TIPS4RM

Leading MATH Success

TIPS4RM Targeted Implementation and Planning Supports for Revised Mathematics

Continuum and Connections

Perimeter, Area, and Volume

Overview

Context Connections

- Positions perimeter, area, and volume in a larger context and shows connections to everyday situations, careers, and tasks
- Identifies relevant manipulatives, technology, and web-based resources for addressing the mathematical theme

Connections Across the Grades

- Outlines the scope and sequence using Grade 6, Grade 7, Grade 8, Grade 9 Applied and Academic, and Grade 10 Applied as organizers
- Includes relevant specific expectations for each grade
- Summarizes prior and future learning

Instruction Connections

- Suggests instructional strategies, with examples, for each of Grade 7, Grade 8, Grade 9 Applied, and Grade 10 Applied
- Includes advice on how to help students develop understanding

Connections Across Strands

- Provides a sampling of connections that can be made across strands, using the theme (perimeter, area, and volume) as an organizer

Developing Proficiency

- Provides questions related to specific expectations for a specific grade/course
- Focuses on specific knowledge, understandings, and skills, as well as on the mathematical processes of Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, and Connecting. *Communicating is part of each question.*
- Presents short-answer questions that are procedural in nature, or identifies the question as problem solving, involving other mathematical processes, as indicated
- Serves as a model for developing other questions relevant to the learning expected for the grade/course

Problem Solving Across the Grades

- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Models a variety of representations and strategies that students may use to solve the problem and that teachers should validate
- Focuses on problem-solving strategies, involving multiple mathematical processes
- Provides an opportunity for students to engage with the problem at many levels
- Provides problems appropriate for students in Grade 7–10. The solutions illustrate that the strategies and knowledge students use may change as they mature and learn more content.

Is This Always True?

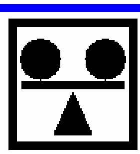
- Presents rich problems to help students develop depth of understanding. Problems may require a body of knowledge not directly related to a specific expectation.
- Focuses on the mathematical processes Reasoning and Proving, and Reflecting

Perimeter, Area, and Volume

Context

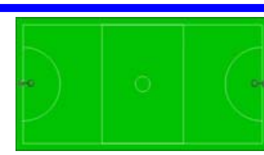
- Stand in any room, in any building, or on any street and you can see the context for a measurement problem.
- Students learn through posing their own measurement problems, drawing from their prior knowledge, and making connections.
- When students learn how to develop measurement formulas for the basic shapes, they are building understanding that can be extended when they encounter a new irregular shape.
- Optimization problems, such as maximizing the area enclosed by a length of fence, require the use and development of formulas.

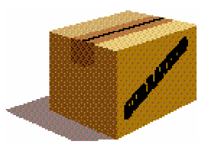
Context Connections

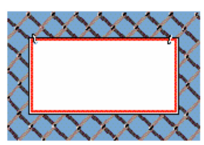

Signs/Logos/Symbols

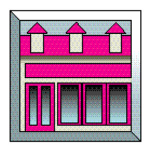

Designing

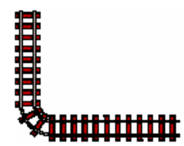

Irregular Surfaces

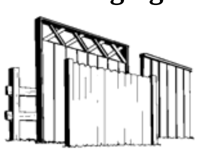

Sports Field

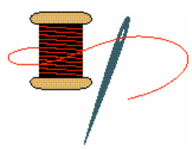

Packaging


Frames


Buildings


Tracks


Fencing


Sewing


Fractions


Other Connections

Manipulatives

- tiles
- cubes
- 3-D models
- nets
- geoboards

Technology

- The Geometer's Sketchpad®
- Fathom
- calculators/graphing calculators
- spreadsheet software

Other Resources

http://standards.nctm.org/document/chapter6/meas.htm
http://mmmproject.org/dp/mainframe.htm
http://www.shodor.org/interactivate/activities/perm/index.html
http://www.oame.on.ca/lmstips/
http:nlvm.usu.edu/en/nav/Vlibrary.html

Connections Across Grades

Selected results of word search using the *Ontario Curriculum Unit Planner*

Search Words: develop, understand, comparison, relate, investigation, pose, surface area, composite, irregular, formula

Grade 6	Grade 7	Grade 8	Grade 9	Grade 10
• construct a rectangle, a square, a triangle, and a parallelogram, using a variety of tools, given the area and/or perimeter; • determine, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula; • develop the formulas for the area of a parallelogram and the area of a triangle, using the area relationships among rectangles, parallelograms, and triangles; • solve problems involving the estimation and calculation of the areas of triangles and the areas of parallelograms; • determine, using concrete materials, the relationship between units used to measure area, and apply the relationship to solve problems that involve conversions from square metres to square centimetres; • determine, through investigation using a variety of tools and strategies, the surface area of rectangular and triangular prisms; • solve problems involving the estimation and calculation of the surface area and volume of triangular and rectangular prisms.	• determine, through investigation using a variety of tools and strategies, the relationship for calculating the area of a trapezoid, and generalize to develop the formula; • solve problems involving the estimation and calculation of the area of a trapezoid; • estimate and calculate the area of composite two-dimensional shapes by decomposing into shapes with known area relationships; • determine, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of right prisms with simple polygonal bases, and generalize to develop the formula; • determine, through investigation using a variety of tools, the surface area of right prisms; • solve problems that involve the surface area and volume of right prisms and that require conversion between metric measures of capacity and volume.	• determine, through investigation using a variety of tools and strategies, the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas; • solve problems involving the estimation and calculation of the circumference and the area of a circle; • determine, through investigation using a variety of tools and strategies, the relationship between the area of the base and height and the volume of a cylinder, and generalize to develop the formula; • determine, through investigation using concrete materials, the surface area of a cylinder; • solve problems involving the surface area and the volume of cylinders, using a variety of strategies.	**<u>Applied and Academic</u>** • determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools, and by examining various values of the area as the side lengths change and the perimeter remains constant; • determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools, and by examining various values of the side lengths and the perimeter as the area stays constant; • solve problems involving the areas and perimeters of composite two-dimensional shapes; • develop, through investigation, the formulas for the volume of a pyramid, a cone, and a sphere. **<u>Applied</u>** • solve problems that require maximizing the area of a rectangle for a fixed perimeter or minimizing the perimeter of a rectangle for a fixed area; • solve problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres. **<u>Academic</u>** • identify, through investigation with a variety of tools, the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area]; • pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures; • determine, through investigation, the relationship for calculating the surface area of a pyramid; • solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.	**<u>Applied</u>** • determine, through investigation, the relationship for calculating the surface area of a pyramid; • solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving combinations of these figures, using the metric system or the imperial system, as appropriate.

Summary of Prior Learning

In earlier years, students:

- build understanding and familiarity with measurement attributes (length, height, width, perimeter, area, etc.);
- become familiar with both standard (metric) and non-standard units of measure, e.g., area of a page is four pencil cases;
- estimate regular and irregular areas using grid paper, e.g., surface area of a puddle;
- distinguish between perimeter and area and understand when each measure should be used (Grade 4);
- become familiar with volume (beginning in Grade 5) and surface area (beginning in Grade 6);
- determine the formulas for perimeter and area of a rectangle, and for the volume of a rectangular prism (Grade 5);
- determine the formulas for area of a triangle and parallelogram, and for the volume of a triangular prism (Grade 6);
- determine the surface area of rectangular and triangular prisms (Grade 6).

In Grade 7, students:

- calculate the area of composite two-dimensional shapes;
- determine the formulas for the area of a trapezoid, and the volume of right prisms;
- determine the surface area of right prisms with polygonal bases.

In Grade 8, students:

- determine the formulas for the area and circumference of a circle, and the volume of a cylinder;
- determine the surface area of a cylinder.

In Grade 9 Applied, students:

- solve problems involving the perimeter and area of composite two-dimensional shapes;
- develop the formulas for the volume of a cone, a pyramid, and a sphere;
- use formulas developed in earlier grades as a good foundation for grade-level investigations.

In Grade 10 Applied, students:

- determine the surface area of a pyramid;
- consolidate understanding of the surface area of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres;
- use imperial measurements, when appropriate.

In later years

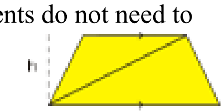
Students' choice of courses will determine the degree to which they apply their understanding of concepts related to area, perimeter, and volume.

Instruction Connections

Suggested Instructional Strategies

Grade 7

- Demonstrate how two different shapes can have the same area but different perimeters by cutting a piece of paper into several pieces and then reassembling and comparing the perimeter and areas.
- Demonstrate how to use transparent grid paper to estimate areas of irregular shapes, e.g., picture of a puddle – experiment with different types of grid paper to develop understanding of units of measure.
- Demonstrate different ways to decompose composite shapes into known shapes.
- Discuss appropriate units of measure, reasons for standard measures, e.g., cm, and selection of appropriate units depending on precision requirements, e.g., Would you measure the length of a pencil and the distance to the next city with the same standard measure? Explain.
- Make connections between the two common formulas for the area of a rectangle (Area = l × w and Area = b × h) – use the formula Area = b × h (base × height), to connect the formulas for the area of a triangle (Grade 6), a parallelogram (Grade 6), and a trapezoid (Grade 7).
- Model how to determine the area of a trapezoid by decomposing a trapezoid into two triangles of the same height, with different bases. Students do not need to memorize a formula when they understand the relationship.



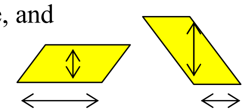
- Use The Geometer's Sketchpad® sketches to facilitate the development of area formulas (diagrams can be made dynamic in GSP®).
- Have students make the net, and express the surface area of right prisms with simple polygonal bases as a sum of areas of different known shapes.
- Use TAPS+® to facilitate understanding of surface area, and to assist in making nets of shapes.

Grade 8

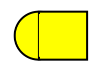
- Ask questions to clarify understanding about measurement, attributes, units of measure, and unit choices.
- Develop the formulas for the circumference and area of a circle through investigation, using paper circles, string/rope, and The Geometer's Sketchpad® – use the formulas only after students have developed them.
- Connect students' knowledge about rectangles and circles to surface area of cylinders. Take apart frozen juice cans so students can see the rectangle, and the circles of the top and bottom, and relate the length of the rectangle to the circumference of the circular top/bottom.
- Connect students' knowledge about the volume of prisms to cylinders to determine the volume of a cylinder, i.e., Volume = (area of the base)(height).

