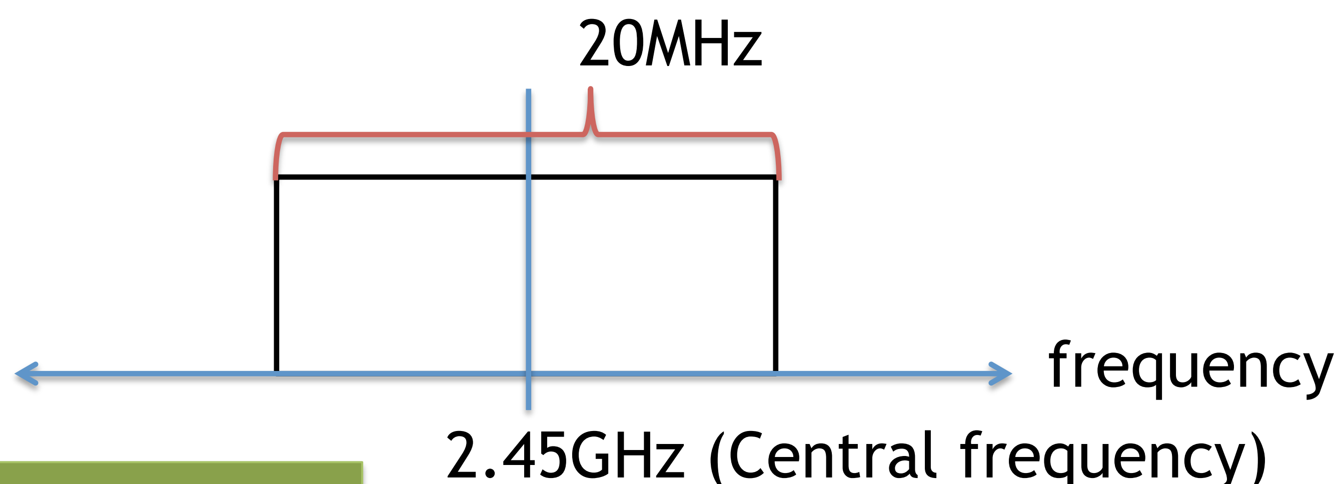
# Orthogonal Frequency Division Modulation (OFDM)

- OFDM diagram
- Inter Symbol Interference
- Packet detection and synchronization
- Related works

# Motivation

- Signal over wireless channel
  - y[n] = Hx[n]
- Work only for narrow-band channels, but not for wide-band channels
  - e.g., 20 MHz for 802.11

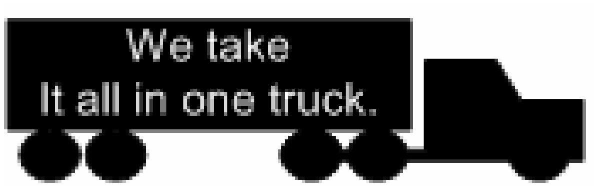


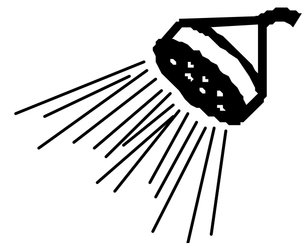

Capacity = BW * log(1+SNR)

