# Overview | Reason About Random Samples

## STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

**4** Model with mathematics.

**5** Use appropriate tools strategically.

* See page 1q to learn how every lesson includes these SMP.

## Objectives

### Content Objectives

- Use proportional reasoning to make inferences about a population from a single sample.
- Understand that it is possible to draw inferences about a population from one random sample or from many random samples.
- Make and compare inferences from different random samples of the same population.

### Language Objectives

- Explain inferences based on single samples by using introductory words, such as *if* and *so,* in speaking and writing.
- Use lesson vocabulary and the words *conclude, conclusion, inference, infer,* and *reasonable* to write and talk about inferences based on one or more random samples.
- Use *better, more*, and *most* to compare random samples and to explain which is more representative and would produce better inferences or estimates.
- Discuss strategies to solve problems that involve inferences based on samples and build on others' responses by explaining when the strategy would work.

## Prior Knowledge

- Determine whether a sample was randomly selected.
- Use proportional reasoning to find an unknown quantity.
- Find the mean and median of a data set.
- Read and interpret box plots and dot plots.

## Vocabulary

### Math Vocabulary

There is no new vocabulary. Review the following key terms.

**box plot** a visual display of a data set on a number line that shows the minimum, the lower quartile, the median, the upper quartile, and the maximum. The sides of the box show the lower and upper quartiles and the line inside the box shows the median. Lines connect the box to the minimum and maximum values.

**estimate (noun)** a close guess made using mathematical thinking.

**estimate (verb)** to give an approximate number or answer based on mathematical thinking.

**mean** the sum of a set of values divided by the number of values. This is often called the *average*.

**median** the middle number, or the halfway point between the two middle numbers, in an ordered set of values.

**population** the entire group of interest. Samples are drawn from populations.

**random sample** a sample in which every element in the population has an equal chance of being selected.

### Academic Vocabulary

**infer** to conclude or decide something based on evidence.

**inference** conclusion or decision made based on evidence.

## Learning Progression

**Earlier In Grade 7,** students solved proportional relationship problems.

**In the previous lesson,** students learned what it means for a sample to be representative of a population. They built understanding that random samples are likely to be representative and explored methods of selecting random samples.

➤

**In this lesson,** students learn that it is possible to draw inferences about a population from one random sample or from many random samples. Students use proportional reasoning to make inferences about a population from a single sample or from multiple samples. They gauge how close to the mean or median any one random sample in the set is.

➤

**Later in Grade 7,** students will make informal inferences about two populations by comparing random samples drawn from each population.

# Overview

## Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

| | MATERIALS | DIFFERENTIATION |
|---|---|---|
| **SESSION 1** **Explore** Random Samples (35–50 min) | | |
| • **Start** (5 min)<br>• **Try It** (5–10 min)<br>• **Discuss It** (10–15 min)<br>• **Connect It** (10–15 min)<br>• **Close: Exit Ticket** (5 min)<br>**Additional Practice** (pages 485–486) | **Math Toolkit** bags, bowls, buttons, cups, index cards, number cubes<br>Presentation Slides | **PREPARE** Interactive Tutorial<br>**RETEACH or REINFORCE** Hands-On Activity<br>**Materials** For each student: 3 craft sticks |
| **SESSION 2** **Develop** Making Inferences from Samples About Populations (45–60 min) | | |
| • **Start** (5 min)<br>• **Try It** (10–15 min)<br>• **Discuss It** (10–15 min)<br>• **Connect It** (15–20 min)<br>• **Close: Exit Ticket** (5 min)<br>**Additional Practice** (pages 491–492) | **Math Toolkit** double number lines, grid paper<br>Presentation Slides | **RETEACH or REINFORCE** Visual Model<br>**Materials** For display: 100 unit cubes (20 each of 5 colors), 1 paper bag<br>**REINFORCE** Fluency & Skills Practice<br>**EXTEND** Deepen Understanding |
| **SESSION 3** **Develop** Making Inferences from Multiple Samples (45–60 min) | | |
| • **Start** (5 min)<br>• **Try It** (10–15 min)<br>• **Discuss It** (10–15 min)<br>• **Connect It** (15–20 min)<br>• **Close: Exit Ticket** (5 min)<br>**Additional Practice** (pages 497–498) | **Math Toolkit** double number lines, grid paper<br>Presentation Slides | **RETEACH or REINFORCE** Visual Model<br>**REINFORCE** Fluency & Skills Practice<br>**EXTEND** Deepen Understanding |
| **SESSION 4** **Refine** Reasoning About Random Samples (45–60 min) | | |
| • **Start** (5 min)<br>• **Monitor & Guide** (15–20 min)<br>• **Group & Differentiate** (20–30 min)<br>• **Close: Exit Ticket** (5 min) | **Math Toolkit** Have items from previous sessions available for students.<br>Presentation Slides | **RETEACH** Visual Model<br>**Materials** For display: 100 unit cubes (20 blue, 80 assorted other colors), 1 paper bag<br>**REINFORCE** Problems 4–7<br>**EXTEND** Challenge<br>**PERSONALIZE** i-Ready |
| **Lesson 23 Quiz** ↖ or **Digital Comprehension Check** | | **RETEACH** Tools for Instruction<br>**REINFORCE** Math Center Activity<br>**EXTEND** Enrichment Activity |

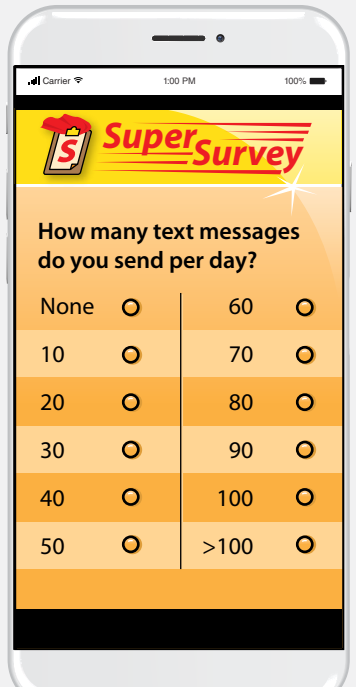
# Overview | Reason About Random Samples

## Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

### SESSION 1 ■ □ □ □

**Try It** Ask students whether they are more likely to make a phone call or send a text message. The first text message was sent in 1992. By 2007, Americans sent and received more texts per month than phone calls. It would have been hard to imagine in the 1990s that texting would become such a prominent mode of communication. Have students share their thoughts about communication in the future. Will people continue to make phone calls and send texts or will they become obsolete? What might replace them?




### SESSION 2 ■ ■ □ □

**Try It** If there is no subway in your area, ask students to raise their hand if they have ever ridden on a subway. If there is a subway in your area, ask students if they ride the subway to school. Select several students to share their experiences. A subway system is an underground train network, often part of a city's public transit system. Record holders among underground train systems include the London Underground, also known as the Tube, which is the oldest, the New York City Subway, which has the most stations, and the Shanghai Metro, which is the longest. Have students share any interesting facts they know about trains or subways.

**Apply It** **Problem 9** Ask students to share if they speak another language at home. In 2015, about 64.7 million U.S. residents age five and older spoke a language other than English at home, which is about 21.5 percent of the entire population. Nearly one in four public school students now speaks another language at home. Ask for volunteers to teach the class how to say a greeting in another language.


### SESSION 3 ■ ■ ■ □

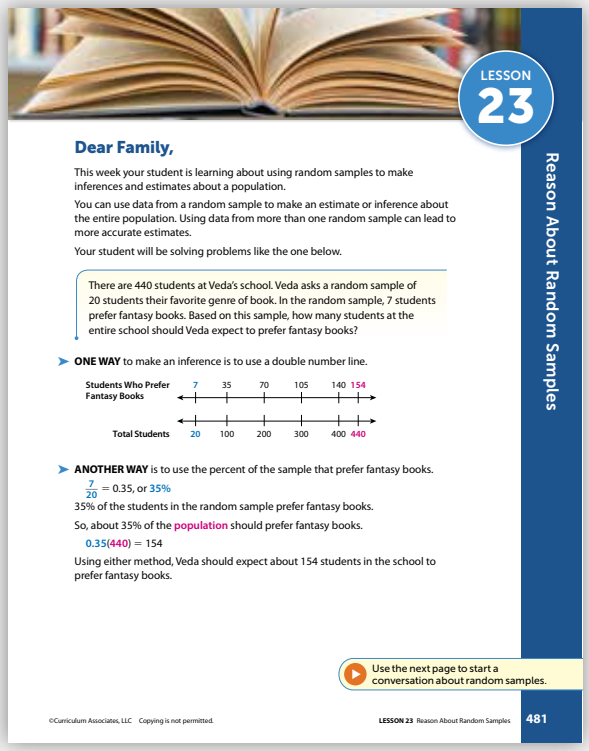
**Apply It** **Problem 7** Ask students to raise their hands if they have ever participated in martial arts training. There are many different styles of martial arts that fall into larger categories. Striking or stand-up martial arts styles teach users how to defend themselves and include boxing, karate, and tae kwon do. Grappling or ground styles teach users how to take their opponent to the ground and include jujutsu, sumo, and wrestling. There are even low-impact and meditative styles of martial arts, which help users develop and practice breathing techniques and fitness, such as tai chi. Ask students who practice martial arts to share something they learned in their training.

### SESSION 4 ■ ■ ■ ■

**Apply It** **Problem 3** There are over 100,000,000 house pets in the United States. Fewer households own cats than dogs, but there are 86 million pet cats compared to only 78.2 million pet dogs. Other common pets include birds, fish, and turtles. Conduct a quick survey about the types of pets that the students have.

## Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 23

Reason About Random Samples

**Dear Family,**

This week your student is learning about using random samples to make inferences and estimates about a population.

You can use data from a random sample to make an estimate or inference about the entire population. Using data from more than one random sample can lead to more accurate estimates.

Your student will be solving problems like the one below.

