METONYMY AS A LENS INTO STUDENT UNDERSTANDING OF SAMPLING DISTRIBUTIONS

Stacey Hancock
Department of Statistics
University of California, Irvine

Joint work with
Jennifer Noll, Portland State University
Aaron Weinberg, Ithaca College

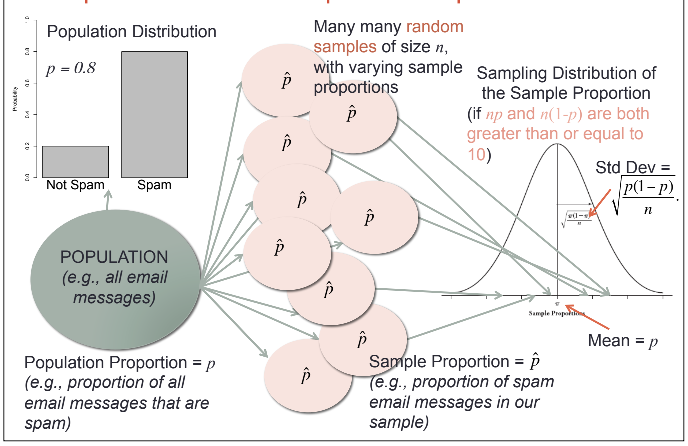
Overview

- Common student misconceptions about sampling distributions in introductory statistics courses
- What is metonymy?
- Student use of metonymy when working with sampling distributions
- Overview of some topics in statistics education research
- Tips and resources for teaching a college introductory statistics course

Sampling Distributions

- A sampling distribution is a probability distribution of sample statistics.
- Sampling distributions allow us to reason about the "unusualness" of an observed sample statistic, which leads to informal statistical inference.
- Sampling distributions form the basis of formal statistical inference (e.g., confidence intervals and hypothesis tests).

Example: Measure variable spam or not spam on each email.

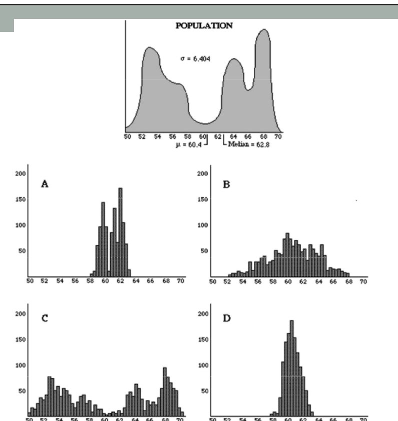


Student Difficulties in Understanding Sampling Distributions

Students often…

- confuse distributions of samples with distributions of sample statistics (sampling distributions);
- apply properties of the population distribution to the sampling distribution;
- experience difficulty making sense of the variability of sample statistics (e.g., sample means) and how this knowledge can be useful in statistical inference;
- aggregate multiple samples into one large sample rather than look at a distribution of statistics from multiple samples.

See, e.g., Chance et al., 2004; Lipson, 2003; Saldanha & Thompson, 2002, 2007.



Source:ARTIST (Assessment Resource Tools for Improving Statistical Thinking)
https://app.gen.umn.edu/artist

What is "metonymy"?

- Metonymies and metaphors are both relationships between two entities, A and B, where B represents A.
- *Metaphor* → A and B are located in two different conceptual domains.
  - e.g., "life is a journey", "the world's a stage"
- *Metonymy* → A and B are located in the same conceptual domain.
  - e.g., "the suits on Wall Street", "a new set of wheels", "Hollywood" to describe the U.S. film industry

Some Types of Metonymy

- "Synecdoche" or "metonymy proper"
  - One part of a concept stands in for the entire concept.
  - Examples:
    - I've got a new set of ***wheels***.
    - The ***strings*** played superbly.
- "Paradigmatic metonymy"
  - A prototype of a concept is used to represent the entire concept.
  - Examples:
    - A triangle is sketched to represent a whole class of triangles.
    - Bell-curve used to represent a class of distributions.

Why study metonymy?

