1-4 Pairs of Angles

Warm Up

Lesson Presentation

Lesson Quiz

Warm Up

Simplify each expression.

1. $90 - (x + 20)$ $70 - x$

2. $180 - (3x - 10)$ $190 - 3x$

Write an algebraic expression for each of the following.

3. 4 more than twice a number $2n + 4$

4. 6 less than half a number $\frac{1}{2}n - 6$

Objectives

Identify adjacent, vertical, complementary, and supplementary angles.

Find measures of pairs of angles.

Vocabulary

adjacent angles

linear pair

complementary angles

supplementary angles

vertical angles

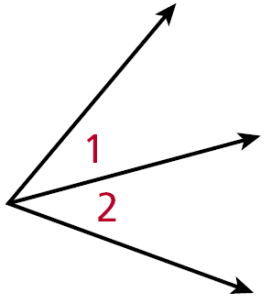
1-4 Pairs of Angles

Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

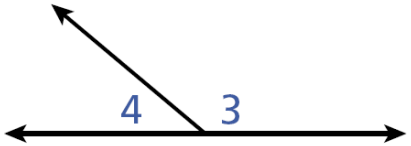
1-4 Pairs of Angles

Pairs of Angles

Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. ∠1 and ∠2 are adjacent angles.



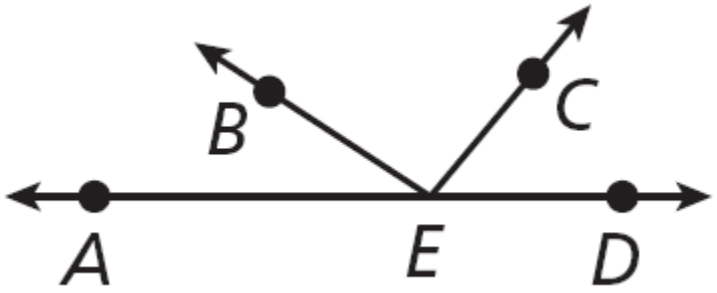
A **linear pair** of angles is a pair of adjacent angles whose noncommon sides are opposite rays. ∠3 and ∠4 form a linear pair.



Example 1A: Identifying Angle Pairs

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

∠AEB* and *∠BED

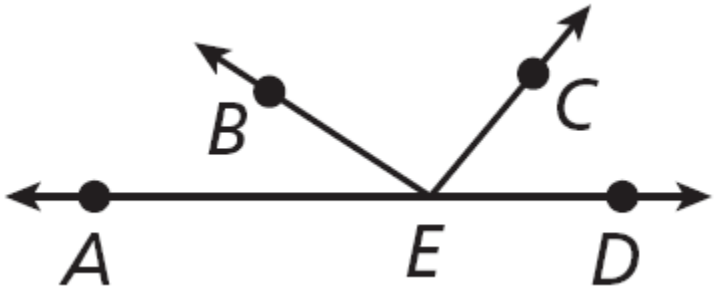


∠AEB and *∠BED* have a common vertex, *E*, a common side, $\overrightarrow{EB}$, and no common interior points. Their noncommon sides, $\overrightarrow{EA}$ and $\overrightarrow{ED}$, are opposite rays. Therefore, *∠AEB* and *∠BED* are adjacent angles and form a linear pair.

Example 1B: Identifying Angle Pairs

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

$\angle AEB$ and $\angle BEC$

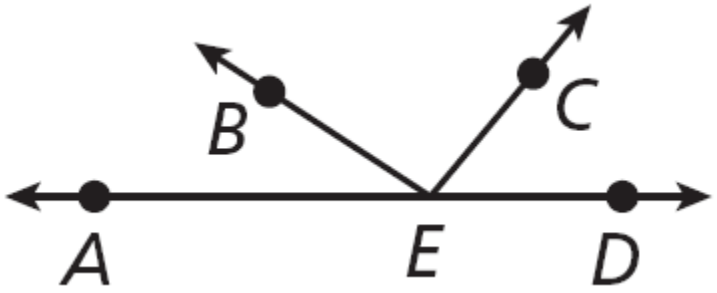


$\angle AEB$ and $\angle BEC$ have a common vertex, E, a common side, $\overrightarrow{EB}$, and no common interior points. Therefore, $\angle AEB$ and $\angle BEC$ are only adjacent angles.

