PARCC Mathematics Update

September 2013 - IHL

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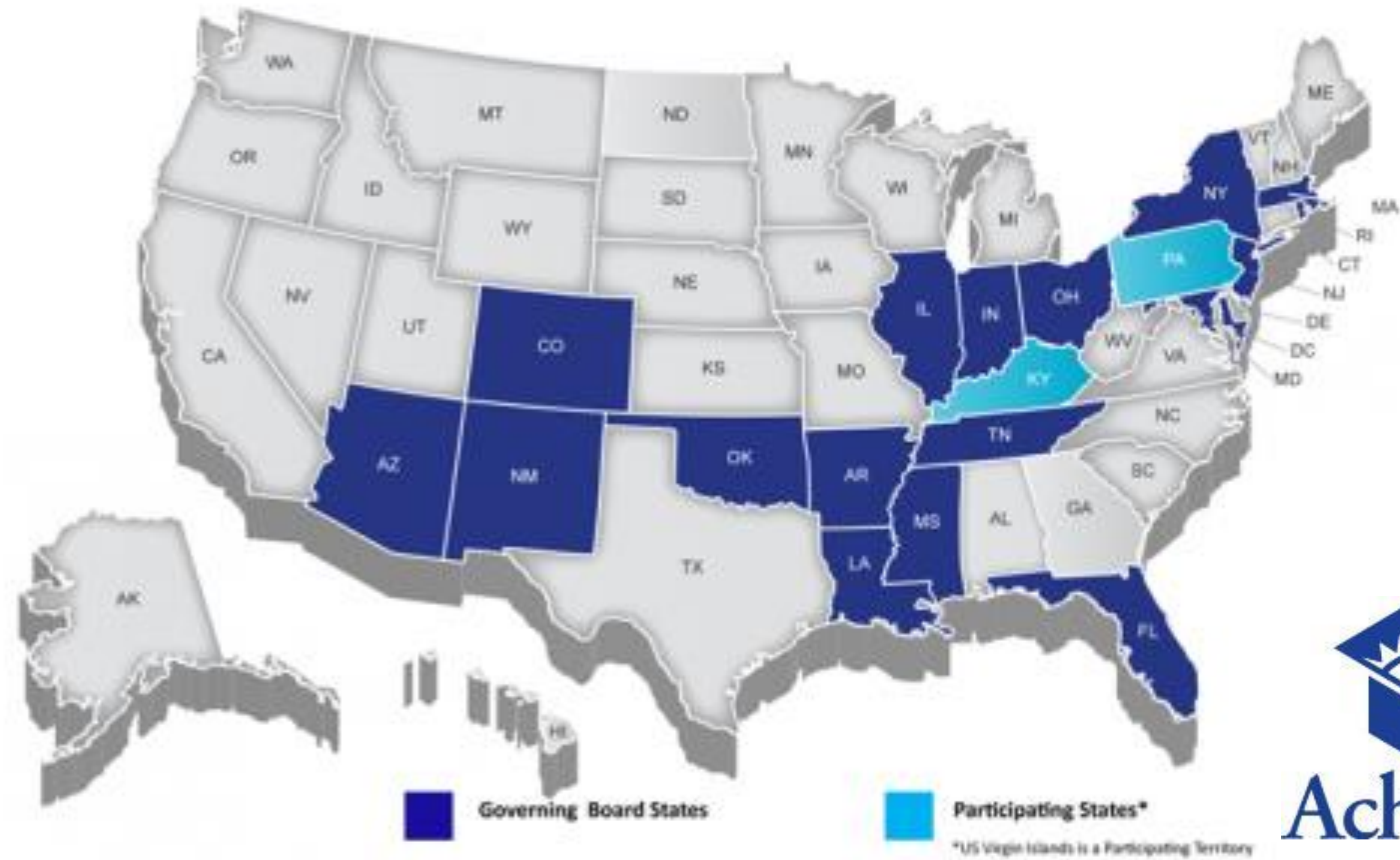