Helping to Develop Understanding

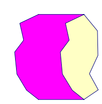
- When grid paper is used to estimate areas, the area is expressed in terms of grid squares. It may be necessary to apply a scale factor to approximate the actual area represented by the drawing or diagram.
- To establish a conceptual basis for understanding perimeter and area, students can use physical models to measure the perimeter (units needed to go around) and area (units needed to cover).
- Height, length, slant height, side, and base can be confusing for students, e.g., any side of a parallelogram can be called a base. Demonstrate that if you cut out the shape and place any base on the floor, the height would be the perpendicular distance from the floor.

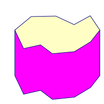


- Shapes can be decomposed (separated) into simple 2-D shapes to find area or perimeter. However, when finding the perimeter of the illustrated shape, students may need to be reminded not to add the interior side of the rectangle and/or the interior diameter of the semi-circle.



- Minimize the need to memorize formulas by focusing on the understanding of the relationships.
- Use manipulatives, e.g., interlocking cubes, to help students understand that the volume of a rectangular prism is the area of its base times its height – students need to understand that this basic formula is true for any right prism.
- For a 2-D polygon, the base can be any side; however, for a 3-D prism the base is the face that 'stacks' to create the prism. This face determines the name of the prism.





- The volume of all right prisms have the same formula, i.e., V = (area of the base)(height). Students investigate this relationship using concrete materials.
- The volume of all right pyramids have the same formula, i.e., V = (area of the base)(height)/3.
- The surface area of pyramids and right prisms are composed of shapes. The net can be used to determine the surface area, e.g., Construct nets for triangular prisms, use prior knowledge about composite 2-D shapes to determine the net's area. A formula is not required.
- Give students multiple opportunities to progress through different representations – concrete→diagrams→symbolic – use formulas only after students have personally developed them.
- Have students write word statements for composite shapes and nets, then replace the words with appropriate variables or formulas.
- Develop all formulas from information that the students already know. Focus on students' conceptual understanding of how formulas are developed rather than expecting memorization.

Suggested Instructional Strategies	Helping to Develop Understanding
Grade 9 Applied • Use concrete materials – students pose problems and "what if" questions. • Investigate the volume of pyramids and cones, using concrete materials. Students should understand the relationship between the corresponding pyramid [cone] and prism [cylinder], i.e., Volume = (area of the base)(height)/3. • Investigate relationships between perimeter and area by graphing data with graphing calculators – students estimate and hypothesize. • Support students in understanding measurement relationships and developing formulas, and in proficiency in their application. **Grade 10 Applied** • Deconstruct 3-D shapes into the nets and determine the surface area by considering the areas of the resulting shapes. • Draw nets using dot paper and technology (TAPS+®) to determine surface area.	

Connections Across Strands

Note

Summary or synthesis of curriculum expectations is in plain font.

Verbatim curriculum expectations are in italics but may not include the examples or phrases such as "through investigation."

Grade 7

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• understand fractions and decimals by connecting them to area • relate multiples and factors to area models • use order of operations in measurement formulas • *represent perfect squares and square roots using a variety of tools, (e.g., geoboards, connecting cubes, grid paper)* • *explain the relationship between exponential notation and the measurement of area and volume*	See Connections Across Grades, p. 3	• sort and classify triangles and quadrilaterals by geometric properties • identify right prisms • understand similarity • *distinguish between and compare similar shapes and congruent shapes* • *create and analyse designs involving translations, reflections, dilatations, and/or simple rotations of two-dimensional shapes* • understand polygons as they relate to tiling	• use patterns • *translate phrases describing simple mathematical relationships into algebraic expressions* • *evaluate algebraic expressions by substituting natural numbers for the variables*	• systematically collect, organize, and analyse data • connect area to bar and circle graphs

Grade 8

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• use order of operations in measurement formulas • use exponents in measurement formulas • develop understanding of squares and square roots through formula for area of circle • develop estimation skills in working with π	See Connections Across Grades, p. 3	• *sort and classify quadrilaterals by geometric properties* • *construct a circle, given its centre and radius, or its centre and a point on the circle, or three points on the circle* • *determine relationships among area, perimeter, corresponding side lengths, and corresponding angles of similar shapes*	• use patterns in algebraic terms • *solve and verify linear equations* [some formulas] *involving a one-variable term and having solutions that are integers* • *evaluate algebraic expressions* [formulas] *with up to three terms, by substituting fractions, decimals, and integers for the variables*	• systematically collect, organize, and analyse primary and secondary data • connect area to bar and circle graphs • *compare two attributes or characteristics, using a scatter plot, and determine whether or not the scatter plot suggests a relationship*

Number Sense and Numeration	Measurement	Geometry and Spatial Sense	Patterning and Algebra	Data Management and Probability
• *solve multi-step problems arising from real-life contexts and involving whole numbers and decimals* • *use estimation when solving problems involving operations with whole numbers, decimals, percents, integers, and fractions* • apply understanding of proportional relationships to geometric shapes and figures		• determine the Pythagorean relationship using area models and diagrams (include the investigation of the areas of semi-circles drawn on the sides of the right triangle) • *solve problems involving right triangles geometrically, using the Pythagorean relationship*	• *translate statements describing mathematical relationships into algebraic expressions and equations* (e.g., the surface area is made up of 3 congruent rectangles plus 2 congruent equilateral triangles)	

Grade 9 Applied

Number Sense and Algebra	Measurement and Geometry	Linear Relations
• simplify numerical expressions involving integers and rational numbers • *describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three* • *substitute into and evaluate algebraic expressions involving exponents* • solve first-degree equations • *substitute into algebraic equations and solve for one variable in the first degree* • *illustrate equivalent ratios, using a variety of tools* • *solve problems involving ratio, rates, and directly proportional relationships in various contexts*	• *relate the geometric representation of the Pythagorean theorem to the algebraic representation* $a^2 + b^2 = c^2$ • *solve problems using the Pythagorean theorem, as required in applications* • investigate the optimal values of measurements of rectangles	• describe the connections between various representations of relations • determine the relationship between variables in measurement formulas, e.g., the radius and area of a circle form a relationship that is non-linear • graph relationships that are determined by measurement formulas, e.g., investigate the relationship between the height and radius of a cone if the volume remains constant • create tables of values, plot points, and graph lines or curves of best fit • use algebraic modelling

Grade 10 Applied

Measurement and Trigonometry	Modelling Linear Relations	Quadratic Relations of the Form $y = ax^2 + bx + c$
• *verify properties of similar triangles* • *solve problems involving the measures of sides and angles in right triangles in real-life applications* • *use the imperial system when solving measurement problems*	• *solve first-degree equations involving one variable, including equations with fractional coefficients* • *determine the value of a variable in the first degree, using a formula*	• factor binomials and trinomials using a variety of tools and strategies (e.g., use an area model) • collect data that can be represented as a quadratic relation (e.g., the length of the side of a square vs. the area of the square)

Developing Proficiency

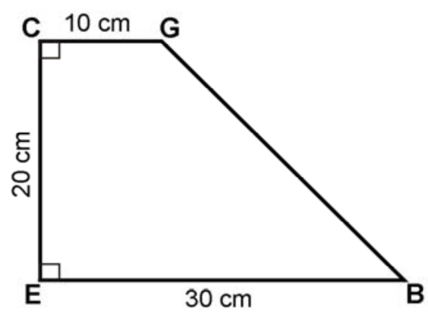
Grade 7

Name:
Date:

Expectation – Measurement, 7m38:
Solve problems involving the estimation and calculation of the area of a trapezoid.

Knowledge and Understanding
(Facts and Procedures)

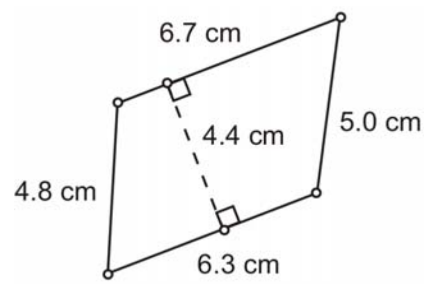
Estimate and calculate the area of the given trapezoid.



Knowledge and Understanding
(Conceptual Understanding, Facts and Procedures)

Calculate the area of the given trapezoid.

Show your work.



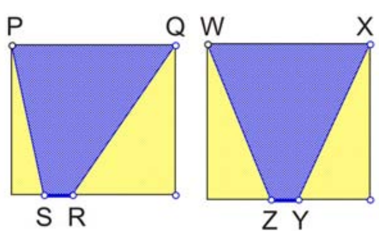
Hint: $A = \frac{(a+b)h}{2}$

Problem Solving
(Reasoning and Proving)

The T-Square Tiling Company makes ceramic floor tiles.
Note: The square tiles that are shown are the same size.

SR and ZY have the <u>same</u> length.

Justify how you know the shaded areas PSRQ and WZYX are the same area.

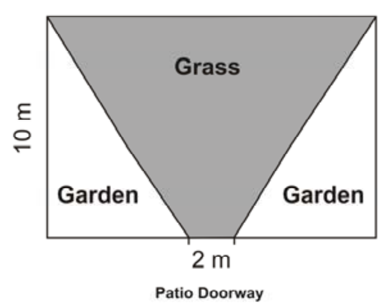


Problem Solving
(Connecting)

Andrea's backyard is rectangular. Its dimensions are 15.0 m by 10.0 m.

Andrea's family is making 2 garden areas and leaving a grassy area from the patio doors to the corners at the back of the yard. The patio doors are 2.0 m wide. Determine the total area of the gardens.

Show your work.



Developing Proficiency

Grade 8

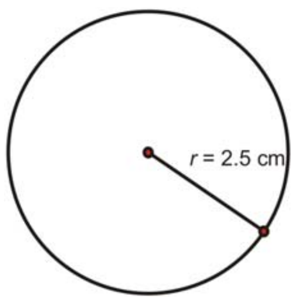
Expectation – Measurement, 8m36:
Solve problems involving the estimation and calculation of the circumference and the area of a circle.

Name:
Date:

Knowledge and Understanding
(Facts and Procedures)

Calculate the circumference and the area of the given circle.

Show your work.

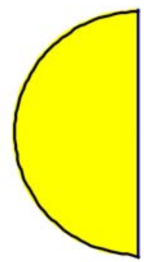


Hint: $C = 2\pi r$ $\quad A = \pi r^2$

Knowledge and Understanding
(Conceptual Understanding, Facts and Procedures)

Determine the **area** and **perimeter** of the shape.

