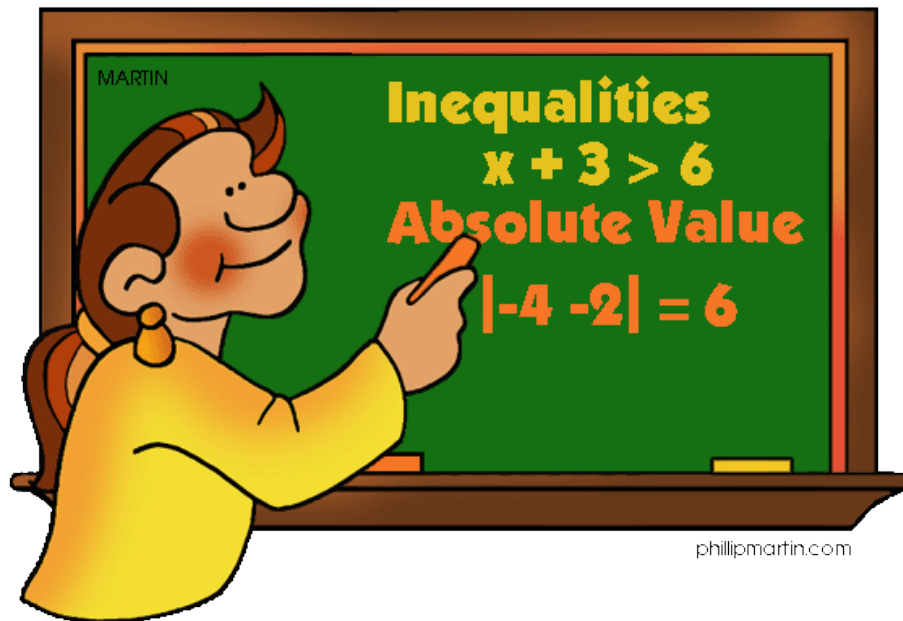# A2T

# Packet #2:    Absolute Value Equations and Inequalities; Quadratic Inequalities; Rational Inequalities



**Name:**_____

**Teacher:**_____

**Pd:** _____

# Table of Contents

# Day 1: Solving Compound Inequalities

**Draw a graph for each inequality.**

1) $x \leq 6$
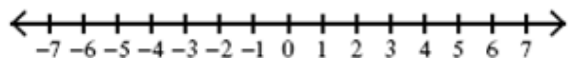
2) $b > -6$
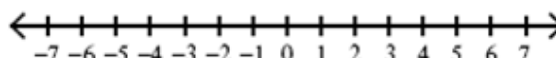
**From each graph, write as an inequality and write in interval notation.**

3)
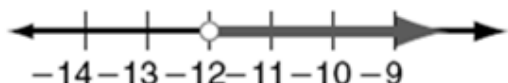
(a) Inequality: _____

(b) Interval Notation: _____

4)

(a) Inequality: _____

(b) Interval Notation: _____

---

**You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region.**

Ex 1: Graph $(x > -1)$ $and$ $(x \leq 3)$

X > -1

X ≤ 3
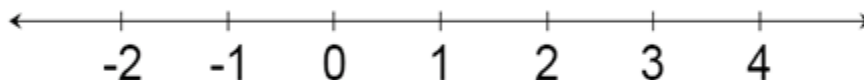
Intersection =
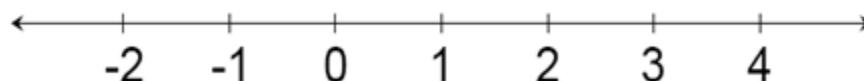
**How do you think we can write this solution as an inequality?** _____

1

**You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combine regions are called the <u>union</u> and show the numbers that are solutions of either inequality.**

Ex 2:  Graph $(x \leq 0)\ or\ (x > 3)$
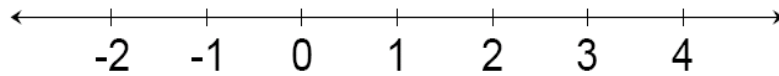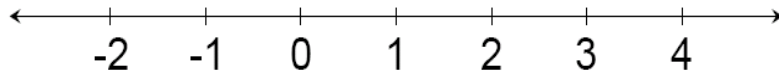


**Union =**
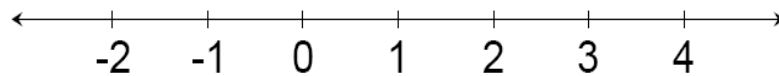


**How do you think we can write this solution as an inequality?** _____

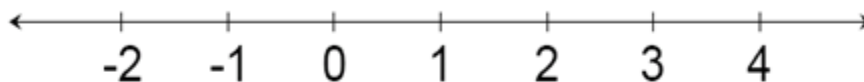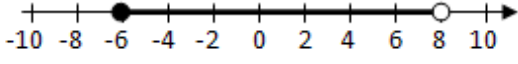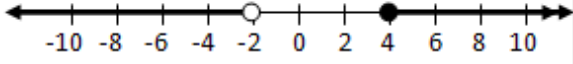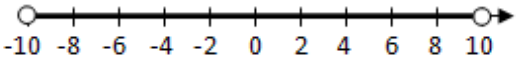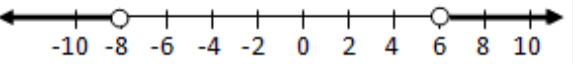<u>**Compound Inequalities**</u>
The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a **<u>compound inequality.</u>**

NOTE the following symbols:

$\Lambda$ means AND                                         $V$ means OR

# Practice: Writing Compound Inequalities from a Graph

| | Graphed Interval | Set Builder Notation | Interval Notation |
|---|---|---|---|
| 1) | -10 -8 -6 -4 -2 0 2 4 6 8 10 | | |
| 2) | -10 -8 -6 -4 -2 0 2 4 6 8 10 | | |
| 3) | -10 -8 -6 -4 -2 0 2 4 6 8 10 | | |
| 4) | -10 -8 -6 -4 -2 0 2 4 6 8 10 | | |

---

**Example 3:** Solve the following compound inequality and graph the solution.

$$10 \leq 2 + 2x < 18$$

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

2  3  4  5  6  7  8  9  10  11  12  13  14

---
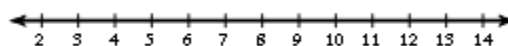
$$1 \leq \frac{v}{5} \leq 2$$

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

0  1  2  3  4  5  6  7  8  9  10  11  12

3
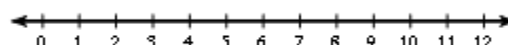
**Example 4:** Solve the following compound inequality and graph the solution.

$2k - 2 \geq -2$ and $-6k + 9 \geq 3$

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____



---
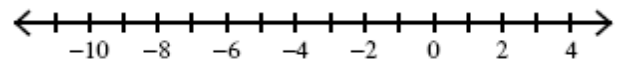
$6p + 10 \geq -2$ and $9 - 4p \geq -15$

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____



4
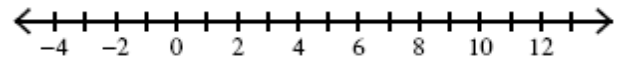
**Example 5:** Solve the following compound inequality and graph the solution.

$-6m - 1 > -43$ or $8 - 8m \le -56$

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

$$\begin{array}{cccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}$$

---
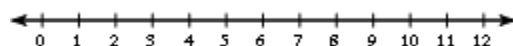
$-6r + 3 > 15$ or $4r + 4 > 16$

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

$$\begin{array}{ccccccccccccccc} -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$
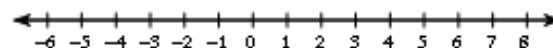
## Challenge

What is the sum of the solutions to the equation $|-5x - 5| = 35$?

(A) $-14$      (B) $-6$      (C) $-2$

(D) $2$      (E) $14$

5
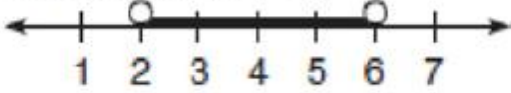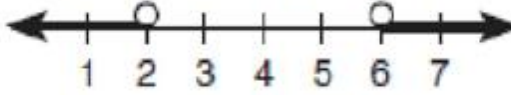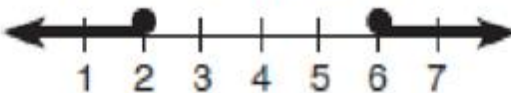
# Summary

There are two types of compound inequalities: AND statements and OR statements. The chart below shows their similarities and differences.

| | AND Statements | | OR Statements |
|---|---|---|---|

**Contrast**

| | How the compound is written | | |
|---|---|---|---|
| $x > 4$ AND $x < 9$ or $4 < x < 9$ | ← | → | $x < 4$ OR $x > 9$ |

| | How the solution looks | | |
|---|---|---|---|
| (number line with 4 and 9) | ← | → | (number line with 4 and 9) |

| | Solution | | |
|---|---|---|---|
| values found in *both* inequalities | ← | → | values found in *either* inequality |

**Compare**

Both types of compound inequalities are solved as two simple inequalities.

Solutions to both types are shown using a number line.

## Exit Ticket

Which graph represents the solution set for
$2x - 4 \leq 8$ and $x + 5 \geq 7$?

1) (number line: open circles at 2 and 6, segment between) 1 2 3 4 5 6 7

2) (number line: closed circles at 2 and 6, segment between) 1 2 3 4 5 6 7

3) (number line: open circles at 2 and 6, shading outward) 1 2 3 4 5 6 7

4) (number line: closed circles at 2 and 6, shading outward) 1 2 3 4 5 6 7

6

Algebra 2 & Trigonmetry

Name_____

# Ms. Williams/Compound Inequalitites

Date_____ Period_____

**Solve each compound inequality and graph its solution.**

1) $-7x - 5 \geq 44$ or $2x - 4 > -14$
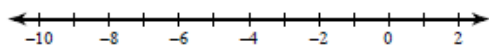


2) $-21 < -8k + 3 \leq 11$
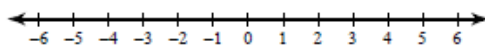


3) $6 - n \geq 4$ or $9n + 7 \geq 97$
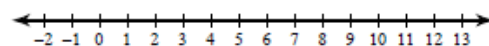


4) $4 - 3n \geq 1$ and $n - 8 \geq -12$
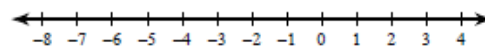


5) $x + 2 < 5$ or $\dfrac{x}{5} > 2$
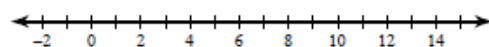


6) $3p \geq -27$ and $\dfrac{p}{6} \leq -1$
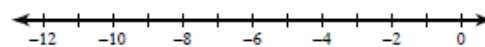
**Directions:** Write a compound inequality from the graph provided and express solution in interval notation.
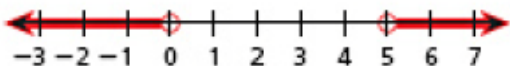
6) 
```
←+—+—+—○—+—+—+—+—○—+—+→
 -3 -2 -1  0  1  2  3  4  5  6  7
```