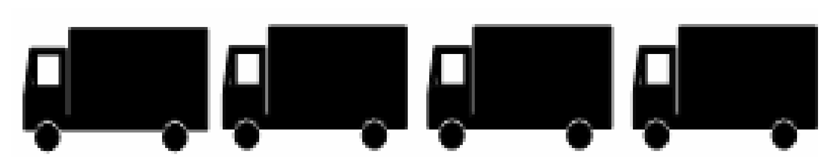
# Basic Concept of OFDM

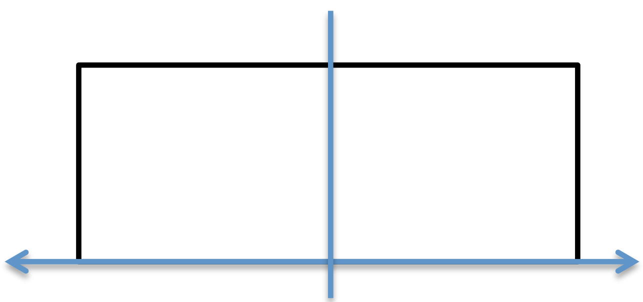
## Wide-band channel

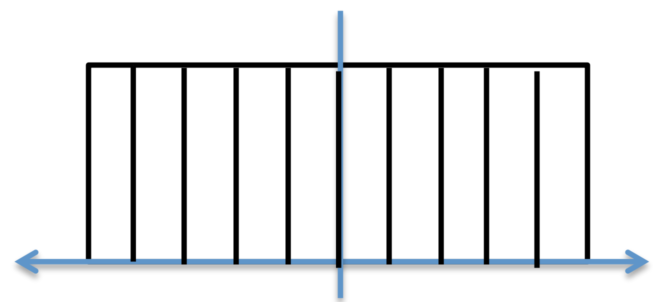
## Multiple narrow-band channels





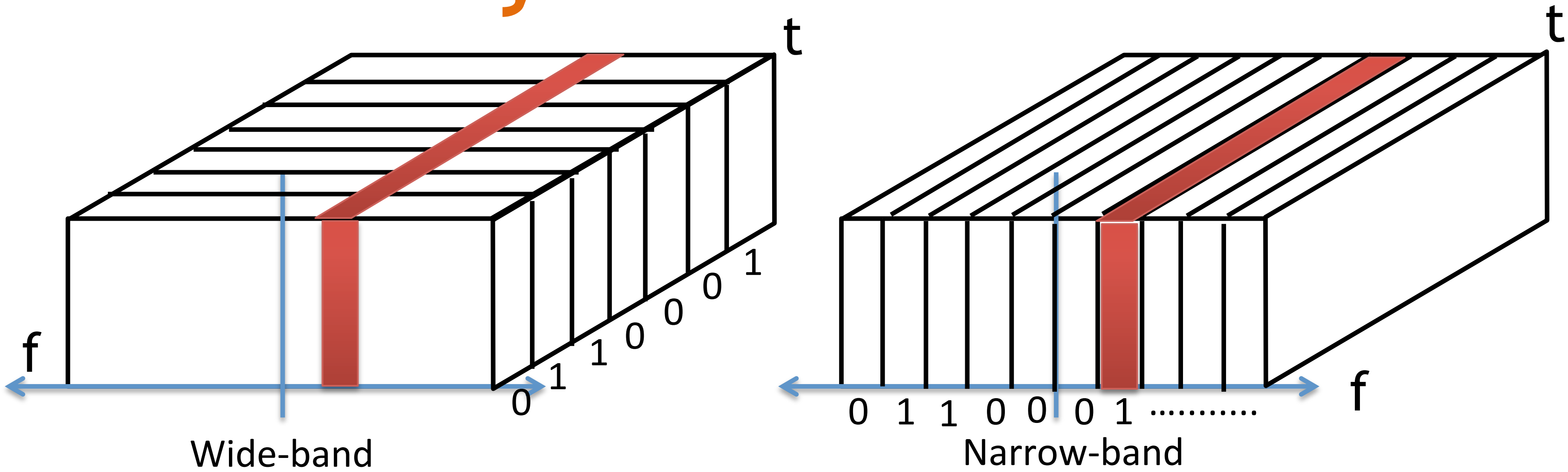




Send a sample using the entire band

Send samples concurrently using multiple **orthogonal sub-channels**

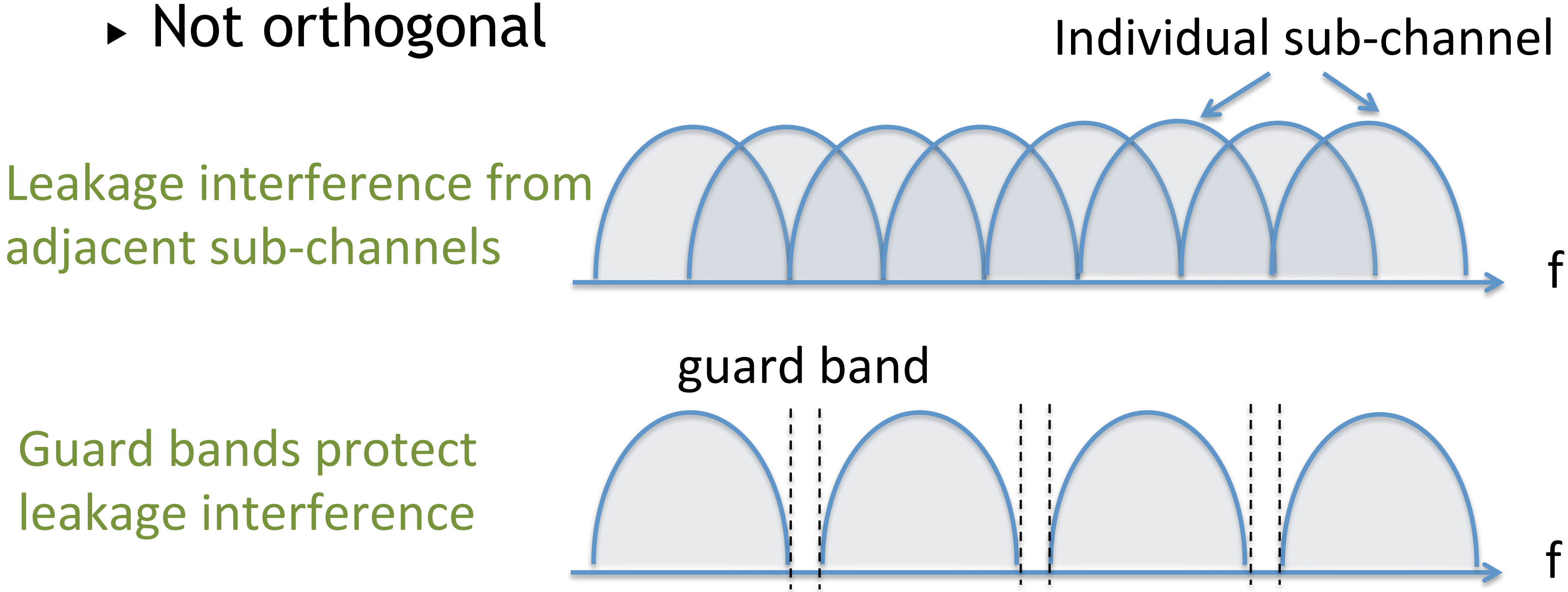
# Why OFDM is better?




- Multiple sub-channels (sub-carriers) carry samples sent at a lower rate
  - Almost same bandwidth with wide-band channel
- Only some of the sub-channels are affected by interferers or multi-path effect

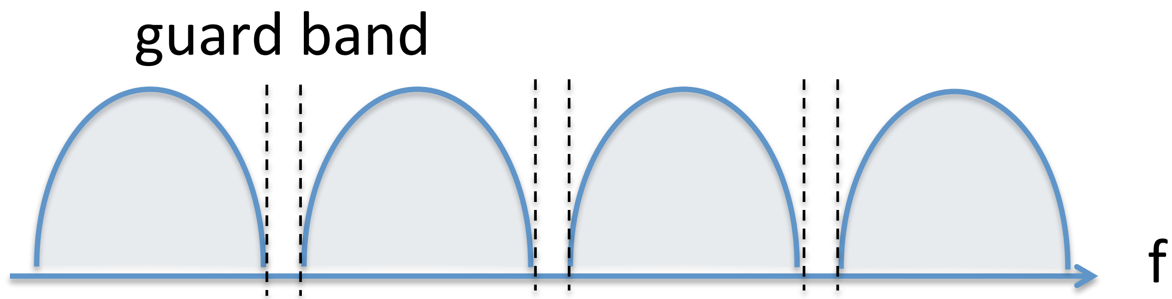
# Importance of Orthogonality

- Why not just use FDM (frequency division multiplexing)
  - Not orthogonal

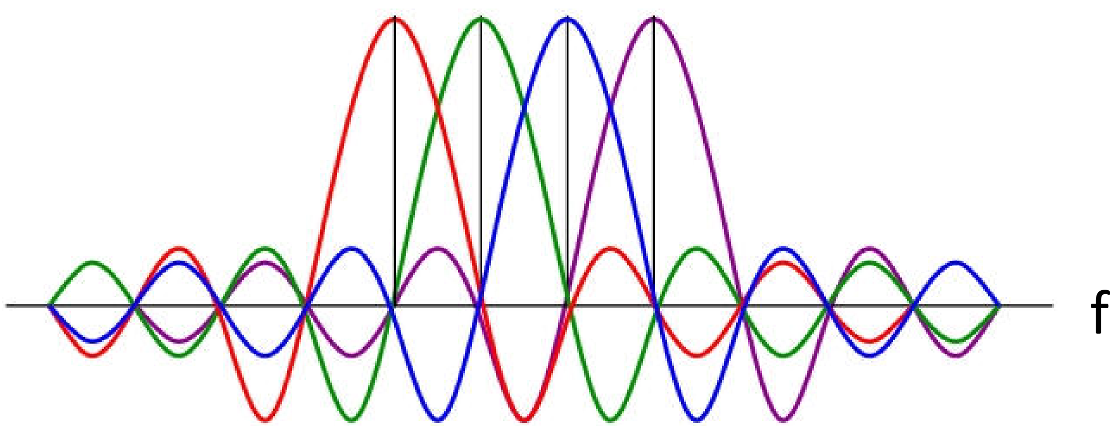



- Need guard bands between adjacent frequency bands → extra overhead and lower throughput

