<u>FIBER-REINFORCED ELASTOMERS</u>: MACROSCOPIC PROPERTIES, MICROSTRUCTURE EVOLUTION, AND STABILITY

Oscar Lopez-Pamies

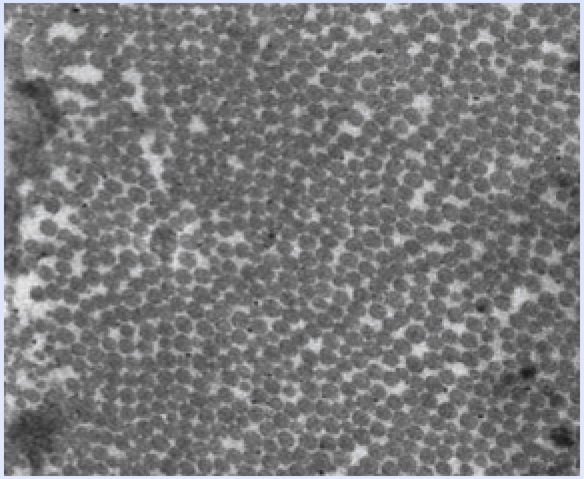
State University of New York, Stony Brook

STONY BROOK

Department of Aerospace Engineering, Iowa State University **February 19, 2010 · Ames, IA**

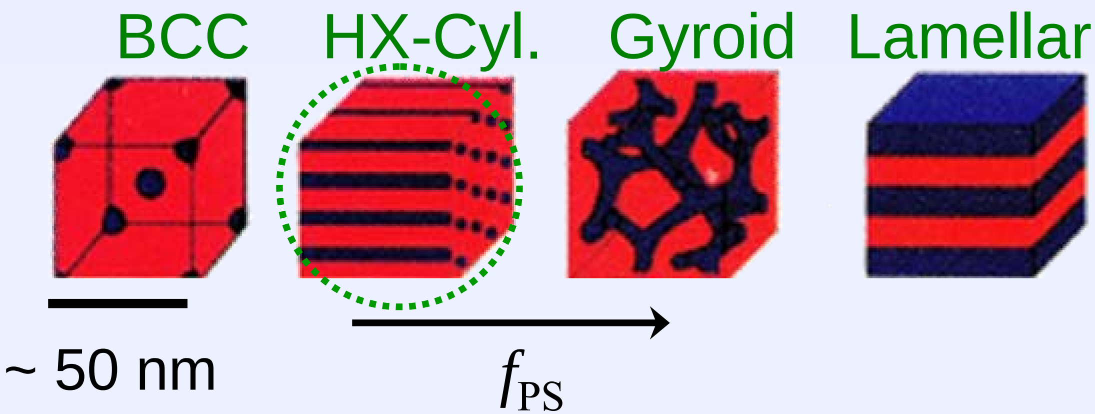
Soft solids reinforced with fibers

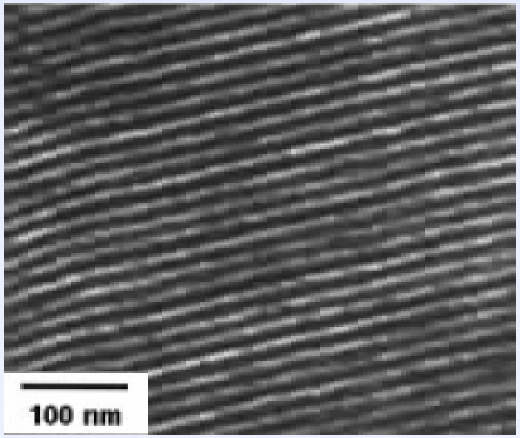
Rubber reinforced with carbon-black and fabric



TEM of Collagen Fibrils in Human Brain Arteries

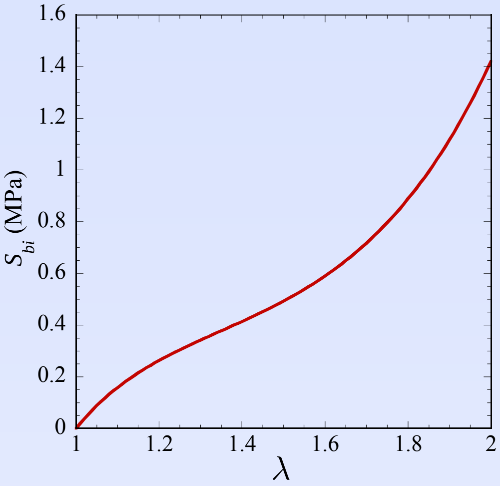
Thermoplastic Elastomers (Self-Assembled Nanodomains)



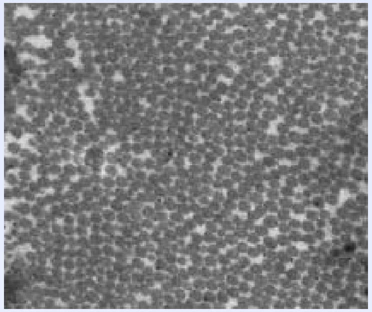


TEM of a triblock copolymer with cylindrical morphology

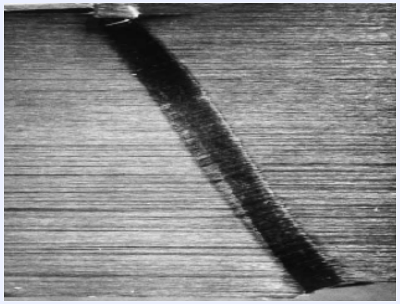
Issues in constitutive modeling of FREs



- Nonlinear constitutive matrix phase and fibers



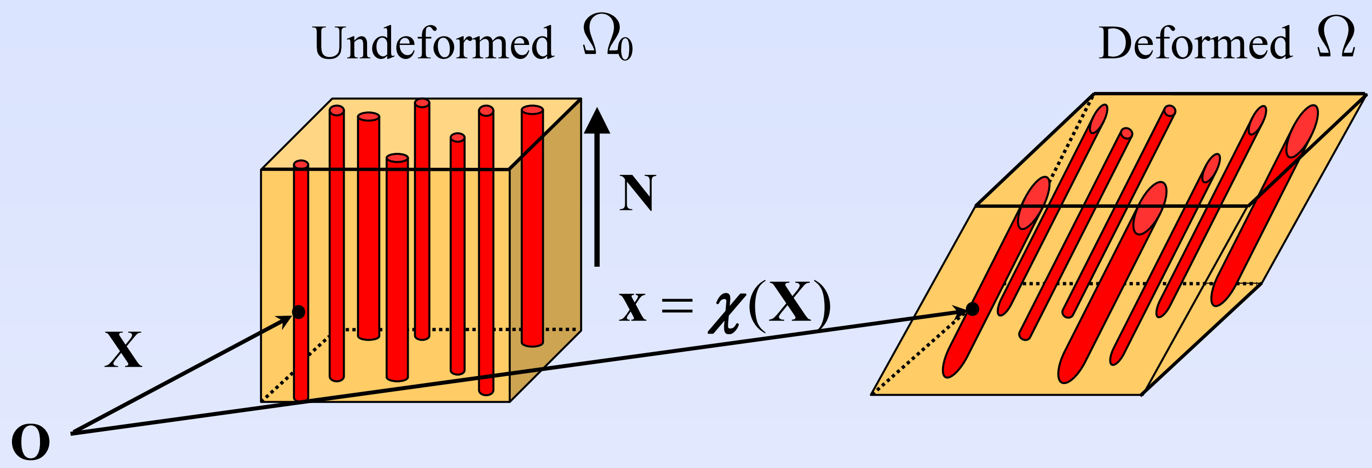
- Complex initial microstructure



- Microstructure evolution (geometric nonlinearity)
- Development of instabilities

Problem setting: Lagrangian formulation

Kinematics



Deformation Gradient

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$$

Right CG Deformation Tensor

$$\mathbf{C} = \mathbf{F}^T \mathbf{F}$$

Local constitutive behavior

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{F}}\left(\mathbf{X}, \mathbf{F}\right)$$

where

$$W\left(\mathbf{X}, \mathbf{F}\right) = \sum_{r=1}^{2} \chi_0^{(r)}\left(\mathbf{X}\right) W^{(r)}\left(\mathbf{F}\right)$$

Stored-energy function of the **matrix**

$$W^{(1)}\left(\mathbf{F}\right)$$

Stored-energy function of the **fibers**

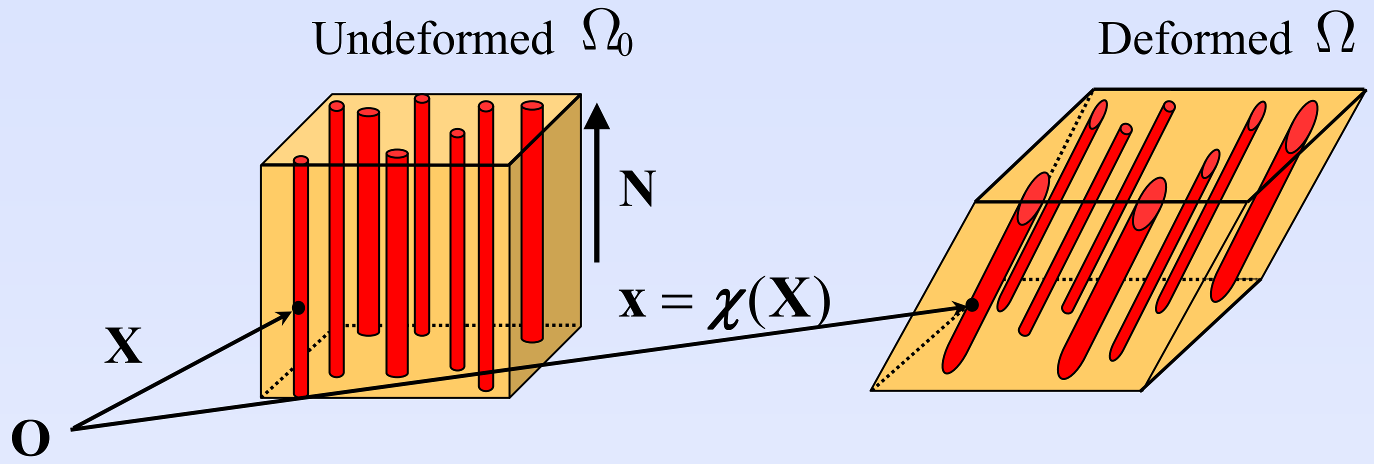
$$W^{(2)}\left(\mathbf{F}\right)$$

Random variable characterizing the microstructure

$$\chi_0^{(r)}\left(\mathbf{X}\right) = \begin{cases} 1 \text{ if } \mathbf{X} \in \text{phase } r \\ 0 \quad\quad\quad \text{otherwise} \end{cases}$$

Problem setting: Lagrangian formulation

Kinematics



Deformation Gradient

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}$$

Right CG Deformation Tensor

$$\mathbf{C} = \mathbf{F}^T \mathbf{F}$$

Local constitutive behavior

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{F}}\left(\mathbf{X}, \mathbf{F}\right)$$

where

$$W\left(\mathbf{X}, \mathbf{F}\right) = \sum_{r=1}^{2} \chi_0^{(r)}\left(\mathbf{X}\right) W^{(r)}\left(\mathbf{F}\right)$$