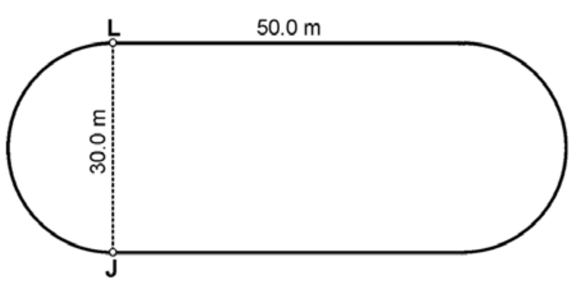
Show *your work*.



Diameter = 6.5 cm

Problem Solving
(Connecting, Reflecting)

Westview School has a track in the school yard.



To run 2 km every day, estimate then calculate how many times you have to go around the track. Compare your estimated and calculated answers.

Show your work.

Problem Solving
(Representing)

The figure has a radius of *r* units.

Develop a formula that represents the perimeter, and show how your formula connects to the diagram.

Developing Proficiency

Grade 9 Applied

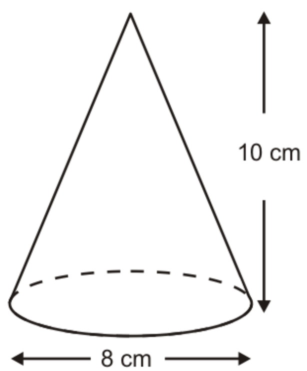
Name:
Date:

Expectation – Measurement and Geometry, MG2.05: Solve problems involving the volumes of prisms, pyramids, cylinders, cones, and spheres.

Knowledge and Understanding
(Facts and Procedures)

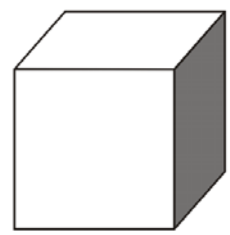
Estimate and determine the volume of a cone that has a diameter of 8.0 cm and a height of 10.0 cm.

Show your work.



Knowledge and Understanding
(Facts and Procedures)

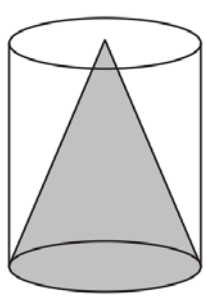
Estimate and determine the volume of the cube of side ½ m.



Problem Solving
(Representing)

Develop a formula to represent the volume of the space remaining if a cone is taken out of a solid cylinder as shown.

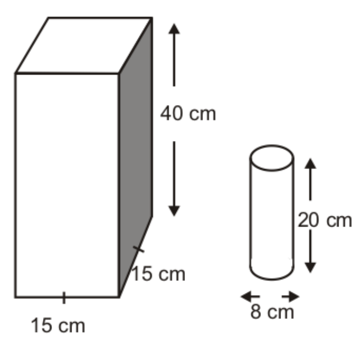
Explain your answer.



Problem Solving
(Connecting, Reflecting, Representing)

Mario has a box of birdseed that is 15 cm × 15 cm × 40 cm. Estimate and calculate how many cylindrical bird feeders with diameter 8 cm and height 20 cm he can fill. Compare your answers.

Show your work.



Developing Proficiency

Grade 10 Applied

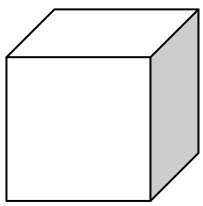
Name:
Date:

Expectation – Measurement and Trigonometry, MT3.04: Solve problems involving the surface areas of prisms, pyramids, and cylinders, and the volumes of prisms, pyramids, cylinders, cones, and spheres, including problems involving combinations of these figures, using the metric system or the imperial system, as appropriate.

Knowledge and Understanding
(Facts and Procedures)

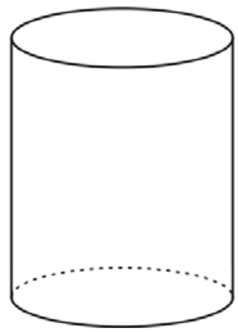
Determine the surface area of the cube of side ½ foot.

Show your work.



Knowledge and Understanding
(Conceptual Understanding)

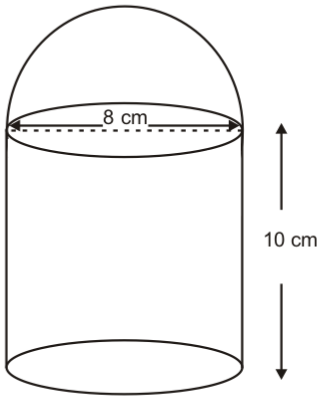
Develop a formula that represents the surface area of an open-topped cylinder. Explain your reasoning.



Problem Solving
(Representing, Selecting Tools and Computational Strategies, Reflecting)

Estimate and determine the volume of a cylinder topped by a hemisphere. The cylinder has a height of 10.0 cm and a diameter of
8.0 cm.

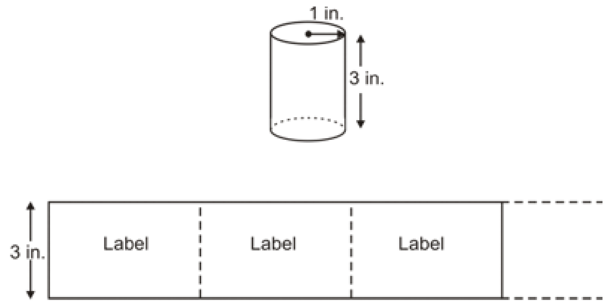
Show your work.



Problem Solving
(Connecting, Reasoning and Proving, Selecting Tools and Computational Strategies)

The Little Can Company makes cylindrical cans with a height of 3 inches and radius of 1 inch. The entire lateral face is covered by a label. The paper for the labels is purchased in rolls 3 inches high. When unrolled the paper is 10 yards long. How many labels can be made from each roll, assuming the label does not overlap on the can?

Show your work.

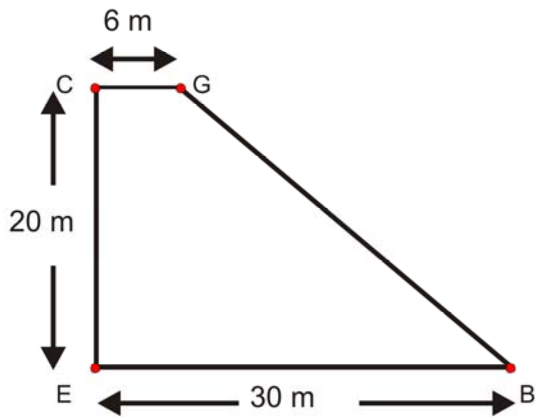


Problem Solving Across the Grades Sample 1

Name:
Date:

Three employees are hired to tar a rectangular parking lot of dimensions 20 m by 30 m. The first employee tars one piece and leaves the remaining shape, shown below, for the other 2 employees to tar equal shares.

Show how they can share the job.



Determine 2 ways of solving this problem. One way must use one single line.
Justify your answer.

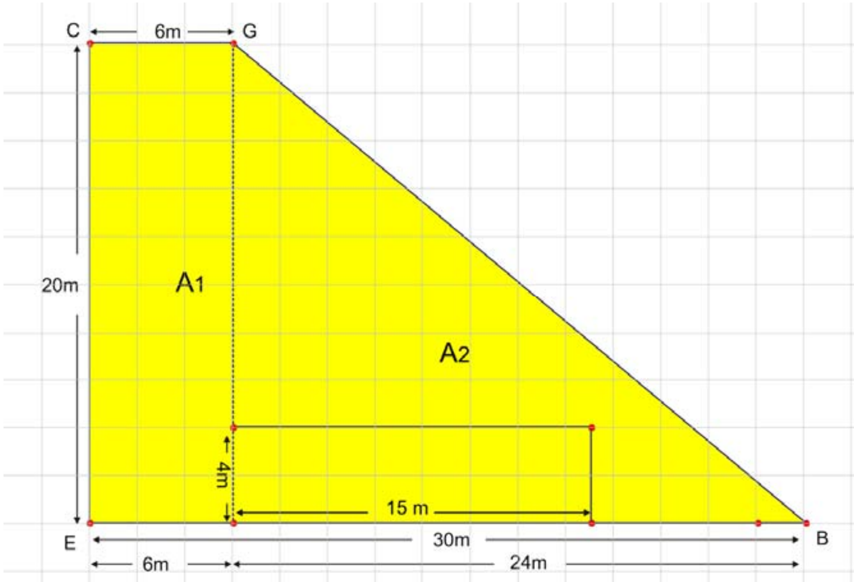
1.	2.

Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

Problem Solving Strategies:
- Draw a diagram
- Guess and check
- Use logic
- Use formulas

1.



$$A_1 = (b)(h) = (6)(20) = 120\,\text{cm}^2$$

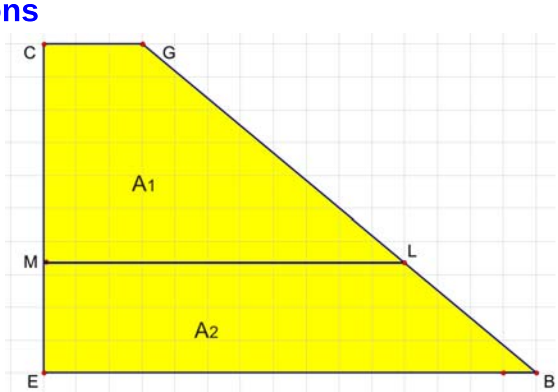
$$A_2 = \frac{bh}{2} = \frac{(24)(20)}{2} = 240\,\text{m}^2$$

A_2 has 120 m^2 more the A_1
$\therefore$ take 60 m^2 from A_2 and add it to A_1.

This can be rectangle of 15 × 4 taken from A_2 (other configurations are possible).

The areas are now equal
$A_1 = 120 + 60 = 180$ m^2
$A_2 = 240 - 60 = 180$ m^2

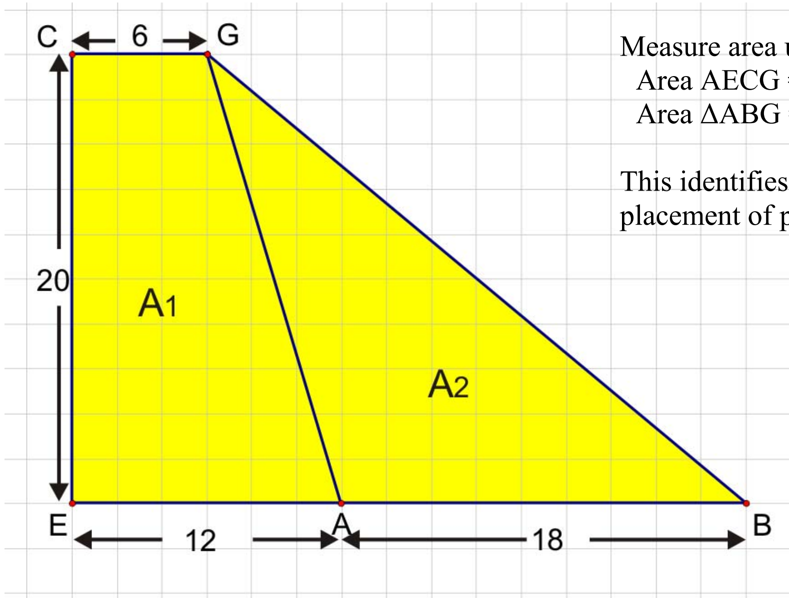
Alternate Configurations



Note
For any solution that requires checking each area, students can calculate the total area and then determine that one piece is $\frac{1}{2}$ the total area.