# Difference between FDM and OFDM



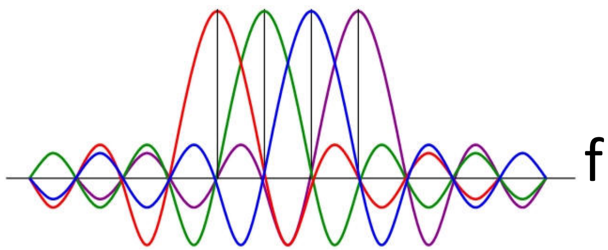

Frequency division multiplexing



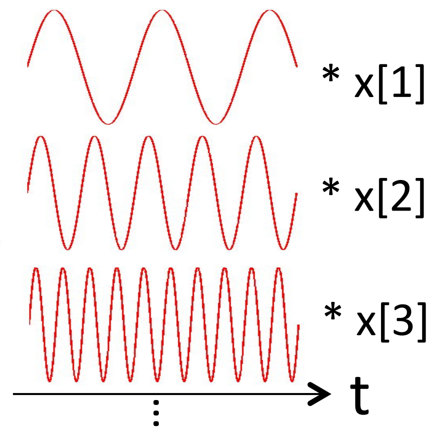

Orthogonal sub-carriers in OFDM

Don't need guard bands

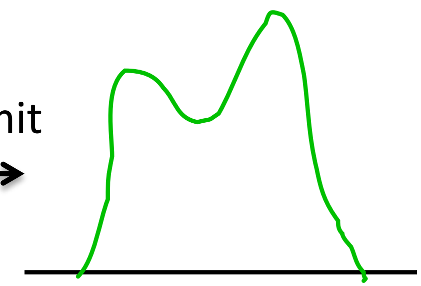
# Orthogonal Frequency Division Modulation




IFFT




transmit

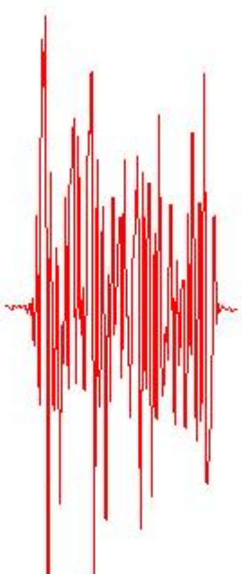


Data coded in frequency domain

Transformation to time domain:
each frequency is a sine wave
In time, all added up