Isotropic matrix

$$W^{(1)}(\mathbf{F}) = \Psi^{(1)}(I_1, I_2)$$

Transversely isotropic fibers

$$W^{(2)}(\mathbf{F}) = \Psi^{(2)}(I_1, I_2, I_4, I_5)$$

where

$$I_1 = \mathrm{tr}\mathbf{C} \qquad I_2 = \frac{1}{2}\left[(\mathrm{tr}\mathbf{C})^2 \text{-} \mathrm{tr}\mathbf{C}^2\right]$$

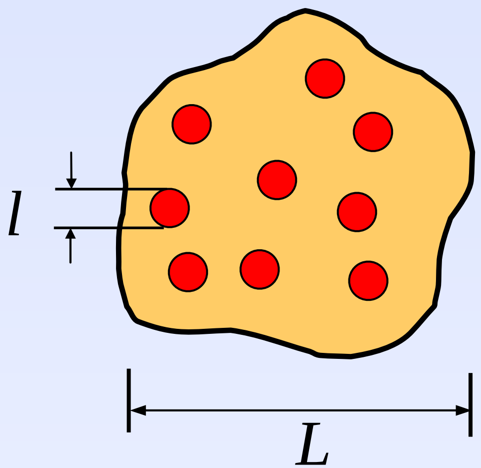
$$I_4 = \mathbf{N} \cdot \mathbf{C}\mathbf{N} \qquad I_5 = \mathbf{N} \cdot \mathbf{C}^2\mathbf{N}$$

Problem setting: macroscopic response

Definition: relation between the volume averages of the stress and deformation gradient over RVE

$$\bar{\mathbf{S}} = \frac{1}{\Omega_0}\int_{\Omega_0} \mathbf{S}\,\mathrm{d}\mathbf{X} \qquad \bar{\mathbf{F}} = \frac{1}{\Omega_0}\int_{\Omega_0} \mathbf{F}\,\mathrm{d}\mathbf{X}$$

Undeformed RVE



Variational Characterization

$$\bar{\mathbf{S}} = \frac{\partial \bar{W}}{\partial \bar{\mathbf{F}}}(\bar{\mathbf{F}})$$

where

$$\bar{W}\left(\bar{\mathbf{F}}\right) = \min_{\mathbf{F}\in K\left(\bar{\mathbf{F}}\right)} \frac{1}{\Omega_0}\int_{\Omega_0} W\left(\mathbf{X},\mathbf{F}\right)\mathrm{d}\mathbf{X}$$

separation of length-scales $\frac{l}{L} << 1$

and

$$K(\bar{\mathbf{F}}) = \left\{ \mathbf{F} \mid \mathbf{F} = \nabla \mathbf{x} \ \text{ in } \ \Omega_0, \ \text{ and } \ \mathbf{x} = \bar{\mathbf{F}}\mathbf{X} \ \text{ on } \ \partial\Omega_0 \right\}$$

Hill (1972), Braides (1985), Müller (1987)

Existing analytical approaches

- **Phenomenological models**

Formulated on the basis of invariants

$$\bar{W}\left(\bar{\mathbf{F}}\right) = F_{iso}\left(\bar{I}_1, \bar{I}_2\right) + G_{ani}\left(\bar{I}_4, \bar{I}_5\right)$$

Merodio, Horgan, Ogden, Pence, Rivlin, Saccomandi, among others

- **Homogenization/Micromechanics models**

Incorporate direct information from microscopic properties

– Estimates for special loading conditions, and special matrix and fiber constituents (He et al., 2006; deBotton et al., 2006)

– "Linear comparison" variational estimates for general loading conditions and isotropic constituents (LP & Ponte Castañeda 2006a,b)

Stability and failure

Classes of instabilities

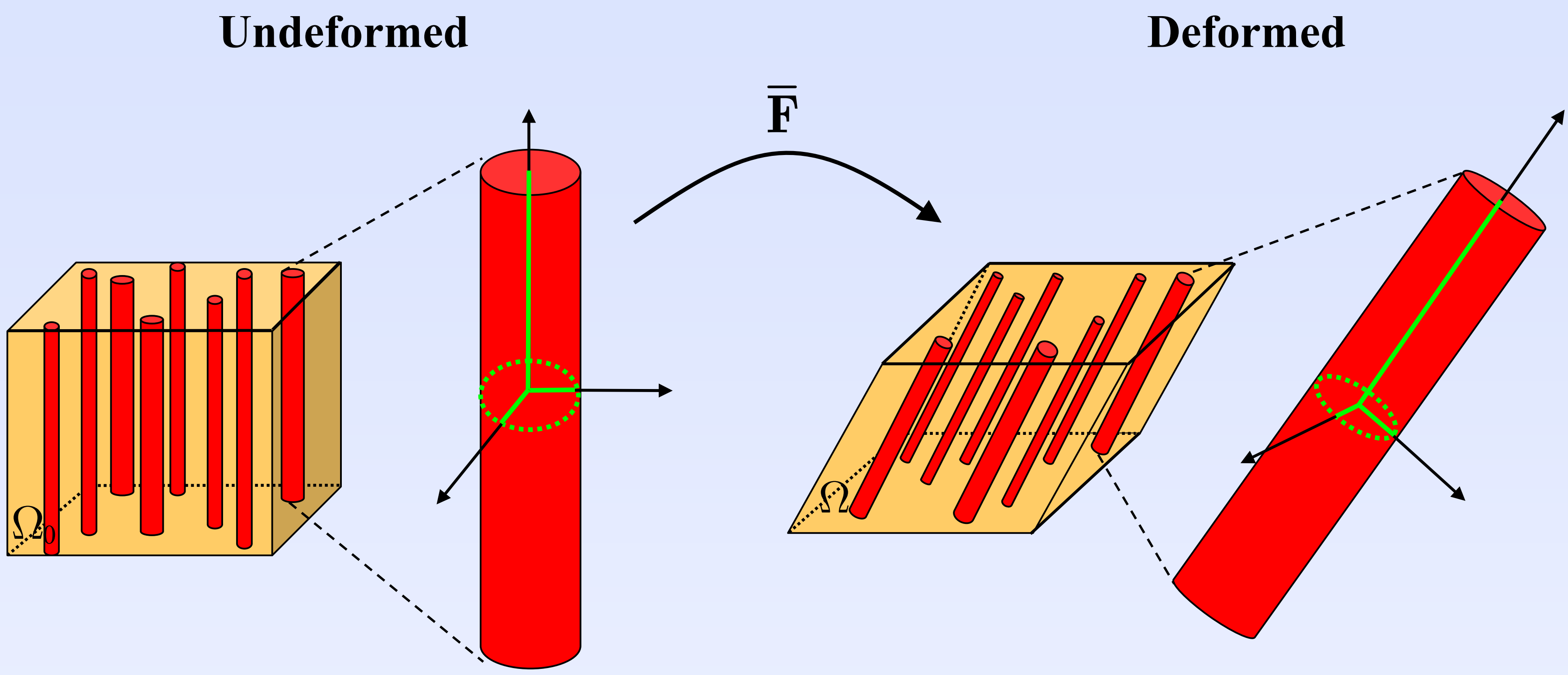
- **Material**
  - fiber failure (local)
  - fiber debonding (local)
  - matrix cavitation (local)
- **Geometrical**
  - short wavelength (local)
  - **long wavelength** (global)

- The loss of strong ellipticity of the macroscopic response of the fiber-reinforced elastomer, as characterized by the effective stored-energy function $\overline{W}(\overline{\mathbf{F}})$, denotes the onset of long wavelength instabilities

$$B(\overline{\mathbf{F}}) = \min_{\substack{\mathrm{u,v} \\ ||\mathrm{u}||=||\mathrm{v}||=1}} \left\{ v_i v_k \frac{\partial^2 \overline{W}}{\partial \overline{F}_{ij} \partial \overline{F}_{kl}} (\overline{\mathbf{F}}) u_j u_l \right\} = 0$$

Geymonat, Müller, Triantafyllidis (1993)

Microstructure evolution



Information on microstructure evolution is important to identify and understand the microscopic mechanisms that govern the macroscopic behavior. For that we need information about the local fields $\mathbf{F}(\mathbf{X})$

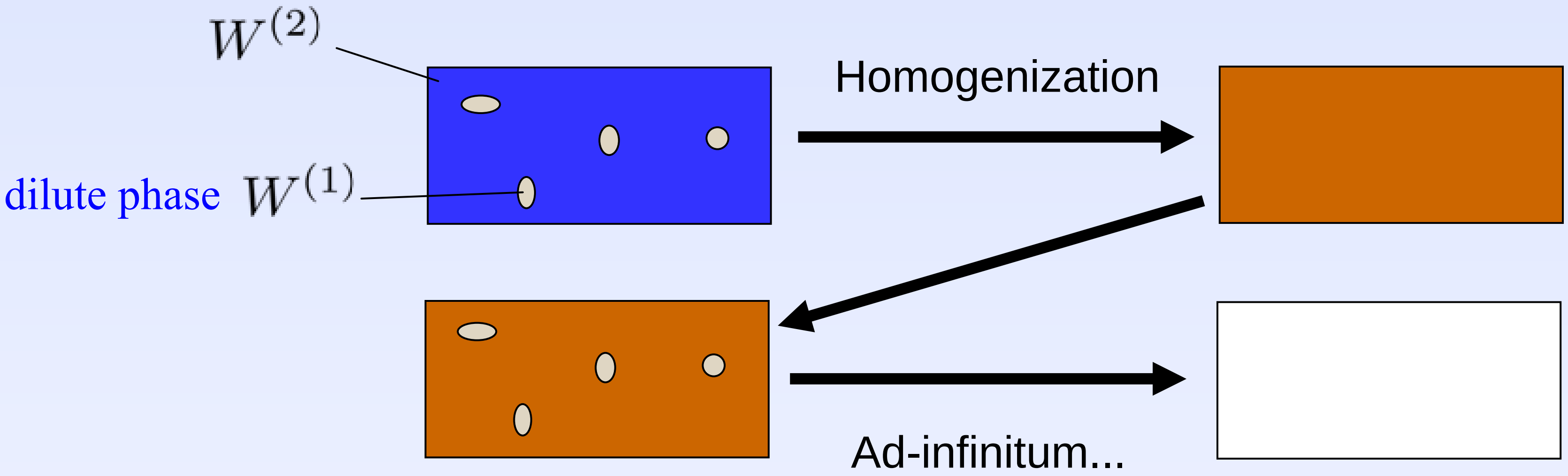
LP (2006)

New Approach:
Iterated Homogenization

Iterated dilute homogenization

Strategy: Construct a particulate distribution of fibers ($\chi_0^{(r)}(\mathbf{X})$) within a hyperelastic material for which it is possible to compute **exactly** the effective stored-energy function $\overline{W}$

