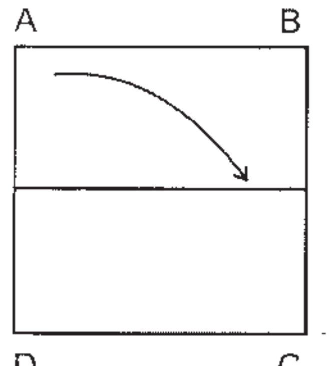


Fig. 6.21
Notice central line

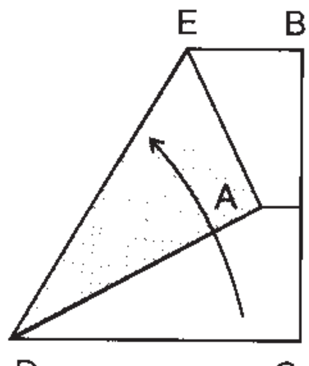


Fig. 6.22
Bring A to touch
Central line and crease DE

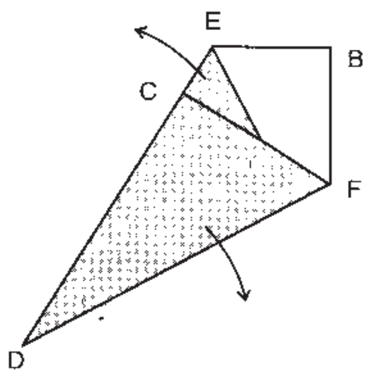


Fig. 6.23
Fold DC upon DE is crease

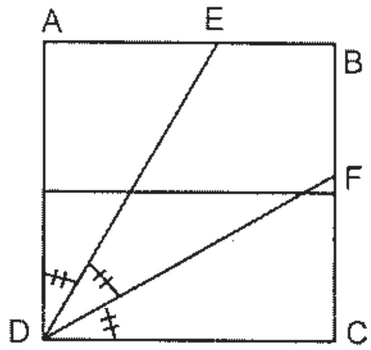


Fig. 6.24
Unfold. $\hat{D}$ is Trisected
Here $A\hat{D}E = E\hat{D}F = F\hat{D}C$

7. Angles in Origami Models

We commence from this chapter giving 'Angle' to Origami Models.

We take popular Origami Models and illustrate the various Angular measurements in them. To create an element of Fun we have framed these into puzzles.

- ☺ **Sail Boat**
- ☺ **Yacht**
- ☺ **Aeroplane**
- ☺ **Bird**
- ☺ **Moth**
- ☺ **Frisbee**

ORIGAMIAN SAYS

Origami Developes Eagerness in Child

Sail Boat

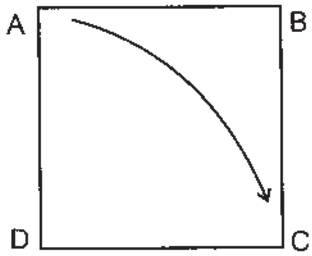


Fig. 7.1

Start with a square
Fold A to C

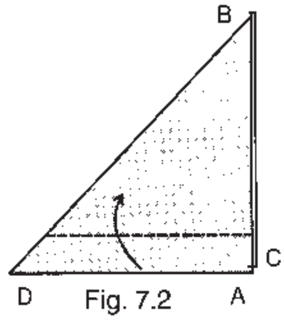


Fig. 7.2

Fold Two Layers at DA up

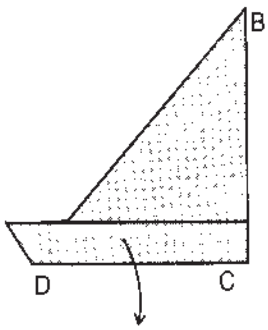


Fig. 7.3
unfold

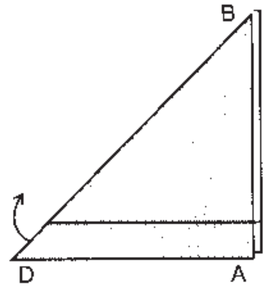


Fig. 7.4
Reverse fold DA

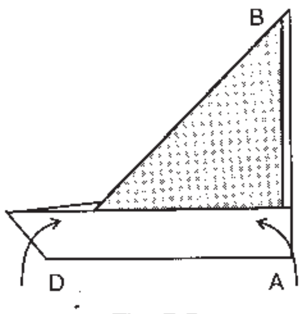


Fig. 7.5
Sail Boat

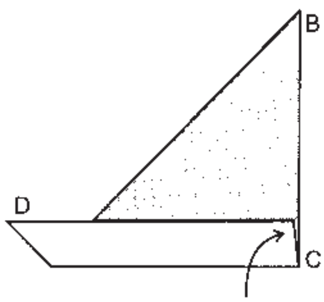


Fig. 7.6

We have started with a Square and folded Sail boat. What are the angles D and B in the Boat?

Yatch

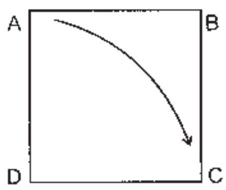


Fig. 7.7
Start from a square

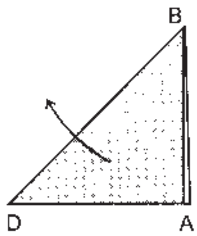


Fig. 7.8
Make a diagonal

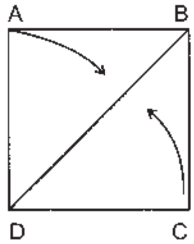


Fig. 7.9

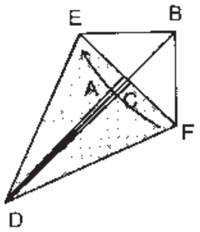


Fig. 7.10
Fold A, C to BD

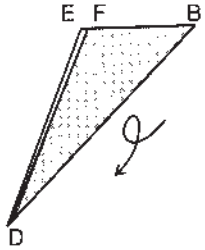


Fig. 7.11
Turn over

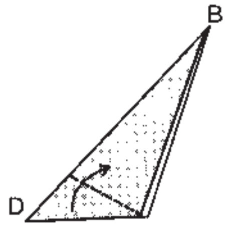


Fig. 7.12
Fold up

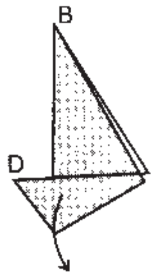


Fig. 7.13
Fold Down

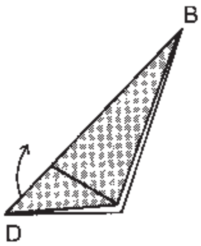


Fig. 7.14
Reverse fold

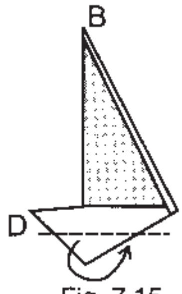


Fig. 7.15
Crease & Fold back

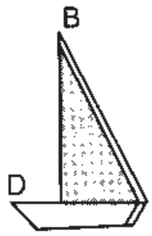


Fig. 7.16
Yacht

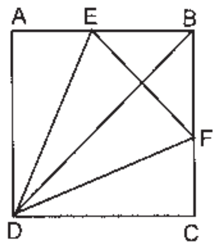


Fig. 7.17

We have folded a Yacht. It looks smart. Now unfold it and see.

Can you give measure of $A\hat{D}E$, $E\hat{D}B$, $B\hat{D}F$, $F\hat{D}C$?

There is an Isosceles Δ whose two sides and angles are equal.
Can you tell those angles?

(For Ans look to page 89)

Aeroplane

Start with a A4 paper

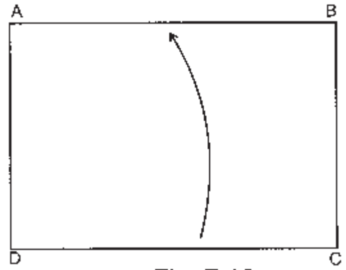


Fig. 7.19

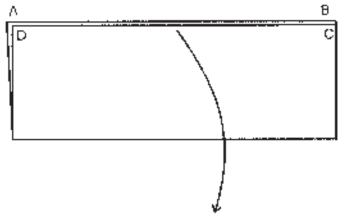


Fig. 7.20

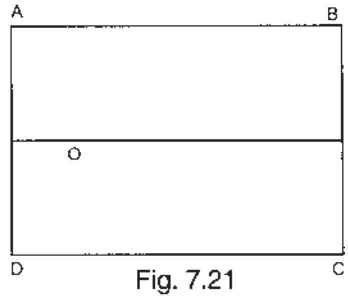


Fig. 7.21

Make a middle line in Rectangle

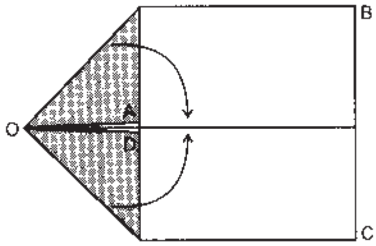


Fig. 7.22

Fold A, B to middle line

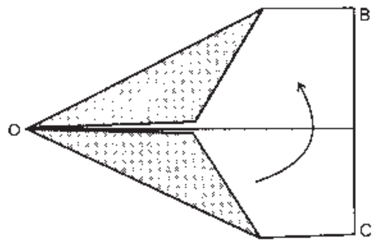


Fig. 7.23

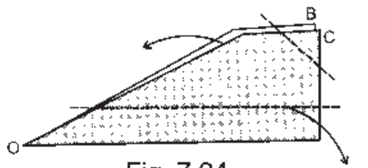


Fig. 7.24

Crease up, about one inch.

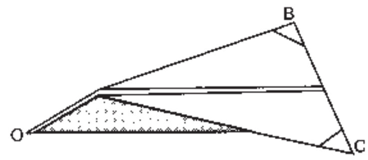


Fig. 7.25

Pull apart B and C

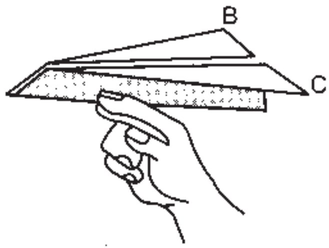


Fig. 7.26

This plane flies well. After playing with it open this model to its original size. Draw on the creases with a pencil.

You can see various lines and angles.

Calculate $G\hat{P}H$

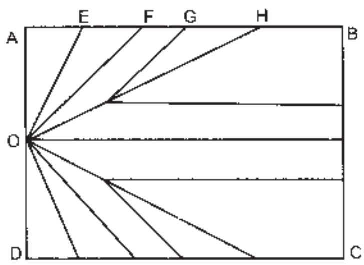


Fig. 7.27

Bird

Start with a square with diagonals folded

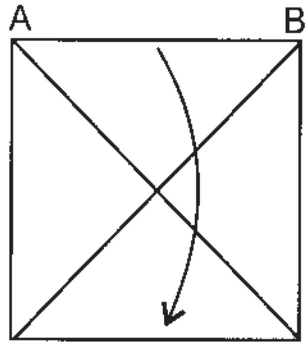


Fig. 7.28

Fold AB upon DC

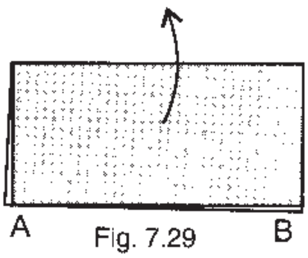


Fig. 7.29

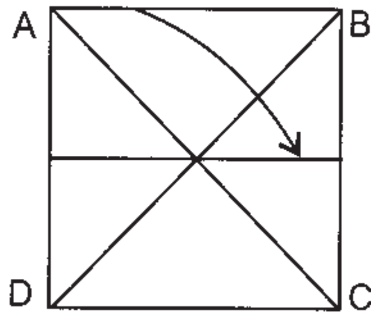


Fig. 7.30

Fold A to middle line

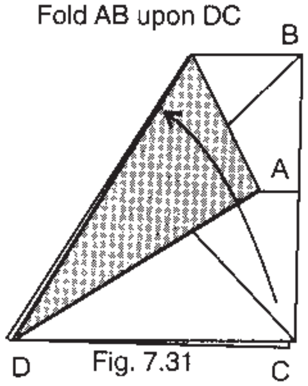


Fig. 7.31

Fold DC up

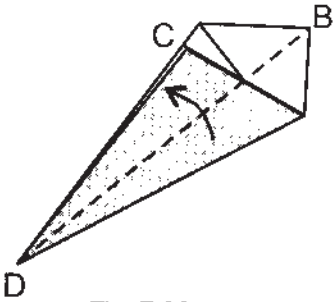


Fig. 7.32

Fold into half

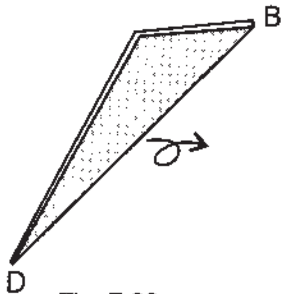


Fig. 7.33

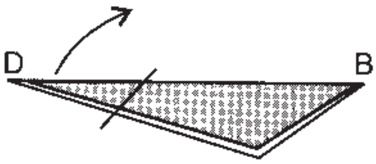


Fig. 7.34

Reverse Fold

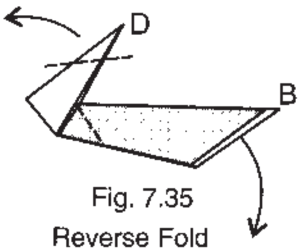


Fig. 7.35

Reverse Fold

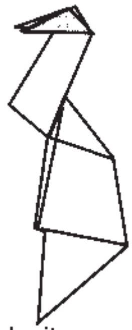


Fig. 7.36

The peacock sits on a support with grace.

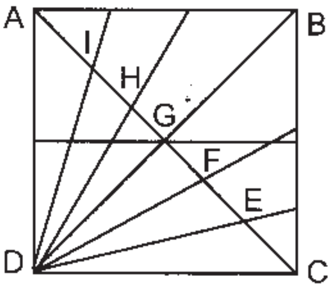


Fig. 7.37

We shall open this model and see what is inside

- Identify all types of Triangles
- Measure the angle DÂI, DÎH, DĤG, DĜF, DÊF and DÊC

(For Ans look to page 89)

Moth

Take a square cut in half. Start with the Rectangle ABCD

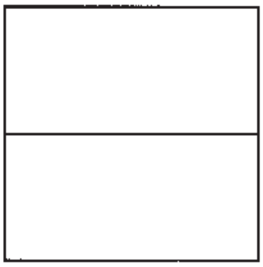


Fig. 7.38

Take one portion

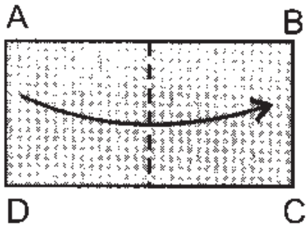


Fig. 7.39

Fold AD upon BC

Fig. 7.40

Fold again

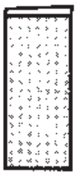


Fig. 7.41

Open

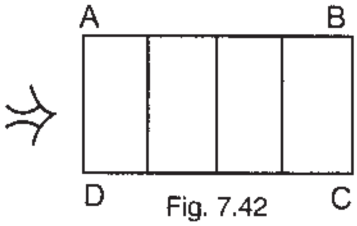


Fig. 7.42

You get four divisions

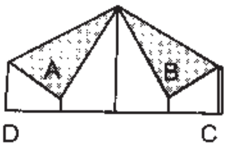


Fig. 7.43

Bring A & B to the lines

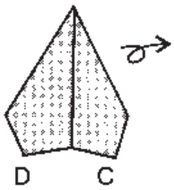


Fig. 7.44

Bring F&G on to the central line

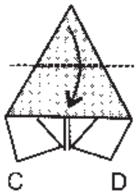


Fig. 7.45

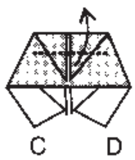


Fig. 7.46

Fold down

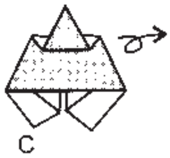


Fig. 7.47

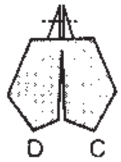


Fig. 7.48

Turn over

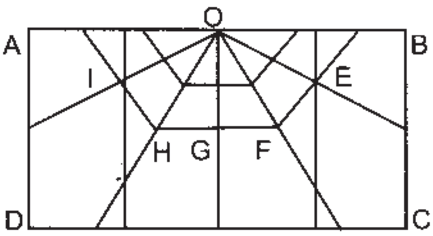


Fig. 7.50

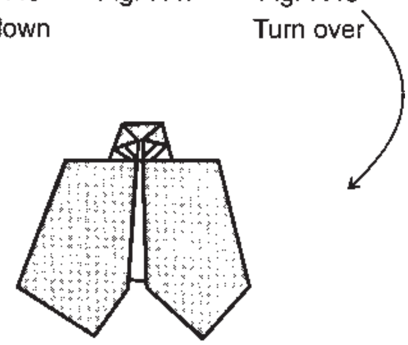


Fig. 7.49

Moth

Open this Butterfly. You see a half Hexagon, Can you find $A\hat{O}E$, $A\hat{O}F$, $A\hat{O}G$, $A\hat{O}H$, $A\hat{O}I$?

(For Ans. look to page 89)

Frisbee

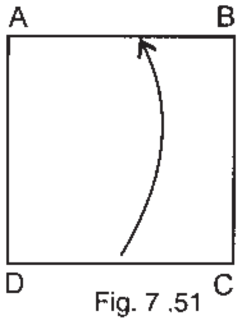


Fig. 7 .51

Take 7 squares of equal size.

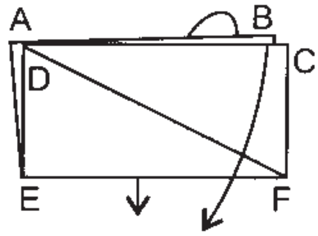


Fig. 7.52

Fold diagonals back to back

You will be surprised to know $E\hat{G}B = 128^{0}\ 57'$ the internal angle of a Heptagon.

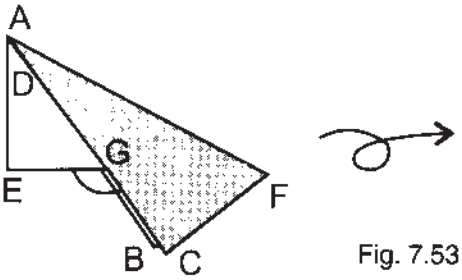


Fig. 7.53

Make seven such units from seven squares.