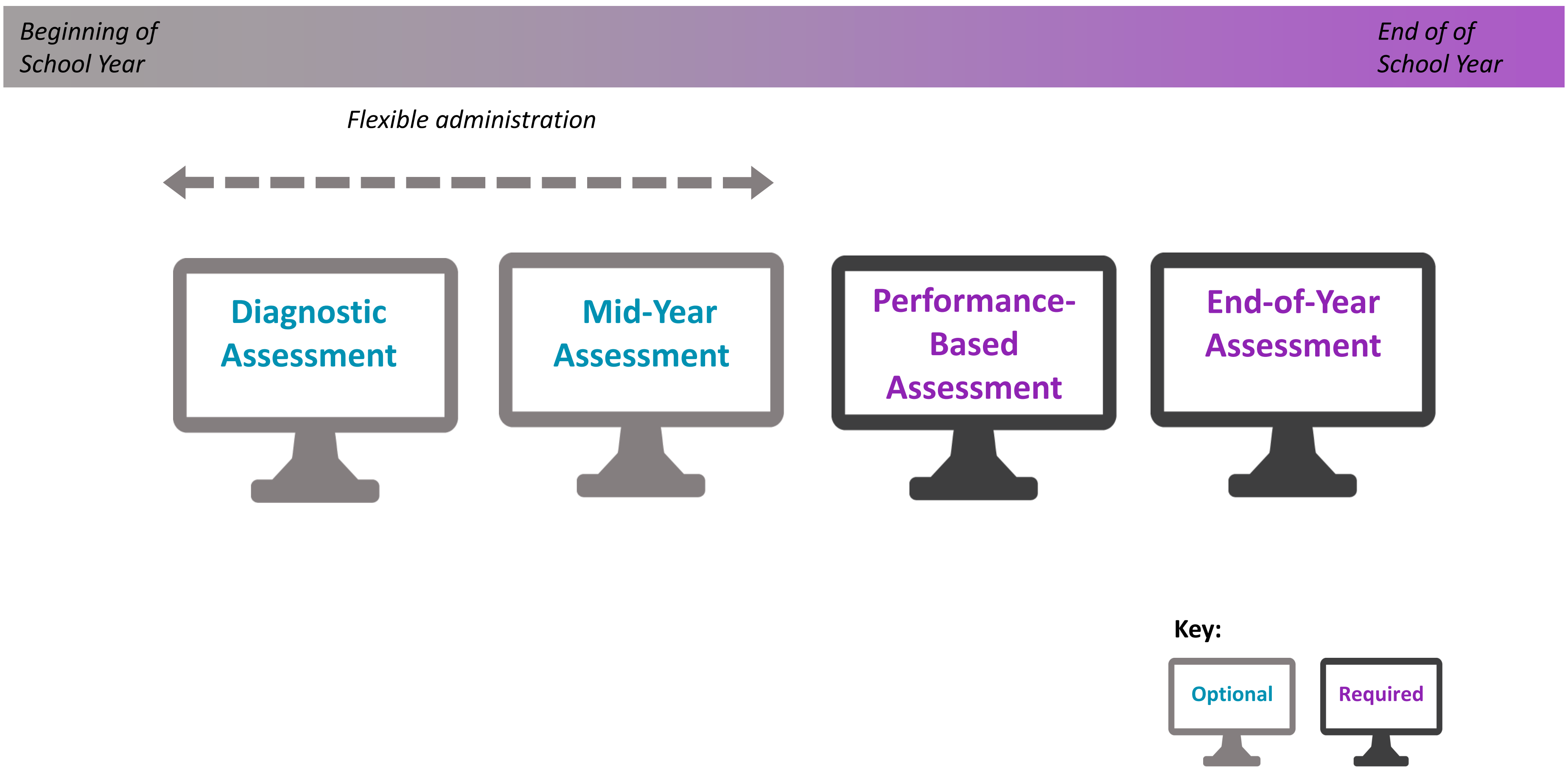
Partnership for Assessment of Readiness for College and Careers (PARCC)