Problem Solving Strategies:
- Make a scale model by selecting a tool (GSP® or graph paper)
- Guess and check
- Use formulas

2.



Measure area using GSP®:
Area AECG = 44.46 cm^2
Area ΔABG = 44.59 cm^2

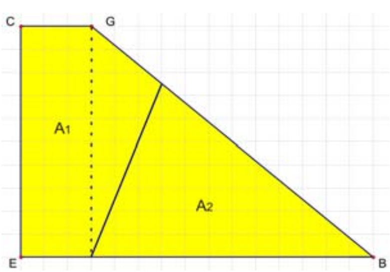
This identifies the placement of point A.

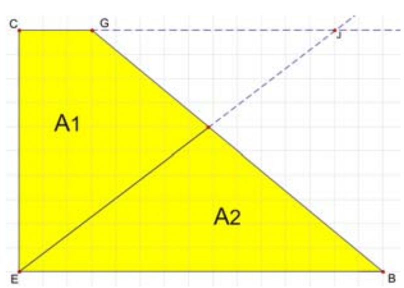
$$A_1 = \frac{(a+b)h}{2}$$
$$= \frac{(12+6)20}{2}$$
$$= (18)(10)$$
$$= 180\ m^2$$

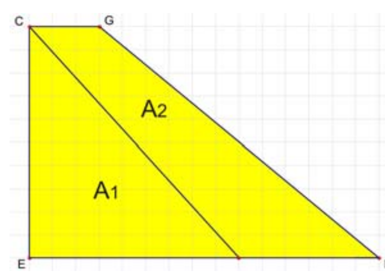
$$A_2 = \frac{(b)(h)}{2}$$
$$A_2 = \frac{(18)(20)}{2}$$
$$= 180\ m^2$$

Note
This result is not exact using GSP® because point A is not on the exact division point. Students must check the areas assuming A is on the correct point, as shown above.

Alternate Configurations



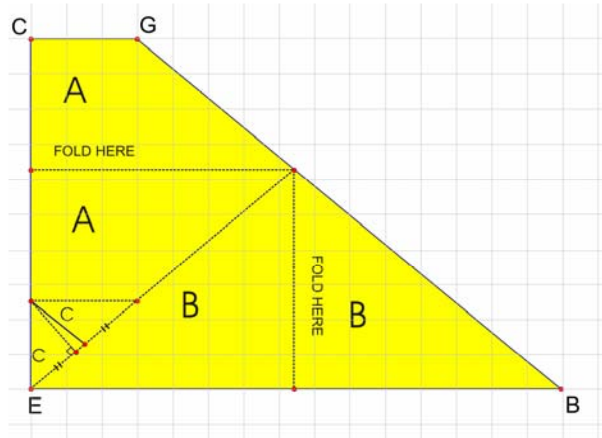




Note
A_1 is 180 m^2, meaning A_2 is also 180 m^2, since the entire area is 360 m^2.

3.

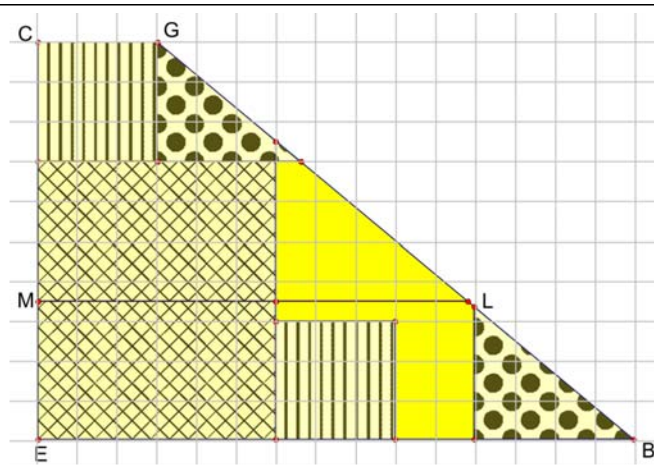
Problem Solving Strategies:
- Use concrete manipulation (paper folding)
- Use logic



Note
Students cut the shape out after making a scale drawing of it on graph paper. Then, they fold the paper onto itself to form congruent shapes. The example shown divides the shape into what appears to be 2 equal parts after 2 folds, forming 2 pairs of congruent shapes, 1 pair labelled A, 1 pair labelled B. The remaining triangles can be divided into 2 equal areas, shown as areas C.

4.

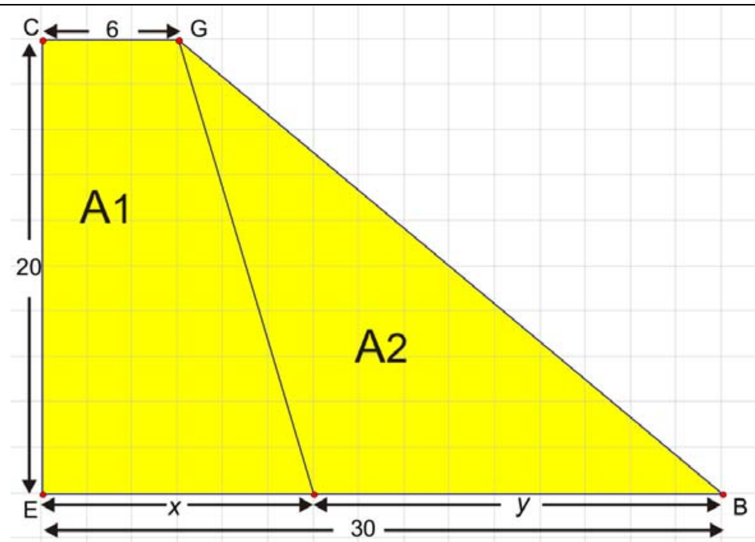
Problem Solving Strategies:
- Use concrete manipulation (cut and paste)
- Use logic



Note
Students cut the shape into 2 pieces along ML line and verify by finding congruent areas on each side (as shown by the checked, striped, and dotted shapes). They check remaining areas by counting squares (approximately $8\frac{1}{2}$ grid squares remaining in each section).

Students' solutions could include any of the Grades 7 and 8 answers.

Problem Solving Strategies:
- Draw diagram
- Use logic
- Create a model (algebraic)
- Graph relationship
- Use technology (GSP® or graphing calculator)



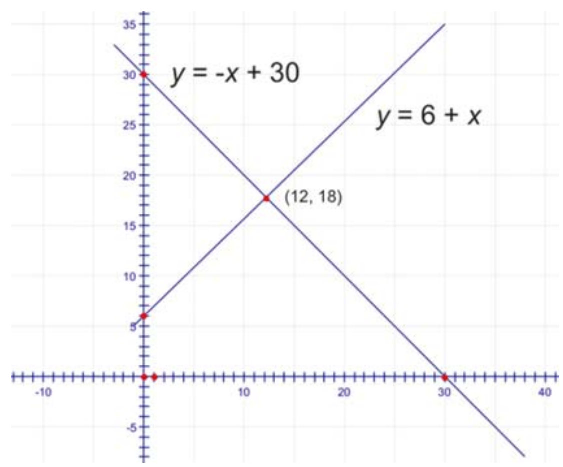
Find Equation 1
$x + y = 30$
$y = -x + 30$

Find $A_1 = \frac{(6+x)(20)}{2}$
$= (6+x)(10)$
$= 60 + 10x$

Find $A_2 = \frac{(y)(20)}{2}$
$= 10y$

Find Equation 2
Since $A_2 = A_1$
$10y = 60 + 10x$
$\therefore y = 6 + x$

Graphical Solution

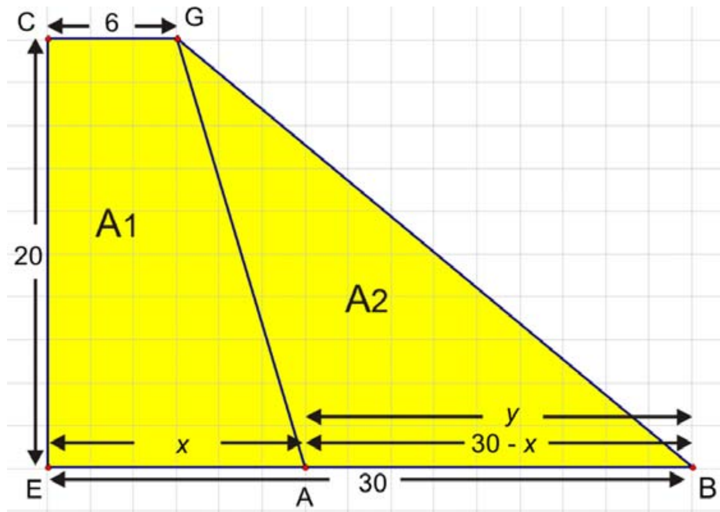


Note
Students graph linear equations 1 and 2, find the point of intersection (12, 18), and interpret its meaning ($x = 12$, $y = 18$, therefore the base of the shape divides into 2 sections of length 12 and 18).
They check by finding the areas A_1 and A_2 using these values.

Students may not generate this solution without support. It is beneficial for students to experience and follow solutions such as this, which incorporate newly learned skills and reinforce various mathematical processes.

Students' solutions could include any of the Grades 7, 8, and 9 answers.

Problem Solving Strategies:
- Draw a diagram
- Create a model (algebraic)
- Use a formula
- Use logic



1.

$$A_1 = A_2$$

$$A_{TRAP} = A_{TRI}$$

$$\frac{(a+b_1)(h)}{2} = \frac{(b_2)(h)}{2}$$

$$\frac{(6+x)(20)}{2} = \frac{(30-x)(20)}{2}$$

$$(6+x)(10) = (30-x)(10)$$

$$60+10x = 300-10x$$

$$20x = 240$$

$$x = 12$$

If $x = 12$

$$y = 30 - x = 18$$

2.

