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The effective porosity and grain size relations in permeability functions

K. Urumović[1] **and K. Urumović Sr.**[*,**]

[1]Croatian Geological Survey, Sachsova 2, P.O. Box 268, 10001 Zagreb, Croatia
[*]formally at: University of Zagreb, Faculty of Mining, Geology and Petroleum Engineering, Zagreb, Croatia
[**]retired

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Correspondence to: K. Urumović (kosta.urumovic@hgi-cgs.hr)

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Abstract

Hydrogeological parameters of coherent and incoherent deposits are deeply dependent of their granulometric characteristics. These relations were shaped in formulas and defaultly used for calculation of hydraulic conductivity, and are valid only for uniform incoherent materials, mostly sands. In this paper, the results of analyses of permeability and specific surface area as a function of granulometric composition of various sediments – from siltey clays to very well graded gravels are presented. The effective porosity and the referential grain size are presented as fundamental granulometric parameters which express an effect of forces operating fluid movement through the saturated porous media. Suggested procedures for calculating referential grain size and determining effective (flow) porosity result with parameters that reliably determine specific surface area and permeability. These procedures ensure successful appliance of Kozeny–Carman model up to the limits of validity of Darcy's law. The value of an effective porosity in function of referential mean grain size has been calibrated within range from 1.5 µm to 6.0 mm. Reliability of these parameters application in KC model was confirmed by very high correlation between predicted and tested hydraulic conductivity – $R^2 = 0.99$ for sandy and gravelly materials and $R^2 = 0.70$ for clayey-siltey materials. Group representation of hydraulic conductivity (ranged from $10^{-12}\,\mathrm{m\,s^{-1}}$ up to $10^{-2}\,\mathrm{m\,s^{-1}}$) presents coefficient of correlation $R^2 = 0.97$, for total sum of 175 samples of various deposits. These results present the new road to researches of porous material's effective porosity, permeability and specific surface area distribution, since these three parameters are critical conditions for successful groundwater flow modelling and contaminant transport. From the practical point of view, it is very important to be able to identify these parameters swiftly, cheaply and very accurately.

1 Introduction

An effect of granular porous media granulometric composition on its transmissivity, accumulation and suction parameters is both permanent scientific challenge and a practical issue. In hydrogeology, attention is particularly devoted to a hydraulic conductivity. Several experimental methods (Hazen, Slichter, Beyer, Terzaghi methods) are used to calculate hydraulic conductivity as a function of certain coefficient and effective grain size. In such calculations, many authors (Kovács, 1981; Vukovic and Soro, 1992; Cheng and Chen, 2007; Odong, 2008; Koch et al., 2011) have regulary used D_{10} – soil particle diameter (mm) that 10 % of all soil particles are finer (smaller) by weight (Hazen, 1892). Such application of an effective grain size is very widely used lately, although some of the authors of used models have used mean grain size as defined by Slichter (1902). Proper use of the mentioned experimental methods is limited for calculating hydraulic conductivity of uniform sand. In case of highly uniform sand, Hazen's effective grain size D_{10} is not much different from mean grain size. Porosity of all uniform sandy deposits is rather level, and can be incorporated in unique coefficient used in most of experimental methods. Diversity of immanent grain size distorts such relations. Since natural materials are mostly non-uniform, Slichter (1902) and Terzaghi (1925) have incorporated porosity function in a hydraulic model. Even so, that practice did not enhance the validity of experimental models for a wide range of grain size and uniformity. More realistic and complete correlation between various granulometric composition can be facilitated through theoretical analysis of water flux in porous media, as conducted by Kozeny (1927) and Carman (1937, 1939) on a laboratory scale. Kozeny and Carman have included both porosity and specific surface area in the flow model, using them as theoretically exact parameters of geometric properties effect on water flow in porous media. Even so, in hydrogeological practice, the Kozeny–Carman (KC) equation is not frequently used. The reason seems to lie in the difficulty to determine the deposit specific surface area that can be either measured or estimated. Calculations of specific surface area using arithmetic mean of grain size in a mixture of spheres of different sizes provided the range of sizes is not too wide (Bear et al., 1968). Errors were caused by grains of extreme size in the whole sample that can distort arithmetic mean value (Arkin and Colton, 1956). That leads to an error in calculating specific surface area, especially in cases of a wide range of grain size composition. These problems have been explained in literature. In case of large grain deposits, distortion was accredited to non-linear loss due to higher flow velocity. Validity of the KC equation was also impaired in case deposits with a higher content of clayey particles. That fact was accredited to an electrochemical correlation between the soil particles and the water (Carrier, 2003). On the whole, a general correlation equation between hydraulic conductivity and gradation incorporating a wide range of soils is not yet available (Boadu, 2000). In means of fine grain deposits, Champuis and Aubertin (2003) recommend determination of specific surface area in laboratory and incorporating gained value in the KC equation.

The objective of this article is to research relations between average mean grain size and effective porosity in function of permeability and specific surface area for a wide range of grain size and uniformity of particles in various soil samples. In hydraulic conductivity calculations Kozeny–Carman equation was used, with means to discover the algorithm for calculating the referential mean grain size that, along with effective porosity, generates harmonious parametric concept of porous media geometrics impact on its transmission capacity.

2 Study area and analyzed deposits

For the purpose of this work, data on researches of sandy and gravely aquifers and clayey-siltey deposits was collected. All of the study sites are located in plain areas of Republic of Croatia (Fig. 1). Northern parts of the Republic of Croatia are covered by thick quaternary deposits with sandy and gravely aquifers (Brkić et al., 2010). Covering aquitards are composed of siltey-clayey deposits.

Analyses of non-coherent deposits were conducted on 36 gravel test samples from six investigation boreholes on the Đurđevac well field (marked as GW on Fig. 1); 19 uniform sand test samples from the investigation boreholes on 2 well fields – Beli Manastir (marked as SU1) and Donji Miholjac (marked as SU2); 28 sand with laminas made from siltey material test samples from 2 investigation boreholes on 2 well fields – Ravnik (marked as FS/SU1) and Osijek (marked as FS/SU2). Appropriate pumping tests were conducted on this test fields to determine the average hydraulic value of aquifers.

Coherent deposits were investigated on 3 sites. Soil samples from exploration boreholes (depth 1.0–30.0 m) were laboratory tested. Analysis of granulometric composition (grain size distribution), hydraulic conductivity and Atterberg limits were used. On the first test field (route of Danube–Sava channel – marked as CI/MI1) all of the mentioned analysis were conducted for every single soil sample. 65 samples of various types of soil were analyzed. On second and third test sites – Ilok (marked as CI/MI2) and Našice (marked as CI/MI3) loess and aquatic loess like sediments were investigated. Laboratory analyses were conducted on 21 samples from 8 investigation boreholes. Particular analyses were conducted on samples from this test site at various depths, and that fact was the reason to correlate mean values for individual boreholes (Urumović, 2013).

3 Methodology

3.1 Hydraulic model

The effect that porosity n and specific surface area a have on fluid movement in porous media can be illustrated by analyzing force field in the representative elementary volume (REV) $\delta V = \delta A \delta s$ (Fig. 2) in the direction of elementary length δs that is perpendicular to elementary plane δA.

