PAIR AND TRIPLET PRODUCTION REVISITED FOR THE RADIOLOGIST*

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THE purpose of this paper is to elucidate in a simple manner the processes of pair and triplet production for the curious radiologist, using concepts known to the theoretical physicist.

INTERACTION OF RADIATION AND MATTER

Electromagnetic radiation interacts with matter by 6 distinct mechanisms. The contribution to photon interaction of each mechanism is dependent on photon energy. These mechanisms in order of increasing photon energy are: (1) coherent scattering; (2) photoelectric effect; (3) Compton scattering; (4) pair production; (5) triplet production; and (6) photodisintegration. The first and last mechanisms are of little or no importance in medical radiology. In the photoelectric effect, a low energy photon (10–90 kev.) is absorbed by an atomic planetary electron that is then ejected from the atom. The third process, Compton scattering, operates over a wide range of photon energies (25 kev. to 35 mev.), but is most important with 100 kev. to 2 mev. photons. In Compton scattering, the incident photon imparts some of its energy to a free or weakly bound electron with the emergence of a scattered lower energy photon. Pair production is the materialization into an electron-positron pair of a photon with energy greater than 1.02 mev. located near an atomic nucleus. Triplet production, similarly, is the materialization into an electron-positron pair of a photon with energy of at least 2.04 mev. located near a free or weakly bound electron. Because of their central importance in medical radiology, both photoelectric effect and Compton scattering are well explained in the standard texts of radiologic physics. Pair and triplet production, on the other hand, are simply given the "black box" treatment. A photon of energy above a certain threshold is considered as entering the black box with the exit of an electron-positron pair. One is told, but not shown, that to conserve momentum, pair and triplet production have to take place near another mass, an atomic nucleus or an electron, respectively (Fig. 1, *a* and *b*). A model of what occurs inside the black box is presented. Also shown is why and how momentum is conserved by the atomic nucleus in pair production and by the electron in triplet production.

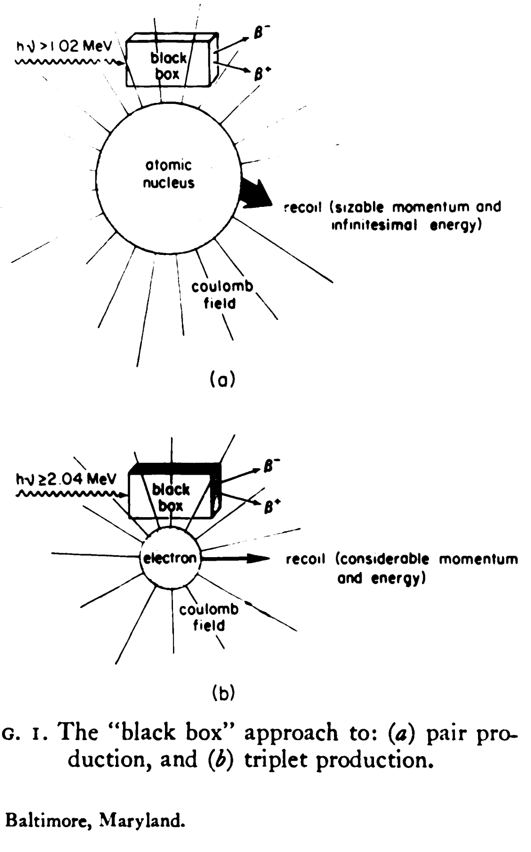

FIG. 1. The "black box" approach to: (*a*) pair production, and (*b*) triplet production.