Equation 1

$$x + y = 30$$

Equation 2

$$A_1 = A_2$$

$$\frac{(20)(x+6)}{2} = \frac{(20)(y)}{2}$$

$$10(x+6) = 10y$$

$$\frac{10(x+6)}{10} = y$$

$$x+6 = y$$

Solve the system.

Substitute the expression for y from Equation 2 into Equation 1.

$$x + (x+6) = 30$$

$$2x + 6 = 30$$

$$2x = 24$$

$$x = 12$$

$$12 + y = 30 \therefore y = 18$$

Therefore, the base of the shape divides into 2 sections of length 12 and 18.

Note

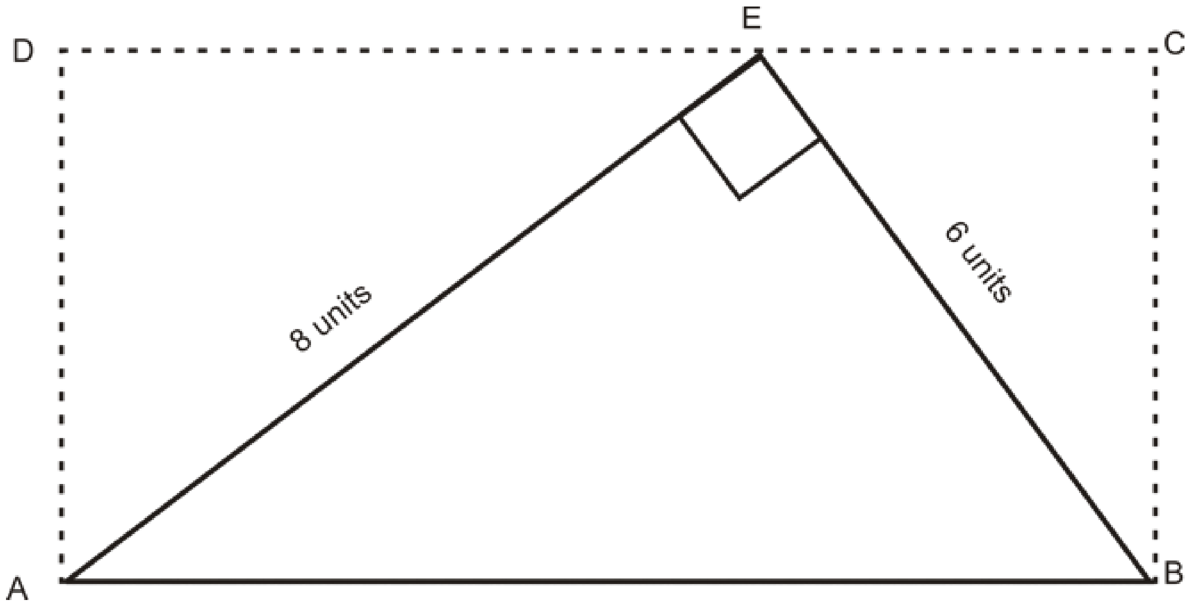
Students verify by finding the area of each section. Students can use a system of equations to solve this algebraically.

Students may not generate this solution without support. It is beneficial for students to experience and follow solutions such as this, which incorporate newly learned skills and reinforce various mathematical processes.

Problem Solving Across the Grades — Sample 2

Name:
Date:

Find 2 different ways to determine the area of the rectangle ABCD:



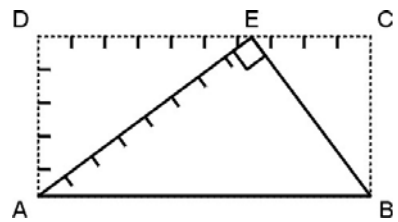
1.	**2.**

Although the teacher may expect students to apply specific mathematical knowledge in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

Problem Solving Strategies:
- Draw a scale diagram and measure
- Use a formula
- Use logic

1.



The area is approximately 10 × 4.6 = 46 units^2 using a scale drawing.

Note
A student who prefers a concrete representation may choose to use a scale drawing to approximate the area.

2.

rectangle ABCD = 2 (ΔABE)

$$= 2\left(\frac{1}{2}\right)(8)(6)$$

$$= 48$$

Therefore, rectangle ABCD = 48 units^2.

Grade 9

Students' solutions could include any of the Grades 7 and 8 answers.

Use Pythagoras to find *x*.

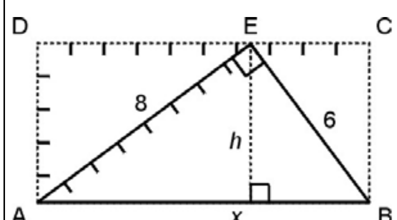
$$x^2 = 8^2 + 6^2$$
$$= 64 + 36$$
$$= 100$$
$$x = 10$$

Compare two area formulas to find *h*.

$$\text{Area } \Delta ABE = \frac{1}{2} \times 8 \times 6 \text{ and area } \Delta ABE = \frac{1}{2} \times 10 \times h$$

$$\therefore \quad 24 = 5 \times h$$

$$h = \frac{24}{5}$$



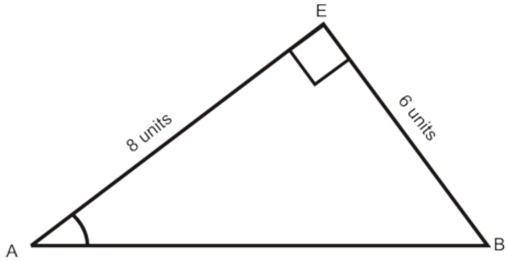
Calculate:

$$\text{Area ABCD} = 10 \times \frac{24}{5}$$
$$= 48 \text{ units}^2$$

Note
Students use the Pythagorean relationship in Grade 9.

Grade 10

1.

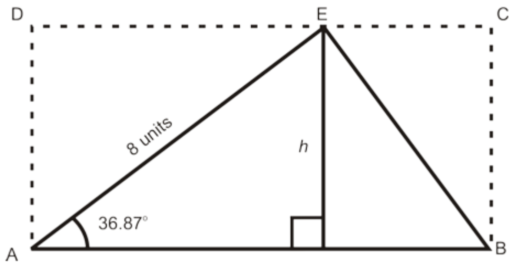


a) Use the Pythagorean relationship to find AB

$$8^2 + 6^2 = (AB)^2$$
$$64 + 36 = (AB)^2$$
$$100 = (AB)^2$$
$$10 = AB$$

b) Use trig to find $\angle A$
In ΔABE,

$$\tan A = \frac{6}{8} = \frac{3}{4} = 0.75$$
$$\tan A = .75$$
$$\therefore \angle A \doteq 36.87^\circ$$



c) Use trig to find *h*:

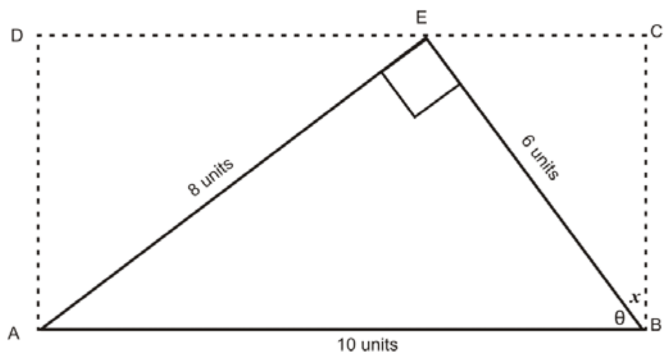
$$\sin 36.87 = \frac{h}{8}$$
$$.6 = \frac{h}{8}$$
$$4.8 = h$$

Height of rectangle ABCD is 4.8

d) Calculate Area of ABCD

$$= (10)(4.8)$$
$$= 48 \text{ units}^2$$

2.



a) use Pythagoras
$AB = 10$

b) $\sin\theta = \frac{8}{10}$
$\theta \doteq 53.13^\circ$

c) $x \doteq 90^\circ - 53.13^\circ$
$= 36.87^\circ$

d) $\cos 36.87 = \frac{CB}{6}$
$CB = 6(0.8)$
$= 4.8$

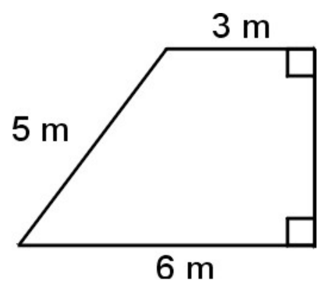
e) Area ABCD $= 10 \times 4.8$
$= 48$

Note
Students can use trigonometry in Grade 10.
Other trigonometry ratios could be used for other, but similar, solutions.

Problem Solving Across the Grades Sample 3

Name:
Date:

Find 2 different ways to find the area of this patch of pavement.



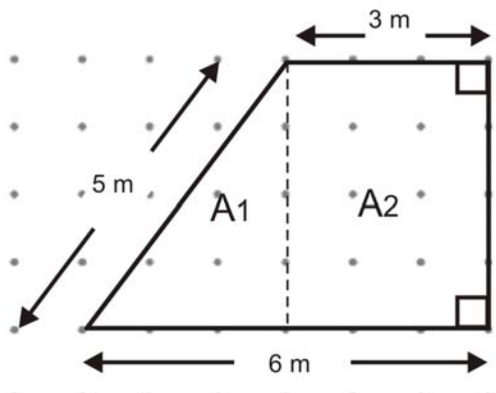
1.	**2.**

Problem Solving Strategies:
- Draw a diagram
- Make a scale model
- Use a formula
- Use technology (GSP®)
- Use tools (geoboard)

Although the teacher may expect students to apply a particular piece of mathematical knowledge, in a problem-solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

1.



$$\begin{aligned} A &= A_1 + A_2 \\ &= \frac{bh}{2} + bh \\ &= \frac{(3)(4)}{2} + 3(4) \\ &= 6 + 12 \\ &= 18\text{ m}^2 \end{aligned}$$

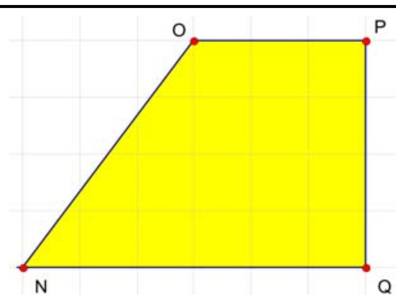
OR

$$\begin{aligned} A &= \frac{(a+b)(h)}{2} \\ &= \frac{(3+6)(4)}{2} \\ &= 18\text{ m}^2 \end{aligned}$$

Note
With the given top and bottom lengths, students use grid paper or a geoboard to determine a diagram that has a slanted side of 5 units, and thus determine the corresponding height. Once students know the height they can apply the area formula for a trapezoid; or separate the shape into a triangle and a rectangle and find each area and the sum of the areas.