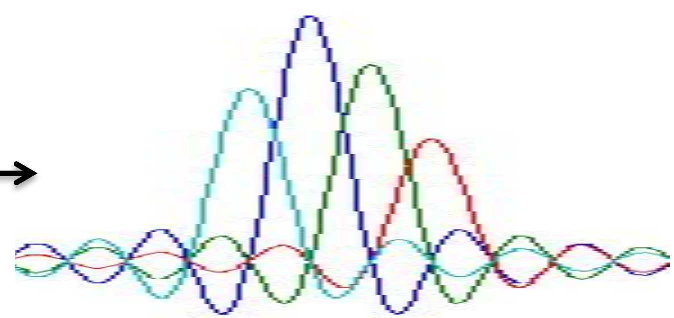
Channel frequency response

receive



Time domain signal

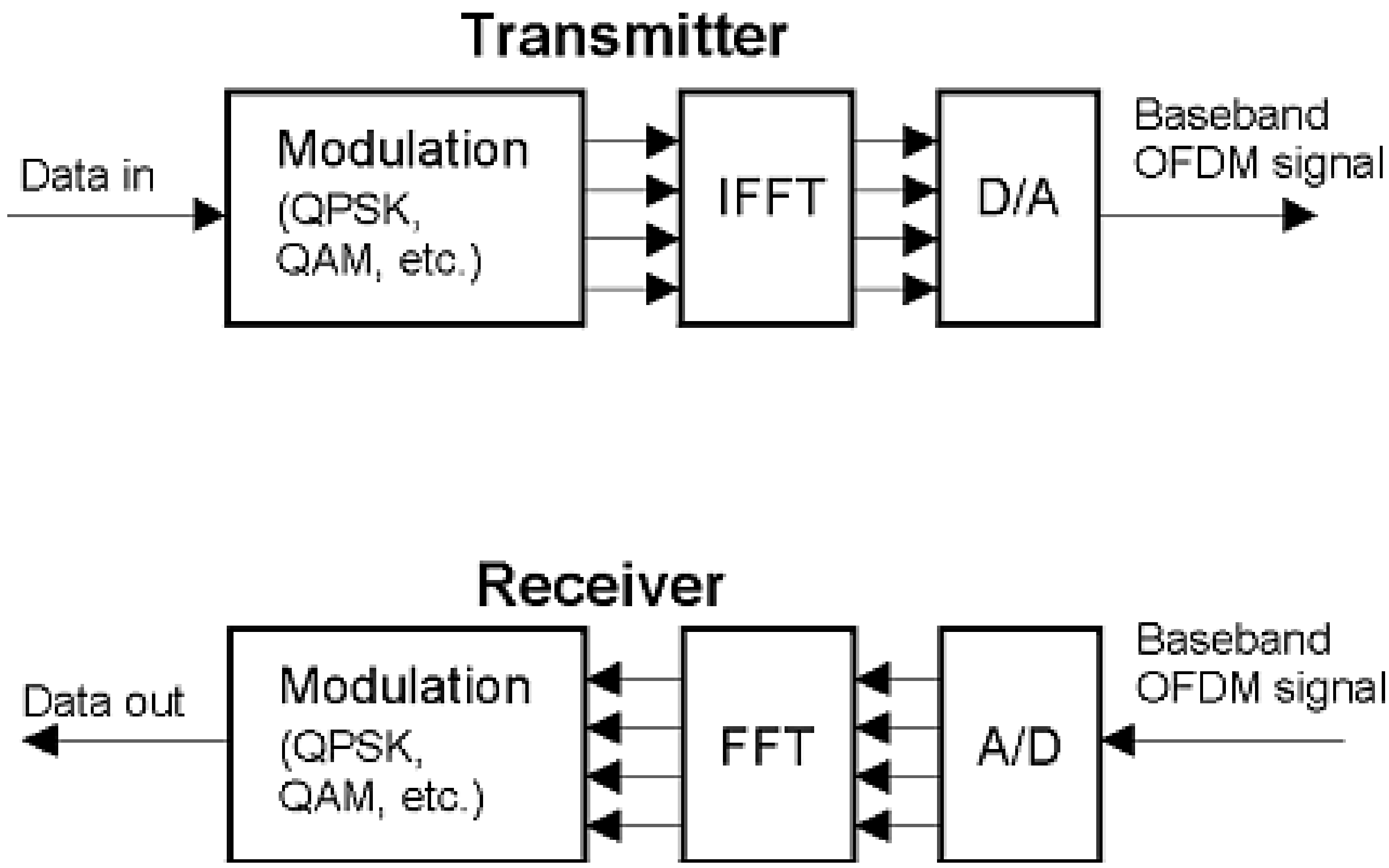
FFT



Frequency domain signal

Decode each subcarrier separately

# OFDM Transmitter and Receiver



# Orthogonality of Sub-carriers

IFFT

Encode: frequency-domain samples → time-domain sample

$$x(t) = \sum_{k=-N/2}^{N/2-1} X[k] e^{j2\pi kt/N}$$

Time-domain

Frequency-domain

$$X[k] = \frac{1}{N} \sum_{t=N/2}^{N/2-1} x(t) e^{-j2\pi kt/N}$$

FFT

Decode: time-domain samples → frequency-domain sample

Orthogonality of any two bins : $\sum_{t=N/2}^{N/2-1} e^{-j2\pi kt/N} e^{-j2\pi pt/N} = 0, \forall p \neq k$

# Example

- Say we use BPSK and 4 sub-carriers to transmit a stream of samples

1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1,…

- Serial to parallel conversion of samples

Frequency-domain signal

| | c1 | c2 | c3 | c4 |
|---|---|---|---|---|
| symbol1 | 1 | 1 | -1 | -1 |
| symbol2 | 1 | 1 | 1 | -1 |
| symbol3 | 1 | -1 | -1 | -1 |
| symbol4 | -1 | 1 | -1 | -1 |
| symbol5 | -1 | 1 | 1 | -1 |
| symbol6 | -1 | -1 | 1 | 1 |

IFFT →

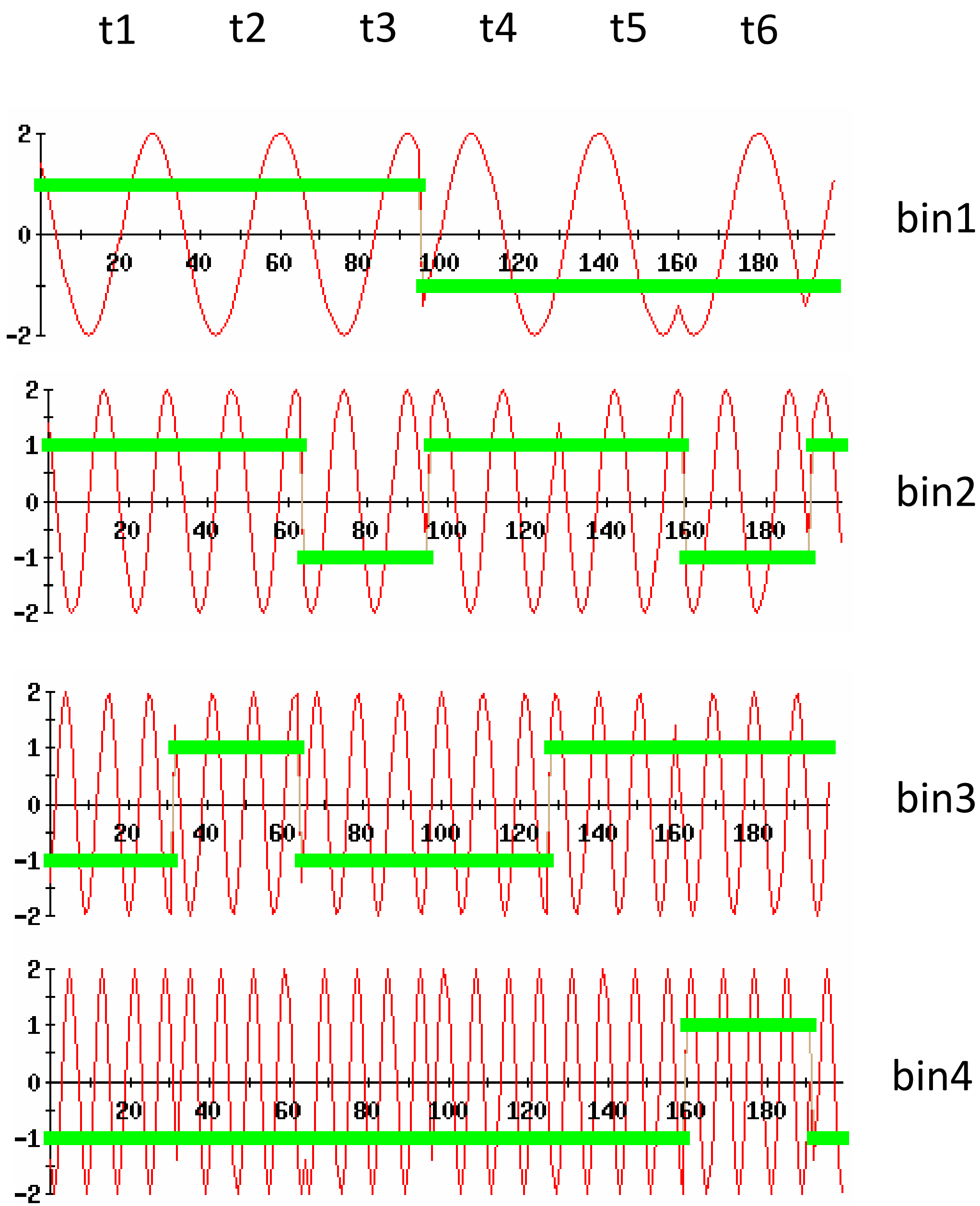
Time-domain signal

| | | | |
|---|---|---|---|
| 0 | 2 - 2i | 0 | 2 + 2i |
| 2 | 0 - 2i | 2 | 0 + 2i |
| -2 | 2 | 2 | 2 |
| -2 | 0 - 2i | -2 | 0 + 2i |
| 0 | -2 - 2i | 0 | -2 + 2i |
| 0 | -2 + 2i | 0 | -2 - 2i |