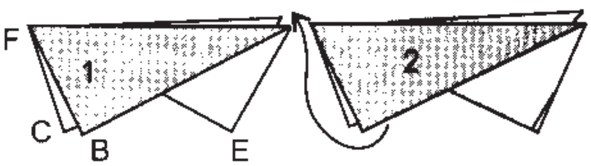


Fig. 7.54

Place No.2 inside the folds of No. 1

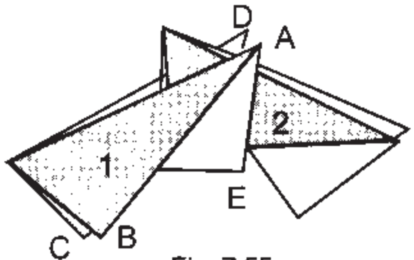


Fig. 7.55

Tuck A, D inside No.2

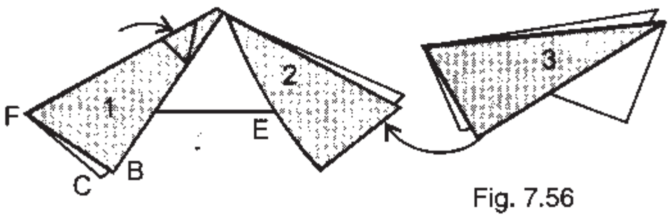


Fig. 7.56

Fold to a side

Place 3 into 2

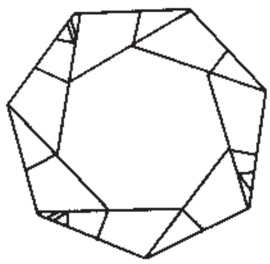


Fig. 7.57

Repeat the same with 5 remaining units

You will be getting two Heptagons.

One inside and another outside.

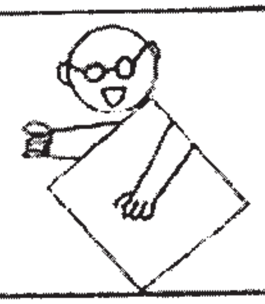
This can be thrown like a Frisbee. Have Fun

8. Fun Filled Square Paper
with Small Squares

Fun with Tea coasters

Folding a plain square paper into myriad shapes is a favorite pastime of origamians. In Spain there is traditional way of paperfolding. Sometimes it is called Moorish Tradition (Muslims of Arab descent) Moorish folders excel in creating patterns with Square paper. Due to religious restriction of non representation of living things in Art, they concentrated in creating Geometric shapes & patterns.

ORIGAMIAN SAYS



Origami Makes Learning Permanant

Sometimes a square paper coloured on one side is folded to produce a pattern. And such squares are assembled to form a screen or carpet which is hung as a decoration. The effect is astounding

Here we give four square foldings in the form if Tea coasters. Coasters are kept beneath Teacups while serving Tea.

Our purpose is to have fun and also to have a peep into maths innate in them.

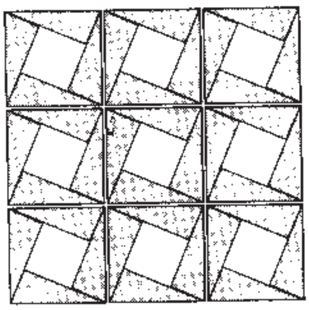


Fig. 8.1

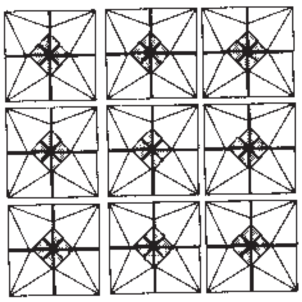


Fig. 8.2

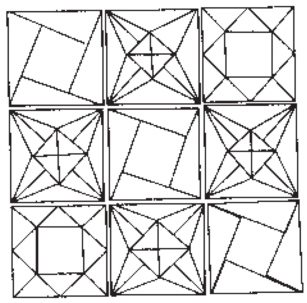


Fig. 8.3

Preparatory Folding

Start from a Square paper 20 cm x 20 cms

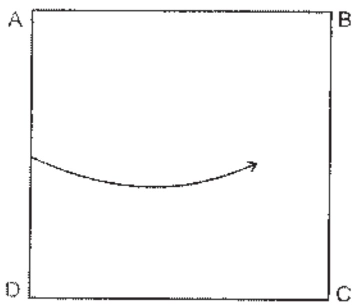


Fig. 8.4

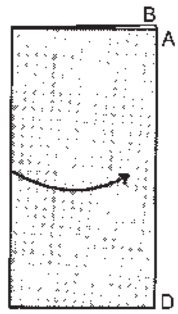


Fig. 8.5

Fig. 8.6

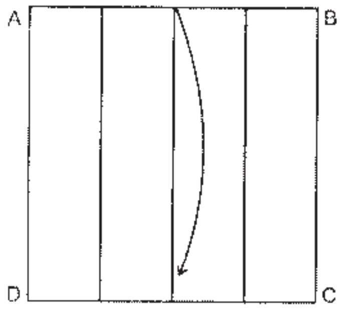


Fig. 8.7

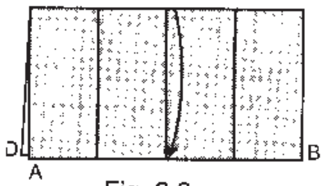


Fig. 8.8

Fig. 8.9

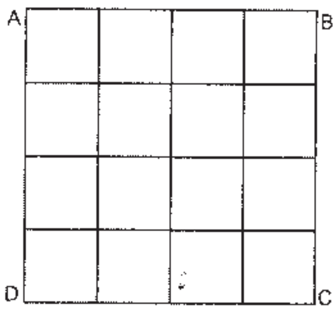


Fig. 7.6

Here you get a 4x4 grid of squares. That means Area ABCD is equal to 16 square units. The unit being the smallest square folded in Original square paper. You can create myriad patterns with these 16 Squares.

Frisbee

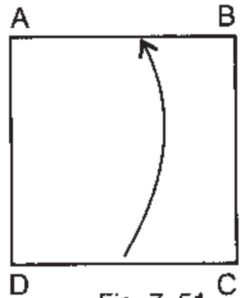


Fig. 7 .51

Take 7 squares of equal size.

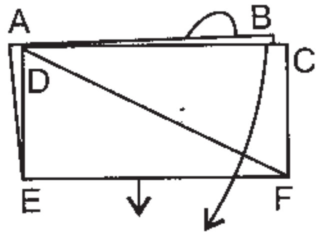


Fig. 7.52

Fold diagonals back to back

You will be surprised to know $E\hat{G}B = 128^{\circ} 57'$ the internal angle of a Heptagon.

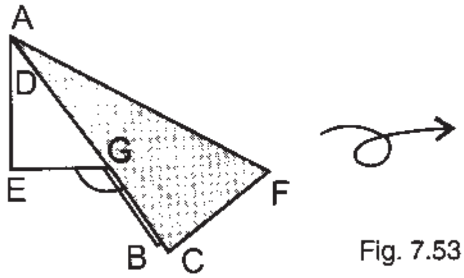


Fig. 7.53

Make seven such units from seven squares.

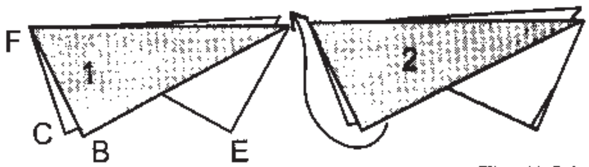


Fig. 7.54

Place No.2 inside the folds of No. 1

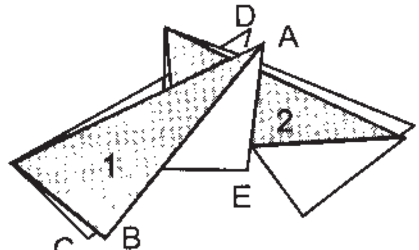


Fig. 7.55

Tuck A, D inside No.2

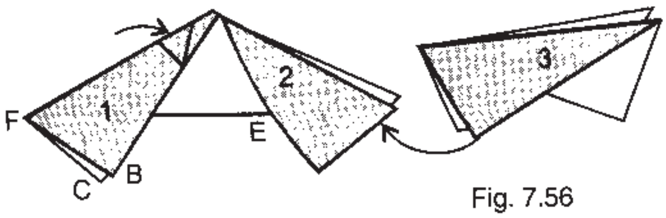


Fig. 7.56

Fold to a side

Place 3 into 2

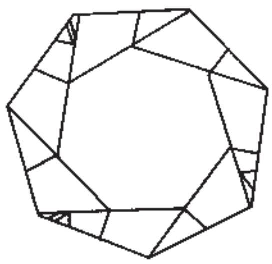


Fig. 7.57

Repeat the same with 5 remaining units
You will be getting two Heptagons.
One inside and another outside.

This can be thrown like a Frisbee. Have Fun

Take a square cut in half. Start with the Rectangle ABCD

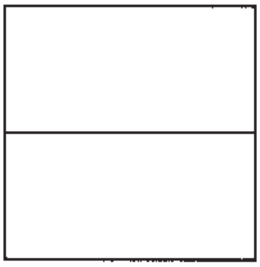


Fig. 7.38

Take one portion

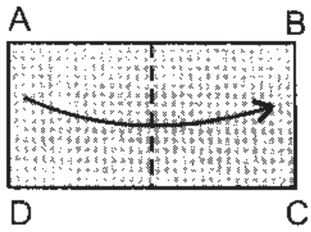


Fig. 7.39

Fold AD upon BC

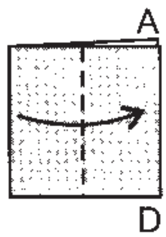


Fig. 7.40

Fold again

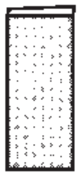


Fig. 7.41

Open

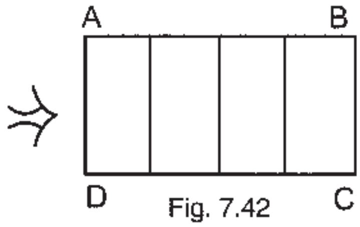


Fig. 7.42

You get four divisions

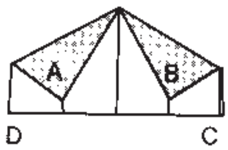


Fig. 7.43

Bring A & B to the lines

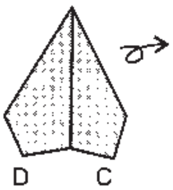


Fig. 7.44

Bring F&G on to the central line

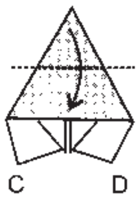


Fig. 7.45

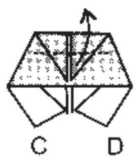


Fig. 7.46

Fold down

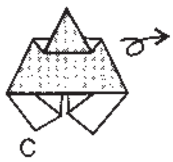


Fig. 7.47

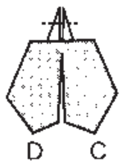


Fig. 7.48

Turn over

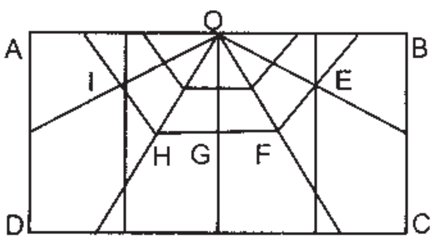


Fig. 7.50

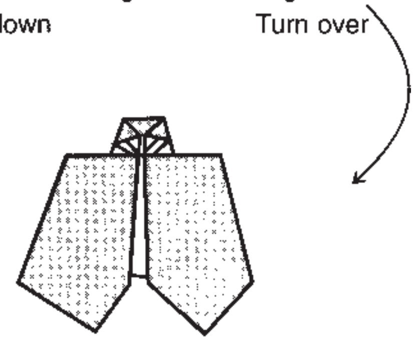


Fig. 7.49

Moth

Open this Butterfly. You see a half Hexagon, Can you find $A\hat{O}E$, $A\hat{O}F$, $A\hat{O}G$, $A\hat{O}H$, $A\hat{O}I$?

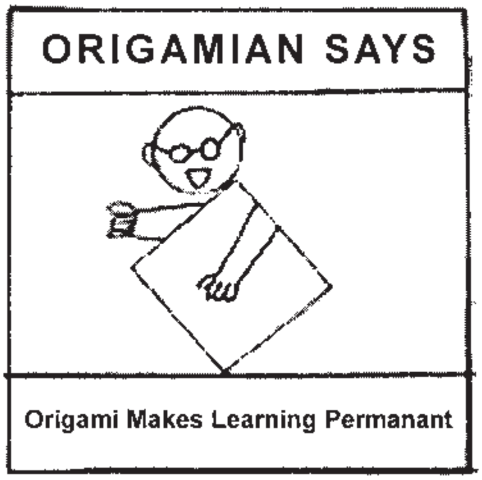
(For Ans. look to page 89)

8. Fun Filled Square Paper

with Small Squares

Fun with Tea coasters

Folding a plain square paper into myriad shapes is a favorite pastime of origamians. In Spain there is traditional way of paperfolding. Sometimes it is called Moorish Tradition (Muslims of Arab descent) Moorish folders excel in creating patterns with Square paper. Due to religious restriction of non representation of living things in Art, they concentrated in creating Geometric shapes & patterns.



Sometimes a square paper coloured on one side is folded to produce a pattern. And such squares are assembled to form a screen or carpet which is hung as a decoration. The effect is astounding

Here we give four square foldings in the form if Tea coasters. Coasters are kept beneath Teacups while serving Tea.

Our purpose is to have fun and also to have a peep into maths innate in them.

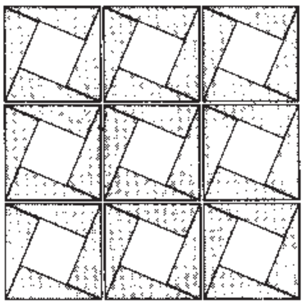


Fig. 8.1

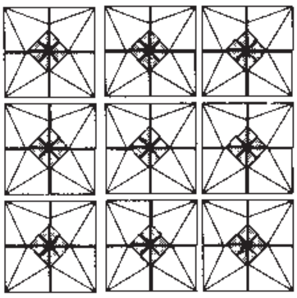


Fig. 8.2

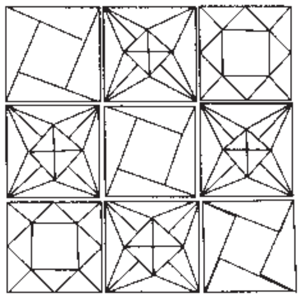


Fig. 8.3

Preparatory Folding

Start from a Square paper 20 cm x 20 cms

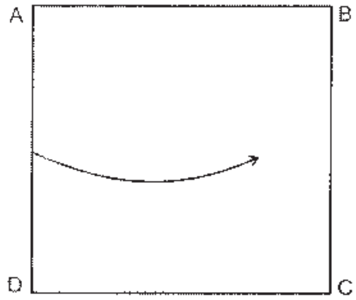


Fig. 8.4

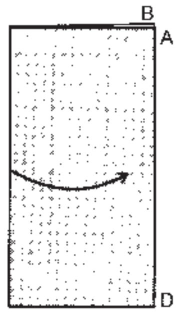


Fig. 8.5

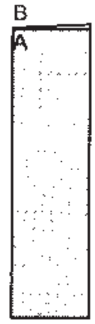


Fig. 8.6

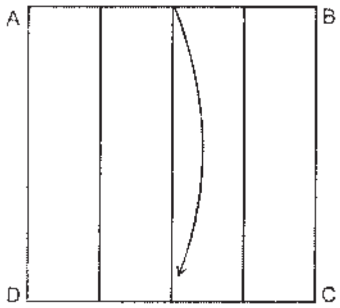


Fig. 8.7

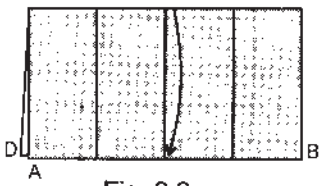


Fig. 8.8

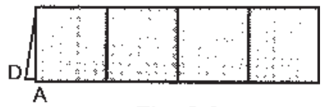


Fig. 8.9

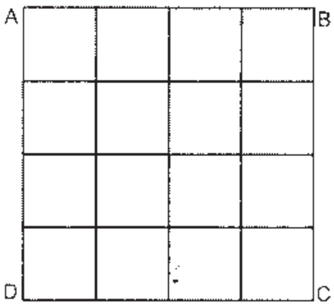


Fig. 7.6

Here you get a 4x4 grid of squares. That means Area ABCD is equal to 16 square units. The unit being the smallest square folded in Original square paper. You can create myriad patterns with these 16 Squares.

Tea coasters-1

Take a square. Mark ABCD on both sides. Using one side coloured paper will be better. Fold BC parallel to AD so that AB=BE crease EF.

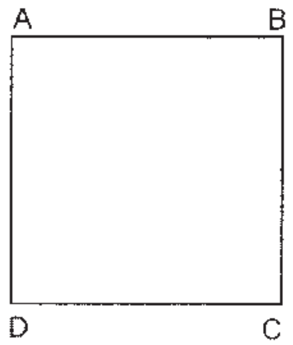


Fig. 8.11

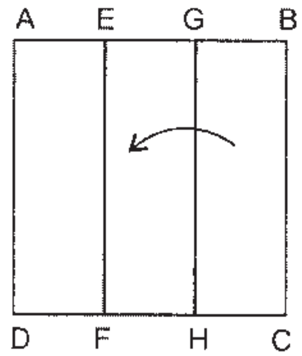


Fig. 812

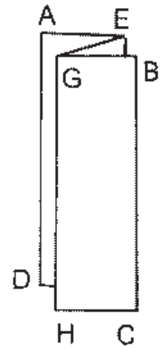


Fig. 8.13

This can be done in Zigzag way also. The three sections should fall equally upon one another. Crease EF & GH

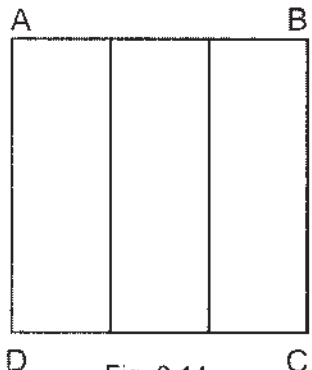


Fig. 8.14

You get this square.

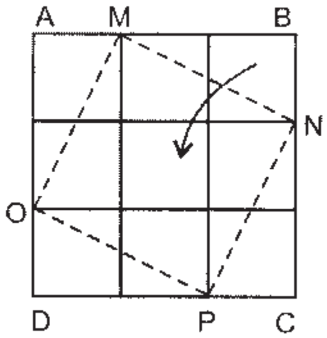


Fig. 8.15

Fold the square across to get 3x3 square grid.