Motion of fluid in pores is caused by forces of pressure and gravity. A force of pressure is transferred on δs between entry plane δA and its parallel exit plane, and the total amount is proportional to gradient $\delta p/\delta s$. A component of gravity force ρg in fluid volume $n\delta A\delta s$ is proportional to sine of the angle that δs makes with its projection on the horizontal plane and equals $\rho g n\delta A\delta s\partial z/\partial s$. These two driving forces are, in fluid motion, confronted by the force of viscosity τ. The force of viscosity is proportional to viscosity coefficient of water μ, average velocity q_s of water flow in direction δs and the effect of geometry of void space given by drag resistance constant r_s in direction of δs and proportional to specific surface area. When the water is flowing, these forces are in balance and whence (Hantush, 1964; Urumović, 2003):

$$-n\delta V\frac{\partial p}{\partial s}-n\delta V\rho g\frac{\partial z}{\partial s}-\delta V\mu r_\mathrm{s}q_\mathrm{s}=0 \tag{1}$$

or:

$$q_\mathrm{s}=-\frac{n\rho g}{r_\mathrm{s}\mu}\frac{\partial(p/\rho g+z)}{\partial s}=-\frac{n\rho g}{r_\mathrm{s}\mu}\frac{\partial h}{\partial s}=-k_\mathrm{s}\frac{\partial h}{\partial s}=-k_\mathrm{s}\frac{\rho g}{\mu}\frac{\partial h}{\partial s} \tag{2}$$

These relations express Darcy's law, as theoretically rigorously described by Hubbert (1956). The attention is here given to permeability, as a property of porous media that is (in Eq. 2) given by relation $k_\mathrm{s}=n/r_\mathrm{s}$, k_s [L^2]. Porosity n is measured as a volume of moving fluid and is connected with specific effect of driving forces of pressure and gravity. Constant r_s expresses an effect of void geometry on the amount of viscosity forces, and represents specific amount of void geometry effect on water retention. Such specific amount is equivalent to a specific surface area a_p, [L^{-1}] inside the porous media, i.e. to a relation of solid grain surface that confronts water flow and saturated void volume that transfers the flow driving force. Following the Hagen Poiseulle law, that is inversely proportional to the hydraulic radius R_H [L]. Since, in isotropic environment, $r_\mathrm{s}\propto a_\mathrm{p}^2$ permeability is:

$$k=\frac{n}{r_\mathrm{s}}=C\frac{n}{a_\mathrm{p}^2}=C\,nR_\mathrm{H}^2, \tag{3}$$

where C represents the non-dimensional coefficient of proportionality that is dependent of the particle shape. $R_\mathrm{H} = 1/a_\mathrm{p}$ represents the hypothetical hydraulic radius of porous media representing the impact of effective voids specific surface area.

3.2 Geometric parameters of permeability

There are four ways of expressing specific surface area based on solid volume expressing surface area, A_s [L^2] as:

a_p [L^{-1}] – specific surface area based on the volume of contented pores V_p;
a_T [L^{-1}] – specific surface area based on the total volume (solids + pores) V_T;
a_m [$\mathrm{L}^2\,\mathrm{M}^{-1}$] – specific surface based on the mass of solids M_s;
a_s [L^{-1}] – specific surface area based on the volume of solids V_s of density ρ_s

All of the mentioned forms of specific surface are related to the hydraulic radius of porous media R_H. Their mutual conversion is expressed by following relations:

$$a_\mathrm{p} = \frac{A_\mathrm{s}}{V_\mathrm{p}} = \frac{a_\mathrm{T}}{n} = \frac{\rho_\mathrm{s}(1-n)}{n} a_\mathrm{m} = \frac{(1-n)}{n} a_\mathrm{s} = \frac{1}{R_\mathrm{H}} \qquad (4)$$

Kozeny (1927) used Eq. (4) with a_T. He developed a theory for a bundle of capillary tubes of equal length. Carman (1937) verified the Kozeny equation end expressed the specific surface per unit mass of solid a_m, so it does not vary with the porosity. Furthermore, Carman (1939) tried to take tortuosity of the porous media into account by introducing an angular deviation of 45° from mean straight trajectory. The best fit with experimental results he obtained with a factor $C = 0.2$ in Eq. (3).

In hydrogeology, specific surface area is often substituted with mean grain diameter D_m. Permeability is given by the relation:

$$k = \frac{n^3}{180(1-n)^2} D_\mathrm{m}^2 \qquad (5)$$

This relation has been achieved by inserting solids specific surface ($a_s = 6/D_m$) from Eq. (4) into Eq. (3) with $C = 0.2$. This solution of the Kozeny–Carman equation (Bear, 1972) is given for uniform sphere particles and for the Carman coefficient $C = 0.2$. That makes effective porosity n (in form of porosity function) and certain effective grain size D_m the critical factors of porous media transmissivity. By grouping them functionally:

$$k = C\frac{n_e}{a_p^2} = n_e\left(\frac{n_e}{(1-n_e)}\frac{D_m}{180}\right)^2 \tag{6}$$

It is obvious that effective porosity n_e, has direct impact on the amount of driving forces and indirectly participates in the conversion of specific surface value into a value of effective mean grain which is the carrier of drag resistance. Both forces affect the moving fluid and that makes effective porosity an active factor only to pores through which the water flows.

3.3 Referential grain size

Many authors present the Kozeny–Carman equation with D_m^2 instead of a_s^2 in Eq. (5) without the whole indication of how to calculate this equivalent mean diameter. In engineering practice, there are three ways to calculate mean of the rated size of adjacent sieves:

$$\text{arithmetic}, d_{i,a} = (d_{i<} + d_{i>})/2, \tag{7}$$

$$\text{geometric}, d_{i,g} = \sqrt{d_{i<} \times d_{i>}}, \tag{8}$$

$$\text{harmonic}, d_{i,h} = 2/[(1/d_{i<}) + (1/d_{i>})]. \tag{9}$$

where $d_{i<}$ [L] is the smallest, and $d_{i>}$ [L] is the largest grain in segment. In all cases, it can be shown that $d_{i,h} < d_{i,g} < d_{i,a}$. However, the difference is not significant. Todd (1959) recommends the use of geometric mean, Bear (1972) prefers harmonic mean and recent authors often follow their recommendations.

The algorithm of integration of all the mentioned grain sizes (Eqs. 7–9) in sieve residue in the whole sample has a crucial effect on the value of mean grain size. An overview of related expert and scientific literature has regularly confirmed the arithmetic sum of mean grain sizes in sieve residue d_i impact on a mean grain (Bear, 1972) (Freeze and Cherry, 1979; Kasenow, 2010) as follows:

$$D_a = 100\left(\sum P_i/d_i\right)^{-1} \tag{10}$$

Here P_i is a percentile of the sieve residue mass in the total mass of the sample. The arithmetic mean was used because of the ease of computation and because of a wide variety of uses to which it can be applied. Correct results of permeability and specific surface were achieved only for uniform deposits of sand and silt (Chapuis and Aubertin, 2003). Major errors were results of applying the Eq. (10) for samples with a wide range of particle sizes. Equivalent annotations were registered in sedimentology and soil science researches. Arkin and Colton (1956) pointed out that the arithmetic mean may be greatly distorted by extreme values and therefore may not be typical. Irani and Callis (1963) advocated the use of geometric rather than arithmetic statistical properties for soil samples. The reason, in part, is that in a natural soil sample there is wide range of particle sizes making the geometrical scale much more suitable then the arithmetic scale. The general mathematical expressions for calculating the geometric particle size diameter D_{lng} of the sample are:

$$D_{\text{lng}} = \text{EXP}\left[\frac{1}{M}\sum m_i \ln(d_{\text{i, g}})\right] \tag{11}$$

or

$$D_{\text{lng}} = \text{EXP}\left[0.01\sum P_i \ln(d_{\text{i, g}})\right] \tag{12}$$

where M [M] represents the mass of the sample, and m_i [M] represents the mass of particular sieve residues, $P_i = 100m_i/M$. It can be shown that $D_{\text{lng}} > D_{\text{aa}}$. That difference is very small when calculated for uniform deposits, but rapidly grows when calculating mean grain of poorly sorted deposits. In case of gravelly sediments, difference may reach up to 2 orders of magnitude.

3.4 Porosity factor

In a permeability model, porosity function, expressed by factors of porous media parameters (Eq. 6), applies only to flow pores (Eq. 2). Following that fact, it was named effective porosity. Effective porosity is not the same as specific yield which is, as draining porosity, determined in a laboratory. Numerical difference between effective porosity and specific yield may not be discernible when analyzing uniform sand, but can significantly rise when analyzing samples containing greater percentage of small size (clay, silt) particles. Presentations of a specific yield in function of granulometric aggregates (Eckis, 1934) or median grain size (Davis and De Wiest, 1966) are not appropriate to use in permeability equations (Eq. 6) for two reasons. First, porosity used in the presentation is not effective porosity n_e, and second, median size is not referential mean grain size, paired with which the effective porosity controls the value of permeability (Eq. 6). The above mentioned "parameter collaboration" requires identification of effective porosity in function of referential grain size as it is presented in this paper, based on the analysis of numerous data on various deposits, from clay to gravel. Starting values of porosity used in this procedure were ranges of an average specific yield values (Fig. 4), according to the data from the US Geol. Survey Water Supply Paper (Morris and Johnson, 1967).

