

Overview | Order Positive and Negative Numbers

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Compare and order positive and negative numbers.
- Interpret numerical inequalities as statements about position of numbers on a number line.
- Write and interpret inequalities in real-world contexts.

Language Objectives

- Use lesson vocabulary and direction and comparison words, such as *left*, *right*, *up*, *down*, *farther*, *best*, and *worst*, to talk about ordering and comparing positive and negative numbers.
- Read and interpret word problems to identify key quantities and compare them using words and inequality statements in discussion and writing.

Prior Knowledge

- Locate positive and negative numbers on the number line.
- Compare and order positive rational numbers.
- Use positive and negative numbers to represent real-world quantities and explain the meaning of 0 in context.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

compare to describe the relationship between the value or size of two numbers or quantities.

inequality a mathematical statement that uses an inequality symbol ($<$, $>$, \leq , \geq) to show the relationship between values of expressions.

negative numbers numbers that are less than 0. They are located to the left of 0 on a horizontal number line and below 0 on a vertical number line.

positive numbers numbers that are greater than 0. They are located to the right of 0 on a horizontal number line and above 0 on a vertical number line.

rational number a number that can be expressed as the fraction $\frac{a}{b}$ or the opposite of $\frac{a}{b}$ where a and b are whole numbers and $b \neq 0$. Rational numbers can also be expressed as a decimal.

Academic Vocabulary

interpret to explain the meaning of something.

lesser less or smaller in value.

relative to compared to something or someone else.

Learning Progression

In earlier grades, students located and labeled positive whole numbers, fractions, and decimals on the number line and compared them using inequality symbols and words. They ordered positive rational numbers.


In the previous lesson, students located positive and negative whole numbers, fractions, and decimals and their opposites on the number line. They described the value of positive and negative rational numbers in relation to 0 and in real-world contexts.

In this lesson, students compare and order positive and negative rational numbers. They interpret inequalities as statements about the relative position of numbers on the number line. They also write inequalities to represent and interpret inequalities in real-world contexts.

Later in Grade 6, students will extend their understanding of locating numbers on the number line to interpret the absolute value of a number. They will also write and graph inequalities with variables and understand that these inequalities have infinitely many solutions.

In Grade 7, students will write inequalities with a variable to represent real-world situations with unknowns. They will solve inequalities that include a variable and graph solutions to inequalities on the number line.

Pacing Guide

Items marked with  are available on the **Teacher Toolbox**.

MATERIALS

DIFFERENTIATION

SESSION 1 **Explore** Ordering Positive and Negative Numbers (35–50 min)


- **Start** (5 min)
- **Try It** (5–10 min)
- **Discuss It** (10–15 min)
- **Connect It** (10–15 min)
- **Close: Exit Ticket** (5 min)

Additional Practice (pages 557–558)



Math Toolkit algebra tiles, number lines, two-color counters

Presentation Slides 

PREPARE Interactive Tutorial 

RETEACH or REINFORCE Hands-On Activity

Materials For each pair: 1 sticky note

SESSION 2 **Develop** Comparing Positive and Negative Numbers (45–60 min)

- **Start** (5 min)
- **Try It** (10–15 min)
- **Discuss It** (10–15 min)
- **Connect It** (15–20 min)
- **Close: Exit Ticket** (5 min)


Additional Practice (pages 563–564)




Math Toolkit graph paper, number lines

Presentation Slides 

RETEACH or REINFORCE Hands-On Activity

Materials For each pair: 1 two-color counter, 1 number cube, Activity Sheet *Number Lines* 

REINFORCE Fluency & Skills Practice 

EXTEND Deepen Understanding

SESSION 3 **Refine** Ordering Positive and Negative Numbers (45–60 min)

- **Start** (5 min)
- **Monitor & Guide** (15–20 min)
- **Group & Differentiate** (20–30 min)
- **Close: Exit Ticket** (5 min)



Math Toolkit Have items from previous sessions available for students.

Presentation Slides 

RETEACH Hands-On Activity


Materials For each pair: 20 index cards


REINFORCE Problems 4–8


EXTEND Challenge

PERSONALIZE 

Lesson 24 Quiz  or
Digital Comprehension Check

RETEACH Tools for Instruction 

REINFORCE Math Center Activity 

EXTEND Enrichment Activity 

Connect to Culture

- Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 ■ □ □

Try It Ask students if they have ever played or watched American football. There are many decisions a coach has to make regarding the team in every football game. People are working on technology that can predict what play a team will run next. In certain circumstances, the prediction is over 90% accurate. Some people may be concerned with how this power of predicting will change how football is played. Ask students if they think football will become a game of beating the computer more than beating the opponent.

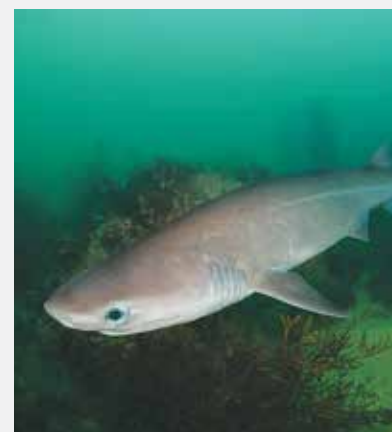
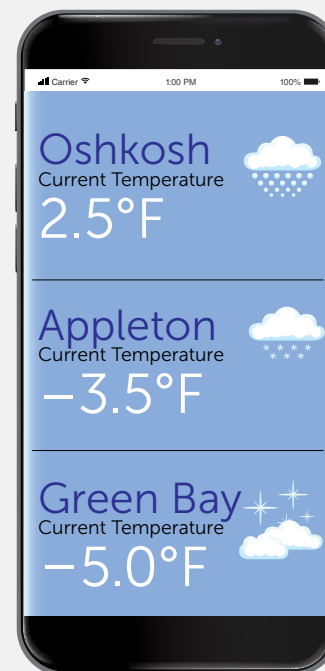
SESSION 2 ■ ■ □

Try It There are many places that typically have very cold climates. The coldest temperature ever recorded was in Antarctica, which was -128.6 degrees Fahrenheit, or -89.2 degrees Celsius. Another place that has experienced a record low temperature is a town in Canada called Snag, in the Yukon province. In 1947 it reached -81 degrees Fahrenheit. The average annual high temperature is 32 degrees Fahrenheit. Survey students about the coldest temperatures they have experienced and discuss the locations where these temperatures occurred.

Apply It Problem 9 One interesting sea creature is the sixgill shark. Most sharks have five gills, but this one has six. It is also one of the largest sharks, excluding the ones that feed on plankton like whales do. The sixgill shark can grow to be up to 18 feet long! These sharks can be found at depths around 8,000 feet below sea level. Another unique fact about them is that they have only the back dorsal fin, so their back is flat in the middle, where other sharks have the iconic large dorsal fin that might stick out of the water as they circle their prey. Encourage a discussion about various sea creatures with which students are familiar.

SESSION 3 ■ ■ ■

Apply It Problem 2 Survey students to determine how often they purchase items from a vending machine. When we think of vending machines, typically we think of buying drinks or snacks. However, there are some vending machines that are much more unique. For example, there is a vending machine in Dubai where you can buy gold. Other examples include vending machines that sell real crabs, mashed potatoes, hair extensions, books, and dog toys and treats. If you are looking for a meal, there is even a vending machine that serves hot pizza, ready to eat!



Connect to Family and Community

- After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.

Order Positive and Negative Numbers

LESSON
24

Dear Family,

This week your student is learning how to compare positive and negative numbers. The farther to the left a number is located on a horizontal number line, the lesser the value of that number.

You can write an inequality to show which of two numbers has the greater or lesser value. For example, $2 > -3$ means that 2 is greater than -3 . Your student will be learning to solve problems like the one below.

A town well extends to an elevation of 255 ft below ground level. Jesse's house well extends to an elevation of -260 ft. Which well is deeper?

► **ONE WAY** to compare the elevations is to use words to describe their relationship in context.

A point 260 ft underground is deeper than a point 255 ft underground.

Both models show that Jesse's well is deeper than the town well.

► **ANOTHER WAY** is to use symbols to write an inequality. Look at the positions of -260 and -255 on a vertical number line.

$-260 < -255$

Use the next page to start a conversation about positive and negative number comparisons.

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LESSON 24 | ORDER POSITIVE AND NEGATIVE NUMBERS

Activity Thinking About Comparing Positive and Negative Numbers Around You

► Do this activity together to investigate comparing positive and negative numbers in the real world.

Did you know when you play golf, the lowest score wins? In fact, your final score could even be negative!

A score of -4 means a person used four fewer strokes than was expected in order to complete the golf course. A score of $+4$ means a person used four more strokes than was expected in order to complete the golf course. Using fewer strokes is better, so a score of -4 is better than $+4$!

Hole	Par	Strokes	Score
1	4	3	-1
2	4	4	-1
3	5	5	-1
4	6	7	0
5	4	6	$+2$
6	5	5	$+2$
7	5	6	$+3$
8	6	6	$+3$
9	4	5	$+4$

Where else do you compare positive and negative numbers in the world around you?

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Connect to Language

- For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Speaking/Writing

Use **Act It Out** to support the problem context before students write a response to Connect It problem 1. Display an arrow beginning with the word *worst* and ending with *best*. Explain that *best* means really good or better than all others and *worst* means really bad or worse than all others. Write four movie titles from worst to best on the arrow. Point as you ask questions to help students compare: *Which movie is the worst? Which movies are better than this? Is this movie better than this? Is this movie worse than this? Which movie is the best?* Next, read the problem. Have students use a similar arrow to order the plays. Then have them work in pairs to explain in writing. Ask: *Which is the worst result? Which plays are better?*

Levels 2–4: Speaking/Writing

Use **Act It Out** to support the problem context before students write a response to Connect It problem 1. Display the words *worst*, *worse*, *best*, and *better*. Ask: *Which words compare two things? Which words compare more than two things?* Call on volunteers to explain *worst* and *best*. Then invite students to order four movie titles with an arrow on the board. Guide other students to use the words to compare the titles. Ask: *Which word describes the favorite movie? [best] And the movie nobody liked? [worst]*

Read the problem with students. Use **Stronger and Clearer Each Time** to support writing. During partner review, encourage students to check whether they listed the plays in the correct order. Monitor as they review comparison words in the explanations.

Levels 3–5: Speaking/Writing

Help students discuss before they write a response to Connect It problem 1. Display the words *worst*, *worse*, *best*, and *better*. Call on volunteers to explain or provide examples.

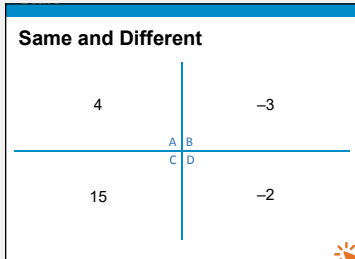
Have students read the problem. Invite volunteers to use the meanings of *worst* and *best* to **Say It Another Way**. Then use **Stronger and Clearer Each Time** to support writing. During partner review, encourage students to ask each other questions to make sure they listed the plays in the correct order, and to check whether they used correct comparison words in their explanations.

Explore Ordering Positive and Negative Numbers

Purpose

- **Explore** the idea that you can use a number line to compare positive and negative numbers.
- **Understand** that the greater of two numbers lies to the right of the other number on a horizontal number line and above the other number on a vertical number line.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are whole-number integers.

A and C are greater than 0.

B and D are less than 0.

A and D are even integers.

B and C are odd integers.

WHY? Support students' facility with positive and negative numbers.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask students what they know about football. After the second read, clarify the terms *worst* and *best* as needed. After the third read, ask: *Do you think the sign of the numbers will be important? Why or why not?*

DISCUSS IT

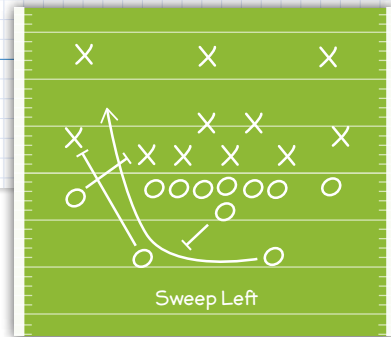
SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- -3 , -4 , and -5 as less than 0.
- 2 and 4 as greater than 0.
- the numbers need to be ordered from least to greatest.