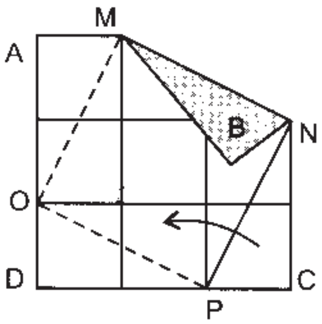


Fig. 8.16

Join MN, NP, PO & OM by folding

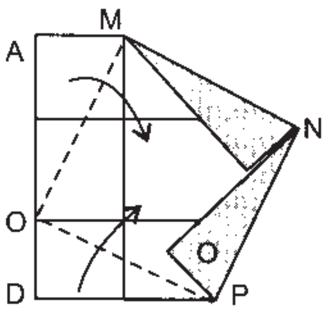


Fig. 8.17

After folding three more times

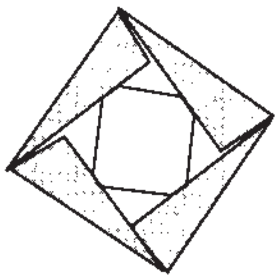
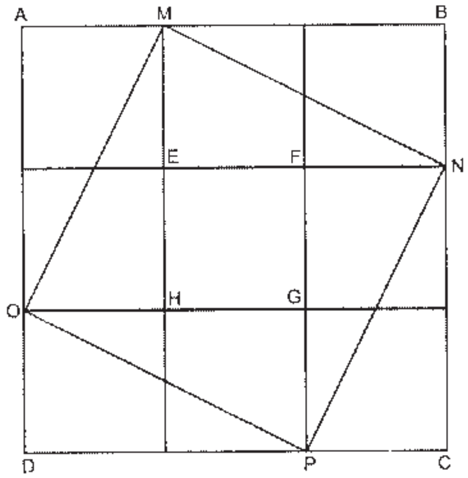


Fig. 8.18

You get a Tea coaste

. . .open this Tea Coaster to find Maths



Mark M, N, O, P. You can draw on all creases with a Pen. Mark points EFGH. Can We find area of MNOP?

Yes we can.

Area MNPO = Δ MEN+ΔNFP+ΔOGP +ΔOHM+ □EFGH

Here EFGH is one square unit in Area. Observe that MN is a diagonal of Rectangle MBNE.

This Rectangle has two squares in it. MN being the diagonal, it bisects this Rectangle.

Therefore ΔMEN = 1/2MBNE= 1 Square unit in Area.

Similarly the areas of Δ NFP, Δ OGP, Δ OHM are one square each.

∴ MNOP = 4 square Unit + □EFGH = 4+1=5 Square units.

Note : Some times Origami models like Boat, have tessellations of right angled triangle instead of squares, In such a case we can count area in terms of triangle units, also. The result will be the same.

MNOP = 5 sq.unit will be equal to 10 Triangle units.

Tea Coaster - 2

Start with a square paper 20cm X 20 cms mark ABCD on both sides.

Results using Paper where one side is coloured look better.

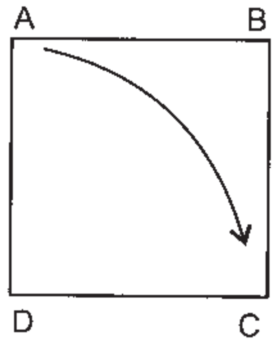


Fig. 8.19
Bring A to C

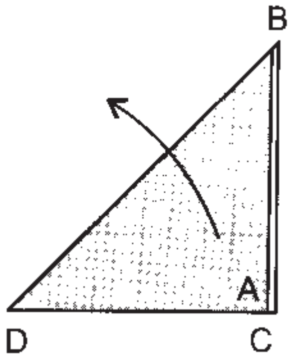


Fig. 8.20
Unfold

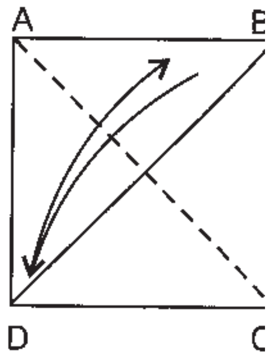


Fig. 8.21
Bring B to D & back

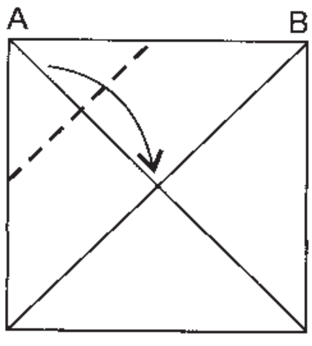


Fig. 8.22
Fold A to Centre

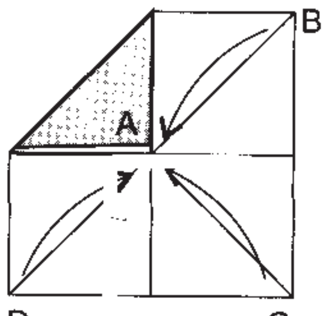


Fig. 8.23
All points to centre

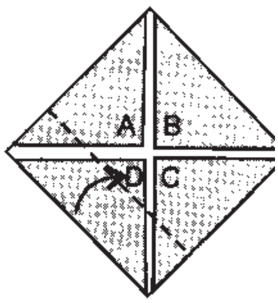


Fig. 8.24
Fold

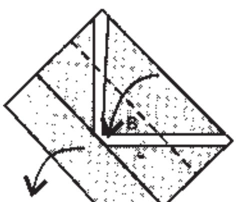


Fig. 8.25

Fold half
towards centre

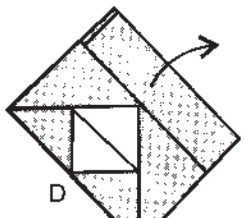


Fig. 8.26

Fold D back

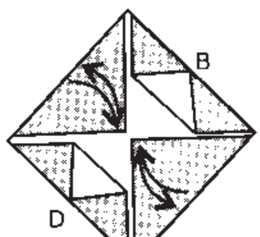


Fig. 8.27

Fold B back Repeat the same
on the other two sides

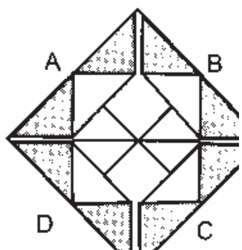


Fig. 8.28

Tea coaster 2

Maths inside Tea Coaster - 2

The Tea Coaster when unfolded looks like this.

How many square units are filled in ABCD?

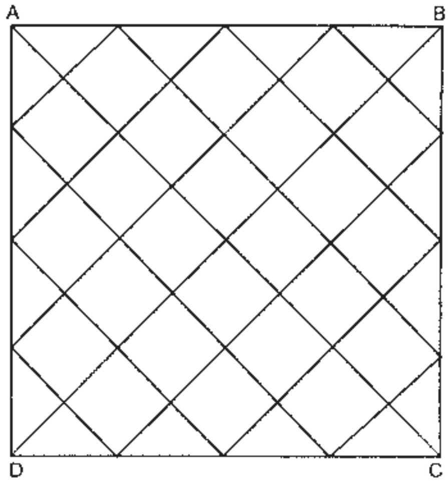


Fig. 8.29

Here we have numbered all the full squares.
They are 24 in number.
What about the remaining portion?
How many unit square areas they cover?

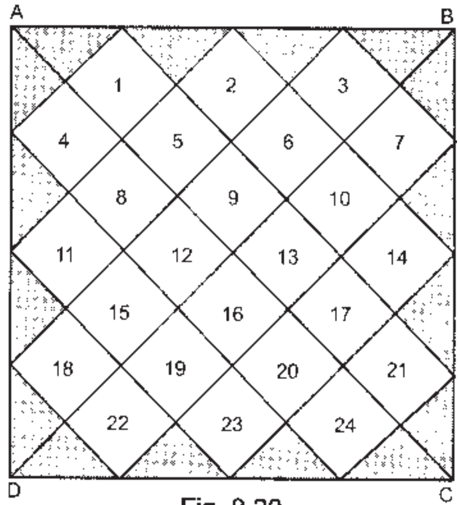


Fig. 8.30

Tea Coaster -3

We start with 4x4 grid folded from s square cut from A4 paper sheet.

We have already seen it in page 21. Mark ABCD, PQRS

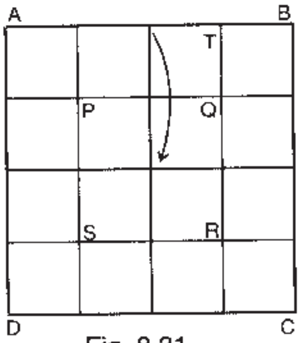


Fig. 8.31

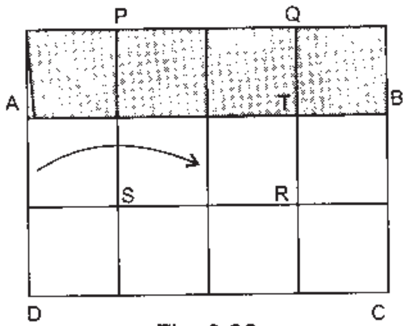


Fig. 8.32

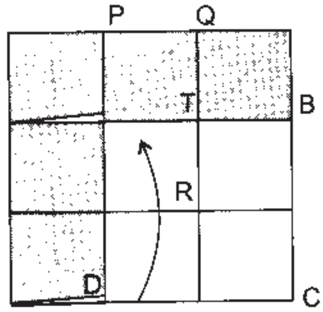


Fig. 8.33

Fig. 8.34

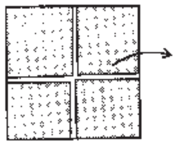


Fig. 8.35

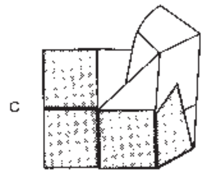


Fig. 8.36

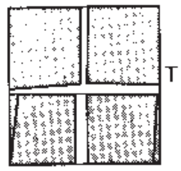


Fig. 8.37

Now unfold B and fold it inside T, so that T square comes up.

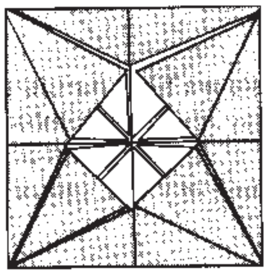


Fig. 8.38

The completed fold at step T is called Purse.

Small things like buttons, Pins, needles can be kept inside.

No. 8 shows Tea coaster which we get by pressing four flaps (like B) diagonally.

We have shown 8 & 9 in larger size.

What is the area of Tea coaster compared to the Original area of one square?

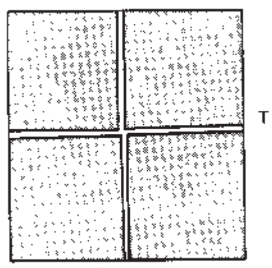


Fig. 8.39

(For Ans. look to page 89)

9. Fun Beyond Measure

☺ **Tessellating a Square**

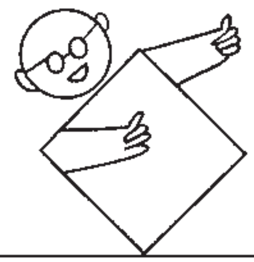
☺ **Formula for Area of Rectangle**

☺ **Formula for Area of Triangle**

☺ **How $2^0=1$**

☺ **To get $\pi = \frac{22}{7}$**

ORIGAMIAN SAYS



Origami Improves Creative Ability

Fun beyond "Measure"

Measuring a Geometric shape - that is finding its Area is an interesting field in Mathematics.

Area is measured in Square units. Why?

To cite a Rectangle we require two elements - Length and Breadth.

To cite a Triangle we require its base and height.

But look to square. Its length(base) is equal to its breadth (height). This is the advantage. We can describe a square by indicating one dimension only.

If we use metric system, area is always squares of one cm units in length.

But to compare areas of geometrical figures, we need not measure them in squares of unit lengths, only.

Why not Tessellate geometric figures with some other figures of equal size?

That means we can fill up two large squares with Triangles, Rectangles or even small squares all of equal size and then compare them. But in paper dividing an area in to squares is easy.

Fun Beyound Measure

In Mathematics this process of filling a given geometric figure with some other geometric figures of smaller but equal size is called tessellation.

To cite an example.

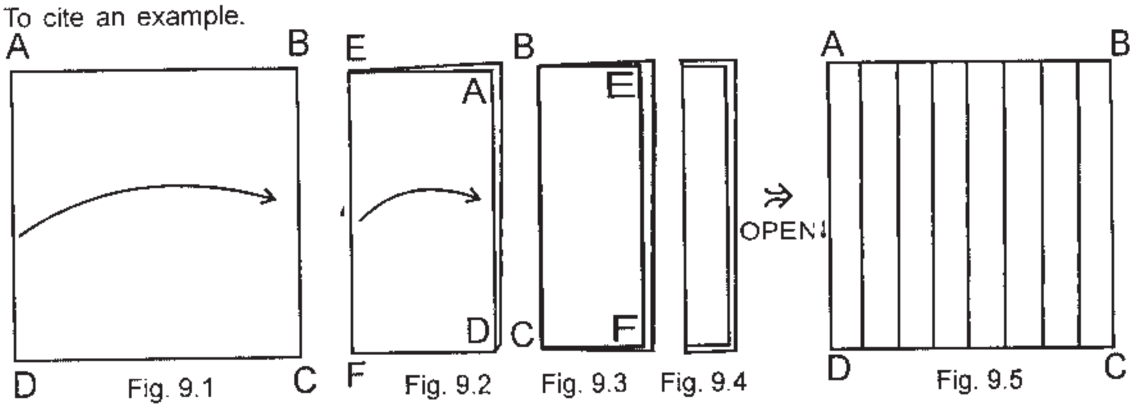


Fig. 9.1 Fig. 9.2 Fig. 9.3 Fig. 9.4 Fig. 9.5

Take a square ABCD fold it in halves thrice. Open the same to see 8 Rectangles. We say ABCD is divided into 8 rectangles.

When we repeat the folds vertically, we get 64 squares in ABCD.

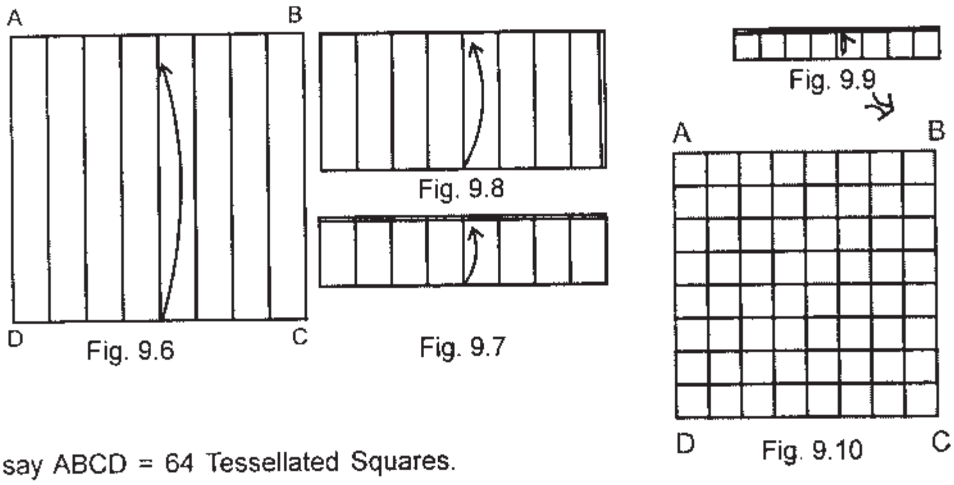


Fig. 9.6 Fig. 9.7 Fig. 9.8 Fig. 9.9 Fig. 9.10

We say ABCD = 64 Tessellated Squares.

There is no need to measure how many cms these small squares measure.

But if we can count them, we can halve ABCD.

We can divide the Area ABCD by just counting and rearranging the small square into required number of parts.

For example to divide ABCD into 8 parts, count 64/ 8 = 8 squares in any manner you like and cut them off. Each piece will have 8 squares, that means each having 1/8 ABCD in area.

A funny way to get formulae for Area of Rectangle

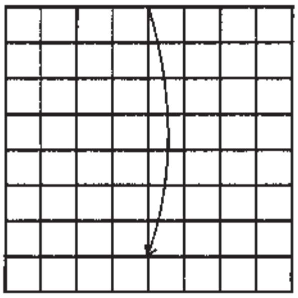


Fig. 9.11

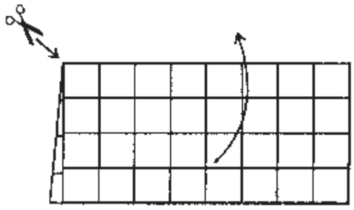


Fig. 9.12
Cut off as shown

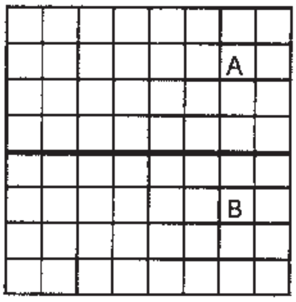


Fig. 9.13
Mark A & B

Start with a 30x30 cm square paper.
Fold the square vertically and horizontally thrice,
You get a Tessellated 64 square paper fold as shown here.

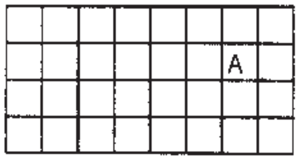


Fig. 9.14
Keep A sepcrately
continue with B

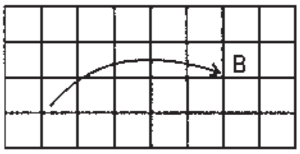


Fig. 9.15
Fold 'B' into half

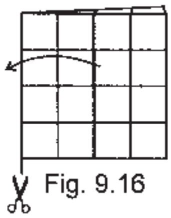


Fig. 9.16

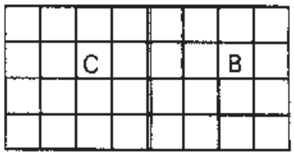
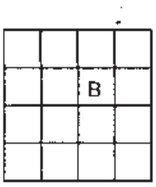


Fig. 9.17
Mark 'C'

Each time you fold in half tear away one portion. That is how you can get A, B, and D, E pieces.



Keep B seperately. Continue with C

Fig. 9.18

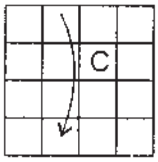


Fig. 9.19

Fold 'C' in half and cut

Mark D

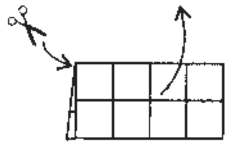


Fig. 9.20

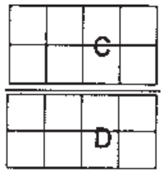


Fig. 9.21

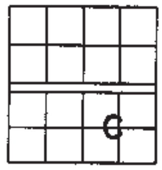


Fig. 9.22

Keep C separately, continue with D

Fig. 9.23

Fold D & cut

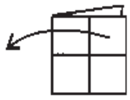


Fig. 9.24

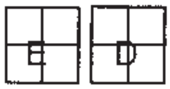


Fig. 9.25

Make E

Fig. 9.26

Fig. 9.27

Count number of unit squares in ABCD.

