TEACHING 3840: Strategies for Reading and Language Arts in the Content Areas of Middle/Secondary Grades

Parabolas, Circles, & Ellipses Unit

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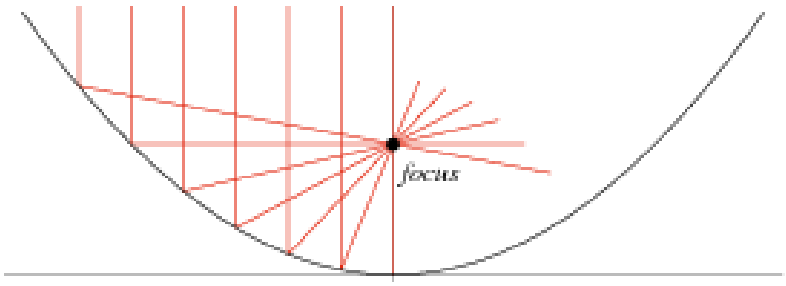
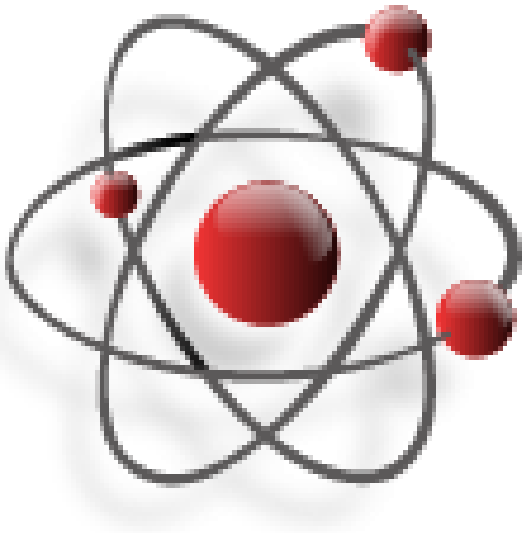
Unit Research





Unit: Parabolas, Circles, & Ellipses

Intended class:	Duration of Unit:
Algebra 2	6 periods at 52 minutes each

Rationale:

Conics are apparent all around us. For example, they can be found in structures, mirrors, and satellite dishes. Conics also represent orbital paths. It is very important to understand three different conics: parabolas, circles, and ellipses because of their appearances in everyday life. It is also necessary to understand the different components of the equations that form three different conic sections to understand how the conics are formed. By manipulating the standard equations into different forms, students are using an essential skill that they will be able to apply to many other types of problems in math. These skills include formula manipulation, solving, and use of the five basic operations (addition, subtraction, multiplication, division, and square root). The five basic operations can be used in many real life applications such as balancing a checkbook or making a budget. Conics problems provide an ideal opportunity to show how simple algebra can be used to model geometry.

Goals:

Students will

- Understand the construction of a specific conic section.
- Make the connection between slicing a cone with a plane and conic sections.
- Visualize each of the conic sections.
- Appreciate real life applications of conic sections.
- Understand the parts of the parabola, such as the vertex, focus, and directrix.
- Grasp how the constants represented by a, b, and c in the standard equation of an ellipse affect the graph.
- Understand the origin of the equations of conic sections.
- Comprehend the geometrical construction of a conic section, given its definition.
- Grasp the meaning of the special points and lines (vertex, focus, foci, directrix, etc.) in a conic section.
- Understand the standard form of an equation of a conic section.

Objectives:

Students will be able to

- Identify which conic section is formed by a specific cut of the cone.
- Match an image of a cone sliced by a plane to its conic section equivalent.
- Identify and Label examples of conic sections used in the real life world from newspapers, magazines, internet resources of their own personal experiences.
- Define the parts of the parabola in their own words.
- Explain how certain parts of the standard equation of an ellipse stretch and shift the graph.
- Describe the equation of a circle in relation to the distance equation.
- Locate the center and radius of a circle given the standard equation.
- Write the appropriate equation of a conic section, given appropriate parts such as focus, vertex, etc.

- Graph a conic section given the standard equation by using their knowledge of quadratic equations.
- Convert a given non-standard equation of a conic section to a standard one.
- Recall the five basic operations to solve problems involving an ellipse.

Standards:

A-Mathematical Processes

A.12.1 Use reason and logic to evaluate information, perceive patterns, identify relationships, formulate questions, pose problems, make and test conjectures, and pursue ideas that lead to further understanding and deeper insight

A.12.2 Communicate logical arguments and clearly show why a result does or does not make sense, why the reasoning is or is not valid, an understanding of the difference between examples that support a conjecture and a proof of the conjecture

A.12.3 Analyze non-routine problems and arrive at solutions by various means, including models and simulations, often starting with provisional conjectures and progressing, directly or indirectly, to a solution, justification, or counter-example

A.12.4 Develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

A.12.6 Read and understand mathematical texts and other instructional materials, and writing about mathematics (e.g., articles in journals)mathematical ideas as they are used in other contexts

B- Number Operations and Relationships

B.12.1 Use complex counting procedures such as union and intersection of sets and arrangements (permutations and combinations) to solve problems

B.12.2 Compare real numbers using order relations (>,<) and transitivity, ordinal scales including logarithmic (e.g., Richter, pH rating), arithmetic differences, and ratios, proportions, percents, rates of change

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

B.12.4 In problem-solving situations involving the application of different number systems (natural, integers, rational, real) select and use appropriate computational procedures, properties (e.g., commutativity, associativity, inverses), and modes of representation (e.g., rationals as repeating decimals, indicated roots as fractional exponents)

B.12.5 Create and critically evaluate numerical arguments presented in a variety of classroom and real-world situations (e.g., political, economic, scientific, social)

B.12.6 Routinely assess the acceptable limits of error when evaluating strategies, testing the reasonableness of results, and using technology to carry out computations

C- Geometry

C.12.1 Identify, describe, and analyze properties of figures, relationships among figures, and relationships among their parts by constructing physical models, drawing precisely with paper-and-pencil, hand calculators, and computer software, using appropriate transformations (e.g., translations, rotations, reflections, enlargements), and using reason and logic

C.12.2 Use geometric models to solve mathematical and real-world problems

C.12.3 Present convincing arguments by means of demonstration, informal proof, counter-examples, or any other logical means to show the truth of statements (e.g., these two triangles are not congruent), and generalizations (e.g., the Pythagorean theorem holds for all right triangles)