Explore Ordering Positive and Negative Numbers



A diagram of a football play

Previously, you learned about positive and negative numbers. In this lesson, you will learn about ordering and comparing positive and negative numbers.

► Use what you know to try to solve the problem below.

A youth football team tries several different plays. The goal of each play is to gain yards. The coach records the result of each play. List the plays from worst to best.

Name of Play	Wedge	Hook	Flag	Draw	Sweep	Toss
Result: Yards Gained (+) or Lost (-)	-3	+4	-5	+2	0	-4

TRY IT



Math Toolkit algebra tiles, number lines, two-color counters

Possible work:

SAMPLE A

A loss of 5 yd is worse than a loss of 4 yd and a loss of 4 yd is worse than a loss of 3 yd.

A gain of 4 yd is better than a gain of 2 yd.

The value 0 comes between the losses and gains.

Worst to best: Flag, Toss, Wedge, Sweep, Draw, Hook

SAMPLE B



The worst play is Flag, then Toss, Wedge, Sweep, and Draw. The best is Hook.

DISCUSS IT

Ask: What did you do first to decide which play is the worst?

Share: The first thing I did was ...



Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- Write, interpret, and explain statements of order for rational numbers in real-world contexts.

Common Misconception Listen for students who order the numbers from least to greatest based on the value of the numeral, with 0 as the least number and -5 as the greatest number. As students share their strategies, ask them to apply their reasoning to a pair of opposites, such as 4 and -4 .

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- comparing the relationship of the numbers using words describing each situation
- **(misconception)** ordering the numbers based on the value of the numeral
- plotting the values on a number line

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to participate actively by looking at the speaker and asking clarifying questions.

Guide students to **Compare and Connect** the representations. If discussion lags, ask students to turn and talk to a partner about the strategies before resuming the class discussion.

ASK How do [student name]'s and [student name]'s representations of the problem compare the numbers given?

LISTEN FOR The numbers can be compared by calculating their distance from 0, or they can be compared by using words that describe the gains or losses so that they can determine the order of plays from worst to best.

CONNECT IT

SMP 2, 4, 5

- Look Back** Look for understanding that the negative numbers are less than the positive numbers, so the play with the most yards lost is the worst and the play with the most yards gained is the best.

DIFFERENTIATION | RETEACH or REINFORCE



Hands-On Activity

Locate positive and negative numbers on a number line.

If students are unsure about locating numbers on a number line, then use this activity to help students visualize the relationship between numbers and the number line.

Materials For each pair: 1 sticky note

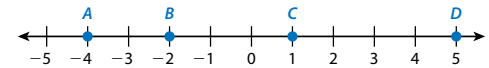
- Display a horizontal number line and place a sticky note with 0 on it at the center of the number line.
- Have half the pairs write a positive number on their sticky note, and have the other half write a negative number on their sticky note.
- Call on a student from one pair to place their number on the number line, locating it in relation to 0. Ask: *Is this number placed correctly?*
- Choose a student from another pair to place their number onto the board, deciding how to locate the number in relation to 0 and the number already placed. Ask: *Is this number placed correctly?*
- Continue until all numbers are placed in the correct order from least to greatest.
- Repeat with a vertical number line.

CONNECT IT

- Look Back** List the plays from worst to best. Explain how you know.
Flag, Toss, Wedge, Sweep, Draw, Hook; Possible explanation: Plot the results on a horizontal number line. The worst result is farthest left because it shows the most yards lost. The plays get better as you move to the right.

- Look Ahead** The goal of a football play is to gain yards. The more yards gained or the fewer yards lost, the better the play is. Number lines can be used to help make these types of comparisons with positive and negative numbers.

- Look at the horizontal number line. Point D is farther to the right from 0 than point C. Which point represents a greater number?



Point D

- Point A is farther to the left from 0 than point B. Which point represents a greater number?

Point B

- Look at the vertical number line. Point G is farther down from 0 than Point F. Which point represents a greater number?

Point F

- Point E is above Point F. Which point represents a lesser number? What is always true when comparing a negative number and a positive number?

Point F; A negative number is always less than a positive number.



- Reflect** How do the values change on a horizontal number line as you move left? How do the values change on a vertical number line as you move up?
Possible answer: On a horizontal number line, values decrease as you move left. On a vertical number line, values increase as you move up.

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- Look Ahead** Point out that vertical and horizontal number lines can be used to order positive and negative numbers. Students should recognize that positive numbers are greater than negative numbers.

CLOSE EXIT TICKET

- Reflect** Look for understanding that the value of a number increases as you move right on a horizontal number line and up on a vertical number line, and the value decreases as you move left on a horizontal number line and down on a vertical number line.

Error Alert If students explain that moving left on a horizontal number line means values become negative and moving up on a vertical number line means values become positive, then have students move from 5 to 4 on a horizontal number line and from -5 to -4 on a vertical number line to encourage a more specific explanation.

Prepare for Ordering Positive and Negative Numbers

Support Vocabulary Development

Assign **Prepare for Ordering Positive and Negative Numbers** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the terms *positive numbers*, *negative numbers*, *rational numbers*, and *inequality*. Encourage students to think about *positive numbers* and *negative numbers* in terms of how they compare to 0 and to consider the different forms in which rational numbers can be written. If students believe that an *inequality* describes two numbers that are simply not equal and struggle to give examples, remind them that *inequality symbols* are used to compare unequal values.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and then prompt a whole-class comparative discussion of the words and examples given.

Have students share examples of negative rational numbers for problem 2 and discuss with a partner how to compare the numbers to 0.

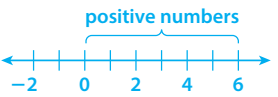
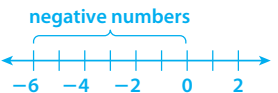
Problem Notes

- Students should recognize that positive numbers are greater than zero, negative numbers are less than zero, rational numbers can be written as decimals or fractions, and inequalities show how any two numbers with different values compare to one another. Students may understand that an inequality can be used to compare negative numbers and positive numbers. Student responses may include inequality statements that include symbols, number line representations, or lists.
- Students should recognize that 0 is greater than any negative number, so their inequality statements should start with $0 >$ and end with the negative rational number of their choice.

Prepare for Ordering Positive and Negative Numbers

- Think about what you know about positive and negative numbers. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

Word	In My Own Words	Example
positive numbers	Numbers that are greater than 0	 A number line with arrows at both ends. Tick marks are labeled -2, 0, 2, 4, and 6. A blue bracket above the line spans from 0 to 6, with the text "positive numbers" written above it.
negative numbers	Numbers that are less than 0	 A number line with arrows at both ends. Tick marks are labeled -6, -4, -2, 0, and 2. A blue bracket above the line spans from -6 to 0, with the text "negative numbers" written above it.
rational numbers	Numbers that can be written as positive or negative fractions	-1.5 is a rational number. It can be written as $-\frac{3}{2}$.
inequality	Two unequal values that are compared using a less than (<) or greater than (>) sign	$5 > 3$

- Choose a negative rational number. Write an inequality using the symbol $>$ to compare your number to 0. Explain your thinking.

Possible answer: $-5.25; 0 > -5.25$; Negative numbers are numbers that are less than 0, so 0 is greater than any negative number.

REAL-WORLD CONNECTION

Elevators sometimes go under the main floor of a building into basement levels. An elevator button to a basement level is often labeled with an L in front of the floor number to indicate it is below the main floor, such as L2. This is similar to negative values where a negative sign (–) is written in front of the number to indicate it is less than 0. Inequalities are also often used to describe height limits on elevators. For example, an elevator typically cannot travel a distance of more than 1,640 feet. Otherwise, the ropes that lift it get too heavy. That means the distance the elevator travels must be less than 1,640 feet. Ask students to think of other real-world situations that include positive and negative numbers or use inequalities.



- 3 Problem 3 provides another look at ordering positive and negative numbers. This problem is similar to the problem about football plays. In both problems, positive and negative numbers need to be ordered from least to greatest. This problem asks for students to order the scores from a trivia game from worst to best.

Students may choose to use a number line to solve.

Suggest that students use **Say It Another Way** by reading the problem out loud and asking a student to paraphrase it. Prompt other students to give a thumbs up or thumbs down to indicate whether they think the paraphrased problem is complete and accurate.

LESSON 24 | SESSION 1

- 3 Some friends play history trivia. Players gain 1 point for a correct answer. Players lose 1 point for an incorrect answer. The player with the greatest score wins. The players' scores are shown in the table.

Player	Score
Brett	-7
Ellema	-1
Felipe	+3
Jennifer	0
Kamal	+2
Riley	-5

- a. List the players from worst score to best score. Show your work.

The negative scores are -7, -1, -5.

A score of -7 is worse than a score of -5, and a score of -5 is worse than a score of -1.

-7 is the worst score, then -5, then -1.

The positive scores are 3, 2.

A score of 3 is better than a score of 2.

3 is the best, then 2 is next best.

A score of 0 comes between the negative scores and positive scores.

SOLUTION Brett, Riley, Ellema, Jennifer, Kamal, Felipe

- b. Check your answer to problem 3a. Show your work.

On the number line, the worst score is the farthest left. The scores get better as you move to the right.



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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2 Apply It**

ACADEMIC VOCABULARY

Elevation is the height above or below a given level, often sea level.

Levels 1–3: Reading/Writing

Help students interpret and respond to Apply It problem 9. Display the lesson term *inequality*. Review that *in-* means *not*, so *inequality* means *not equal*. Adapt **Three Reads** to help students make sense of the problem. For Read 1, review the vocabulary. Have students draw a vertical line. Label *sea level*. Help them draw arrows to show *below* and *higher*. After Read 2, help students tell what they need to find. For Read 3, help students add the quantities to the sketches and write their responses:

- The ____ can dive to a lower elevation.

Levels 2–4: Reading/Writing

Help students interpret and respond to Apply It problem 9. Display the lesson term *inequality*. Explain that *in-* means *not*. Write an example of an *inequality* and invite students to explain the relationship. Adapt **Three Reads** to help students make sense of the problem. For Read 1, call on volunteers to tell what the problem is about. For Read 2, have students tell what they need to find. For Read 3, help students tell how they will use the quantities and write the answer:

- The ____ can dive to a lower elevation because ____.

Levels 3–5: Reading/Writing

Help students make sense of Apply It problem 9. Display the lesson term *inequality*. Have students use the meaning of *in-* to define the term with a partner. Adapt **Three Reads** by having students work together in pairs to answer the questions. Then have them take turns paraphrasing the problem. Call on volunteers to confirm understanding.

Allow time for students to write an inequality statement, and then encourage them to add a written explanation of the inequality statement using context from the problem.

Develop Comparing Positive and Negative Numbers

Purpose

- **Develop** strategies for writing and interpreting comparison statements for two rational numbers.
- **Recognize** that you can write a comparison using $<$ or $>$ based on the relative positions of the two numbers on the number line.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

- A 12° below 0°F is a negative quantity.
- B 10° from 0°F is a negative quantity.
- C 5° above 0°F is a negative quantity.

Possible Solutions

- A is always true.
- B is sometimes true.
- C is never true.

WHY? Support students' understanding of determining positive and negative numbers when compared to 0.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of *interpret* in math.

HOW? Read Analyze It. Explain that *interpret* in this case means *to explain in the context, or situation, presented in the word problem*. Ask: *What phrase does Analyze It use to explain what the inequality statement means in the real world?* [warmer than] Ask students to turn and talk about why it is important to interpret the inequality statement in terms of the situation.

TRY IT

SMP 1, 2, 4, 5, 6

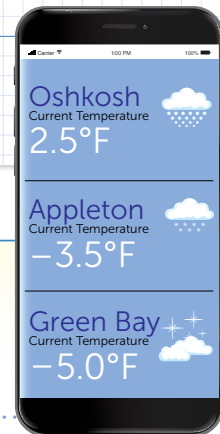
Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding that there are three different comparisons of the three temperatures, and for each pair you can write the comparison as less than or greater than, for a total of six comparisons.

Develop Comparing Positive and Negative Numbers

► Read and try to solve the problem below.

On a winter day, Adnan looks up the current temperatures in three nearby cities. Adnan chooses two of the temperatures and writes a comparison. What are all the possible comparisons he can write? You can use words and/or symbols.