Example 1C: Identifying Angle Pairs

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

$\angle DEC$ and $\angle AEB$

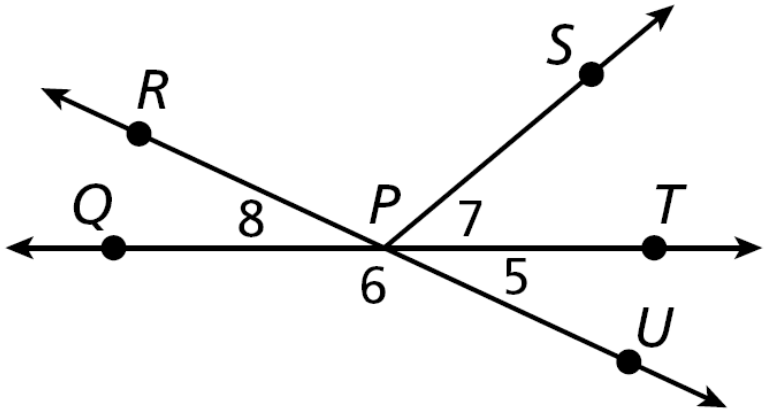


$\angle DEC$ and $\angle AEB$ share E but do not have a common side, so $\angle DEC$ and $\angle AEB$ are not adjacent angles.

Check It Out! Example 1a

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

∠5 and ∠6

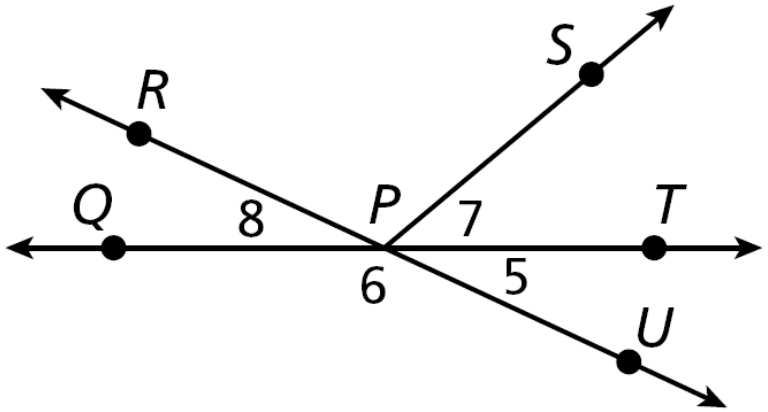


∠5 and ∠6 are adjacent angles. Their noncommon sides, $\overrightarrow{EA}$ and $\overrightarrow{ED}$, are opposite rays, so ∠5 and ∠6 also form a linear pair.

Check It Out! Example 1b

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

∠7 and ∠*SPU*

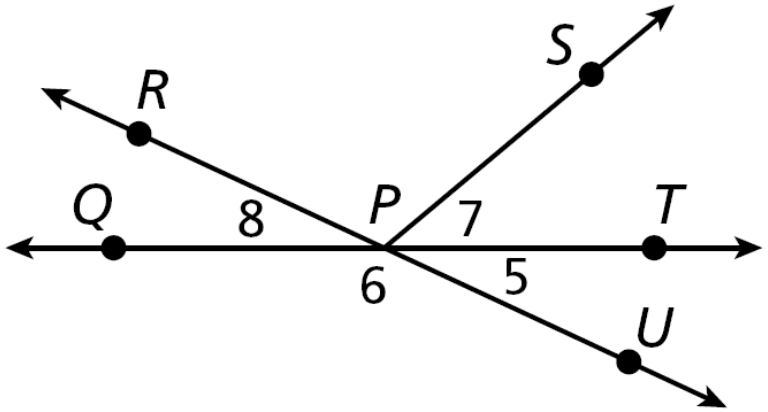


∠7 and ∠*SPU* have a common vertex, *P*, but do not have a common side. So ∠7 and ∠*SPU* are not adjacent angles.