C.12.4 Use the two-dimensional rectangular coordinate system and algebraic procedures to describe and characterize geometric properties and relationships such as slope, intercepts, parallelism, and perpendicularity

C.12.5 Identify and demonstrate an understanding of the three ratios used in right-triangle trigonometry (sine, cosine, tangent)

D-Measurement

D.12.1 Identify, describe, and use derived attributes (e.g., density, speed, acceleration, pressure) to represent and solve problem situations

D.12.2 Select and use tools with appropriate degree of precision to determine measurements directly within specified degrees of accuracy and error (tolerance)

D.12.3 Determine measurements indirectly, using estimation, proportional reasoning, including those involving squaring and cubing (e.g., reasoning that areas of circles are proportional to the squares of their radii), techniques of algebra, geometry, and right triangle trigonometry, formulas in applications (e.g., for compound interest, distance formula), geometric formulas to derive lengths, areas, or volumes of shapes and objects (e.g., cones, parallelograms, cylinders, pyramids), geometric relationships and properties of circles and polygons (e.g., size of central angles, area of a sector of a circle), and conversion constants to relate measures in one system to another (e.g., meters to feet, dollars to Deutschmarks

E- Statistics and Probability

E.12.1 Work with data in the context of real-world situations by formulating hypotheses that lead to collection and analysis of one- and two-variable data, designing a data collection plan that considers random sampling, control groups, the role of assumptions, etc., conducting an investigation based on that plan, and using technology to generate displays, summary statistics, and presentations

E.12.2 Organize and display data from statistical investigations using frequency distributions, percentiles, quartiles, deciles, line of best fit (estimated regression line), and matrices

E.12.3 Interpret and analyze information from organized and displayed data when given measures of dispersion, including standard deviation and variance, measures of reliability, and measures of correlation

E.12.4 Analyze, evaluate, and critique the methods and conclusions of statistical experiments reported in

journals, magazines, news media, advertising, etc.

E.12.5 Determine the likelihood of occurrence of complex events by using a variety of strategies (e.g., combinations) to identify possible outcomes, conducting an experiment, designing and conducting simulations, and applying theoretical probability

F- Algebraic Relationships

F.12.1 Analyze and generalize patterns of change (e.g., direct and inverse variation) and numerical sequences, and then represent them with algebraic expressions and equations

F.12.2 Use mathematical functions (e.g., linear, exponential, quadratic, power) in a variety of ways, including recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function, translating different forms of representing them (e.g., tables, graphs, functional notation, formulas), describing the relationships among variable quantities in a problem, and using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, minimum)

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities numerically, graphically, including use of appropriate technology, and symbolically, including use of the quadratic formula

F.12.4 Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities

Materials:

- Construction paper (enough for 4 per person), scissors, tape, paper and pencils for webbing (students will provide), white board and markers for class webbing, introduction PowerPoint with pictures of conic sections, graphing calculator, parabola worksheets, blank paper, circular objects (bracelet, pencil eraser, CD, circular protractor, etc. enough for one per student), circles note sheet, cubing sheet with rubric, KWL chart for ellipses, ellipses note sheet, Jeopardy PowerPoint, Unit Test

Sources:

- **http://www.pmsd.org/LinkClick.aspx?fileticket=0%2FmUWPUh9F8%3D&tabid=669&mid=1526**
- **http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Erbas/emat6690/Insunit/conicsunit.html**
- **http://dpi.wi.gov/standards/matintro.html**

Day 1: Introduction to Conic Sections

Intended Class:	**Lesson Duration:**
Algebra 2	52 minute class period

Rationale:

The different conic sections can be seen in everyday life. The purpose of this day's activities would be to give the students a visual understanding of the origin of each conic section. This will teach new vocabulary and encourage critical thinking skills when "slicing" the conic to create the parabola, circle, ellipse, and hyperbola. Visualization and critical thinking skills are important for the cognitive development of students, as well as essential skills in everyday life for planning ahead, organization, and creating strategies.

Goals:

Students will learn how to

- Understand how to construct a specific conic section.
- Make the connection between slicing a cone with a plane and conic sections.
- Be able to visualize each of the conic sections.

Objectives:

Students will be able to

- Create a web using "Conic Sections" as the center and using each of the conic sections as branches off of that, as well as identifying the characteristics of each conic section for further branching.
- Write a comparison/contrast paper analyzing the characteristics and properties of the conic sections using the webbing done in class.
- Analyze and evaluate mathematical thinking and strategies of others given other students' webbing
- Define parabola, ellipse, circle, and hyperbola through comparing and contrasting each section in their comparison/contrast paper.
- Describe how planes and cones are related to conic sections in their comparison/contrast paper.
- Use the language of mathematics to express mathematical ideas, such as ellipse, cone, parabola, hyperbola, plane, and circle, precisely.

Standard Alignment:

A-Mathematical Processes

A.12.2 Communicate logical arguments and clearly show why a result does or does not make sense, why the reasoning is or is not valid, an understanding of the difference between examples that support a conjecture and a proof of the conjecture

A.12.4 Develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

B- Number Operations and Relationships

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

C- Geometry

C.12.1 Identify, describe, and analyze properties of figures, relationships among figures, and relationships among their parts by constructing physical models, drawing precisely with paper-and-pencil, hand calculators, and computer software, using appropriate transformations (e.g., translations, rotations, reflections, enlargements), and using reason and logic

C.12.2 Use geometric models to solve mathematical and real-world problems

C.12.4 Use the two-dimensional rectangular coordinate system and algebraic procedures to describe and characterize geometric properties and relationships such as slope, intercepts, parallelism, and perpendicularity

F- Algebraic Relationships

F.12.2 Use mathematical functions (e.g., linear, exponential, quadratic, power) in a variety of ways, including recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function, translating different forms of representing them (e.g., tables, graphs, functional notation, formulas), describing the relationships among variable quantities in a problem, and using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, minimum)

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities numerically, graphically, including use of appropriate technology, and symbolically, including use of the quadratic formula

F.12.4 Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities

Materials:

- Construction paper (enough for 4 per person), Scissors**,** Tape
- paper and pencils for webbing (students will provide)**,** white board and markers for class webbing
- Introduction PowerPoint with pictures of conic sections
- Rubric for the comparison/contrast paper

Preparation:

Show students the introduction PowerPoint, which is made up of pictures of conic sections and pictures of a plane and cone. At each slide the student will write their own definition of the term using the picture provided. Ask for student volunteers to share their definitions of each of the terms. Using student definitions create a class definition of each term; the teacher will type ideas into the PowerPoint at the end with each of the ideas quoted by the student who shared it.