There are 440 students at Veda's school. Veda asks a random sample of 20 students their favorite genre of book. In the random sample, 7 students prefer fantasy books. Based on this sample, how many students at the entire school should Veda expect to prefer fantasy books?

➤ **ONE WAY** to make an inference is to use a double number line.

| | | | | | | |
|---|---|---|---|---|---|---|
| Students Who Prefer Fantasy Books | 7 | 35 | 70 | 105 | 140 | 154 |
| Total Students | 20 | 100 | 200 | 300 | 400 | 440 |

➤ **ANOTHER WAY** is to use the percent of the sample that prefer fantasy books.

$\frac{7}{20} = 0.35$, or 35%

35% of the students in the random sample prefer fantasy books.

So, about 35% of the population should prefer fantasy books.

$0.35(440) = 154$

Using either method, Veda should expect about 154 students in the school to prefer fantasy books.

Use the next page to start a conversation about random samples.







**Activity** Thinking About Random Samples Around You

➤ **Do this activity together to investigate random samples in the real world.**

Have you ever tried to guess the number of small objects in a jar? Your guess may have been way off. However, if you had been able to average the guesses from lots of people, your guess might have been close to the true number.

*Wisdom of crowds* is the idea that collecting information from many different people can result in a better decision than relying on information from one person, even an expert! When you have more sample guesses, you can make a better guess.

? What are some times when you might want to use more than one sample to make an inference?





## Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

**DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS** Use with **Session 1** **Discuss It**

### MATH TERMS

An *estimate* is a close guess made using mathematical thinking.

To *estimate* is to give an approximate number or answer based on mathematical thinking.

**Levels 1–3:** Listening/Speaking

Prepare students to participate in Discuss It. Display the Math Terms and read them aloud. Point out the difference in pronunciation. Have students repeat the terms. Encourage them to tell which term defines an action and have them pronounce the term. Then read Discuss It and help partners use the terms to share their conclusions. Ask: *How did Vivian estimate? How did Edward estimate? Which estimate is more representative?*

- *Vivian estimated using ____.*
- *Edward estimated using ____.*
- *I concluded ____ 's estimate is more representative.*

**Levels 2–4:** Listening/Speaking

Prepare students to participate in Discuss It. Display the Math Terms and read them aloud. Have students tell if they notice a difference in pronunciation. Call on volunteers to identify which names a thing and which names an action. Then read Discuss It with students. Have partners use the terms to explain their conclusions. Ask: *How did Vivian estimate? How did Edward estimate? What do you conclude about the estimates?* Then have students use *more* to restate and explain:

- *The ____ representative sample is ____.*
- *I concluded this because ____.*

**Levels 3–5:** Listening/Speaking

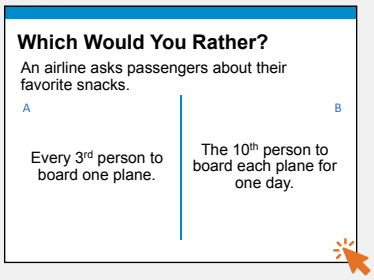
Prepare students to participate in Discuss It. Display the Math Terms and invite students to read them aloud. Listen for correct pronunciation and provide feedback as needed. Then ask students to explain the difference in meaning and pronunciation. Have students turn to partners to read Discuss It. Ask partners to use the terms as they share and explain their conclusions. Next, encourage them to say if they agree and, if so, to build on by adding phrases or sentences that include *conclude, conclusion,* and *more.*

# Explore Random Samples

## Purpose

- **Explore** the idea that a random sample can be used to make a specific inference about a population.
- **Understand** that different ways of selecting a sample can affect how representative a sample is.

## START CONNECT TO PRIOR KNOWLEDGE




**Possible Solutions**

**A can be found more quickly.**

**B may be more representative of all of an airline's passengers.**

**WHY?** Support students' thinking about random samples.

## TRY IT SMP 1, 2, 4, 5, 6

### Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. When paraphrasing, encourage students to consider what is similar and different in the two parts of the problem.

## DISCUSS IT SMP 2, 3, 6

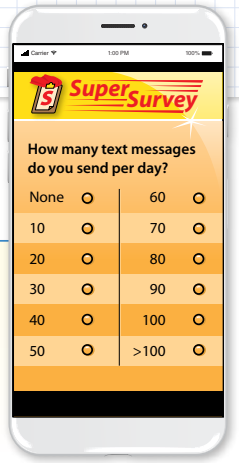
### Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the students in the band are a subset of all of the students in the seventh grade.
- Edward's sample will likely be larger than Vivian's sample since it is a random sample of the entire population, not a subset of the population.

LESSON 23 | SESSION 1 ■ □ □ □

## Explore Random Samples




Previously, you learned about random samples. In this lesson, you will learn about using random samples to make estimates.

➤ **Use what you know to try to solve the problem below.**

Vivian surveys a random sample of Grade 7 students in the school band. She asks the students how many text messages they send each day. The median of her sample data is 50.

Edward surveys a random sample of all Grade 7 students. He asks the students how many text messages they send each day. The median of his sample data is 60. Whose result is more likely to be representative of all the Grade 7 students? Why?

**TRY IT**

**Math Toolkit** bags, bowls, buttons, cups, index cards, number cubes

**Possible work:**

**SAMPLE A**

**Edward's result is more likely to be representative. Vivian's sample is only from students in the grade that are in the band. So, she cannot generalize her results to the entire grade.**

**SAMPLE B**

**Edward's result is more likely to be representative. He took a random sample of the whole grade and used that to make a statement about the population.**

**DISCUSS IT**

***Ask:*** How did you reach that conclusion?

***Share:*** First, I thought . . .

**Learning Target** SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

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**Common Misconception** Listen for students who use the results of the sample rather than thinking about how each survey was conducted to determine whether the sample is representative of the whole grade. For example, a student who claims that Edward's sample is more representative because 60 is greater than 50. As students share their strategies, ask students to consider whether they could answer the question without knowing the results of each sample.

### Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- reason about Vivian's sample, concluding that a random sample of the band is likely to be representative of the band, not the entire population
- **(misconception)** use results of the sample to reason that Edward's sample is more representative
- reason about Edward's sample, concluding that his sample gave every Grade 7 student an equal chance to be selected, so it is more likely to be representative of all Grade 7 students

### Facilitate Whole Class Discussion

Call on students to share selected strategies. As they listen to the presentations, remind students that one way to agree and build on ideas is to add details to help listeners understand more about the strategy or solution.

Guide students to **Compare and Connect** the representations. To engage all students, ask them to turn and talk to answer the question below.

> **ASK** *How do [student's name] and [student's name] support their choice?*
>
> **LISTEN FOR** They both compare Vivian's sample to Edward's sample to explain which is more likely to be representative.

## CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that for a sample to be representative of a population, it must be taken randomly from across the entire population, not from within a subset of the population.

**DIFFERENTIATION | RETEACH or REINFORCE**

### Hands-On Activity

**Model samples of a population.**

*If students are unsure about why a random sample tends to be more representative than a non-random sample, then use this activity to explore samples.*

**Materials** For each student: 3 craft sticks

- Have students write the name of a color on each craft stick and form groups. Have them combine their craft sticks to form a population and then have each student select a craft stick at random.
- Ask: *Why do your selected sticks form a random sample?* [They are a subset of the population that we selected at random.]
- Have students record their sample data, return their sample to the population, and select craft sticks that have color names with exactly 4 letters.
- Ask: *Why is this sample not a random sample?* [The members of the sample were not chosen at random.]
- Have students record their sample data and compare how a specific characteristic is represented in each sample and the population.
- Repeat the activity with other samples, such as colors that start with the letter *p*, and other characteristics, such as the color purple.
- Extend the activity by guiding students to discuss why a random sample is more likely to be representative of a population than a nonrandom sample.



## CONNECT IT

1 **Look Back** Is Vivian's or Edward's result more likely to be representative of the whole grade? Why?

Edward's; Possible explanation: His random sample is from the whole grade instead of just the students in band in the grade.

2 **Look Ahead** While Vivian and Edward surveyed different populations, they both surveyed random samples. You can use data from a random sample to make an estimate or an inference about a population.

**a.** Sofia surveys a random sample of students in her school. She finds that 20 people in her random sample have pierced ears. Is it reasonable for Sofia to estimate that 20 people in the population of students at her school have pierced ears? Explain.

No; Possible explanation: A sample is just part of the population. If there are 20 people with pierced ears in the sample, there should be more people with pierced ears in the whole population.

**b.** Sofia also finds that 50% of the students in her random sample wear glasses. Why is it a more reasonable inference that about 50% of the students in her school wear glasses than exactly 50% of the students in her school wear glasses?

Possible answer: A random sample should look similar to the population it is drawn from, but it may not be exactly the same. So, you are more likely to be right if you infer that about 50% wear glasses.

3 **Reflect** How is making an inference about a population different from knowing something for certain about a population?

Possible answer: When you make an inference about a population, you use information from a sample to make an estimate about the population as a whole. You do not know for sure that your inference is correct.



2 **Look Ahead** Point out that the result of the sample should apply to the whole population at about the same rate. Students should recognize that since an inference is an estimate about the whole population, they do not have the information needed to make an exact statement about the population.

## CLOSE EXIT TICKET

3 **Reflect** Look for understanding that when using a sample to make inferences about a population, the inferences are estimates that are unlikely to match the population exactly.

**Common Misconception** If students think that the only way to make an inference about a population is to use an exact calculation, such as surveying every member of a population, then have pairs of students discuss the ways they reasoned about the populations in the session. Pairs can discuss this question: *Did you need to know the number of text messages for all the Grade 7 students to reason about the larger population?*

# Prepare for Reasoning About Random Samples

## Support Vocabulary Development

Assign **Prepare for Reasoning About Random Samples** as extra practice in class or as homework.

*If you have students complete this in class, then use the guidance below.*

Ask students to consider the term *random sample*. Suggest that they think about what it means for a sample of a population to be chosen randomly.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of definitions, examples, and non-examples.