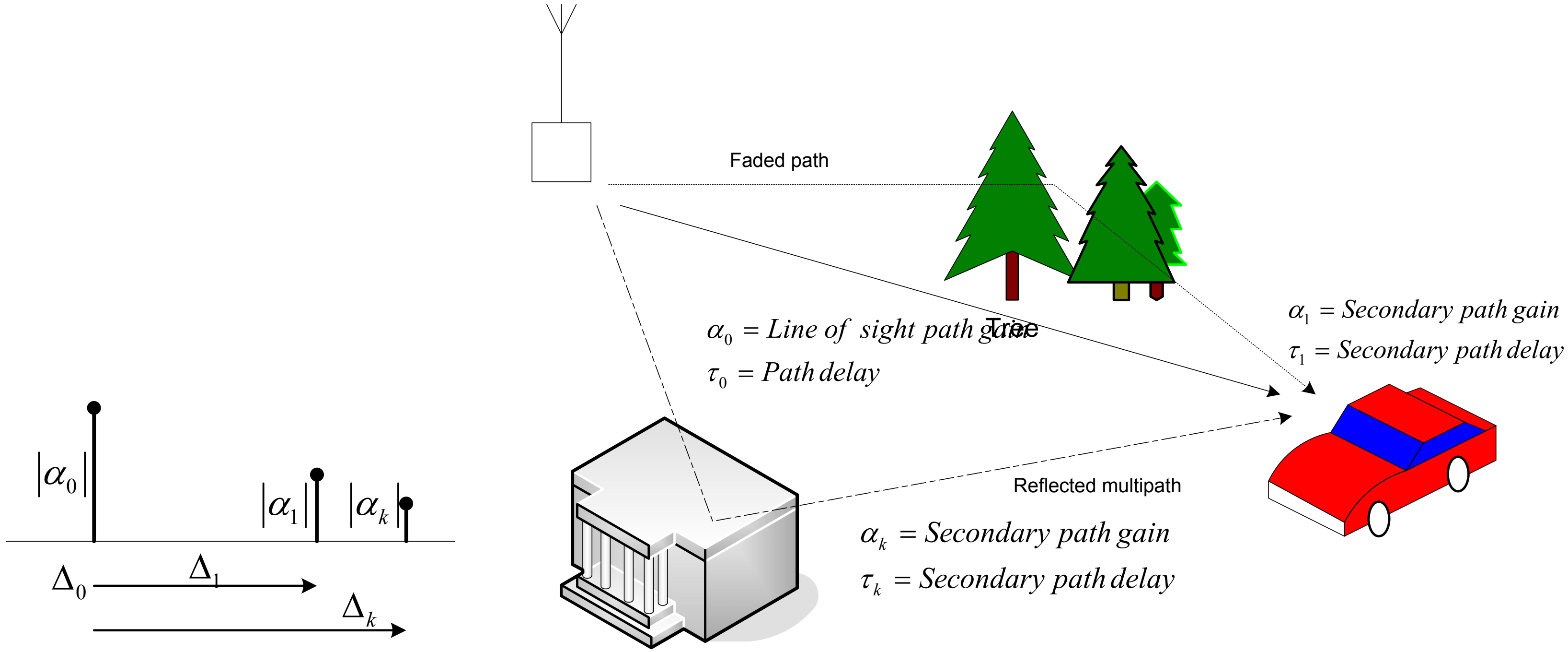
- Parallel to serial conversion, and transmit time-domain samples

0, 2 - 2i, 0, 2 + 2i, 2, 0 - 2i, 2, 0 + 2i, -2, 2, 2, 2, -2, 0 - 2i, -2,
0 + 2i, 0, -2 - 2i, 0, -2 + 2i, 0, -2 + 2i, 0, -2 - 2i, …

| | | | | |
|---|---|---|---|---|
| symbol1 | 1 | 1 | -1 | -1 |
| symbol2 | 1 | 1 | 1 | -1 |
| symbol3 | 1 | -1 | -1 | -1 |
| symbol4 | -1 | 1 | -1 | -1 |
| symbol5 | -1 | 1 | 1 | -1 |
| symbol6 | -1 | -1 | 1 | 1 |