The teacher will ask questions afterwards:

- What do these definitions have in common? What do they have different?

- Do you disagree with any of the definitions? Why?

Assistance:

The teacher will then show students how to create a cone using construction paper, tape, and scissors. Students will create five cones; two will be formed from one piece of paper (to create the hyperbola). The teacher will demonstrate to the students how to cut each of their cones in order to form the four conic sections (show how the plane "slices" the cone). Students will then cut their four cones accordingly and observe what shape/section is formed for each.

The teacher will ask questions afterwards:

- Which of the terms we saw in the PowerPoint corresponds to cone 1? Cone 2? , etc.
- How did we slice the cones differently to form the circle versus the ellipse? The parabola versus the hyperbola?

Reflection:

Using the information that they gathered from the cone activity and the definitions they created from the PowerPoint, students will form their own webbing; "conic sections" will be in the middle and each of the four conic sections will be a bubble around it, with sub-bubbles around each of those. After about 15 minutes students will share to the class what they included in their webbing and the teacher will create a webbing of the class' ideas. Students can add to their webbing as the discussion takes place.

Questions to further the webbing process:

- How did we slice the cone to form each of these conic sections?
- What characteristics does each of these sections have?
- What does each of the sections remind you of?
- Where have you seen these sections in the real world?

Students will write a comparison/contrast paper outside of class on the conic sections using the webbing done in class. They will also need to include what cones and planes have to do with conic sections.

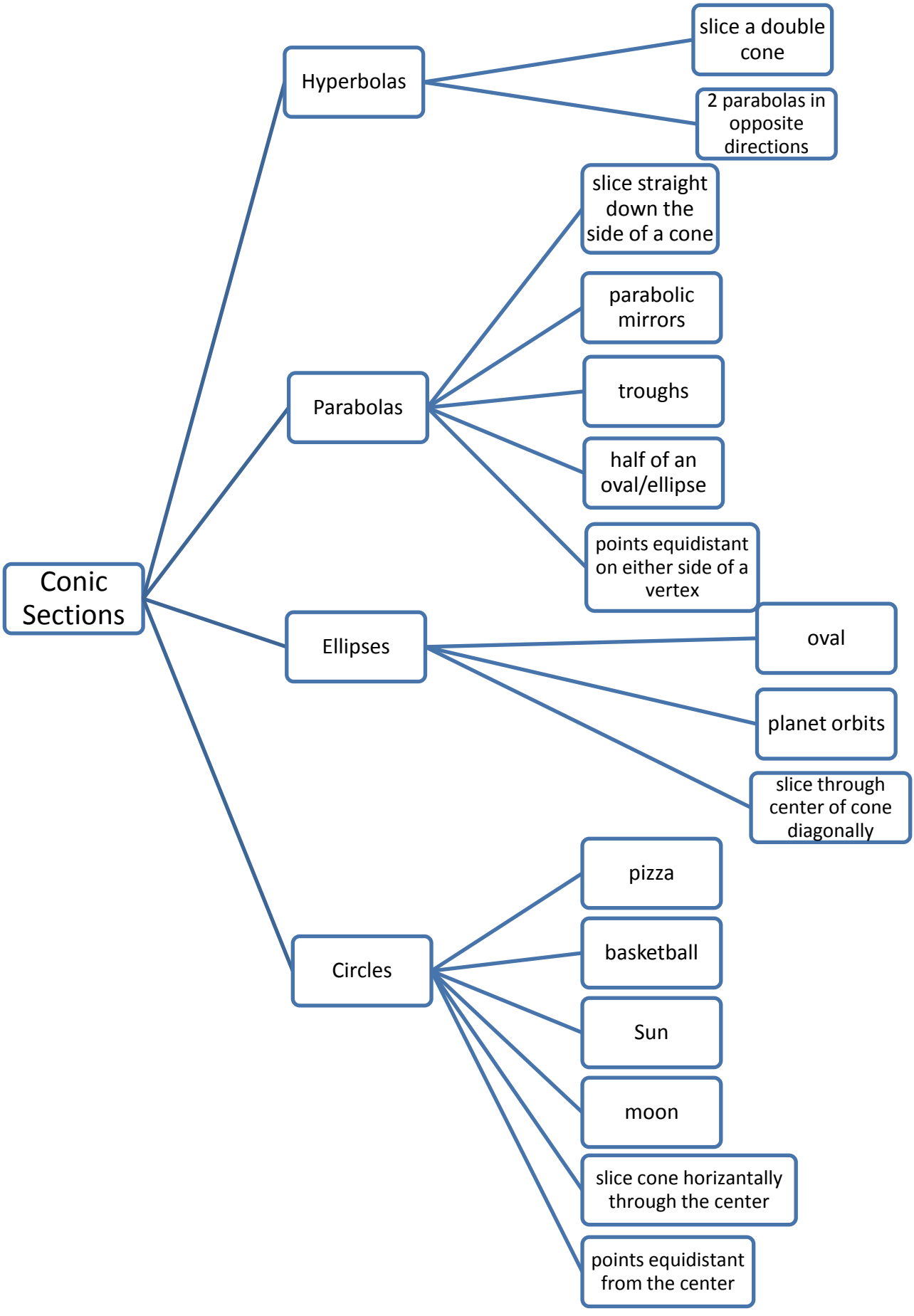
Assessment used:

Formative: The teacher will learn what prior knowledge and how well they picked up on the PowerPoint by the definitions that they provide during "Preparation". The teacher will learn how well the students are connecting the terms with the conic sections formed in the activity through the questioning in "Assistance". In the same section the teacher will learn if students are making correct conclusions about each of the sections. The teacher will learn overall how deep of an understanding the class overall had for each of the sections through questioning in "Reflection".

Summative: Comparison/contrast paper for the conic sections and how planes and cones are related to them.