Have students look at the description of José's sample in problem 2 and discuss with a partner whether the sample is representative.

## Problem Notes

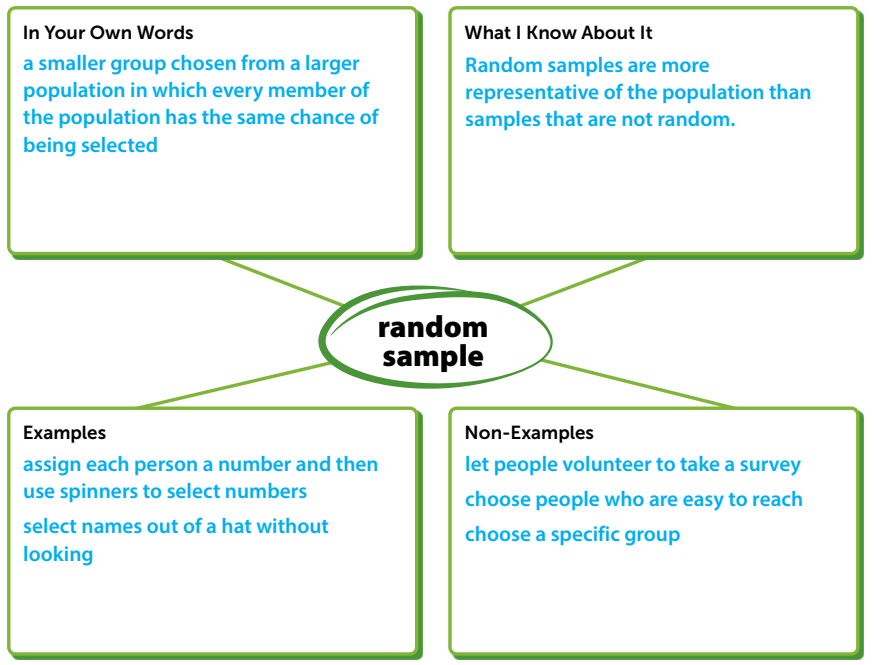
1. Students should understand that a random sample is a subset of a population and that it is treated as representative of the whole population. Student responses might include selecting names from an alphabetized list. Students should recognize that to be representative of the population, every member of the population must have an equal chance of being selected as part of the sample.

2. Students should recognize that students who are not in the book club cannot be selected as part of José's sample, so they do not have an equal chance of being selected.



## Prepare for Reasoning About Random Samples

1. Think about what you know about random samples. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**




2. José is interested in the favorite activities of the students in his school. He surveys all the members of the school's book club. Did José survey a random sample of all the students in his school? Explain.

   **No; Possible explanation: José's sample is not random because all the students he surveys are part of one group, the book club. Not every student in his school has an equal chance of being chosen.**



### REAL-WORLD CONNECTION

Television, radio, and other media platforms survey people across the country about what watch and listen to. The Nielsen ratings system chooses about 37,000 households that are as proportional to the census data as possible to best represent the approximately 120 million households in the United States. The data can then be used to make inferences about the total number of viewers for a particular show without having to know what each individual in the country has watched. Data can then be used to make projections about future viewership of a similar program and help set prices for ad agencies according to the number of expected viewers. Ask students to think of other real-world examples when surveying a sample of a population is useful.

# Additional Practice

3 Problem 3 provides another look at how to choose a random sample of a population. This problem is similar to the problem about surveying students to find the number of texts they send. In both problems, two surveys are conducted with different methods for sampling. This problem asks which sample is more representative of the larger population. The correct answer can be justified by explaining what is more representative about Jordan's sample or what is less representative about Dara's sample.

Suggest that students use **Say It Another Way** to help them make sense of the problem.



3 On the last day of school, all of the students in Jordan and Dara's middle school compete in a long jump contest. Jordan surveys a random sample from all students in the school about how far they can long jump. The mean of her sample data is 11 ft 6 in.

Dara surveys a random sample of Grade 8 students in the school. The mean of his sample data is 12 ft 2 in.

**a.** Whose answer do you think is more likely to be representative of all the students in the school? Explain.

Jordan's; Possible explanation: Jordan's random sample is from the whole school instead of just the eighth grade.

**b.** Justify your answer in a different way.

Possible answer: Dara's sample is only from the Grade 8 students in the school. So, his results can only be generalized to the eighth grade.



## DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

### **Levels 1–3:** Reading/Speaking

Modify **Three Reads** to help students interpret Apply It problem 7. Read the first three sentences, pausing to ask: *What do students in the sample need to answer? How many students say* yes*? And* no*? How many students are undecided or don't know? How many students are in Grade 8?* Next, read the question. Have students tell what they need to find out. Circle *about* and ask students if they need to find an exact answer or an estimate. Read the complete problem. Have students read the quantities they will use to solve the problem. Allow time for students to work on the problem. Then ask: *What is your estimate?*

- *My estimate is about ____.*

### **Levels 2–4:** Reading/Speaking

Modify **Three Reads** to help students interpret Apply It problem 7. Read the first three sentences with students. Ask: *What do students in the sample need to answer? How many students say* yes*? How many say* no*? How many students are undecided? What does* undecided *mean? How many Grade 8 students are there?* Next, read the question with students. Have them tell what they need to find out. Ask students if they need to find an exact answer or an estimate. Then ask: *How do you know?* Read the complete problem. Have students read the quantities they will use to solve the problem. Allow time for students to work on the problem. Then ask: *What is your estimate?* Encourage them to use the word *about* to talk about their estimates.

### **Levels 3–5:** Reading/Speaking

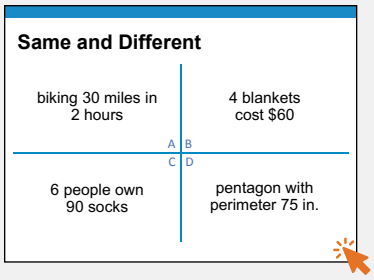
Modify **Three Reads** to help students interpret Apply It problem 7. Have students read the first three sentences and talk about the information. Invite volunteers to explain the meaning of *undecided* and use the meaning to talk about the answers in the sample. Next, have students read the question and tell what they need to find out. Encourage them to use *estimate, close, approximation*, *exact*, and *precise* to describe what they need to find. Have students read the complete problem and summarize the information they will use to solve the problem. Allow time for them to work on the problem. Then have students turn and talk to partners about their findings.

# Develop Making Inferences from Samples About Populations

## Purpose

- **Develop** strategies for making inferences from a single random sample about a population.
- **Recognize** that you can use a random sample of a population to make inferences about the characteristics of that population.

## START CONNECT TO PRIOR KNOWLEDGE




**Possible Solutions**

All have a unit rate of 15.

A and D both involve distances.

C is the only one involving money.

**WHY?** Support students' facility with recognizing proportional relationships.

## DEVELOP ACADEMIC LANGUAGE

**WHY?** Understand sentences with introductory clauses.

**HOW?** Ask: *What do people use to make inferences?* Explain that people support their inferences by describing the information they used to make the inference. Have students share how the question in Try It asks about an inference and how the comma divides the question in two parts. Ask: *What part expresses the inference? What part tells what you need to use to make the inference?*

## TRY IT

SMP 1, 2, 4, 5, 6

### Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Make sure students notice that the frequency table includes important information that is part of the problem.

LESSON 23 | SESSION 2

## Develop Making Inferences from Samples About Populations

➤ **Read and try to solve the problem below.**

There are 406 students at Destiny's school. Destiny surveys 20 randomly chosen students from her school about how they get to school.

Based on the data from the sample, about how many students in the school should Destiny expect to take the subway?

| Transportation Method | Frequency |
|---|---|
| Subway | 8 |
| Walk | 4 |
| Bus | 4 |
| Bike | 2 |
| Car | 2 |

TRY IT

Math Toolkit double number lines, grid paper

Possible work:

SAMPLE A

| Students Taking Subway | Total Students |
|---|---|
| 8 | 20 |
| 1 | 2.5 |
| 162.4 | 406 |

Since you cannot have 0.4 of a student, Destiny should expect that about 162 students take the subway to school.

SAMPLE B

Find how many times greater 406 is than 20.

$20x = 406$

$x = 20.3$

The population is 20.3 times as large as the sample.

$8(20.3) = 162.4$

Destiny should expect that about 162 students take the subway to school.

DISCUSS IT

***Ask:*** How do you know your answer is reasonable?

***Share:*** I know my answer is reasonable because . . .



## DISCUSS IT

SMP 2, 3, 6

### Support Partner Discussion

After students work on Try It, encourage them to explain their work and respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions, such as:

- *How does the data in your answer relate to the numbers in the problem?*
- *Why can you use the information in your solution to make an inference about the students in Destiny's school?*

**Error Alert** If students confuse sample and population data, such as by dividing 20 by 406, then have them describe what each number in the problem represents. Ask: *What does the information in the chart tell you about the 20 students surveyed? How does the information about the surveyed students relate to all the students at Destiny's school?*

## Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table of ratios equivalent to 8 : 20
- equation showing how many times greater the population is than the sample
- double number line comparing the number of students who take the subway to the total number of students
- percent of the students in the sample who take the subway

## Facilitate Whole Class Discussion

Call on students to share selected strategies. As they present, prompt students to refer to their models or diagrams as they justify their solutions.

Guide students to **Compare and Connect** the representations. Allow individual think time before asking students to respond to the question below.

**ASK** *How do all of the strategies use proportional reasoning?*

**LISTEN FOR** All the strategies used the ratio 8 : 20 to find the equivalent ratio of subway riders in the population.

## Model It

**If students presented these models,** have students connect these models to those presented in class.

**If no student presented at least one of these models,** have students first analyze key features of the models, and then connect them to the models presented in class.