# Multi-Path Effect

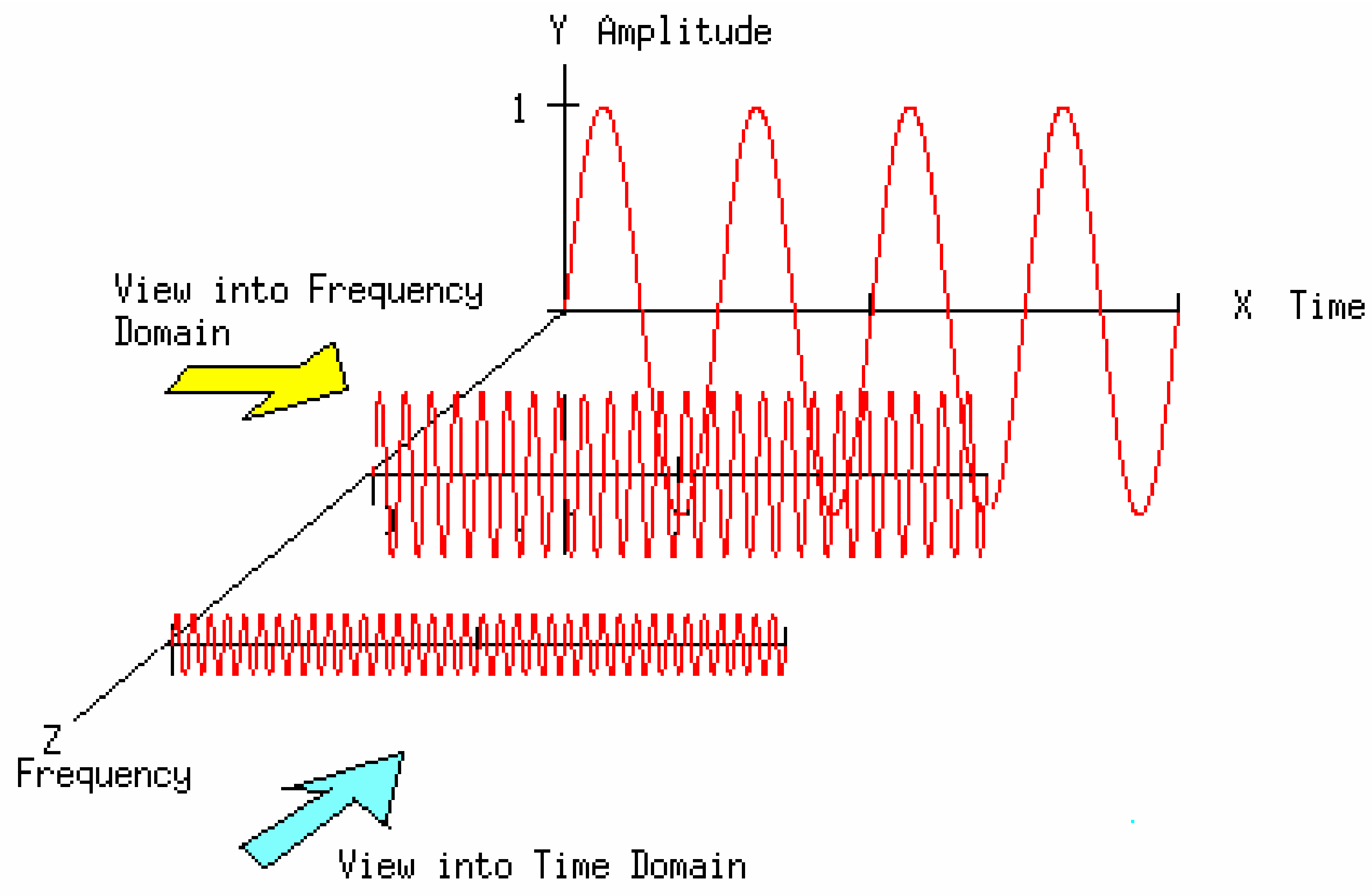



$$y(t) = h(0)x(t) + h(1)x(t-1) + h(2)x(t-2) + \cdots$$
$$= \sum_{\Delta} h(\Delta)x(t-\Delta) = h(t) \otimes x(t)$$