Sources:

- http://www.pmsd.org/LinkClick.aspx?fileticket=0%2FmUWPUh9F8%3D&tabid=669&mid=1526
- http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Erbas/emat6690/Insunit/conicsunit.html
- http://www.readwritethink.org/files/resources/lesson_images/lesson275/compcon_rubric.pdf


Conic Sections
Hyperbolas
slice a double cone
2 parabolas in opposite directions
Parabolas
slice straight down the side of a cone
parabolic mirrors
troughs
half of an oval/ellipse
points equidistant on either side of a vertex
Ellipses
oval
planet orbits
slice through center of cone diagonally
Circles
pizza
basketball
Sun
moon
slice cone horizantally through the center
points equidistant from the center

Name:_____

Comparison and Contrast Rubric

CATEGORY	4	3	2	1
Purpose & Supporting Details	The paper compares and contrasts items clearly. The paper points to specific examples to illustrate the comparison. The paper includes only the information relevant to the comparison.	The paper compares and contrasts items clearly, but the supporting information is general. The paper includes only the information relevant to the comparison.	The paper compares and contrasts items clearly, but the supporting information is incomplete. The paper may include information that is not relevant to the comparison.	The paper compares or contrasts, but does not include both. There is no supporting information or support is incomplete.
Organization & Structure	The paper breaks the information into whole-to-whole, similarities -to-differences, or point-by-point structure. It follows a consistent order when discussing the comparison.	The paper breaks the information into whole-to-whole, similarities -to-differences, or point-by-point structure but does not follow a consistent order when discussing the comparison.	The paper breaks the information into whole-to-whole, similarities -to-differences, or point-by-point structure, but some information is in the wrong section. Some details are not in a logical or expected order, and this distracts the reader.	Many details are not in a logical or expected order. There is little sense that the writing is organized.
Transitions	The paper moves smoothly from one idea to the next. The paper uses comparison and contrast transition words to show relationships between ideas. The paper uses a variety of sentence structures and transitions.	The paper moves from one idea to the next, but there is little variety. The paper uses comparison and contrast transition words to show relationships between ideas.	Some transitions work well; but connections between other ideas are fuzzy.	The transitions between ideas are unclear or nonexistent.
Grammar & Spelling (Conventions)	Writer makes no errors in grammar or spelling that distract the reader from the content.	Writer makes 1-2 errors in grammar or spelling that distract the reader from the content.	Writer makes 3-4 errors in grammar or spelling that distract the reader from the content.	Writer makes more than 4 errors in grammar or spelling that distract the reader from the content.

Day 2: Parabolas

Intended Class:	Lesson Duration:
Algebra 2	52 minute class period

Rationale:

Students need to know about parabolas because there are many real-life applications to parabolas that they will encounter throughout their lifetime. For example, any object in free fall, be it a piece of chalk, a skate board rider or a ball, will follow a parabolic trajectory. Parabolic shapes are easy to find.

- Many homes outside the cable area have parabolic antennas to receive television transmissions from a satellite far out in space.
- Parabolic reflectors are used to build solar ovens, searchlights and radio telescopes. Reflectors and satellite dishes are in the shape of parabolic surfaces
- Parabolic surfaces called paraboloids figure in optics and other technological applications.
- The headlamps of your car are in this shape as well. In fact the bulb is placed at a special point called the focus of the parabola.

By learning the basics of a parabola students will be broadening their understanding of exactly how these parabolas work, how we can construct them, and how useful they are in everyday life.

Goals:

Students will

- Familiarize themselves with the parts of a parabola.
- Understand how the arbitrary numbers represented by a, b, and c in the standard equation of an ellipse affect the overall shape of the graph.

Objectives:

Students will be able to

- Compare and contrast when a parabola opens upward or downward
- identify the axis of symmetry
- graph a parabola on a graphing calculator
- Manipulate quadratic equations to solve for “y”, the variable representing the vertical axis
- Find the vertex of a specific parabola given an equation
- Work with peers, communicating collectively to compare and contrast the graphs of various parabolas provided in the Lab Sheet.

Standard Alignment:

A-Mathematical Processes

A.12.4 Develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

B- Number Operations and Relationships

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

C- Geometry

C.12.1 Identify, describe, and analyze properties of figures, relationships among figures, and relationships among their parts by constructing physical models, drawing precisely with paper-and-pencil, hand calculators, and computer software, using appropriate transformations (e.g., translations, rotations, reflections, enlargements), and using reason and logic

F- Algebraic Relationships

F.12.2 Use mathematical functions (e.g., linear, exponential, quadratic, power) in a variety of ways, including recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function, translating different forms of representing them (e.g., tables, graphs, functional notation, formulas), describing the relationships among variable quantities in a problem, and using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, minimum)

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities numerically, graphically, including use of appropriate technology, and symbolically, including use of the quadratic formula

F.12.4 Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities

Materials:

- graphing calculator
- worksheet

Preparation:

The teacher will write the 5 term words (*Parabola, Axis of symmetry, Roots, Turning point/vertex , Maximum/minimum point*) on the board and ask the students to define each term the best they can as a quick write. They will be given 5 minutes to come up with their own definitions. Then we will discuss in class what we believe the terms mean and come up with class definitions for each term. When students are sharing their definition, I will ask them how they came about their definition, hoping to hear things like sight words, for example, when the word “point” is contained in the term. They may already know what symmetry means which would help them understand what axis of symmetry is. These five terms are important because the students will be asked to find these terms, given an

equation.

Assistance:

The teacher and students will work together on the first part of the handout that asks them to fill in the blank, some being the terms. We will discuss what they believe what the answer is and why. Then they will work with a partner using graphing calculators to see the difference of a graph of a parabola when a, b, or c are changed from the standard parabola equation, $ax^2 + b x + c = y$ on the second part of the handout. Here is where they will see the effects a, b, and c cause on the position/shape of the graph. They will be asked to write what they see and their conclusion about the differences next to a sketch of the graphs and equations that were given. The teacher will be walking around helping the students out during this time. When everyone is done, we will discuss as a class what the correct answers are.