- Lakoff and Johnson (1980) view metaphor and metonymy as cognitive constructs, and suggest that metaphor and metonymy actually structure our thinking.
- We learn new ideas by connecting the new ideas to familiar ideas that we already understand.
- Metaphor and metonymy may provide us with insight into how students are making those connections and developing understanding of new concepts.

Conceptual Domain of Distribution

Representations	Attributes	Classes of Distributions	Inferential Interpretations
Graphs (box plots, histograms, scatterplots, dot plots, pie charts, etc.), tables, and spreadsheet formulas	Local features (individual data points, outliers) and global features (measures of center, spread, shape)	Empirical versus theoretical, types of distributions (normal, uniform, binomial, chi-square, etc.), distribution of sample, distribution of population, and sampling distribution	Signals in noise, p-value, and empirical rule; variability within and between samples; connections between probability models and statistics

Methods

- Data collected at two institutions:
  - Site 1: medium-sized private comprehensive college
  - Site 2: small private liberal arts-based research university
- Undergraduate introductory statistics course.
  - Site 1: primarily math minors
  - Site 2: primarily geography and environmental science majors
- Class sizes around 25 students, primarily traditional undergraduate students.

Methods

- Both courses were non-traditional in their use of textbooks aimed at leading students through activities used to "construct" statistical ideas.
- Data included written pre- and post-assessment of statistical concepts given to all students; as well as semi-structured interviews with 11 selected students.

Sources for assessment tools:

- ARTIST (Assessment Resource Tools for Improving Statistical Thinking) https://app.gen.umn.edu/artist
- CAOS (Comprehensive Assessment of Outcomes in a first Statistics course) https://app.gen.umn.edu/artist/caos.html

Interview Tasks

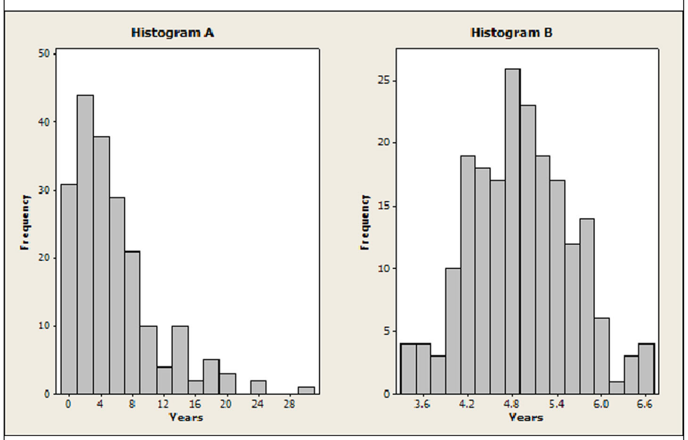
1. Definitions.
   - Students asked to define and give examples of:
     "distribution"
     "distribution of the population"
     "distribution of a sample"
     "sampling distribution"
   - Also asked to compare sample distribution to sampling distribution.

Interview Tasks

2. Tire task.

A new type of tire is tested to see how long its tread lasts. In one study, a researcher sampled 200 tires, measured how long each one lasts, and graphed her results in Histogram A (below). In a separate study, 200 people each looked at 50 tires; they each found the average life of their 50 tires and then they graphed their 50-tire averages. Histogram B shows the sample averages of the 200 people.

- Students then asked a series of questions about this situation, e.g.,
  - which distribution has more variability?
  - approximately what is the mean life span of the tires?
  - if the standard deviation of the population was 5 years, explain what this tells you about the life of the tires?
  - if the mean of 50 tires is 6 years, do you think this is evidence that Michelin's tires really last longer?



Interview Tasks

3. Platypus task.

Ecologists are studying the breeding habits of the Australian River platypus. These platypuses lay between one and seven eggs each year. Based on historical research, the ecologists estimate that the number of eggs occurs with the following frequencies:

Eggs	Relative Frequency
1	35% of all nests
2	25%
3	15%
4	10%
5	5%
6	5%
7	5%

One day, the ecologists go out on one stretch of the Brisbane River, and they find ten platypus nests, with an average nest size of 4 eggs. They want to determine whether this is unusual. How could you construct a sampling distribution to help them determine this?