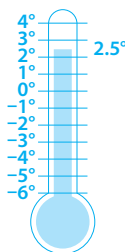


TRY IT

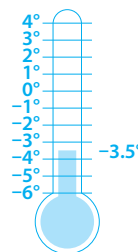
Math Toolkit graph paper, number lines

Possible work:

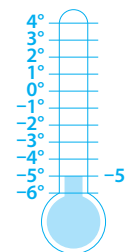
SAMPLE A



2.5°F is above -3.5°F .
 2.5°F is above -5°F .

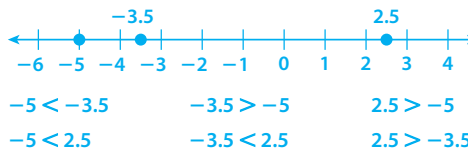


-3.5°F is below 2.5°F .
 -3.5°F is above -5°F .



-5 is below 2.5°F .
 -5°F is below -3.5°F .

SAMPLE B



DISCUSS IT

Ask: How can you determine that you have found all the possible comparisons?
Share: I know I found all the comparisons because ...

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Why did you choose the model or strategy you used?*
- *How did your model help you determine the number of possible comparisons?*

Common Misconception Listen for students who think that each comparison can only be written one way. As students share their strategies, ask students to look at a pair of positive numbers, such as 3 and 5, and write all of the possible comparisons for the pair. Remind students that two numbers can be compared so that either number appears first in the inequality. If the order of numbers is different, the inequality symbol is also different.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- horizontal or vertical number lines
- inequalities
- analysis with words

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to build on ideas they agree with by saying why the idea makes sense or why the solution is reasonable.

Guide students to **Compare and Connect** the representations. To emphasize how to write a comparison between positive and negative numbers, call on two or three other students to rephrase so that students hear the idea in more than one way.

ASK *Where does [student name]'s model compare each pair of temperatures?*

LISTEN FOR Temperatures are plotted on a number line, compared using inequalities, or compared using words such as *warmer* and *colder* to interpret numbers in a real-world situation.

Model Its & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *What is the greatest temperature? How can this temperature be represented in each model?*

LISTEN FOR The greatest number is 2.5 and it is located at the top of the number line, before the greater than symbol, or is described as being warmer than another temperature.

For the number line, prompt students to identify how the model is labeled.

- *Where are the positive numbers placed on the number line? The negative numbers?*

For the inequalities, prompt students to connect the words to the symbols in the expression.

- *How could you write each comparison as a less-than inequality?*

For the analysis, prompt students to connect the word *warmer* or *colder* to the phrases *greater than* or *less than* and to the symbols in the expression.

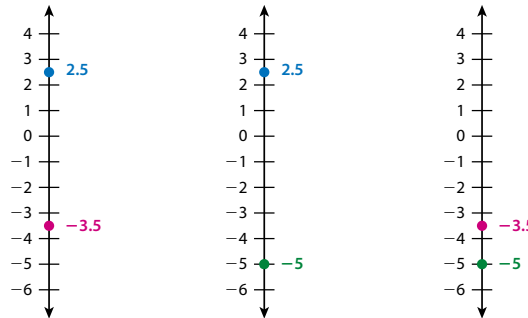
- *How could you use the word "warmer" to compare two temperatures?*

Explore different ways to compare positive and negative numbers.

On a winter day, Adnan looks up the current temperatures in three nearby cities. The temperatures are 2.5°F, −3.5°F, and −5°F. Adnan chooses two of the temperatures and writes a comparison. What are all the possible comparisons he can write? You can use words and/or symbols.

Model It

You can use a number line to compare positive and negative numbers.



Model It

You can write an inequality to compare positive and negative numbers.

$2.5 > -3.5$ $2.5 > -5$ $-3.5 > -5$

Analyze It

You can use words to interpret the meaning of an inequality in a real-world situation.

- 2.5°F is warmer than −3.5°F.
- 2.5°F is warmer than −5°F.
- −3.5°F is warmer than −5°F.



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DIFFERENTIATION | EXTEND



Deepen Understanding

Making Sense of Quantities and the Relationships Between Them

SMP 2

Prompt students to think about the pattern and structure present in positive and negative numbers and inequalities.

ASK *If the value of a is greater than the value of b , and the value of b is greater than the value of c , what is the relationship between a and c ?*

LISTEN FOR Student responses should indicate that a is greater than c . If you were to plot all three variables on a horizontal number line, a would be to the right of b , and b would be to the right of c . Therefore, a is to the right of c .

ASK *Does the temperature increase or decrease from 0 to 2.5? Does the temperature increase or decrease from −5 to −3.5?*

LISTEN FOR Student responses should indicate that 2.5 is greater than 0, so it is an increase in temperature. −3.5 is greater than −5, so it is also an increase in temperature.

ASK *When comparing numbers, how do you know which one is greater?*

LISTEN FOR Student responses should indicate that positive numbers are greater than negative numbers. Those to the right on a number line are greater than those on the left.

Develop Comparing Positive and Negative Numbers

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about comparing positive and negative numbers.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- A number line can compare numbers based on the location of the numbers, and an inequality uses $<$ or $>$ to make the same comparison.
- Warmer temperatures are greater than colder temperatures.
- -3.5 degrees is colder than 2.5 degrees.

Facilitate Whole Class Discussion

- 4 Look for understanding that there are six possible inequalities, since each number can be written as either less than or greater than another number.

ASK How can you compare two negative numbers?

LISTEN FOR You could plot the numbers on a number line and determine which number is farther from 0 to the left or below.

- 5 Look for the idea that an inequality can decontextualize numbers to highlight the relationship between them before recontextualizing the numbers in the problem.

ASK Why can a number line be used to write an inequality for any pair of numbers?

LISTEN FOR All rational numbers, regardless of sign, can be plotted on a number line, which can be used to write an inequality.

- 6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- Use the problem from the previous page to help you understand how to compare positive and negative numbers.

- 1 Look at the two **Model Its**. How can you use a number line to help you write an inequality?
Possible answer: A number line shows which numbers are greater or lesser by where the numbers are located on the number line. An inequality uses a greater than or less than symbol to make the same comparison.
- 2 Look at **Analyze It**. How can the inequalities help to determine which of two temperatures is warmer?
The warmer temperatures are the temperatures that are represented with a greater number.
- 3 Use the symbol $<$ to rewrite the comparison between -3.5°F and 2.5°F . Then interpret the meaning of the inequality using the words *colder than*.
 $-3.5^\circ\text{F} < 2.5^\circ\text{F}$; -3.5°F is colder than 2.5°F .
- 4 What are all the possible inequality statements Adnan might write? Use $<$ and $>$.
 $-3.5 < 2.5$, $2.5 > -3.5$, $-5 < -3.5$, $-3.5 > -5$, $-5 < 2.5$, $2.5 > -5$
- 5 When given a pair of numbers in a real-world situation, how can you compare the numbers using $<$ and $>$? How can an inequality help you interpret the comparison in the real-world situation?
Possible answer: Plot the numbers on a number line. The number that is above or to the right is greater. Then write the comparison using $<$ or $>$. An inequality helps you think about what the numbers represent when compared to each other in the real-world situation.
- 6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to compare positive and negative numbers.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE



Hands-On Activity

Use number lines to write inequalities.

If students are unsure about writing inequalities, then use this activity to show how to use number lines to compare integers.

Materials For each pair: 1 two-color counter, 1 number cube, Activity Sheet *Number Lines*

- Pair students and distribute an Activity Sheet, a number cube, and a counter. Tell students that one color represents a positive sign and the other color represents a negative sign. Have students mark the top number line to show -6 to 6 .
- For each round, have each student roll the number cube and flip the counter to generate an integer. Then have students mark the locations of the integers on the number line.
- Have pairs decide which integer is greater. Below the number line, they should write two inequalities, one using $<$ and the other using $>$.
- Ask: How do you know which number to write first when writing an inequality using a less than symbol? [The lesser number is to the left and the greater number is to the right.]
- Repeat as time allows.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, drawing equal-sized sections of number lines can be difficult, and precise measures here are not necessary, as the model is being used to think through a problem.

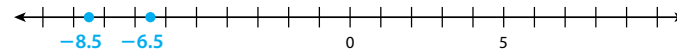
- 7 Students may recognize that -6.5 is greater because it is the farther to the right on the number line.
- 8 **A, D, and F are correct.** Students may solve the problem by plotting the pair of numbers on a number line.
- B** is not correct. This answer is a result of thinking that numbers on the left are greater than numbers on the right on a horizontal number line.
- C** is not correct. This answer is a result of thinking that numbers to the right are less than numbers to the left on a horizontal number line.
- E** is not correct. This answer is a result of thinking that numbers lying below are greater than numbers lying above on a vertical number line.

LESSON 24 | SESSION 2

Apply It

► Use what you learned to solve these problems.

- 7 Plot and label the numbers -6.5 and -8.5 on the number line. Then write an inequality using the symbol $>$ to compare the two numbers.

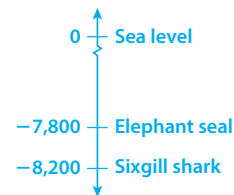


SOLUTION $-6.5 > -8.5$

- 8 Which of the following statements are true? Select all that apply.

- A** $4 > -17$ because 4 is to the right of -17 on a horizontal number line.
- B** $4 > -17$ because 4 is to the left of -17 on a horizontal number line.
- C** $4 > -17$ because -17 is to the right of 4 on a horizontal number line.
- D** $4 > -17$ because 4 is above -17 on a vertical number line.
- E** $4 > -17$ because 4 is below -17 on a vertical number line.
- F** $4 > -17$ because -17 is below 4 on a vertical number line.

- 9 Notah is studying ocean animals. He learns that the sixgill shark can dive to an elevation of about $-8,200$ ft relative to sea level and the elephant seal can dive to an elevation of about $-7,800$ ft. Write an inequality to compare these elevations. Which animal can dive to a lower elevation? Show your work. **Possible work:**



SOLUTION $-7,800 > -8,200$ (or $-8,200 < -7,800$);

The sixgill shark can dive to a lower elevation.



A sixgill shark

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CLOSE EXIT TICKET

- 9 See **Connect to Culture** to support student engagement. Students' solutions should show an understanding that:
- positive and negative numbers can be plotted on a number line.
 - the relative positions of two numbers on a number line can be used to write a comparison using $<$ or $>$ and interpret the results.

Error Alert If a student's answer is $-8,200 > -7,800$, then review how placement on a number line affects an inequality. Numbers to the left or below are less than numbers to the right or above. Also review with students the meaning of the inequality symbols $<$ (less than) and $>$ (greater than).

Practice Comparing Positive and Negative Numbers

Problem Notes

Assign **Practice Comparing Positive and Negative Numbers** as extra practice in class or as homework.

- 1 a. *Basic*
 - b. Students may recognize that they could also write the inequality as $-60 < -40$. *Medium*
- 2 Students may recognize that -25 is less than -15 because it is farther left on the horizontal number line. *Medium*

Practice Comparing Positive and Negative Numbers

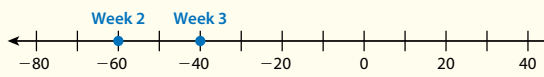
► Study the Example showing how to compare two negative numbers. Then solve problems 1–5.

Example

The table shows the amount of money Savanna either withdraws (–) or deposits (+) into her bank account over 5 weeks. Write an inequality to compare the withdrawals for Week 2 and Week 3.

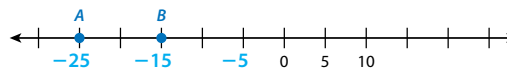
Week	Week 1	Week 2	Week 3	Week 4	Week 5
Amount	+\$40	–\$40	–\$60	+\$100	–\$80

Plot the amounts on a number line.



-60 is to the left of -40 . So, $-60 < -40$.

- 1 a. Compare the two amounts in the Example using the symbol $>$.
 $-40 > -60$
 - b. Does using $>$ for the inequality change which amount represents withdrawing more money? Explain.
No; Possible explanation: You can compare the amounts using $<$ or $>$, but the amount that is farther to the left on the number line does not change. -60 is farther from 0 than -40 , so it represents withdrawing more money.
- 2 Write an inequality that compares the value of point A and the value of point B. Show your work. **Possible work:** See number line.



SOLUTION $-25 < -15$ (or $-15 > -25$)

Vocabulary

inequality

a mathematical statement that uses an inequality symbol to show the relationship between values of expressions.