Reflection:

Quick Write: Students will be asked to write about what they learned, what they don't quite understand, and what they would like to know more about. Then they will be asked to write a paragraph using the terms for parabolas. They will also be asked to write down the effects the variables a, b, and c in the standard equation have on the graph from the top of their heads, no notes. They can have about 10 minutes for this. With the remaining time, the teacher can ask volunteers to share their quick write. And after a student shares, you can discuss their questions about what they didn't understand. If one student didn't quite understand part of the lesson, chances are some other students didn't either. About two students should be able to share before class is over. Students will be asked to hand in their quick writes in order for the teacher to see where each student is in their understanding.

Assessment used:

Formative:

Observation: When the teacher is walking around during the graph handout time, they can make sure the students are on the right track by asking some questions to push them in the right direction. Example of question to ask students: How is this graph different from the basic equation's graph line?

Self-assessment: They are self-assessing in their quick write during reflection.

Summative:

Quick writes will be graded on whether or not students used the term correctly or not. 5 points will be awarded if all terms are used correctly, and an additional 3 points if they correctly stated at least 3 of the effects on the graph caused by a, b, and c. B wasn't mention during the lesson plan so if they state the effects b has on the graph they will receive bonus points. In other words, students have a chance of getting 10/8 on the assignment

Sources:

- http://math.buffalostate.edu/~it/projects/Dueger.pdf
- http://www.pmsd.org/LinkClick.aspx?fileticket=0%2FmUWPUh9F8%3D&tabid=669&mid=1526

(write terms on the board)

Parabola

Axis of symmetry

Roots

Turning point/vertex

Maximum/minimum point

Example of a student's quickwrite

Parabola-A symmetrical open plane curve, u-shaped graph

Axis of symmetry- the line that splits the parabola up the middle

Roots- points on a graph? ** students most likely have not been exposed to this word before**

Turning point/vertex – the point where the parabola changes direction.

Maximum/minimum point –point that's the highest /lowest on the graph

Day 3: Circles

Intended Class:	Lesson Duration:
Algebra 2	52 minute class period

Rationale:

Circles are found in everyday life so it is important for students to understand their properties and geometric construction. It is important to understand the construction of the equation of a circle to understand its properties and how it is formed. While learning about the standard equation of a circle, students will review many of the essential skills needed to succeed in mathematics. These skills include manipulation, solving, addition, subtraction, multiplication, division, and square root. These skills can be applied to many everyday activities throughout student's lives.

These paragraphs should be read to students at the beginning of the lesson to make them aware of the need to understand circles:

Now what makes the circle so special and why? Well, it is the precise definition of this conic section that has caused this shape to have such a monumental effect on the progress of mankind. For how would man have progressed without the wheel? Precisely because the points of the circle are all the same distance from the center, is the smooth roll of the wheel possible. After all, how efficient would your car or dune buggy be, for that matter, if the wheels were in the shape of hexagons?

The centers of flowers, eyes, and many more things are circular and we see them in our everyday life.

If you're into sports, then a sudden lack of circles would completely ruin your sporting activities. In many sports, balls are necessary (football, tennis, ping pong, golf, and so many more). Without circles there would be no spheres, and these sports simply wouldn't be possible for anybody to play. Planets, the movement of the planets, natural cycles, and natural shapes - there are circles absolutely everywhere.

Goals:

Students will

- Understand the terms circle, center, radius, diameter, circumference, tangent, and secant.
- Make the connection between the distance formula and the standard equation of a circle.
- Fully understand the equation of a circle; its parts and how they affect the graph.

Objectives:

Students will be able to:

- Define circle, center, radius, diameter, circumference, tangent, and secant in their own words.
- Describe a circular object using the cubing method.
- Manipulate the distance formula to get the standard equation of a circle.
- Rearrange equations to get the standard equation of the circle.
- Graph the circle, given its equation.

- Write the equation of and graph a circle, given its center and a tangent line.
- Write the equation of and graph a circle, given the endpoints of the diameter.

Standard Alignment:

A-Mathematical Processes

A.12.4 Develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

B- Number Operations and Relationships

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

C- Geometry

C.12.1 Identify, describe, and analyze properties of figures, relationships among figures, and relationships among their parts by constructing physical models, drawing precisely with paper-and-pencil, hand calculators, and computer software, using appropriate transformations (e.g., translations, rotations, reflections, enlargements), and using reason and logic

C.12.2 Use geometric models to solve mathematical and real-world problems

F- Algebraic Relationships

F.12.2 Use mathematical functions (e.g., linear, exponential, quadratic, power) in a variety of ways, including recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function, translating different forms of representing them (e.g., tables, graphs, functional notation, formulas), describing the relationships among variable quantities in a problem, and using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, minimum)

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities numerically, graphically, including use of appropriate technology, and symbolically, including use of the quadratic formula

Materials:

- Paper
- Pencil
- Circular objects (bracelet, pencil eraser, CD, circular protractor, ect.)- # of students
- Note Sheet
- Cubing Sheet with Rubric

Preparation:

Students will do a quick write with the keywords: circle, center, radius, diameter, circumference,

tangent, and secant, explaining each of the terms in their own words. [5 minutes] We will then write definitions for these words as a class on the board and correct any misconceptions they had in their notesheets. [5-10 minutes] (sample expected definitions)

Circle- set of all points in a plane that are equidistant from the center.

Center- point inside the circle. All points on the circle are the same distance from the center point.

Radius- distance from the center to any point on the circle.

Diameter- length of the line going across the circle and passing through the center. It is twice the radius.

Circumference- distance around the circle.

Tangent- a line passing a circle and touching it at just one point.

Secant- a line that intersects a circle at two points.

Assistance:

The teacher will bring an assortment of circular objects (bracelet, pencil eraser, CD, circular protractor, ect.). Each student will pick one of these objects and place it on their desk with paper and a writing utensil. They will then write notes about their object using the 'cubing' technique. They will be instructed to relate the keywords to their object by describing the relation of the object to their desk and paper. For the comparison portion, they should collaborate with the person next to them to compare the two objects. [15 minutes]

When they are finished note taking, they will partner up with someone other than the person they compared with to discuss their notes. The students will be instructed to let their partner read their notes without showing them the object. The students should give each other constructive criticism about what they could describe better, or how they could use the keywords more effectively. [5 minutes] The students will be expected to compile their notes into a cohesive paper following cubing form and explaining their object fully. This will be due in the next class.