Step 1: iterated-dilute homogenization

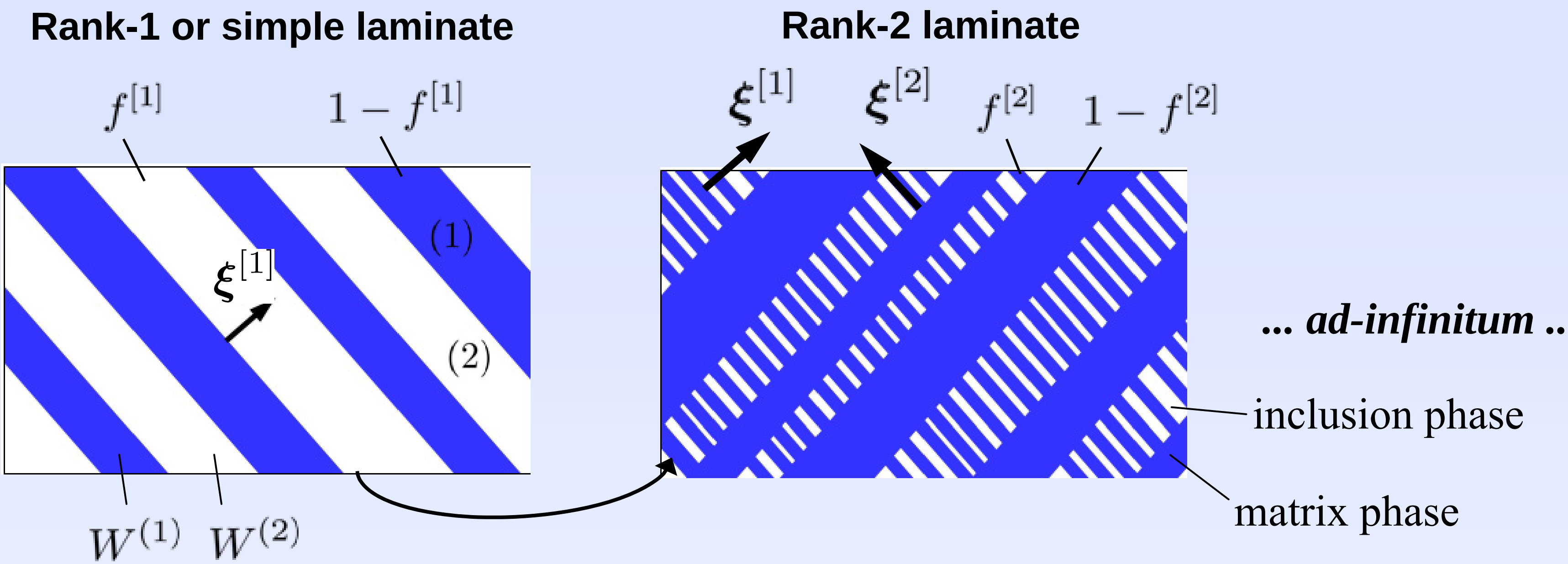


$$c_0 \frac{\partial \overline{W}}{\partial c_0} - H\left[W^{(1)}, \overline{W}; \overline{\mathbf{F}}\right] = 0, \quad \overline{W}(\overline{\mathbf{F}}, 1) = W^{(2)}$$

LP, *J. App. Mech.* (2010)

Auxiliary dilute problem: sequential laminates

Step 2: sequential laminates



When the **matrix** phase is dilute:

$$H\left[W^{(1)}, \bar{W}, \bar{\mathbf{F}}\right] = \bar{W} + \max_{\boldsymbol{\omega}} \int_S \left[\boldsymbol{\omega} \cdot \frac{\partial \bar{W}}{\partial \bar{\mathbf{F}}} \boldsymbol{\xi} - W^{(1)}\left(\bar{\mathbf{F}} + \boldsymbol{\omega} \otimes \boldsymbol{\xi}\right)\right] \underbrace{\nu(\boldsymbol{\xi})} \, \mathrm{d}S$$

distributional function related to the two-point statistics of fiber distribution in the undeformed configuration

Idiart, *JMPS* (2008)

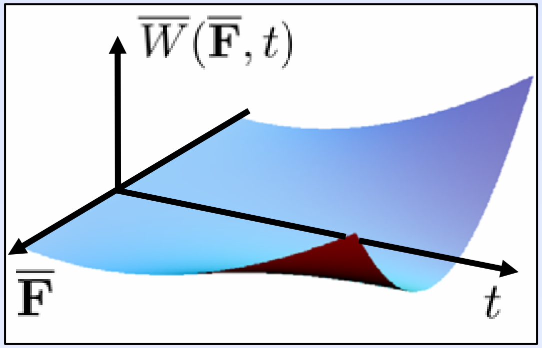
Iterated homogenization framework

- The effective stored-energy function $\overline{W}$ can be finally shown to be given by the following **Hamilton-Jacobi equation**

$$c_0 \frac{\partial \overline{W}}{\partial c_0} - \overline{W} - \max_{\boldsymbol{\omega}} \int_S \left[\boldsymbol{\omega} \cdot \frac{\partial \overline{W}}{\partial \overline{\mathbf{F}}} \boldsymbol{\xi} - W^{(1)} \left(\overline{\mathbf{F}} + \boldsymbol{\omega} \otimes \boldsymbol{\xi} \right) \right] \nu(\boldsymbol{\xi}) \, \mathrm{d}S = 0$$

subject to the initial condition

$$\overline{W}(\overline{\mathbf{F}}, 1) = W^{(2)}(\overline{\mathbf{F}})$$



the **time** variable is $t \doteq -\ln c_0$ (initial fiber concentration)

the **spatial** variable is $\overline{\mathbf{F}}$

the **Hamiltonian** is

$$H\left[W^{(1)}, \overline{W}, \overline{\mathbf{F}} \right] = \overline{W} + \max_{\boldsymbol{\omega}} \int_S \left[\boldsymbol{\omega} \cdot \frac{\partial \overline{W}}{\partial \overline{\mathbf{F}}} \boldsymbol{\xi} - W^{(1)} \left(\overline{\mathbf{F}} + \boldsymbol{\omega} \otimes \boldsymbol{\xi} \right) \right] \nu(\boldsymbol{\xi}) \, \mathrm{d}S$$

LP & Idiart, *J. Eng. Math.* (2010)

Iterated homogenization framework: local fields

- Consider the following **perturbed problem** for $\overline{W}_\tau$

$$c_0 \frac{\partial \overline{W}_\tau}{\partial c_0} - \overline{W}_\tau - \max_{\boldsymbol{\omega}} \int_S \left[\boldsymbol{\omega} \cdot \frac{\partial \overline{W}_\tau}{\partial \overline{\mathbf{F}}} \boldsymbol{\xi} - W^{(1)} \left(\overline{\mathbf{F}} + \boldsymbol{\omega} \otimes \boldsymbol{\xi} \right) \right] \nu(\boldsymbol{\xi})\, \mathrm{d}S = 0$$

subject to the initial condition

$$\overline{W}(\overline{\mathbf{F}}, 1) = W^{(2)}(\overline{\mathbf{F}}) + \tau U(\overline{\mathbf{F}})$$

Then, the following identity is true

$$\frac{1}{\Omega_0^{(2)}} \int_{\Omega_0^{(2)}} U\left(\mathbf{F}(\mathbf{X}) \right) \mathrm{d}\mathbf{X} = \frac{1}{c_0} \frac{\partial \overline{W}_\tau}{\partial \tau} \bigg|_{\tau=0}$$

$U\left(\cdot\right)$ can be any function of interest, e.g., the deformation gradient

LP & Idiart, *J. Eng. Math.* (2010)

Remarks on the iterated homogenization approach

- The proposed IH method provides solutions for $\overline{W}$ in terms of $W^{(1)}$ and $W^{(2)}$ and the **one-** and **two-point statistics** of the random distribution of fibers

- In the limit of small deformations as $\overline{\mathbf{F}} \rightarrow \mathbf{I}$, the IH formulation **reduces to the HS lower bound** for fiber-reinforced random media

- In the further limit of dilute fiber concentration $c_0 \rightarrow 0$, the IH formulation recovers the **exact result of Eshelby** for a dilute distribution of ellipsodial fibers

- The proposed IH method provides access to **local fields**, which in turn permits the study of the **evolution of microstructure** and the **onset of instabilities**

- The computations amount to solving appropriate **Hamilton-Jacobi equations,** which are fairly tractable

Application to Fiber-Reinforced Neo-Hookean Solids

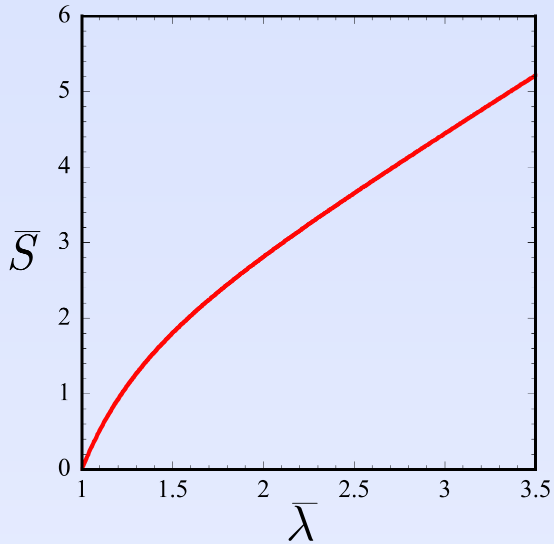
Fiber-reinforced Neo-Hookean solids

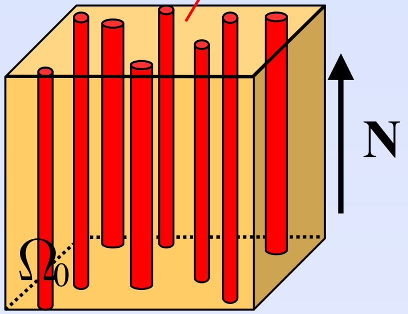
Neo-Hookean matrix

$$W^{(1)}(\mathbf{F}) = \Psi^{(1)}(I_1) = \frac{\mu^{(1)}}{2}\left(I_1 - 3\right)$$

Stiffer Neo-Hookean fibers

$$W^{(2)}(\mathbf{F}) = \Psi^{(2)}(I_1) = \frac{\mu^{(2)}}{2}\left(I_1 - 3\right)$$





Macroscopic stored-energy function

$$\overline{W}(\overline{\mathbf{F}}, c_0) = \overline{\Psi}(\overline{I}_1, \overline{I}_2, \overline{I}_4, \overline{I}_5, c_0)$$

Hamilton-Jacobi equation

$$c_0 \frac{\partial \overline{\Psi}}{\partial c_0} + \frac{\mu^{(1)}}{2}\left(\overline{I}_1 - 3\right) - \overline{\Psi} + \mu^{(1)}\left(\overline{I}_1 - \overline{I}_4 - \frac{2}{\sqrt{\overline{I}_4}}\right)\left(\frac{1}{2} - \frac{1}{\mu^{(1)}}\frac{\partial \overline{\Psi}}{\partial \overline{I}_1}\right)^2 = 0$$

subject to the initial condition $\overline{\Psi}(\overline{I}_1, \overline{I}_2, \overline{I}_4, \overline{I}_5, 1) = \Psi^{(2)}(\overline{I}_1, \overline{I}_4)$