Fluency & Skills Practice

Comparing Positive and Negative Numbers

In this activity, students compare positive and negative numbers. They also order positive and negative integers, fractions, and decimals, from least to greatest.

Comparing Positive and Negative Numbers

► Write $<$ or $>$ to make each comparison true.

1 $7 \bigcirc 10$ 2 $7 \bigcirc -10$ 3 $-7 \bigcirc -10$

4 $\frac{2}{3} \bigcirc -\frac{1}{2}$ 5 $-50 \bigcirc 0.3$ 6 $-12 \bigcirc -35$

7 $-5 \bigcirc 4.5$ 8 $\frac{1}{2} \bigcirc -80$ 9 $-\frac{1}{4} \bigcirc -1.4$

► Write each set of numbers in order from least to greatest.

10 $5, -2, -1.4$ 11 $3.4, 7, -3.5, -3$ 12 $-2.1, -2, -3, 0$

13 $-\frac{3}{4}, -2, -\frac{1}{2}, 2$ 14 $5, 0, -6, -0.1$ 15 $7.5, -200, -1.5, -8$

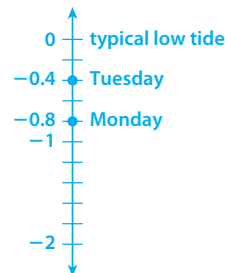
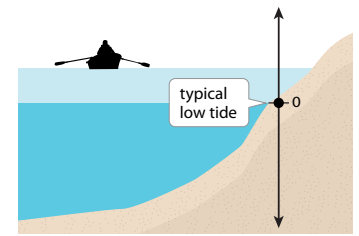
16 $\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}$ 17 $1.2, -2.1, -21, 0.12$ 18 $0.1, -0.2, 0.55, -0.31$

19 Describe how to determine which of two negative numbers is greater. Give an example.

- 3 a. Students may also use a horizontal number line. **Medium**
- b. Students may recognize that Tuesday has the higher low tide because -0.4 is greater than -0.8 . **Medium**
- 4 Students should recognize that the number to the left of the less than symbol ($<$) is the lesser number and will be located farther left on a horizontal number line. **Basic**
- 5 Students may recognize that a negative score will always be less than a positive score. **Challenge**

LESSON 24 | SESSION 2

- 3 The typical level of a low tide at a beach is the 0 point on a number line. Each day's high and low tides are measured relative to the typical low tide. On Monday morning, low tide is at -0.8 ft. On Tuesday morning, low tide is at -0.4 ft.
- a. Write an inequality to compare the low tides on Monday and Tuesday mornings. Show your work. **Possible work:**



SOLUTION $-0.8 < -0.4$ (or $-0.4 > -0.8$)

- b. Which day has a higher low tide? Explain.
Tuesday; Possible explanation: The value for Tuesday is above the value for Monday on a vertical number line.
- 4 Consider the inequality $-3 < -2\frac{1}{2}$. What does the inequality tell you about the location of -3 compared to the location of $-2\frac{1}{2}$ on a horizontal number line? Use *to the right* and *to the left* in your answer.
 -3 lies to the left of $-2\frac{1}{2}$ and $-2\frac{1}{2}$ lies to the right of -3 .
- 5 In golf, the winner is the person with the lowest score. At the end of a round of golf, Jada's score is positive. Isabel's score is negative. Can you determine who wins? If so, tell who wins and why. If not, explain why not.
Yes; Isabel wins; Possible explanation: Isabel's score is negative, and Jada's score is positive. That means that Isabel's score is to the left of Jada's score on a number line. So, Isabel's score is less than Jada's score.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3 Apply It**

Levels 1–3: Reading/Listening

Help students make sense of Apply It problem 2. Display a picture of a vending machine. Read the problem. Emphasize the idea that the cafeteria worker counts and records how the number of sandwiches changes from the start to the end of each day. Explain that each number on the table shows that change. Have students take turns stating the daily change:

- On [day], there were ____ fewer/more sandwiches at the end of the day.

Next, have students compare the numbers for Monday and Thursday and tell which number is greater. Invite others to tell if they agree or disagree with the answers.

Levels 2–4: Reading/Speaking

Help students make sense of Apply It problem 2. Have students read the problem with partners and **Say It Another Way**. Monitor and provide support as students explain what the cafeteria worker counts and records each day.

Then have them take turns explaining rows from the table. Ask: *Which days had less sandwiches at the end? Which day had a greater change, Monday or Thursday?* Invite other students to comment on the answers:

- I agree because ____.
- I disagree because ____.

Levels 3–5: Reading/Speaking

Help students make sense of Apply It problem 2. First, adapt **Notice and Wonder** by having partners work together to examine the table and write at least three things they notice and wonder. Then have them read the problem together and talk about what they need to find.

Allow time for students to work on the problem individually. Then have them share their answers with their partner. Invite listeners who agree to add details or explain how they got the same answer. For students who disagree, ask them to explain why they have a different answer.

Refine Ordering Positive and Negative Numbers

Purpose

- **Refine** strategies for comparing and ordering positive and negative numbers.
- **Refine** understanding of using a number line to order and compare rational numbers.

START CHECK FOR UNDERSTANDING

List the following numbers from least to greatest:

1.5, 25, -12, -8, 5, -30, $-\frac{1}{4}$

Solution

$-30, -12,$
 $-8, -\frac{1}{4}, 1.5,$
 $5, 25$

WHY? Confirm students' understanding of ordering positive and negative numbers, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

Refine Ordering Positive and Negative Numbers

► Complete the Example below. Then solve problems 1–9.

Example

Order the following rational numbers from least to greatest.

$\frac{1}{4}, -1.25, -\frac{3}{4}, 0.5, 1, -\frac{3}{2}$

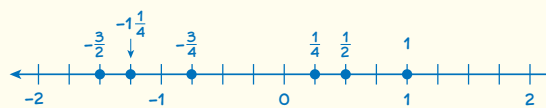
Look at how you could use a number line to order rational numbers.

Write the decimals as fractions.

$$-1.25 = -1\frac{1}{4}$$

$$0.5 = \frac{1}{2}$$

Plot the numbers on a number line.



SOLUTION $-\frac{3}{2}, -1.25, -\frac{3}{4}, \frac{1}{4}, 0.5, 1$

CONSIDER THIS ...

You can write all the rational numbers as fractions or write them all as decimals.

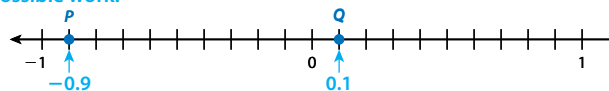
PAIR/SHARE

How would the order change if you changed $-\frac{3}{2}$ to $\frac{3}{2}$?

Apply It

- Write two inequalities that compare the value of point *P* and the value of point *Q*. Show your work.

Possible work:



SOLUTION $-0.9 < 0.1; 0.1 > -0.9$

CONSIDER THIS ...

What does each tick mark on the number line represent?

PAIR/SHARE

What inequality can you write to compare the value of point *P* to -1 ?

START ERROR ANALYSIS

If the error is ...	Students may ...	To support understanding ...
$\frac{1}{4}$, 1.5, 5, -8, -12, 25, -30	have compared the magnitude of the numbers.	Ask students to write an inequality comparing -30 and 25 . Ask students to consider how the negative sign might affect which number is greater.
$\frac{1}{4}$, -8, -12, -30, 1.5, 5, 25	have confused values of negative numbers and thought, for example, that -8 is less than -12 because 8 is less than 12 .	Ask students to draw a number line and plot all the numbers. Prompt them to articulate which direction on the number line shows greater or lesser numbers. Prompt them to recognize that numbers farther left on a horizontal number line are less than numbers to the right.
25, 5, 1.5, $-\frac{1}{4}$, -8, -12, -30	have ordered the numbers from greatest to least.	Prompt students to review the terms <i>greatest</i> and <i>least</i> by pointing out that <i>greater</i> means <i>more than</i> and <i>least</i> means <i>less than</i> .

Example

Guide students in understanding the Example. Ask:

- *On a number line, what direction are the greater numbers? What direction are the lesser numbers?*
- *How does a number line help you solve this problem? How else might you solve it?*

Help all students focus on the Example and responses to the questions by prompting students to check their understanding by trying to paraphrase the speaker's ideas. Remind them that they can ask questions about ideas that are unclear.

Look for understanding that positive and negative numbers can be ordered using a number line.

Apply It

- 1 Students should recognize that each tick mark on the number line represents 0.1, so P has a value of -0.9 and Q has a value of 0.1 . **DOK 1**
- 2 See **Connect to Culture** to support student engagement. Students may use a number line to compare the change on Monday and the change on Thursday. **DOK 2**
- 3 **C is correct.** Students may solve the problem by plotting all of the distances on a number line and then writing inequalities to compare them.
 - A** is not correct. This answer uses a less than symbol ($<$) rather than a greater than symbol ($>$) when comparing -4 and -8 .
 - B** is not correct. This answer switches the less than symbol ($<$) and greater than ($>$) symbol.
 - D** is not correct. This answer uses the great than symbol ($>$) rather than less than symbol ($<$) when comparing -8 and -6 .

DOK 3

LESSON 24 | SESSION 3

- 2 A vending machine in a cafeteria sells sandwiches. The machine is restocked once during the day. At the end of each day, a cafeteria worker records how many more (+) or fewer (–) sandwiches are in the machine than there were at the start of the day. The table shows the changes for one week.

Day	Change in Number of Sandwiches
Monday	–3
Tuesday	+4
Wednesday	–5
Thursday	–2
Friday	0

Write an inequality to compare the changes for Monday and Thursday. Tell what your inequality means in terms of the situation. Show your work.

Possible work:

-3 is farther to the left of 0 on a horizontal number line than -2 is, so -3 is less than -2 .

SOLUTION $-3 < -2$; The number of sandwiches in the machine is farther below the starting amount on Monday than on Thursday.

- 3 An elevation of -4 m is higher than an elevation of -8 m. An elevation of -8 m is lower than an elevation of -6 m. Which set of inequalities correctly expresses these relationships?
 - A** $-4 < -8$ and $-8 < -6$
 - B** $-4 < -8$ and $-8 > -6$
 - C** $-4 > -8$ and $-8 < -6$
 - D** $-4 > -8$ and $-8 > -6$

Anders chose B as the correct answer. How might he have gotten that answer?

Possible answer: Anders ignored the negative signs when he wrote the inequalities. He used the inequalities $4 < 8$ and $8 > 6$.

CONSIDER THIS . . .

The number of sandwiches at the end of a day is the result of some sandwiches being sold and the machine being restocked with more sandwiches.

PAIR/SHARE

What does the 0 in the row for Friday mean in this situation?

CONSIDER THIS . . .

How can you plot the elevations on a vertical number line or a horizontal number line to help write the inequalities?

PAIR/SHARE

What is a different way you can write the correct pair of inequalities?

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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 7

Meeting Proficiency

- **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Resources for Differentiation are found on the next page.

Refine Ordering Positive and Negative Numbers

Apply It

- 4 Students may write inequalities comparing the elevation of Lake Eyre and the elevation of each location in the U.S. **DOK 2**
- 5 Students may write the inequality using words to recognize that if you change the sign of both integers in an inequality, the inequality symbol also changes. **DOK 3**
- 6
 - a. Point *K* has a value of -2 , which is less than -1 .
 - b. Point *B* is farther to the right than point *W*.
 - c. Point *S* is a negative number.
 - d. Point *W* is farther to the left of -0.5 on the number line.

DOK 2

- 4 Sea level has an elevation of 0 ft. Lake Eyre is the lowest point in Australia. It has an elevation of -15 m relative to sea level. Which of the following U.S. locations, if any, have a lower elevation than Lake Eyre? Explain.

Location	Elevation (m)
Death Valley, California	-86
New Orleans, Louisiana	-2.4
Imperial, California	-18
Ouachita River, Arkansas	17

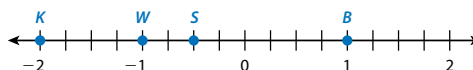


Lake Eyre, Australia

Death Valley and Imperial, California; Possible explanation: On a vertical number line, -86 and -18 are below -15 , so these two values represent lower elevations than the elevation of Lake Eyre.