The teacher and students will go through the note sheet which includes:

- manipulating the distance formula to get the standard equation of a circle,
- rearranging equations to get the standard equation of the circle,
- recognizing the center and radius of a circle given the standard equation,
- graphing the circle, given its equation,
- writing the equation of and graphing a circle given it's center and a tangent line, and
- writing the equation of and graphing a circle given the endpoints of the diameter.

While going through the note sheet, the teacher will ask for the students to brainstorm about how to

solve each type of problem instead of just going through the examples together. [25-30 minutes]

Reflection:

Students will reflect in a journal entry. They will be encouraged to write freely, but will prompted by these questions:

- What did you learn about circles today?
- What do you have questions about/need to review about circles?
- What do you have questions about/need to review about conic sections?
- What are you excited to learn about conic sections in the future?

Students will also need to complete their cubing paper.

Assessment used:

Baseline- The quick write activity at the beginning of the lesson gives the teacher a good idea what knowledge the students have about each one of the keywords and what will need more work.

Formative- While making class definitions, the teacher will assess the mood of the class when compiling each definition to decide if none, some, or all of the class understands.

While going through the note sheet, the teacher will be able to call on students and ask them questions. This will give them a chance to assess if the student grasps the concept or needs more explanation.

Summative- The reflection journal will give students a chance to express what they have learned in this section as well as any problems they are having with the conic sections material up to this point. Using this, the teacher can assess what may need extra review.

The cubing paper will complete the cubing activity and give the teacher an idea of how well the student understands the concepts of circles and is able to describe and relate them.

Sources:

Definitions-adapted from **http://www.mathopenref.com/circle.html**

Cubing worksheet information adapted from Professor Omachonu's Cubing Sheet

Problems from note sheet adapted from Algebra 2 Textbook by Glencoe

Day 4: Ellipses

Intended Class:	**Lesson Duration:**
Algebra 2	52 minute class period

Rationale:

The ellipse is an important tool of algebra and geometry. Manipulation of the elliptical formula is an essential skill that students will be able to apply to many other types of problems in mathematics. These skills include formula manipulation, solving, and use of the five basic operations (addition, subtraction, multiplication, division, and square root). The five basic operations can be used in many real-life applications, such as balancing a checkbook or making a budget. Important real world applications of the ellipse include planetary orbits, uses in machinery, and lithotripsy, a surgical procedure used to remove kidney stones.

Goals:

Students will be able to

- Understand the origin of the equation of an ellipse
- Familiarize themselves with real world applications of an ellipse
- Apply various skills to solve problems involving an ellipse

Objectives:

Students will

- Describe the following vocabulary words: Ellipse, focus, center, major axis, and minor axis in their own words on the KWL chart
- Write the general equation of an ellipse on the KWL chart
- Apply the general equation of an ellipse to specific examples to solve for the standard form of the equation
- Graph an ellipse given an equation
- Give the foci, vertices, co-vertices, and center of an ellipse when provided with an equation
- Give the equation of an ellipse when provided with the vertices and foci

Standard Alignment:

A-Mathematical Processes

A.12.4 Develop effective oral and written presentations employing correct mathematical terminology, notation, symbols, and conventions for mathematical arguments and display of data

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

A.12.6 Read and understand mathematical texts and other instructional materials, and writing about mathematics (e.g., articles in journals)mathematical ideas as they are used in other contexts

B- Number Operations and Relationships

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

C- Geometry

C.12.2 Use geometric models to solve mathematical and real-world problems

F- Algebraic Relationships

F.12.2 Use mathematical functions (e.g., linear, exponential, quadratic, power) in a variety of ways, including recognizing that a variety of mathematical and real-world phenomena can be modeled by the same type of function, translating different forms of representing them (e.g., tables, graphs, functional notation, formulas), describing the relationships among variable quantities in a problem, and using appropriate technology to interpret properties of their graphical representations (e.g., intercepts, slopes, rates of change, changes in rates of change, maximum, minimum)

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities numerically, graphically, including use of appropriate technology, and symbolically, including use of the quadratic formula

F.12.4 Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities

Materials:

- Pencil
- Paper
- KWL Chart
- Note sheet

Preparation:

The students will be broken into groups of 2-4 and asked to complete the K column of the KWL chart with definitions in their own words or with a picture. After roughly five minutes, we will discuss as a class what we already know about an ellipse. It is important for students to work together so that any simple misconceptions can be cleared within their groups. Students will then be asked to fill out the W column of the KWL chart, what they would like to know..

Assistance:

The lesson will be presented to the students while they follow along with the provided note sheet. These notes focus on vocabulary, skills, and application of these skills. Within these notes, the students will learn the general equation of an ellipse, how to find the foci and vertices, orientation of the ellipse, and how to graph the ellipse. We will use the following word identification strategies in the lesson:

- Sight Word recognition: Students will be able to recognize words they already know; i.e. Center
- Analogy: Students will be able to determine the difference between the major axis and minor axis by using prior knowledge of the words major and minor. Also, since they will have already learned the word *vertex* in a previous lesson, they will be able to determine the difference

between *vertex* and *covertex* on the ellipse.
- Morphemic Analysis: Students will be able to use their understanding of the word *ellipse* to comprehend the meaning of something being *elliptical*.

Throughout the lesson, students will use various skills to find the foci, vertices, covertices, and equation of any particular ellipse. Also, the students will be asked to graph any particular ellipse given an equation, vertices, or foci.

Reflection:

Students will fill out the L column of their KWL charts with the correct definitions or sketches provided in class. In addition, students will be asked to complete the back side of the KWL chart, which asks students to draw an ellipse and label each of the vocab words they learned. At the bottom, they are asked to write the general equation of an ellipse.

With the small group they worked with at the beginning of class, students will be presented this problem: *Mercury follows an elliptical orbit around the sun. Draw a sketch of Mercury's orbit. Write an equation for the orbit.* (hint: You need more information, so feel free to ask questions.)

[Extra information for teacher's use:

-Mercury is 29 miles from the center of the sun.

-At its farthest point, Mercury is 43.8 million miles from the center of sun.

-Assume the center of this orbit lies at the origin.]

Students will need to think critically about this problem to discover what information they need. If they're struggling with the missing information, we will ask pointed questions to prompt the correct questions from them.

Key for this problem on attached sheet

Assessment used:

Baseline- The 'K' section of the KWL chart will allow the teacher to assess how much previous knowledge the students have about the keywords and concepts.