time-domain

$$\Leftrightarrow \quad Y(f) = H(f)X(f)$$

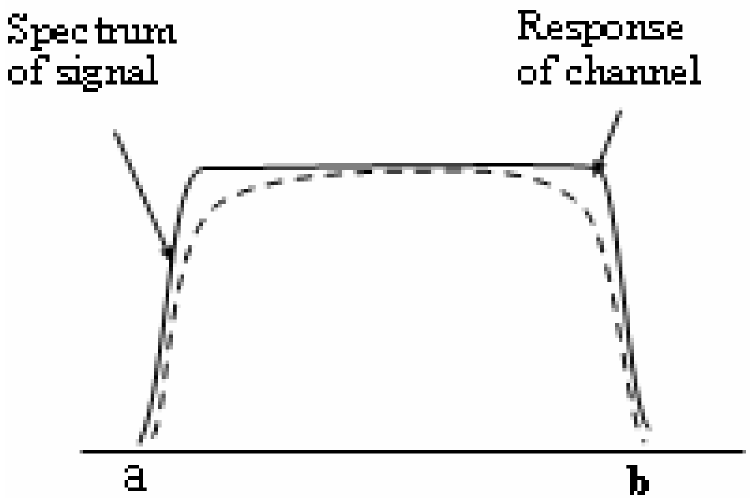
frequency-domain

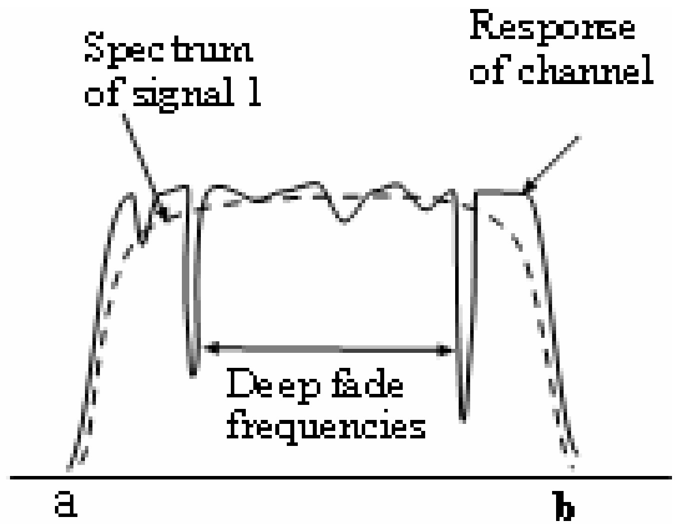


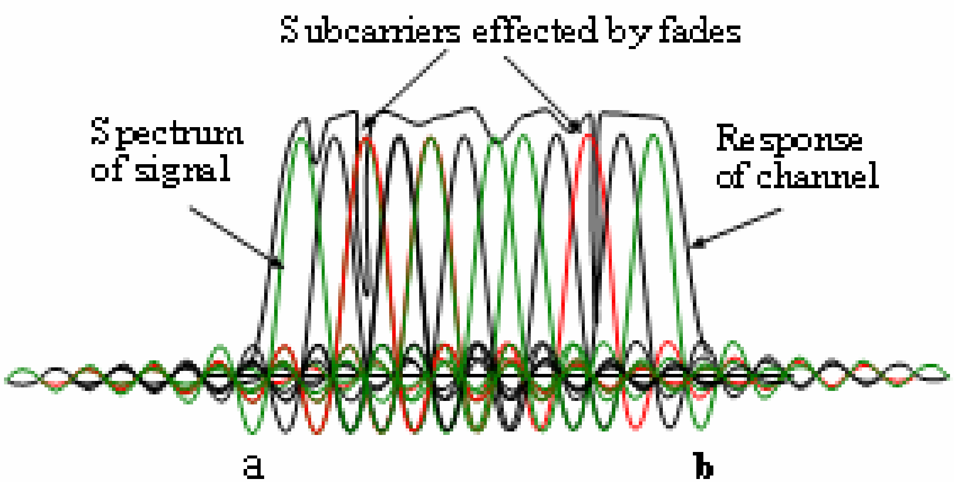

Current symbol + delayed-version symbol

→ Signals are deconstructive in only certain frequencies

# Frequency Selective Fading



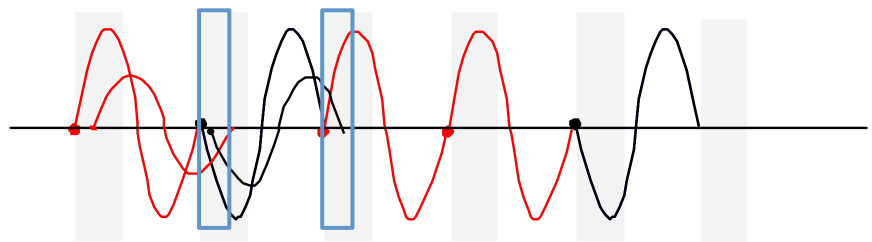



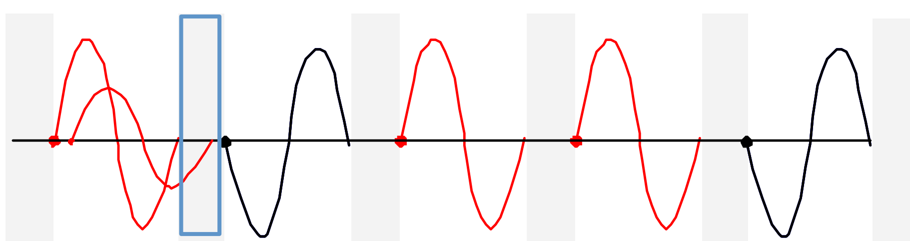

Frequency selective fading: Only some sub-carriers get affected

# Inter Symbol Interference (ISI)

- The delayed version of a symbol overlaps with the adjacent symbol



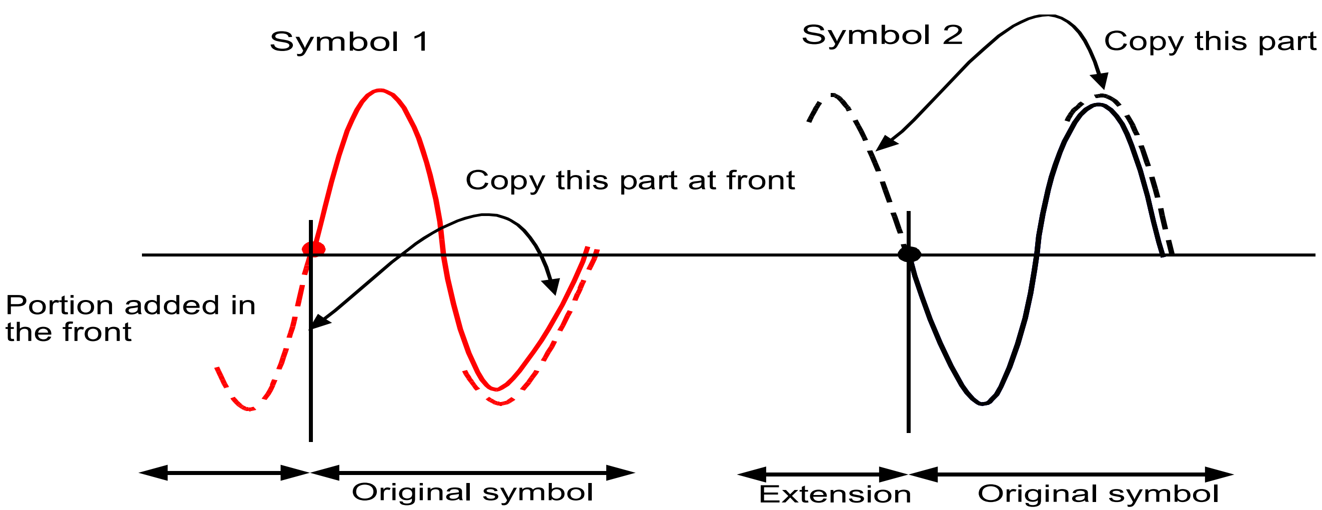
- One simple solution to avoid this is to introduce a guard-band



Guard band

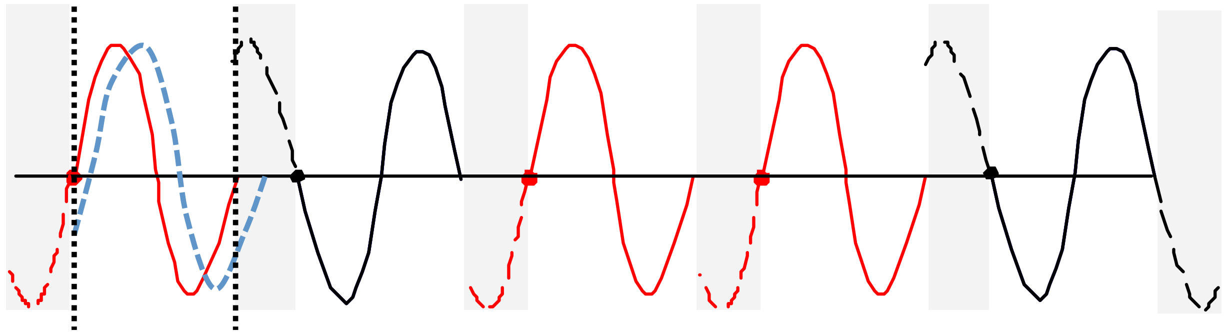
# Cyclic Prefix (CP)

- However, we don't know the delay spread exactly
  - The hardware doesn't allow blank space because it needs to send out signals continuously
- Solution: Cyclic Prefix
  - Make the symbol period longer by copying the tail and glue it in the front

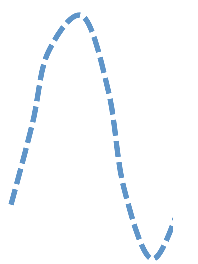
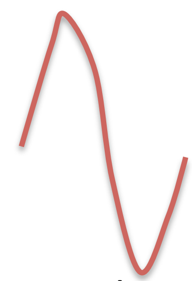



In 802.11, CP:data = 1:4

# Cyclic Prefix (CP)



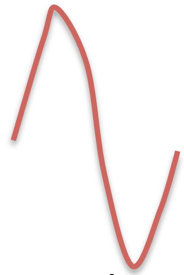
- Because of the usage of FFT, the signal is periodic

FFT(  ) = exp(-2j$\pi_\Delta$f)*FFT(  )

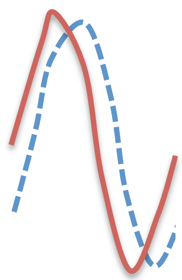
delayed version — original signal

- Delay in the time domain corresponds to rotation in the frequency domain
  - Can still obtain the correct signal in the frequency domain by compensating this rotation