Starting values of porosity used in this procedure were ranges of an average specific yield values (Fig. 4), according to the data from the US Geol. Survey Water Supply Paper (Morris and Johnson, 1967). Reputation of the laboratory and a large number of analyses (33 samples of gravel, 287 of sand and 266 of silt and clay) provided a high quality base for identification of mean value of specific yield range. The value of effective porosity is slightly lower than the value of specific yield. This value is related to the referential mean grain size (D_{lng}), forming the function of drag resistance effect in the water flow through a porous media (Eq. 6, Fig. 3).

The reliable reconstruction of effective porosity range (Fig. 5) was ensured through strong impact of discussed form of porosity function (Fig. 3) and exact calculation of referential mean grain size (Eqs. 11 and 12). These relations simultaneously verified the applicability of Kozeny–Carman equation for wide range of granulometric composition.

Identification of effective porosity rate has been achieved due to reliable guidelines-test fields for non-coherent deposits were properly studied and investigated, and laboratory analysis of hydraulic conductivity was conducted on numerous soil samples.

4 Results and verification

Reliable verification of analyzed parameter relations for a wide range of granulometric composition was conducted by using the Kozeny–Carman equation and analyses of researched deposits hydraulic conductivity in situ as well as in the laboratory. Hydraulic conductivity K [$\mathrm{LT^{-1}}$] given through the KC equation (according to Eq. 6) is:

$$K = \frac{\rho g}{\mu} \frac{n_{\mathrm{e}}^3}{180(1-n_{\mathrm{e}})^2} D_{\mathrm{m}}^2 = 0.0625 \frac{n_{\mathrm{e}}^3}{(1-n_{\mathrm{e}})^2} D_{\mathrm{m}}^2 (\mathrm{m\,s^{-1}}), \tag{13}$$

where ρ [$\mathrm{M\,L^{-3}}$] represents the density and µ [$\mathrm{M\,L^{-1}\,T^{-1}}$] represents the viscosity of water, with g [$\mathrm{MLT^{-2}}$] being gravity. Coefficient 0.0625 is correct for a diameter of the mean grain D_{m} expressed in mm and a water temperature of 10 °C. Hazen's (1892) non-dimensional temperature correction factor $\tau = 0.70 + 0.03T$ (T – temperature in °C) was used to present an effect of temperature difference, ensuring error less than 2 % for $T < 30\,°\mathrm{C}$.

The Kozeny–Carman equation is, actually, a special form of Darcy's law, so it should be applicable for every possible natural sample of porous media. Hydraulic testing of natural deposits represents a specific question in correlation investigations. Non coherent deposits make it almost impossible to ensure laboratory testing of content and distribution of particles as well as the consolidation of material in its natural, undisturbed state. Identification of average hydraulic conductivity calculated by analyzing the pumping tests data was used for correlation in non-coherent deposits. Test sites were chosen to fulfill the following criteria: borehole core must be of 100 % natural lithological compound, and analysis of particle size distribution must be conducted on the core samples. If exploration borehole was located in the vicinity of the tested well, hydraulic conductivity of local scale was used. If there were more boreholes on a greater distance from the pumped well, hydraulic conductivity of a regional scale was determined and used for correlation. Congruously to the test data scale, values of the predicted K, obtained from the grain size distribution analysis, were averaged. Siltey and clayey samples were processed in a specific way. If one specific sample was analyzed in the laboratory (grain size analysis and hydraulic conductivity), the results were, both literally and functionally, on a laboratory scale.

Criteria for evaluating the acceptable accuracy of predicted hydraulic conductivity, expressed by its correlation with a tested K value, should not be equal for different types of materials. Chapuis and Aubertin (2003) of the *École Polytechnique de Montréal*, have disclosed a very interesting study. They have concluded that acceptable accuracy of a predicted value of K for clayey materials is a K value that is between $1/3$ and 3 times the measured K value, which is within the expected margin of variation for the laboratory permeability test. That relation referred to a calculation of K by the Kozeny–Carman equation using a specific surface area determined in the laboratory. Such criteria can definitely be an acceptable accuracy limit for calculating the K using referential grain size. In the case of siltey, non-plastic soils, three specimens of the same sample may give K values ranging between $1/2$ and 2 times the mean value and an excellent precision (K value within ±20 %) can be reached with sand and gravel when the special procedure is applied (Chapuis and Aubertin, 2003). These criteria were accepted for hydraulic conductivity calculation using the KC equation by applying effective porosity and referential mean grain size. The accepted criteria require a high level of accuracy of determining referential mean grain size and effective porosity concerning their role in Eq. (13).

In the process of verification, the results acquired using the KC equation were matched with the results of the hydraulic tests. The average local K values of sandy aquifers were identified (pumping test data) and compared to the average sample K value. Verification of K values for the gravelly aquifer is of a regional scale, since the boreholes that provided high quality core were located at a distance of 150–500 m from the pumped well. The tested value of hydraulic conductivity was determined by analyzing a series of successive steady states. The third case was of a laboratory scale where K values of coherent materials were analyzed. The hydraulic conductivity values of siltey-clayey samples as well as granulometric parameters were a result of laboratory testing. These were the procedures to which the criteria for correlating predicted and tested K values were customized.

4.1 Incohesive deposit

Validity limits of the Eq. (13) are rarely discussed in hydrogeological circles. Arithmetic sum of proportions of arithmetic, geometric or harmonic mean size of grain between each pair of sieve sizes (Eqs. 7–10) is commonly used to calculate the mean grain size of a sample. In papers and reports on applying the KC formula, non-plastic silt commonly represents the lower validity limit. The upper validity limit is 3 mm grain (Carrier, 2003; Odong, 2008). Common view is that best results are achieved for analyzing uniform sands.

4.1.1 Sandy aquifer

Results of the analysis for four specific sandy aquifers are presented in this subheading. Two of those aquifers consist of uniform sand of different depths, and two consist of fine sand with siltey laminas of different depths.

Predicted hydraulic conductivity was calculated using mean grain size determined by four different methods: arithmetic sum of arithmetic, geometric and harmonic mean between each pair of sieve sizes (Eqs. 7–10) and total geometric mean (Eqs. 8 and 11). In the case of uniform, mid-grain sands, a very high accuracy was achieved (Table 1, Fig. 6).

It is interesting to point out that the use of D_{aa}, D_{ag} and D_{ah} results in a mild underestimating of hydraulic conductivity value, and D_{lng} with mild overestimating of hydraulic conductivity value. That can be interpreted by a slight washout of the core sample, since it is difficult to avoid a washout of the core while drilling through the layers of uniform sand. An especially interesting fact is that the use of grain size D_{40} (Table 1, Fig. 6) provided remarkable results with practically negligible error.

Analyses of samples from fine sandy aquifers with siltey laminas (Figs. 7 and 8) resulted with regularly underestimated K values. Laminas of silt were so thin that it was not possible to isolate content of sand in the samples (Fig. 8).

In such specific cases, grain size D_{40} or even D_{50} present hydraulic properties of sand much better than the calculated mean grain size of the whole sample. Thin laminas of silt, through which horizontal flow is negligible, have a strong impact on the grain size distribution curve. These distortions are considerably weaker if referential geometric mean grain size, D_{lng} is used in the calculations.

4.1.2 Gravelly aquifer

Predicted K values of the gravelly aquifer were analyzed at the same level as those of the sandy aquifer. Due to clarity, only K values based on D_{lng}, D_{aa}, D_{ah} and D_{40} (Table 2, Fig. 9) are presented.

A high quality drilling core from six exploration boreholes as well as a particle size distribution analysis data of relevant core samples was at disposal. All of the boreholes were scattered around the pumped well at test field GW. Borehole SPB-2 is situated on the border part of well field where a part of an aquifer is of sandy development and that is the reason why that data is not incorporated in average K value in correlation.