Overall stress-strain relation

Closed-form solution for $\overline{W}$

$$\overline{W}(\overline{\mathbf{F}}) = \overline{\Psi}(\overline{I}_1, \overline{I}_4) = f\left(\overline{I}_1\right) + g\left(\overline{I}_4\right)$$

where

$$f(\overline{I}_1) = \frac{\tilde{\mu}}{2}\left(\overline{I}_1 - 3\right) \quad \text{and} \quad g(\overline{I}_4) = \frac{\bar{\mu} - \tilde{\mu}}{2} \frac{\left(\sqrt{\overline{I}_4} + 2\right)\left(\sqrt{\overline{I}_4} - 1\right)}{\sqrt{\overline{I}_4}}$$

with

$$\bar{\mu} = (1 - c_0)\mu^{(1)} + c\mu^{(2)} \quad \text{and} \quad \tilde{\mu} = \frac{(1 - c_0)\mu^{(1)} + (1 + c_0)\mu^{(2)}}{(1 + c_0)\mu^{(1)} + (1 - c_0)\mu^{(2)}}\mu^{(1)}$$

Note I: separable functional form

Note II: no dependence on the second $\overline{I}_2$ nor fifth $\overline{I}_5$ invariants

Overall stress-strain relation

$$\overline{\mathbf{S}} = \frac{\partial \overline{W}}{\partial \overline{\mathbf{F}}}(\overline{\mathbf{F}}) - \overline{p}\overline{\mathbf{F}}^{-T} = \tilde{\mu}\overline{\mathbf{F}} + (\bar{\mu} - \tilde{\mu})\left[1 - \overline{I}_4^{-3/2}\right]\overline{\mathbf{F}}\mathbf{N} \otimes \mathbf{N} - \overline{p}\overline{\mathbf{F}}^{-T}$$

LP & Idiart, *J. Eng. Math.* (2010)

Macroscopic Instabilities

Classes of instabilities

- Material
  - fiber failure (local)
  - fiber debonding (local)
  - matrix cavitation (local)
- Geometrical
  - short wavelength (local)
  - long wavelength (global)

Along an arbitrary loading path, the material first becomes unstable at a critical deformation $\overline{\mathbf{F}}_{cr}$ *with* $\overline{I}_4^{cr} = \overline{\mathbf{F}}_{cr}\mathbf{N} \cdot \overline{\mathbf{F}}_{cr}\mathbf{N}$ *such that*

$$\overline{I}_4^{cr} = \left(1 - \frac{\tilde{\mu}}{\overline{\mu}}\right)^{2/3}$$

Note: $\tilde{\mu} \leq \overline{\mu} \Rightarrow \overline{I}_4^{cr} \leq 1$

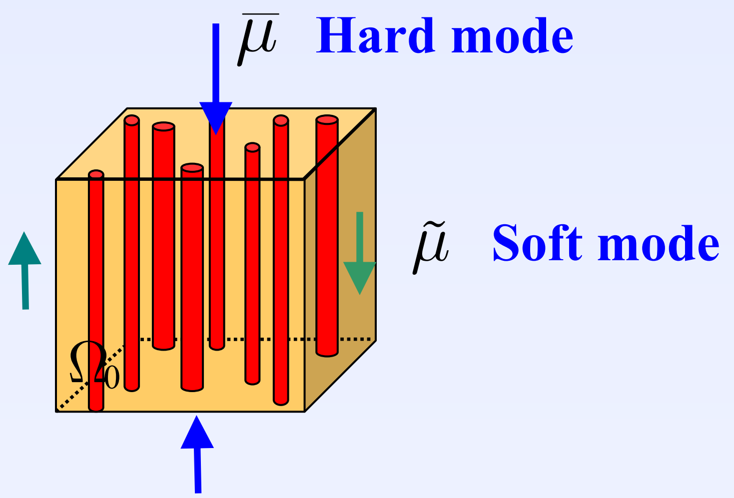
LP & Idiart, *J. Eng. Math.* (2010)

Macroscopic Instabilities

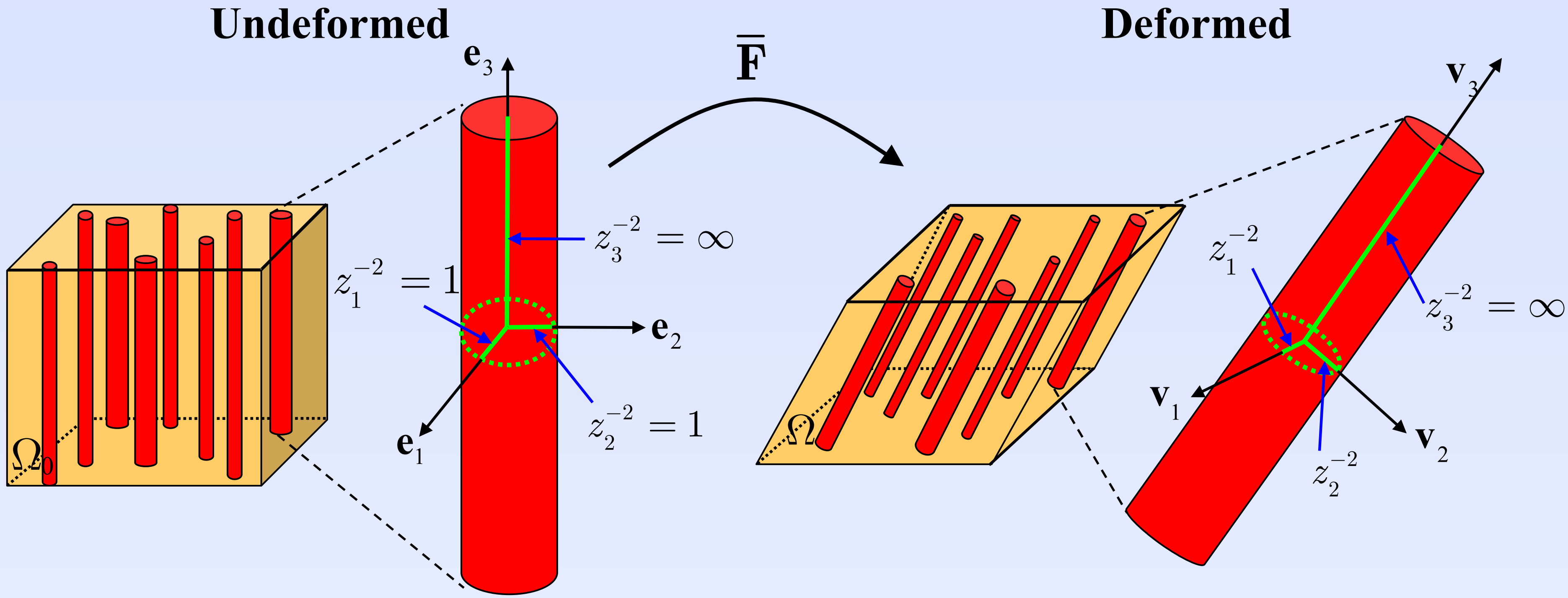
Observations

- Macroscopic instabilities may only occur when the deformation in the fiber direction, as measured by $\bar{I}_4$, reaches a sufficiently large **compressive** value $\bar{I}_4^{cr} \leq 1$

- The condition states that instabilities may develop whenever the compressive deformation along the fiber direction reaches a critical value determined by the ratio of **hard-to-soft** modes of deformation



Microstructure evolution



The **average shape** and **orientation** of the fibers in the deformed configuration are characterized by the Eulerian ellipsoid

$$E = \left\{ \mathbf{x} \mid \mathbf{x} \cdot \mathbf{Z}^T \mathbf{Z} \mathbf{x} \leq \mathbf{1} \right\}$$

where

$$\mathbf{Z} = \left(\mathbf{I} - \mathbf{N} \otimes \mathbf{N} \right) \left(\overline{\mathbf{F}}^{(2)} \right)^{-1}$$

Eigenvalues of $\mathbf{Z}^T \mathbf{Z}$

z_1, z_2, z_3

Eigenvectors of $\mathbf{Z}^T \mathbf{Z}$

$\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$

LP (2006), LP & Idiart (2010)

Microstructure evolution

$\overline{\mathbf{F}}^{(2)}$ is the **average deformation gradient in the fibers**. In the IH framework, it is solution of the pde

$$c_0 \frac{\partial \overline{\mathbf{F}}^{(2)}}{\partial c_0} - \frac{\partial \overline{\mathbf{F}}^{(2)}}{\partial \overline{\mathbf{F}}} \int_S \boldsymbol{\omega} \otimes \boldsymbol{\xi} \; \mathrm{d}S = 0$$

subject to the initial condition $\overline{\mathbf{F}}^{(2)}(\overline{\mathbf{F}}, 1) = \overline{\mathbf{F}}$

Closed-form solution

$$\overline{\mathbf{F}}^{(2)} = \gamma_1 \left[\overline{\mathbf{F}} - \overline{\mathbf{F}}\mathbf{N} \otimes \mathbf{N} \right] - \frac{2\overline{\nu} - \gamma_1}{\sqrt{\overline{I}_4}} \left[\overline{\mathbf{F}}^{-T} - \overline{\mathbf{F}}^{-T}\mathbf{N} \otimes \mathbf{N} \right] + \frac{2\overline{\nu} - \gamma_1}{\sqrt{\overline{I}_4}} \overline{\mathbf{F}}\mathbf{N} \otimes \mathbf{u} + \overline{\mathbf{F}}\mathbf{N} \otimes \mathbf{N}$$