Check It Out! Example 1c

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

∠7 and ∠8



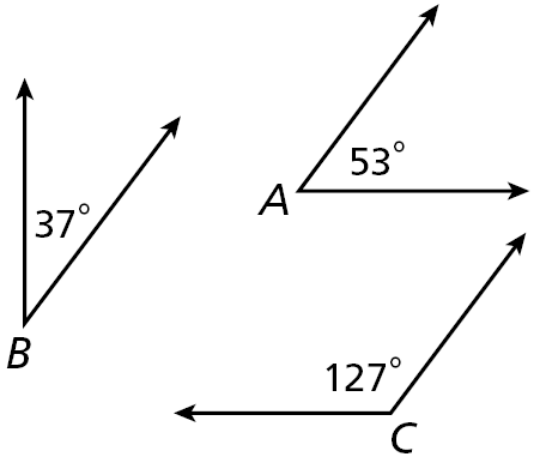
∠7 and ∠8 have a common vertex, *P*, but do not have a common side. So ∠7 and ∠8 are not adjacent angles.

1-4 Pairs of Angles

Complementary and Supplementary Angles

Complementary angles are two angles whose measures have a sum of 90°.
$\angle A$ and $\angle B$ are complementary.

Supplementary angles are two angles whose measures have a sum of 180°.
$\angle A$ and $\angle C$ are supplementary.



You can find the complement of an angle that measures $x°$ by subtracting its measure from 90°, or $(90 - x)°$.

You can find the supplement of an angle that measures $x°$ by subtracting its measure from 180°, or $(180 - x)°$.

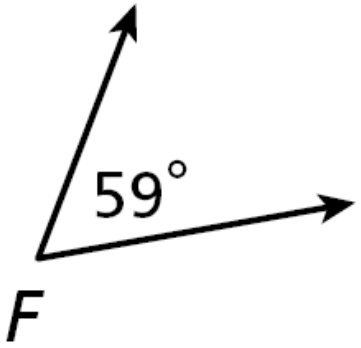
Example 2: Finding the Measures of Complements and Supplements

Find the measure of each of the following.

A. complement of $\angle F$

$(90 - x)°$

$90° - 59° = 31°$

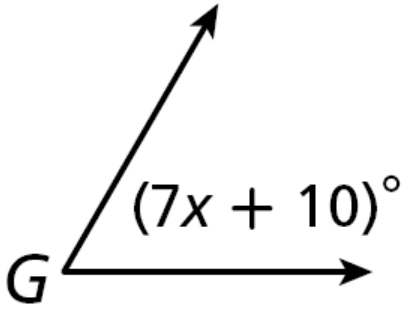


B. supplement of $\angle G$

$(180 - x)°$

$180 - (7x+10)° = 180° - 7x - 10$

$= (170 - 7x)°$



Check It Out! Example 2

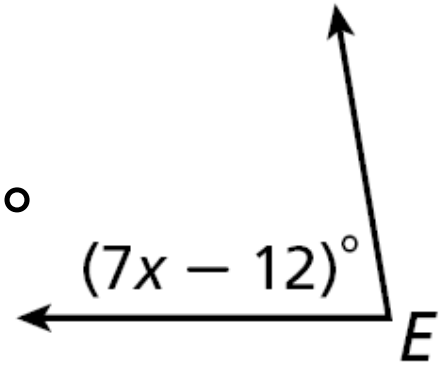
Find the measure of each of the following.

a. complement of $\angle E$

$(90 - x)°$

$90° - (7x - 12)° = 90° - 7x° + 12°$

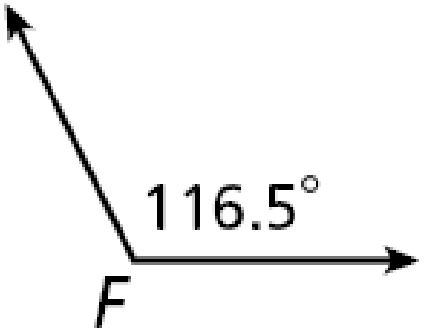
$= (102 - 7x)°$



b. supplement of $\angle F$

$(180 - x)°$

$180° - 116.5° = 63\frac{1}{2}°$



Example 3: Using Complements and Supplements to Solve Problems

An angle is 10° more than 3 times the measure of its complement. Find the measure of the complement.