**ASK** *How do both models use the ratio 8 : 20?*

**LISTEN FOR** The first uses the ratio to find equivalent ratios and the second uses the ratio to find a percent.

**For the double number line,** prompt students to think about why the relationship is proportional.

- *How do you know you can use a double number line to represent the relationship?*
- *How do you know the relationship is proportional?*

**For the percent,** prompt students to think about why the percent applies to the sample and to the total population.

- *What does the percent represent?*
- *How do you know you can use the sample to make an inference about the population?*



➤ **Explore different ways to use a random sample to make inferences about a population.**

There are 406 students at Destiny's school. Destiny surveys 20 randomly chosen students from her school about how they get to school.

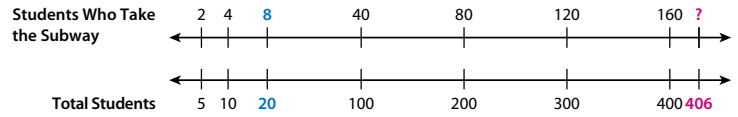
Based on the data from the sample, about how many students in the school should Destiny expect to take the subway?

| Transportation Method | Frequency |
|---|---|
| Subway | 8 |
| Walk | 4 |
| Bus | 4 |
| Bike | 2 |
| Car | 2 |

### Model It

You can use a double number line to make an inference about the population.

The data come from a random sample, so you can expect them to be representative of the population.




### Model It

You can use percents to make an inference about a population.

Find the percent of students in the sample that take the subway.

$\frac{8}{20} = 0.4$, or 40%

So, you can infer that 40% of all the students in the population take the subway.

0.4(406)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**DIFFERENTIATION | EXTEND**

SMP 4

### Deepen Understanding

**Using Double Number Lines to Model Other Data**

Prompt students to think about changing the model to answer different questions.

**ASK** *How could you use the model to make an inference about a similar population with a different number of total students?*

**LISTEN FOR** You could use the given ratios or find more ratios until the bottom number line has a number close to the new total number of students.

**ASK** *How could you change the model to find the number of students who take the bus?*

**LISTEN FOR** Since $\frac{1}{4}$ of the number of students who take the subway take the bus, you could divide all of the numbers on the top number line by 4.

**ASK** *How could you change the model to find the number of students who do not take the subway?*

**LISTEN FOR** Since 50% more students do not take the subway, you could increase all of the numbers on the top number line by 50%.

# Develop Making Inferences from Samples About Populations

## CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about making inferences about a population using a random sample.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5. During discussions, prompt students to use precise academic language and call on volunteers to reword vague or unclear statements.

### Monitor and Confirm Understanding 1 – 3

- The labels for the two number lines show the quantities of subway riders and total students as equivalent fractions.
- $8 \div 20 = 0.40$
- 40% of 406 is equal to 162.4, which is 162 when rounded to the nearest whole number.

### Facilitate Whole Class Discussion

4 Look for the idea that though results from different samples may be different, they are still valid.

> **ASK** *How could three samples of the same population give three different results?*
>
> **LISTEN FOR** Sampling involves an element of chance, so the results of different samples are likely to be close to, but not exactly the same as, each other and the population.

5 Look for the understanding that you can apply proportional reasoning from a random sample to a population.

> **ASK** *Why is a proportional relationship between characteristics of a random sample and a population likely?*
>
> **LISTEN FOR** Since a random sample is likely to be representative, and a representative sample has similar characteristics to a population, a proportional relationship between characteristics of a random sample and a population is likely.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

## CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use a random sample to make inferences about a population.**

1 Look at the first **Model It**. What two quantities does it show a proportional relationship between?

the number of students who take the subway and the total number of students

2 Look at 0.4(406) in the second **Model It**. How is the random sample represented in the expression?

The 0.4 is the fraction of students in the random sample who take the subway.

3 Based on Destiny's random sample, about how many students can she expect to take the subway? Is this the same as the result of finding 40% of 406? Explain.

162; No; Possible explanation: Since 40% of 406 is 162.4, and you cannot have a partial student, the result is rounded to the nearest whole number.

4 Destiny's friends Xavier and Querida also each survey 20 students selected at random from the school. Based on their samples, Xavier infers that about 142 students take the subway to school and Querida infers that about 183 students do. Explain why both of their inferences are reasonable.

All three inferences are based on random samples of 20 students, so all are reasonable. Random samples from the same population will not always be exactly the same, so they might lead to different inferences.

5 How can you use proportional reasoning to make an inference about a population from a random sample of that population?

Possible answer: The percent of the random sample and the percent of the population with a certain characteristic should be about the same. So, you can find that percent of the population to make an inference.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to use a random sample to make inferences about a population.

Responses will vary. Check student responses.

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### DIFFERENTIATION | RETEACH or REINFORCE

#### Visual Model

**Draw inferences from random samples.**

*If students are unsure about how a sample is representative of a population, then use this activity to show how characteristics in a random sample and a population are proportional.*

**Materials** For display: 100 unit cubes (20 each of 5 colors), 1 paper bag

- Place the cubes in the bag. Tell students the bag holds a population of 100 cubes. Allow each student to select a cube from the bag at random until you have a sample of at least 10 cubes. Display the results in a frequency table.
- Say: *Since a random sample is likely to have similar characteristics to the population it comes from, we can infer that the population is like the sample, but larger.*
- Ask: *What ratio represents the number of [color] cubes in the sample? Use that ratio to make an inference about the number of [color] cubes in the population.* [Answer depends on experiment.] Have students use proportional reasoning to infer the number of each color cube in the population.
- Reveal the contents of the bag and guide students to compare their sample and inference to the population. They should identify which characteristics in the sample were most representative of the population.

## Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, students who choose to use a double number line need not scale the number lines precisely to show understanding of how each sample relates to the population.

**7** Students may draw a double number line and match the total number of students with the corresponding number who plan to take computer science.

**8** **D is correct.** This is $\frac{30}{50} = 60\%$.

**A** is not correct. This answer is 50% of 30.

**B** is not correct. This answer is the number of students who said they play a sport.

**C** is not correct. This is the size of the random sample.



### Apply It

➤ **Use what you learned to solve these problems.**

**7** A random sample of Grade 8 students at a school are asked whether they plan to take computer science in high school. Of those asked, 15 students plan to take computer science, 5 do not, and 7 are unsure. There are 326 Grade 8 students in the school. Based on the sample, about how many Grade 8 students in the school plan to take computer science in high school? Show your work. Possible work:

A total of 15 + 5 + 7, or 27, students are sampled. So, $\frac{15}{27}$ of the sample plan to take computer science. That means about $\frac{15}{27}$ of the population of 326 Grade 8 students plan to take computer science.

$\frac{15}{27}(326) = 181.\overline{1}$

**SOLUTION** About 181 Grade 8 students plan to take computer science.

**8** Yolanda asks a random sample of 50 students at her school what they do after school. Of the 50 students, 30 say they play a sport. Based on the sample, about what percent of students at Yolanda's school play a sport after school?

**A** 15%  **B** 30%

**C** 50%  **D** 60%

**9** There are 5,119 households in Salvador's town. Salvador surveys a random sample of 20 households from his town. He finds that more than one language is spoken in 4 of the households. Based on the sample, in about how many households in the town should Salvador expect more than one language to be spoken? Show your work. Possible work:

In the sample, $\frac{4}{20}$ of households speak more than one language at home.

$\frac{4}{20} \cdot 5{,}119 = \frac{20{,}476}{20}$, or $1{,}023\frac{4}{5}$


**SOLUTION** Salvador should expect about 1,024 households in the town to speak more than one language at home.

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## CLOSE EXIT TICKET

**9** See ***Connect to Culture*** to support student engagement. Students' solutions should show an understanding of:

- using a random sample to make an inference about the entire population.
- applying the fraction of homes that speak more than one language in the sample to the entire population as an estimate.

**Error Alert** If students state that $1{,}023\frac{4}{5}$ households speak more than one language at home, then have them interpret the calculation using the context of the problem to make sense of the answer. Ask students if it makes sense for Salvador to receive a fraction as an answer to his survey question and allow them to adjust their answers as needed.

# Practice Making Inferences from Samples About Populations

## Problem Notes

Assign **Practice Making Inferences from Samples About Populations** as extra practice in class or as homework.

1 **a.** ***Basic***

**b.** Students should understand that an inference based on a sample is an estimate of a larger population. ***Medium***



## Practice Making Inferences from Samples About Populations

➤ **Study the Example showing how to use a random sample to make an inference about a population. Then solve problems 1–4.**

**Example**

**Garrett is running for class president. He wants to know if he is likely to win, so his friend Jacob surveys a random sample of 10 students in his school. Of the 10 students, 7 say they will vote for Garrett. Suppose all 233 students in the school vote in the election for class president. About how many students should Garrett expect to vote for him?**

Since $\frac{7}{10}$ of students in the sample say they will vote for Garrett, he should expect about $\frac{7}{10}$ of the population to vote for him.

$\frac{7}{10}(233) = 163\frac{1}{10}$

Garrett should expect about 163 students to vote for him.

1 Jacob conducts another survey of students in the school in the Example. This time, he surveys a random sample of 30 students.

**a.** In Jacob's sample, 24 students say they will vote for Garrett. Based on this sample, about how many students in the school should Garrett expect to vote for him? Show your work. Possible work:

$\frac{24}{30} = \frac{8}{10}$, or 0.8

$0.8(233) = 186.4$

SOLUTION Garrett should expect about 186 students to vote for him.

**b.** Using Jacob's surveys, can Garrett know for certain how many students plan to vote for him? Explain why or why not.

No; Possible explanation: Making an inference is not the same as knowing for sure. His samples might not be perfectly representative of the population.

**Vocabulary**

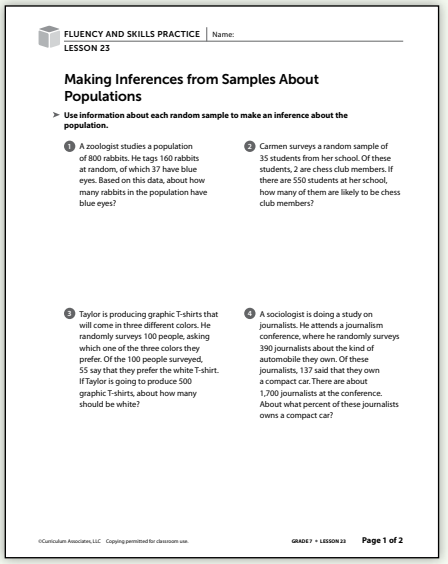
**random sample**
a sample in which every element in the population has an equal chance of being selected.