Also count number of squares along length and breadth.

Fill the details in the Table.

	Geometric shapes	Length	Breadth	Total No. of Squares insidethe shape	LxB
A	Rectangle	8	4	32	32
B	Square	4	4	16	16
C	Rectangle	4	2	8	8
D	Square	2	2	4	4

You will observe that total number of squares inside a rectangle or a square being equal to L × B

Hence Area of Rectangle = L x B= Length x Breadth

Paper folding to confirm the formula for Area of a Triangle

Area formula for triangle = 1/2 Base x Height.

We prove this formula in class room, and workout many problems.

Here is a simple paper folding method to give proof.

We are required to fold 64 squares in a particular way for this.

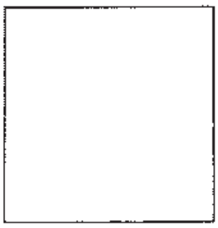


Fig. 9.28
Start 10"x10" paper

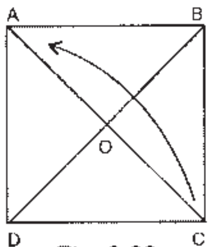


Fig. 9.29
Fold Diagonals

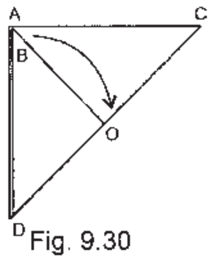


Fig. 9.30
Fold C to A

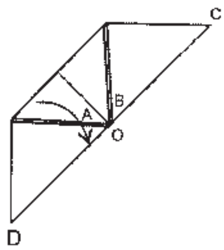


Fig. 9.31
Fold A & B to 'O'

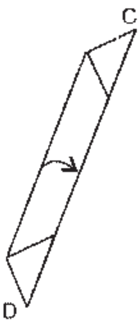


Fig. 9.32
Fold twice horizontally

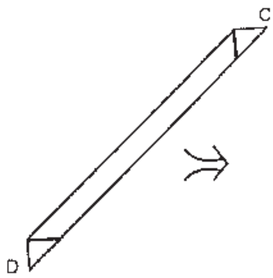


Fig. 9.33
Open

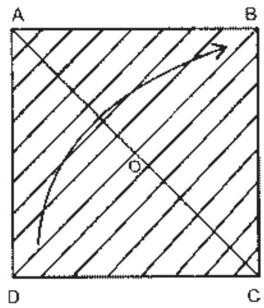


Fig. 9.34
Repeat steps 1-6 on the other diagonal AC.

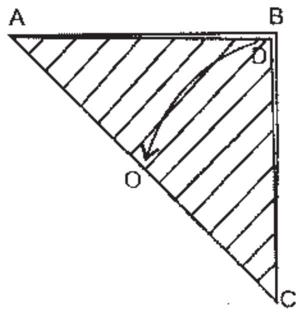


Fig. 9.35
Fold to D to B

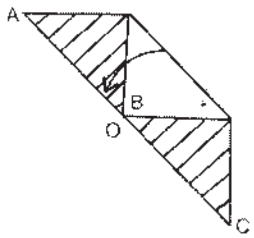


Fig. 9.36

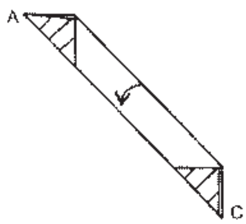


Fig. 9.37

Fold twice horizontally

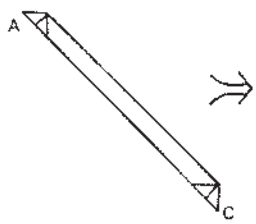


Fig. 9.38

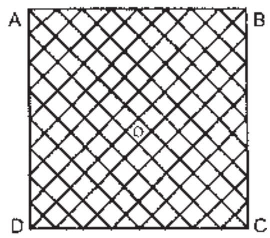


Fig. 9.39

Unfold to get a grid

The square ABCD is now filled with many small squares & triangles. Fold D to B. Fold X, Y (This is arbitrary) so that XYC becomes a scalene triangle

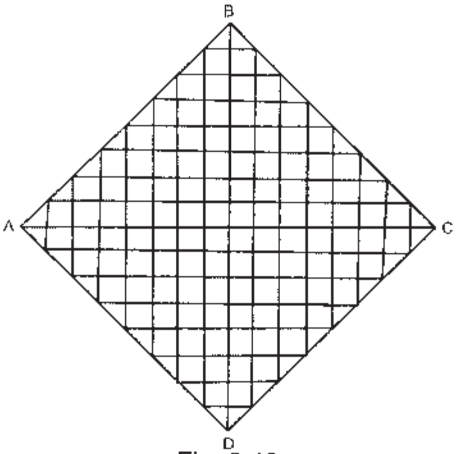


Fig. 9.40

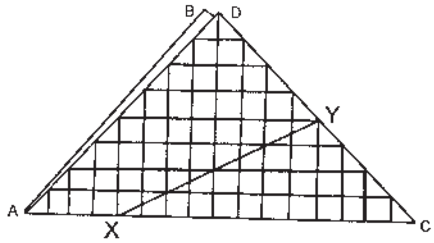


Fig. 9.41

Now what is the Area of XYC? Look at Fig 13, which is Enlarged Drawing.
We can actually count number of squares inside XYC.

Full Squares	= 18
4 Triangles	= 2
Broken squares upon XY	= 4

(Each broken part has its counter part- match them)

Total	= 24

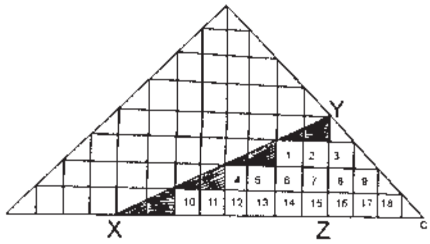


Fig. 9.42

Now Measure XC = 12 square lengths

YZ = 4 squares lengths

Area of XYC = ½ Base Height

$= \frac{1}{2}(XC)\cdot(YZ) = \frac{1}{2}(12)(4)$

= 24 squares

Notice that the actual count of squares is also 24.

Please note that :

a) We have not used a scale to measure the length
b) The length is measured in terms of squares of equal size filling the triangle
c) In the same method find formulae for Trapezium & Parallelogram

What's Fun in folding paper anyway? To Show $2^0=1$

Yes there is much to know about folding itself.

Take a paper ABCD. Fold it inhalf. unfold, What you see in fig 3. ABCD has been divided into two parts 1 & 2.

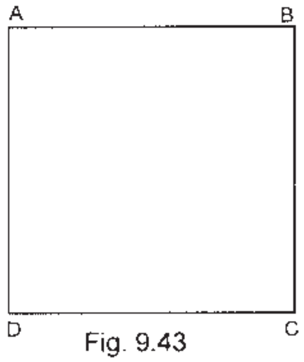


Fig. 9.43

Fig. 9.44

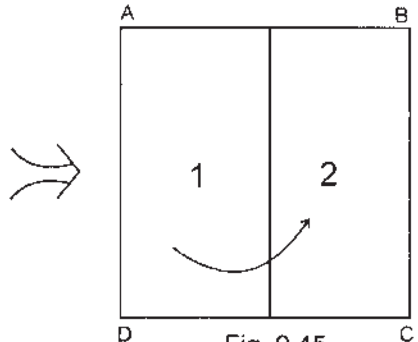


Fig. 9.45

Fold again. Start from Fig. 9.44. Fold in half. Open and look at it. You get 4 parts (Fig. 9.48) Let us do this process again and again and make a table.

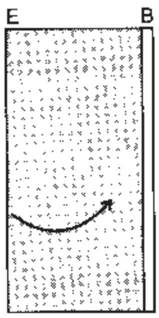


Fig. 9.46

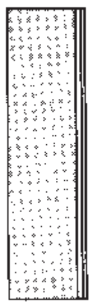


Fig. 9.47

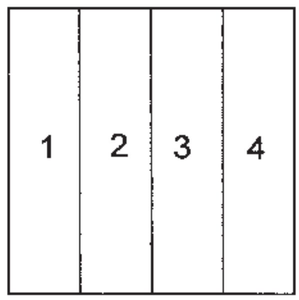


Fig. 9.48

No. of folds	No. of parts	
1	2	2^1
2	4	2^2
3	8	2^3
..	..	..
..	..	..
n		2^n

The relationship is obvious.'n' folds produce 2^n parts.

Here is the catch. What happens when we DO NOT Fold, there will only be one part. Therefore **$2^n=2^0=1$**

To find the value of π through Paper folding

Measuring circumference and diameter of a given circle is challenge in class room. Many a time their ratio will not be $\pi = 22/7$, hence unsatisfactory to the students. Here is a method which gives value for π.

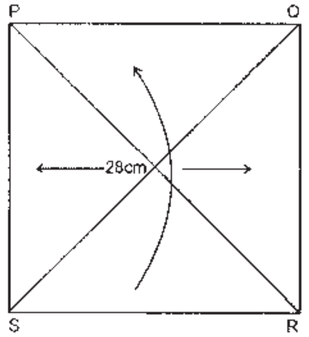


Fig. 9.49

Start with a square paper with 28cm side. Fold the diagonals

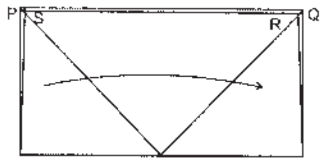


Fig. 9.50

Lay SR upon PQ and crease. Fold PS on RQ.

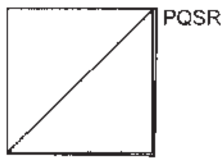


Fig. 9.51

Now the paper is folded to its 1/4 size.

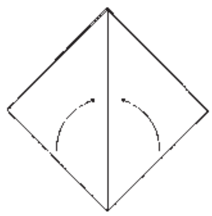


Fig. 9.52

Fold the two sides to the central line.

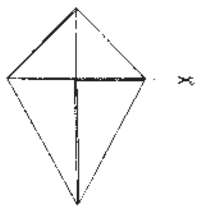


Fig. 9.53

Cut off the triangular portion.

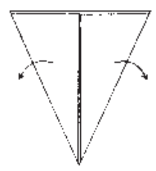


Fig. 9.54
Unfold ,mark ABCD

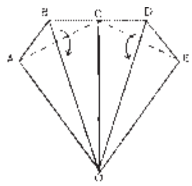


Fig. 9.55
Unfold CDE

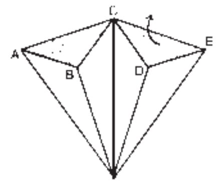


Fig. 9.56
Fold B, D down so that AC, CE are creased triangles ABC and CDE are formed

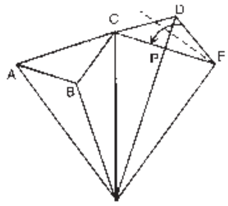


Fig. 9.57

Fold D to CE now DEC gets bisected and OD is cut at P

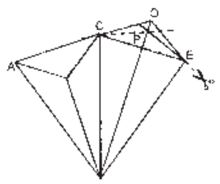


Fig. 9.58

Cut off EP,PC. Repeat same cuts in triangle ABC

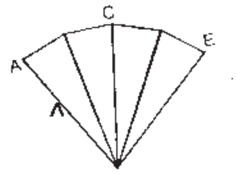


Fig. 9.59

Unfold this Diamond shape

The finished Shape is 16 sided Polygon, which looks almost a circle. Now measure any side of this Polygon. The side will be 5.5cm. Hence approximating this polygon to a circle we get,

$$\frac{Circumference}{Diameter} = \frac{5.5x16}{28} = \frac{88}{28} = \frac{22}{7} = \pi$$

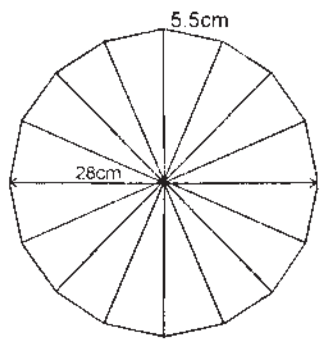


Fig. 9.60

10. Algebraic Identities and Origami Models

☺ $(a-b)^2 = a^2-2ab+b^2$

☺ $(a+b)^2 = a^2+2ab+b^2$

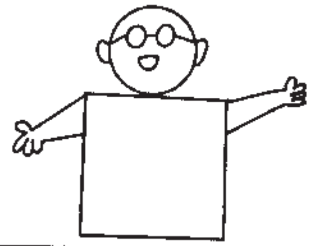
☺ $(x+a)(x+b)=x^2+x(a+b)+ab$

☺ $(x+a)(x-b)=x^2+x(a-b)-ab$

☺ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

☺ $(a^2-b^2) = (a+b)(a-b)$

ORIGAMIAN SAY



Origami Developes Spirit to Innovate

Algebraic Identities

Number is dimensionless. We said earlier that we indicate any quantity - be it money, ar volume, length, breadth, weight, etc., with numbers.

And number is an abstraction.

Algebra is a step further.

The abstract number is represented by a letter 'a' or 'x' or 'y' or 'z'..............

Algebra is an important branch of Mathematics. It provides generalised formulae for multiplicat for different quantities.

Many times it is confusing to multiply all these quantities.

Besides, conceptual clarity is difficult to get.

Here is a funny way wherein, you make an origami model like a boat, a cock or a lamp a arrive at Algebraic Identities like $(a+b)^2$, $(a-b)^2$ and $(a+b+c)^2$.

We have to note that while 'a' in algebra represents a quantity, it also represents a length Geometry. Similarly a^2 is a x a as well as a square figure with side = a in length.

Rooster and $(a-b)^2 = a^2-2ab+b^2$

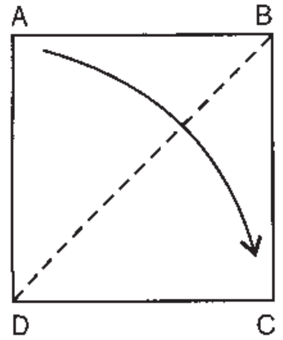


Fig. 10.1
Take a Square paper

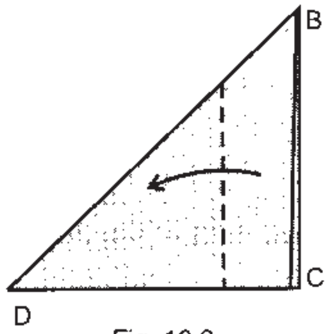


Fig. 10.2
Fold A to C. Fold a small vertical portion to the side.

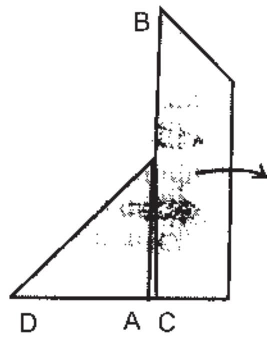


Fig. 10.3
Unfold

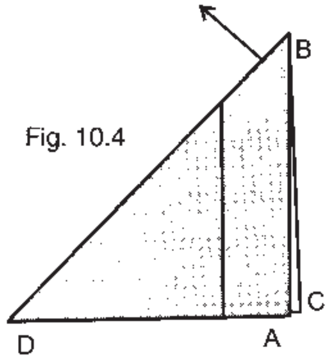


Fig. 10.4
Reverse Fold at B

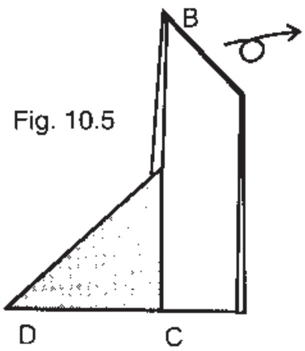


Fig. 10.5
Trun over

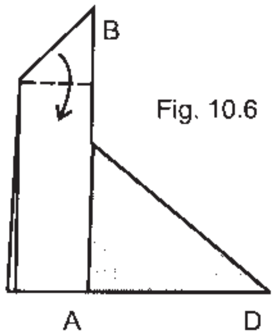


Fig. 10.6
Crease 'B' Down

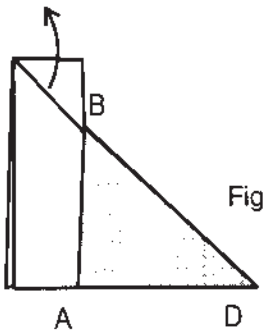


Fig. 10.7
Reverse fold at B

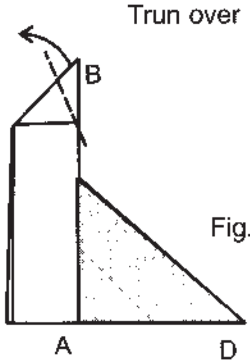


Fig. 10.8
Fold B up

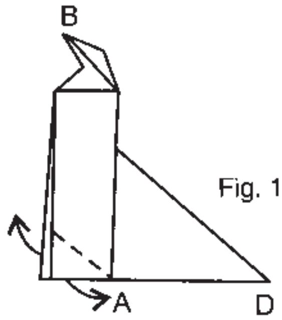


Fig. 10.9
Pull Sides

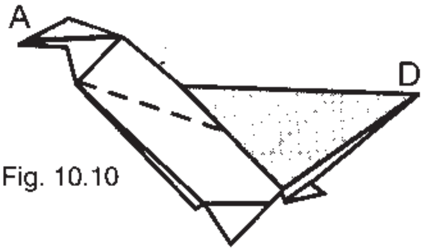


Fig. 10.10
Sink the neck to the sides

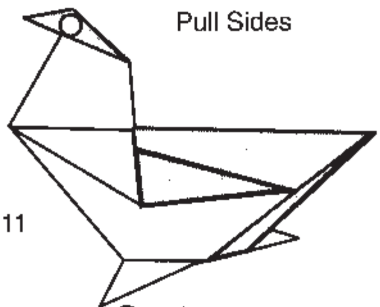


Fig. 10.11
Rooster

10. Algebraic Identities and Origami Models

☺ $(a-b)^2 = a^2-2ab+b^2$

☺ $(a+b)^2 = a^2+2ab+b^2$

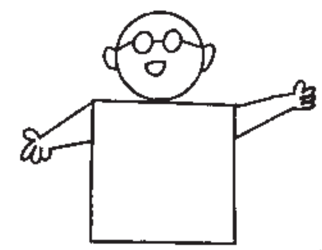
☺ $(x+a)(x+b) = x^2+x(a+b)+ab$

☺ $(x+a)(x-b) = x^2+x(a-b)-ab$

☺ $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

☺ $(a^2-b^2) = (a+b)(a-b)$

ORIGAMIAN SAYS



Origami Developes Spirit to Innovate

Algebraic Identities

Number is dimensionless. We said earlier that we indicate any quantity - be it money, area, volume, length, breadth, weight, etc., with numbers.