Achieve

PARCC Assessment Design

Mathematics, Grades 3–8 and High School End-of-Course



A Little History of PARCC Mathematics

- PARCC states developed Claims for Mathematics based on the **CCSSM.**
- PARCC states developed the Model Content Frameworks to provide guidance to key elements of excellent instruction aligned with the Standards.
- The blueprints for the PARCC Mathematics Assessments have been developed using the CCSS, Claims and Model Content Frameworks.
- Cognitive Complexity Framework development in partnership with item development contractors.
- Performance Level Descriptors released.
- Phase 1 (first half of item bank) of item development has ended.
- Phase 2 (second half of item bank) of item development is beginning.
- Forms Construction beginning the November for the Spring 2014 FT.

Claims Driving Design: Mathematics

Students are on-track or ready for college and careers

Sub-claim A: Students **solve problems involving the major content** for their grade level with connections to practices

Sub-Claim B: Students **solve problems involving the additional and supporting content** for their grade level with connections to practices

Sub-claim C: Students **express mathematical reasoning** by constructing mathematical arguments and critiques

Sub-Claim D: Students solve real world problems engaging particularly in the **modeling practice**

Sub-Claim E: Student **demonstrate fluency** in areas set forth in the Standards for Content in grades 3-6

PARCC Model Content Frameworks

Approach of the Model Content Frameworks for Mathematics

- PARCC Model Content Frameworks provide a deep analysis of the CCSS, leading to more guidance on how focus, coherence, content and practices all work together.
- They **focus on framing the critical advances in the standards:**
  - Focus and coherence
  - Content knowledge, conceptual understanding, and expertise
  - Content and mathematical practices
- Model Content Frameworks for grades 3-8, Algebra I, Geometry, Algebra II, Mathematics I, Mathematics II, Mathematics III

Model Content Frameworks Grade 3 Example

Key: ■ Major Clusters; □ Supporting Clusters; ○ Additional Clusters

Operations and Algebraic Thinking	
■	Represent and solve problems involving multiplication and division.
■	Understand properties of multiplication and the relationship between multiplication and division.
■	Multiply and divide within 100.
■	Solve problems involving the four operations, and identify and explain patterns in arithmetic.
Number and Operations in Base Ten	
○	Use place value understanding and properties of operations to perform multi-digit arithmetic.
Number and Operations — Fractions	
■	Develop understanding of fractions as numbers.
Measurement and Data	
■	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
□	Represent and interpret data.
■	Geometric measurement: understand concepts of area and relate area to multiplication and addition.
○	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Geometry	
□	Reason with shapes and their attributes.

How PARCC has been presenting Evidence-Centered Design (ECD)

Claims

Design begins with the inferences (**claims**) we want to make about students

Evidence

In order to support **claims**, we must gather **evidence**

Tasks

Tasks are designed to elicit specific **evidence** from students in support of **claims**

ECD is a deliberate and systematic approach to assessment development that will help to **establish the validity** of the assessments, **increase the comparability** of year-to year results, and **increase efficiencies/reduce costs**.

Claims Structure: Mathematics

Master Claim: On-Track for college and career readiness. The degree to which a student is college and career ready (or "on-track" to being ready) in mathematics. The student solves grade-level /course-level problems in mathematics as set forth in the Standards for Mathematical Content with connections to the Standards for Mathematical Practice.

Total Exam Score Points: 82 (Grades 3-8), 97 or 107(HS)

Sub-Claim A: Major Content[1] with Connections to Practices

The student solves problems involving the Major Content[1] for her grade/course with connections to the Standards for Mathematical Practice.

~37 pts (3-8), ~42 pts (HS)

Sub-Claim B: Additional & Supporting Content[2] with Connections to Practices

The student solves problems involving the Additional and Supporting Content[2] for her grade/course with connections to the Standards for Mathematical Practice.

~14 pts (3-8), ~23 pts (HS)

Sub-Claim C: Highlighted Practices MP.3,6 with Connections to Content[3] (expressing mathematical reasoning)

The student expresses grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and/or attending to precision when making mathematical statements.

14 pts (3-8),
14 pts (HS)
4 pts (Alg II/Math 3 CCR)

Sub-Claim D: Highlighted Practice MP.4 with Connections to Content (modeling/application)

The student solves real-world problems with a degree of difficulty appropriate to the grade/course by applying knowledge and skills articulated in the standards for the current grade/course (or for more complex problems, knowledge and skills articulated in the standards for previous grades/courses), *engaging particularly in the Modeling practice*, and where helpful making sense of problems and persevering to solve them (MP. 1),reasoning abstractly and quantitatively (MP. 2), using appropriate tools strategically (MP.5), looking for and making use of structure (MP.7), and/or looking for and expressing regularity in repeated reasoning (MP.8).

