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Parabolic Channel Design

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Abstract - The parabolic shape is often a suitable shape for channel cross sections due to its various advantages over conventional shapes such as the trapezoidal, triangular and rectangular shapes. Methods have been developed for the design of optimal parabolic section shapes which often do not take freeboard or maximum or minimum side slope into consideration. It is well known that often the maximum side slope of a channel is predetermined by the field conditions of the project. Hence there is need for methods that take maximum allowable side slope into consideration. With this requirement in mind, a method is developed for the design of a parabolic channel with consideration of freeboard and maximum allowable side slope. This is compared with another method that produces a hydraulically optimal design for given project conditions without taking maximum side slope into consideration. A table is provided as a design aid.

Index Terms - channel design; cross section; irrigation; optimal; parabolic

1 INTRODUCTION

The parabolic cross section shape is for many situations the best practical shape for an open channel. One of its advantages is the ability to maintain a higher velocity at low discharge which reduces the tendency to deposit sediment. Another is its greater depth at low discharges which enables it to carry floating and semi-floating debris more easily than a flat-bottomed channel. Moreover, small parabolic channels tend to suffer less damage from cattle because they do not walk in them, and from cyclists because they do not ride in them as often happens to other conventional channels. Furthermore, Mironenko et. al. ([13]) and Chahar ([5]) stated that since river beds, unlined channels, and irrigation furrows all tend to approximate a stable parabolic shape, unlined channels are made more hydraulically stable when they are initially constructed in a parabolic shape. The channel side slopes along the cross section are always less than the maximum allowable slide slope which occurs at the water surface. Laycock ([11]) also states that in the form of precast concrete segments, the parabola has an inherent structural strength, especially if the sides are unsupported, and practically, it is easy to design the profile of varying thickness so that the strength is at the root of the cantilevered sides, where it is needed.

Loganathan ([12]) and Chahar ([5]) showed that the optimal parabolic cross section has a side slope of 1 / 0.513 at the water surface. However, Laycock ([11]) pointed out that, in order to increase strength and ease of handling, small precast segments must have a narrower top width than the hydraulically most efficient section, and cast-in situ concrete channels should have a wider top width to flatten the side slopes for safety reasons and to make construction without formwork easier. He argued that people and animals have great difficulty getting out of large, smooth-lined channels with side slopes greater than 1/2, and that constructing channels without formwork greatly reduces the construction costs of the channel. Montanes ([14]) also stated that a side slope of 60^{o} (or 1/0.577) is too steep to be stable in most types of ground except rock. Thus, although the most hydraulically efficient parabolic section has a water surface side slope of 1/0.513, often what is needed in the field is a channel which has a surface water side slope less than this value or greater than this value.

Hussein [9], Anwar and Clarke ([1]) and Anwar and de Vries ([2]) suggested methods for the design of power-law channels where the variable to be optimized is the exponent m for the power-law channel and not the side slope at the water surface. However, these tend to produce U-shaped cross sectional designs which tend to lose some of the advantages of the first order parabolic channel.

In this paper, a method is proposed for the design of a first order parabolic channel with consideration of freeboard and maximum allowable side slope. Another method is shown that only considers freeboard and not the maximum allowable side slope. The second method optimizes the channel section using the optimum side slope value at the water surface of 1/0.513. A design example is used to compare the two methods.

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2 GEOMETRIC PROPERTIES OF THE PARABOLIC SECTION

A parabolic canal is described by the equation ([13],[12],[5]):

$$Y = aX^2 \tag{1}$$

Where Y = vertical co-ordinate; X = abscissa; and a = shape parameter (Figure 1).

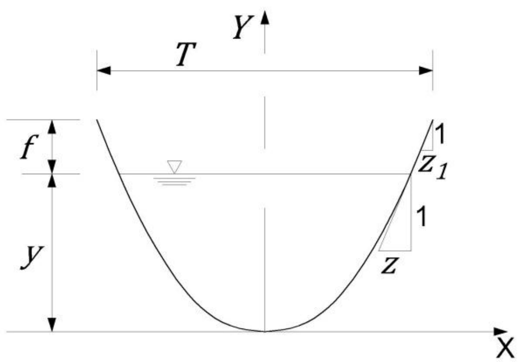

Figure 1 Parabolic channel cross section

It can be shown that the total area A_t is given by

$$A_t = \frac{8}{3}(y+f)^2 z_1 \tag{2}$$

and flow area A as

$$A = \frac{8}{3} z y^2 \tag{3}$$

in which y = the flow depth (m); f = freeboard (m); $1/z_1$ is the side slope at the top bank level, and 1/z is the side slope at the water surface level.