And number is an abstraction.

Algebra is a step further.

The abstract number is represented by a letter 'a' or 'x' or 'y' or 'z'...............

Algebra is an important branch of Mathematics. It provides generalised formulae for multiplication for different quantities.

Many times it is confusing to multiply all these quantities.

Besides, conceptual clarity is difficult to get.

Here is a funny way wherein, you make an origami model like a boat, a cock or a lamp and arrive at Algebraic Identities like $(a+b)^2$, $(a-b)^2$ and $(a+b+c)^2$.

We have to note that while 'a' in algebra represents a quantity, it also represents a length in Geometry. Similarly a^2 is a x a as well as a square figure with side = a in length.

Unfold this Rooster. You get a square with these lines.

Mark MNOP and Q. Ignore other lines.

We have squares. MBNO, POQD; Rectangles AMOP, ONCQ

Let AB=a, MB=b

Then we have AM = PO = (a-b) ;AMOP=AM.MO=(a-b). b

NC=OQ=(a-b)   MBNO=b^2

Now Area of POQD = Area ABCD - Area AMOP - Area ONCQ - Area MBN

$(a-b)^2 = a^2 - (a-b)\,b - b(a-b) - b^2$

$= a^2 - ab + b^2 - ab + b^2 - b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

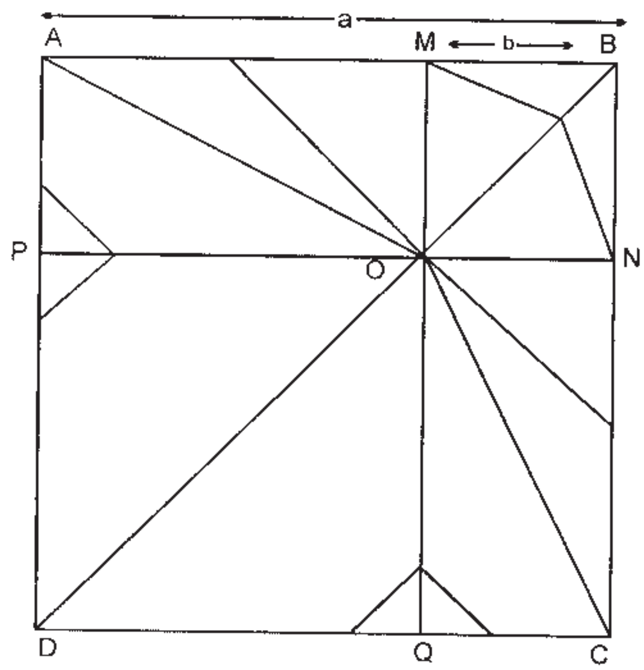


Fig. 10.12

Sail Boat and $(a+b)^2 = a^2+2ab+b^2$

We have folded a sail boat in page 34.

Do the same now.

Start from a Square 15x15 cm and get a boat.

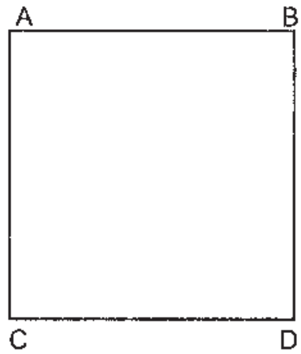


Fig. 10.13

Unfold the Boat. You get lines as shown in Fig. 3. mark ABCD, MNOPQ.

Let AM= a, MB = b

Then AB=AM+MB=a+b=AP+PD

Area ABCD = Square AMOP+Square OQCN + Rectangle MBQO+ Rectangle POND

$$AB \times AD = a^2 + b^2 + ab+ab$$

$$(a+b)(a+b) = a^2+2ab+b^2$$

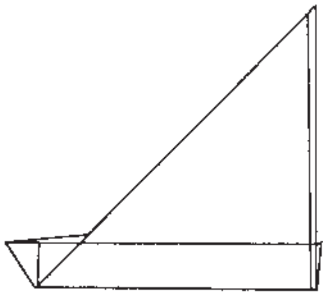


Fig. 10.14

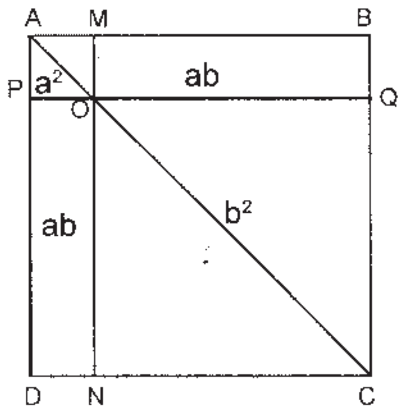


Fig. 10.15

Half Tray and Algebraic Identity $(x+a)(x+b)=x^2+x(a+b)+ab$

Start with a Rectangular paper 10x5 cms.

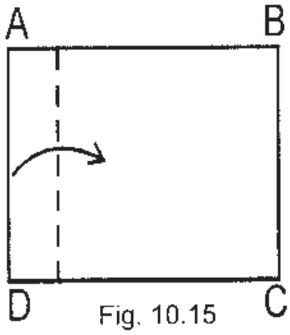


Fig. 10.15

Fold 2cms to a side

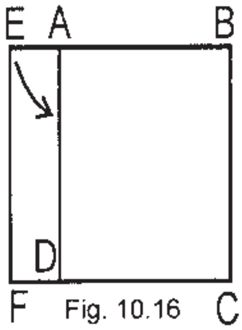


Fig. 10.16

Lay EA on AD

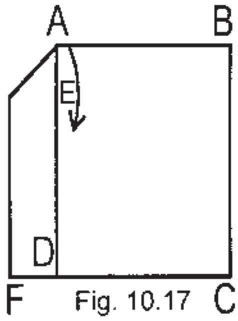


Fig. 10.17

Fold AB down

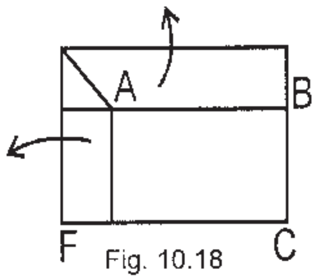


Fig. 10.18

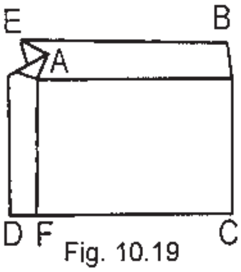


Fig. 10.19

Lift EB & AD
You get a projaction at A

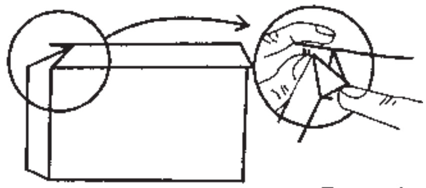


Fig. 10.20

Press A

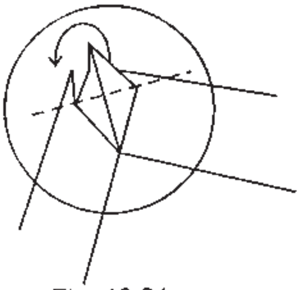


Fig. 10.21

Fold behind

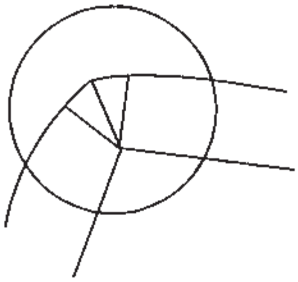


Fig. 10.22

Corner gets locked

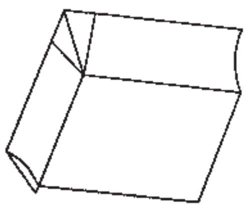
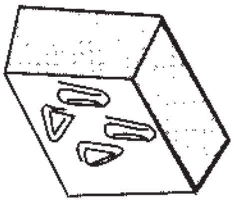


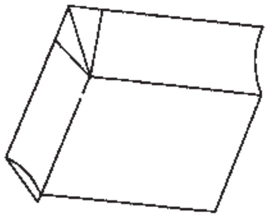
Fig. 10.23

Fig. 10.24



This is half tray used to keep Dot pins, Paper clips etc

Half Tray and - $(x+a)(x+b)=x^2+x(a+b)+ab$(contd...)



When we open this Half Tray we see these lines.

Fig. 10.25

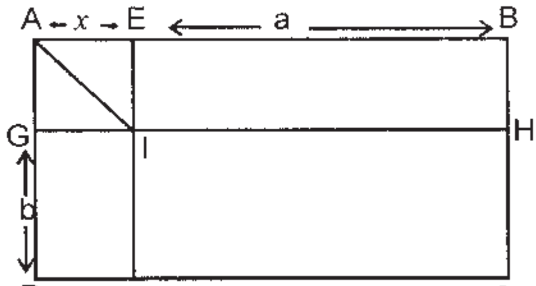


Fig. 10.26

The Rectangle ABCD is divided into four major portions.

Let AE=x EB=a GD=b AG=x

Area of ABCD = AB x AD = (AE+EB)(AG+GD)

= (x+a)(x+b)=AEIG+EBHI+GIFD+IHCF

= $(x+a)(x+b)=x^2+ax+bx+ab=x^2+x(a+b)+ab$

Grocers Paper Packet and $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Suppose you go to a Village Grocery store and ask for 2gm of Asafoetida. The shopkeeper puts 2 gms of smelly substance on a square paper and quickly folds it into a packet and gives it you. The same Type of packet we shall fold to get $(a+b+c)^2$

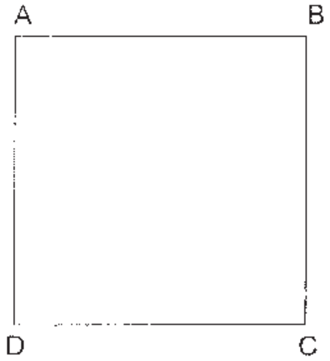


Fig. 10.27

Start from a Square

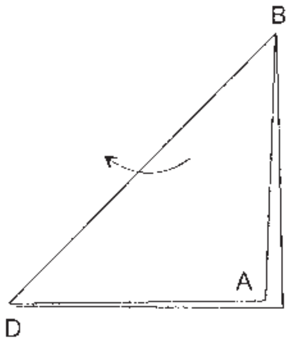


Fig. 10.28

Fold Diagonal

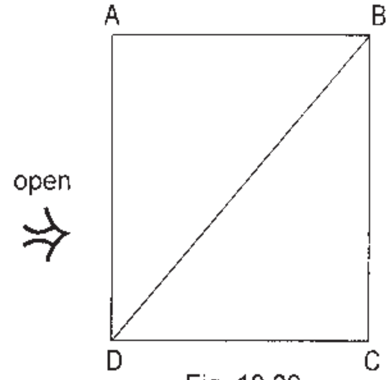


Fig. 10.29

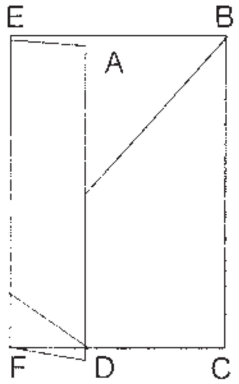


Fig. 10.30

Fold EAFD arbitrarily

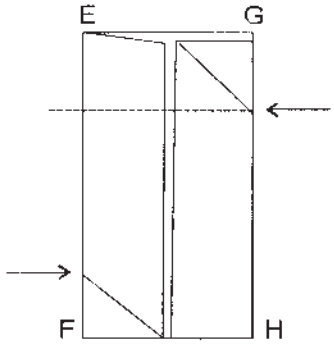


Fig. 10.31

Fold BC upon AD

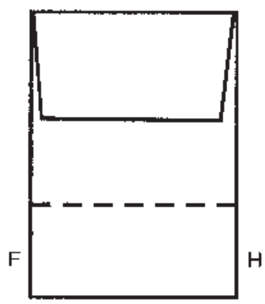


Fig. 10.32

Fold EG Down

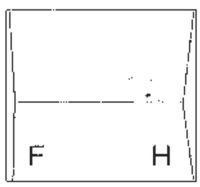


Fig. 10.33

Fold FH upon EG

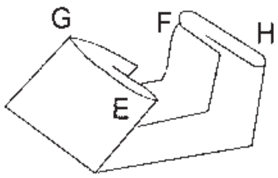


Fig. 10.34

It looks like
this side ways

Fig. 10.35

Tuck one inside the other

Paper Packet and $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$ (contd...)

When you unfold this paper packet you see these lines.

Here Let AE =a; EG=b; GB=c

Then AB = AE+EG+GB

= (a+b+c)

Area of ABCD = AB x AD

= $(a+b+c)^2$

= $a^2+b^2+c^2+2ac+2ab+2bc$

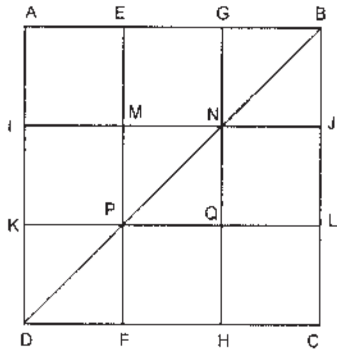


Fig. 10.36

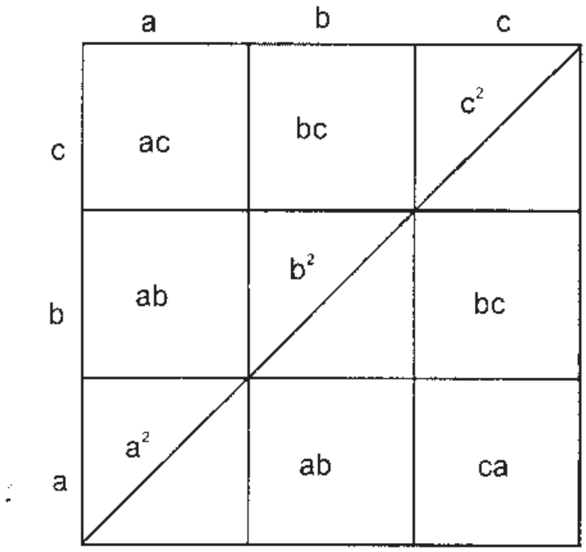


Fig. 10.37

Algebraic Identites - Two Puzzles

Almost all identities studied up to High School can be visualised as partitions of areas in a Square sheet of paper. Without much effort the square can be folded to segregated areas. It will be a good exercise to hone student's skills to solve these two puzzles.

a) You are given a square paper ABCD. Fold lines to illustrate $(x+a)(x-b) = x^2 + x\,(a-b) - ab$.

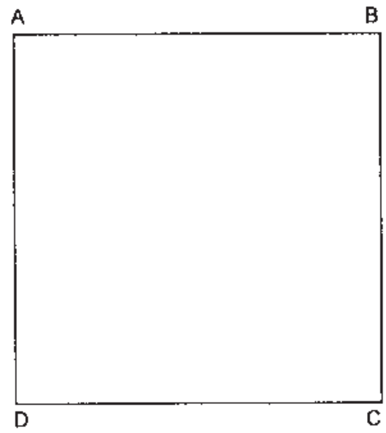


Fig. 10.38

b) In the square ABCD, fold creases to show $(a+b)(a-b) = a^2 - b^2$.

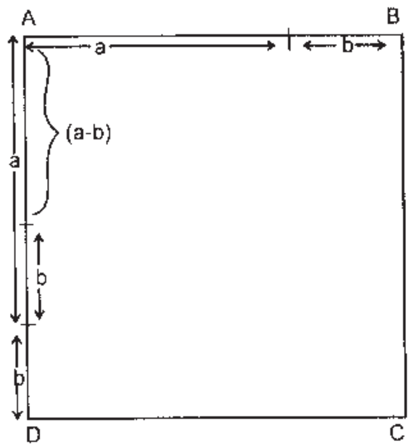
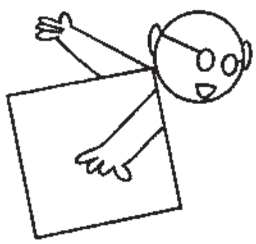


Fig. 10.39

11. Theorems on Triangles in Origami Models

- ☺ **Pythagoras Theorem**
- ☺ **Extended to an Acute Angle**
- ☺ **Extended to an Obtuse Angle**

ORIGAMIAN SAYS



Origami Developes Observational Ability

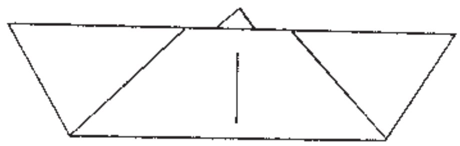
Paper Boat and Theorems on Triangles

Almost every one knows how to make a paper Boat! Origami models like Boat, Box etc have entered our folklore.

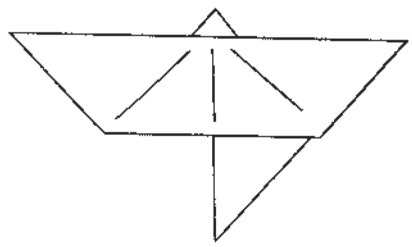
Let us illustrate the proof of various theorems by folding an ordinary boat from a Square Paper.

We have chosen ordinary boat because there are many kinds of Boats in Origami.

This chapter covers major theorems on Triangles found in our text books.



Boat



Sword Boat

Double boat

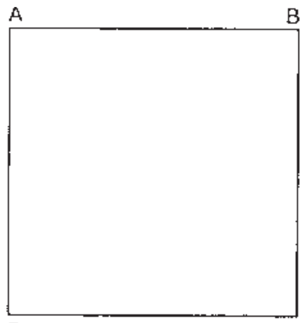


Fig. 11.1

Take a square

Fig. 11.2

Fold in half

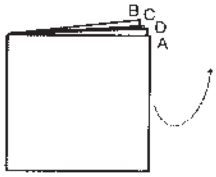


Fig. 11.3

Fold in half

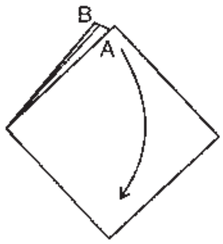


Fig. 11.4

Bring A down to front

Pythagoras Theorem states that in any Right angled triangle the square upon the hypotenuse is equal to the sum of the squares on the other two sides.