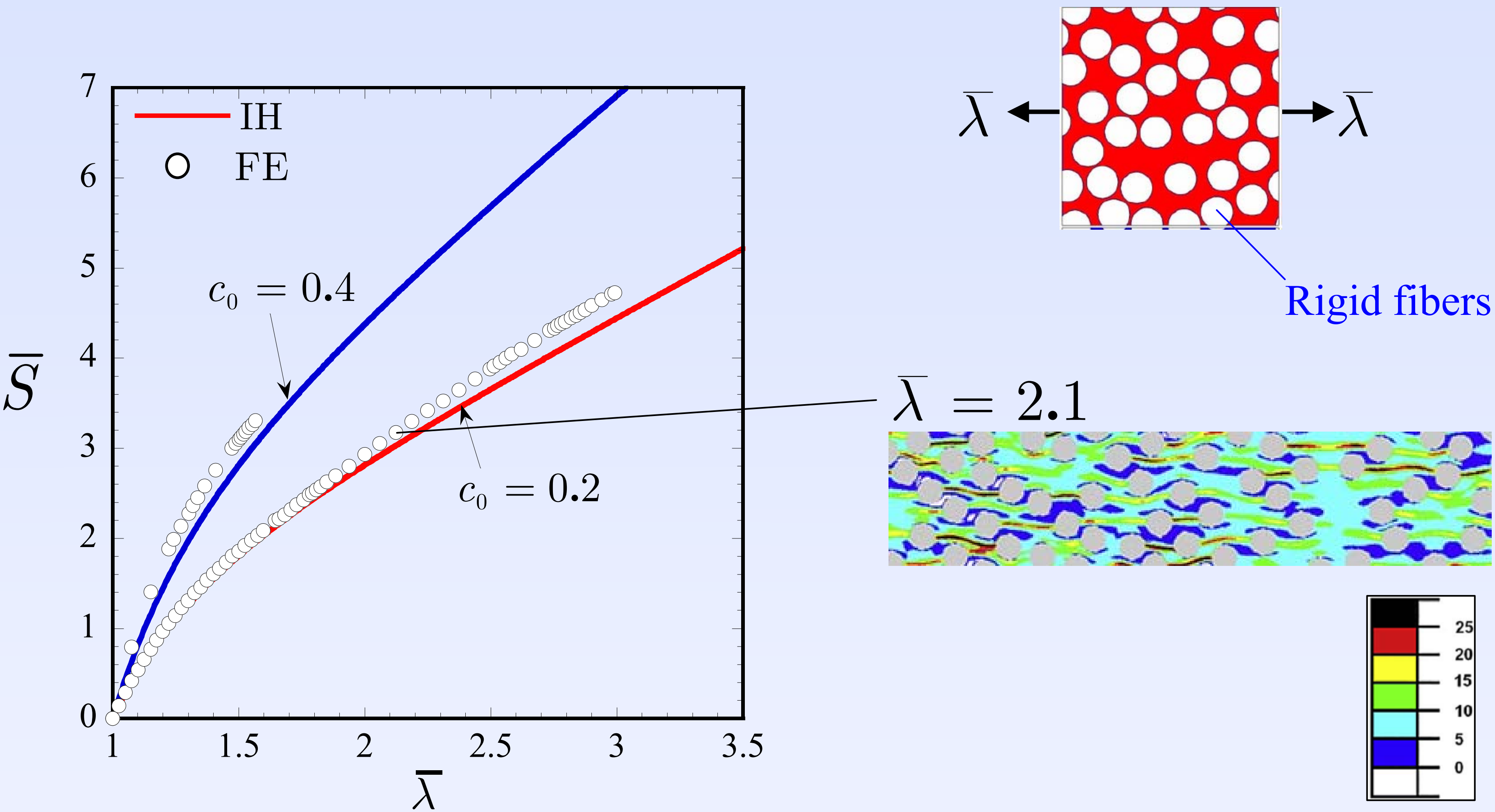
where $\mathbf{u} = (\mathbf{I} - \mathbf{N} \otimes \mathbf{N})\overline{\mathbf{F}}^T \overline{\mathbf{F}}\mathbf{N}$, and

$$\gamma_1 = \overline{\nu} + \frac{\sqrt{\sqrt{\overline{I}_4} + \overline{\nu}^2 \left(\overline{I}_1 \overline{I}_4 - \overline{I}_5 - 2\sqrt{\overline{I}_4} \right)}}{\sqrt{\overline{I}_1 \overline{I}_4 - \overline{I}_5 + 2\sqrt{\overline{I}_4}}}, \qquad \overline{\nu} = \frac{\mu^{(1)}}{(1 + c_0)\mu^{(1)} + (1 - c_0)\mu^{(2)}}$$

LP, Idiart, & Li (2010)

Sample Results

IH vs. FEM: in-plane stress-strain response



Moraleda et al. (2009)

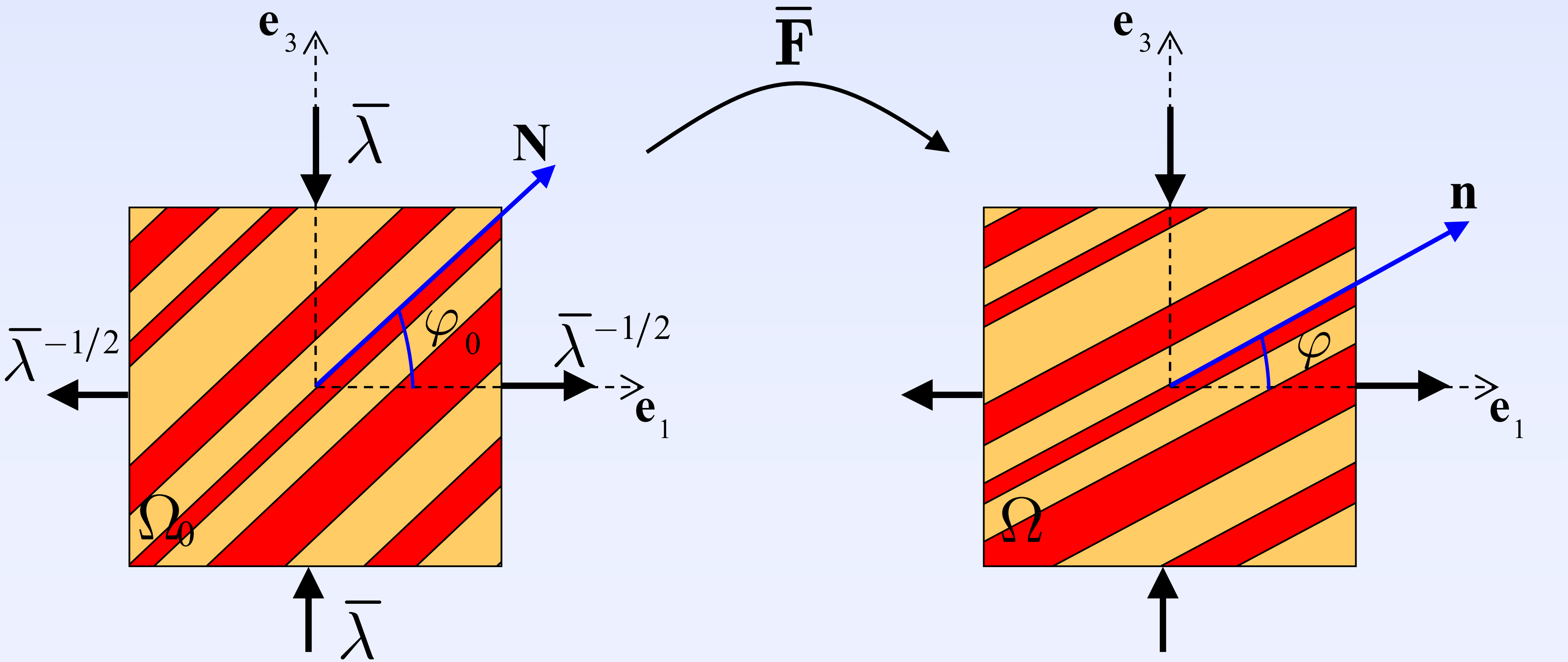
Axisymmetric compression at an angle φ_0

Applied Loading

$$\bar{\mathbf{F}} = \begin{pmatrix} \bar{\lambda}^{-1/2} & 0 & 0 \\ 0 & \bar{\lambda}^{-1/2} & 0 \\ 0 & 0 & \bar{\lambda} \end{pmatrix}$$

Initial fiber orientation

$$\mathbf{N} = \cos\varphi_0\, \mathbf{e}_1 + \sin\varphi_0\, \mathbf{e}_3$$



Axisymmetric compression at an angle φ_0

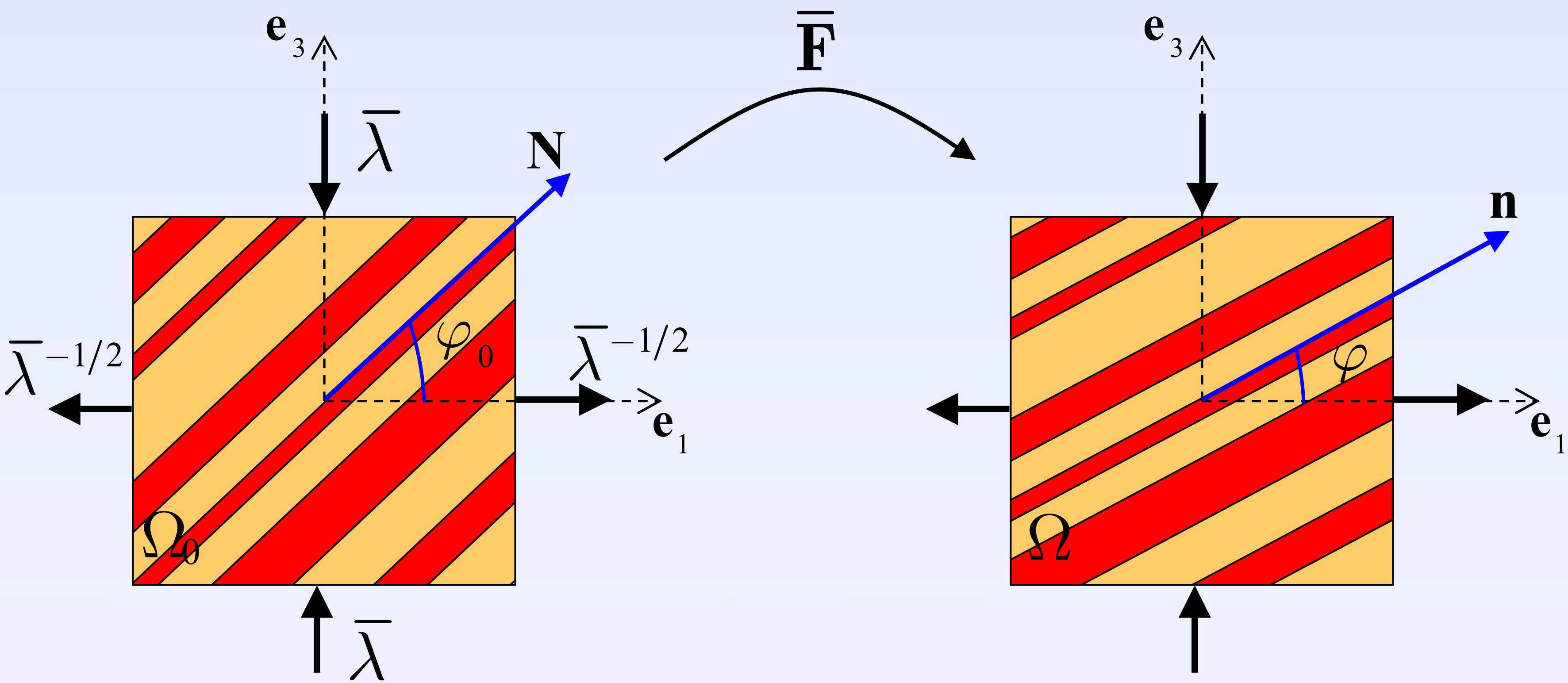
Applied Loading

$$\bar{\mathbf{F}} = \begin{pmatrix} \bar{\lambda}^{-1/2} & 0 & 0 \\ 0 & \bar{\lambda}^{-1/2} & 0 \\ 0 & 0 & \bar{\lambda} \end{pmatrix}$$