- 5 Doug says that $-7 > -5$ because $7 > 5$. Do you agree? Explain.
No; Possible explanation: -7 is 7 units to the left of 0 and -5 is 5 units to the left of 0. This means that -7 is the lesser value.

- 6 Tell whether each statement about the points on the number line is *True* or *False*.



	True	False
a. The value of point <i>K</i> is greater than -1 .	<input type="radio"/>	<input checked="" type="radio"/>
b. The value of point <i>B</i> is greater than the value of point <i>W</i> .	<input checked="" type="radio"/>	<input type="radio"/>
c. The value of point <i>S</i> is less than 1.	<input checked="" type="radio"/>	<input type="radio"/>
d. The value of point <i>W</i> is less than -0.5 .	<input checked="" type="radio"/>	<input type="radio"/>

DIFFERENTIATION

RETEACH



Hands-On Activity

Play a game to determine the greater number.

Students approaching proficiency with ordering positive and negative numbers will benefit from this game using inequalities.

Materials For each pair: 20 index cards

- Pair students and give each pair 20 index cards. Have students write one positive or negative rational number on each of the cards, such as -7 , 3.5 , or $-\frac{1}{3}$. Prompt students to include integers, fractions, and decimals.
- Have each student to draw a card from the deck.
- Ask: *Which of you has the greater number?*
- The student with the greater number gets a point. Have students place the selected cards in a different pile.
- Have pairs repeat until one students reaches 10 points. Have students shuffle the cards as needed.

7 Students may also convert all of the numbers to fractions before drawing the number line.
DOK 2

8 **B, C, D, and E are correct.** Students may solve the problem by plotting all of the numbers on a number line and choosing the numbers that are between -5 and 2 .

A is not correct. This answer is less than 2 but not greater than -5 .

F is not correct. This answer is greater than -5 but greater than 2 instead of less than 2 .

DOK 2

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that positive and negative numbers can be ordered using a vertical number line and the number line can be used to write inequalities.

Error Alert If students write the inequality $-0.5 < -1.5$, then prompt them to draw a vertical number line and plot both points to see that -1.5 is below -0.5 .

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they turn and talk to a partner to discuss the definition of *interpret*.

SELF CHECK Have students review and check off any new skills on the Unit 6 Opener.

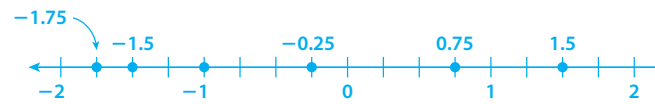
LESSON 24 | SESSION 3

7 Order the following rational numbers from least to greatest. Show your work.

$-1.5, \frac{3}{4}, -\frac{1}{4}, -1.75, -1, 1.5$

Possible work:

$\frac{3}{4} = 0.75$ $-\frac{1}{4} = -0.25$



SOLUTION $-1.75, -1.5, -1, -\frac{1}{4}, \frac{3}{4}, 1.5$

8 Lilia wants to replace both question marks with the same number so that the inequalities correctly compare the numbers.

$? > -5$ and $? < 2$

Which of these numbers could Lilia use? Select all that apply.

- A** -7
- B** -4
- C** -2
- D** 0
- E** 1
- F** 5

9 **Math Journal** Choose two of the rational numbers shown below. Write two inequalities to compare the numbers, using $<$ and $>$. Then describe the location of one number compared to the other on a vertical number line. Use *above* and/or *below* in the description.

$-\frac{3}{4}, -1.5, \frac{1}{4}, -1\frac{1}{4}, -0.5$

Possible answer: $-\frac{3}{4}$ and -1.5 ; $-\frac{3}{4} > -1.5$ and $-1.5 < -\frac{3}{4}$

On a vertical number line, $-\frac{3}{4}$ lies above -1.5 ; -1.5 lies below $-\frac{3}{4}$.

End of Lesson Checklist

- INTERACTIVE GLOSSARY** Write a new entry for *interpret*. Write at least one synonym for *interpret*.
- SELF CHECK** Go back to the Unit 6 Opener and see what you can check off.

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REINFORCE



Problems 4–8
Order positive and negative numbers.

Students meeting proficiency will benefit from additional work with ordering positive and negative numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND



Challenge
Order positive and negative numbers in the real world.

Students extending beyond proficiency will benefit from exploring real-world applications of ordering positive and negative numbers.

- Group or pair students. Ask: *What real-world situations might use ordering of positive and negative numbers? What would those numbers represent?* [Possible answer: Measure the increase or decrease of the speed of a car; positive numbers represent an increase in speed and negative numbers a decrease in speed.]
- Have one student to suggest possible positive and negative numbers to compare, and have the other student(s) order the numbers and articulate what they could represent.

PERSONALIZE



Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Understand Absolute Value

? **UNDERSTAND:** What is the absolute value of a number?

MATH FOCUS

This Understand lesson extends students' understanding of ordering rational numbers to finding absolute value. Foundational understanding established in this lesson supports students in developing strategies to solve and graph inequalities.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 2, 3, and 7 are integrated into the Understand lesson structure.*

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Understand absolute value as a distance from 0 on the number line.
- Interpret absolute value as a magnitude in a real-world situation.
- Make comparisons between values regarding absolute value.

Language Objectives

- Define and explain absolute value by using lesson vocabulary and the words *never*, *always*, *increase*, *decrease*, *farther*, and *closer* in speaking and writing.
- Use antonyms or near antonyms, such as *above*, *below*, *have*, *owe*, *gained*, and *lost*, to interpret absolute value in real-world situations.
- Use a number line and phrases such as *distance from 0*, *to the left*, *to the right*, *greater than*, and *less than* to explain how ordering absolute values compares to ordering rational numbers.

Prior Knowledge

- Locate rational numbers on the number line.
- Order and compare rational numbers.

Vocabulary

Math Vocabulary

absolute value a number's distance from 0 on the number line. Absolute value is never negative.

Review the following key terms.

inequality a mathematical statement that uses an inequality symbol ($<$, $>$, \leq , \geq) to show the relationship between values of expressions.

negative numbers numbers that are less than 0. They are located to the left of 0 on a horizontal number line and below 0 on a vertical number line.

opposite numbers numbers that are the same distance from 0 on the number line but in opposite directions. Opposite numbers have the same numeral but opposite signs.

positive numbers numbers that are greater than 0. They are located to the right of 0 on a horizontal number line and above 0 on a vertical number line.

Academic Vocabulary

notation system of symbols or characters used to represent information.

Learning Progression

In Grade 5, students located and labeled positive whole numbers, fractions, and decimals on the number line and compared them using inequality symbols and words. They ordered positive rational numbers.


Earlier in Grade 6, students compared and ordered rational numbers. They used inequalities to compare numerical values. They also described the value of positive and negative rational numbers in relation to 0.

In this lesson, students understand that absolute value is a distance from 0 on the number line and that opposites always have the same absolute value. They interpret absolute value as the magnitude of a positive or negative real-world quantity. They distinguish between comparing two numbers or real-world quantities and comparing their absolute values.

Later in Grade 6, students will apply the concept of absolute value as they graph ordered pairs in the four-quadrant coordinate plane and find the distance between coordinates on the points on a horizontal or vertical line.

In Grade 7, students will use absolute value concepts as they learn to add and subtract with negative numbers and they will see that the distance between two rational numbers on a number line is the absolute value of their difference.

Pacing Guide

Items marked with  are available on the **Teacher Toolbox**.

MATERIALS


DIFFERENTIATION

SESSION 1 Explore Absolute Value (35–50 min)


- **Start** (5 min)
- **Model It** (5 min)
- **Discuss It** (5–10 min)
- **Model It** (5–10 min)
- **Discuss It** (10–15 min)
- **Close: Exit Ticket** (5 min)

Additional Practice (pages 573–574)

Presentation Slides 

PREPARE Interactive Tutorial 

RETEACH or REINFORCE Visual Model

Materials For each student: Activity Sheet
Number Lines 


SESSION 2 Develop Understanding of Absolute Value (45–60 min)


- **Start** (5 min)
- **Model It: Compare Absolute Values** (5 min)
- **Discuss It** (5–10 min)
- **Model It: Interpret Absolute Value** (5 min)
- **Discuss It** (10–15 min)
- **Connect It** (10–15 min)
- **Close: Exit Ticket** (5 min)

Additional Practice (pages 577–578)

Presentation Slides 


RETEACH or REINFORCE Hands-On Activity

Materials For each pair: 2 two-color counters,
Activity Sheet *Number Lines* 


REINFORCE Fluency & Skills Practice 


SESSION 3 Refine Ideas About Absolute Value (45–60 min)


- **Start** (5 min)
- **Apply It** (35–50 min)
- **Close: Exit Ticket** (5 min)

Presentation Slides 

Lesson 25 Quiz  or
Digital Comprehension Check

RETEACH Tools for Instruction 

REINFORCE Math Center Activity 

EXTEND Enrichment Activity 

Connect to Culture

- Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 ■ □ □

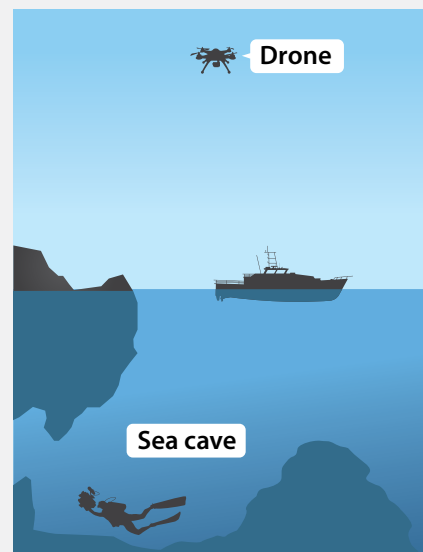
Model It Problem 1 Underwater drones, called ROVs (Remotely Operated Vehicles), are robotic systems that can explore the depths of rivers, lakes, and oceans from the shore or by divers in the water. These drones are able to navigate under water using propellers and can record and transmit both video and high-definition photos to onshore computers. In recent years, drones have been developed that are capable of diving under water, floating on the water's surface, and flying through the air. These types of drones can be used to help with search and rescue operations as well as with underwater engineering inspections. Beaches can be monitored for erosion, oil spills, and other types of pollution. They can also be used to explore both air and water conditions in extreme weather.

SESSION 2 ■ ■ □

Model It Problem 4 The first video game was developed in 1958 by a physicist. Ask students about their favorite video game. Elicit a discussion about what they think makes video games fun. One of the common responses for why people enjoy playing video games is the challenge of winning, which is often measured in points. Achieving the highest number of points in a video game can be a personal goal that is rewarding. Video games can seem simple to play, but creating and designing them is often challenging. Hours of research and work goes into creating fun and exciting video games that are frequently played today.

SESSION 3 ■ ■ ■

Model It Problem 4 SCUBA stands for *Self-Contained Underwater Breathing Apparatus*. The first aqualung was invented in 1943 by Jacques Costeau. Since then, technology has continued to improve so that people can spend more time at deeper elevations to explore under water. The record for the deepest dive was more than 1,090 feet! At this elevation, it takes only 15 minutes to descend but about 15 hours to ascend. The longest open saltwater dive was just under six days! The diver's team brought down food and water and played football and checkers with him to pass the time.



Connect to Family and Community

- After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.

LESSON
25

Understand Absolute Value

Dear Family,

This week your student is learning about absolute value. The **absolute value** of a number is its distance from 0 on the number line.

The symbol $|5|$ is read as the *absolute value of 5*.

$|5| = 5$ because the distance from 5 to 0 is 5 units.
 $|-5| = 5$ because the distance from -5 to 0 is 5 units. } This means $|5| = |-5|$.

Your student will be comparing absolute values such as the ones below.

Use $<$, $>$, or $=$ to compare the absolute values of -6 and -4 .

► **ONE WAY** to compare absolute values is by using a number line.

It is a greater distance from -6 to 0 than it is from -4 to 0.
 $|-6| > |-4|$

► **ANOTHER WAY** is to interpret the absolute values in a real-world situation. Think of -6 and -4 as representing debts of \$6 and \$4. Since $|-6| = 6$ and $|-4| = 4$, the absolute values of -6 and -4 represent the amounts owed, \$6 and \$4. A person with a debt of \$6 owes more money than a person with a debt of \$4.
 $|-6| > |-4|$
 Using either model, you can see that $|-6| > |-4|$.