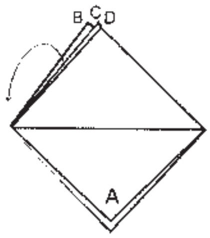


Fig. 11.5

Fold BCD to the back

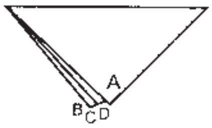


Fig. 11.6

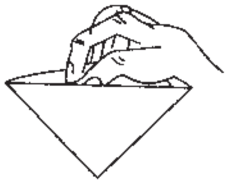


Fig. 11.7

Put finger into the pocket and press

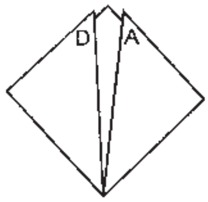


Fig. 11.8

Unfold this Boat

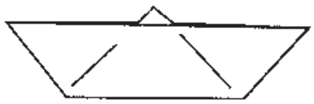


Fig. 11.9

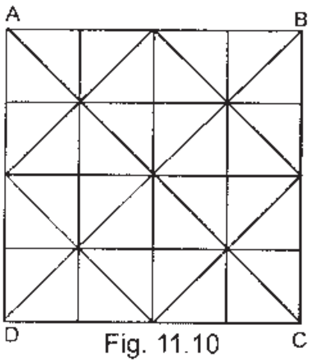


Fig. 11.10

You get this pattern in the Paper

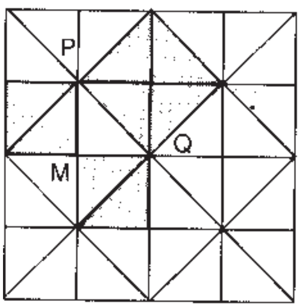


Fig. 11.11

This pattern has appeared without effort on our part. We can see squares, Diagonals, Right angle triangles etc. We can also observe that all Right angled triangles are equal in size.Now choose one Right angled Δ MPQ arbitratily. We can observe that MQ, PM and PQ have squares. upon them. PM & MQ having two RA Δs each in their squares. add up to four Right angled Δs in the square upon PQ.

This is the **Pythagorean Relationship.**

$$PQ^2=PM^2+MQ^2$$

Addendum to paper boat & Pythagoras' Theorems

Triangle illustrated in the boat is Δ MPQ in which

MP = MQ (Isosceles Right Angle Triangle)

By considering Δ MPQ as shown in Fig. 11.12

We can prove Pythagoras Theorems with a General Right Angled Triangle also.

PM and MQ are the sides and PQ is the

Hypotenuse of the triangle PMQ. We have to prove

$PQ^2 = PM^2 + MQ^2$

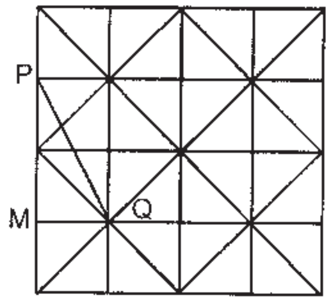


Fig. 11.12

Please observe that a square MNOP (Fig. 11.13) stands upon PM and square MQLK stands upon MQ. Considering MQLK as one unit square

We have PM^2 = MNOP = 4 units (as shown in Fig. 11.14)

MQ^2 = MQLK = 1 unit

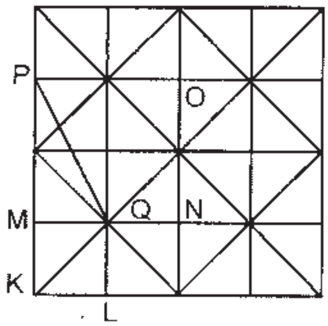


Fig. 11.13

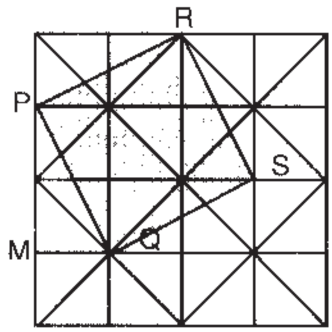


Fig. 11.14

We can construct a square upon PQ (Fig. 11.14) with points at R and S. Fold PR,RS and QS. Look at Fig.3 to see PQRS upon PQ.

You may recqllect that it is the same figure as in page 43 where you calculated area of the Tea coasters. Hence we can adopt the value we counted there. This is 5 unit square.

Hence $PM^2 + MQ^2 = PQ^2$

4 + 1 = 5 (Relation stands proved).

4 units + 1 unit = PQ^2 = 5 units.

Sword Boat

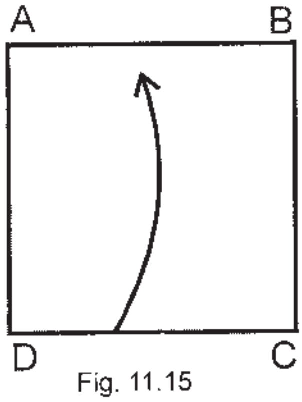


Fig. 11.15
Start from a square paper

Fig. 11.16
Fold DC upon AB

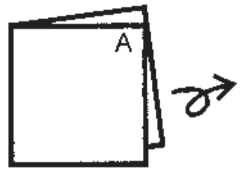


Fig. 11.17
Fold AD to BC

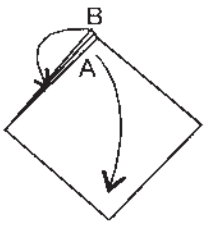


Fig. 11.18
Turn over Fold A and B down

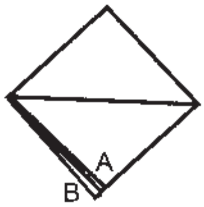


Fig. 11.19

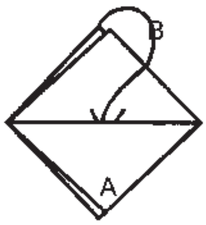


Fig. 11.20

Put CD together behind A into the pocket

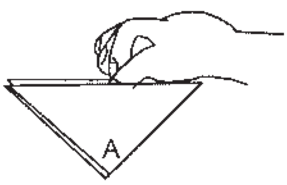


Fig. 11.21

Put thumb and press to
form a square

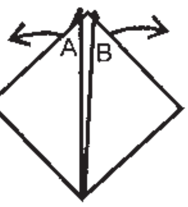


Fig. 11.22
Pull A and B

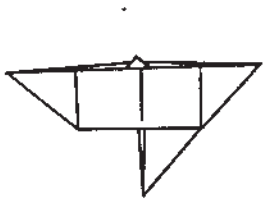


Fig. 11.23

The sword boat when unfolded gives the same 4x4 grid as ordinary boat.

Paper Boat and "Extension Theorem of Pythagoras"

We have illustrated Pythagoras theorem in the previous page. Pythagoras theorem has its extensions when the triangle is not a right angle Δ. We can prove Pythagoras Theorems for acute angle and obtuse angle triangles or Δ also, through Paper Boat folding.

For that we start with a boat. Unfold it to get this pattern.

The pattern here does not contain acute angle triangle. Hence we shall fold as follows. (Mark ABCD, P&O and start)

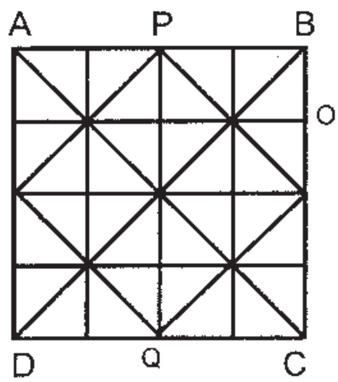


Fig. 11.24

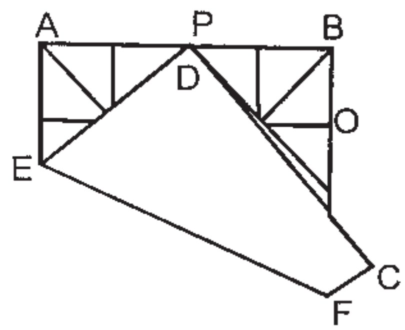


Fig. 11.25
Bring D to P and crease along EF

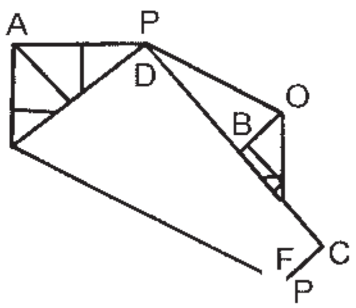


Fig. 11.26
Crease joining P and O so that PB is upon DC

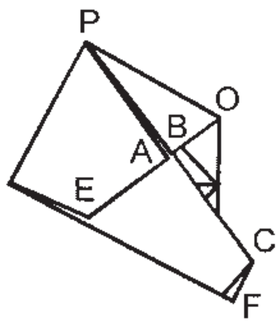


Fig. 11.27
Fold PA upon DC

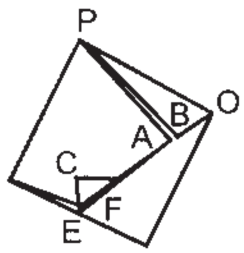


Fig. 11.28
Fold CF upon EABO

The result of these fold is to get a small square as shown in Fig. 11.28, unfold the same.

Extension of Pythagoras Theorems (Obtuse angle)

> **The Theorem States : In any triangle the square on the side opposite to an obtuse angle is equal to the sum of the squares on the other two sides twice the rectangle contained by one of these sides and projection of the other upon it.**

For clarity mark all lines with a pencil. Mark M, N, Q, R, S, T and X. Join MQ. Here Δ MQN is the obtuse angle, MX is the projection of MN upon QN. As per the theorem,

$$MN^2 = MQ^2+QN^2+2QN.XQ$$

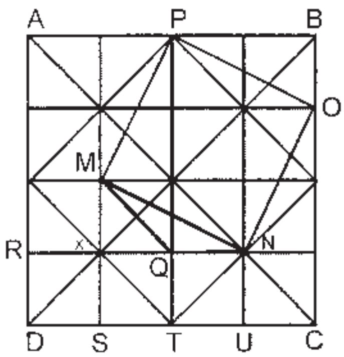


Fig. 11.29

When we unfold square from Fig. 11.28 we get this.

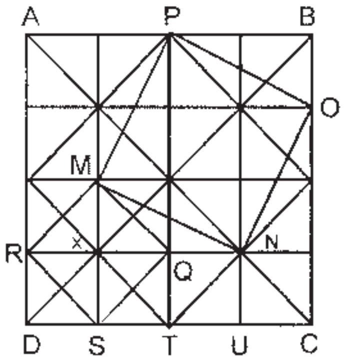


Fig. 11.30

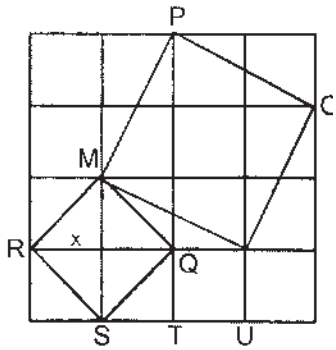


Fig. 11.31

Essential lines

Observe that square MNOP sits on MN and a square QNUT is upon QN

MQSR also forms a square. For clarity see Fig. 11.30, Where we have joined MR, RS & SQ

In Fig. 11.31 we have shown only the essential lines which you can mark by a sketch pen.

Consider square QNUT= unit square. Then square MQSR=2 units

QN.XQ= Square UNIT = 1 Unit; As already shown earlier on page 51, MNOP is the same as in Tea-coasters.

Therefore MNOP = 5 Sq. unit.

Substituting the values, we get $MN^2 = MQ^2 + QN^2 + 2QN.XQ$

$$MN^2 = 2 + 1 + 2(1) = 5$$

Thus the equation is tallied.

Double Boat

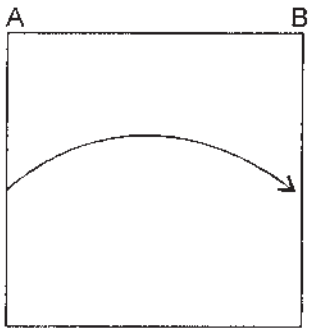


Fig. 11.32

Start with a square
Fold AD upon BC

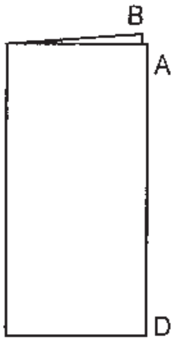


Fig. 11.33

Fold in half

Fig. 11.34

Fold in half

Fig. 11.35

Repeat the same horizontally. You get 8x8 squares. Lines have been avoided from step 7 for purpose of clarity

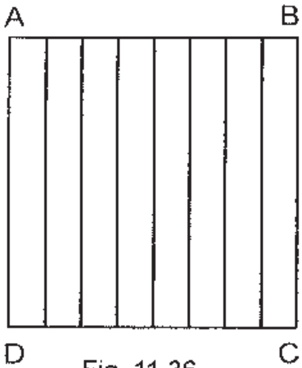


Fig. 11.36

Now unfold to get
8 equal divisions

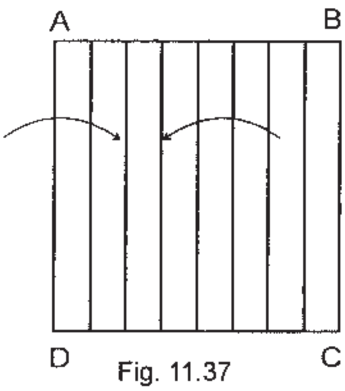


Fig. 11.37

Fold AD and
BC to centre

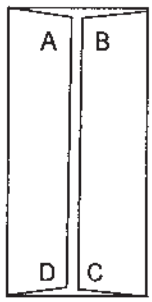


Fig. 11.38

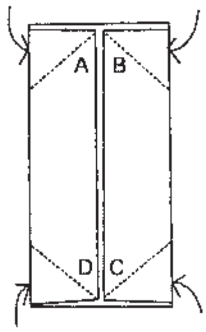


Fig. 11.39

Sink corners inside

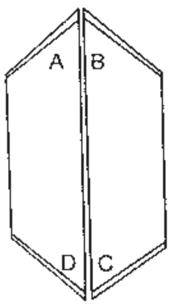


Fig. 11.40

Turn over

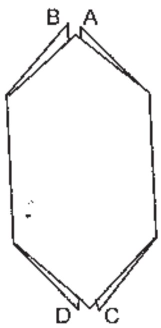


Fig. 11.41

Turn horizontal

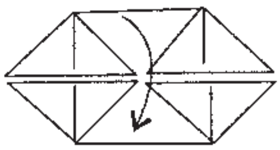


Fig. 11.42

Fold down

Fig. 11.43

Double Boat

Extension of Pythagoras Theoram (Acute Angle)

> The Theorem States : In an Acute angled triangle the square on the side opposite to the acute angle is equal to the sum of the squares on the other two sides diminished by twice the rectangle contained by one of these sides and the projection of the other upon it.

The same folded figure ABCD in the previous case for obtuse angle (page73) can also be used to illustrate this model.

$\hat{MNQ}$ being the acute angle in Δ MQN (MQ Opposite to N), as per the theorem we have

$MQ^2 = MN^2 + QN^2 - 2NX.NQ$

By substituting values of RHS as in previous page

$MQ^2 = 5 + 1 - 2\,(2) = 2$

The square on MQ has an area of 2 small squares. Hence equation is tallied.

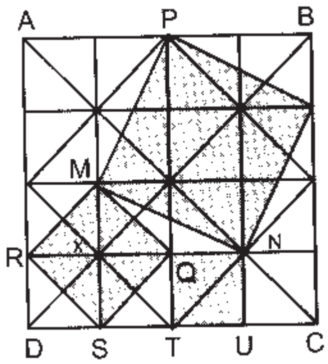
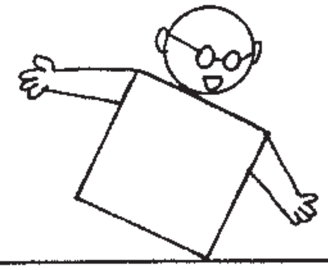


Fig. 11.44

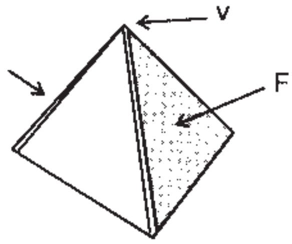
12. Fun with Solids – 3D from 2D

When a 2 dimensional Origami model can be so much fun can 3 dimensional models be lesser! 3D origami models made in myriad colours and hung in class rooms make great decoration. Platonic Solids are so called because they were studied by Plato. He associated five elements (Water, Air, Fire, Earth, Ether) with regular solids - Pyramid, Octahedron, Cube, Dodecahedron, Icosahedrons. He considered that these were building blocks of the Universe.It is true that these solid patterns are widely used in our life, in both natural, man-made objects.Solid Geometry is taught in high schools. Hence learning to make Platonic solids helps understand its Mathematics.We have collected various methods by which platonic solids can be quickly folded with paper.This form of Origami is called **Modular Origami**.

ORIGAMIAN SAYS

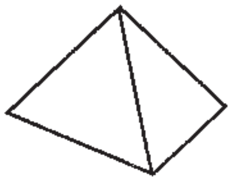


Origami Adds Interest in Mathematics

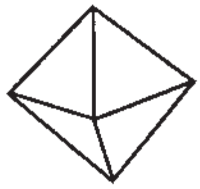


E= edge, V= vertex, F= face

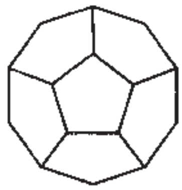
In any geometrical solid, there is a definite relationship between E, V & F. This relationship was discovered by Euler as **F+V=2+E**



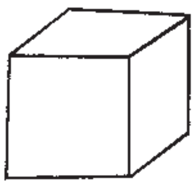
Pyramid



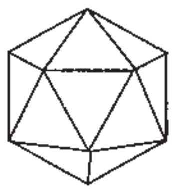
Octahedron



Dodecahedron



Cube



Icosahedron

Basic Fold-Paper strip from A4 size paper

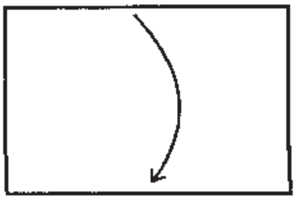


Fig. 12.1

Take A4 size paper

Fig. 12.2

Fold down & Cut

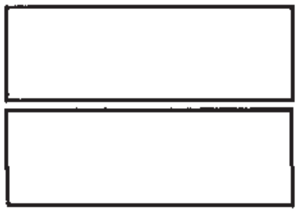


Fig. 12.3

You get two strips

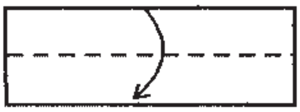


Fig. 12.4

Take a strip

Fig. 12.5

Fold in middle

Fig. 12.6

Fold in half AB
(about 10 cm)

Fig. 12.7

Bring B to central line, & crease. Fold A to C and continue folds

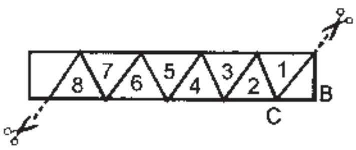


Fig. 12.8

Cut off extras on both sides

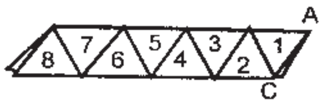


Fig. 12.9

In a strip cut from A4 Size Paper you get exactly eight Equilateral Triangles.