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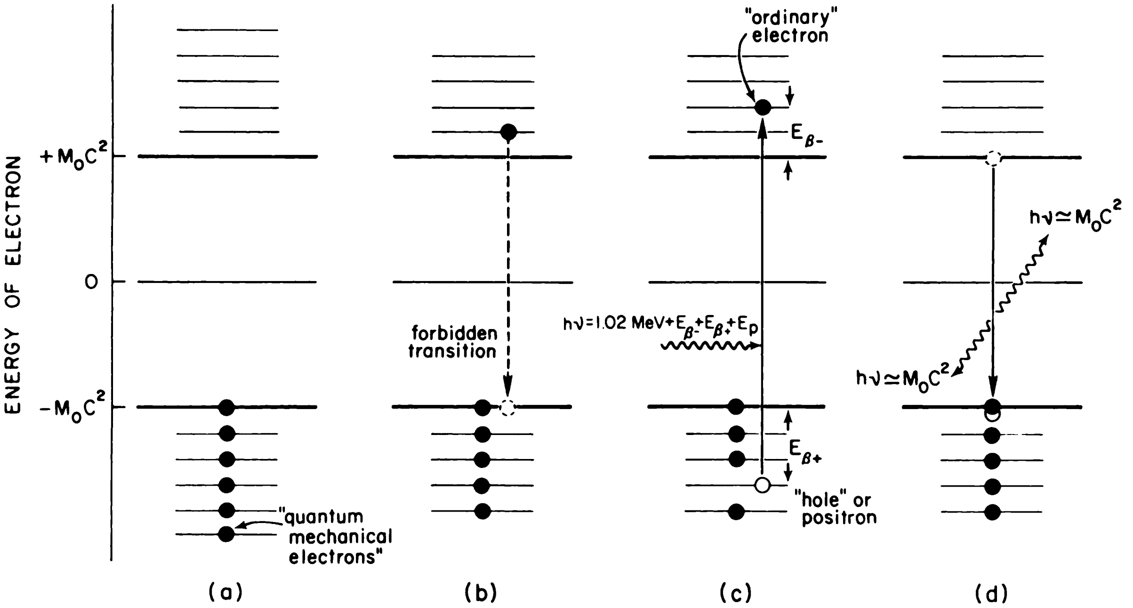

Fig. 2. Energy level diagram of the free electron according to the Dirac theory. (*a*) Dirac's electron-vacuum in which all the negative energy states are filled with "quantum mechanical electrons." (*b*) "Ordinary" electrons are not allowed to transit to oblivion. All the negative energy states are filled and each state can only accommodate a single electron. (*c*) Photon annihilation and creation of an electron-positron pair. Incident photon energy,

$$h\nu = 1.02\text{ mev.} + E_{\beta-} + E_{\beta+} + E_p,$$

where $E_{\beta-}$, $E_{\beta+}$ = kinetic energy of the electron and positron respectively and E_p = energy transferred with the excess momentum to the nucleus or electron. (*d*) Positron-electron annihilation with creation of 2 photons.

QUANTUM MECHANICAL MODEL OF THE ELECTRON-POSITRON PAIR

Pair and triplet production are experimentally observed phenomena having no analogy in classical physics. They can be described, however, with quantum mechanics using the Dirac[2] relativistic wave equation for an electron interacting with an electromagnetic field. Solution of the Dirac equation shows that free electrons can exist in 2 sets of quantum energy states. One set of quantum states has positive energy from $+m_0c^2$ to $+\infty$ and the other set of quantum states has negative energy from $-m_0c^2$ to $-\infty$, where m_0 = an electron rest mass, c = velocity of light, and m_0c^2 = an electron rest energy. The energy band from $-m_0c^2$ to $+m_0c^2$ has no permissible quantum states and is a forbidden zone to free electrons (Fig. 2*a*). The electrons in the quantum states of positive energy are interpreted as the "ordinary" electrons, while electrons in the quantum states of negative energy are "quantum mechanical entities" with properties having no classical analogy. The present Dirac theory is faced with an impending catastrophe! What if an electron transits from the positive to the negative energy bands of quantum states? This means that an "ordinary" electron would become a "quantum mechanical entity" and spontaneously drop out of sight! Spontaneous electron self-annihilation with release of a photon(s), however, has never been experimentally observed (Fig. 2*b*). To avoid this serious difficulty, 2 fundamental assumptions have to be made;[2,3] (1) all negative energy quantum states ($-m_0c^2$ to $-\infty$) for free electrons are filled, with a maximum of 1 electron per quantum state, in accordance with the Pauli exclusion principle. Since only 1 electron can occupy a given quantum state, no electron can transit from the positive energy band to the filled negative energy states; (2) the "quantum mechanical" electrons filling up the negative energy states do not produce external (coulomb) fields, and do not contribute to the total charge, momentum or energy of a system. It is assumed, however, that an external electromagnetic field can act on these "quantum mechanical entities." A "vacuum" can be considered as that electron distribution where all negative energy quantum states are filled and none of the positive energy states are occupied (Fig. 2*a*).