Current fiber orientation

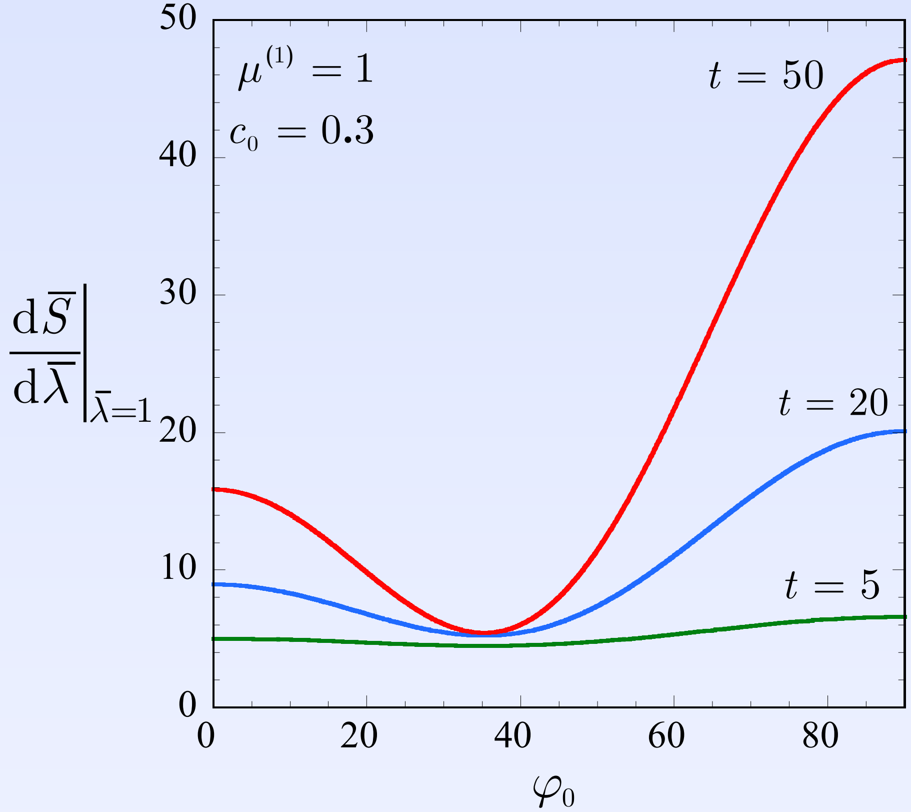
$$\mathbf{n} = \cos\varphi\,\mathbf{e}_1 + \sin\varphi\,\mathbf{e}_3$$

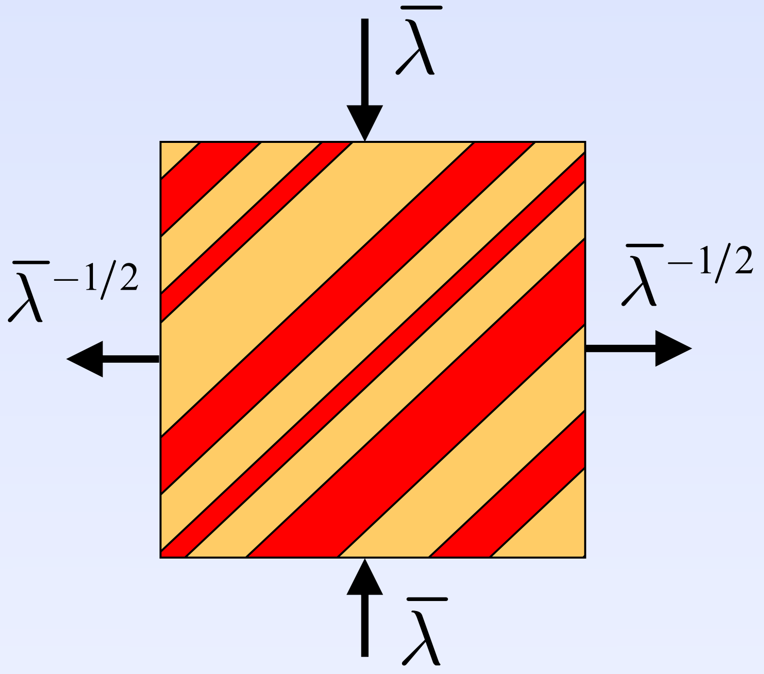
$$\varphi = \mathrm{Arcos}\left[\frac{\cos\varphi_0}{\sqrt{\cos^2\varphi_0 + \bar{\lambda}^3\cos^2\varphi_0}}\right]$$



Axisymmetric compression at an angle φ_0

Small-deformation response

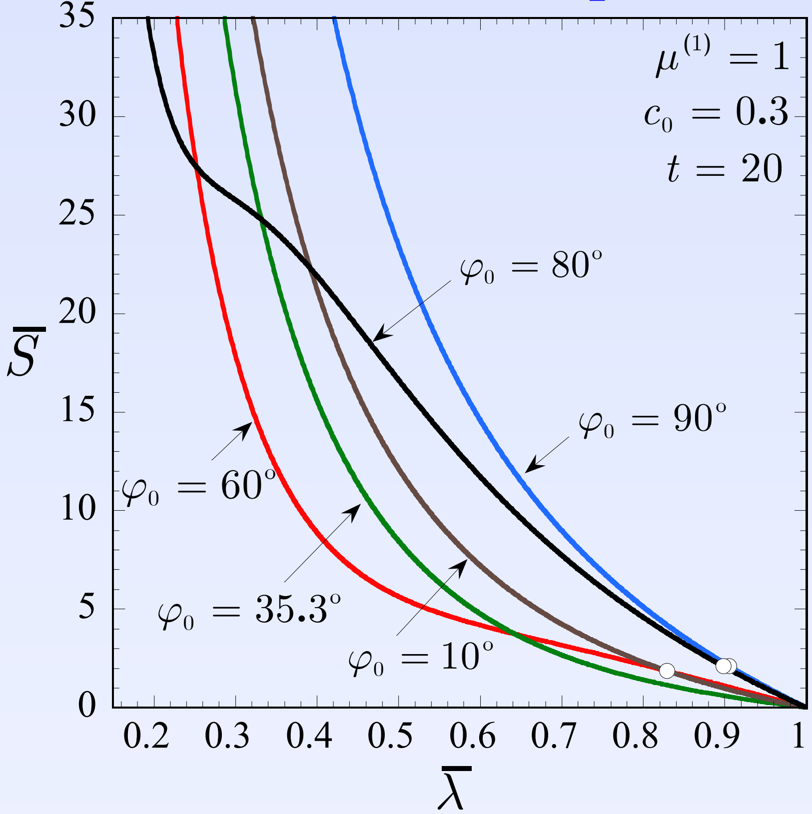




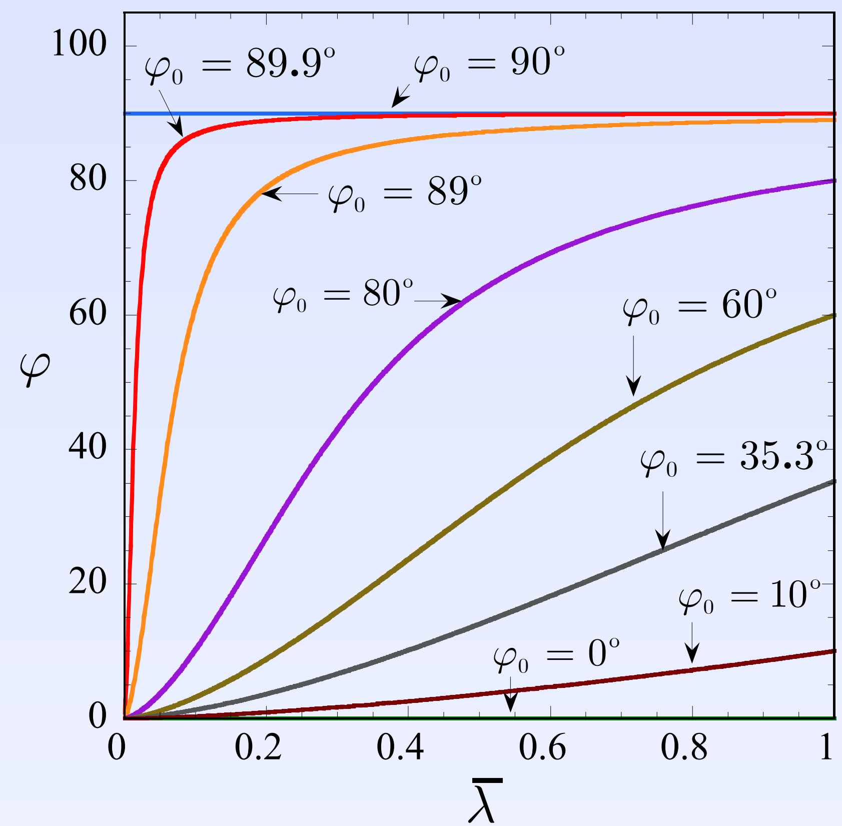
$t \doteq \frac{\mu^{(2)}}{\mu^{(1)}}$ heterogeneity contrast

Axisymmetric compression at an angle φ_0

Large-deformation stress-strain response



Evolution of fiber orientation



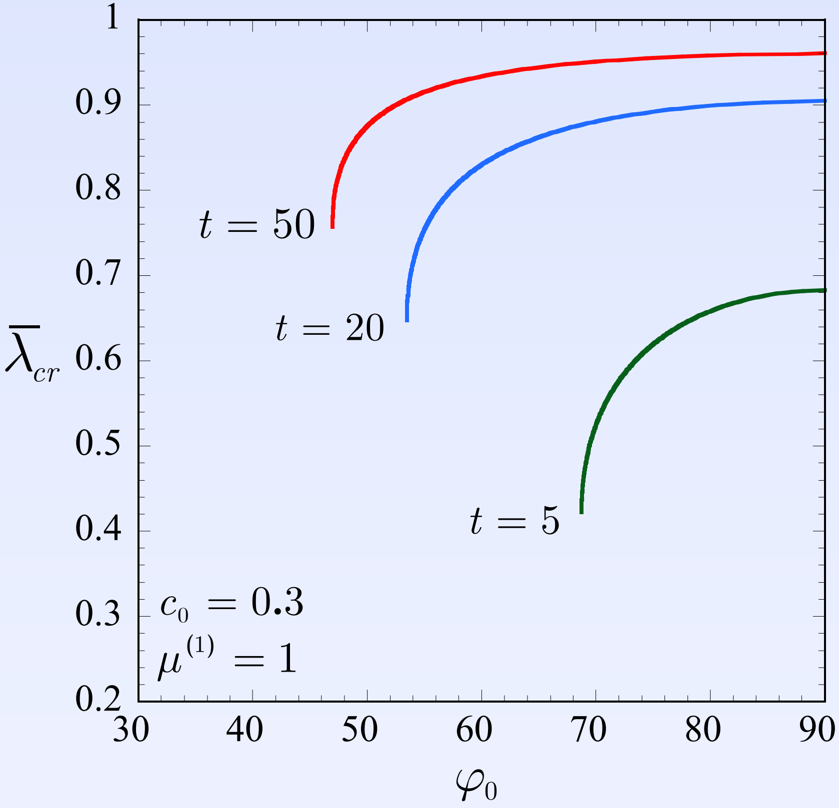
Note: Macroscopic instability at ○

LP, Idiart, & Li (2010)