Observed Metonymies for Distribution

- *Normal distribution as a paradigmatic metonymy*: Student uses the normal distribution as a prototype for all distributions.
  - Supports: developing ideas about sample size, area under the curve, and probability; make inferences based on area under curve where statistic falls relative to tails.
  - Hinders: erroneous conclusions when apply properties of a normal distribution to distributions that are not normal (e.g., empirical rule)

Tire Task Student Observations

- Note that the distribution of tires is not normal, yet 7 of the 11 student interviews applied the empirical rule to this distribution.
  - Four of the seven later added the stipulation that the population needed to be unimodal and symmetric (though they had Histogram A in front of them).
- Example:
  - "I could pretend that this [graph A] is part of a sampling distribution, even though I know it's not. …So, if I assume that the population fits a normal curve, then I could use the empirical rule which tells me that 95% of the data is going to be within two standard deviations of the center… I just realized I'm kind of making assumptions this entire time.... but if I did assume that it was skewed to the right I'd be totally lost."

Observed Metonymies for Distribution

- *Proper metonymy for sampling distribution*: Sampling distribution referred to as a compilation of many *samples* (rather than many sample statistics).
  - Supports: allows students to express a complicated idea in fewer words; idea of repeated sampling and more data to make better inferences.
  - Hinders: miss how variability among sample statistics and distribution of sample statistics support statistical inference

Student Definition Observations

- Of the 11 students interviewed, 4 defined sampling distributions using some form of metonymic shortening.
- Examples:
  - A sampling distribution is "the difference from sample to sample." [Site 2]
  - "A sampling distribution is when you have a bunch of different samples and then distribute it over a graph." [This student later in the discussion said, "so you have a bunch of means."] [Site 2]
  - "A sampling distribution is when you compile the results of a bunch of samples into one graph to get a better representation of the overall population." [Site 1]
  - "A sampling distribution is a distribution of many distributions of a sample." [This student later said we were "showing the relative frequency of averages of other samples."] [Site 1]

Using Sampling Distributions for Inference

- Students were asked to compare a sample of new tires to the information presented in histograms, where the new sample had a sample mean of 6 years. Asked if the new tires had a higher mean lifespan.
- Student responses [Site 2]:
  - "Well in this case they tested more tires [points to histogram B] and like six was not as frequent, but in histogram A they tested less tires but it [6] was frequent. So I think there's some evidence but I'm not sure if it's strong evidence."
  - "Because it's more, more data technically because you have two hundred people looking at fifty tires each so it's ten thousand tires that are being looked at instead of two hundred…. This one [Graph A] only uses two hundred tires and this one [Graph B] uses ten thousand tires and like averaged it. So I feel like histogram B would indicate more if it was unusual."

Using Sampling Distributions for Inference

- Platypus task – student told ecologists collect ten platypus nests with an average nest size of 4 eggs. Ask how they could construct a sampling distribution to determine if this is unusual.
- Student response [Site 1]:
  - "I'm just trying to decide whether my sample is going to be a single platypus nest or not. I'm leaning towards yes…. They did 10 samples [referring to the researchers in the problem] so I'll do 10 too. So, actually, I'll just do 1,000. … Essentially I'm just running a lot of trials and then we can look at all of the response values….If I looked at 1,000 platypus nests, what's the probability that the nest is going to have 4 eggs."

Using Sampling Distributions for Inference

- Student response continued [Site 1]:
  - Student: I feel like I'm not accounting for the 10 platypus nests….I mean, that's how many they looked at. So I guess they kind of have their own little sampling distribution going on, don't they? Unless I look at each nest as an observational unit and the number of nests they look at as the sample, which would be a different…Actually, I could do that if I upped the number of observations to 10. So now every trial that I run I have to switch it to an average. Now I'm taking ten tests and finding the average.
  - Student: Now I'm not looking at an actual observation anymore. I'm looking at an average.
  - Interviewer: Is that bad?
  - Student: It's bad…because there's never going to be an average of 7 because 35% of them are going to be 1.