Consider pair and triplet production in terms of the above model. The absorption of an incident photon (an external electromagnetic field) with an energy greater than $2m_0c^2 = 1.02$ mev. by an electron in a negative energy quantum state, raises that electron to a quantum state in the positive energy band. This transition is interpreted physically as the disappearance of the photon and the appearance of an "ordinary" electron with a negative charge, and a given momentum and positive kinetic energy, $E_{\beta-}$ (Fig. 2*c*). The vacated and empty quantum state in the negative energy band is like a "hole" in a sea of electrons and is interpreted as the appearance of a positive electron, with a positive kinetic energy, $E_{\beta+}$, since it represents the absence of a negative electron with a negative energy (Fig. 2*c*). This new particle, the positron, was predicted and described theoretically approximately 3 years prior to its experimental discovery in cosmic rays.[1] This model describes the disappearance of a photon and the simultaneous appearance of both an electron and positron, the substance of both pair and triplet production.

The reverse transition, with an ordinary electron falling into a "hole" in the negative energy band is interpreted as the electron-positron pair annihilation. The transition probability is highest for an electron in its lowest positive energy state (*i.e.*, near $+m_0c^2$) or with near zero kinetic energy to fall into a "hole" of energy near $-m_0c^2$, *i.e.*, a positron of approximately zero kinetic energy. The rest energy and whatever small kinetic energy the electron and positron have are released as 2 photons, each with an energy of approximately m_0c^2 (Fig. 2*d*). The 2 photons move in opposite directions as required to conserve momentum, since annihilation usually takes place away from a third mass such as a nucleus. This quantum mechanical model describes the occurrences inside the black box in pair and triplet production and in electron-positron annihilation. Although charge was shown to be conserved, energy and momentum were not.

NONCONSERVATION OF MOMENTUM WITH PAIR AND TRIPLET PRODUCTION TAKING PLACE OUT IN SPACE

Conservation of momentum in any process where there is "materialization" of electromagnetic waves or photons, as in pair and triplet production, is difficult. This difficulty stems from the fact that photons of all the "particles" of the same total energy (rest and kinetic) have the greatest momentum. This is best demonstrated as follows.

The momentum of a material particle of rest mass m_0 is.

$$P_m = mv = \frac{m_0}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}} v, \qquad (1)$$

where c is the speed of light and v is the velocity of the material particle.

The energy of the material particle is:

$$E_m = mc_2 = \frac{m_0}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}} c_2. \qquad (2)$$

The ratio gives the momentum per unit energy for the material particle:

$$\left(\frac{P}{E}\right)_m = \frac{v}{c^2}.$$

The momentum of a photon or electromagnetic "particle" is:

$$P_\phi = \frac{h\nu}{c}, \qquad (3)$$

and its energy is:

$$E_\phi = h\nu. \qquad (4)$$

The ratio gives the photon momentum per unit energy:

$$\left(\frac{P}{E}\right)_\phi = \frac{1}{c}.$$

Since $c > v$ always, then

$$\left(\frac{P}{E}\right)_\phi > \left(\frac{P}{E}\right)_m.$$

If energy is conserved $E_\phi = E_m$, then the momenta are $P_\phi > P_m$ and they cannot be conserved. Momentum, however, must be conserved and therefore a photon out in space cannot undergo pair or triplet production. An additional mass such as an atomic nucleus or electron is needed to absorb the excess photon momentum in pair and triplet production, respectively.

PHOTON THRESHOLD ENERGY FOR PAIR AND TRIPLET PRODUCTION

The black box inside which photon "materialization" occurs is mechanically coupled via the strong coulomb field adjacent to the atomic nucleus or electron. It is via this strong mechanical coupling that momentum is transferred to the nucleus or electron. It is important to note that it is not physically possible to transfer momentum without any kinetic energy, however small that energy may be. This fact brings up the notion of threshold energy for the incident photon in pair and triplet production. The conservation of energy equation is:

$$E_\phi = m_{\beta+}c^2 + m_{\beta-}c^2 + E_P,$$

where E_ϕ = energy of incident photon, $m_{\beta+}c^2$ and $m_{\beta-}c^2$ are the total (rest and kinetic) energies of the positron and electron, respectively, and E_p is the kinetic energy transferred with the excess photon momentum to the atomic nucleus or electron.

Consider the photon energy threshold for pair production. It is assumed that the positron and electron kinetic energy are zero, *i.e.*, $v_{\beta+} = v_{\beta-} = 0$ in Equation 2. This, in reality, would result in a very short lifetime of the pair, for in such circumstances the probability for annihilation is maximal. The excess photon momentum which is transferred to the atomic nucleus may be sizable but the kinetic energy, E_p, transferred to the nucleus is infinitesimally small due to the very large nuclear mass with respect to the electron mass. Hence the threshold for pair production is infinitesimally greater than $2m_0c^2$ but *not equal* to $2m_0c^2$. This difference is very important conceptually, if not practically, in pair production. This is more apparent when one considers triplet production.