Platonic Solid - Triangular Pyramid

Fig. 12.10
Cut strip from A4 paper

Fig. 12.11
Fold Eight Eq. Triangle
Mark 1 to 5

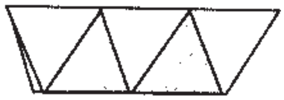


Fig. 12.12
Take 1 to 5

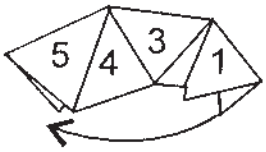


Fig. 12.13
Insert No. 1 Δ in 5
and press the edges.

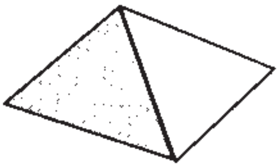


Fig. 12.14

Pyramid is ready

No. of Faces - 4
No. of Edges - 6
No. of Vertical - 4

Verify V+F = 2+E

Platonic Solid - Octohedron

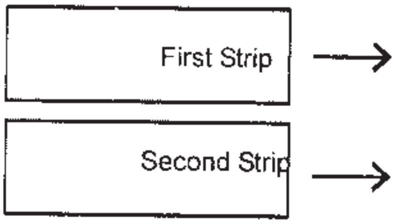


Fig. 12.15

Fold 8 Equilateral Δ s

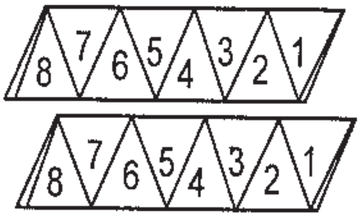


Fig. 12.16

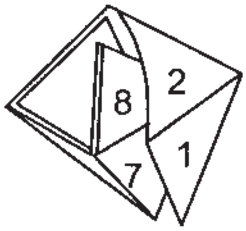


Fig. 12.17
Insert 7 & 8 inside
1 and 2

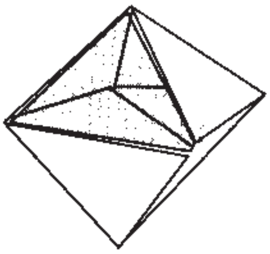


Fig. 12.18

Wrap second strip
around first model

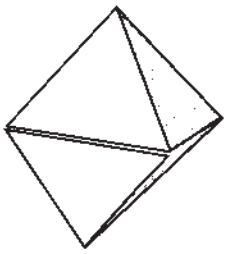


Fig. 12.19

No. of Faces - 8
No. of Edges - 12
No. of Vertical - 6

Verify V+F = 2+E

Platonic Solid - Cube

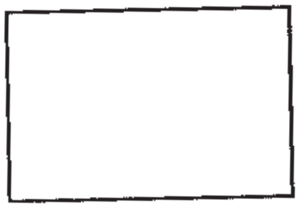


Fig. 12.20

Cut A4 in two halves

Fig. 12.21

Fold them in half

Fig. 12.22

You get strip A & B

Fig. 12.23

Fold a rightangle triangle mark the base line

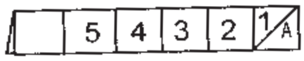


Fig. 12.24

Make five squares of equal length. Mark them 1 to 5

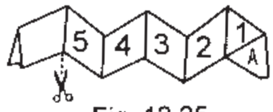


Fig. 12.25

Cut off extra length

Fig. 12.26

Insert 1 inside 5

Fig. 12.27

This forms a square ring make B strip into a square ring

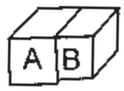


Fig. 12.28

Insert A ring into B

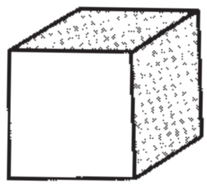


Fig. 12.29

Cube

No. of Faces - 6

No. of Edges - 8

No. of Vertical - 12

Verify V+F = 2+E

Platonic Solid - Dodecahedron

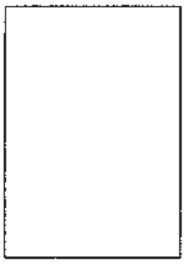


Fig. 12.30
Start with A4 paper
Fold mid lines.

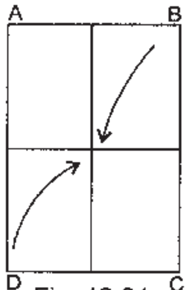


Fig. 12.31
Bring B & D to centre

Fig. 12.32
Fold A & C to centre

Fig. 12.33
[illegible]

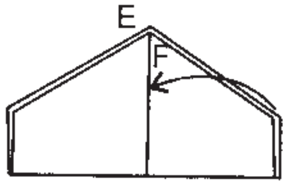


Fig. 12.34
Bring right edge
horizontal to base

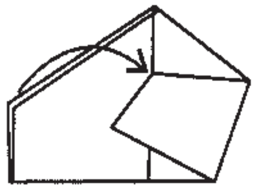


Fig. 12.35
Repeat left edge

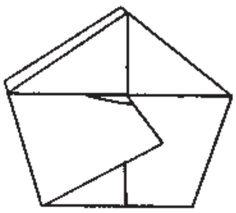


Fig. 12.36
Pentagon

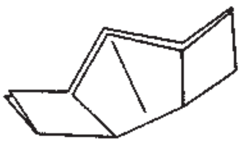


Fig. 12.37

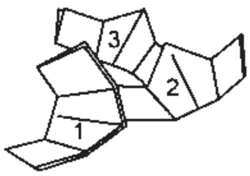


Fig. 12.38

Make three models and join them as shown in Fig. 12.38. This is one unit.

No. of Faces - 12
No. of Edges - 30
No. of Vertices - 20
Verify V + F = 2 + E

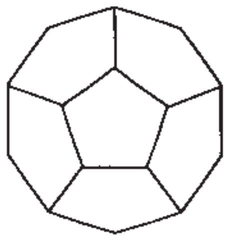


Fig. 12.39

Join four such units as in Fig. 12.38 to get a Dodecahedron.

Platonic solids - Icosahedron

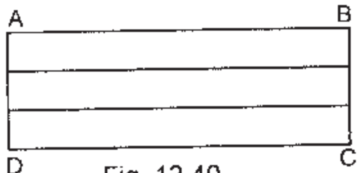


Fig. 12.40

Start with a 1:3 paper ABCD.Fold horizontally into 3 equal parts as explained in page no 89.

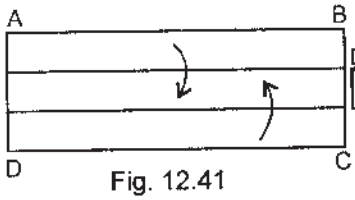


Fig. 12.41

Fold AB & DC to the central part.

Fig. 12.42

You get a 3 layered strip

Fig. 12.43

Fold this into equilateral triangles as in page 77

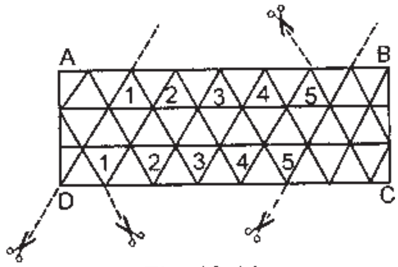


Fig. 12.44

Mark 1 to 5 Cutoff extra triangles

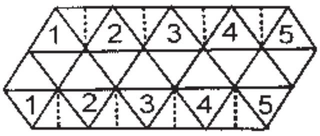


Fig. 12.45
This is base of Icosahedron

Sink fold the dotted triangles and paste them with adhesive. Finish upper part first & then paste lower part to get a Icosahedron

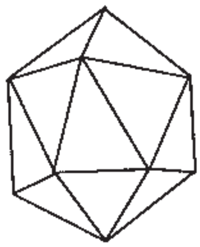


Fig. 12.46

Icosahedron

13. Fun with Paper Trays

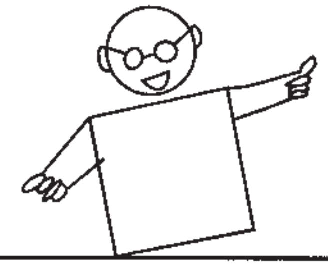
☺ **Tray with volume 4 cubic units**

☺ **Tray with volume 6 cubic units**

☺ **Tray with volume 24 cubic units**

☺ **Tray with 32 cubic units**

ORIGAMIAN SAYS



Origami Developes Greater Understanding

Fun with Paper Trays and Mathematics

We know how to tessellate a given square paper into smaller squares. So that the area of each smaller square is equal to 1/64 of larger square. We can use this square as one unit and measure length, breadth and height of the Origami Trays we make with given square paper.

We have to use a large square paper, say 30 cms X 30 cm.

In the following pages we have given procedure to make four trays using the standard 30 cm X 30 cm square paper. Always start by folding 64 Squares in this. We will be making trays with volumes of 32 cubic units, 24 cubic units, 8 cubic units and 4 cubic units. That means it is in proportion 4:8:24:32 or1:2:6:8. You can do many things with these trays. You can use them to demonstrate

(a) Fractions

(b) Proportions

(c) Ratios

(d) Fill them with salt powder or sugar powder or sand. Pour them for one tray to another to compare volumes. This in itself is a play.

Making a Tray with 4x4 paper - Volume 4 Cubic Units

Start with 8 x 8 = 64 Sq.paper. 4x4 paper means four unit squares each for length and also breadth.

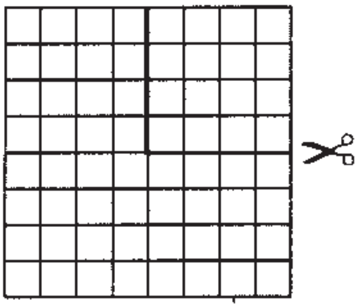


Fig. 13.1

64 squares

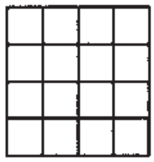


Fig. 13.2

4x4 paper

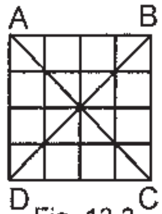


Fig. 13.3

Fold diagonals

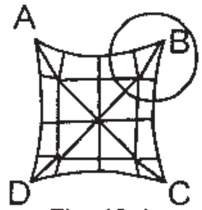


Fig. 13.4

Crease edges along diagonals

Fig. 13.5

Fig. 13.6

Mid line

Fig. 13.7

Fig. 13.8

Push with a finger

Fig. 13.9

Fold inside

Fig. 13.10

Corner gets locked.

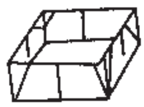


Fig. 13.11

Repate on all corners.

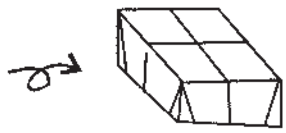


Fig. 13.12

You see 2x2 base area and 4 cubic units volume.

Note that we have locked corners to get this tray.

Making a Tray with 4x8 paper - Volume 8 Cubic Units

This is the continuation of our series in making Trays. After folding the 4x8 paper You get a Tray with base area 4x2 & ht of one square.

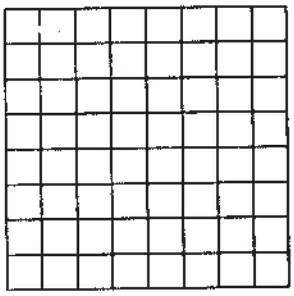


Fig. 13.13

Start with a square tessellate into 64 squares. Cut in the middle

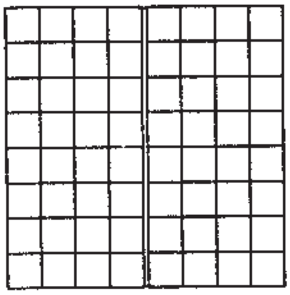


Fig. 13.14

You get two pieces

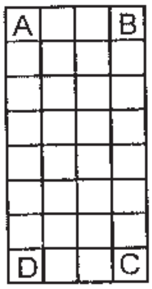


Fig. 13.15

Take one piece and mark ABCD

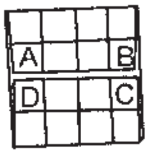


Fig. 13.16

Fold AB,CD to the centre

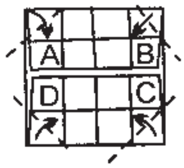


Fig. 13.17

Fold corners

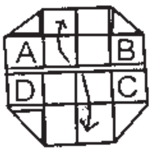


Fig. 13.18

Fold back AB,DC

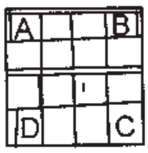


Fig. 13.19

Lift corners up

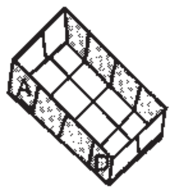


Fig. 13.20

Tray with base area 2x4, height of 1 unit. volume is of 8 cubic units.

Note that we have locked sides for this tray also.

Making a Tray with 8x8 paper - Volume 24 Cubic Units

Start with 30cm x 30cm paper. Tesselate into 64 squares. The tray we get after folding steps from 1 to 8 is 6x4 Tray. The base of this tray is covered by 6x4=24 squares. The height of this tray is one unit. Here volume of this tray = Area of Base x ht = 24 x 1= 24 cubic units. Follow symbols to fold.

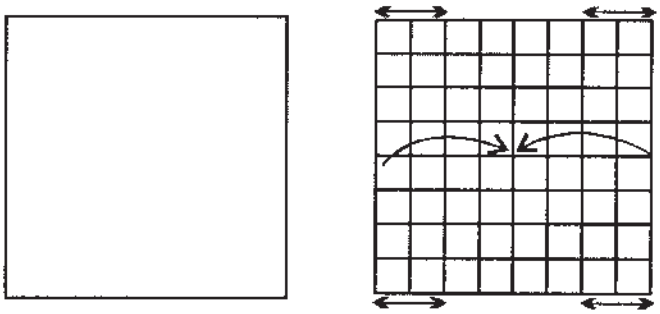


Fig. 13.21

Fill the square with 64 small squares

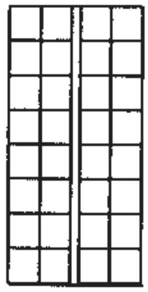


Fig. 13.22

For greater clarity grid lines are not shown. Fold one square at corners.

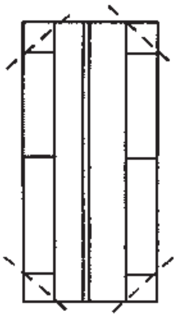


Fig. 13.23

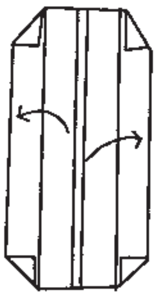


Fig. 13.24

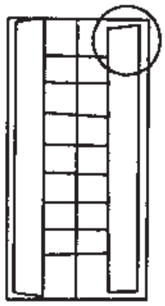


Fig. 13.25

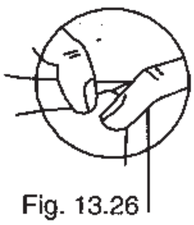


Fig. 13.26

Insert fingers and lift the sides

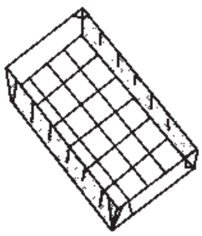


Fig. 13.27

Finished tray

Turn over

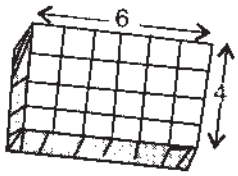


Fig. 13.28

Bottom area

Making a Tray with 8x8 paper - Volume 32 Cubic Units

Here we start with a 30x30 cm Square. Tessellate it into 64 squares. After folding as shown from 1 to 11, we get a tray with base area 4x4 units and height 2 square units. Therefore, volume of this tray is = 4x4x2 = 32 cubic units.

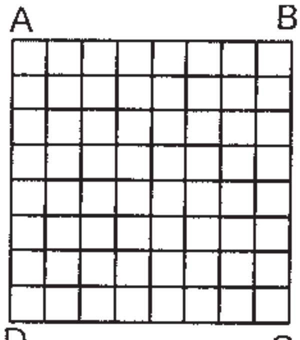


Fig. 13.29

64 squares

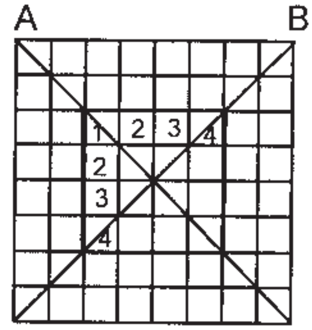


Fig. 13.30

Fold diagonal and mark 4x4 areas.

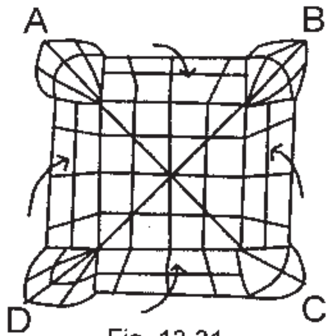


Fig. 13.31

Lift sides

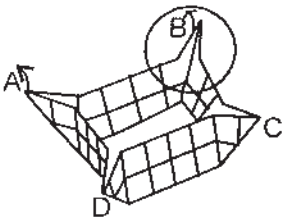


Fig. 13.32

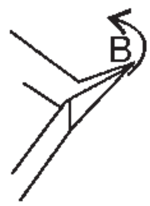


Fig. 13.33


Fig. 13.34


Fig. 13.35

Folding and locking corners


Fig. 13.36


Fig. 13.37

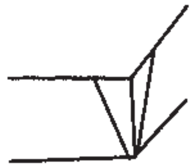


Fig. 13.38
Corner locked

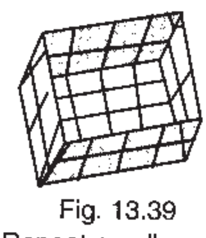


Fig. 13.39
Repeat on all corners

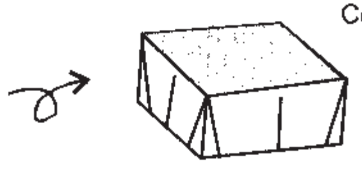
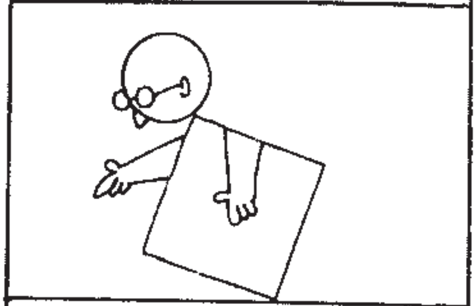


Fig. 13.40

14. Dividing a Paper into Three Equal Parts

Dividing a piece of paper into three equal number of parts is challenging. In certain books Trigonometric approximations are used for this purpose. But here we give easy methods for the same. The Mathematics involved is explained at the end.