It can also be shown that z_1 and z are related by the equation

$$z = z_1\sqrt{1+k} \tag{4}$$

where

$$k = \frac{f}{y} \tag{5}$$

Thus flow area A can be expressed as

$$A = \frac{8}{3} y^2 z_1 \sqrt{1+k} \tag{6}$$

Top width T is given by

$$T = 4z_1 y(1+k) \tag{7}$$

Wetted perimeter, W, is given by

$$W = y\,J(z) \tag{8}$$

where

$$J(z) = 2z^2\left[\frac{1}{z}\sqrt{1+\frac{1}{z^2}} + ln\left(\frac{1}{z} + \sqrt{1+\frac{1}{z^2}}\right)\right] \tag{9}$$

3 A NON-OPTIMAL DESIGN METHOD

Manning's equation is the most used equation for uniform flow design of open channels and it is used in this analysis. The Manning's equation can be expressed as follows

$$Q = \frac{1}{n}\frac{A^{5/3}}{W^{2/3}}\sqrt{S} \tag{10}$$

in which Q = flow discharge (m^3/s); n = Manning's roughness coefficient; and S = longitudinal bed slope (%).

Substituting (3) and (8) into (10) gives

$$\frac{y}{\left(\frac{Qn}{\sqrt{S}}\right)^{\frac{3}{8}}} = \frac{[J(z)]^{\frac{1}{4}}}{\left(\frac{8}{3}z\right)^{\frac{5}{8}}} \tag{11}$$

The LHS of (11) is the nondimensional expression for flow depth ([5]). Thus

$$Y_* = G(z) \tag{12}$$

where Y_* is nondimensional flow depth and

$$G(z) = \frac{[J(z)]^{\frac{1}{4}}}{\left(\frac{8}{3}z\right)^{\frac{5}{8}}} \tag{13}$$

TABLE 1 gives values of Y_* for different values of z_1 and k.

TABLE 1(a)

VALUES OF Y_* FOR DIFFERENT VALUES OF z_1 AND k

z_1	Y_*						
	$k = 0$	$k = 0.1$	$k = 0.2$	$k = 0.3$	$k = 0.4$	$k = 0.5$	$k = 0.6$
0.1	2.7401	2.6616	2.5920	2.5297	2.4735	2.4223	2.3756
0.2	1.8090	1.7595	1.7157	1.6766	1.6413	1.6093	1.5800
0.3	1.4359	1.3986	1.3657	1.3363	1.3098	1.2857	1.2638
0.4	1.2290	1.1987	1.1719	1.1480	1.1265	1.1069	1.0891
0.5	1.0956	1.0698	1.0470	1.0266	1.0083	0.9916	0.9764
0.6	1.0014	0.9788	0.9587	0.9408	0.9247	0.9100	0.8966
0.7	0.9307	0.9104	0.8924	0.8762	0.8617	0.8485	0.8364
0.8	0.8753	0.8567	0.8402	0.8255	0.8121	0.8000	0.7889
0.9	0.8304	0.8131	0.7979	0.7842	0.7718	0.7605	0.7502
1.0	0.7930	0.7768	0.7625	0.7497	0.7381	0.7275	0.7178
1.1	0.7612	0.7460	0.7324	0.7203	0.7093	0.6993	0.6901
1.2	0.7337	0.7193	0.7064	0.6948	0.6844	0.6748	0.6660
1.3	0.7096	0.6958	0.6835	0.6725	0.6624	0.6533	0.6449
1.4	0.6883	0.6751	0.6632	0.6526	0.6430	0.6341	0.6260
1.5	0.6692	0.6564	0.6451	0.6348	0.6255	0.6170	0.6091
1.6	0.6520	0.6396	0.6286	0.6187	0.6097	0.6014	0.5938
1.7	0.6363	0.6243	0.6136	0.6040	0.5953	0.5872	0.5799
1.8	0.6220	0.6103	0.5999	0.5906	0.5820	0.5742	0.5671
1.9	0.6088	0.5975	0.5873	0.5782	0.5699	0.5623	0.5553
2.0	0.5966	0.5855	0.5756	0.5667	0.5586	0.5512	0.5443
2.1	0.5853	0.5745	0.5648	0.5561	0.5481	0.5409	0.5342
2.2	0.5747	0.5641	0.5547	0.5461	0.5383	0.5312	0.5247
2.3	0.5648	0.5545	0.5452	0.5368	0.5292	0.5222	0.5158
2.4	0.5556	0.5454	0.5363	0.5281	0.5206	0.5137	0.5074
2.5	0.5468	0.5368	0.5279	0.5198	0.5125	0.5057	0.4995
2.6	0.5386	0.5288	0.5200	0.5120	0.5048	0.4982	0.4921
2.7	0.5308	0.5211	0.5125	0.5047	0.4976	0.4911	0.4850
2.8	0.5234	0.5139	0.5054	0.4977	0.4907	0.4843	0.4783
2.9	0.5164	0.5070	0.4986	0.4911	0.4842	0.4778	0.4720
3.0	0.5097	0.5005	0.4922	0.4847	0.4779	0.4717	0.4659