Step 1 Let $m\angle A = x°$. Then $\angle B$, its complement measures $(90 - x)°$.

Step 2 Write and solve an equation.

$x = 3(90 - x) + 10$ *Substitute x for m∠A and 90 – x for m∠B.*

$x = 270 - 3x + 10$ *Distrib. Prop.*

$x = 280 - 3x$ *Combine like terms.*

$4x = 280$ *Divide both sides by 4.*

$x = 70$ *Simplify.*

The measure of the complement, $\angle B$, is $(90 - 70)° = 20°$.

Check It Out! Example 3

An angle's measure is 12° more than $\frac{1}{2}$ the measure of its supplement. Find the measure of the angle.

$x = 0.5(180 - x) + 12$	*Substitute x for m∠A and 180 - x for m∠B.*
$x = 90 - 0.5x + 12$	*Distrib. Prop.*
$x = 102 - 0.5x$	*Combine like terms.*
$1.5x = 102$	*Divide both sides by 1.5.*
$x = 68$	*Simplify.*

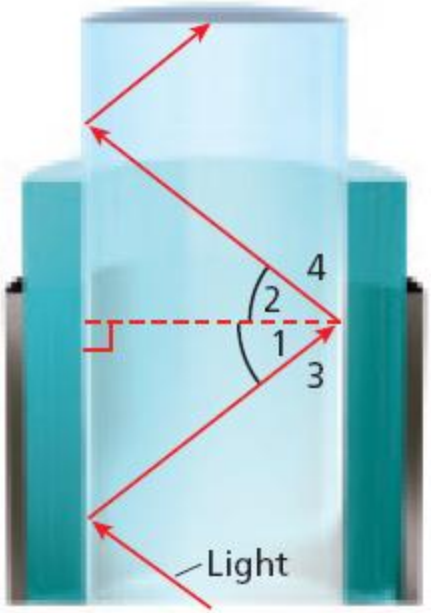
The measure of the angle is 68°.

Example 4: Problem-Solving Application

PROBLEM SOLVING

Light passing through a fiber optic cable reflects off the walls of the cable in such a way that $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.

If $m\angle 1 = 47°$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

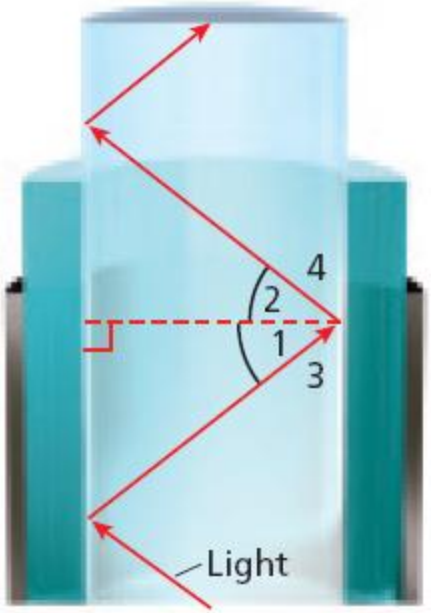


1 Understand the Problem

The **answers** are the measures of ∠2, ∠3, and ∠4.

List the important information:

- ∠1 ≅ ∠2
- ∠1 and ∠3 are complementary, and ∠2 and ∠4 are complementary.
- m∠1 = 47°



2 Make a Plan

If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

If $\angle 3$ and $\angle 1$ are complementary, then $m\angle 3 = (90 - 47)°$.