Predicted K values of particular samples and two boreholes (SPB-3, SPB-5) mean value are presented graphically in Fig. 9. Mean predicted $K(D_{\text{lng}})$ of borehole SPB-3 is only 10 % smaller than the tested value. This borehole's core quality is presented by a core segment from 23.0 to 30.0 m depth (Fig. 10).

The highest deviation of the predicted $K(D_{\text{lng}})$ value is recorded in the borehole SPB-5 core – average $K(D_{\text{lng}})$ is 71 % higher value than K_{t}. However, the most important fact is that the geometric mean $K(D_{\text{lng}})$ of all boreholes (Table 2) in the tested area is only 5 % higher than K_{t}. Both values are of the same regional significance. Namely, $K(D_{\text{lng}})$ presents: (1) the result of total geometric mean size of all the grains in the sample, (2) hydraulic conductivity of all the samples in the borehole and (3) all the boreholes on the test field. The tested hydraulic conductivity K_{t} is identified by analyzing series of successive cones of depression achieved in that area during the pumping test. As opposed to that, $K(D_{\text{aa}})$ and $K(D_{\text{ah}})$ show lower values by 2–3 orders of magnitude, manifestly showing the degeneration of arithmetic algorithm for calculating mean grain size for a wide range of particle sizes. From a practical point of view, an interesting fact is that very good results are achieved using grain size D_{40}.

4.1.3 The correlation of predicted and tested K values of incoherent deposit

The graphical correlation between hydraulically tested and predicted hydraulic conductivity calculated by using specific methods of mean grain size calculation, illustrates accordance of results of these methods only in the case of uniform mid-grain sandy deposits. In this example, relatively homogenous mid-grain sands were analyzed, and relations would probably be of similar accuracy in a wide grain size range of uniform, homogenous, incoherent deposits. An occurrence of siltey laminas in small-grain sandy samples excessively reduces arithmetic mean of the grain size. Fine particles have the most intensive effect in gravel, where extremely wide range of particle sizes occurs. This wide range of particle sizes in gravel deposits is a product of a natural state, and in that case, only mean grain size D_{lng} represents the real effective grain size, so the predicted $K(D_{\text{lng}})$ only slightly defers from K_{t}. Accordingly, in all types

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of incoherent deposits, D_{lng} represents the correct size of a referential mean grain. Depletion occurs only in case of sandy samples intercalated with thin laminas of silt. Yet, the analyses of those samples show that $K(D_{\mathrm{lng}})$ is of the same order of magnitude as K_{t}. The numerical correlation between the predicted $(K(D_{\mathrm{lng}}))$ and the tested (K_{t}) hydraulic conductivity for all analyzed incoherent deposits show a very high correlation coefficient $R^2 = 0.998$. Also, it is interesting to register a very high accuracy of $K(D_{40})$, achieving an extremely high correlation coefficient $R^2 = 1.000$ (Table 4).

It can be assumed that effective grain D_{40} very correctly represents the true referential grain size of incoherent deposits, even in case of sand intercalated by laminas of silt.

4.2 Cohesive deposit

Use of the KC equation for calculating hydraulic conductivity of cohesive materials using particle size has frequently been disputed in numerous papers and reports. The reasons being: varied particle size, high proportions of fine fractions in deposits (Young and Mulligan, 2004), electrochemical reaction between the soil particles and water, large content of particles such as mica (Carrier, 2003) etc. All of these factors also affect effective porosity, and some of them affect the mean grain size. The question is: does (and/or how much) effective porosity and referential mean grain with its size and distribution incorporate effect of the mentioned factors?

The first problem is determining mean grain from the granulometric data, especially since the size of the smallest grain is unknown. The grain size curve always has a minimal measurable particle size d_{min}, and in clay sample, there can be a relatively large content of particles smaller than the measurable one. As an equivalent size $D_{\mathrm{i,\,min}}$ (size corresponding to mean size of particles smaller than the minimum size), with respect to specific surface and permeability, Chapuis and Légaré (1992) used relation:

$$D_{\mathrm{i,min}} = \sqrt{d_{\mathrm{min}}^2/3}. \tag{14}$$

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Total sum of 86 samples of clayey-siltey deposits from three exploration fields were analyzed. All the samples were permeability tested in the laboratory only once, so there was no control of the test accuracy. Yet, due to a large number of tests, results were acceptable for correlating with the predicted K values. However, a few problems remained concerning selecting appropriate samples for correlation. In the mean values correlation of samples from the individual boreholes in loess and aquatic loess-like deposits (test fields CI/MI2 and CI/MI3), boreholes with less than two tested samples were excluded. In correlation of the predicted and tested K values of the same single sample, samples with anomalies connected to the short period of testing, laminated samples etc. were also excluded. The correlation was conducted for 61 samples of various clayey deposits sampled from the Danube–Sava channel boreholes (test field CI/MI1).

Validity of the aquitard's predicted K values was analyzed by using geometrical and three forms of arithmetical mean value. All of the arithmetical mean values provided hydraulic conductivity values one order of magnitude lower than the tested one. On average, arithmetical mean values are outside acceptable limits of accuracy (Table 3).

Good results were achieved by using referential geometrical mean grain size – the predicted values of hydraulic conductivity were very close to the tested value, e.g. within the set limits of the accuracy criteria. Graphical correlation (Fig. 13) illustrates concentrating $K(D_{\mathrm{lng}})$ values in the vicinity of the tested value K_{t}, and most of the results are within range $1/3K_{\mathrm{t}} < K(D_{\mathrm{lng}}) < 3K_{\mathrm{t}}$. The numerical correlation confirms their high correlativity, $R^2 = 0.696$. That is a very high value, especially concerning the fact that some of deviations may have been a result of an error in conducting of the laboratory permeability test. Achieved results confirm earlier conclusions that the total geometric mean grain D_{lng} truly represents an effective mean grain of the siltey-clayey deposits. Also, it was used as reliable point of reference for verification of the porosity curve $n_{\mathrm{e}} = f(D_{\mathrm{lng}})$, presented in Fig. 5.

5 Verification of the KC model using effective porosity and referential mean grain-size

Universality of the hydraulic model is realized only when it presents a continuum of flow conditions from large to imperceptible. That is conditioned by its theoretical validity and credibility of the used parameters. The theoretical validity of the Kozeny–Carman model was tested multiple times on many occasions, but its use was primarily limited by unavailability of the specific surface area and porosity, and was also further complicated by the conversion between specific surface area and diameter of the effective grain size. In that sense, the verification of the KC model universality is conditioned by the versatility of the referential grain size formulation and its connection to effective porosity.

The effective grain size formulation is simple only in the case of very uniform deposits of sand and coarse silt, when the arithmetic mean is successfully used. In the cases of extreme uniformity of sand, divergence between the mean grain and the grain defined by percentage of particles that pass through the sieve is very small. Because of that, many authors recommend the use of Hazen's effective grain size D_{10}. However, along with the rise of uniformity coefficient, the above mentioned formulation becomes inappropriate. This problem is universally solved by applying the total geometric mean value of grain diameter (Eq. 12). In this process, problems are related only to credibility of the sample for grain size distribution analysis. Such technological problems are especially present with samples of incoherent materials from borehole logs. That was the cause for searching effective grain size as various percentages of particles passing through the sieves. Grain size D_{40} proved to be the closest value to an referential grain size and was incorporated in the final correlations.

Pearson's correlation was conducted for numerical and logarithmic values of hydraulic conductivity of all samples, grouped in three basic data groups (Table 4): non-coherent materials (gravel and sand); coherent materials (silt and clay); group of all the analyzed samples. Verification of the results for non-coherent materials group was conducted for 8 more samples from the USGS laboratory (Morris and Johnson, 1967). Verification of the results for coherent materials was conducted by analyses of two more samples from the USGS laboratory. Correlation results of the last mentioned group are presented in Fig. 14.