## Fluency & Skills Practice

### *Making Inferences from Samples About Populations*

In this activity, students make inferences about populations by interpreting survey reuslts from samples.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 23

Making Inferences from Samples About Populations

➤ Use information about each random sample to make an inference about the population.

1 A zoologist studies a population of 800 rabbits. He tags 160 rabbits at random, of which 37 have blue eyes. Based on this data, about how many rabbits in the population have blue eyes?

2 Carmen surveys a random sample of 35 students from her school. Of these students, 2 are chess club members. If there are 550 students at her school, how many of them are likely to be chess club members?

3 Taylor is producing graphic T-shirts that will come in three different colors. He randomly surveys 100 people, asking which one of the three colors they prefer. Of the 100 people surveyed, 55 say that they prefer the white T-shirt. If Taylor is going to produce 500 graphic T-shirts, about how many should be white?

4 A sociologist is doing a study on journalists. He attends a journalism conference, where he randomly surveys 390 journalists about the kind of automobile they own. Of these journalists, 137 said that they own a compact car. There are about 1,700 journalists at the conference. About what percent of these journalists owns a compact car?

2 Students should recognize that the sample size for the number of tickets purchased both Friday afternoon and Friday evening was 60 tickets. ***Challenge***

3 **B** **is correct.** This answer is the percent of students taking painting in the sample, which is about the same percentage as the population.

**E** **is correct.** This answer reflects that the inference is an estimate and uses the correct proportional relationship to make an inference about a total.

**A** is not correct. This answer cannot be determined from the results of the survey.

**C** is not correct. This answer results from using a sample size of 30 students rather than 60.

**D** is not correct. This answer does not reflect that an inference is an estimate of a population.

***Medium***

4 Students should recognize that the sample size is 80 students. ***Medium***



2 Mindy works at a movie theater. One Friday, she collects a random sample of the type of tickets sold in the afternoon and the evening. She estimates that when 400 tickets are sold on a Friday evening, about 100 of them will be senior tickets. Is Mindy's estimate reasonable? Explain.

| Time of Day | Adult | Senior |
|---|---|---|
| Afternoon | 12 | 48 |
| Evening | 45 | 15 |

Yes; Possible explanation: There were 60 tickets sold in the evening and 15 were senior tickets. Since $15 \div 60 = 0.25$, Mindy can reasonably estimate that when 400 tickets are sold, about 25% of them will be senior tickets. Since 0.25(400) equals 100, her estimate is reasonable.

3 Students at a certain high school have to take an arts or technology class. A random sample of 60 students from the high school are surveyed. Each student is asked which class they take. Based on the survey results, which of the following statements are true? Select all that apply.

| Class | Number of Students |
|---|---|
| Dance | 19 |
| Electronics | 8 |
| Music | 7 |
| Painting | 15 |
| Photography | 11 |

**A** There are many excellent dancers at the high school.

(B) About 25% of the students at the high school take painting.

**C** Of every 30 students in the high school, about 11 of them take photography.

**D** Next year, 7 out of every 60 students at the high school will take music.

(E) In a group of 120 students from the high school, about 16 of the students likely take electronics.

4 Moses writes a paper on fruit for health class. He surveys a random sample of students at his school about their favorite fruit. In his sample 46 students say strawberries. The other 34 students in his sample say a different fruit. There are 506 students in Moses's school. What inference can Moses make about the number of students in his school who would say strawberries are their favorite fruit? Show your work. Possible work:

$\frac{46}{46 + 34} = \frac{46}{80}$, or 57.5%

Find 57.5% of 506.

0.575(506) = 290.95

SOLUTION About 291 students would say strawberries are their favorite fruit.



## DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Apply It

### Levels 1–3: Reading/Speaking

Use **Notice and Wonder** to prepare students to respond to Apply It problem 7. Have students look at the plot. Help them read the title and encourage them to tell what they notice about the shape of the data. Then help them tell what they wonder. Next read the problem. Explain that *staff* are the people who work at the center. Emphasize the plural form of the verbs and pronoun as you ask: *What **are** the staff deciding? Who **do they** survey?* Have partners circle the answers in the text. Then ask them to tell what measures Riley and Brian used to infer. Next, ask: *Which measure gives a more reasonable inference?*

- *The ____ will provide a more reasonable ____.*

### Levels 2–4: Reading/Speaking

Use **Notice and Wonder** to prepare students to respond to Apply It problem 7. Have students look at the plot and tell what they notice about the data. Then have them tell what they wonder. Read the problem with students. Explain *staff* as needed. Allow time for students to connect the text to the plot. Then ask: *What are the staff deciding? Who do they survey?* Have partners refer to the text to answer. Ask them what they notice about the verbs. Then have them tell what measures Riley and Brian used to infer. Ask: *Which measure gives a more reasonable inference?* Next, help students build on and tell if the measure will always give a reasonable inference. Guide them to use *always* or *only sometimes* in their conversations.

### Levels 3–5: Reading/Speaking

Use **Notice and Wonder** to prepare students to respond to Apply It problem 7. Have students look at the plot and tell what they notice and wonder. Then have them read the problem. Call on volunteers to summarize the information from the first paragraph. Ask other students to listen for the meaning of the word *staff* and to check for the correct use of verbs and pronouns. Next, give students time to work on the problem.

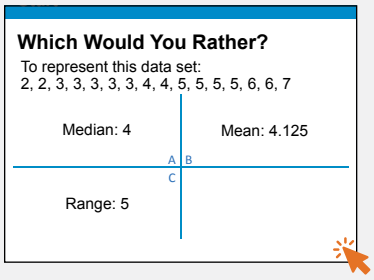
Have students share answers and discuss strategies. Then have students meet with students who used similar strategies. Ask questions to help them build on: *Do you think your strategy will always work? Why or why not?*

# **Develop** Making Inferences from Multiple Samples

## Purpose

- **Develop** strategies for making inferences from multiple samples.
- **Recognize** that multiple samples can provide more information about the population than a single sample does.

## START CONNECT TO PRIOR KNOWLEDGE




**Possible Solutions**

A; 4 is a number that is in the data set, so I would use the median.

B; There are no outliers, so I would use the mean.

**WHY?** Support students' facility in choosing a measure of central tendency to describe a data set.

## DEVELOP ACADEMIC LANGUAGE

**WHY?** Support students to build on a strategy they agree with by discussing when it works.

**HOW?** Have students share their strategies and why they chose them. Group students who chose similar strategies together. Prompt students to build onto ideas they agree with by giving reasons that the strategy will always work or will work sometimes. Have each group report back and discuss why strategies may always work or only sometimes work.

## TRY IT

SMP 1, 2, 4, 5, 6

### Make Sense of the Problem

Before students work on Try It, use **Three Reads** to help them make sense of the problem. Draw students' attention to the dot plot and help them understand how Mr. Seda's class found the data that is displayed in the plot.

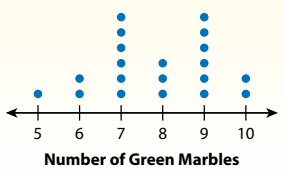
LESSON 23 | SESSION 3 ■ ■ ■ □

## **Develop** Making Inferences from Multiple Samples

➤ **Read and try to solve the problem below.**

Mr. Seda has a bag with 500 marbles. The marbles are either green or yellow. He has 20 students in his class take turns selecting 10 marbles from the bag without looking. Each student records the number of green marbles and then returns the marbles to the bag.

What is a reasonable estimate for the number of green marbles in the bag?




**TRY IT**

**Math Toolkit** double number lines, grid paper

**Possible work:**

**SAMPLE A**

Mean: $\frac{5 + 2(6) + 6(7) + 3(8) + 6(9) + 2(10)}{20} = \frac{157}{20}$

$= 7.85$

$\frac{7.85}{10} = \frac{392.5}{500}$

A reasonable estimate for the number of green marbles in the bag is 393.

**SAMPLE B**

There are 20 data points, so the median is between the 10th and 11th points. Both of those are 8. So, the median is 8.

$\frac{8}{10} = 80\%$

$0.8(500) = 400$

A reasonable estimate for the number of green marbles in the bag is 400.

**DISCUSS IT**

***Ask:*** Why did you choose that strategy to find a reasonable estimate?

***Share:*** I chose that strategy because . . .



## DISCUSS IT

SMP 2, 3, 6

### Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How can you use the results of one sample to make an inference?*
- *How does the data set shown by the dot plot represent multiple samples?*
- *How could you combine the results of multiple samples? Will that help you make an inference?*

**Common Misconception** Listen for students who make an inference using the result of just one student's sample. As students share their strategies, ask them if they have used all of the available information to make their inference. Guide them to consider applying a measure of central tendency to make an inference.

## Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- (common misconception) an inference made from a single sample
- double number lines relating the mean or median to the total number of marbles
- percent representing the mean or median number of green marbles in the samples
- equation relating the mean or median to the total number of marbles

## Facilitate Whole Class Discussion

Call on students to share selected strategies. As students present, remind them that a good explanation describes what you did and why you decided to do it.

Guide students to **Compare and Connect** the representations. Allow students individual think time about the various strategies before starting the discussion to compare them.

> **ASK** *How did each strategy summarize the information in the dot plot?*
>
> **LISTEN FOR** Each used a measure of central tendency, such as the mean or median, to represent the information with a single number.

## Analyze It

**If students presented these models,** have students connect these models to those presented in class.

**If no student presented at least one of these models,** have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How is the data represented in each analysis? How is it the same or different?*
>
> **LISTEN FOR** They both use a measure of central tendency to represent the data. One uses the median, which is greater than the mean used by the other.