▶ Use the next page to start a conversation about absolute value.

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LESSON 25 | UNDERSTAND ABSOLUTE VALUE

Activity Thinking About Absolute Value Around You

► Do this activity together to investigate absolute value in the real world.

Bocce is a game where players throw bocce balls to get as close to a smaller target ball as possible. It does not matter if your bocce ball rolls past the target ball or if it stops before. Either way, you measure the distance from your bocce ball to the target ball.

In this way, playing bocce is like using absolute value. You can think about the target ball as the 0 point on a number line. It does not matter if your bocce ball ends up past the target ball at $+5$ or in front of the target ball at -5 , because the distance from the target ball is still 5.

Where else do you see absolute value in the world around you?

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Connect to Language

- For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1 Discuss It**

Levels 1–3: Speaking/Listening

Use a **Co-Constructed Word Bank** to help students respond to the question in the second Discuss It. Have students turn to partners to discuss what they did or read about in Model It. Then help them suggest words they can use to talk about absolute value. Add relevant words to the word bank, including *opposite*, *positive*, *negative*, *zero*, and *distance*. Have partners use sketches, gestures, and everyday language to show an example of each term.

Read the Discuss It question. Reword by asking: *How can you explain absolute value using the word distance?* Provide a sentence starter:

- The absolute value of a number is _____.

Levels 2–4: Speaking/Listening

Use a **Co-Constructed Word Bank** to help students respond to the question in the second Discuss It. Have students turn to partners to paraphrase Model It. Then have them suggest words they can use to talk about absolute value. Add relevant words to the word bank if needed.

Read the Discuss It question with students. Have students circle *related*. Ask students to read the question again and call on volunteers to **Say It Another Way** to explain what the question is asking without using the term *related*. Students might use other academic terms, such as *connected* or *relationship*. Work with students to write a sentence starter to support their responses.

Levels 3–5: Speaking/Listening

Use a **Co-Constructed Word Bank** to help students respond to the question in the second Discuss It. Have students turn to partners to paraphrase Model It. Then have them suggest words they can use to talk about absolute value. Add relevant words to the word bank.

Have students read the question with a partner and adapt **Say It Another Way** by having students write down their paraphrase before sharing it with their partner. Have partners work together to write a sentence starter to support their responses.

Explore Absolute Value

Purpose

- **Explore** the idea that absolute value tells the distance a number is from 0 on a number line.
- **Understand** that distance from 0 on a number line is always positive.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

- A A negative number is greater than a positive number.
- B A positive number is greater than a negative number.
- C A positive number is greater than a positive number.

Possible Solutions

A is never true.

B is always true.

C is sometimes true.

WHY? Prepare students to explore distance from zero by accessing prior knowledge of comparing positive and negative numbers.

MODEL IT

SMP 2

Read the *Understand* question at the top of the page. Remind students that they know how to identify positive numbers and negative numbers on a number line.

1 – 3 See **Connect to Culture** to support student engagement. Tell students that they are going to use what they know about positive and negative numbers to think about distance from 0 on the number line. After volunteers read the problems, encourage students to ask questions such as, *Can you explain more about ____?* or *What does ____ mean?* if they need to hear additional information.

Common Misconception If students do not believe that two different numbers can be the same distance from 0, then have them use a ruler to visualize the concept. Prompt students to draw a horizontal number line, labeling 0 near the middle of the line. Then have students visualize the distances $|-3|$ and $|3|$ by marking and labeling -3 at three centimeters to the left of 0 and 3 at three centimeters to the right of 0.

Explore Absolute Value

UNDERSTAND: What is the absolute value of a number?

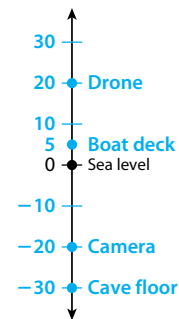
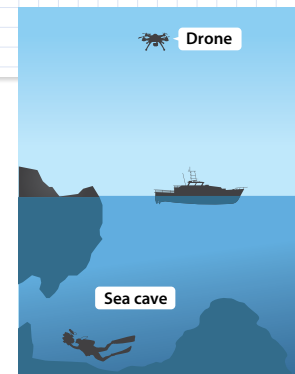
Model It

► Complete the problems about distance from 0.

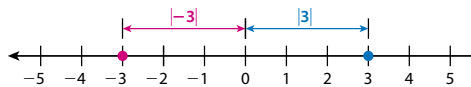
- 1 A scientist standing on the deck of a boat uses a drone, and a scuba diver uses a camera to explore a sea cave. The table shows the elevations of four objects relative to sea level.

Object	Camera	Cave floor	Drone	Boat deck
Elevation	−20 ft	−30 ft	20 ft	5 ft

- a. Use the number line to show the elevations of the objects from the table. Label each object at its elevation. **See number line.**
- b. Are any of the objects the same distance from sea level? If so, how far from sea level are they?
The camera and drone are both 20 ft from sea level.
- c. Another object is 3 ft from sea level. Is the object's elevation *positive*, *negative*, or could it be *either*? Explain.
Either; if the object is 3 ft above sea level, its elevation is 3 ft. If the object is 3 ft below sea level, its elevation is −3 ft.



- 2 The **absolute value** of a number is its distance from 0. The notation $|-3|$ is read as *the absolute value of −3* and represents the distance of -3 from 0.



$|3| = 3$ because the distance from 0 to 3 is 3 units.
 $|-3| = 3$ because the distance from 0 to -3 is 3 units.

- Learning Targets** SMP 2, SMP 3, SMP 7
- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
 - Distinguish comparisons of absolute value from statements about order.

DISCUSS IT

Ask: How is absolute value related to zero on the number line?
Share: I think two numbers will have the same absolute value when ...

DISCUSS IT

SMP 3, 7

Support Partner Discussion

After students complete problems 1 and 2, have them respond to Discuss It with a partner.

Listen for understanding that:

- a negative number and a positive number could be the same distance from 0.
- the distance from 0 can be found by using the distance above or below 0 on a vertical number line or the distance to the left or right from 0 on a horizontal number line. Opposites have the same absolute value.

Facilitate Whole Class Discussion

Prompt students to **Compare and Connect** thinking about absolute value on a vertical number line and a horizontal number line.

ASK How are the distances from sea level to the camera and the drone similar to the distances from 0 to -3 and 3? How are these distances different?

LISTEN FOR In each case, the distances are in different directions from 0 on the number line. The camera and drone are the same distance from 0, and -3 and 3 are both the same distance from 0.

MODEL IT

SMP 3, 7

- 3 Call on students to give reasons to explain why their answers for part b are reasonable.
- 4 Prompt students to compare the patterns in the values of the numbers and the absolute values of the numbers for this problem and for problem 3. Allow extra think time as they reason through how the values change.
- 5 To engage all students, ask them to turn and talk to partners about the generalizations presented in parts a and b.

Common Misconception If students believe that numbers that are farther away from 0 on the left side of the number line have a lesser absolute value, then prompt them to complete this sentence: *The farther a number is from 0, the _____ the number's distance from 0.*

DISCUSS IT

Support Partner Discussion

After students complete problem 5, have them respond to Discuss It with a partner.

Listen for understanding of:

- absolute value is a measure of the distance from 0.
- the distance between 0 and 0 is 0.

DIFFERENTIATION | RETEACH or REINFORCE



Visual Model
Model Absolute Value

If students are unsure about absolute values of negative numbers, then use this activity to help demonstrate measuring distance in a negative direction.

Materials For each student: Activity Sheet Number Lines

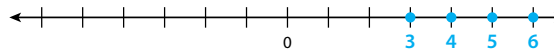
- Have students label a number line from -5 to 5 and plot points at -4 and 4.
- Have students use jumps from 0 to each point plotted. Ask: *How many jumps did it take to move from 0 to 4? From 0 to -4? [4; 4] What is the distance from 0 to -4? Write a numerical equation showing the absolute value of -4. [4; |-4| = 4]*
- Repeat with other number lines and opposites.

LESSON 25 | SESSION 1

Model It

Complete the problems about absolute value.

- 3 a. Plot and label the numbers 3, 4, 5, and 6 on the number line. Do the values of the numbers *increase* or *decrease* as the numbers go from 3 to 6?



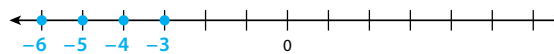
The values increase.

- b. Write the absolute value of each number. Do the absolute values of the numbers *increase* or *decrease* the numbers go from 3 to 6?

$|3| = 3$ $|4| = 4$ $|5| = 5$ $|6| = 6$

The absolute values increase.

- 4 a. Plot and label the numbers -3, -4, -5, and -6 on the number line. Do the values of the numbers *increase* or *decrease* as the numbers go from -3 to -6?



The values decrease.

- b. Write the absolute value of each number. Do the absolute values of the numbers *increase* or *decrease* as the numbers go from -3 to -6?

$|-3| = 3$ $|-4| = 4$ $|-5| = 5$ $|-6| = 6$

The absolute values increase.

- 5 Write *lesser* or *greater* to complete each statement.
 - a. The farther a number is from 0, the greater the number's absolute value.
 - b. The closer a number is to 0, the lesser the number's absolute value.

- 6 **Reflect** Is the absolute value of a number ever negative? Explain your reasoning.

No; Distance is always greater than or equal to 0, so absolute value is always positive.

DISCUSS IT

Ask: How are distance and absolute value related?

Share: I think the absolute value of 0 is ... because ...

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Facilitate Whole Class Discussion

Prompt students to **Compare and Connect** the number line that shows absolute values of positive numbers and the number line that shows absolute values of negative numbers.

ASK How are the number lines and absolute values similar? How are they different?

LISTEN FOR Both number lines show the same absolute values. One shows absolute values measured to the right of 0. The other shows absolute values measured to the left of 0.

CLOSE EXIT TICKET

- 6 **Reflect** Look for understanding of absolute value never being negative because distance from 0 on a number line is always positive.

Common Misconception If students try to reason that absolute value can be positive or negative, then remind them that absolute value measures distance, not direction. It does not matter if you are moving four spaces left, right, up or down on a horizontal or vertical number line, the distance remains 4.

Prepare for Understanding Absolute Value

Support Vocabulary Development

Assign **Prepare for Understanding Absolute Value** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *opposite numbers* by discussing what the word *opposite* means and how the term can be applied to values and numbers.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the words, illustrations, examples, and non-examples given.

Have students look at the number line in problem 2 and discuss possible strategies for locating opposite numbers on a number line.

Problem Notes

- Students should understand that opposite numbers have the same distance from 0 on the number line but are on opposite sides of 0. Student responses may include a number line, examples (such as 3 and -3 or -1.3 and $+1.3$), and non-examples (such as -2 and 4).
- Students may recognize that since *J* and *Y* are the only pair of points that appear to be equidistant from 0 and on opposite sides of 0, they are opposite numbers.

Prepare for Understanding Absolute Value

- Think about what you know about opposite numbers. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

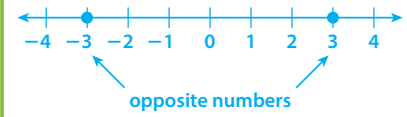
Possible answers:

In My Own Words

Opposite numbers are the same distance from 0 on the number line, but on opposite sides of 0.

Opposite numbers have opposite signs.

My Illustrations



opposite numbers

Examples

3 and -3 are opposite numbers.

$-\frac{1}{2}$ is the opposite of $\frac{1}{2}$.

The opposite of -1.3 is $+1.3$.

Non-Examples

-2 and 4 are not opposite numbers, even though one is negative and one is positive.

$\frac{2}{3}$ and $\frac{3}{2}$ are not opposite numbers.

- Look at the number line. Which pair of points appears to show a pair of opposite numbers? Explain your reasoning.

Points *J* and *Y*; The points appear to be the same distance from 0 and in opposite directions from 0.



REAL-WORLD CONNECTION

When equipment is designed by a product development engineer, the concept of absolute value is applied to the measurements to ensure the pieces meet safety standards. A metal piece for an engine of a helicopter must be the right size or it might not function properly. To prevent malfunction, pieces are measured using precision guidelines. For example, a piece that is expected to be 120 mm long could still be considered precise if the absolute value of the difference between the actual length and 120 mm is less than 0.02 mm. If the length is greater than 120.02 mm or less than 119.98 mm, the piece cannot be used. Ask students to think of other real-world examples involving measurements that must meet specific precision requirements.