2.

$m\ \overline{NO} = 5.01\text{ cm}$
Area ONQP = 18.09 cm^2

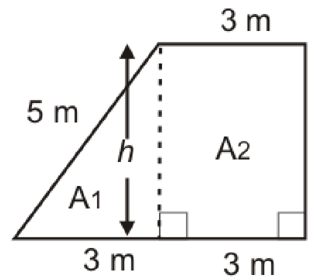


Note
Students adjust a drawing with GSP® using all the given information, including the slanted side of 5 units. Students use GSP® to measure the area.

Students' solutions could include any of the Grades 7 and 8 answers.

Problem Solving Strategies:
- Draw a diagram
- Use a formula

1.



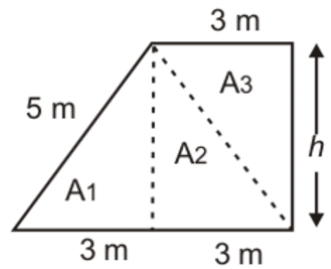
Use the Pythagorean relationship to determine that $h = 4$ m.

$$A_1 = \frac{1}{2} \times 3 \times 4 = 6\,m^2$$

$$A_2 = 3 \times 4 = 12\,m^2$$

Total area = 6 + 12 = 18 m^2

2.

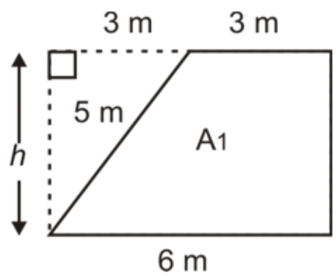


Use the Pythagorean relationship to determine that $h = 4$ m.

$$A_1 = A_2 = A_3 = \frac{1}{2} \times 3 \times 4 = 6\,m^2$$

Total area = $A_1 + A_2 + A_3 = 3 \times 6 = 18\ m^2$

3.



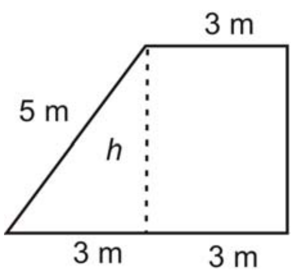
Use the Pythagorean relationship to determine that $h = 4$ m.

A_1 (rectangle) = $6 \times 4 = 24\ m^2$

A_2 (triangle) = $\frac{1}{2} \times 3 \times 4 = 6\,m^2$

A = 24 – 6 = 18 m^2

4.



Use the Pythagorean relationship to determine that $h = 4$ m.

$$\begin{aligned} A\ \text{(trapezoid)} &= \frac{1}{2}(a+b) \times h \\ &= \frac{1}{2}(3+6) \times 4 \\ &= \frac{1}{2}(9) \times 4 \\ &= 18\,m^2 \end{aligned}$$

Note
Students in Grades 9 and 10 can use the Pythagorean relationship.

Problem Solving Across the Grades   Sample 4

Name:
Date:

The diagram is a square inside a circle and a square outside the circle.

Which has the greater area:

- the space between the circle and the inside square (Diagram A)?

or

- the space between the circle and the outside square (Diagram B)?

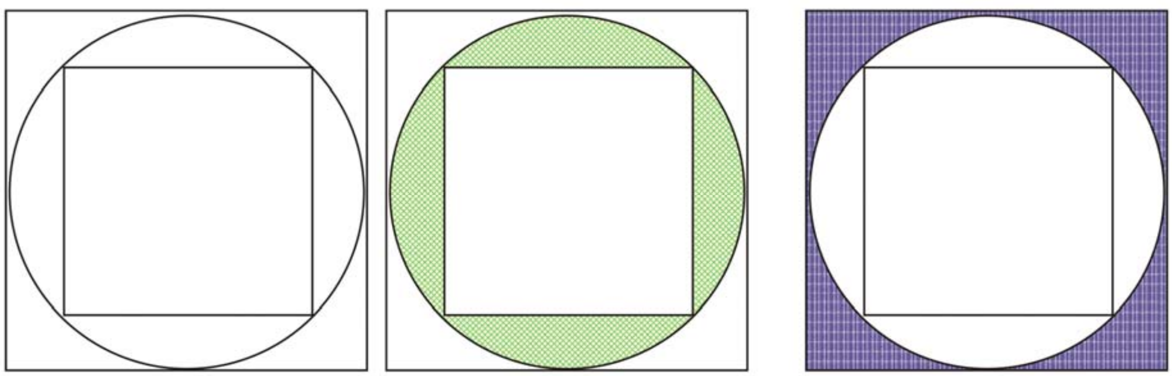


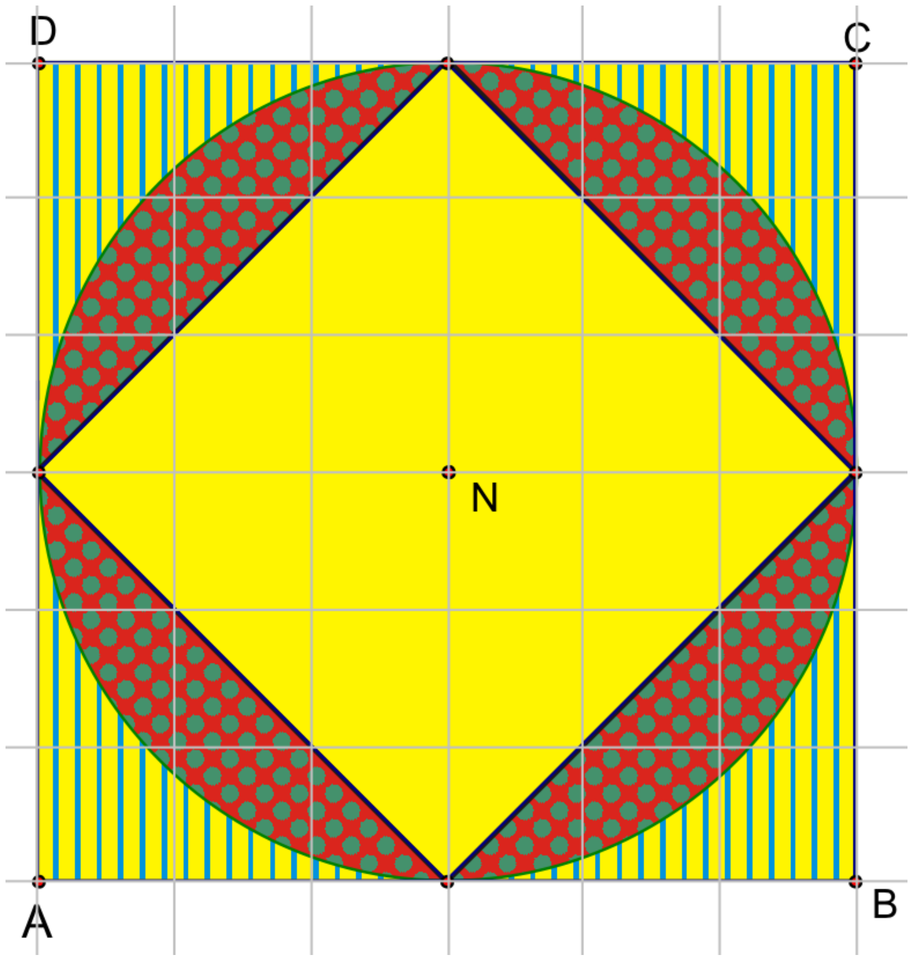
Diagram A   **Diagram B**

Problem Solving Strategies:
- Make a scale model
- Use concrete material (cut and paste)
- Use logic

Although the teacher may expect students to apply a particular piece of mathematical knowledge, in a problem solving context, students may find some unexpected way to solve the problem.

Have a variety of tools available from which students can choose to assist them with their thinking and communication.

1.



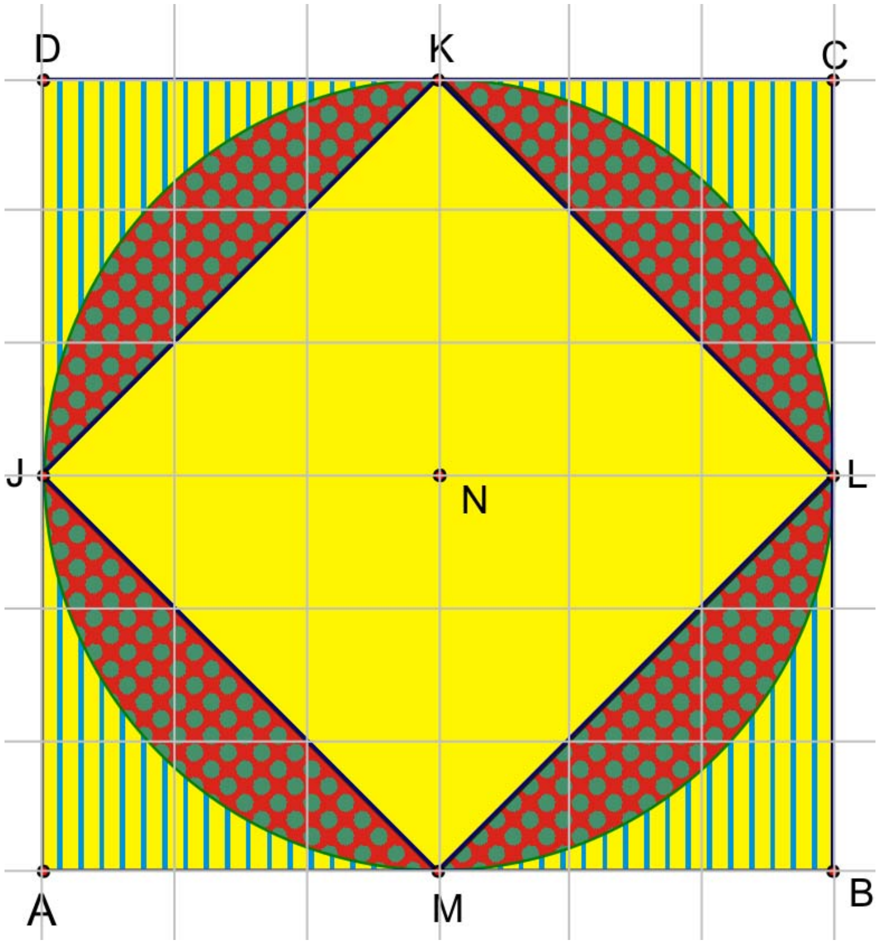
Note

This question may be easier to do if students rotate the inside square 45°.