12 pts (3-8),
18 pts (HS)
6 pts (Alg II/Math 3 CCR)

Sub-Claim E: Fluency in applicable grades (3-6)

The student demonstrates fluency as set forth in the Standards for Mathematical Content in her grade.

7-9 pts (3-6)

[1] For the purposes of the PARCC Mathematics assessments, the Major Content in a grade/course is determined by that grade level's Major Clusters as identified in the *PARCC Model Content Frameworks v.3.0* for Mathematics. Note that tasks on PARCC assessments providing evidence for this claim will sometimes require the student to apply the knowledge, skills, and understandings from across several Major Clusters.
[2] The Additional and Supporting Content in a grade/course is determined by that grade level's Additional and Supporting Clusters as identified in the *PARCC Model Content Frameworks v.3.0* for Mathematics.
[3] For 3 – 8, Sub-Claim C includes only Major Content. For High School, Sub-Claim C includes Major, Additional and Supporting Content.

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Overview of Evidence Statements: Types of Evidence Statements

Several types of evidence statements are being used to describe what a task should be assessing, including:

- Those using **exact standards language**
- Those transparently **derived from exact standards** language, e.g., by splitting a content standard
- **Integrative evidence statements** that express plausible direct implications of the standards without going beyond the standards to create new requirements
- **Sub-claim C and D evidence statements**, which put MP.3, 4, 6 as primary with connections to content

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be assessing, including:

1. Those using **exact standards language**

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to Mathematical Practices
8.EE.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 1/3^3 = 1/27$.	i) Tasks do not have a context. ii) Tasks center on the properties and equivalence, not on simplification. For example, a task might ask a student to classify expressions according to whether or not they are equivalent to a given expression.	MP.7

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be assessing, including:

2. Those transparently **derived from exact standards** language, e.g., by splitting a content standard

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
8.F.5-1	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).	i) Pool should contain tasks with and without contexts.	MP.2, MP.5
8.F.5-2	Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	i) Pool should contain tasks with and without contexts.	MP.2, MP.5, MP.7

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be assessing, including:

3. **Integrative evidence statements** that express plausible direct implications of the standards without going beyond the standards to create new requirements

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
4.Int.1	Solve one-step word problems involving adding or subtracting two four-digit numbers.	The given numbers are such as to require an efficient/standard algorithm (e.g., 7263 + 4875, 7263 – 4875, 7406 – 4637). The given numbers do not suggest any obvious *ad hoc* or mental strategy (as would be present for example in a case such as16,999 + 3,501 or 7300 – 6301, for example). i) Grade 4 expectations in CCSSM are limited to whole numbers less than or equal to 1,000,000; for purposes of assessment, both of the given numbers should be limited to 4 digits.	MP.1

Overview of Evidence Statements: Examples

Several types of evidence statements are being used to describe what a task should be assessing, including:

4. **Sub-claim C & Sub-claim D Evidence Statements**, which put MP.3, 4, 6 as primary with connections to content

Key	Evidence Statement Text	Clarifications, limits, emphases, and other information intended to ensure appropriate variety in tasks	Relationship to MP
HS.C.5.11	Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.11, involving any of the function types measured in the standards.	i) For example, students might be asked how many positive solutions there are to the equation $e^x = x+2$ or the equation $e^x = x+1$, explaining how they know. The student might use technology strategically to plot both sides of the equation without prompting.	MP.3

How PARCC has been presenting Evidence-Centered Design (ECD)

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Tasks

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ECD is a deliberate and systematic approach to assessment development that will help to **establish the validity** of the assessments, **increase the comparability** of year-to year results, and **increase efficiencies/reduce costs**.

Overview of Task Types

- The PARCC assessments for mathematics will involve three primary types of tasks: Type I, II, and III.
- Each task type is described on the basis of several factors, principally the purpose of the task in generating evidence for certain sub-claims.