# Cyclic Prefix (CP)

w/o multipath y(t) → FFT(  ) → $Y[k] = H[k]X[k]$

original signal

w multipath y(t) → FFT(  ) → $Y[k] = \alpha(1+\exp(-2j\pi_{\Delta}k))*X[k]$

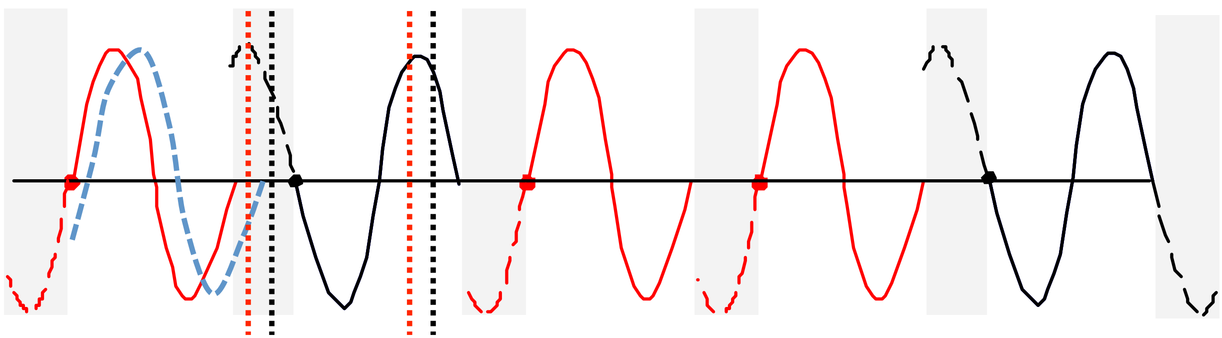
$= H'[k]X[k]$

original signal + delayed-version signal

Lump the phase shift in H

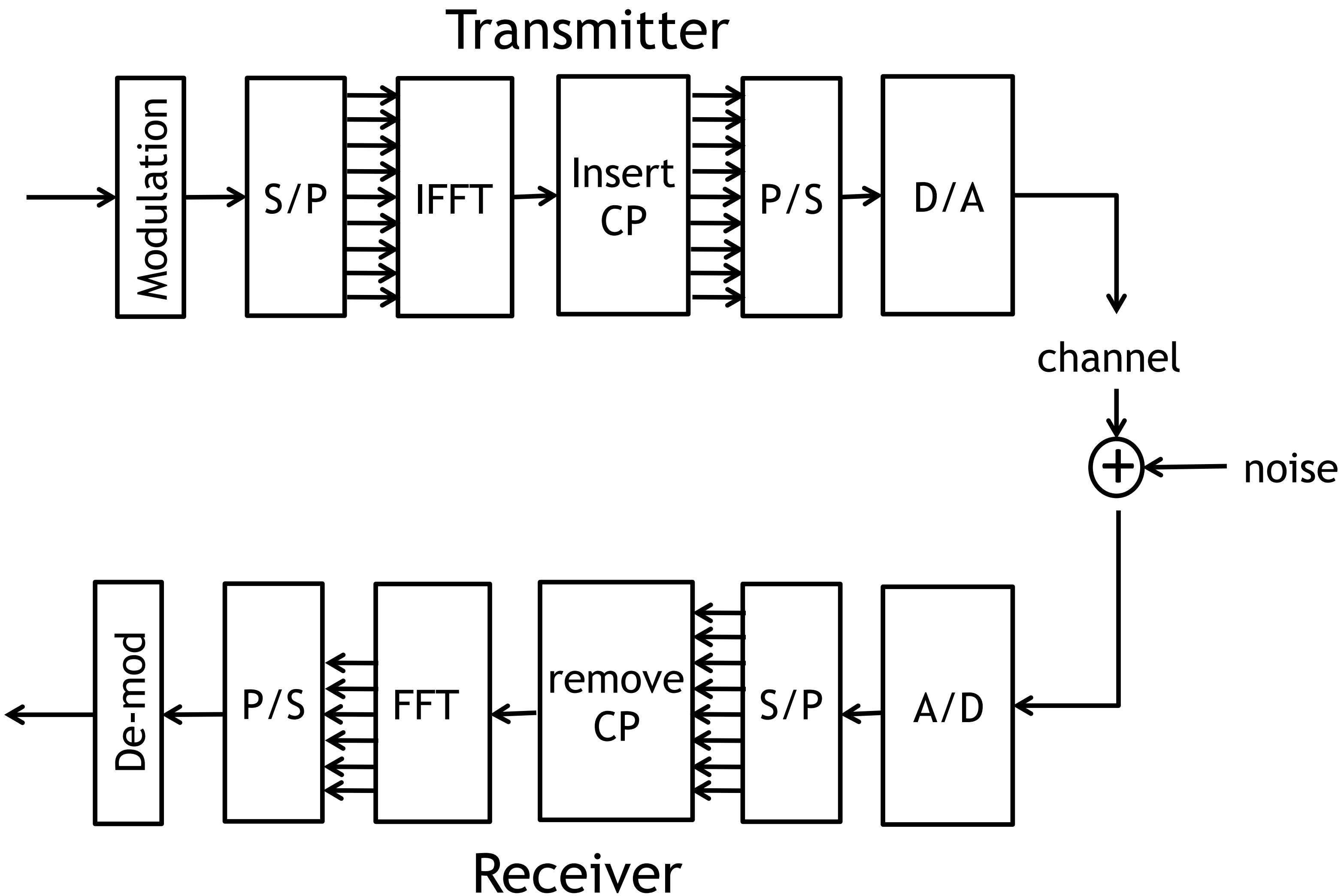
# Side Benefit of CP

- Allow the signal to be decoded even if the packet is detected after some delay

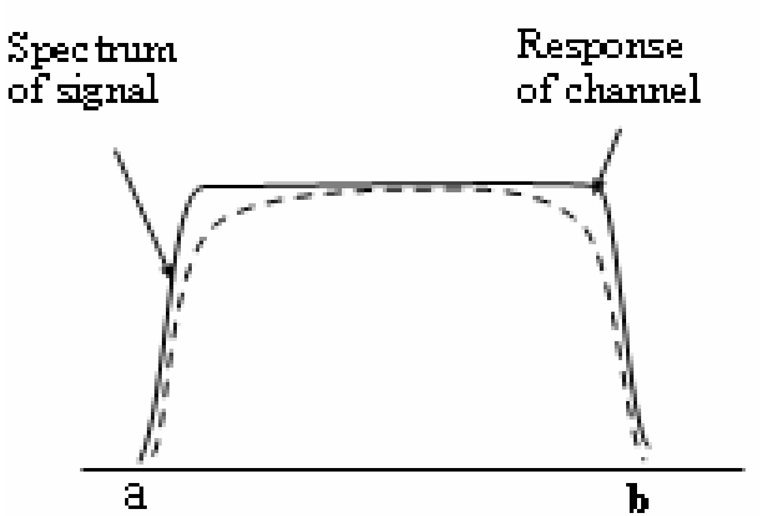


undecodable    decodable

# OFDM Diagram



# Unoccupied Subcarriers




- Edge sub-carriers are more vulnerable to errors under discrete FFT
  - Frequency might be shifted due to noise or multi-path
- Leave them unused
  - In 802.11, only 48 of 64 bins are occupied bins
- Is it really worth to use OFDM when it costs so many overheads (CP, unoccupied bins)?