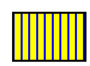
Students could cut out the areas in question and see how they "fit" together. This will provide an acceptable answer and justification, particularly if done with several-sized circles.

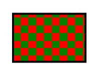
2.

Problem Solving Strategies:
- Make a scale model
- Use technology (GSP®)



Area DABC = 100.00 cm^2
Area JMLK = 49.95 cm^2
Area ⊙NL = 78.22 cm^2

(Area DABC) – (Area ⊙NL) = 21.78 cm^2 

(Area ⊙NL) – (Area JMLK) = 28.27 cm^2 

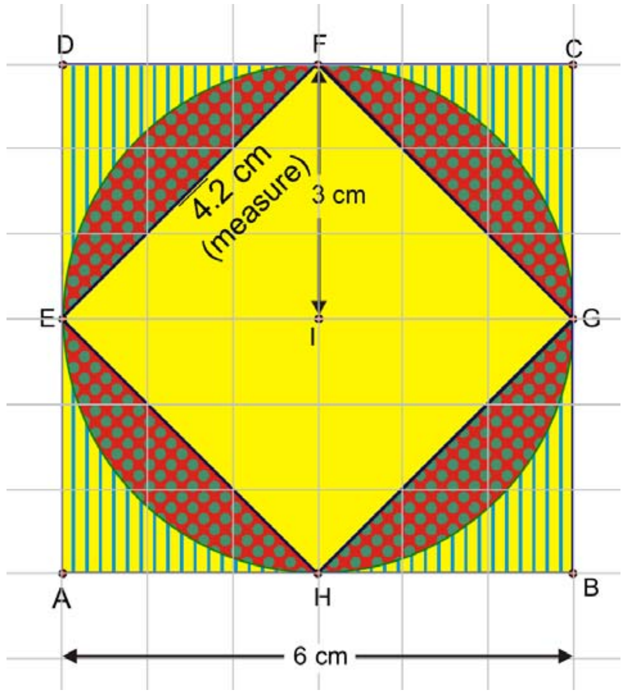
28.27 > 21.78

Therefore, the area between the circle and inside square is greater than the area between the circle and the outside square.

Note
⊙NL is the GSP® symbol which refers to the area of the circle with radius NL.

Grade 8

Problem Solving Strategies:
- Make a scale diagram and measure
- Use formulas
- Use logic



$$\text{Area DABC} = (6)(6)$$
$$= 36\ \text{cm}^2$$

$$\text{Area EFGH} \doteq (4.2)(4.2)$$
$$= 17.64\ \text{cm}^{2*}$$

$$\text{Area of circle} = \pi r^2$$
$$\doteq (3.14)(3)^2$$
$$= 28.26\ \text{cm}^2$$

$$\text{Area DABC} - \text{Area circle} = 36 - 28.26$$
$$= 7.72\ \text{cm}^2$$
$$\text{Area of circle} - \text{Area EFGH} = 28.26 - 17.64$$
$$= 10.62\ \text{cm}^2$$

Area between circle and inside square is greater than the area between the circle and the outside square.

*Alternate solution could include the proof that the inside square (EFGH) is exactly half the area of the outside square (DABC), thus no measuring is required.

Note

This provides an accurate answer and justification, particularly if it is done with several circles of different sizes. Students could use GSP® to measure and calculate, as well.

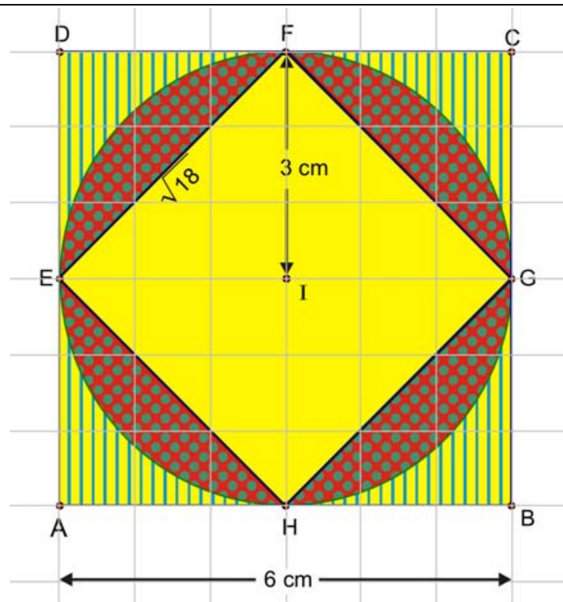
In Grade 8 students know the formula for the area of a circle.

Grade 9

Sample Solutions

Students' solutions could include any of the Grades 7 and 8 answers.

Problem Solving Strategies:
- Use formulas



Area ABCD
= $(l)(w)$
=(6)(6)
= 36 cm^2

Length of side of inside square h:

$a^2 + b^2 = h^2$

$3^2 + 3^2 = h^2$

$9 + 9 = h^2$

$18 = h^2$

$\sqrt{18} = h$

Area EFGH $= \left(\sqrt{18}\right)\left(\sqrt{18}\right)$

$= 18$ cm^2

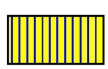
Alternate solution could include the proof that the inside square (EFGH) is exactly half the area of the outside square (DABC), thus students would not need the Pythagorean relationship to solve.

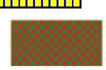
Area ⊙IG

$= \pi r^2$

$= (3.14)(3)^2$

$= 28.26$ cm^2

(Area ABCD) – (Area ⊙IG) = 7.74 cm^2 

(Area ⊙IG) – (Area EHGF) = 10.26 cm^2 

$10.26 > 7.74$

Therefore, the area between the circle and the outside square is smaller than the area between the circle and the inside square.

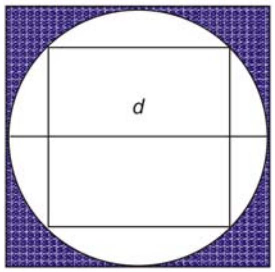
Note

In Grade 9 students use the Pythagorean relationship to find the exact length of the side of the inside square.

Students' solutions could include any of the Grades 7, 8, 9 answers.

Problem Solving Strategies:
- use algebra to consider the general case

1. General Case (using diameter *d*)



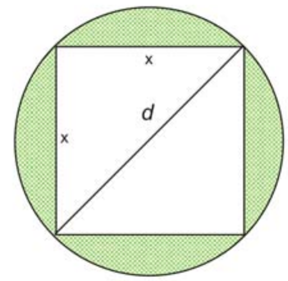
$$\text{Area of circle} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{d^2\pi}{4}$$

Area of outside square = d^2

Shaded area

$$\begin{aligned} A &= d^2 - \frac{d^2\pi}{4} \\ &= d^2\left(1-\frac{\pi}{4}\right) \\ &= d^2\left(\frac{4-\pi}{4}\right) \\ &\doteq d^2\left(\frac{0.86}{4}\right) \\ &= 0.215d^2 \end{aligned}$$

$$\begin{aligned} x^2 + x^2 &= d^2 \\ 2x^2 &= d^2 \\ x^2 &= \frac{d^2}{2} \\ x &= \frac{d}{\sqrt{2}} \end{aligned}$$



$$\text{Area of inside square} = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$$

or

Area of inside square is $\frac{1}{2}$ the area of outside square.

$$\text{Area of circle} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{d^2\pi}{4}$$

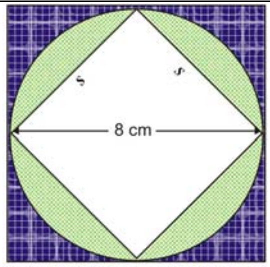
Shaded area

$$\begin{aligned} A &= \frac{d^2\pi}{4} - \frac{d^2}{2} \\ &= d^2\left(\frac{\pi}{4}-\frac{1}{2}\right) \\ &= d^2\left(\frac{\pi-2}{4}\right) \\ &\doteq d^2(0.285) \\ &= 0.285d^2 \end{aligned}$$

Compare $0.215d^2$ to $0.285d^2$. Since $0.285d^2$ is larger, the area between the circle and the inside square is greater than the area between the circle and the outside square.

1. General Case (using diameter *d*) (continued)

Check general case with a specific example:



Area of large square $8 \times 8 = 64$ cm^2

Area of circle
$\pi r^2 \doteq (3.14)(4)^2$
$= 50.26$

Shaded area
= 64 – 50.26
= 13.76

Check using general case:
A = $0.215(8)^2 = 13.76$

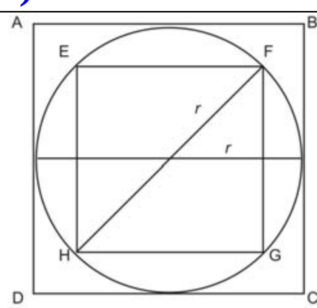
Area of circle ≐ 50.26
Side of small square:
$2s^2 = 64$ cm^2
$s^2 = 32$
$s = \sqrt{32}$
$\doteq 5.66$

Area of small square:
$s^2 = \left(\sqrt{32}\right)^2$
$= \left(\sqrt{32}\right)^2$
$= 32$ (or $\frac{1}{2}$ large square).

Shaded area
50.26 – 32
= 18.26

Check using general case:
A = $(0.285)(8)^2 = 18.26$

2. General Case (using radius *r*)



Area ABCD
$= (2r)^2$
$= 4r^2$

Area of circle
$= \pi r^2$

Length HG
$(2r)^2 = 2(\text{HG})^2$
$4r^2 = 2(\text{HG})^2$
$\frac{2r}{\sqrt{2}} = \text{HG}$

Area of HGFE
$= \left(\frac{2}{\sqrt{2}} r\right)^2$
$= \frac{4r^2}{2}$
$= 2r^2$

Area in corners
$= 4r^2 - \pi r^2$
$= 0.86r^2$

Area in circle portions
$= \pi r^2 - 2r^2$
$\doteq 1.14r^2$

Compare $1.14r^2$ to $0.86r^2$. $1.14r^2$ is larger. Therefore, the area between the circle and the inside square is greater than the area between the circle and the outside square.

Note

In Grade 10 students could apply their algebra skills (e.g., factoring) and/or trigonometry. Students could check the general case using a specific example.