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

7) 
```
          5
←+—+—○—+—+—+—+—+—●—+—+—+→
-10 -8 -6 -4 -2  0  2  4  6  8  10
```

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

8) 
```
←+—+—+—○—+—●—+—+—+—+—+→
 -3 -2 -1  0  1  2  3  4  5  6  7
```

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

9) 
```
←+—+—+—○—+—+—○—+—+—+→
 -5 -4 -3 -2 -1  0  1  2  3  4  5
```

Inequality: _____

Interval Notation: _____

Set Builder Notation: _____

8
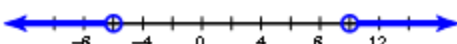
# Day 2: Solving Absolute Value Equations

<u>**Warm – Up:**</u>

Solve each compound inequality and graph its solution.

$$n - 5 > 5 \text{ or } \frac{n}{6} < -1$$
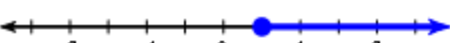
A) $n > 10$ or $n < -6$ :
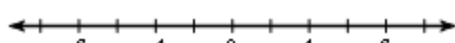
B) $n \leq 0$ or $n \geq 2$ :

C) $n \geq 2$ :

D) No solution. :

---

## Graphical Definition of Absolute Value: The absolute value of a number is the number's distance from zero on the number line.

How far away from zero are the following?
-4 and 4

Notice that both 4 and -4 are a distance of 4 units away from zero. This means that |4| and |-4| are both 4.
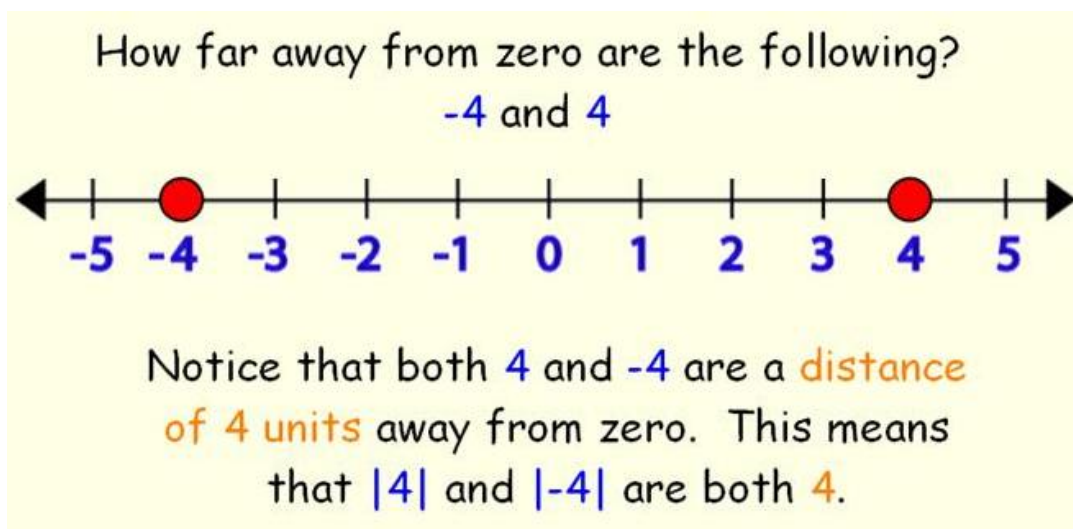
<u>Examples:</u>

$|-5| = \_\_$ $\qquad |5| = \_\_\_ \qquad |0| = \_\_\_$

Please note that "just making the inside positive" does no work when there are algebraic expressions inside the absolute value symbols.

Examples:

| $\lvert -2x \rvert$ | $\lvert x + 5 \rvert$ | $\lvert x - 5 \rvert$ |
| Does not always equal $2x$ | Does not always equal $x + 5$ | Generally does not equal $x + 5$ |

**Example 1**:
*Determine, by inspection (this means just looking at what you have and figuring it out!) the TWO solutions to each of the following absolute value equations.*
*Think about what values would have to be inside the absolute value symbol in order to make the statement true.*

| a) $\lvert x \rvert = 6$ | b) $\lvert x - 6 \rvert = 10$ | c) $\lvert x + 1 \rvert = 4$ | d) $\lvert x \rvert = -8$ |
|---|---|---|---|
| {___,___} | {___,___} | {___,___} | {___,___} |

## Solving Absolute Value Equations Algebraically

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

**Example 2: What is the solution set of the equation $\lvert 2x - 3 \rvert = 17$?**

**Practice:  What is the solution set of the equation  $|4x - 2| = 10$?**

---

**Example 3: What is the solution set of the equation  $|5n - 4| + 18 = 8$?**

**Example 4: What is the solution set of the equation $|2x + 5| = x + 4$ ?**

***

**Practice: What is the solution set of the equation $|4 - x| = 3x$ ?**

**Example 5: What is the solution set of the equation** $|y - 4| - 3y = 6$ ?

Practice: What is the solution set of the equation $|x + 6| - 18 = 2x$ ?