Separate sub-group was formed by non-coherent material data from all five CRO test fields by using the effective grain size D_{40}. Correlation has provided very high correlation coefficients. The lowest values of correlation coefficients have been achieved for siltey-clayey materials group, but their values (Table 4) certainly confirm validity of the presented relations. It is very important to point out that test data used in this research refer to standard, serial tests, and that specific tests would probably result in even stronger correlativity.

Graphical correlation between the tested and the predicted hydraulic conductivity (Fig. 14) illustrates universality of the KC model (if applying referential mean grain size D_{lng} and an effective porosity n_{e}) in a wide range of flow conditions. Very high values of correlation coefficients R^2 (Table 4) confirm its relations in porous media conditions, on a laboratory scale.

6 Conclusions

The following conclusions can be drawn from this study:

1. Geometric mean size of all particles contained in the sample D_{lng}, unambiguously affects its permeability and specific surface area of coherent and non-coherent deposits, regardless of the grain size and distribution of specific particles. In that sense, D_{lng} represents the referential grain size of the sample.

2. The distribution of an effective porosity in function of referential grain size $n_{\mathrm{e}} = f(D_{\mathrm{lng}})$ is presented graphically for all types of clastic deposits. The graph was constructed following literature data and was calibrated according to congruence between the tested hydraulic conductivity and its predicted value calculated by applying the Kozeny–Carman equation. So, this effective porosity presents flow

porosity and is slightly lower than the specific yield which is commonly stated in standard literature.

3. Successful appliance of the KC flow model confirms its validity in a range of hydraulic conductivity between 10^{-12} and $10^{-2}\,\mathrm{m\,s^{-1}}$. Simultaneously, the value of an effective porosity and its relative referential grain size D_{lng} in a range between 1.5 µm up to 6 mm has been verified. It can be concluded that, through presented parameters, the range of applying the Kozeny–Carman model for calculating permeability and specific surface area is extended up to the limits of Darcy's law validity.
4. Value of the referent mean grain size is, in cases of analyzed non-coherent samples, very close to the value of effective grain size D_{40} (read from grain size distribution curve).

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References

Arkin, H. and Colton, R. R.: Statistical Methods, 4th Edn., Barnes and Noble Inc., New York, 1956.

Bear, J.: Dyinamics of Fluid in Porous Media, Elsevier, New York, 1972.

Bear, J., Zaslavsky, D., and Irmay, S.: Physical Principles of Water Percolation and Seepage, Publ. No. XXXIX, UNESCO, Arid Zone Research, Paris, 1968.

Boadu, F. K.: Hydraulic conductivity of soils from grain-size distribution: new models, J. Geotech. Geoenviron., 126, 739–746, 2000.

Brkić, Ž., Larva, O., and Urumović, K.: The quantitative status of groundwater in alluvial aquifers in norther Croatia, Geologia Croatica, Jornal of the Croatian Geological Survey and the Croatian Geological Society, 63, 283–298, 2010.

Carman, P. C.: Fluid flow through granular beds, Transactions of the Institute of Chemical Engineering, 15, 150–166, 1937.

Carman, P. C.: Permeability of saturated sand, soil and clay, The Journal of Agricultural Science, 29, 263–273, 1939.

Carrier III, W. D.: Goodbye, Hazen, hello, Kozeny–Carman, J. Geotech. Geoenviron., 129, 1054–1056, 2003.

Chapuis, R. P. and Aubertin, M.: Predicting the Coefficient of Permeability of Soils Using the Kozeny–Carman Equation, Département des génies civil, géologique et des mines, École Polytechnique de Montréal, Montreal, 2003.

Chapuis, R. P. and Légaré, P.-P.: A simple method for determining the surface area of fine aggregates and fillers in bituminous mixtures, in: Effects of Aggregates and Mineral Filters on Asphalt Mixture Performance, edited by: Meininger, R. C., ASTM, Philadelphia, ASTM STP 1147, 177–186, 1992.

Cheng, C. and Chen, X.: Evaluation of methods for determination of hydraulic properties on an aquifer-aquitard system hydrologically connected to river, Hydrogeol. J., 15, 669–678, 2007.

Davis, S. N. and De Wiest, R. J. M.: Hydrogeology, John Wiley & Sons, New York, 1966.

Eckis, R. P.: South Coastal Basin Investigation, Geology, and Ground Water Storage Capacity of Valley Fill, California Division of Water Resources Bulletin, Sacramento, 45 pp., 1934.

Freeze, R. A. and Cherry, J. A.: Groundwater, Prentice-Hall Inc., Engelwood Cliffs, New Jersey, 1979.

Hantush, Mahdi S.: Hydraulic of wells, Academic Press, New York, 1964.

Hazen, A.: Some Physical Properties of Sands and Gravels, With Special Reference to Their Use in Filtration, Pub. Doc. No 34, 539–556, Massachusetts State Board of Health, 1892.

Hubbert, M. K.: Darcy's law and the field equations of the flow of underground fluids, AIME Petroleum Transactions, 207, 222–239, 1956.

Irani, R. R. and Callis, C. F.: Particle Size: Measurement, Interpretation and Application, John Wiley & Sons, New York, 1963.

Kasenow, M.: Applied Ground-Water Hydrology and Well Hydraulics, Water Resources Publications, LLC, Highlands Ranch, Colorado, 1997.

Kasenow, M.: Determination of Hydraulic Conductivity from Grain Size Analysis, Water Resources Publications, LLC, Highlands Ranch, Colorado, 2010.

Koch, K., Kemna, A., Irving, J., and Holliger, K.: Impact of changes in grain size and pore space on the hydraulic conductivity and spectral induced polarization response of sand, Hydrol. Earth Syst. Sci., 15, 1785–1794, doi:10.5194/hess-15-1785-2011, 2011.

Kovacz, G: Seepage Hydraulics, Developments in Water Science, Elsevier Science Publishers, Amsterdam, 1981.

Kozeny, J.: Uber Kapillare Leitung des Wassers im Boden, Akad. Wiss. Wien, 136, 271–306, 1927 (in German).

Morris, D. A. and Johnson, A. I.: Summary of Hydrologic and Physical Properties of Rock and Soil Materials, as Analyzed by the Hydrologic Laboratory of US Geological Survey 1948–60, Water Supply Paper 1839-D, US Geological Survey, Washington, 42 pp., 1967.

Odong, J.: Evaluation of empirical formulae for determination of hodraulic conductivity based on grain-size analysis, J. Am. Sci., 3, 1–6, 2008.

Slichter, C. S.: The Motions of Underground Waters, Water Supply and Irrigation Paper, US Geological Survey, Washington, 1902.

Terzaghi, K.: Principles of soil mechanics. Engineering News-Record, 95, Nos. 19–23, 25–27, pp. 742–746, 796–800, 832–836, 874–878, 912–915, 987–990, 1026-1029, 1064–1068, 1925.

Todd, D. K.: Ground Water Hydrology, John Wiley & Sons, New York, 1959.

Urumović, K.: Parameter quantification of clastic sediments hydrogeologic properties based on test fields in northern Croatia, Ph.D. thesis, unpubl, Univrsity of Zagreb, Zagreb, RGNf, 164 pp., July 2013.

Urumović Sr., K.: Physical Principles of Groundwater Dynamics, Faculty of Mining, Geology and Oil Engineering, Zagreb, 2003 (in croatian).

Vukovic, M. and Soro, A.: Determination of Hydraulic Conductivity of Porous Media from Grain Size Composition, Water Resources Publications, Littleton, Colorado, 1992.

Young, R. N. and Mulligan, C. N.: Natural Attenuation of Contaminants in Soils, Lewis Publishers, Boca Raton, 2004.

Table 1. Average difference between predicted hydraulic conductivity calculated using Kozeny–Carman equation end tested one on well fields.