Observations

- We hypothesize that one of the reasons the paradigmatic metonymy of using a normal distribution as a prototype for all distributions is the emphasis we normally place on the normal distribution in an introductory statistics course.
  - Suggests maybe we should incorporate more small-sample methods and other types of distributions into the introductory statistics course, including randomization/permutation methods.

Observations

- Proper metonymy of a sampling distribution as a compilation of many samples is not necessarily problematic, but instructors should be aware of this metonymy.
  - Instructors may often use this metonymic shortening, assuming that students understand the implied definition.

Limitations and Future Work

- Data only from a pilot study with 11 interviews.
- Original interviews not designed to study metonymy and language.
- Still need to analyze written assessments. Connect written assessment work of those students interviewed.
- Future work to include comparative study of how students new to statistics use language in explaining statistical concepts to how "experts" in the field use language.

Statistical Literacy, Reasoning, and Thinking

Ben-Zvi and Garfield (2004):

- ***Statistical literacy*** includes basic and important skills that may be used in understanding statistical information or research results.
- ***Statistical reasoning*** may be defined as the way people reason with statistical ideas and make sense of statistical information.
- ***Statistical thinking*** involves an understanding of why and how statistical investigations are conducted and the "big ideas" that underlie statistical investigations.

College Introductory Statistics Course

Guidelines for Assessment and Instruction in Statistics Education (GAISE) recommendations:

1. Emphasize statistical literacy and develop statistical thinking;
2. Use real data;
3. Stress conceptual understanding rather than mere knowledge of procedures;
4. Foster active learning in the classroom;
5. Use technology for developing conceptual understanding and analyzing data;
6. Use assessments to improve and evaluate student learning.

Seven Topics that Every Citizen Should Know about Statistics and Probability (Utts, 2003)

1. Cause and effect → can we conclude it? Observational studies versus randomized experiments.
2. Statistical significance versus practical importance.
3. The difference between finding "no effect" or "no difference" and finding no *statistically significant* effect or difference.
4. Common sources of bias in surveys and experiments, e.g., poor wording of questions, volunteer response, socially desirable answers.

Seven Topics that Every Citizen Should Know about Statistics and Probability (Utts, 2003)

5. Coincidences and seemingly very improbable events are not uncommon because there are so many possibilities.
6. "Confusion of the inverse" – the probability that you test positive given you have a disease is not the same as the probability you have the disease given you test positive!
7. Understanding that variability is natural, and that "normal" is not the same as "average."

A Few "Big Ideas" in Current Statistics Education Research

- Randomization and simulation-based curriculums
- Teaching and learning in the K-12 curriculum
- Incorporating "big data" and "data science" into statistics courses
- Hybrid, online and flipped statistics classrooms
- Technology in statistics education

Statistics Education Journals

- *Journal of Statistics Education*
  - http://www.amstat.org/publications/jse
- *Statistics Education Research Journal*
  - http://iase-web.org/Publications.php?p=SERJ
- *Technology Innovations in Statistics Education*
  - http://www.escholarship.org/uc/uclastat_cts_tise
- *Teaching Statistics*
  - http://www.teachingstatistics.co.uk/

Statistics Education Professional Societies

- International Association for Statistics Education
  - http://iase-web.org
  - Part of the International Statistical Institute (ISI), but can be joined independently.
  - Publishes SERJ
- American Statistical Association (ASA) – Section on Statistical Education
  - http://www.amstat.org/sections/educ/
- Mathematical Association of America (MAA) – Special Interest Group on Statistics Education
  - http://sigmaa.maa.org/stat-ed/