**Challenge:**

**Solve |x - 3| = |x + 2|**

**Summary:**

Example: $|2x - 1| + 3 = 4x$

| | |
|---|---|
| $|2x - 1| + 3 = 4x$ | Isolate the absolute value. |
| $\underline{\phantom{|2x-1|+3} -3 = -3}$ | |
| $|2x - 1| = 4x - 3$ | |

| | | |
|---|---|---|
| $2x - 1 = 4x - 3$ | $2x - 1 = -(4x - 3)$ | Change to two derived equations. Please note at this point there is no more absolute value! Solve. |
| $\underline{+1 = +1}$ | $2x - 1 = -4x + 3$ | |
| $2x = 4x - 2$ | $\underline{+1 = +1}$ | |
| $\underline{-4x = -4x}$ | $2x = -4x + 4$ | |
| $\dfrac{-2x}{-2} = \dfrac{-2}{-2}$ | $\underline{+4x = +4x}$ | |
| $x = 1$ | $\dfrac{6x}{6} = \dfrac{4}{6}$ | |
| | $x = \dfrac{2}{3}$ | |

| Check | Check | Check each in the |
|---|---|---|
| $|2x - 1| + 3 = 4x$ | $|2x - 1| + 3 = 4x$ | ORIGINAL equation. |
| $|2(1) - 1| + 3 = 4(1)$ | $\left|2\left(\dfrac{2}{3}\right) - 1\right| + 3 = 4\left(\dfrac{2}{3}\right)$ | Use parentheses around each substitution. |
| $|2 - 1| + 3 = 4$ | $\left|\dfrac{4}{3} - 1\right| + 3 = \dfrac{8}{3}$ | |
| $|1| + 3 = 4$ | $\left|\dfrac{1}{3}\right| + 3 = \dfrac{8}{3}$ | |
| $1 + 3 = 4$ | $\dfrac{1}{3} + 3 = \dfrac{8}{3}$ | |
| $4 = 4$ | $\dfrac{10}{3} = \dfrac{8}{3}$ | |
| | REJECT | |

**In this case, one solution gets rejected. The solution set is {4}.**

---

**Exit Ticket:**

What is the solution set of $|4n + 8| = 16$?

(1) $\{-6\}$            (3) $\{-6, 2\}$

(2) $\{2\}$              (4) $\{\}$

# Day 3: Solving Absolute Value Inequalities

Solve for $x$: $|2x - 6| - x = 3$.
(1) $x = 1$ or $x = 9$    (3) $x = 1$
(2) $x = 9$ or $x = -1$    (4) $x = 9$

---

Yesterday we discussed that the absolute value of a number is the number's distance from zero on the number line.

So, |a| is defined as the distance from a to 0.

$|x| < 4$                                                                    $|x| > 4$
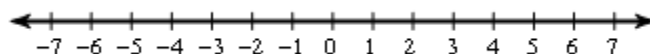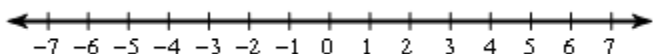
                                          

So,

---

Use these facts to solve:
- Less ThAND
  - Re-write as a compound **AND** statement
  - Interval and Graph will be between two numbers

- GreatOR
  - Re-write as an **OR** statement
  - Interval and Graph will be Union of two sets

Solve and graph each of the following inequalities:
Example 1:  $|2x + 3| < 7$

| Step 1: Is the absolute value isolated? | |
| Step 2: Is the number on the other side negative? | |
| Step 3: Set up a compound inequality | |
| Step 4: Solve the compound inequality and graph. $\longleftrightarrow$ | |

Example 2:  $\left| \frac{n}{2} - 4 \right| \geq 3$

| Step 1: Is the absolute value isolated? | |
| Step 2: Is the number on the other side negative? | |
| Step 3: Set up a compound inequality | |
| Step 4: Solve the compound inequality and graph. $\longleftrightarrow$ | |

16

Solve and graph each of the following inequalities:

Practice:  $|x - 2| > 7$



$\left|\dfrac{5d\ +\ 2}{3}\right| \le 4$

Solve and graph each of the following inequalities:

## Example 3: $7 + |x - 2| < 18$

| Step 1: Is the absolute value isolated? | |
|---|---|
| Step 2: Is the number on the other side negative? | |
| Step 3: Set up a compound inequality | |
| Step 4: Solve the compound inequality and graph. | |

## Example 2: $-2|3x + 2| \geq -16$

| Step 1: Is the absolute value isolated? | |
|---|---|
| Step 2: Is the number on the other side negative? | |
| Step 3: Set up a compound inequality | |
| Step 4: Solve the compound inequality and graph. | |

Solve and graph each of the following inequalities:

Practice: $-3|x - 8| < 24$



$4 + |3m - 6| < 22$

## Special Cases:

- o If the **Absolute value** is greater than a negative number
  - o This is ALWAYS TRUE
  - o Solution is (-∞, ∞) or All Real Numbers

## $|3x - 4| + 9 > 5$

| Step 1: Is the absolute value isolated? | |
|---|---|
| Step 2: Is the number on the other side negative? | |

- o If the **Absolute value** is less than zero
  - o This is NEVER TRUE
  - o No Solution or { }

## $|5x + 6| + 4 < 1$

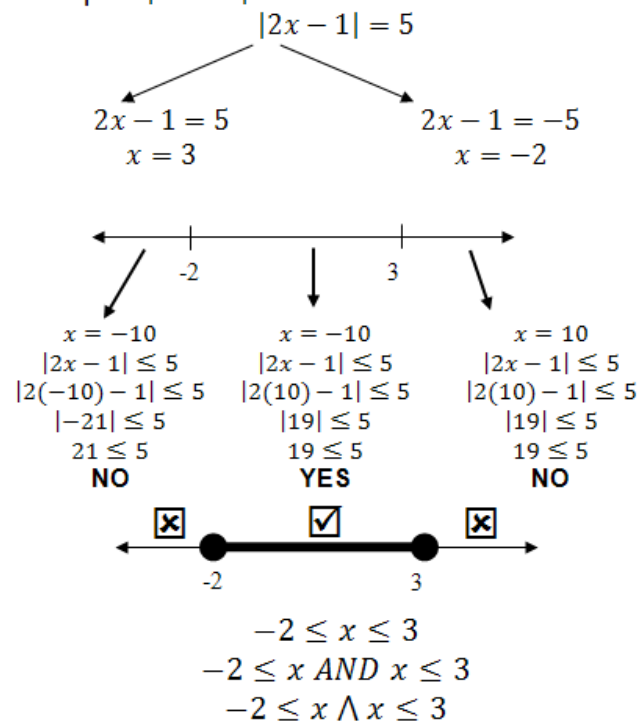| Step 1: Is the absolute value isolated? | |
|---|---|
| Step 2: Is the number on the other side negative? | |

