1. Section 11.2 The Parabola

Definition 1.1. *A* **parabola** *is the set of all points, $P = (x, y)$, that are equidistant from a fixed point called the* **focus** *and a fixed line called the* **directrix**.

Theorem 1.1. $(a > 0)$ *If a parabola has vertex at the origin and...*

(1) focus at $(0, a)$ and directrix at $y = -a$, then the equation for the parabola is

$$x^2 = 4ay$$

(2) focus at $(0, -a)$ and directrix at $y = a$, then the equation for the parabola is

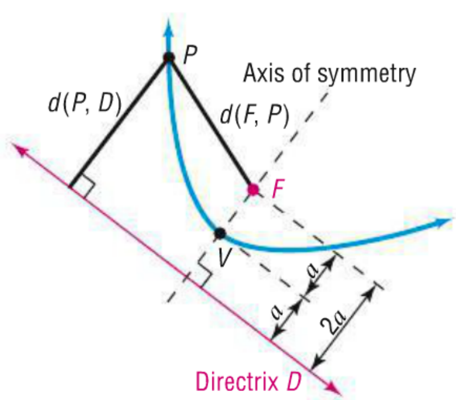
$$x^2 = -4ay$$

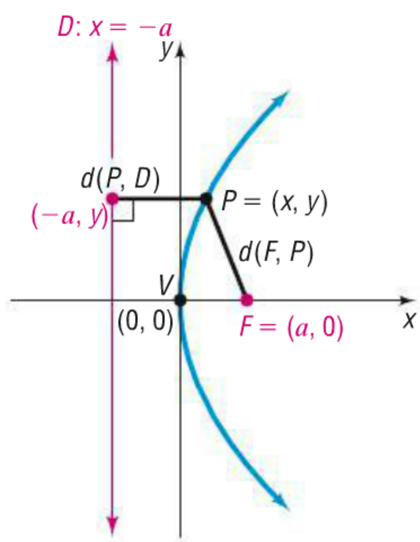
(3) focus at $(a, 0)$ and directrix at $x = -a$, then the equation for the parabola is

$$y^2 = 4ax$$

(4) focus at $(-a, 0)$ and directrix at $x = a$, then the equation for the parabola is

$$y^2 = -4ax$$





Example 1.1.

(a) Make a rough sketch of the graph of the parabola with focus at $(0, 3)$ *and directrix at* $y = -3$.

(b) What if the focus is at $(0, -3)$ *and the directrix is at* $y = 3$*?*

Example 1.2.

(a) Make a rough sketch of the graph of the parabola with focus at $(3, 0)$ *and directrix at* $x = -3$.

(b) What if the focus is at $(-3, 0)$ *and the directrix is at* $x = 3$*?*

Remark 1.1. *The vertex and the focus lie on the axis of symmetry. The distance, a, from the vertex to the focus equals the distance from the vertex to the directrix. The vertex is midway between the focus and the directrix.*

Example 1.3. *Find the directrix of the parabola given by* $y^2 = 6x$

(A) $x = 3/2$

(B) $x = -3/2$

(C) $y = 3/2$

(D) $y = -3/2$

Definition 1.2. *The* **latus rectum** *of a parabola is the chord through the focus parallel to the directrix. The length of the latus rectum is* $4a$*. The endpoints of the latus rectum determine the width of the parabola around the focus.*

Example 1.4. *Sketch the graph of* $(y-3)^2 = 6(x+2)$*, when* $y^2 = 6x$ *is shifted* 2 *units left and* 3 *units up.*

The endpoints of the latus rectum are ____________________________________.

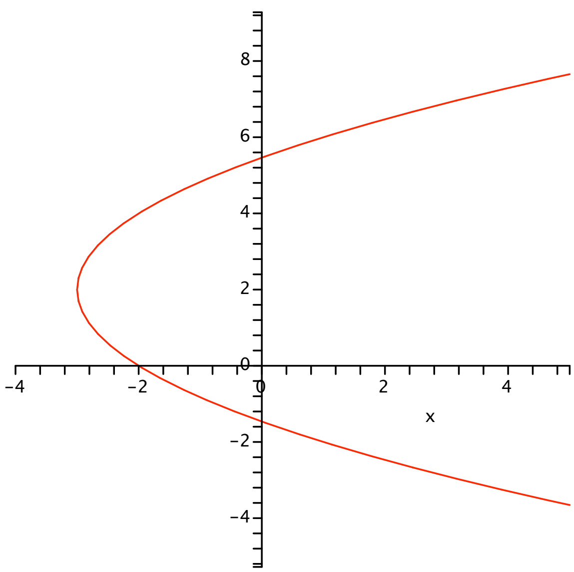
2. Equations: vertex at (h, k), $a > 0$

Equation	$(x-h)^2 = 4a(y-k)$	$(x-h)^2 = -4a(y-k)$
Axis of Symmetry		
Opens		
Focus		
Directrix		
Endpoints of Latus Rectum		
Sketch		

Equation	$(y-k)^2 = 4a(x-h)$	$(y-k)^2 = -4a(x-h)$
Axis of Symmetry		
Opens		
Focus		
Directrix		
Endpoints of Latus Rectum		
Sketch		

Remark 2.1. *The equations above give you the equations in Theorem* 1.1 *when the vertex is at* $(0,0)$*; in other words* $h = 0$ *and* $k = 0$*.*

Example 2.1. *Find the equation of the parabola graphed below.*



Example 2.2.

(a) Find the equation of the parabola with focus $(-1, -2)$ *and vertex at* $(1, -2)$.
(b) Find the two points that define the latus rectum.

Example 2.3. *Find the equation of the parabola with vertex* $(-1, -2)$ *and directrix* $x = 3$.

Example 2.4. *Find the equation of the parabola with focus* $(-1,-2)$ *and directrix* $y = 3$.

(A) $(y - \frac{1}{2})^2 = -10(x + 1)$

(B) $(y - \frac{1}{2})^2 = -20(x + 1)$

(C) $(x + 1)^2 = -10(y - \frac{1}{2})$

(D) $(x + 1)^2 = -20(y - \frac{1}{2})$

Example 2.5. *Find the focus, directrix and vertex of the parabola with the equation* $2(y + 2)^2 = x + 3$

Example 2.6. *Find the focus, directrix and vertex of the parabola with the equation* $2x + y^2 + 4y + 1 = 0$

Example 2.7. *A reflecting telescope contains a mirror shaped like a paraboloid of revolution. If the mirror is* 4 *inches across at its opening and is* 3 *inches deep, where will the collected light be concentrated?*

The light will be concentrated at a point __________*inch from the base of the mirror along its axis of symmetry.*

Example 2.8. *A bridge is built in the shape of a parabolic arch. The bridge arch has a span of* 192 *feet and a maximum height of* 35 *feet. Find the height of the arch at* 20 *feet from its center. Round your answer to the nearest tenth.*