Variety of equivalent grain size		Predicted hydraulic conductivity calculated using							Tested K $\times 10^{-4}$ ($\mathrm{m\,s^{-1}}$)	Kind of sand
		Diameter form grain-size distribution curves			Mean grain size					
		D_{30}	D_{40}	D_{50}	D_{aa}	D_{ag}	D_{ah}	D_{lng}		
		Difference between predicted and tested hydraulic conductivity (%)								
Well fields	SU-1	−16.5	−0.1	14.3	−0.7	−4.2	−7.8	16.6	2.55	Medium uniform
	SU-2	−17.6	−0.5	25.8	−5.3	−9.7	−14.0	12.0	2.78	
	FS/SU-1	−34.8	−7.8	18.1	−85.3	−86.9	−88.4	−32.2	1.16	Fine to medium
	FS/SU-2	−52.4	−32.7	−12.1	−43.4	−47.8	−52.1	−18.9	1.40	

Table 2. Mean hydraulic conductivity of gravelly aquifer (test field GW).

Borehole	Calculation manner of the effective mean grain-size in samples								Tested hydraulic conductivity K_t $(\mathrm{m\,s^{-1}})$
	$K(D_{lng})$		$K(D_{aa})$		$K(D_{ah})$		$K(D_{40})$		
	Calculation manner of the predicted average hydraulic conductivity in the borehole								
	Geom.	Arithm.	Geom.	Arithm.	Geom.	Arithm.	Geom.	Arithm.	
SPB-1	2.5×10^{-3}	3.5×10^{-3}	9.0×10^{-6}	1.1×10^{-5}	7.2×10^{-6}	9.1×10^{-6}	1.1×10^{-3}	2.5×10^{-3}	1.8×10^{-3}
SPB-3	1.6×10^{-3}	2.5×10^{-3}	2.7×10^{-6}	4.1×10^{-6}	2.2×10^{-6}	3.3×10^{-6}	6.4×10^{-4}	1.6×10^{-3}	
SPB-4	1.3×10^{-3}	2.2×10^{-3}	1.8×10^{-6}	2.1×10^{-6}	1.5×10^{-6}	1.8×10^{-6}	5.2×10^{-4}	1.1×10^{-3}	
SPB-5	3.0×10^{-3}	4.1×10^{-3}	6.9×10^{-6}	1.0×10^{-5}	5.7×10^{-6}	8.3×10^{-6}	1.6×10^{-3}	4.6×10^{-3}	
SPB-6	1.2×10^{-3}	1.4×10^{-3}	2.7×10^{-6}	2.9×10^{-6}	2.2×10^{-6}	2.4×10^{-6}	7.1×10^{-4}	8.8×10^{-4}	
Aver.	1.8×10^{-3}	2.8×10^{-3}	3.6×10^{-6}	5.5×10^{-6}	3.0×10^{-6}	4.4×10^{-6}	8.8×10^{-4}	2.0×10^{-3}	
K/K_t	1.05	1.59	0.0021	0.0031	0.0017	0.0025	0.50	1.11	

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Table 3. Average relations and diffrence between the tested (K_t) and the predicted ($K(D)$) hydraulic conductivity depending on used mean grain of coherent deposits.

Average relation and difference	Used mean grain			
	Geometric mean		Arithmetic mean	
	D_{lng}	D_{ah}	D_{ag}	D_{aa}
$K(D)/K_t$	0.69	0.084	0.085	0.087
Difference %	−44	−1087	−1078	−1046

Table 4. Numerical results of correlations between the tested and the peredicted K calculated using the Kozeny–Carman equation (for all samples from test fields in Croatia and a few samples from US Geol. Survey laboratory, Morris and Johnson, 1967).

Sample locations	Materials	Effective grain size	Pearson's correlation coeffecients				
			Mark	Nominal values		Log values	
				R	R^2	R	R^2
CRO test fileds	Gravel, sand	D_{lng}	R_1	0.999	0.998	0.998	0.976
	Gravel, sand	D_{40}	R_2	1.000	1.000	0.995	0.990
Togeather CRO + USGS lab.	Gravel, sand	D_{lng}	R_3	0.997	0.994	0.993	0.985
CRO test fileds	Silt, clay	D_{lng}	R_4	0.74	0.547	0.834	0.696
	Gravel, sand, silt, clay	D_{lng}	R_5	0.999	0.999	0.971	0.942
All togeather CRO + USGS lab.	Gravel, sand, silt, clay	D_{lng}	R_6	0.997	0.995	0.985	0.971

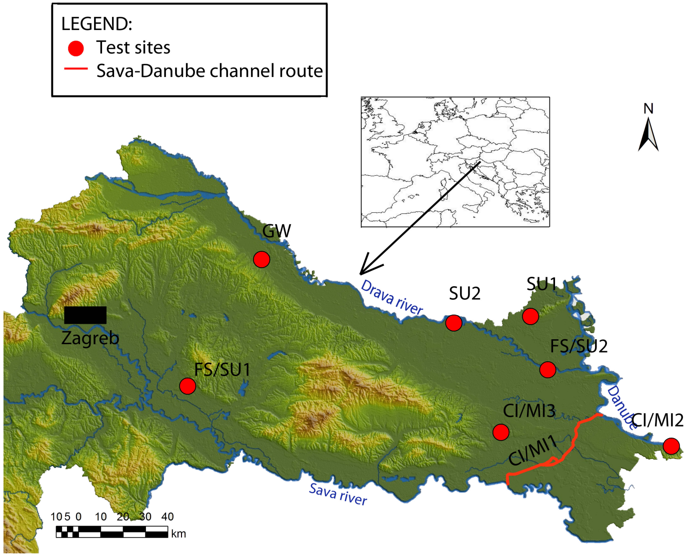

Figure 1. Situation map of Northern Croatia with test sites locations.

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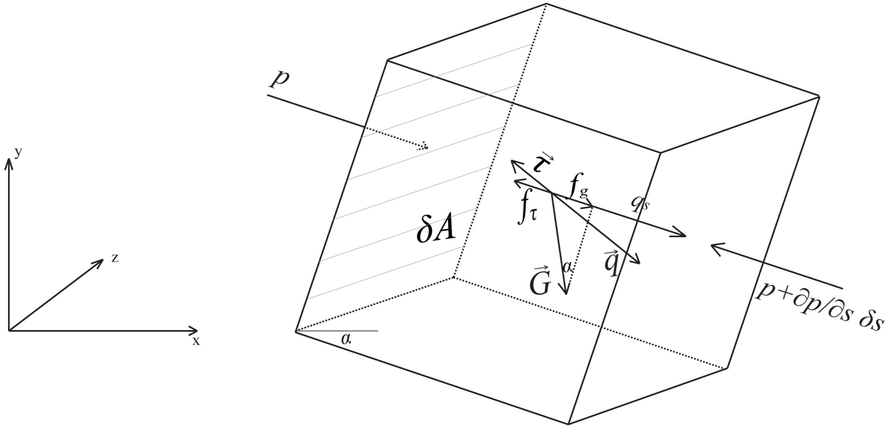

Figure 2. Definition sketch of liquid driving and opposed viscous forces for elemental volume.

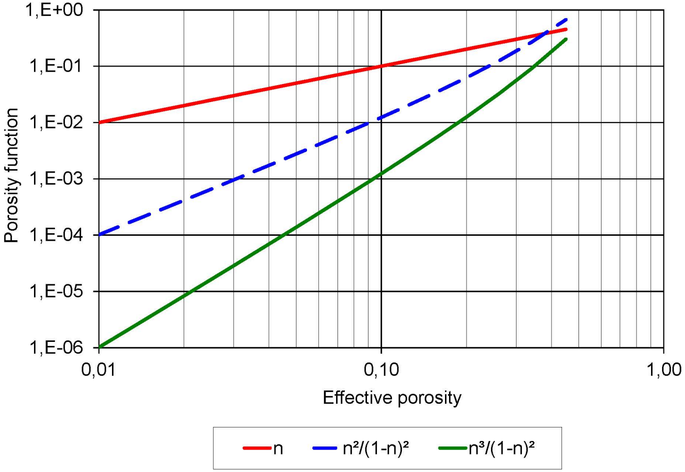

Figure 3. Efect of driving (n) and drag resistance ($n^2/(1-n)^2$) factor on porosity function ($n^3/(1-n)^2$).