## Challenge

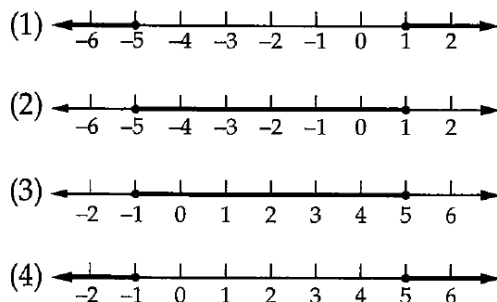Solve and graph the following inequality.

$$x^2 - x - 12 \geq 0$$

## Summary:

Example: $|2x - 1| \leq 5$

$$|2x - 1| = 5$$

$$2x - 1 = 5 \qquad\qquad 2x - 1 = -5$$
$$x = 3 \qquad\qquad\qquad x = -2$$

-2          3

| $x = -10$ | $x = -10$ | $x = 10$ |
|---|---|---|
| $|2x - 1| \leq 5$ | $|2x - 1| \leq 5$ | $|2x - 1| \leq 5$ |
| $|2(-10) - 1| \leq 5$ | $|2(10) - 1| \leq 5$ | $|2(10) - 1| \leq 5$ |
| $|-21| \leq 5$ | $|19| \leq 5$ | $|19| \leq 5$ |
| $21 \leq 5$ | $19 \leq 5$ | $19 \leq 5$ |
| **NO** | **YES** | **NO** |

-2          3

$$-2 \leq x \leq 3$$
$$-2 \leq x \ AND \ x \leq 3$$
$$-2 \leq x \wedge x \leq 3$$

1. PRETEND the problem is an absolute value equation. This gives you the BOUNDARY POINTS of the solution set. (Do not check... I will not give you a problem that doesn't "work".)

2. Graph these boundary points on a number line.

3. Pick ANY value in EACH REGION of the number line and test it in the ORIGINAL inequality. If it is true, put a check. If it is false, put an "x".

4. The solution set is where the "yes" is/are. Graph it, and write the solution set using AND or OR notation, whichever is appropriate for the problem.

5. Use open circles for < and closed circles for ≤.

## Exit Ticket

Which is the graph of the solution set of $|10x - 20| \geq 30$?

(1) ![number line from -6 to 2]
-6  -5  -4  -3  -2  -1  0  1  2

(2) ![number line from -6 to 2]
-6  -5  -4  -3  -2  -1  0  1  2

(3) ![number line from -2 to 6]
-2  -1  0  1  2  3  4  5  6

(4) ![number line from -2 to 6]
-2  -1  0  1  2  3  4  5  6

21

The solution set of $|8x - 4| > 20$ is
(1) $\{x|\ x < -2 \text{ or } x > 3\}$
(2) $\{x|\ x < -3 \text{ or } x > 2\}$
(3) $\{x|\ -2 < x < 3\}$
(4) $\{x|\ -3 < x < 2\}$

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## Solving Quadratic inequalities by factoring

Set the quadratic to 0, with the 0 on the RIGHT side of the inequality.
Factor the quadratic and solve it.

- If the inequality is < or ≤, then the solution set is all of the values BETWEEN the roots.

    $root_1 < x < root_2$
    $root_1 \leq x \leq root_2$

- If the inequality is >, then the solution set is all of the values OUTSIDE OF the roots.
    $x < root_1 \ \ OR \ \ x > root_2$
    $x \leq root_1 \ \ OR \ \ x \geq root_2$

Example:  What is the solution set of the inequality $-2x^2 + 3x + 5 > 0$ ?
$-2x^2 + 3x + 5 > 0$
$-1(2x^2 - 3x - 5) > 0$
$2x^2 - 3x - 5 < 0$
$(2x - 5)(x + 1) < 0$
$roots\ are \frac{5}{2} and -1$
$\{x|-1 < x < 2.5\}$

Quadratic Inequalities are solved and graphed almost exactly like absolute value inequalities.

Find the solution set for the inequality and graph the solution set.

$$x^2 - 2x - 15 < 0$$

| | |
|---|---|
| **Step 1:** Is the quadratic inequality in standard form ? | |
| **Step 2:** Factor the quadratic and solve the quadratic for the roots.<br>These will be the _critical_ points. | |
| **Step 3:** Is the inequality a conjunction or a disjunction? |  |
| **Step 4: Write your answer** | |

**Practice:** Find the solution set for the inequality and graph the solution set.

$$x^2 + 6x + 8 \geq 0$$

$x^2 - 2x - 20 > 4$

| Step 1: Is the quadratic inequality in standard form ? | |
|---|---|
| Step 2: Factor the quadratic and solve the quadratic for the roots. These will be the ___critical___ points. | |
| Step 3: Is the inequality a conjunction or a disjunction? |  |
| Step 4: Write your answer | |