Formative- The 'W' section of the KWL chart will allow the teacher to assess how much the students learned from the discussion, and what they want to learn more about.

While going through the note sheet, the teacher will be able to call on students and ask them questions. This will give them a chance to assess if the student grasps the concept or needs more explanation.

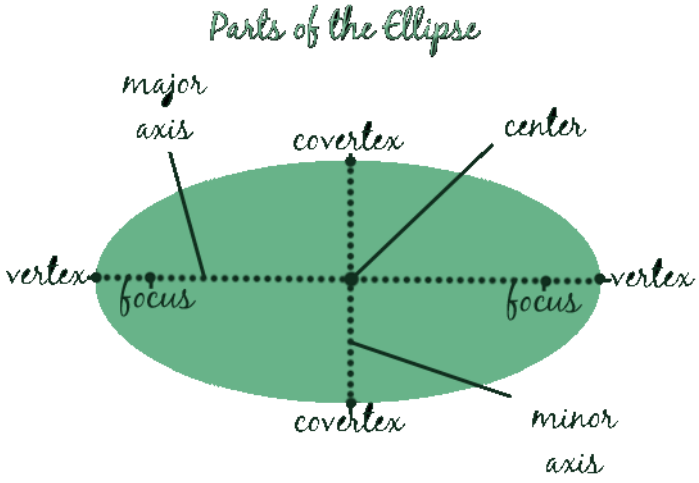
Summative- The 'L' section of the KWL chart will allow the teacher to assess whether the students have learned throughout the lesson. This will give them an opportunity to assess if the group grasps the keywords and concepts or needs to review.

The back of the KWL sheet will allow the teacher to assess if the students can properly label each part of an ellipse and write its equation.

The ‘mercury’ problem will test student’s comprehension of the skills and concepts given during the lesson and their ability to apply them.
Sources:
Examples adapted from Algebra 2 Textbook by Glencoe Note Sheet images and information adapted from http://www2.norwalk-city.k12.oh.us/wordpress/halgebra2/2009/05/19/104-ellipses/ Note Sheet adapted from http://faculty.atu.edu/sjordan/Precalculus%20notes/Conic%20Sections--Ellipses.pdf Graph paper from http://www.senocular.com/pub/kirupa/rotatevectorthread_files/cartplane.gif

Ellipses

An ellipse is the set of points in a plane for which the sum of the distances from two fixed points is a given constant, 2a. The two fixed points are called the focal points or foci of the ellipse; the line passing through the foci is the major axis. The points of intersection of the axis and the ellipse are the vertices, and the perpendicular bisector of the major axis is the minor axis.



Equation of Ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

(x,y) = coordinates of any point on the ellipse

a = the horizontal from center to a vertex

b = vertical from center to a vertex

Center is at (h,k), intersection of the major axis and minor axis

Major axis = longer segment, foci always lie on it

Minor axis = short segment

Vertex = The point at either end of the major axis

Covertex = The point at either end of the minor axis

Foci = Points that "define" the ellipse. The sum of the distances of any point on the ellipse to the foci is constant.

Use the distance formula and the definition of an ellipse to find the equation of the ellipse.

Foci at (8,0) and (-8,0). major axis of length 24.

$$\sqrt{(x+8)^2+y^2}+\sqrt{(x-8)^2+y^2}=24$$

$$\sqrt{(x+8)^2+y^2}=24-\sqrt{(x-8)^2+y^2}$$

$$(x+8)^2+y^2=576-48\sqrt{(x-8)^2+y^2}+(x-8)^2+y^2$$

$$x^2+16x+64+y^2=576-48\sqrt{(x-8)^2+y^2}+x^2-16x+64+y^2$$

$$32x-576=-48\sqrt{(x-8)^2+y^2}$$

$$2x-36=-3\sqrt{(x-8)^2+y^2}$$

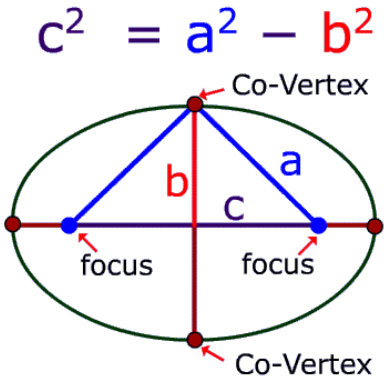
$$4x^2-144x+1296=9((x-8)^2+y^2)$$

$$4x^2-144x+1296=9x^2-144x+576+9y^2$$

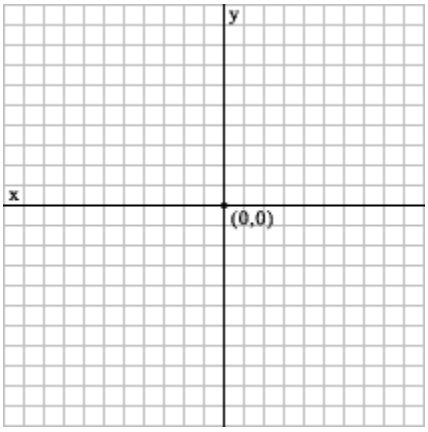
$$5x^2+9y^2=720$$

$$\frac{x^2}{144}+\frac{y^2}{80}=1$$

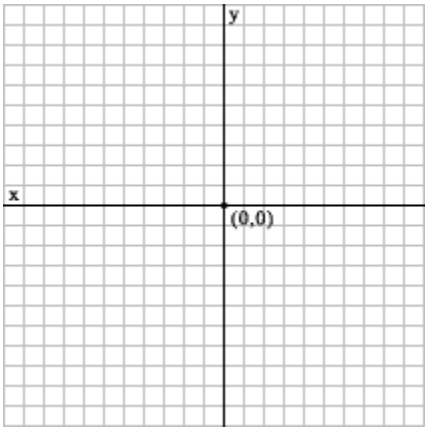
The ellipse at the right shows a more detailed sketch of the foci. The sum of the distances from the foci to any point on the ellipse is 2*a* units. The distance from the center to either focus is *c* units. We can calculate the value of *b* by using the *Pythagorean* Theorem, $b^2 = a^2 - c^2$.