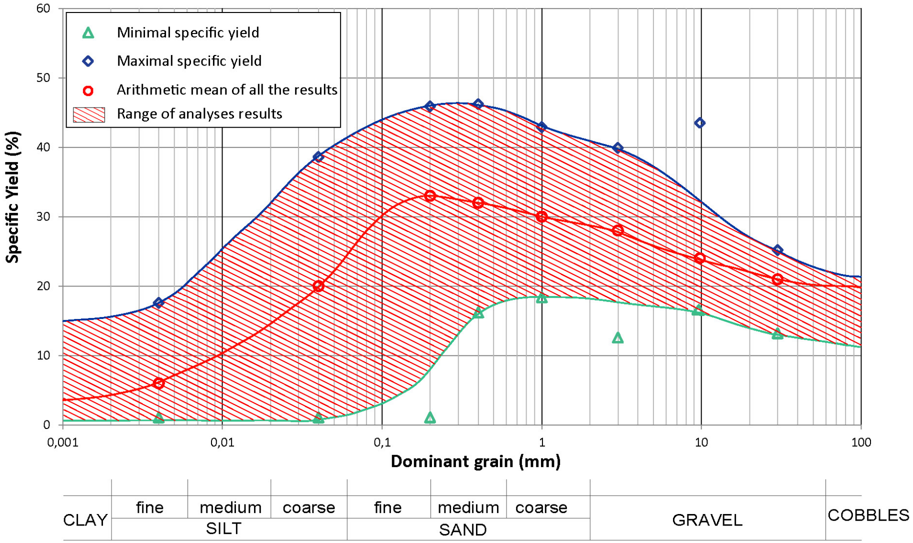

Figure 4. Range and arithmetic mean of specific yield values for 586 analyses in the Hydrol. Lab. of the US Geol. Survey (from Morris and Johnson, 1967).

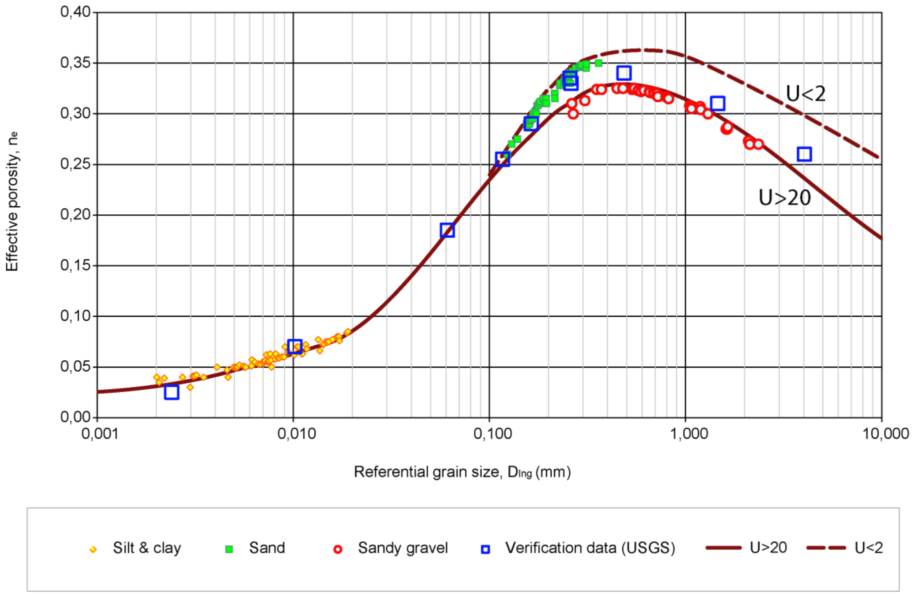

Figure 5. Effective porosity (n_e) in function of referential mean grain D_{lng}.
Note: dot line devides uniform grain deposits $U = D_{60}/D_{10} < 2$, and medim uniform grain deposits $2 < U < 20$. Verified samples of non-uniform grain deposits of sand and gravel ($U > 20$) lie below the full line.

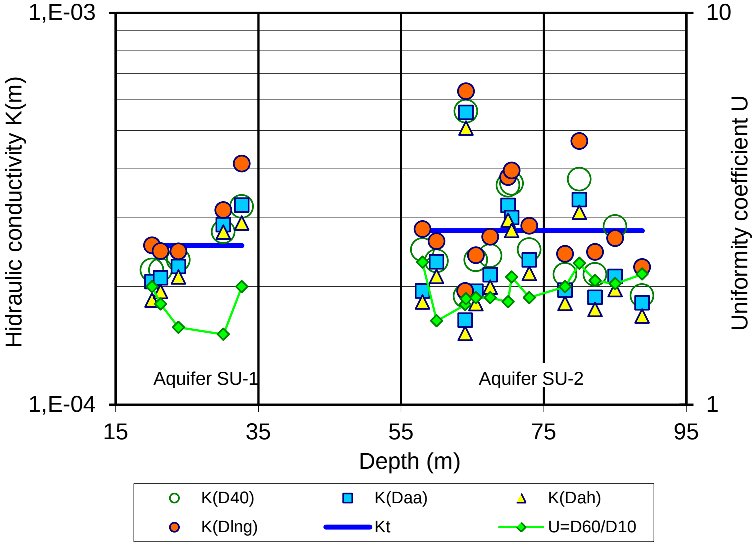

Figure 6. Results of predicted hydraulic conductivity calculated using Kozeny–Carman equation for samples from medium uniform sandy aquifers.

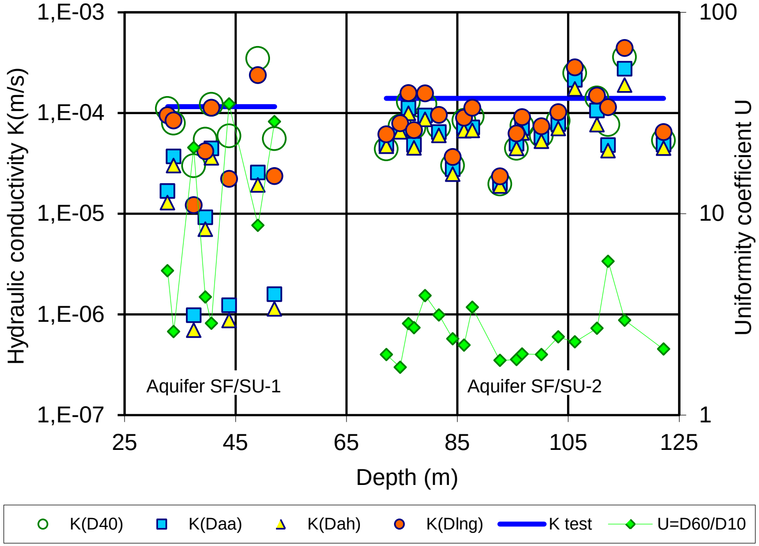

Figure 7. Results of predicted hydraulic conductivity calculated using KC equation for samples from fine sandy aquifers with thin siltey laminas.

Figure 8. Fine sand sample with thin silty laminas from test field SF/SU1 (see Fig. 7).

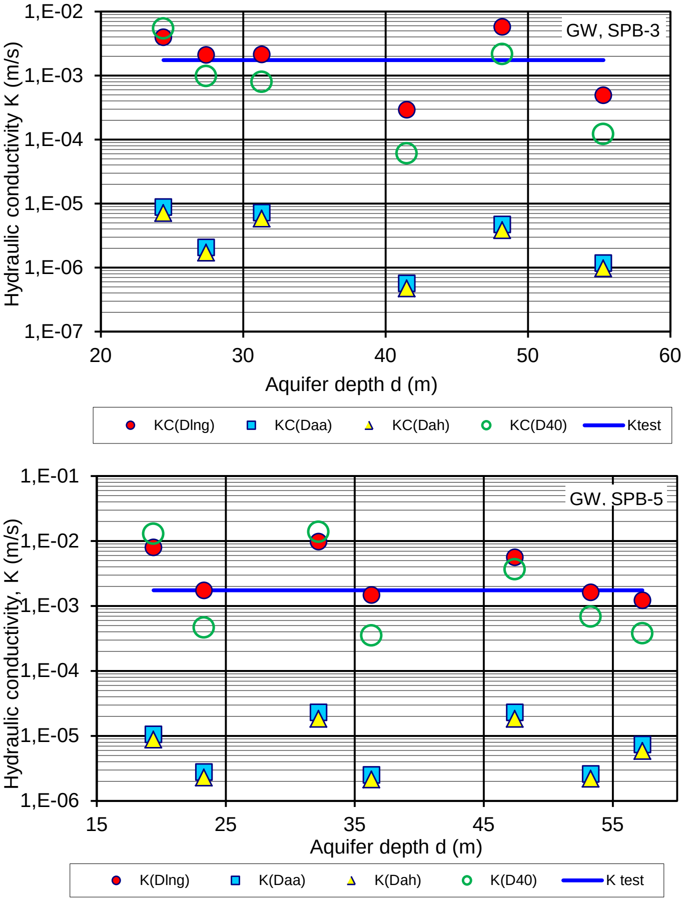

Figure 9. Results of predicted hydraulic conductivity calculated using KC equation for samples from gravely aquifer (test field GW).