**Practice:** Find the solution set for the inequality and graph the solution set.

$x^2 - 3x - 3 \leq 7$

$2x^2 > -7x - 3$

| Step 1: Is the quadratic inequality in standard form ? | |
|---|---|
| Step 2: Factor the quadratic and solve the quadratic for the roots.<br>These will be the *critical* points. | |
| Step 3:  Is the inequality a conjunction or a disjunction? |  |
| Step 4: Write your answer | |

**Practice:**  Find the solution set for the inequality and graph the solution set.
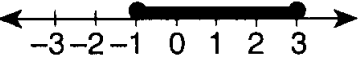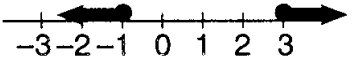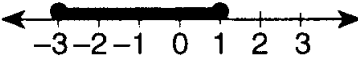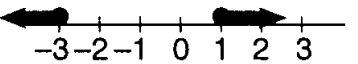$3x^2 > -4x - 1$

1. The solution set for the inequality $x^2 + 4x - 5 \geq 0$ is

1) $-5 \leq x \leq 1$
2) $x \leq -1$ or $x \geq 5$
3) $x \leq -5$ or $x \geq 1$
4) $-1 \leq x \leq 5$

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2. What is the solution set for the inequality $x^2 - 2x - 3 \leq 0$?

1) 
   -3 -2 -1   0   1   2   3

2) 
   -3 -2 -1   0   1   2   3

3) 
   -3 -2 -1   0   1   2   3

4) 
   -3 -2 -1   0   1   2   3

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## Challenge:

Solve and Graph: $\dfrac{3}{x-2} \leq -1$

## Summary:

| | |
|---|---|
| $x^2 - x - 6 < 0$ <br><br> $(x - 3)(x + 2) < 0$ <br><br> For the product of these binomials to be negative, either: <br> 1. $(x - 3)$ must be negative AND $(x + 2)$ must be positive; or <br> 2. $(x - 3)$ must be positive AND $(x + 2)$ must be negative | CASE 1 <br><br> $x - 3 < 0$   AND   $x + 2 > 0$ <br> $x < 3$         $x > -2$ <br><br> CASE 2 <br><br> $x - 3 > 0$   AND   $x + 2 > 0$ <br> $x > 3$         $x < -2$ <br><br> The answer is the first case, $-2 < x < 3$. The second case is not possible, as $x$ cannot be both greater than 3 and less than -2. |

## Key Concept

If the quadratic is $ax^2 + bx + c < 0$, then the solution set is $\{x \mid r_1 < x < r_2\}$
If the quadratic is $ax^2 + bx + c \leq 0$, then the solution set is $\{x \mid r_1 \leq x \leq r_2\}$
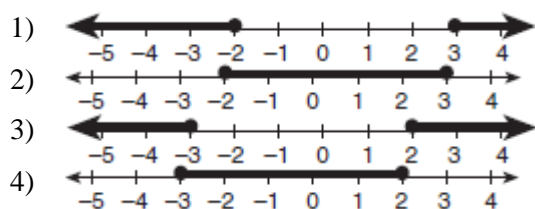If the quadratic is $ax^2 + bx + c > 0$, then the solution set is $\{x \mid x < r_1 \text{ or } x > r_2\}$
If the quadratic is $ax^2 + bx + c \geq 0$, then the solution set is $\{x \mid x \leq r_1 \text{ or } x \geq r_2\}$

# Day 5: Solving Rational Inequalities

## Warm – Up:
Which graph represents the solution of the inequality $x^2 - x - 6 \geq 0$?

1)

2)

3)

4)

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\*\*\* Inequalities are usually solved with the same procedures that are used to solve equations.

\*\*\*Remember that we divide or multiply by a negative number, the inequality is reversed.

## Example 1: Solving Simple Rational Inequalities (No Variable in Denominator)

$$-\frac{17}{8} > \frac{1}{2} - \frac{3v}{2}$$

| | |
|---|---|
| **Step 1:** Is there a variable in your denominator? | |
| **Step 2:** Find the LCD of your denominators | LCD = |
| **Step 3:** Multiply each term by the LCD | |
| **Step 4:** Solve the inequality. | |

Practice: Solve the Inequalities below.

Practice 1: $\dfrac{5}{3} + \dfrac{p}{3} \leq \dfrac{23}{12}$

Practice 2: $\dfrac{x + 7}{8} > \dfrac{x - 3}{10}$

# Example 2: Solving Rational Inequalities (Variables in Denominator)

Solve and Graph the following inequality: $\dfrac{x+1}{x-5} \leq 0$

| | |
|---|---|
| **Step 1:** Is there a variable in your denominator? | |
| **Step 2:** Write the inequality in the correct form. **One side must be zero** and the other side can have only one fraction, so simplify the fractions if there is more than one fraction. | |
| **Step 3:** Find the key or critical values. To find the key/critical values, set the numerator and denominator of the fraction equal to zero and solve. | |
| **Step 4:** Make a sign analysis chart. To make a sign analysis chart, use the key/critical values found in Step 2 to divide the number line into sections. | |
| **Step 5:** Perform the sign analysis. To do the sign analysis, pick one number from each of the sections created in Step 3 and plug that number into the polynomial to determine the sign of the resulting answer. | |
| **Step 6:** Use the sign analysis chart to determine which sections satisfy the inequality. | |
| **Step 7:** Write the final answer. | |