TABLE 1(b)

z_1	Y_*			
	k = 0.7	k = 0.8	k = 0.9	k = 1.0
0.1	2.3325	2.2927	2.2557	2.2212
0.2	1.5531	1.5283	1.5052	1.4837
0.3	1.2436	1.2250	1.2078	1.1917
0.4	1.0727	1.0576	1.0436	1.0305
0.5	0.9624	0.9495	0.9375	0.9264
0.6	0.8843	0.8729	0.8623	0.8524
0.7	0.8253	0.8149	0.8054	0.7964
0.8	0.7787	0.7692	0.7604	0.7521
0.9	0.7406	0.7318	0.7236	0.7159
1.0	0.7088	0.7005	0.6927	0.6855
1.1	0.6816	0.6737	0.6663	0.6594
1.2	0.6579	0.6504	0.6433	0.6368
1.3	0.6371	0.6298	0.6231	0.6168
1.4	0.6185	0.6116	0.6051	0.5990
1.5	0.6019	0.5951	0.5888	0.5830
1.6	0.5868	0.5803	0.5742	0.5684
1.7	0.5730	0.5667	0.5607	0.5552
1.8	0.5604	0.5542	0.5484	0.5430
1.9	0.5488	0.5427	0.5371	0.5318
2.0	0.5380	0.5321	0.5266	0.5214
2.1	0.5279	0.5222	0.5168	0.5117
2.2	0.5186	0.5129	0.5076	0.5026
2.3	0.5098	0.5042	0.4990	0.4942
2.4	0.5015	0.4961	0.4910	0.4862
2.5	0.4938	0.4884	0.4834	0.4787
2.6	0.4864	0.4811	0.4762	0.4716
2.7	0.4795	0.4743	0.4694	0.4648
2.8	0.4729	0.4677	0.4629	0.4585
2.9	0.4666	0.4615	0.4568	0.4524
3.0	0.4606	0.4556	0.4510	0.4466

3.1 DESIGN PROCEDURE FOR THE NON-OPTIMAL SECTION

The design procedure of an open channel is often an iterative one. For a given specific project, the design procedure can be summarized in the following steps:

1. Choose the Manning's roughness coefficient n, allowable maximum side slope $1/z_1$ for the particular type of lining, and freeboard required.

2. For the given Q and S_o, and chosen n, compute the length scale $L = (nQ/\sqrt{S})^{3/8}$.

3. Using the chosen z_1 and for a chosen value of k_o, find the nondimensional depth Y_* from table 1.

4. Find the flow depth of the channel section by finding the product of L and Y_*.

5. Compute the resulting freeboard $f_o = k_o Y_*$.

6. If computed $f_0 = f$, go to (5), else, set $k_1 = f/Y_*$, start at (3) and repeat.

7. Compute flow area using (6); z using (4); wetted perimeter using (8) and (9); total cross sectional area using (2); and top width using (7).

8. End

Mironenko et. al [13] gives values of z_1 for different materials; Laycock ([11]) gives values of freeboard for different normal flow discharges; and Cuenca (1989) gives values of Manning's roughness coefficient n for different types of lining materials.