Figure 10. Gravel borehole core from 23 to 30 m depth et borehole SPB-3 (see Fig. 9).

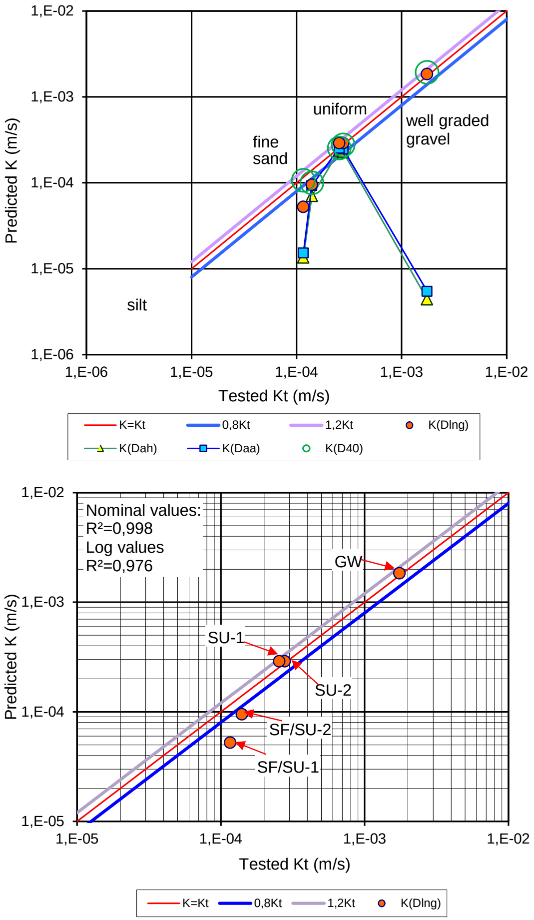

Figure 11. Graphical correlation between predicted K and tested K_t of sand and gravel deposits.

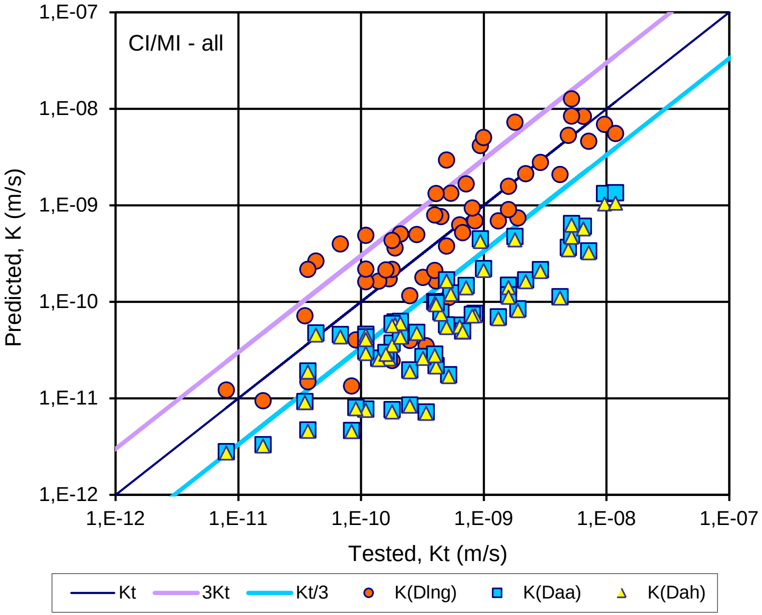

Figure 12. Graphical correlation between the tested (K_t) and the predicted hydraulic conductivity using geometric $K(D_{\text{lng}})$, and arithmetic ($K(D_{\text{aa}})$ and K (D_{ah})) mean grain size for siltey-clayey samples.

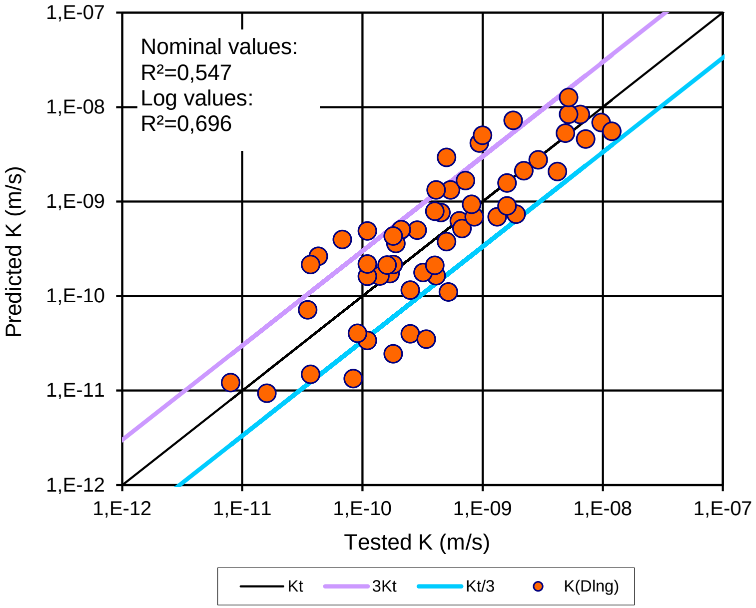

Figure 13. Verification of graphical and numerical correlation between the tested (K_t) and the predicted hydraulic conductivity $K(D_{\text{lng}})$ using referential geometric mean grain size for clayey-siltey samples.

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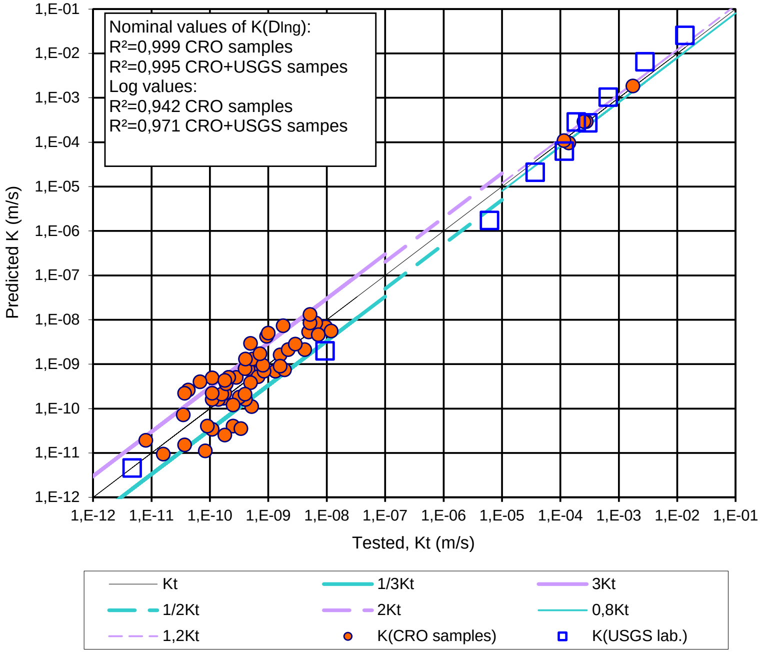

Figure 14. Verification of graphical and numerical correlation between the tested (K_t) and the predicted hydraulic conductivity using referemtial geometric mean size $K(D_{\text{lng}})$ for all samples.