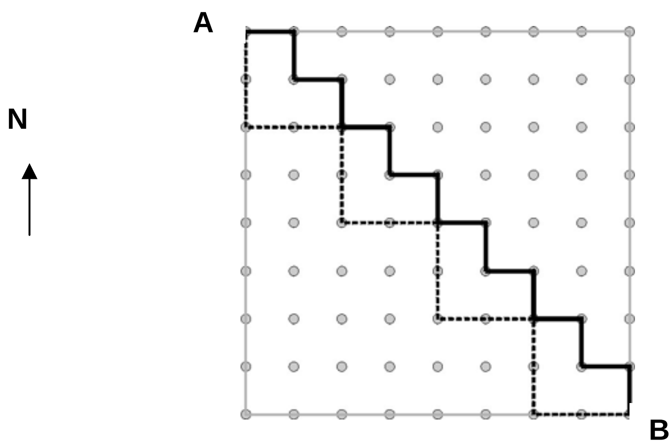
Students may not generate this solution without support. It is beneficial for students to experience and follow solutions such as this, which incorporate newly learned skills and reinforce various mathematical processes.

Is This Always True?
(Reflecting, Reasoning and Proving)

Grades 7–10

Name:
Date:

Trevor travels from A to B by walking only south or east on the streets shown. The first time he follows the route indicated by the solid lines and determines that his walk was 16 blocks long. The second time Trevor walks from A to B by following streets south or east only, he follows the route indicated by the broken lines and determines that his walk was again 16 blocks long. Will his trip always be 16 blocks long? Show your work and explain your answer.

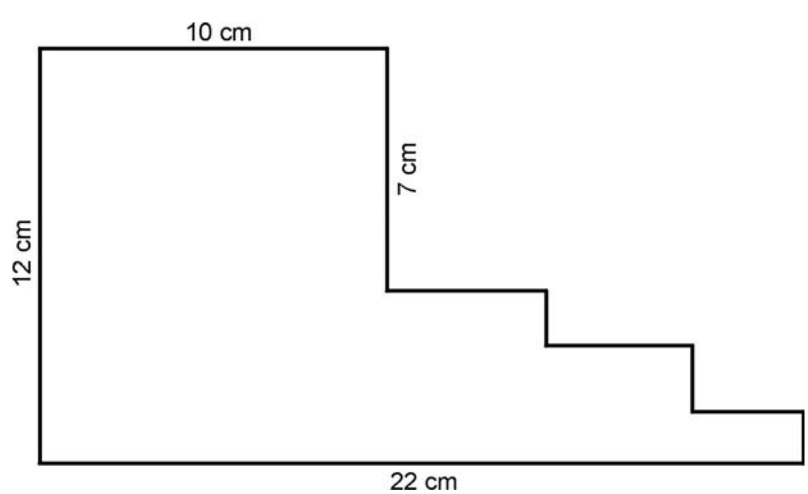


1.

Sample Solutions

Yes. Trevor will have to travel 8 blocks east and 8 blocks south, or a total of 16 blocks, regardless of the order he chooses to do the east and south parts of the trip. This concept can be connected to finding perimeter of a step shape, where sizes of the steps are not known. Perimeter is $2 \times 12 + 2 \times 22 = 68$ cm since the 4 vertical steps on the right add to 12 and the 4 upper horizontal lengths add to 22.

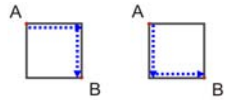
Problem Solving Strategies:
- Draw a diagram
- Act it out
- Use logic
- Consider extensions and variations to the problem



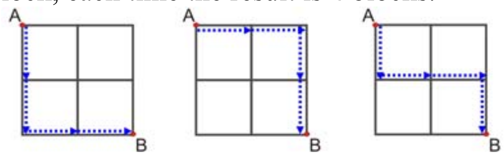
2.

Problem Solving Strategies:
- Look at a simpler problem
- Draw a diagram
- Act it out
- Make an organized list
- Look for a pattern

- If students choose a 1 × 1 block, the result is 2 blocks.

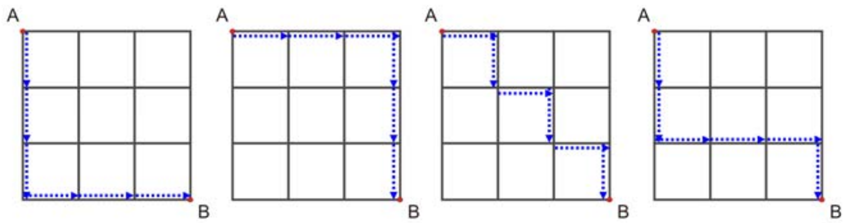


- If students choose a 2 × 2 block, each time the result is 4 blocks.



(more examples are possible)

- If students choose a 3 × 3 block, each time the result is 6 blocks.



(more examples are possible)

At this point one might generalize that it appears that, regardless of the route chosen, the total number of blocks needed to go from point A to point B will be the same. Furthermore, it appears that the total number of blocks is 2*b*, where *b* is the number of horizontal or vertical blocks.

Number of Horizontal and Vertical Blocks *b*	Total Number of Blocks Travelled
1	2
2	4
3	6
⋮	
8	16
b	2*b*

Is This Always True?
(Reflecting, Reasoning and Proving)

Grades 7–10

Name:
Date:

1. Shelley says that the circle with diameter 4 has a smaller area than the square with side 4 and a larger area than the square with a diagonal 4.
   Is this true for any number that Shelley chooses? Give reasons for your answer.

2. Is it always true that the circumference of a circle is more than 6 times the radius of the circle? Explain.

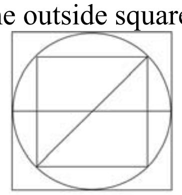
Sample Solutions

1.

Grade 7

Yes. This diagram shows a circle. It does not matter what the diameter of the circle is, a square whose diagonal is equal to the diameter means the square fits inside the circle, so its area is smaller than the circle. The circle fits inside the square whose side is equal to the diameter, so the area of the circle is smaller than the area of the outside square.

Problem Solving Strategies:
- Draw a diagram
- Use logic



Grades 8–10

Students can choose specific examples and do the calculation to show the relevant areas and compare. GSP® can be used to show this, as well.

Problem Solving Strategies:
- Draw a diagram
- Use technology (GSP®)
- Use formulas

2.

Grade 7

Yes. I can show this several ways.

Problem Solving Strategies:
- Use concrete materials (string, measurement tools)
- Use technology (GSP®)
- Work backwards

I could choose a few examples of circles that have different radii, measure the circumference of each using string, and see if it is more than 6 times its radius.

I could make a circle and measure the radius, and cut a piece of string whose length is 6 times the radius, then see if I can place that string along the circumference with space left over.

I can use Geometer's Sketchpad® and measure the radius and circumference of a circle. Then I divide the length of the circumference by the length of the radius and see if I get a number larger than 6. If I animate this sketch, I can see that I always get the same number when I do this calculation.

I can place string along the circumference of the circle and cut it the exact length. Then I can cut off pieces equal to the radius. If I have 6 pieces plus some left over I will know the circumference is more than 6 times the radius.

Grade 8

I could use algebra as follows: $C = 2\pi r$ and $2\pi \doteq 2 \times 3.1415 = 6.283$, which is more than 6.
Therefore, $C > 6r$

Problem Solving Strategies:
- Use formula
- Use algebra
- Use logic

Grades 9 and 10

Any of the above solutions.

Is This Always True?
(Reflecting, Reasoning, and Proving)

Grades 9 and 10 Applied

Name:
Date:

Is it always true that the largest rectangle with a given perimeter is a square?
Explain your answer.

Note: This can be explored in Grades 7 and 8

Sample Solutions

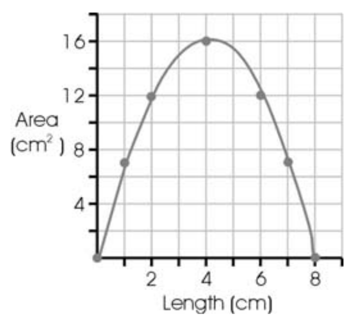
Problem Solving Strategies:
- Look at a simpler problem
- Draw a diagram
- Make a graph
- Use technology (GSP®)
- Make an organized list
- Use logic

Yes. I can look at a specific example of a rectangle with a perimeter of 16 cm. The largest number for area of a rectangle whose perimeter is 16 cm can be determined in a table of values. If perimeter is 16 cm, length plus width is 8 cm.

Length (cm)	Width (cm)	Area (cm^2)
1	7	7
2	6	12
3	5	15
4	4	16
5	3	15
6	2	12
7	1	7

The largest area is 16 cm^2. This happens when the length = width = 4 cm. This is a square.

In a graph of area vs. length, the highest point on the graph identifies the maximum area.



The highest point on the graph occurs when $x = 4$. This means that the length is 4. So, the width must also be 4, and this is a square.

If I do this same problem in the same way with a perimeter of 32 cm, I think I would get a larger square. I could double all of the numbers in the length and the width column in the chart, and change the area column. Once again the largest area would be a square. The side length would be 8 cm and the area would be 64 cm^2.

I could use a perimeter of 8, divide all of the numbers in the length and the width column in the chart by 2, and change the area column. Again, the largest area would be a square. The side length would be 2 cm and the area would be 4 cm^2. So the largest rectangle with a given perimeter is always a square.

Note: There is a GSP® sketch that shows the graph and the changing rectangle for any perimeter which demonstrates that for any perimeter the largest rectangle possible is a square.

Is It Always True?

Grade 7–10

(Reflecting, Reasoning and Proving)

Name:
Date:

Is it always true that 2 triangles of equal area are congruent? Provide reasons for your answer.

Sample Solutions

Problem Solving Strategies:
- Use a counter example
- Draw a diagram
- Use logic

No. I can provide a counter-example:
Triangle ABC has a base of 10 cm and a height of 8 cm.
Its area is 40 cm^2.

Triangle DEF has a base of 5 cm and a height of 16 cm.
Its area is also 40 cm^2.

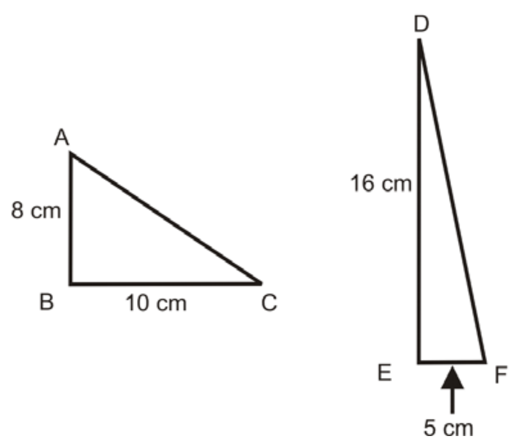
Although they have the same area, these triangles are not congruent.

We can show they are not congruent by:

- considering the lengths of the sides. All 3 sides of a triangle must be the same length if the triangles are congruent. These triangles do not have equal side lengths.

or

- drawing a sketch of the triangles. ABC and DEF are not the same shape and size, thus they are not congruent.



Note: All congruent triangles have equal area, but not all triangles of equal area are congruent.