If $\angle 4$ and $\angle 2$ are complementary, then $m\angle 4 = (90 - 47)°$.

3 Solve

By the Transitive Property of Equality, if $m\angle 1 = 47°$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 47°$.

Since $\angle 3$ and $\angle 1$ are complementary, $m\angle 3 = 43°$. Similarly, since $\angle 2$ and $\angle 4$ are complementary, $m\angle 4 = 43°$.

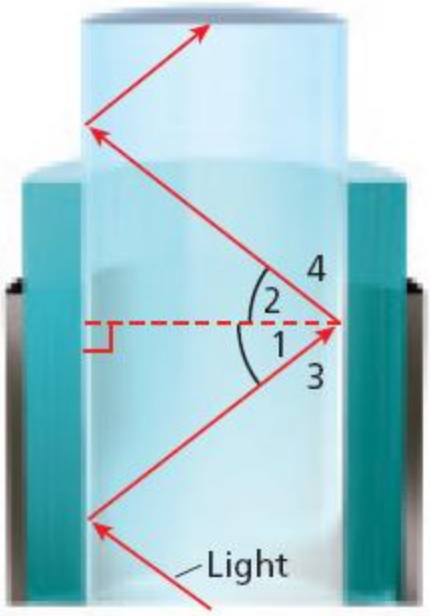
4 Look Back

The answer makes sense because 47° + 43° = 90°, so ∠1 and ∠3 are complementary, and ∠2 and ∠4 are complementary.

Thus m∠2 = 47°, m∠3 = 43°, and m∠4 =43°.

Check It Out! Example 4

What if...? Suppose m∠3 = 27.6°. Find m∠1, m∠2, and m∠4.

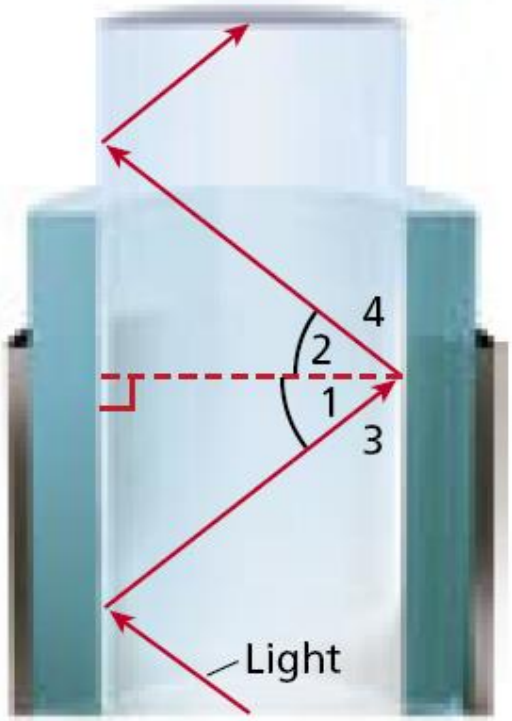


1 Understand the Problem

The **answers** are the measures of ∠1, ∠2, and ∠4.

List the important information:

- ∠1 ≅ ∠2
- ∠1 and ∠3 are complementary, and ∠2 and ∠4 are complementary.
- m∠3 = 27.6°



2 Make a Plan

If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

If $\angle 3$ and $\angle 1$ are complementary, then $m\angle 1 = (90 - 27.6)°$.

If $\angle 4$ and $\angle 2$ are complementary, then $m\angle 4 = (90 - 27.6)°$.

3 Solve

By the Transitive Property of Equality, if $m\angle 1 = 62.4°$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 62.4°$.