**For the mean,** prompt students to think about how the mean represents the data from all of the samples.

- *Can the mean be a reasonable measure for all of the samples when it does not equal any of them? Why?*

**For the median,** prompt students to consider the meaning of the outliers in the box plot.

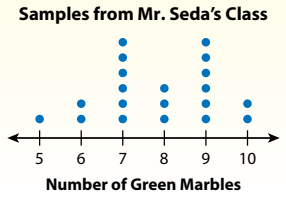
- *How can you use a box plot to reason about outliers?*
- *How does the median compare to the outliers?*



➤ **Explore different ways to make inferences about a population from multiple random samples.**

Mr. Seda has a bag with 500 marbles. Some marbles are green, and the rest are yellow. He has 20 students in his class each select 10 marbles from the bag without looking. Each student records the number of green marbles and then returns the marbles to the bag.

What is a reasonable estimate for the number of green marbles in the bag?




### Analyze It

You can use the mean of the samples to make an inference.

Each sample has 10 marbles. The mean of the number of green marbles in each sample is 7.85.

You can expect the fraction $\frac{\text{green marbles}}{\text{total marbles}}$ to be about the same for the population and the average of the samples.

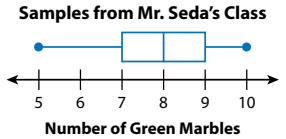
Let $g$ represent the number of green marbles in the population.

$$\frac{7.85}{10} = \frac{g}{500}$$

### Analyze It

You can use the median of the samples to make an inference.

Use a box plot to organize the samples.




Each sample has 10 marbles. The median number of green marbles in each sample is 8.

Let $g$ represent the number of green marbles in the population.

$$\frac{8}{10} = \frac{g}{500}$$

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**DIFFERENTIATION | EXTEND**

SMP 5

### Deepen Understanding

**Using Dot Plots and Box Plots Strategically**

Prompt students to think about what information can be understood about the data set by using different displays; such as a dot plot and a box plot.

> **ASK** *How does each plot show outliers in the data set?*
>
> **LISTEN FOR** In the dot plot, there will be numbers that have few or no dots between the main set of data and a number at either end that has dots. The box plot shows a longer whisker between the box and the endpoint when there is an outlier.

> **ASK** *What aspects of the data set are easier to see in the dot plot? In the box plot?*
>
> **LISTEN FOR** The number of data points, the symmetry of the data, and the data points that occur most often are easier to see in a dot plot. The median number in the data and the way the data are distributed are easier to see in a box plot.

**Generalize** Encourage students to describe when they might choose each plot to model and solve a problem. Knowing how the data will be analyzed to solve the problem will help them determine which model would be easier to use to find that information.

# Develop Making Inferences from Multiple Samples

## CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to use multiple samples to make an inference about a population.

Before students begin to record and expand on their work in Analyze It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

### Monitor and Confirm Understanding 1 – 3

- If a data set is not symmetrical, the median more accurately represents the data set than the mean. If the data set is symmetrical, both may represent the data set well.
- A single random sample may be an outlier and lead to an inaccurate conclusion about the population. Using multiple random samples helps prevent outliers from skewing conclusions.

### Facilitate Whole Class Discussion

4 Look for understanding that a single sample could be an outlier that leads to less accurate inference than is likely from several samples.

> **ASK** *How does this sample compare to the others? How will that affect an inference you make from that sample?*
>
> **LISTEN FOR** There are less green marbles in this sample than most others, so you would make an inference that there are less green marbles than there probably are.

5 Look for the idea that having data from multiple samples will result in a more accurate inference.

> **ASK** *How is the mean of multiple samples different from a single sample? How does that affect an inference made from the mean?*
>
> **LISTEN FOR** The mean of multiple samples is more likely to accurately represent a population than a single sample, so an inference based on the mean is also more likely to be accurate.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

## CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use multiple random samples to make an inference about a population.**

1 Look at the **Analyze Its**. How many green marbles do you estimate are in the bag when you use the mean? What about when you use the median? 393; 400

2 Why are the estimates different even though both use a measure of center?
Possible answer: Mean and median are different ways to measure the center of the data. The mean is the average of the data points, where the median tells you the halfway point of the data set.

3 Are both estimates reasonable? Why?
Yes; Possible explanation: Both the estimates are reasonable because both the mean and the median describe the center of the data well. The estimate based on the mean might be slightly better because the data is fairly symmetrical.

4 The first student who selected a sample got 6 green marbles in his sample. Why might making an inference from just this one sample be misleading?
Possible answer: Based on the other samples, 6 is not a typical value. It is significantly less than the center of the sample values, so it would likely give an estimate that is too low.

5 How can using more than one sample help you make a better inference about a population than using only one sample?
Possible answer: When you use only one sample, you do not know if it describes the whole population well. When you use multiple samples, any outliers are likely to get balanced out. So, then you can make an inference based on data that better describes the whole population.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.
Responses will vary. Check student responses.

495

## DIFFERENTIATION | RETEACH or REINFORCE

### Visual Model

**Relate data in a dot plot to variations in inferences using a number line.**

*If students are unsure about how each sample relates to the entire population, then use this activity to compare inferences based on different samples.*

- Assign each student a data point from the dot plot. Ask: *What does your data point represent?* [The number of green marbles chosen in a sample of 10 marbles.] Ask: *How can you use your data point to make an inference about the marbles in the bag?* [I can use proportional reasoning to find an equivalent ratio for 500 total marbles.]
- Display a number line from 0 to 500, marked at intervals of 50. Have each student use their data point to make an inference about the number of green marbles in the bag and plot their inference on the number line.
- Have students discuss what they notice about the different inferences and the variety among them.
- Plot the inferences based on the mean and median. Have students compare the inferences based on single samples to the inferences based on measures of central tendency to help them see why the latter are more likely to be accurate.

## Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, students' explanations or work may not include every calculation used to answer the question.

**7** See ***Connect to Culture*** to support student engagement. Students should notice that since the data is not symmetrical the median will more accurately reflect the data.

**8** **B** **is correct.** This answer uses Ramón's survey for the lesser estimate and Aimee's survey for the greater estimate.

**A** is not correct. This answer uses Caleb's survey for the lesser estimate and Aimee's survey for the greater estimate.

**C** is not correct. This answer is the number responding favorably from the surveys using a percent sign.

**D** is not correct. This answer uses the mean of the data as the lesser estimate and the median as the greater estimate.

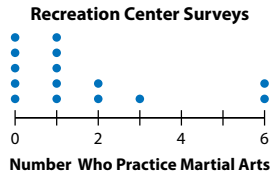


### Apply It

➤ **Use what you learned to solve these problems.**

**7** Staff at a recreation center are deciding whether to offer martial arts classes. Each of 15 staff members surveys a random sample of 8 recreation center members about whether they practice martial arts.

Riley uses the median of the results and infers that $\frac{1}{8}$ of the recreation center members practice martial arts. Brian uses the mean and infers that $\frac{1}{5}$ do. Explain why Riley's inference is more reasonable than Brian's.




Possible explanation: Since the data are not symmetrical, the median is a better measure of center and provides a better basis for an inference.

**8** Caleb, Aimee, and Ramón all survey random samples of 40 students in their school about whether they babysit their younger siblings. They use the results to make an inference about what percent of students in the school babysit their younger siblings. Which range of inferences is most reasonable?

**A** 27.5% to 30%

**B** 20% to 30%

**C** 8% to 12%

**D** 10.3% to 11%

| Student Who Conducted Survey | Number of Students Who Babysit Siblings |
|---|---|
| Caleb | 𝍸 𝍸 \| |
| Aimee | 𝍸 𝍸 \|\| |
| Ramón | 𝍸 \|\|\| |

**9** Mason has a bag with 171 beads. Some beads are red, some are orange, and the rest are yellow. He selects 8 beads without looking, counts how many are orange, and then puts the beads back in the bag. He does this 12 times. What is a reasonable estimate for the number of orange beads in the bag? Show your work. Possible work:

| Number of Orange Beads | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| Frequency | 2 | 5 | 3 | 1 | 1 |

Median: 3

$\frac{3}{8} = 0.375$ $0.375(171) = 64.125$

**SOLUTION** Possible answer: A reasonable estimate is 64 orange beads.



## CLOSE EXIT TICKET

**9** Students may use the mean, about 4.17, which would result in an inference of about 89, which is also reasonable. Students' solutions should show an understanding that:

- the mean or median can be used to consolidate the data from many samples.
- the ratio of the mean (or median) to the number of beads from each sample should be the same as the number of orange beads in the bag to the total number of beads in the bag.

**Error Alert** If students write the mean, or median, divided by the total number of samples,12, rather than by the number of beads in each sample, then suggest that students read the problem again and note what each number represents.

# Practice Making Inferences from Multiple Samples

## Problem Notes

Assign **Practice Making Inferences from Multiple Samples** as extra practice in class or as homework.

1. Students may use an approximation of the mean, such as 6.7 or 6.73, to make an inference about the number of green jewels in the box. ***Medium***
2. Students may consider different cases of how the mean and median relate to each other, such as when they are equal, when the difference between the mean and median is small, and when the difference is large and think about how those different cases lead to different inferences. ***Basic***

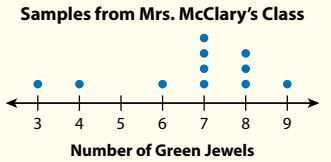


## Practice Making Inferences from Multiple Samples

➤ **Study the Example showing how to use multiple random samples to make an inference about a population. Then solve problems 1–4.**

### Example

**A box in Mrs. McClary's class has 200 plastic jewels. She has 11 students each select a random sample of 10 jewels, count the number of green jewels, and then return all 10 jewels to the box. Estimate the number of green jewels in the box.**




The median number of green jewels in each sample is 7. There are 10 jewels in a sample. There are 200 jewels in the population.

$\frac{7}{10} \times 200 = 140$

A reasonable estimate for the number of green jewels in the box is 140.