4 OPTIMAL DESIGN METHOD

In line with Loganathan ([12]), minimize total area A_t

$$A_t = \frac{8}{3}(y+f)^2 z_1 \tag{14}$$

subject to

$$y = 0.5417LJ^{1/4}z^{-5/8} \tag{15}$$

in which

$$L = \left(\frac{nQ}{\sqrt{S}}\right)^{3/8} \tag{16}$$

Substituting (15) into (14) gives

$$A_{t_*} = 0.7825J^{1/2}z^{-1/4}(1+k)^{3/2} \tag{17}$$

in which

$$A_{t_*} = \frac{A_t}{L^2} \tag{18}$$

and

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$$k = \frac{f}{y} \tag{19}$$

Flow depth in (15) can be expressed in nondimensional form as

$$Y_* = 0.5417 J^{1/4} z^{-5/8} \tag{20}$$

in which

$$Y_* = \frac{y}{L} \tag{21}$$

Eq. (17) reaches its unconstrained minimum at $z = 0.514$ for any given k. Thus, the unconstrained nondimensional values for flow depth, flow area, and wetted perimeter of the optimal parabolic channel section are:

$$Y_* = 1.0806 \quad A_* = 1.6003 \quad W_* = 3.2400 \tag{22}$$

It can be shown that T_* is given by the equation:

$$T_* = \frac{T}{L} = 4zY_*\sqrt{1+k} \tag{23}$$

4.1 DESIGN PROCEDURE FOR THE OPTIMAL SECTION

The design procedure for an optimal parabolic channel section can be summarized as follows:

1. For the required discharge Q and given longitudinal slope S, choose the Manning's roughness coefficient n, and freeboard required.
2. Compute the length scale $L = (nQ/\sqrt{S})^{3/8}$.
3. Compute flow depth $y = 1.08388 * L$.
4. Compute $k = f / y$.
5. Compute z_1 using (4) and $z = 0.514$.
6. Compute flow area using (6), total area using (2), wetted perimeter using (8) and (9), top width using (7), and total depth $h = f + y$.
7. End

5 APPLICATION

The example used by Mironenko et. al. ([13]) is used here. The problem is re-phrased as:

Design a parabolic channel section to carry 1.0 m³/s of water. The channel will be built in a firm clay soil with a longitudinal slope of 0.1% and will have $n = 0.035$.

5.1 Solution using the non-optimal method

Iteration 1:

(1) From Mironenko ([13]), $z_1 = 1.5$ for firm clay soil. From Laycock ([11]), $f = 0.5$ m; (2) $L = 1.0388$; (3) assuming $k_o = 0.3$, $Y_* = 0.6348$; (4) $y = L * Y_* = 1.0388 * 0.6348 = 0.66$ m; (5) $f_o = k_o * y = 0.3 * 0.66 = 0.20$ m; (6) $f_o \neq f$

Iteration 2:

(3) $k_1 = f / y = 0.5 / 0.66 = 0.8$, $Y_* = 0.5951$; (4) $y = L * Y_* = 1.0388 * 0.5951 = 0.62$ m; (5) $f_1 = k_1 * y = 0.8 * 0.62 = 0.47$ m; (6) Because $f_1 \approx f$, the iteration is terminated at this point. (7) $A = 2.06$ m² ; z = 2.0; $W = 5.16$ m; $A_t = 5.02$ m²; $T = 6.70$ m.

5.2 Solution using the optimal method

(1) $Q = 1.0$ m³/s, $S = 0.1\%$, $n = 0.035$, $f = 0.5$ m; (2) $L = 1.0388$; (3) y = 1.0806*1.0388 = 1.12 m (4) $k = 0.5 /1.12 = 0.446$ (5) $z_1 = 0.427$ (5) $A = 1.6003 * 1.0388^2 = 1.73$ m², $A_t = 2.78$ m², $W = 3.2400 * 1.0388 = 3.37$ m, $T = 2.77$ m.

It is clear that the results of example (2) are more optimal than the results of example (1). Flow area, wetted perimeter, top width and total cross sectional area obtained by the first method are all larger than those obtained by the second method.

6 SUMMARY AND CONCLUSIONS

Several methods have been developed for the design of optimal parabolic channels. However, rarely has maximum and, in some cases, minimum allowable side slopes been taken into account even though in practice there is often need to take this design parameters into consideration. Sometimes the side slope is chosen based on the angle of repose of material for better stability or for vehicles to cross the channel during no-flow periods ([5]). Also, improving strength and ease of handling of small precast segments is done by deliberately making the side slopes steeper. Furthermore, safety is enhanced to both animals and people by making side slopes of large channels as flat as possible. Thus the value of side slope maybe be predetermined and has to be taken into consideration in the design process. This paper has proposed a method by which this problem can be addressed. However, the proposed method results in a non-optimal section as shown by comparing the results of the two methods. This is because the side slope parameter z is the governing parameter for a parabola and when this parameter is prevented from assuming its optimal value of 0.514 at the normal flow depth, the section obtained is less optimal.

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