Source: Appendix D of the PARCC Task Development ITN on page 17

Overview of PARCC Mathematics Task Types

Task Type	Description of Task Type
I. Tasks assessing *concepts, skills and procedures*	• Balance of conceptual understanding, fluency, and application • Can involve any or all mathematical practice standards • Machine scorable including innovative, computer-based formats • Will appear on the End of Year and Performance Based Assessment components • Sub-claims A, B and E
II. Tasks assessing *expressing mathematical reasoning*	• Each task calls for written arguments / justifications, critique of reasoning, or precision in mathematical statements (MP.3, 6). • Can involve other mathematical practice standards • May include a mix of machine scored and hand scored responses • Included on the Performance Based Assessment component • Sub-claim C
III. Tasks assessing *modeling / applications*	• Each task calls for modeling/application in a real-world context or scenario (MP.4) • Can involve other mathematical practice standards • May include a mix of machine scored and hand scored responses • Included on the Performance Based Assessment component • Sub-claim D

For more information see PARCC Task Development ITN Appendix D.

Design of PARCC Math Summative Assessment

- Performance Based Assessment (PBA)
  - Type I items (Machine-scorable)
  - Type II items (Mathematical Reasoning/Hand-Scored – scoring rubrics are drafted but PLD development will inform final rubrics)
  - Type III items (Mathematical Modeling/Hand-Scored and/or Machine-scored - scoring rubrics are drafted but PLD development will inform final rubrics)
- End-of-Year Assessment (EOY)
  - Type I items only (All Machine-scorable)

Math: High School Type I Sample Item

- Item has two possible solutions
- Students have to recognize the nature of the equation to know how to solve
- Technology prevents guessing and working backward

Solve the following equation:

$$(3x - 2)^2 = 6x - 4.$$

When you are finished, enter the solution(s) below.

Solution 1: $\frac{\square}{\square}$

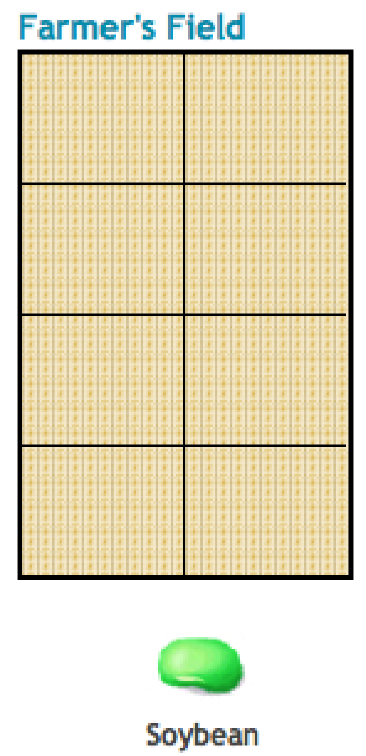
Click + to enter another solution or click Done.

Math: Grade 3 Type II Sample Item

SAMPLE ITEM

Part A

A farmer plants 3/4 of the field with soybeans.
Drag the soybean to the field as many times as needed to show the fraction of the field that is planted with soybeans.



- This a fairly traditional fraction task in a computer-based setting.
- Unlike traditional multiple choice, it is difficult to guess the correct answer or use a choice elimination strategy and there is more than one correct solution.
- Unlike paper and pencil tests, students can create a visual representation even though the task is scored automatically.

Math: Grade 3 Type II Sample Item

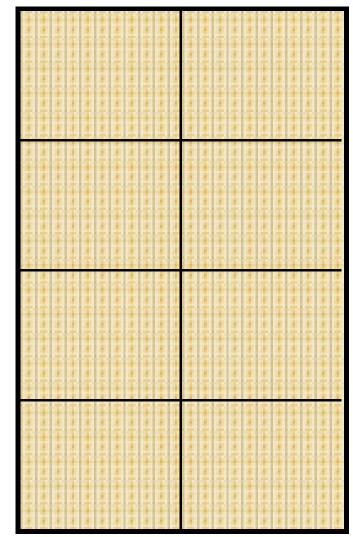
SAMPLE ITEM

Part B

Type a fraction different than 3/4 in the boxes that also represents the fractional part of the farmer's field that is planted with soybeans.

$\frac{3}{4} = \frac{\square}{\square}$

Farmer's Field



Reset

Explain why the two fractions above are equal.

- Second part of multi-step problem, and, unlike traditional multiple choice, it is difficult to guess the correct answer or use a choice elimination strategy.

Math: High School Type III Sample Item

- This task is a Type III sample item assessing Mathematical Modeling
- In **Part a**, students extend a sequence established by the context. This sequence sets up the parts of the task that follow.