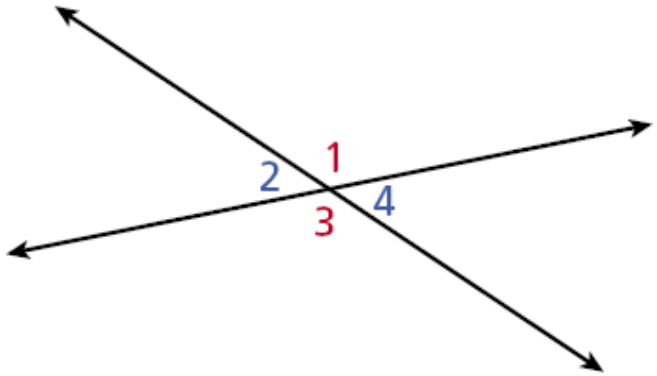
Since $\angle 3$ and $\angle 1$ are complementary, $m\angle 3 = 27.6°$. Similarly, since $\angle 2$ and $\angle 4$ are complementary, $m\angle 4 = 27.6°$.

4 Look Back

The answer makes sense because 27.6° + 62.4° = 90°, so $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.

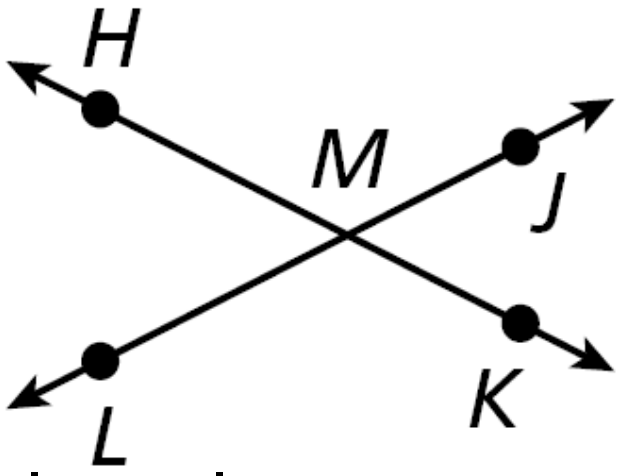
Thus $m\angle 1 = m\angle 2 = 62.4°$; $m\angle 4 = 27.6°$.

Another angle pair relationship exists between two angles whose sides form two pairs of opposite rays. **Vertical angles** are two nonadjacent angles formed by two intersecting lines. ∠**1** and ∠**3** are vertical angles, as are ∠**2** and ∠**4**.



Example 5: Identifying Vertical Angles

Name the pairs of vertical angles.



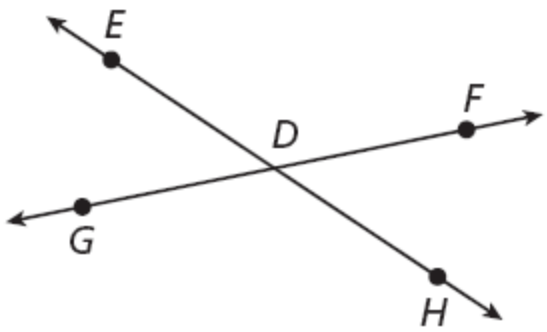
$\angle HML$ and $\angle JMK$ are vertical angles.
$\angle HMJ$ and $\angle LMK$ are vertical angles.

Check $m\angle HML \approx m\angle JMK \approx 60°$.
$m\angle HMJ \approx m\angle LMK \approx 120°$.

Check It Out! Example 5

Name a pair of vertical angles. Do they appear to have the same measure? Check by measuring with a protractor.

$\angle EDG$ and $\angle FDH$ are vertical angles and appear to have the same measure.



Check $m\angle EDG \approx m\angle FDH \approx 45°$

Lesson Quiz: Part I

m∠*A* = 64.1°, and m∠*B* =(4*x* – 30)°. Find the measure of each of the following.

1. supplement of ∠A 115.9°

2. complement of ∠*B* (120 – 4*x*) °

3. Determine whether this statement is true or false. If false, explain why. *If two angles are complementary and congruent, then the measure of each is 90°.*

False; each is 45°.

Lesson Quiz: Part II

m∠*XYZ* = 2*x*° and m∠*PQR* = (8*x* - 20)°.

4. If ∠*XYZ* and ∠*PQR* are supplementary, find the measure of each angle.

40°; 140°

5. If ∠*XYZ* and ∠*PQR* are complementary, find the measure of each angle.

22°; 68°