**Example 3:** Solve and Graph the following inequality: $\dfrac{x}{x-4} - \dfrac{3}{x-4} \leq 2$

| | |
|---|---|
| **Step 1:** Is there a variable in your denominator? | |
| **Step 2:** Write the inequality in the correct form. **One side must be zero** and the other side can have only one fraction, so simplify the fractions if there is more than one fraction. | |
| **Step 3:** Find the key or critical values. To find the key/critical values, set the numerator and denominator of the fraction equal to zero and solve. | |
| **Step 4:** Make a sign analysis chart. To make a sign analysis chart, use the key/critical values found in Step 2 to divide the number line into sections. | |
| **Step 5:** Perform the sign analysis. To do the sign analysis, pick one number from each of the sections created in Step 3 and plug that number into the polynomial to determine the sign of the resulting answer. | |
| **Step 6:** Use the sign analysis chart to determine which sections satisfy the inequality. | |
| **Step 7:** Write the final answer. | |

**Example 4:** Solve and Graph the following inequality: $6 - \dfrac{5}{p+2} > 2 - \dfrac{9}{p+2}$

| | |
|---|---|
| **Step 1:** Is there a variable in your denominator? | |
| **Step 2:** Write the inequality in the correct form. **One side must be zero** and the other side can have only one fraction, so simplify the fractions if there is more than one fraction. | |
| **Step 3:** Find the key or critical values. To find the key/critical values, set the numerator and denominator of the fraction equal to zero and solve. | |
| **Step 4:** Make a sign analysis chart. To make a sign analysis chart, use the key/critical values found in Step 2 to divide the number line into sections. | |
| **Step 5:** Perform the sign analysis. To do the sign analysis, pick one number from each of the sections created in Step 3 and plug that number into the polynomial to determine the sign of the resulting answer. | |
| **Step 6:** Use the sign analysis chart to determine which sections satisfy the inequality. | |
| **Step 7:** Write the final answer. | |

**Example 5:** Solve and Graph the following inequality: $\frac{x-8}{x} \le 3 - x$

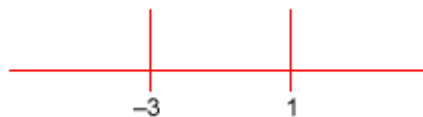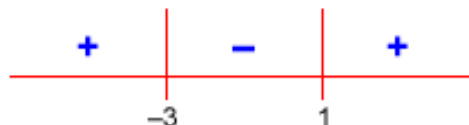| | |
|---|---|
| **Step 1:** Is there a variable in your denominator? | |
| **Step 2:** Write the inequality in the correct form. **One side must be zero** and the other side can have only one fraction, so simplify the fractions if there is more than one fraction. | |
| **Step 3:** Find the key or critical values. To find the key/critical values, set the numerator and denominator of the fraction equal to zero and solve. | |
| **Step 4:** Make a sign analysis chart. To make a sign analysis chart, use the key/critical values found in Step 2 to divide the number line into sections. | |
| **Step 5:** Perform the sign analysis. To do the sign analysis, pick one number from each of the sections created in Step 3 and plug that number into the polynomial to determine the sign of the resulting answer. | |
| **Step 6:** Use the sign analysis chart to determine which sections satisfy the inequality. | |
| **Step 7:** Write the final answer. | |

Solve, graph the solution, and express the solution in interval notation. $\dfrac{3x+1}{x-1} \geq 2$

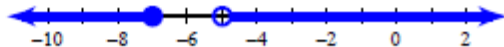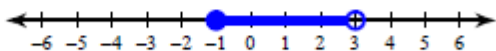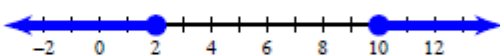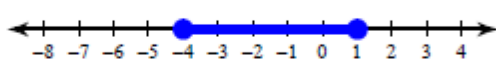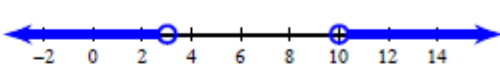| | |
|---|---|
| **Step 1**: Write the inequality in the correct form. One side must be zero and the other side can have only one fraction, so simplify the fractions if there is more than one fraction. | $\dfrac{3x+1}{x-1} - 2 \geq 0$ <br><br> $\dfrac{3x+1-2(x-1)}{x-1} \geq 0$ <br><br> $\dfrac{x+3}{x-1} \geq 0$ |
| **Step 2**: Find the key or critical values. To find the key/critical values, set the numerator and denominator of the fraction equal to zero and solve. | $x+3=0 \quad$ and $\quad x-1=0$ <br><br> $x=-3 \qquad\qquad x=1$ |
| **Step 3**: Make a sign analysis chart. To make a sign analysis chart, use the key/critical values found in Step 2 to divide the number line into sections. |  |
| **Step 4**: Perform the sign analysis. To do the sign analysis, pick one number from each of the sections created in Step 3 and plug that number into the polynomial to determine the sign of the resulting answer. <br><br> **Remember:** <br> Same Signs → Positive *Answer* <br> Different Signs → Negative *Answer* |  |
| **Step 5**: Use the sign analysis chart to determine which sections satisfy the inequality. In this case, we have greater than or equal to zero, so we want all of the positive sections. Notice that x ≠ 1 because it would make the original problem undefined, so you must use an open circle at x = 1 instead of a closed circle to draw the graph. |  |
| **Step 6**: Use interval notation to write the final answer. | $(-\infty, -3] \cup (1, \infty)$ |

33

# HOMEWORK ANSWERS
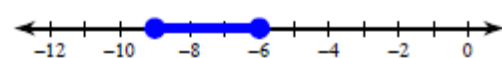
1) $x \le -7$ or $x > -5$ :

2) $-1 \le k < 3$ :
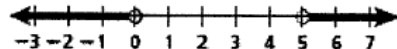
3) $n \le 2$ or $n \ge 10$ :

4) $-4 \le n \le 1$ :

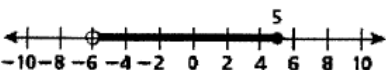5) $x < 3$ or $x > 10$ :

6) $-9 \le p \le -6$ :

6)

Inequality: $X < 0$ or $X > 5$

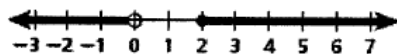Interval Notation: $(-\infty, 0) \cup (5, \infty)$

Set Builder Notation: $\{X \mid X < 0 \text{ or } X > 5\}$

7.