1. Use the mean to find an estimate for the number of green jewels in the box in the Example. Show your work. **Possible work:**

$\frac{3 + 4 + 6 + 4(7) + 3(8) + 9}{11} = 6.\overline{72}$

$\frac{6.\overline{72}}{10} \approx 67.2\%$

$0.672(200) = 134.4$

**SOLUTION** A reasonable estimate for the number of green jewels is 134.

2. Describe a situation when an inference from the mean and an inference from the median might be very different.

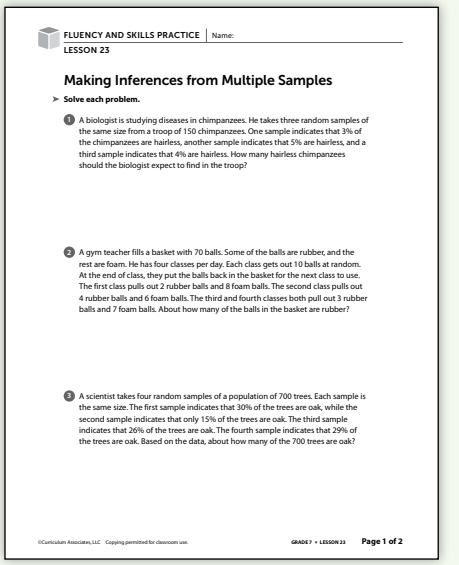
**Possible answer: They might be very different when there is an outlier. The outlier could change the mean of the samples a lot, but it would only change the median of the samples a little.**



## Fluency & Skills Practice

### *Making Inferences from Multiple Samples*

In this activity, students make inferences about populations by interpreting survey results from multiple samples.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 23

**Making Inferences from Multiple Samples**

➤ **Solve each problem.**

1. A biologist is studying diseases in chimpanzees. He takes three random samples of the same size from a troop of 150 chimpanzees. One sample indicates that 3% of the chimpanzees are hairless, another sample indicates that 5% are hairless, and a third sample indicates that 4% are hairless. How many hairless chimpanzees should the biologist expect to find in the troop?

2. A gym teacher fills a basket with 70 balls. Some of the balls are rubber, and the rest are foam. He has four classes per day. Each class gets out 10 balls at random. At the end of class, they put the balls back in the basket for the next class to use. The first class pulls out 2 rubber balls and 8 foam balls. The second class pulls out 4 rubber balls and 6 foam balls. The third and fourth classes both pull out 3 rubber balls and 7 foam balls. About how many of the balls in the basket are rubber?

3. A scientist takes four random samples of a population of 700 trees. Each sample is the same size. The first sample indicates that 30% of the trees are oak, while the second sample indicates that only 15% of the trees are oak. The third sample indicates that 26% of the trees are oak. The fourth sample indicates that 29% of the trees are oak. Based on the data, about how many of the 700 trees are oak?


GRADE 7 • LESSON 23 Page 1 of 2

3 Students may also find the mean, which is the same as the median and should result in the same inference. ***Medium***

4 **B** **is correct.** This answer reflects that the average number of blue marbles is greater than the average number of any of the other color marbles.

**D** **is correct.** This answer uses the sample with the least number of green marbles and the sample with the greatest number of green marbles to make inferences about the number of green marbles in the jar.

**A** is not correct. This answer is the total number of red marbles pulled for all four samples.

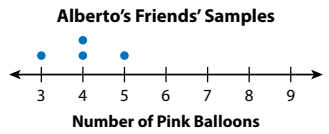
**C** is not correct. This answer does not reflect that the average number of white marbles in the samples is less than the average number of green marbles in the samples.

**E** is not correct. This answer looks only at the results from Sample 1.

***Challenge***



3 Alberto buys a bag of 600 balloons. Some of the balloons are pink, and the rest are green. He wants to estimate the number of pink balloons in the bag. He asks 4 friends to each select a random sample of 10 balloons. Each friend returns the sample before the next friend selects a sample.




Based on the data, what is a reasonable estimate for the number of pink balloons in the bag? Show your work.

Possible work:

The median is 4, and there are 10 balloons in each sample.

$\frac{4}{10} \cdot 600 = 240$

**SOLUTION** A reasonable estimate for the number of pink balloons is 240.

4 A jar holds 3,000 marbles of four different colors. Neena tries to guess the number of marbles of each color. She is allowed to take 4 random samples of 200 marbles each.

| | Blue | Green | Red | White |
|---|---|---|---|---|
| **Sample 1** | 82 | 42 | 36 | 40 |
| **Sample 2** | 70 | 51 | 49 | 30 |
| **Sample 3** | 58 | 65 | 45 | 32 |
| **Sample 4** | 65 | 62 | 38 | 35 |

Based on Neena's data, which of the following statements are true? Select all that apply.

**A** The total number of red marbles in the jar is approximately 168.

**B** There are likely more blue marbles in the jar than any other color.

**C** There are probably more white marbles than green marbles in the jar.

**D** Approximately 21% to 33% of the marbles in the jar are green.

**E** There are probably about twice as many blue marbles as green marbles in the jar.

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## DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

**Levels 1–3:** Speaking/Writing

Prepare students to respond to Apply It problem 8. Review the Math and Academic Vocabulary from the lesson. Then help students talk about the problems they solved in the lesson. Begin a **Co-Constructed Word Bank** with representative words, such as *survey, group, representative, data,* and *size*. Read the problem. Provide examples to help students choose a population, for example, a group of students or a group of tourists. Then help them come up with possible topics for a survey. Have students decide the name of the person or persons making the survey. Guide partners as they write a word problem about a survey, the population, the answers to the survey, and the inference.

**Levels 2–4:** Speaking/Writing

Prepare students to respond to Apply It problem 8. Review the vocabulary from the lesson. Then have students talk about the problems they solved in the lesson. Help them make a **Co-Constructed Word Bank** with representative words from the problems. Then read the problem with students. Have them brainstorm examples of populations. Next, have students turn to partners to choose a population and come up with possible topics for a survey. Have students state the survey, the population, the answers to the survey, and the inference. Then have them draft their word problems. Encourage students to meet with other partners to share their problems. Have partners review to check if the information is clear and complete.

**Levels 3–5:** Speaking/Writing

Prepare students to respond to Apply It problem 8. Review the vocabulary from the lesson. Then have students talk about the problems they solved in the lesson. Then have students read the problem. Have students make a **Co-Constructed Word Bank** with words they can use to write their problem. Next, have them choose a population and come up with possible topics for a survey. Have students identify the survey, the population, the answers to the survey, and the inference. Then have them draft the problem. Encourage students to meet with other partners to review their problems. Have partners give feedback on each other's problems and the use of precise language and then revise accordingly.

# **Refine** Reasoning About Random Samples

## Purpose

- **Refine** strategies for making inferences about a population from a single random sample and multiple samples.
- **Refine** understanding of how to use proportional relationships and reasoning to make inferences about populations from samples.

## START CHECK FOR UNDERSTANDING

A jar has 150 buttons. In a randomly selected sample of 10 buttons, 4 were red and 6 were blue. About how many buttons in the whole jar are red?

**Solution**

60

**WHY?** Confirm students' understanding of using a sample to estimate population totals, identifying common errors to address as needed.

## MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 23 | SESSION 4

## **Refine** Reasoning About Random Samples

➤ **Complete the Example below. Then solve problems 1–8.**

### Example

**There are 150 cards in a bag. Each card is either black or red. Carter selects 5 cards at random, records the number of red cards, and returns the cards. He does this until he has 12 samples of 5 cards.**

**Carter's Results: 0, 1, 1, 1, 2, 2, 2, 2, 3, 3, 5**

**Kwame does the same, but he uses 10 cards in his samples.**

**Kwame's Results: 1, 2, 3, 3, 4, 4, 4, 4, 5, 5, 7**

**What is a reasonable estimate for the number of red cards in the bag?**

Look at how you could show your work using percents.

The median number of red cards in Carter's samples is 2.

$\frac{2}{5} = 40\%$

The median number of red cards in Kwame's samples is 4.

$\frac{4}{10} = 40\%$

**SOLUTION** A reasonable estimate for the number of red cards is 60.

**CONSIDER THIS . . .**
To find the percent of cards that are red, divide the number that are red by the number of cards in the sample.

**PAIR/SHARE**
Why would it be inaccurate to make an estimate using the median of Carter and Kwame's combined samples?

### Apply It

1 A school has 800 students. The staff plans to order a bag for each student. They ask 120 randomly selected students which color bag they prefer. Based on this sample, how many bags of each color should the staff order? Show your work. Possible work:

| Color | Quantity |
|---|---|
| Blue | 48 |
| Black | 54 |
| Gray | 18 |

Blue: $\frac{48}{120} \cdot 800 = 320$ Gray: $\frac{18}{120} \cdot 800 = 120$

Black: $\frac{54}{120} \cdot 800 = 360$

**SOLUTION** The staff should order 320 blue, 360 black, and 120 gray bags.

**CONSIDER THIS . . .**
The sum of the quantities of each color bag ordered should be 800.

**PAIR/SHARE**
How could you solve this problem with a different strategy?



## START ERROR ANALYSIS

| If the error is . . . | Students may . . . | To support understanding . . . |
|---|---|---|
| 4 | have chosen the number of red buttons in the sample rather than finding the number of red buttons in the jar. | Have students model the problem with a double number line. Prompt students to articulate how the quantities 4, 10, and 150 are related. |
| 40 | have calculated the percent of red buttons from the sample. | Ask students to represent 40% as a fraction and tell what the 40 and 100 mean in the context of the problem. Then ask them to think about how to use a fraction equivalent to the percent to predict the number of red buttons from the jar of 150 buttons. |
| 90 | have made an inference about the number of blue buttons in the jar instead of red. | Ask students to reread the problem carefully, striking through information about blue buttons and circling information about red buttons and the total number of buttons. |

## Example

Guide students in understanding the Example. Ask:

- *How can you find the median number of red cards in Carter's sample? In Kwame's sample?*
- *Why is the denominator for Carter's sample, 5, different from the denominator for Kwame's sample, 10?*
- *How did you apply the median of 40% to estimate the number of red cards in the bag?*

Help all students focus on the Example and responses to the questions by reminding them that they can ask questions about ideas that are not clear during math discussions.