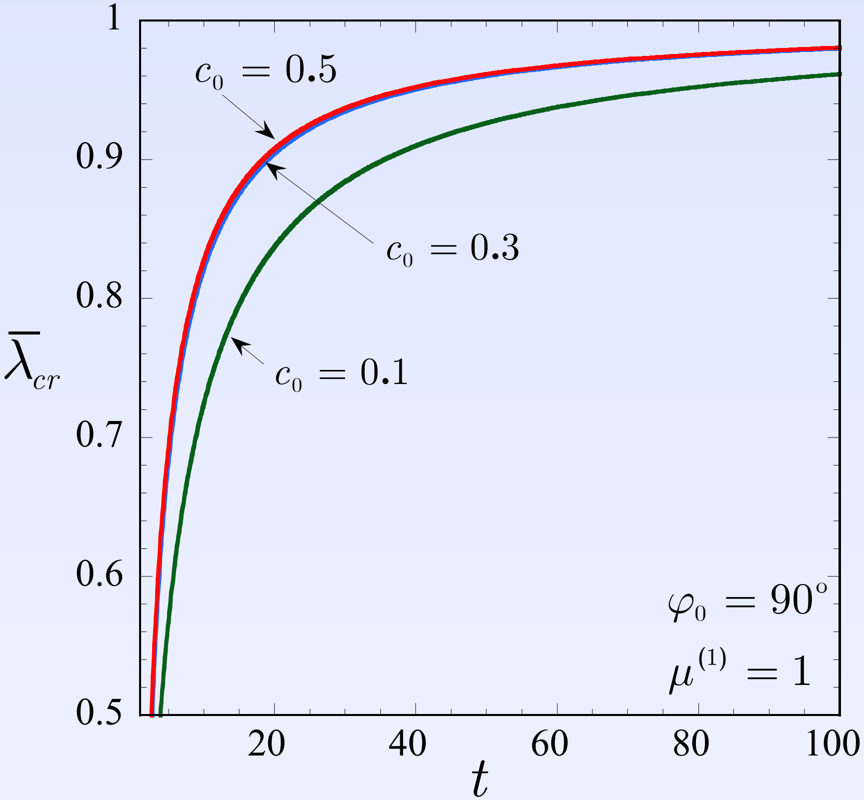
Axisymmetric compression at an angle φ_0

Onset of macroscopic instabilities at $\overline{\lambda}_{cr}$

Effect of fiber orientation



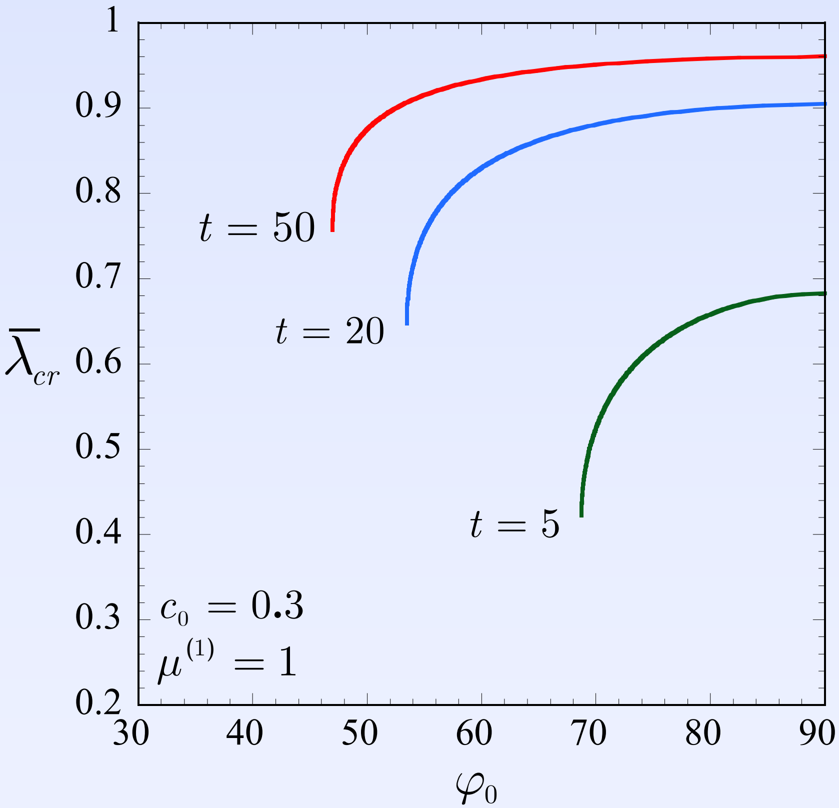
Effect of fiber-to matrix contrast



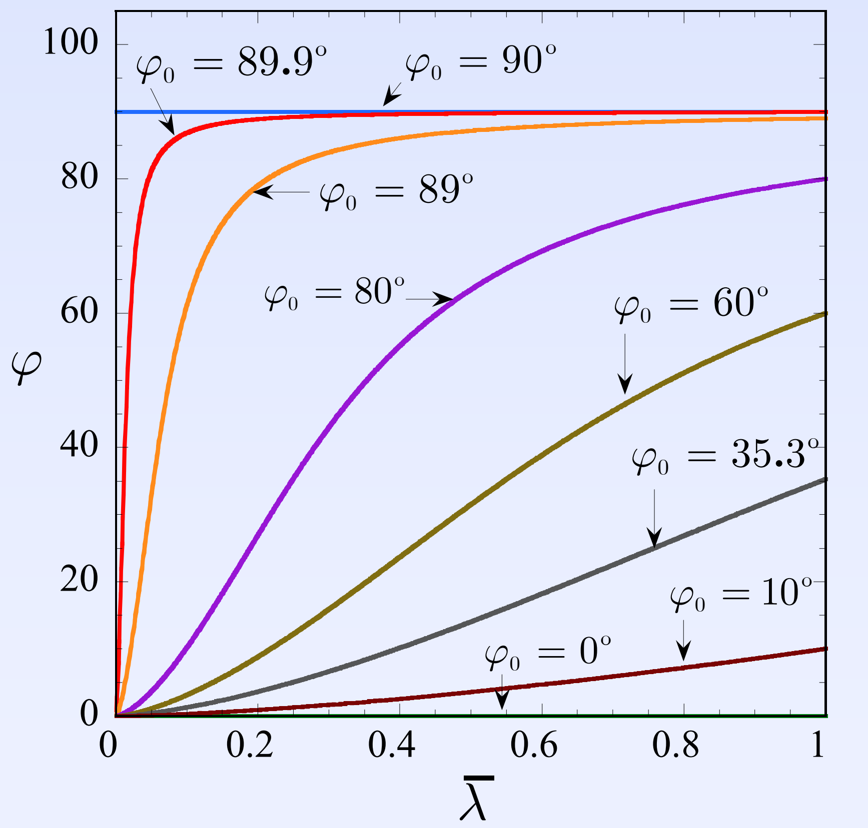
Axisymmetric compression at an angle φ_0

Onset of macroscopic instabilities at $\bar{\lambda}_{cr}$

Effect of fiber orientation



Evolution of fiber orientation



Remarks

The **rotation of the fibers** — which depends critically on the relative orientation between the loading axes and the fiber direction — can act as a dominant **geometric softening mechanism**.

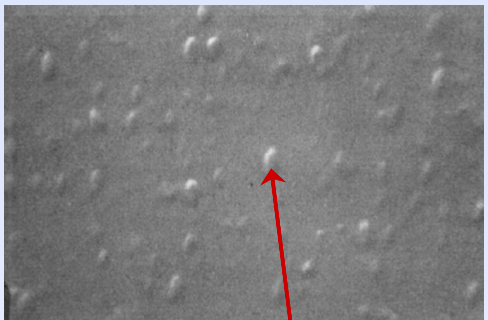
- It was found that the long axes of **the fibers rotate away from the axis of maximum compressive loading** towards the axis of maximum tension.

- Loadings with predominant compression along the fibers lead to larger rotation of the fibers, which in turn lead to larger geometric **softening of the constitutive response**, and in some cases — when the heterogeneity contrast between the matrix and the fibers is sufficiently high — also to the **loss of macroscopic stability**.

Remarks

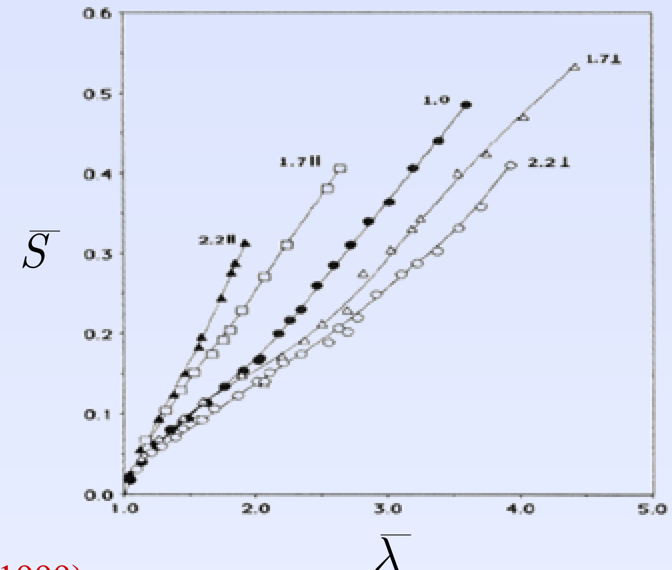
The results of this work can help understanding the behavior of many **other solids with oriented microstructures**

Reinforced Elastomers (Wang and Mark 1990)



2 μm

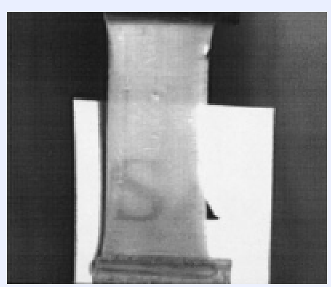
Polystyrene Ellipsoids

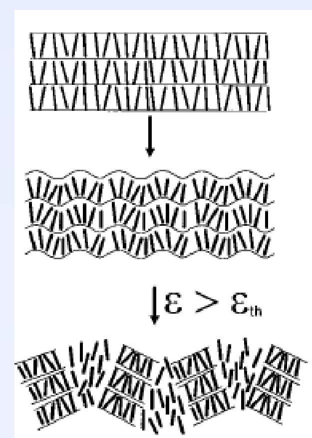


Liquid crystal elastomers (Nishikawa and Finkelmann1999)