Cellular growth

About the task | CCSSM Alignment | **Part a** | Part b | Part c | Part d | Scoring

In a cellular regeneration experiment, Jaydon Laboratory found that for cells put in containers with a particular growth medium, the number of cells at the end of each week was double the number of cells at the end of the previous week.

The data for the first 6 weeks of the experiment are shown in the table. Fill in the blanks to complete the table for weeks 7-10.

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480
7	
8	
9	
10	

Submit Answer

Math: High School Type III Sample Item

- In **Part b**, students create a recursive expression that can be used to model the sequence of growth; they then consider limitations on the domain to fit the context.

Cellular growth

About the task | CCSSM Alignment | Part a | **Part b** | Part c | Part d | Scoring

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

Assume that as the experiment continues, the number of cells at the end of each week continues to be double the number of cells at the end of the previous week. Let w_n represent the number of cells in the growth medium in week n. Drag the tiles to write a recursive definition for the sequence that represents the number of cells in the growth medium at the end of each week.

an integer	a real number	2	15	30
+	-	•	÷	w_{n-1}
$n \geq 0$	$n \geq 1$	$n \geq 2$	$n \geq 6$	$n \geq 15$

$w_1 =$ ____

$w_n =$ ____ ____ ____ ,

where n is ____ , such that ____

Math: High School Type III Sample Item

- In **Part c**, students choose appropriate statements that could be used to model the situation.
- The use of a multiple-answer, multiple-choice format allows insights into student thinking.

Cellular growth

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

Let w_n represent the number of cells in the growth medium at the end of week n. Which of these statements are true about the explicit formula for w_n?

Select all that apply.

- ☐ $w_n = 15 + 15 \cdot 2(n - 1)$
- ☐ $w_n = 15 + 15 \cdot 2(n)$
- ☐ $w_n = 15 \cdot 2^{n-1}$
- ☐ $w_n = \frac{1}{2} \cdot 15 \cdot 2^{n-1}$
- ☐ $w_n = \frac{1}{2} \cdot 15 \cdot 2^{n}$
- ☐ $n \geq 1$, where n is an integer
- ☐ $n \geq 1$, where n is a real number
- ☐ n can be any real number

Submit Answer

Math: High School Type III Sample Item

- In **Part d**, students are required to use either the explicit or recursive model they constructed to answer a question about what number of weeks might have resulted in a particular number of cells.

Cellular growth

Write your answers to the following problem in your answer booklet.

Consider the table of data about the cellular regeneration experiment.

Week	Number of cells in medium
1	15
2	30
3	60
4	120
5	240
6	480

a. If the number of cells continues to grow according to the pattern shown in the table, at what week number will the number of cells exceed one billion?

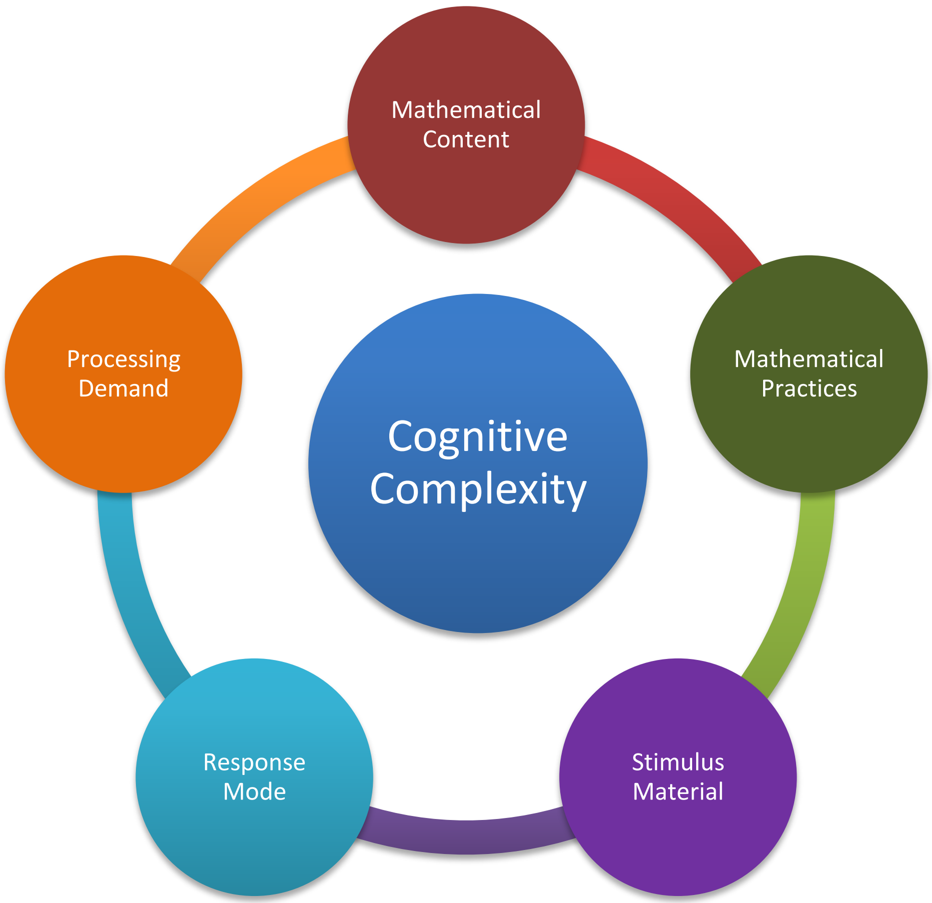
b. Explain how the process you used to find the week number relates to either the recursive model or the explicit model you constructed in the previous questions.