Look for understanding that they can use proportional reasoning to make inferences about the number of red cards in the bag.

## Apply It

**1** Students should recognize that they need to divide the quantity of tote bags in each color by the total quantity of all tote bags in order to make their inferences. ***DOK 2***

**2** Students may also use the median to make an inference. Since the median is 6, they may estimate that 30%, or 240 balls, will be purple. ***DOK 2***

**3** See ***Connect to Culture*** to support student engagement.

**D** **is correct.** This answer applies the ratio $\frac{9}{10}$ to the population of 140 to make an inference.

**A** is not correct. This answer is the number of students in the survey who have a cat.

**B** is not correct. This answer uses the number of survey answers, 25, as the sample size.

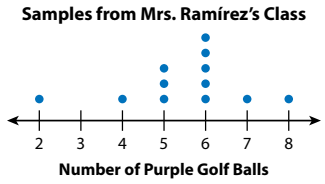
**C** is not correct. This answer is an inference about the number of students expected to have a dog.

***DOK 3***



**2** A box contains a mix of 800 golf balls. Some are white and the rest are purple. Each student in Mrs. Ramírez's class selects a random sample of 20 golf balls from the box, counts the purple golf balls, and returns the sample to the box.

**CONSIDER THIS . . .**
You could use the mean or the median to make an estimate.




What is a reasonable estimate for the number of purple golf balls in the box? Show your work.

Possible work:

Mean: $\frac{2 + 4 + 3(5) + 5(6) + 7 + 8}{12} = 5.5$

$\frac{5.5}{20}(800) = 220$

**PAIR/SHARE**
How would your confidence in your estimate change if there were 25 students in Ms. Ramírez's class?

**SOLUTION** Possible answer: A reasonable estimate for the number of purple golf balls in the box is 220.

**3** Mr. Shen selects 20 of his 140 students at random to survey about their pets. His survey indicates that 8 students have a dog, 9 students have a cat, 3 students have another kind of pet, and 5 students have no pet. How many of his students should Mr. Shen expect to have a cat?

**CONSIDER THIS . . .**
Some students have more than one pet.

**A** 9

**B** 50

**C** 56

**D** 63

Nadia chose B as the correct answer. How might she have gotten that answer?

Possible answer: Nadia may have found 8 + 9 + 3 + 5 and used 25 as the sample size. That counts students with more than one pet more than once.

**PAIR/SHARE**
How can you check your answer?



## GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

**Approaching Proficiency**
- **RETEACH** Visual Model
- **REINFORCE** Problems 4, 5, 6

**Meeting Proficiency**
- **REINFORCE** Problems 4–7

**Extending Beyond Proficiency**
- **REINFORCE** Problems 4–7
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

**Resources for Differentiation** are found on the next page.

# Refine Reasoning About Random Samples

## Apply It

4 Students may conclude that since the results of Esteban's sample are more widely spread out, the inferences that could be made from the sample would less accurately describe the actual number of white marbles in the bag. ***DOK 3***

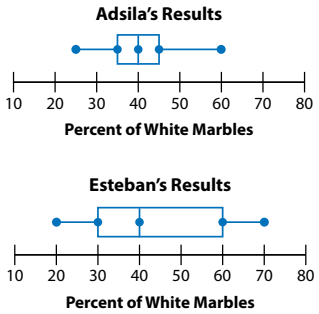
5 Students may use the median to infer that 540 people in the population have brown eyes. ***DOK 2***

6 **a.** Students may look at the range between each of the samples for each color to think about the variation. The variation for blue is 17, for green is 8, for red is 18, and for white is 11. ***DOK 1***

**b.** Students may reason that by having less variation in the data set, the results of an inference based on the data will more accurately represent the actual population. ***DOK 3***

4 Adsila and Esteban have a bag containing 250 marbles. Each marble is red, black, or white. Adsila and Esteban each select a sample of marbles from the bag, record the percent that are white, and return all the marbles to the bag. They each collect 15 samples. Their results are shown in the box plots. If you could only use one sample to make an estimate of the number of white marbles in the bag, would you rather use one of Adsila's samples or one of Esteban's? Why?

Adsila's; Possible explanation: The data from her samples are more tightly clustered around the center, so there is less chance of getting a misleading sample.




5 Emma has 3 random samples of the same size from a population of 650 people. In one sample, 78% of people have brown eyes. In another, 85% have brown eyes. In the third, 83% have brown eyes. About how many people with brown eyes should she expect are in the population? Show your work. Possible work:

The mean is 82%.

0.82(650) = 533

**SOLUTION** Possible answer: She should expect about 533 members of the population to have brown eyes.

6 A jar contains 1,800 marbles. James enters a contest to guess the number of any one color of marbles in the jar. He is allowed to take 4 random samples of 200 marbles each.

| | Blue | Green | Red | White |
|---|---|---|---|---|
| **Sample 1** | 41 | 37 | 63 | 49 |
| **Sample 2** | 58 | 34 | 54 | 54 |
| **Sample 3** | 52 | 29 | 59 | 60 |
| **Sample 4** | 47 | 30 | 72 | 51 |

**a.** In James's samples, the color red has the most variation and the color green has the least variation.

**b.** James can choose which color of marble to make his guess about. Which color do you think he should choose? Why?

Possible answer: James should choose green; Green has the least variation in his samples. So, his inference about green is most likely to be close to the real number.



## DIFFERENTIATION

### RETEACH

### Visual Model

**Compare random samples of different sizes.**

*Students approaching proficiency with making inferences from multiple random samples of a population will benefit from exploring the effect sample size has on accuracy of inferences.*

**Materials** For display: 100 unit cubes (20 blue, 80 assorted other colors), 1 paper bag

- Place the 100 cubes in the paper bag. Display a frequency table with spaces for 0–4 blue cubes. Have 5 students select a sample of 4 cubes, record the number of blue cubes, and return the cubes to the bag. Mix the cubes in the bag before each student's selection.
- When all students have selected their samples, have students work together to find the median of the data and to make an inference from the samples to the bag of 100 cubes. Have a volunteer record their work.
- Next, display a frequency table with spaces for 0–10 blue cubes. Have 5 students select a sample of 10 cubes, record the number of blue cubes in the table, and return the cubes to the bag. Mix the cubes in the bag before each student's selection.
- When all students have selected their samples, have students work together to find the median of the data and to make inferences from the samples to the bag of 100 cubes. Have a volunteer record their work.
- Compare the data and inferences from both samples. Ask students to consider how changing the sample size changed the data they could work with. Would they have been able to make the inference they did using a sample size of 4?
- To extend the activity, ask students to choose another sample size that they think will let them make a more precise inference. Then repeat.

**7** **B and D are correct.** Students may solve the problem by writing the population of each type of fish as a percent of the sample size and comparing the percents as needed.

**A** is not correct. This answer is the total number of fish that were sampled.

**C** is not correct. This answer switches the whitefish and trout in its comparison.

**E** is not correct. This answer does not take into account that the sample size in summer is twice as large as the sample size in winter.

*DOK 3*

## CLOSE EXIT TICKET

**8** **Math Journal** Look for a sample size, the results of the sample, and the size of the general population in the word problem. Also look for proportional reasoning to make an inference.

**Common Misconception** If students do not use language that indicates the inference is an estimate, such as *about,* then ask if they know for certain what the population is like based on an inference from a sample.

## End of Lesson Checklist

**INTERACTIVE GLOSSARY** Support students by suggesting they review the Model It from Session 2.

**SELF CHECK** Have students review and check off any new skills on the Unit 5 Opener.



**7** Aniyah studies the fish populations in a lake. She catches fish, tags them, identifies the type, and returns them to the lake. She takes two random samples in the winter and two in the summer. She organizes her data in the table at the right. Which inferences about the fish populations in the lake are reasonable? Select all that apply.

| | Trout | Whitefish | Walleye | Sample Size |
|---|---|---|---|---|
| Winter | 42 | 44 | 14 | 100 |
| | 46 | 42 | 12 | 100 |
| Summer | 91 | 84 | 25 | 200 |
| | 85 | 89 | 26 | 200 |


**A** The total number of fish in the lake is 600.

**B** The walleye population comprises anywhere from 12% to 14% of the total population in both the winter and summer.

**C** The number of whitefish in the lake is greater than the number of trout.

**D** The ratios of the populations of trout, whitefish, and walleye are relatively stable from the winter to the summer.

**E** The populations of trout, whitefish, and walleye are approximately twice as large in the summer as in the winter.

**8** **Math Journal** Write a word problem about taking a random sample and making an inference about the population. Then explain how to solve your problem.

Possible answer: Ian surveys 20 randomly selected students at his school about their favorite type of music. There are 7 students who say their favorite music type is pop and 600 students in the school. About how many students in the school should Ian expect have pop as their favorite type of music?

To solve, multiply $\frac{7}{20}$ by 600 to get 210, so the answer is about 210 students.

## End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *inference*. Tell what you do when you make an *inference* about a population from a random sample.
- ☐ **SELF CHECK** Go back to the Unit 5 Opener and see what you can check off.



## REINFORCE

### Problems 4–7

**Use random samples to make inferences about a population.**

*Students meeting proficiency will benefit from additional work with reasoning about random samples by solving problems in a variety of formats.*

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

## EXTEND

### Challenge

**Match inferences and random samples.**

*Students extending beyond proficiency will benefit from thinking of situations with matching samples and inferences.*

- Have each student write an inference about a population of colored marbles on a sheet of paper. Then have them give a sample that supports the inference and two samples do not support the inference on separate sheets of paper.
- Have students trade papers with a partner. Each student must determine which sample supports the inference.
- As time allows, have students repeat the process, trading papers with other partners.

## PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.