Inequality: $-6 < X \le 5$

Interval Notation: $(-6, 5]$

Set Builder Notation: $\{X \mid -6 < X \le 5\}$

8)

Inequality: $X < 0$ or $X \ge 2$

Interval Notation: $(-\infty, 0) \cup [2, \infty)$

Set Builder Notation: $\{X \mid X < 0 \text{ or } X \ge 2\}$

9)

Inequality: $-2 < X < 1$

Interval Notation: $(-2, 1)$

Set Builder Notation: $\{X \mid -2 < X < 1\}$

## 1-4 Solving Absolute Value Equations and Inequalities (pages 16–17)

**Developing Skills**

3. $\{-7, 17\}$      4. $\{-2, -14\}$      5. $\{-1, 6\}$

6. $\{-3, 7\}$      7. $\{1, 7\}$      8. $\{3, -4\}$

9. $\{5, 9\}$      10. $\{3, -3\}$      11. $\{2, -10\}$

12. $\{-3, 8\}$      13. $\varnothing$      14. $\{-3, 17\}$

15. $x < -9$ or $x > 9$,
$\{\ldots, -12, -11, -10, 10, 11, 12, \ldots\}$

16. $x < -9$ or $x > 5$, $\{\ldots, -12, -11, -10, 6, 7, 8, \ldots\}$

17. $-11 \le b \le -1$, $\{-11, -10, -9, \ldots, -3, -2, -1\}$

18. $-1 < y < 7$, $\{0, 1, 2, 3, 4, 5, 6\}$

19. $y < -19$ or $y > 7$,
$\{\ldots, -22, -21, -20, 8, 9, 10, \ldots\}$

20. $b \le -1$ or $b \ge 8$, $\{\ldots, -3, -2, -1, 8, 9, 10, \ldots\}$

21. $-3 < x < 7$, $\{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

22. The set of integers

23. $0 < b < 10$, $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

24. $b < -3$ or $b > 14$, $\{\ldots, -6, -5, -4, 15, 16, 17, \ldots\}$

25. $\varnothing$

26. $-3 \le b \le 17$, $\{-3, -2, -1, \ldots, 15, 16, 17\}$

27. $\{253, 254, 255, 256, 257, 258, 259\}$, $253 \le x \le 259$

28. $\{150, 151, 152, 153, \ldots, 297, 298, 299, 300\}$,
$150 \le t \le 300$

29. $|c - 200| \le 28$, solution $= 172 \le c \le 228$,
$\{172, 173, 174, \ldots, 226, 227, 228\}$

## 3-1 The Real Numbers and Absolute Value (page 83)

In 15–26, answers will be graphs of number lines.

15. $-7 < x < 7$      16. $a \ge 8$ or $a \le 2$

17. $y > 2$ or $y < -7$      18. $-1 \le b \le 2$

19. $a < -9$ or $a > -1$      20. $-4 < x < 2$

21. $x > \frac{2}{5}$ or $x < -\frac{1}{5}$      22. $\{\ \}$ or $\varnothing$

23. all real numbers      24. $x = \frac{-4}{5}$

25. all real numbers      26. all real numbers

## 1-8 Quadratic Inequalities *(page 35)*

**Developing Skills**

3. $-3 < x < -2, \varnothing$
4. $x < -6$ or $x > 1, \{\ldots, -9, -8, -7, 2, 3, 4, \ldots\}$
5. $1 \le x \le 2, \{1, 2\}$
6. $x < 2$ or $x > 5, \{\ldots, -1, 0, 1, 6, 7, 8, \ldots\}$
7. $-2 < x < 3, \{-1, 0, 1, 2\}$
8. $x \le -2$ or $x \ge 10,$
   $\{\ldots, -4, -3, -2, 10, 11, 12, \ldots\}$
9. $-4 < x < 3, \{-3, -2, -1, 0, 1, 2\}$
10. $x < 1$ or $x > 5, \{\ldots, -2, -1, 0, 6, 7, 8, \ldots\}$
11. $x \le 0$ or $x \ge 2, \{\ldots, -2, -1, 0, 2, 3, 4, \ldots\}$
12. $-2 < x < 3, \{-1, 0, 1, 2\}$
13. $x < 2$ or $x > 2, \{\ldots, -1, 0, 1, 3, 4, 5, \ldots\}$
14. The set of integers
15. $-2 < x < 1, \{-1, 0\}$
16. $-3 \le x \le 4, \{-3, -2, -1, 0, 1, 2, 3, 4\}$
17. $x < -3$ or $x > 4, \{\ldots, -6, -5, -4, 5, 6, 7, \ldots\}$

## Day 5 Answers:

## 2-8 Solving Rational Inequalities
   *(pages 73–74)*

3. $a < -24$
4. $y < 8$
5. $b > \frac{2}{5}$
6. $d < 2$
7. $a > \frac{153}{5}$
8. $0 < x < 1$
9. $0 < y < 4$
10. $a < -2$ or $a > -1$
11. $\frac{5}{3} < x < 4$
12. $x < 0$ or $x > \frac{1}{2}$
13. $-7 < x < -5$
14. $-5 < a < -1$