ORIGAMIAN SAYS



Origami Develops Curiocity and Inquisitiveness

To Divide a given Rectangle into three equal parts

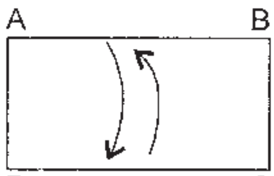


Fig. 14.1

Start with a rectangular paper ABCD. Lay AB upon DC and fold back.

Fig. 14.2

A line appears

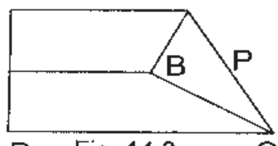


Fig. 14.3

Fold B to this line

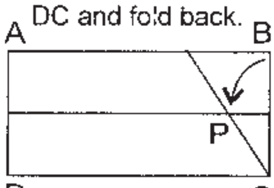


Fig. 14.4

This fold cuts the line at P. Fold B to P.

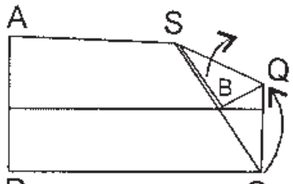


Fig. 14.5

This Fold creates a Crease SQ. Fold DC so that C touches Q. Fold back SB.

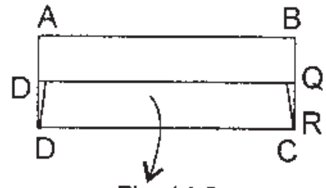


Fig. 14.6

This crease creates point R

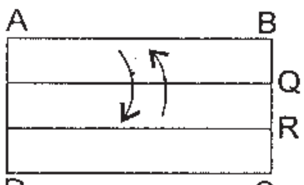


Fig. 14.7

Fold AB to touch R and fold back.Now BQ = QR=RC.

Turn the page to know the proof.

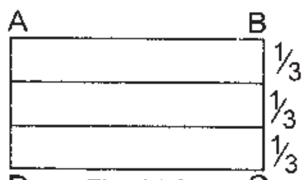


Fig. 14.8

ABCD is Divided into 3 equal parts

Maths in the folds - Dividing paper in to 3 equal parts (Contd.)

Mark all folds with a pencil. Join QP (See Fig. 14.9)

Now we have two triangles ΔSBC and ΔPQC.

These two are similar, because $S\hat{B}C = Q\hat{P}C$ (we have folded B to P) $\hat{C}$ is common

hence, $P\hat{Q}C = B\hat{S}C$

In ΔSBC to ΔPQC the sides are to be in equal ratio

$$\frac{SB}{PQ} = \frac{BC}{PC} = \frac{SC}{QC}$$

i.e.,

$$BC = \frac{SC x PC}{QC} = \frac{(SP+PC) \times PC}{QC} = \frac{2.PC.PC}{QC} = \frac{2PC^2}{QC}$$

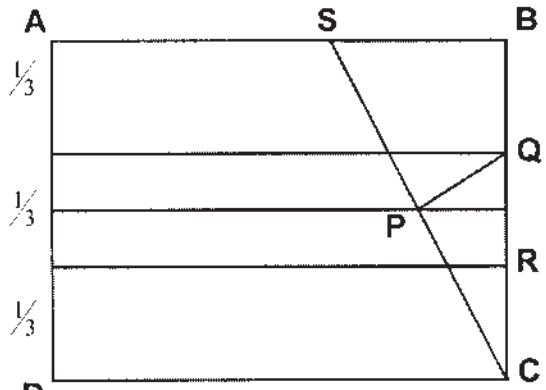


Fig. 14.9

In Δ PQC $QC^2 = PQ^2 + PC^2$ and by folding, we know BQ = PQ. substituting these

$$BC = \frac{2(QC^2 - PQ^2)}{QC} = \frac{2(QC+PQ)(QC-PQ)}{QC} = \frac{2BC(QC-PQ)}{QC}$$

Therefore 2QC - 2PQ = QC

2PQ = QC OR 2BQ = QC, ie., BQ = 1/2QC

Since, QR = RC (by folding)

BQ = QR = RC = 1/3 BC

Dividing paper in to required no of equal parts

Preparatory folding - Start with a square or rectangular paper

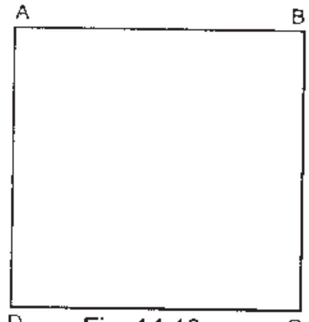


Fig. 14.10

Fold DC to AB

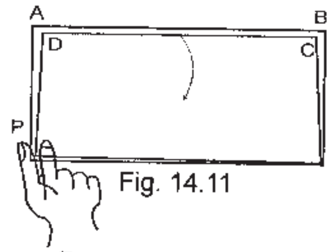


Fig. 14.11

Press at mid point

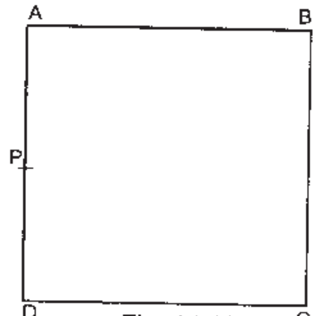


Fig. 14.12

Mark point P

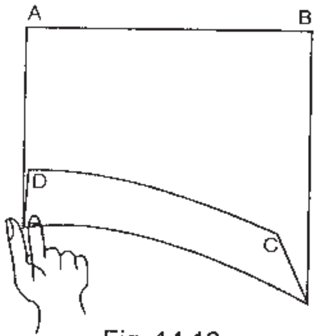


Fig. 14.13

Fold D to P

Press at Q mid point of PD

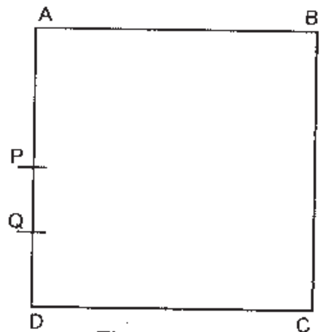


Fig. 14.14

Mark Q fold D to Q

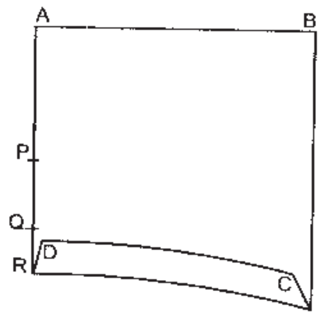


Fig. 14.15

Press at R mid point of QD

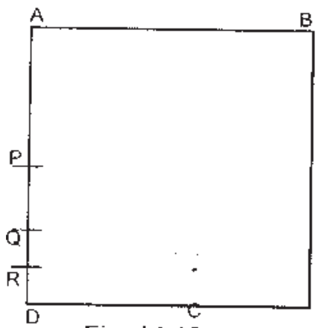


Fig. 14.16

Mark R fold Q to P

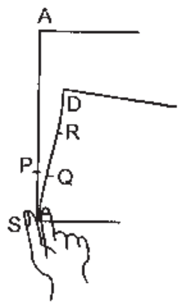


Fig. 14.17

Press at S, mid point of PQ

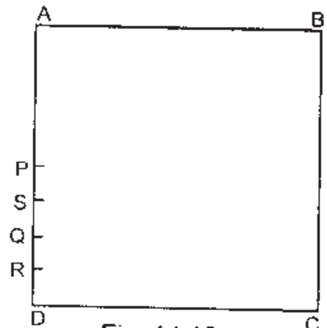


Fig. 14.18

You have got points PSQR on AD which are at equal distance. Mark the points in the same way on AP

Dividing paper in to required no of equal parts Contd.

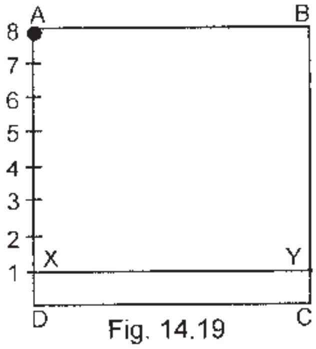


Fig. 14.19

Having marked equidistant points on AD, choose required pointst to make division in ABCD.Fold XY II DC at point 1

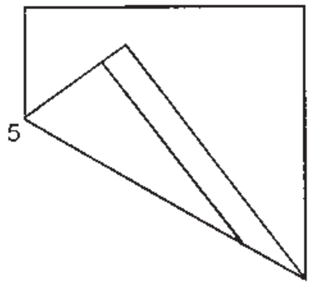


Fig. 14.20

Join point 5 and C by making a fold .

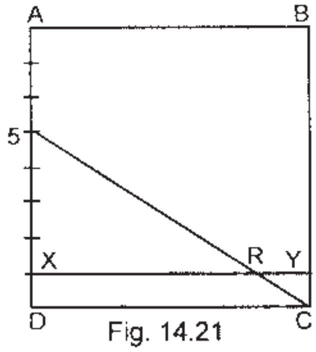


Fig. 14.21

Line 5C cuts XY at R. RY = 1/5 DC.

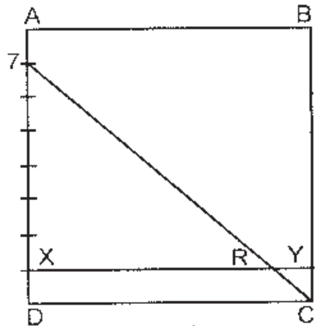


Fig. 14.22

When you join 7 and C RY = 1/7DC.

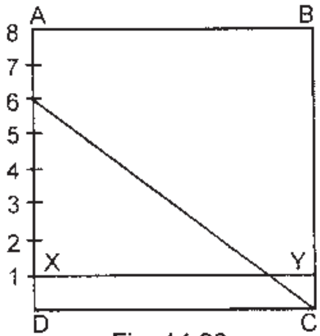


Fig. 14.23

Join 6 and C.RY=1/6DC

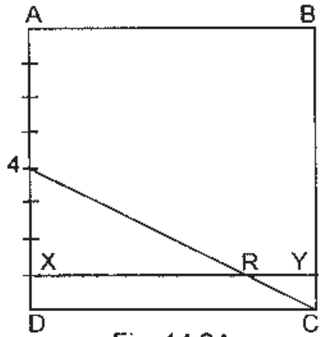


Fig. 14.24

Join 4 and C.RY=1/4DC

After getting 1/4 or 1/6 part, the whole paper has to be folded to RY, so that DC gets divided into required parts.

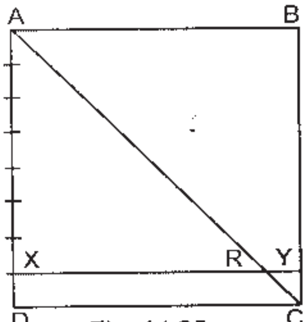


Fig. 14.25

Join 8 andC. RY=1/8DC

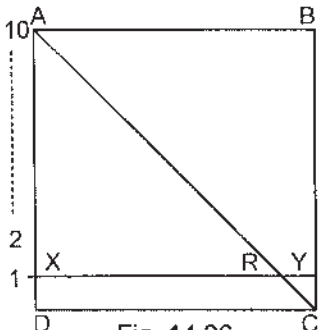


Fig. 14.26

If A is the 10th point then AC cuts XY at 1/10DC.

Dividing into Required Number of Parts all

When ABCD is divided into 5 parts , we fold through points 5&C.This creates Triangales D5C & RYC.These are similar. (See Fig. 14.27)

Therefore $\frac{DC}{RY} = \frac{D5}{YC} = \frac{5units}{1unit}$

So, $RY = \frac{1}{5}DC$

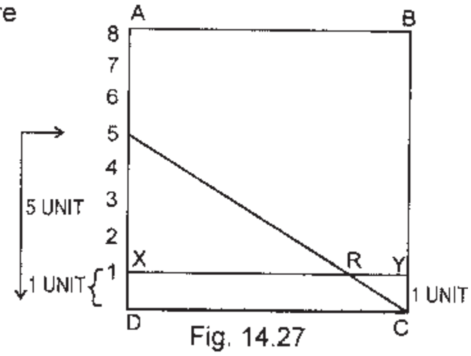


Fig. 14.27

We illustrate here, a sequence of folds to divide ABCD into five parts after getting RY = 1/5 DC.

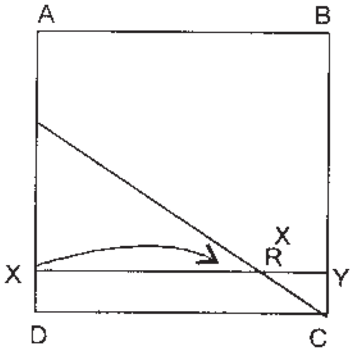


Fig. 14.28

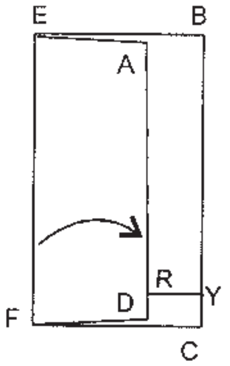


Fig. 14.29

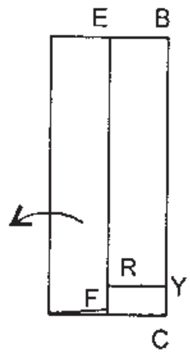


Fig. 14.30

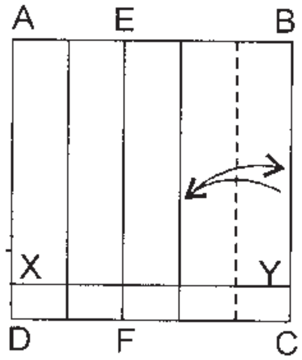


Fig. 14.31

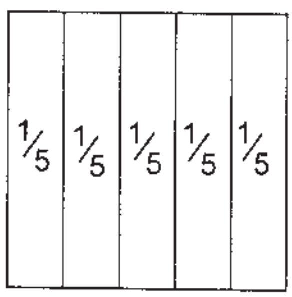
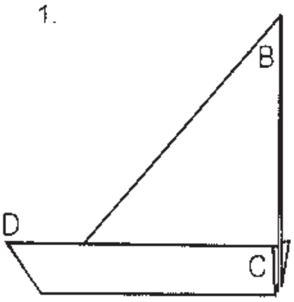


Fig. 14.32

Answer : Page 34

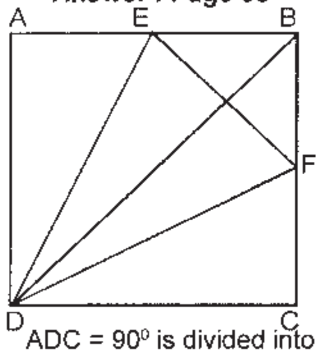
1.



$\angle D = 45^0 = \angle B$

Answer : Page 35

2.



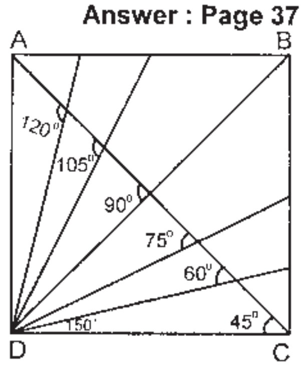
ADC = 90^0 is divided into four equal parts.

$\angle ADE = \angle EDB = 22\frac{1}{2}^0$,

$\angle BDF = \angle FDC = 22\frac{1}{2}^0$

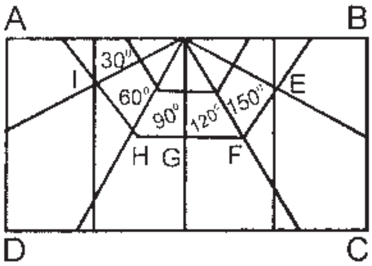
Answer : Page 37

3.



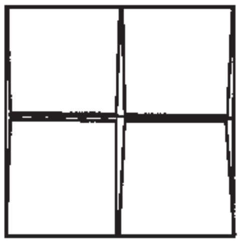
Answer : Page 38

4.



Answer : Page 46

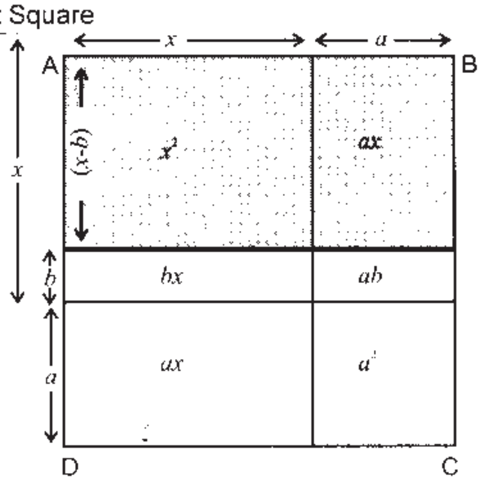
5.

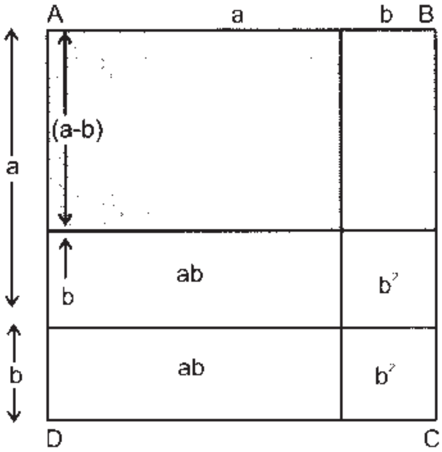


Area of the purse is ¼ of original square.

6. Answers to question in page number No. 45

Eight Square





Area ABCD

$(x+a)(x-b) = (x+a)^2 - bx - ax - ab - a^2$

$= x^2 + a^2 + 2ax - bx - ax - ab - a^2$

$= x^2 + ax - bx - ab$

$= x^2 + x(a - b) - ab$

Here ABCD = $(a+b)^2$

$(a+b)(a-b) = (a+b)^2 - 2ab - b^2 - b^2$

$= (a^2 + 2ab + b^2) - 2ab - 2b^2$

$= a^2 - b^2$

7. Ans to Page 64 (a)

8. Ans to Page 64 (b)