The conservation of energy for triplet production is:

$$m'_{e-}c^2 + E_\phi = m_{\beta+}c^2 + m_{\beta-}c^2 + E_P, \qquad (5)$$

where $m'_{e-}c^2$ is the total (rest and kinetic) energy of the electron prior to triplet production. No generality is lost if it is assumed that the electron is stationary and the first term on the left of Equation 5 becomes m_0c^2. The kinetic energy transferred to the stationary electron along with the excess photon momentum in triplet production is finite and *not* negligible, $E_p = m_{e-}c^2$. Intuitively, it can be assumed that at threshold energy, the stationary electron and the positron-electron pair are all ejected in the forward direction (*i.e.*, direction of incident photon) and that the 3 particles with equal masses share the energy (and momentum) equally.[5] The conservation of energy in Equation 5 becomes:

$$m_0c^2 + E_\phi = 3mc^2. \qquad (6)$$

Conservation of momentum for triplet production using Equations 1 and 3 is:

$$\frac{E_\phi}{c} = 3mv = 3m\beta c, \qquad (7)$$

where $\beta = v/c$.

Eliminating E_ϕ from Equations 6 and 7,

$$m_0c^2 + 3m\beta c^2 = 3mc^2$$

$$m_0 + 3\frac{m_0}{(1-\beta^2)^{1/2}}\beta = 3\frac{m_0}{(1-\beta^2)^{1/2}}$$

$$(1-\beta^2)^{1/2} + 3\beta = 3.$$

Squaring both sides: $\beta = \frac{4}{5}$

$$m = \frac{m_0}{\left(1-\frac{16}{25}\right)^{1/2}} = \frac{5}{3}m_0.$$

Substituting in Equation 7

$$E_\phi = 3\left(\frac{5}{3}m_0\right)\frac{4}{5}c^2 = 4m_0c^2.$$

Thus the incident photon energy threshold for triplet production is $4m_0c^2$.

PROPERTIES OF MULTIBODY SYSTEMS

Pair and triplet production are multibody systems involving the interaction of an incident photon, an electron-positron pair and an atomic nucleus or electron. Mathematically, only 2 body systems can be solved in "closed form," where all variables are assigned exact values. For example, the Compton scattering is a 2 body system, where all energies, momenta and scattering angles, can be assigned exact values determined by using conservation laws of energy and momentum. There is, therefore, an inherent indetermination in pair and triplet production and the energies, momenta and scattering angles cannot be assigned exact values. For this reason, "intuitive assumptions" had to be made about the momenta, energies and scattering angles of the ejected particles, to calculate the threshold energy for triplet production. This indetermination in pair and triplet production is not "because momentum is not conserved" as some have suggested.[4] When the atomic nucleus or stationary electron is included in the system, momentum is conserved but the indetermination persists.

It should be noted that from a theoretical standpoint, electron-positron pairs could result from the collision of particles (nuclei, protons or electrons) with sufficient energy. Even photon-photon collision could result in an electron-positron pair.

The dependence of pair and triplet production on incident photon energy, atomic number of the absorber, etc., are well summarized in the radiologic physics literature and will not be discussed here.

SUMMARY

Pair and triplet production have electron-positron pair formation in common. Electron-positron pair formation can be considered, in a model presented, as the transition of a "special" electron from a negative energy quantum state to that of positive energy becoming an "ordinary" electron. The transition is across a forbidden energy zone $2m_0c^2$ wide (rest mass of the electron-positron pair). The vacated negative energy quantum state is the positron, since it represents the absence of a negative electron.

The photon of all "particles" has the highest momentum per unit energy such that when a photon "materializes," as in electron-positron pair formation, another mass has to be present in the system to conserve momentum. The mass may be an atomic nucleus as in pair production or an electron as in triplet production. The excess photon momentum is transferred to the nucleus or electron via their respective coulomb fields as mechanical coupling. In pair production, even though the momentum transferred may be sizable, the associated kinetic energy is infinitesimally small due to the very large nuclear mass. Hence the incident photon energy threshold for pair production in infinitesimally greater than $2m_0c^2$. With triplet production the excess photon momentum is transferred to an electron, and the associated kinetic energy transferred is considerable. The calculated incident photon energy threshold is $4m_0c^2$ after making a few "intuitive" assumptions on the system. These assumptions are necessary since both pair and triplet production are multibody systems which cannot be solved so that all variables are assigned exact values.

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