- 3 a. Student models should show appropriate spacing and labeling of tick marks and objects with the two positive values appearing above 0 and the two negative values appearing below 0.
- b. Student models should show that the airplane and submarine are each 10 units from 0.
- 4 a. The absolute value of 40 is 40.
- b. The absolute value of -40 is 40. Prompt students to discuss why the absolute value of 40 and -40 are the same and both positive values.
- 5 a. Student models should show appropriate labels and each tick mark representing more than 1 unit. Student answers should show understanding of the values of the numbers decreasing from -2 to -8 .
- b. Student answers should show positive absolute values for each equation and an understanding that the absolute values of the numbers increase from $|-2|$ to $|-8|$.

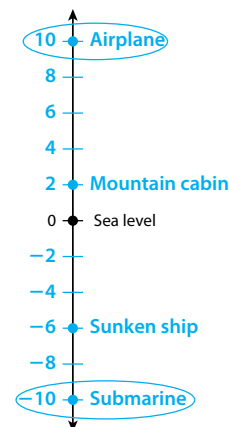
LESSON 25 | SESSION 1

Complete problems 3–5.

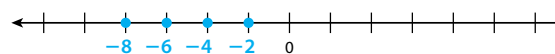
- 3 The table shows the elevations of four objects relative to sea level.

Object	Elevation (km)
Mountain cabin	2
Submarine	-10
Sunken ship	-6
Airplane	10

- a. Use the number line to show the elevations of the four objects. Label each object at its elevation.
See number line.
- b. Circle the two objects on your number line that are the same distance from 0. See number line.



- 4 The notation $|40|$ means the absolute value of 40.
- a. $|40| = 40$ because the distance from 0 to 40 is 40.
- b. $|-40| = 40$ because the distance from 0 to -40 is 40.
- 5 a. Plot and label the numbers -2 , -4 , -6 , and -8 on the number line. Do the values of the numbers increase or decrease as the numbers go from -2 to -8 ?



The values decrease.

- b. Find the absolute value of each number. Do the absolute values of the numbers increase or decrease as the numbers go from -2 to -8 ?

$|-2| = 2$ $|-4| = 4$ $|-6| = 6$ $|-8| = 8$

The absolute values increase.

Vocabulary
absolute value
a number's distance from 0 on the number line. Absolute value is never negative.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with Session 2 Model It

ACADEMIC VOCABULARY

Magnitude is the size, extent, or importance of something.

Levels 1–3: Reading/Speaking

Support students as they interpret Model It problem 3. Define and discuss the term *magnitude*. Point out the Spanish cognate *magnitud*, if applicable.

Use **Act It Out** to have volunteers role play the contexts of owing and earning \$20. Guide students to understand that although the amount is the same, the magnitude of the amount is different for each volunteer. Next, have students work in pairs to identify and underline the words that express magnitude in problems 3a–c.

Levels 2–4: Reading/Speaking

Support students as they interpret Model It problem 3. Display the Academic Vocabulary and have students find the word in the problem and replace it with a synonym. Then read the example. Ask students to identify the quantities and the words that explain what the quantities mean in a real-world situation.

Next, have students identify the words and phrases in problems 3a–c that express the magnitude of the given situation. [*owe, win, lose*] Have students turn and talk to a partner about what each word or phrase means in the situation.

Levels 3–5: Reading/Speaking

Support students as they interpret Model It problem 3. Display the Academic Vocabulary and have students find the word in the problem. Call on volunteers to **Say It Another Way**.

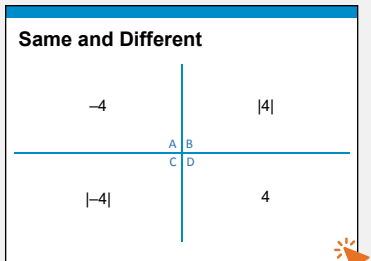
Next, have students read problems 3a–c with a partner. Ask students to respond individually using words and phrases from the problem that express the magnitude of a negative number as an absolute value. Then have partners work together to come up with other words and phrases that express absolute value in real-world situations.

Develop Understanding of Absolute Value

Purpose

- **Develop** the idea that comparing two numbers is different from comparing their absolute values.
- **Understand** how absolute values describe the size, or magnitude, of a quantity.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All involve the numeral 4.

A is the only one equal to -4.

A and C are values of negative 4.

B and D are values of positive 4.

WHY? Support students' facility with evaluating and comparing absolute value expressions.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of *magnitude*.

HOW? Explain that *magnitude* is the size, extent, or importance of something. Have students use prior knowledge or a thesaurus to list synonyms for the word. (*intensity, immensity, importance, significance*). During Model It, call on volunteers to identify words and phrases from each problem that express the magnitude of a negative number as an absolute value.

MODEL IT

SMP 2

- 1 – 2 As students complete the problems, ask them to turn and talk about comparing two numbers and comparing their absolute values.

Common Misconception If students think a greater absolute value always means a greater number, then allow students to shade the distance from 0 to -9 and 0 to 5 in problem 1a. Ask which distance is greater and how the inequality $|-9| > |5|$ reflects that.

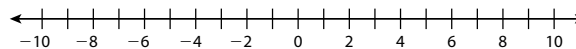
UNDERSTAND: What is the absolute value of a number?

Develop Understanding of Absolute Value

Model It: Compare Absolute Values

► Try these two problems about comparing absolute values.

- 1 Use the number line to help you compare the numbers and compare their absolute values. Write $<$, $>$, or $=$ in each circle to make a true statement. Explain how you know.



a. $-9 < 5$ $|-9| > |5|$

Possible explanation: The value of -9 is less than 5 because -9 is to the left of 5. $|-9|$ is greater than $|5|$ because the distance from -9 to 0 is greater than the distance from 5 to 0.

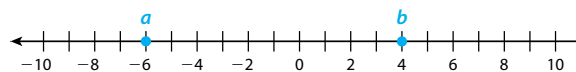
b. $-1 < 2$ $|-1| < |2|$

Possible explanation: The value of -1 is less than 2 because -1 is to the left of 2. $|-1|$ is also less than $|2|$ because the distance from -1 to 0 is less than the distance from 2 to 0.

c. $-8 < 8$ $|-8| = |8|$

Possible explanation: The distance from -8 to 0 is the same as the distance from 8 to 0.

- 2 Plot and label points for two numbers a and b so that $a < b$ and $|a| > |b|$. Explain your thinking.



See number line for possible numbers; Possible explanation: -6 is to the left of 4, so -6 is less than 4. -6 is farther from 0 than 4 is, so $|-6|$ is greater than $|4|$.

DISCUSS IT

Ask: How does a number line help you determine which absolute value is greater?

Share: I think that when you compare two numbers and then compare their absolute values, the inequality symbols can be different because ...

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DISCUSS IT

SMP 3, 7

Support Partner Discussion

After students complete problems 1 and 2, have them respond to Discuss It with a partner. Remind students to paraphrase and then check to see if they understand the speaker's ideas.

Facilitate Whole Class Discussion

For each problem, have students use **Notice and Wonder** to enhance understanding of comparing two numbers and then comparing their absolute values.

ASK How is comparing two numbers on the number line different from comparing their absolute values on the number line?

LISTEN FOR Greater numbers are to the right and lesser numbers are to the left on the number line. Greater absolute values are farther away from 0 and lesser absolute values are closer to 0 regardless of the direction.

ASK When does comparing two numbers and comparing their absolute values result in using the same inequality symbols?

LISTEN FOR The lesser number must be closer to 0 than the greater number.



MODEL IT

SMP 2

3 – 4 See **Connect to Culture** to support student engagement. As students complete the problems, prompt students to give reasons why the phrase that describes the quantity is reasonable in the context.

DISCUSS IT

SMP 3, 7

Support Partner Discussion

After students complete problems 3 and 4, have them respond to Discuss It with a partner. Encourage them to rephrase the question and clarify understanding with each other before discussion of concepts begins.

Facilitate Whole Class Discussion

For each problem, have students pause and ask for questions or comments to verify understanding from classmates.

ASK How does absolute value help you describe the magnitude of negative quantities?

LISTEN FOR Absolute value describes how far the quantity is from zero and can be used for several situations, including those where money is owed and where point values fall below zero.

DIFFERENTIATION | RETEACH or REINFORCE



Hands-On Activity

Use counters to model absolute value.

If students are unsure about using absolute value to compare quantities, then use this activity to help them understand how absolute value relates to distance from 0.

Materials For each pair: 2 two-color counters, Activity Sheet *Number Lines*

- Display this problem: *The Red Team is -4 yards from where they started. The Yellow Team is 3 yards from where they started.* Have students place tick marks about 1 inch apart on a number line, then use counters to mark -4 and 3 using the red and yellow sides to model where each team is now.
- Ask: *Which team is farther from where they started? How do you know?* [Red Team; -4 is 4 units from 0 and 3 is 3 units from 0] Have students write an inequality to represent the situation. [$|-4| > |3|$ or $|3| < |-4|$]
- Repeat with other quantities as time allows.

LESSON 25 | SESSION 2

Model It: Interpret Absolute Value

► Try these two problems about interpreting absolute value.

3 The absolute value of a number may be used to describe the size, or magnitude, of a real-world quantity. Complete each equation and sentence.

a. $|-20| = 20$ $-\$20$ means you owe \$ 20.

b. $|+10| = 10$ A score of +10 points means you win 10 points.

c. $|-10| = 10$ A score of -10 points means you lose 10 points.

4 In each turn of a game, a player either wins or loses points. After the first turn, Jacob's score is -250 points and Indira's score is -300 points. Circle the inequality that makes a correct comparison. Then write a sentence to tell what the inequality means in this situation.

a. $-300 > -250$ $-300 < -250$

Possible answer: Indira's score is lower than Jacob's score.

b. $|-300| > |-250|$ $|-300| < |-250|$

Possible answer: Indira lost more points than Jacob lost.

DISCUSS IT

Ask: How would you interpret the absolute value of a negative temperature?

Share: I think you can use positive numbers to describe negative quantities because ...

CONNECT IT

► Complete the problems below.

5 A whale starts at an elevation of -200 ft relative to sea level and then swims to an elevation of -150 ft. Write an inequality using absolute value notation to compare the distances below sea level. Explain your reasoning.

$|-200| > |-150|$; This inequality shows that 200 ft below sea level is farther below sea level than 150 ft because $|-200| = 200$ and $|-150| = 150$.

6 Luis says $|4|$ is greater than $|-5|$ because 4 is positive, -5 is negative, and any positive number is greater than any negative number. Do you agree? Explain.

No; Possible explanation: $|-5|$ is not a negative number. Luis is comparing two positive numbers. $|4| = 4$ and $|-5| = 5$. Since $4 < 5$, $|4|$ is less than $|-5|$.

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CONNECT IT

SMP 2, 3, 7

5 Student responses should show understanding that -200 is farther from 0 than -150, so the whale is farther away from sea level at 200 feet below than it is at 150 feet below.

CLOSE EXIT TICKET

6 Look for understanding that positive numbers are always greater than negative numbers, but the size of the value is what matters when comparing absolute values.

Common Misconception If students believe absolute values of positive numbers are always greater than absolute values of negative numbers, then have them paraphrase the definition of *absolute value*. Discuss whether it is possible for distance to be negative.

Practice Comparing Absolute Values

Problem Notes

Assign **Practice Comparing Absolute Values** as extra practice in class or as homework.

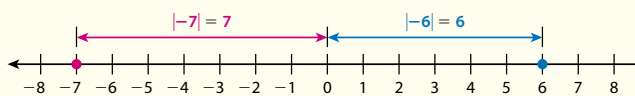
- 1 Students may select an integer from -8 to -3 and recognize that the selected value is less than -2 because it is to the left of -2 , but its absolute value is greater than $|-2|$ because it is farther away from 0 than -2 . **Medium**
- 2
 - a. Students should recognize that -3 is less than 5 because it is to the left of 5 and its absolute value is less than 5 because it is closer to 0 than 5. **Basic**
 - b. Students should recognize that 4 is greater than -4 because it is to the right of -4 and its absolute value is equal to $|-4|$ because it is the same distance from 0 as -4 . **Basic**

Practice Comparing Absolute Values

► Study how the Example shows comparing two numbers and their absolute values. Then solve problems 1–5.