Statistics Education Conferences

- U.S. Conference on Teaching Statistics (USCOTS)
  - Held every two years, next one this May in PN!
- International Conference on Teaching Statistics (ICOTS)
  - Held every four years, alternating continents – next in Kyoto, Japan, 2018
- Electronic Conference on Teaching Statistics (eCOTS)
  - Alternates years with USCOTS
- International Research Forums on Statistical Reasoning, Thinking, and Literacy (SRTL)
  - Held every two years. http://srtl.fos.auckland.ac.nz/

www.causeweb.org

- Have regular webinars on
  - teaching and learning statistics
  - highlights from the most recent issue of the *Journal of Statistics Education*
  - activities to use in class
- Lecture examples, labs, homeworks and projects.
- Data sets and how to use them.
- Curriculum development.
- Cartoons and "statistical art."
- Information on upcoming and past USCOTS and eCOTS conferences.
- And much more…

Other Resources

- Assessment Resource Tools for Improving Statistical Thinking (ARTIST)
  - http://app.gen.umn.edu/artist/
- Change Agents for Teaching and Learning Statistics (CATALST)
  - http://www.tc.umn.edu/~catalst/
- WWW Resources for Teaching Statistics
  - http://it.stlawu.edu/~rlock/tise98/onepage.html

News Stories

- ***Chance News*** reviews current stories in the news that involve probability and statistics concepts and is freely available:
  http://test.causeweb.org/wiki/chance/index.php/Main_Page
- Stats.org "checks out the numbers behind the news", with lots of links to current news stories.
- If you see something interesting, bookmark it! I have a folder dedicated to interesting news stories I can use in class.
  - e.g., "Student science experiment finds plants won't grow near Wi-Fi router", "When cheeseburger = walking, will we eat less?", "On Facebook, you are what you 'like', Cambridge study finds"

References

- Ben-Zvi, D. and Garfield, J. (2004). Statistical literacy, reasoning, and thinking: Goals, definitions, and challenges. In J. Ben-Zvi and Garfield, D. (Eds.), ***The challenge of developing statistical literacy, reasoning and thinking*** (pp. 3-15). Dordrecht, The Netherlands: Kluwer.
- Garfield, J. and Ben-Zvi, D. (2008). ***Developing Students' Statistical Reasoning: Connecting Research and Teaching Practice***. Springer.
- Noll, J. and Hancock. S. (2014). Proper and paradigmatic metonymy as a lens for characterizing student concpetions of distributions and sampling. *Educational Studies in Mathematics.* DOI 10.1007/s10649-014-9547-1
- Utts, J. (2003). What educated citizens should know about statistics and probability. *The American Statistican*, Vol. 57, No. 2, pp. 74-79.

References

- Chance, B., DelMas, R., & Garfield, J. (2004). Reasoning about sampling distributions. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 295–323). Dordrecht: Kluwer.
- Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. Chicago: The University of Chicago Press.
- Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.
- Lipson, K. (2003). The role of the sampling distribution in understanding statistical inference. *Mathematics Education Research Journal*, 15(3), 270–287.
- Saldanha, L., & Thompson, P. (2002). Conceptions of sample and their relationship to statistical inference. *Educational Studies in Mathematics*, 51(3), 257–270.
- Saldanha, L., & Thompson, P. (2007). Exploring connections between sampling distribution and statistical inference: An analysis of students' engagement and thinking in the context of instruction involving repeated sampling. *International Electronic Journal of Mathematics Education*, 2(3), 270–297.

Where to Find Data?

- http://www.ics.uci.edu/~staceyah/data.html
  Includes…
  - US Census data
  - World Bank economic data
  - Election data
  - Energy data
  - Environmental data
  - Links to more data links…
- Many ***introductory statistics textbooks*** have their data sets freely available online, often with nice descriptions:
  - *Mind on Statistics* by Utts and Heckard
  - *Introduction to the Practice of Staistics* by Moore and McCabe
  - *Workshop Statistics* by Rossman and Chance
    - www.rossmanchance.com – Applets

Statistics Blogs in the US

- Andrew Gelman's statistics blog: http://andrewgelman.com/
- Nate Silver's now famous statistics on politics:
  http://fivethirtyeight.com/
- FlowingData data visualization blog: http://flowingdata.com/