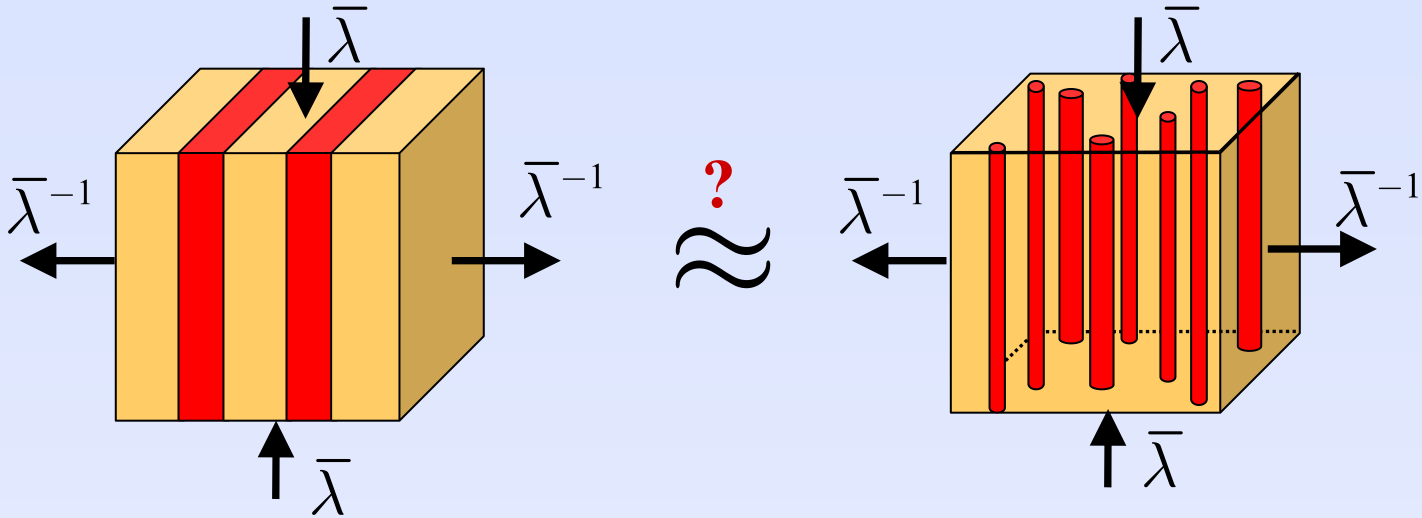
Transparent

Opaque





Contact with earlier work with laminates



- For the **aligned plane-strain loading of a laminate** the onset of macroscopic instabilities occurs at

$$\bar{\lambda}_{cr}^{Lam} = \left(1 - \frac{\tilde{\mu}^{L}}{\bar{\mu}}\right)^{1/4}$$

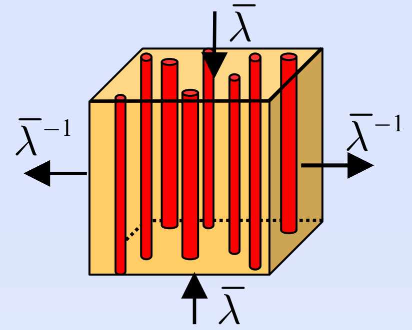
where

$$\bar{\mu} = (1-c)\mu^{(1)} + c\mu^{(2)} \qquad \text{and} \qquad \tilde{\mu}^{L} = \left(\frac{1-c}{\mu^{(1)}} + \frac{c}{\mu^{(2)}}\right)^{-1}$$

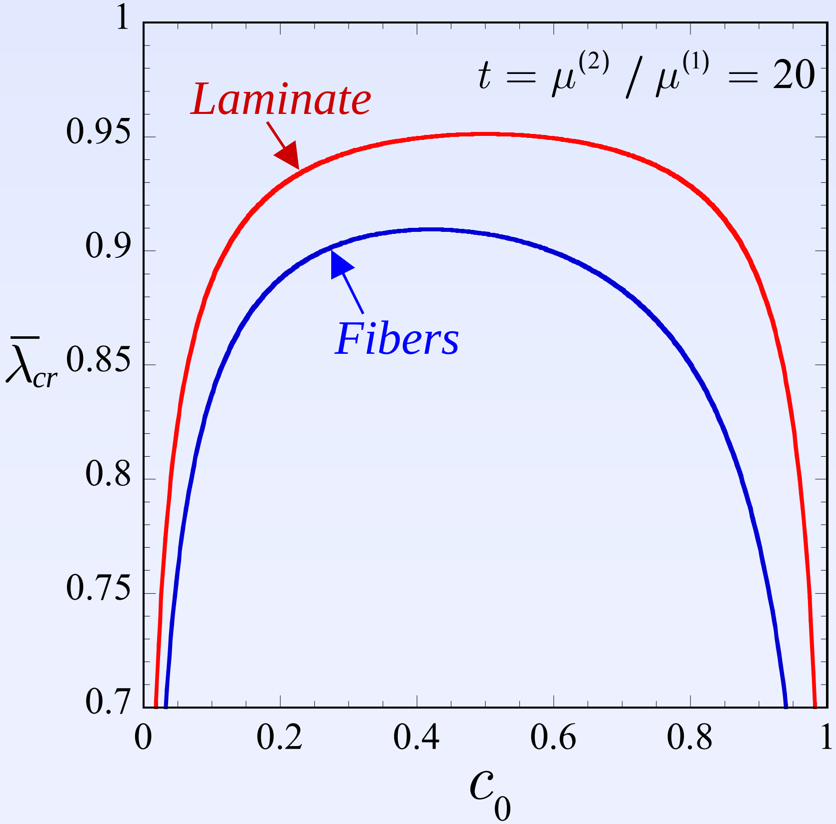
Rosen (1965); Triantafyllidis and Maker (1985)

Contact with earlier work with laminates

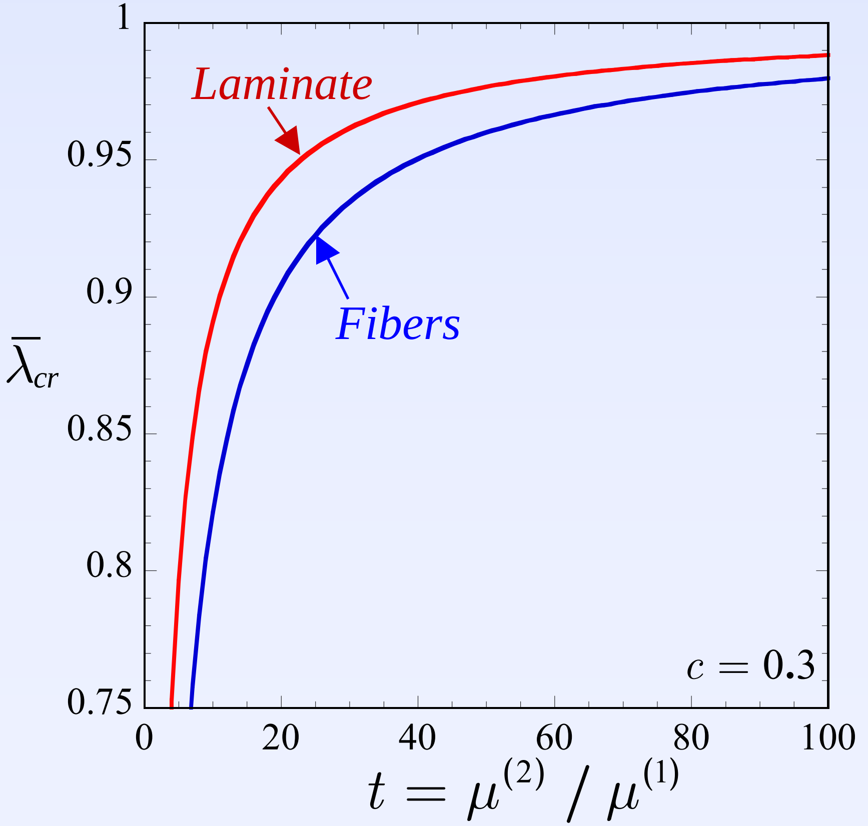
Aligned plane-strain loading conditions



Effect of **fiber concentration** c_0



Effect of **contrast** $t = \mu^{(2)} / \mu^{(1)}$



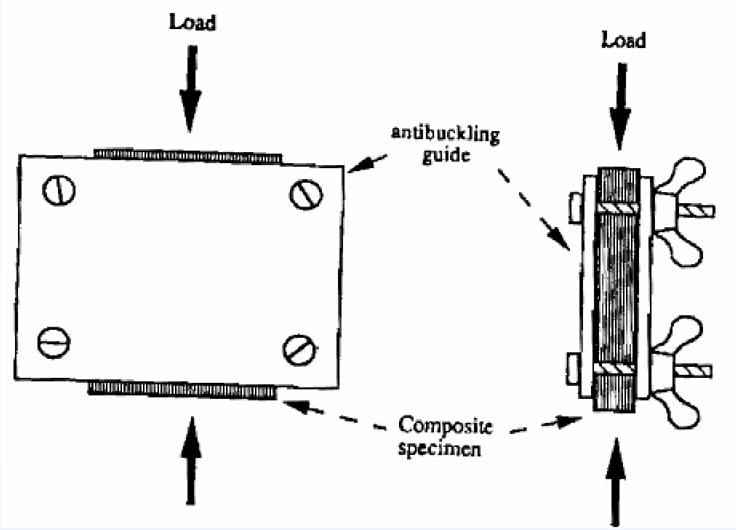
Contact with experiments

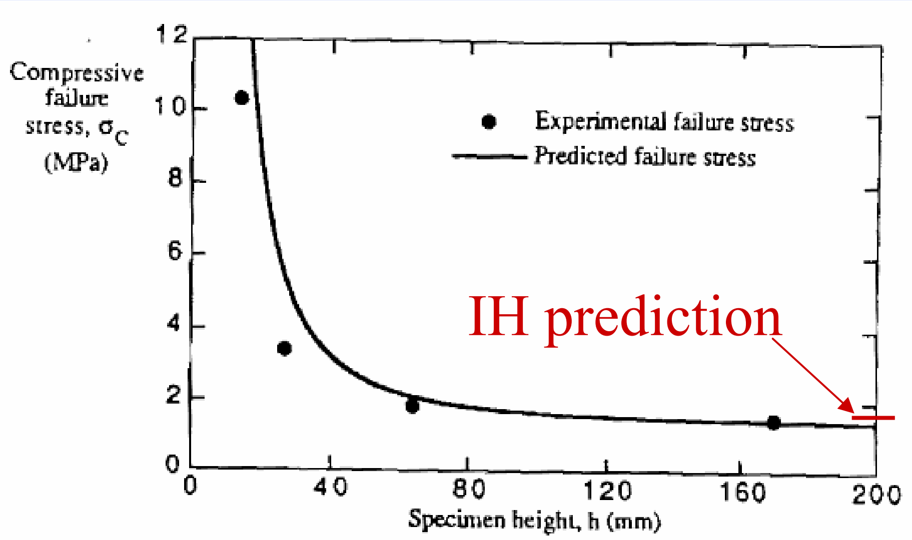
Matrix: **Silicone** — Young's Modulus $E^{(1)} = 2.9$ MPa

Fibers: **Spaghetti** — Young's Modulus $E^{(2)} = 69$ MPa

Volume fraction $c_0 = 31\%$

Experimental setup





Jelf & Fleck (1992)

Final remarks

- An iterated homogenization approach in finite elasticity has been proposed to construct exact (realizable) constitutive models for fiber-reinforced hyperelastic solids
- Because the proposed formulation grants access to local fields, it can be used to thoroughly study the onset of failure and the evolution of microstructure in fiber-reinforced soft solids with random microstructures
- The required analysis reduces to the study of tractable Hamilton-Jacobi equations
- As a first application, closed-form results were derived for fiber-reinforced Neo-Hookean elastomers
- These ideas can be generalized to more complex systems of soft heterogeneous media with random microstructures