Example

Use the numbers -7 and 6 . Which number has the greater value? Which number has the greater absolute value?



6 is to the right of -7 on the number line, so 6 is greater than -7 .

-7 is 7 units from 0.

6 is 6 units from 0.

So, -7 has the greater absolute value.

$-7 < 6$ and $|-7| > |6|$.

- 1 Choose a number less than -2 that is on the number line in the Example. Is your number's absolute value *greater than 2* or *less than 2*? Explain how you know.
Greater than 2; Possible answer: -6 ; $-6 < -2$ because it is farther to the left on the number line. $|-6| = 6$ and $|-2| = 2$. So, $|-6| > |-2|$.

- 2 Use the number line from the Example to help you compare the numbers and compare their absolute values. Write $<$, $>$, or $=$ in each circle to make a true statement. Explain how you know.

a. -3 $<$ 5 $|-3|$ $<$ $|5|$

Possible explanation: -3 is less than 5 because -3 is to the left of 5. $|-3|$ is less than $|5|$ because the distance from -3 to 0 is less than the distance from 5 to 0.

b. 4 $>$ -4 $|4|$ $=$ $|-4|$

Possible explanation: 4 is greater than -4 because -4 is to the left of 4. $|4|$ is equal to $|-4|$ because the distance from 4 to 0 is the same as the distance from -4 to 0.

Vocabulary

absolute value

a number's distance from 0 on the number line. Absolute value is never negative.

Fluency & Skills Practice

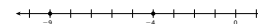
Understanding Absolute Value

In this activity, students explore ideas about absolute value using a number line and a real-world situation.

FLUENCY AND SKILLS PRACTICE Name: _____
LESSON 25

Understanding Absolute Value

1 Answer the questions about this number line.



Which is greater, -9 or -4 ? Explain.

Which is greater, $|-9|$ or $|-4|$? Explain.

2 A football team tries to move the ball forward as many yards as possible on each play, but sometimes they end up behind where they started. The distances, in yards, that a team moves on its first five plays are 2, -1 , 4, 3, and -5 . A positive number indicates moving the ball forward, and a negative number indicates moving the ball backward.

Which number in the list is the greatest?

What is a better question to ask to find out which play went the farthest from where the team started?

The coach considers any play that moves the team more than 4 yards from where they started a "big play." Which play(s) are big plays?

3 When does it make sense to compare the absolute values of numbers rather than the numbers themselves?

- 3 Students may write amounts owed as positive values and select the greatest positive value as the greatest amount owed. **Medium**
- 4 a. Students may recognize that -24 is the lower temperature because it is to the left of -21 on a number line. **Medium**
- b. Students may recognize that $|-24|$ is greater than $|-21|$ because -21 is closer to 0 than -24 . **Medium**
- 5 Students may recognize that the only way to satisfy $c < d$ and $|c| > |d|$ is graphing c as a negative number that is a greater distance away from 0 than d . **Challenge**

LESSON 25 | SESSION 2

- 3 Sophia, Malcolm, and Oren are playing a money game. Their bank balances are shown in the table. Complete the table by writing the absolute value of each bank balance to show how much each player owes. Who owes the greatest amount?



Player	Bank Balance	Amount Owed
Sophia	-\$150	\$150
Malcolm	-\$325	\$325
Oren	-\$275	\$275

Malcolm owes the greatest amount.

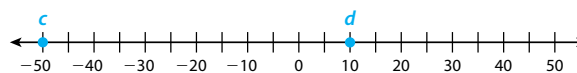
- 4 The temperature on Monday is -24°C . The temperature on Tuesday is -21°C . Circle the inequality that makes a correct comparison. Then write a sentence to tell what the inequality means in this situation.
- a. $-24 < -21$ $-24 > -21$

Possible answer: The temperature on Monday is lower than the temperature on Tuesday.

- b. $|-24| < |-21|$ $|-24| > |-21|$

Possible answer: The temperature on Monday is farther below 0°C than the temperature on Tuesday is.

- 5 Plot and label points for two numbers c and d so that $c < d$ and $|c| > |d|$. Explain your thinking.



See number line for possible numbers; Possible explanation: -50 is to the left of 10 , so -50 is less than 10 . -50 is farther from 0 than 10 is, so $|-50|$ is greater than $|10|$.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with Session 3 Apply It

Levels 1–3: Reading/Writing

Support students as they interpret and respond to Apply It problem 4. Read the first paragraph and provide sentence starters to help students describe quantities:

- The sea level is _____.
- The ocean floor is at _____.
- _____ is at _____.

Explain that *sea level* means 0. Ask questions to help students label each location on their models. Support students as they write and compare distances in problem 4b, using sentence frames such as these:

- The distance from the _____ to 0 is _____.
- The _____ is closer to 0 because _____.

Levels 2–4: Reading/Writing

Support students as they interpret and respond to Apply It problem 4. Read the first paragraph with students. Model how to describe a quantity and have students describe remaining quantities in the problem. Then have students reread the paragraph and think about a model for the situation. Ask: *Where will you put “sea level” on your model?* Allow time for students to work on their models. Then have them turn to partners to compare and connect models.

Next, have partners read problem 4b and tell what they need to compare. Provide a sentence frame to support writing:

- The distance from _____ to _____ is _____ because _____.

Levels 3–5: Reading/Writing

Support students as they interpret and respond to Apply It problem 4. Adapt **Three Reads** to help students make sense of the problem. Read the problem and call on volunteers to tell what the problem is about. After the second read, ask them to describe the quantities in the problem. After the third read, have students think about a model. Ask: *What are the quantities you need to represent? Where would “sea level” be in the model?*

After students model the problem, have them turn to partners to read and discuss problem 4b, using their models to justify their answers. Then allow time for students to write their answers in complete sentences.

Refine Ideas About Absolute Value

Purpose

- **Refine** understanding of how to find and compare the absolute value of two or more numbers.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

- A A negative number is greater than a positive number.
- B A positive number is greater than a negative number.
- C The absolute value of a negative number is greater than the absolute value of a positive number.



Possible Solutions

A is never true.

B is always true.

C is sometimes true.

WHY? Reinforce that comparing absolute values is different from comparing values of the numbers themselves.

APPLY IT

SMP 2, 3, 7

Have students work independently or with a partner for problems 1–3.

- 1 Deduce** Look for understanding that -14 is the only even number that is less than -12 and has an absolute value between 9 and 15 . Ask students to explain how they used the clues in the problem to narrow down the correct number. Prompt discussion with questions such as:

- *In what order did you use the clues to determine Jia's number?*
- *How did you justify that the number you found was correct?*
- *What other even numbers did you consider before identifying the correct even number?*

UNDERSTAND: What is the absolute value of a number?

Refine Ideas About Absolute Value

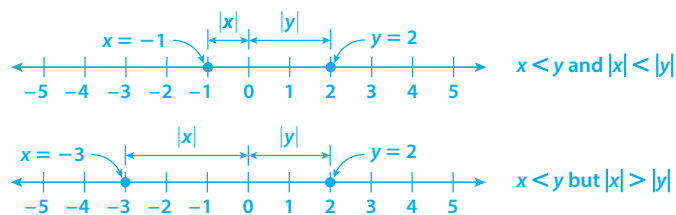
Apply It Math Toolkit number lines

➤ Complete problems 1–5.

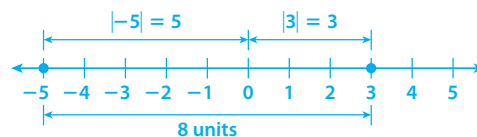
- 1 Deduce** Jia is thinking of a number. She gives three clues about the number: the number is even, the number is less than -12 , and the absolute value of the number is between 9 and 15 . What is Jia's number? Explain how you know.
 -14 ; **Possible explanation:** The number is negative because it is less than -12 . Only the negative numbers between -9 and -15 have an absolute value between 9 and 15 . The even numbers between -9 and -15 are -10 , -12 , and -14 . The number -14 is the only one of these even numbers that is also less than -12 .

- 2 Analyze** Ian says that if $x < y$, then $|x| < |y|$. Is Ian's statement *always true*, *sometimes true*, or *never true*? Use a model to help explain your thinking.

Sometimes true; Possible explanation:



- 3 Apply** Mrs. Shen writes the expression $|-5| + |3|$ on the board. Show or explain why the sum $|-5| + |3|$ is the distance between -5 and 3 on a number line.
Possible explanation: $|-5|$ is the distance from -5 to 0 and $|3|$ is the distance from 0 to 3 . The sum of these two distances, 8 , is the distance from -5 to 3 .



- 2 Analyze** Look for understanding that sometimes a negative number is closer to 0 than a positive number and sometimes it is farther away. Have students share their solutions and completed models. Use the following to start a discussion:
 - *Explain how you set up and labeled your models.*
 - *How do your models show each inequality?*
 - *How did you use your models to analyze Ian's statement?*
- 3 Apply** Look for understanding that finding the absolute values of -5 and 3 before adding results in a sum that is the distance between -5 and 3 on the number line. Ask students to explain how they determined that the statement is true.

Common Misconception If students believe that the sum of two absolute values will always be the distance between the two values, then allow students to investigate a variety of examples, such as $|-4| + |-3|$, $|2| + |-8|$, and $|6| + |1|$. Encourage students to evaluate each expression and model each on a number line to make comparisons and generalize that the distance between the two values is only the same as the sum of the two absolute values when one number is negative and the other is positive.

- 4 See **Connect to Culture** to support student engagement. Before students begin, read the first part of the problem aloud and engage them in a discussion about objects and people relative to sea level. Have students read the directions for Parts A and B, and then have them rephrase to confirm that they understand each part of the task.

As students work on their own, walk around to assess that they draw an appropriately spaced model with clear labels for the tour guide, sea level, the diver, and the ocean floor.

Have students share their model and inequality with a partner and explain how they determined which person was closer to sea level.

CLOSE EXIT TICKET

- 5 **Math Journal** Look for understanding that ordering numbers is based on their relative positions on a number line and ordering absolute values is based on distances from 0 on a number line.

Error Alert If students confuse ordering the numbers with ordering their absolute values, then prompt them to use two different number lines to help them complete each task. Elicit from students that the positions of the numbers on the first number line help with ordering the numbers and the distances from 0 on the second number line help with the ordering the absolute values.

✓ End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they sketch a number line, label -4 and 3 on the number line, and use a highlighter or pencil to shade and compare absolute values.

LESSON 25 | SESSION 3

- 4 A tour group is going sea diving. The ocean floor is at -18 ft relative to sea level. One diver is already at -11 ft. The tour guide is keeping watch on a platform 5 ft above sea level, directly above the diver.

PART A Draw a model of the situation. **Possible model:**



PART B Write an absolute value inequality comparing the distances of the tour guide and the diver to sea level. Who is closer to sea level? Explain how you know.

$|5| < |-11|$ (or $|-11| > |5|$); **The tour guide;** **Possible explanation:** The distance from the tour guide to 0 is $|5|$, or 5. The distance from the diver to 0 is $|-11|$, or 11. Since $5 < 11$, the tour guide is closer to sea level.

- 5 **Math Journal** Order the numbers $5, -7, -9,$ and -2 from least to greatest. Then order the absolute values $|5|, |-7|, |-9|,$ and $|-2|$ from least to greatest. Explain how absolute value affects which values are lesser and which values are greater.

$-9, -7, -2, 5; |-2|, |5|, |-7|, |-9|$; **Possible explanation:** The absolute value of a number is its distance from 0. So, when you order the absolute values of numbers, it does not matter whether the number is positive or negative. Only the distance from 0 matters.



✓ End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for *absolute value*. Explain why the absolute value of -4 is greater than the absolute value of 3.

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Short Response Scoring Rubric (2 points)

Problem 4

Points	Expectations
PART A	
2	The model shows the 4 given values in the correct order AND is labeled correctly.
1	The model shows the 4 given numerical values listed in the correct order OR shows all 4 descriptions in the situation without numerical values in the correct order.
PART B	
2	The absolute value inequality is correct AND the explanation justifies that the tour guide is closer to sea level than the diver.
1	The absolute value inequality is correct OR the explanation tells why the tour guide is closer to sea level than the diver.