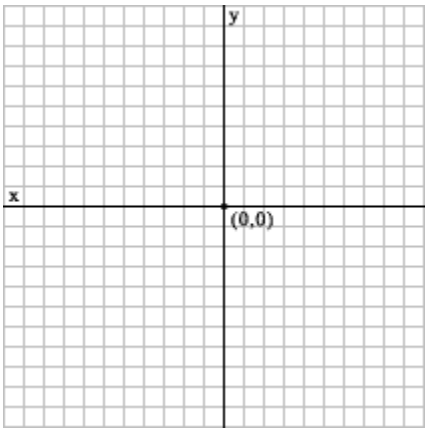
Graph the ellipse given by $\frac{(x+2)^2}{49} + \frac{(y-3)^2}{1} = 1$, and find the center, vertices, co-vertices and foci.



Find the coordinate of the foci and the lengths of the major and minor axes of an ellipse whose equation is$16x^2 + 4y^2 = 144$. Then graph the ellipse.



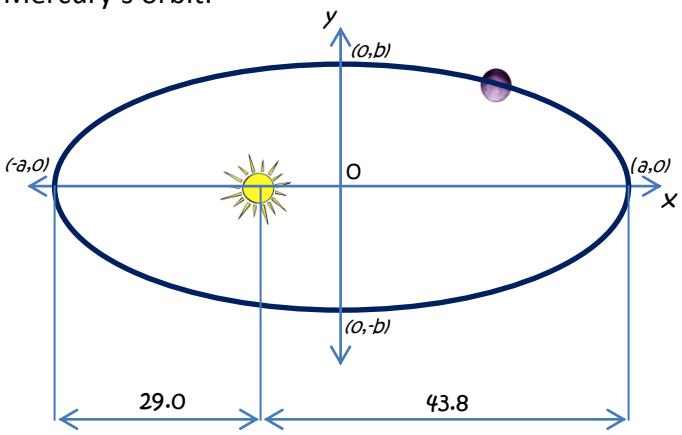
Find the equation of the ellipse with vertices at (7,0) and (-7,0) and foci at (4,0) and (-4,0). Then graph the ellipse.



Mercury follows an elliptical orbit around the sun. Draw a sketch of Mercury's orbit. Write an equation for the orbit. (hint: You need more information, so feel free to ask questions.)

[Extra information for teacher's use: Mercury is 29 miles from the center of the sun. At its farthest point, Mercury is 43.8 million miles from the center of sun. Assume the center of this orbit lies at the origin.]

a. Draw a sketch of Mercury's orbit.



b. Write an equation for the orbit.

The length of the major axis is 29.0 + 43.8 or 72.8 million miles. Use this information to find the values of a.

$2a=72{,}800{,}000$

$a=36{,}400{,}000$

The distance from the center of the orbit to the center of the sun is

36.4 – 29.0 or 7.4 million miles. This is the value of c, the distance from the center to the focus.

$b^2 = c^2 + a^2$

$= (36{,}400{,}0000)^2 - (7{,}400{,}000)^2$

$= 1{,}324{,}960{,}000{,}000{,}000 - 54{,}760{,}000{,}000{,}000$

$= 1{,}270{,}200{,}000{,}000{,}000$ or 1.2702×10^{15}

Now we can write the equation

$$\frac{x^2}{\quad} + \frac{y^2}{\quad} = 1$$

$a^2 \quad b^2$

$$\frac{x^2}{(36{,}400{,}000)^2} + \frac{y^2}{1{,}270{,}200{,}000{,}000{,}000} = 1$$

$$\frac{x^2}{1.32496 \times 10^{15}} + \frac{y^2}{1.2702 \times 10^{15}} = 1$$

An equation for the orbit is $\frac{x^2}{1.32496 \times 10^{15}} + \frac{y^2}{1.2702 \times 10^{15}} = 1$

Day 5: Unit Review/ Jeopardy	
Intended Class:	**Lesson Duration:**
Algebra 2	52 minute class period
Rationale:	
Through a fun game of jeopardy students will be motivated to recall what they have learned.	
Goals:	
Students will be able to • Review their knowledge of conic sections for the upcoming exam.	
Objectives:	
Students will be able to • State the general equations of conic sections. • Describe direction, stretches, and shifts in the graphs of conic sections, given the equation. • Give the center and radius of a circle, when provided with an equation. Write the equation of a circle, given its center and tangent or endpoints of diameter. • Find the center, foci, and lengths of axes of an ellipse, given an equation. • Describe each of the conic sections. • Describe vocabulary words, given the definition. • Use the distance formula to find the distance between two points.	
Standard Alignment:	
A-Mathematical Processes	

A.12.1 Use reason and logic to evaluate information, perceive patterns, identify relationships, formulate questions, pose problems, make and test conjectures, and pursue ideas that lead to further understanding and deeper insight

A.12.5 Organize work and present mathematical procedures and results clearly, systematically, succinctly, and correctly

B- Number Operations and Relationships

B.12.3 Perform and explain operations on real numbers (add, subtract, multiply, divide, raise to a power, extract a root, take opposites and reciprocals, determine absolute value)

C- Geometry

C.12.2 Use geometric models to solve mathematical and real-world problems

F- Algebraic Relationships

F.12.3 Solve linear and quadratic equations, linear inequalities, and systems of linear equations and inequalities numerically, graphically, including use of appropriate technology, and symbolically, including use of the quadratic formula

F.12.4 Model and solve a variety of mathematical and real-world problems by using algebraic expressions, equations, and inequalities

Materials:

- Jeopardy Review PowerPoint
- White board/Markers to keep score

Procedure:

When students enter class, they will be numbered off into groups of about five and told to sit with their group. The teacher will explain the game of Jeopardy as it will be used in this context and the groups will start to play. This will include a random group choosing a category and point amount. When the question comes up, each group will work together to come up with an answer to the problem. They will then 'buzz in.' If they are correct, they will get the points and choose the next category and points. If they are incorrect, the other groups will have an opportunity to get the points. The winning group will get bragging rights until the next Jeopardy review.

Assessment used:

Formative- This activity give the teacher a final opportunity to assess whether the majority of the students understand the concepts of conic sections. If they notice that students do not understand or that the same few students are constantly answering, they should go over each problem in depth as a more descriptive review.

Sources:

Some ideas adapted from- http://teachers.net/lessons/posts/4198.html