PARCC Cognitive Complexity Framework

- Blooms?
- Webb's DOK?
- CCSS demand a new type of cognitive complexity framework.
- PARCC partnered with the Item Development contractors to develop a new cognitive complexity framework.
- New framework is based on multiple dimensions.

Factors that determine the Cognitive Complexity of PARCC Mathematics Items

1. Mathematical Content
2. Mathematical Practices
3. Stimulus Material
4. Response Mode
5. Processing Demand

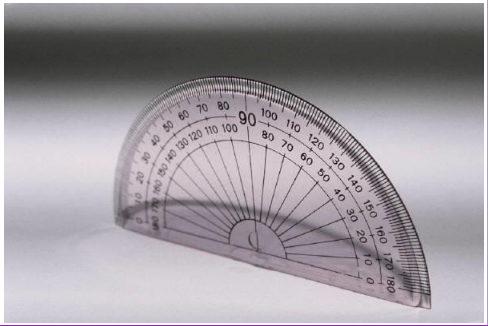


For further reading on the PARCC Cognitive Complexity Framework see, " **Proposed Sources of Cognitive Complexity in PARCC Items and Tasks: Mathematics** " Aug. 31, 2012

1. Mathematical Content

At each grade level, there is a range in the level of demand in the content standards--from low to moderate to high complexity. Within Mathematical Content, complexity is affected by:

- Numbers: Whole numbers vs. fractions
- Expressions and Equations: The types of numbers or operations in an expression or equation (3/7, √)
- Diagrams, graphs, or other concrete representations: may contribute to greater overall complexity than simpler graphs such as scatterplots.
- Problem structures: Word problems with underlying algebraic structures vs. word problems with underlying arithmetic structures.



2. Mathematical Practices

MPs involve what students are asked to do with mathematical content, such as engage in application and analysis of the content. The actions that students perform on mathematical objects also contribute to Mathematical Practices complexity.

Low Complexity

- Items at this level primarily involve recalling or recognizing concepts or procedures specified in the Standards.

High Complexity

- High complexity items make heavy demands on students, because students are expected to use reasoning, planning, synthesis, analysis, judgment, and creative thought. They may be expected to justify mathematical statements or construct a formal mathematical argument.

3. Stimulus Material

This dimension of cognitive complexity accounts for the number of different pieces of stimulus material in an item, as well as the role of technology tools in the item.

Low Complexity

- Low complexity involves a single piece of (or no) stimulus material (e.g., table, graph, figure, etc.) OR single online tool (generally, incremental technology)

High Complexity

- High complexity involves two pieces of stimulus material with online tool(s) OR three pieces of stimulus material with or without online tools.

4. Response Mode

The way in which examinees are required to complete assessment activities influences an item's cognitive complexity.

- **Low cognitive complexity** response modes in mathematics involve primarily selecting responses and producing short responses, rather than generating more extended responses.

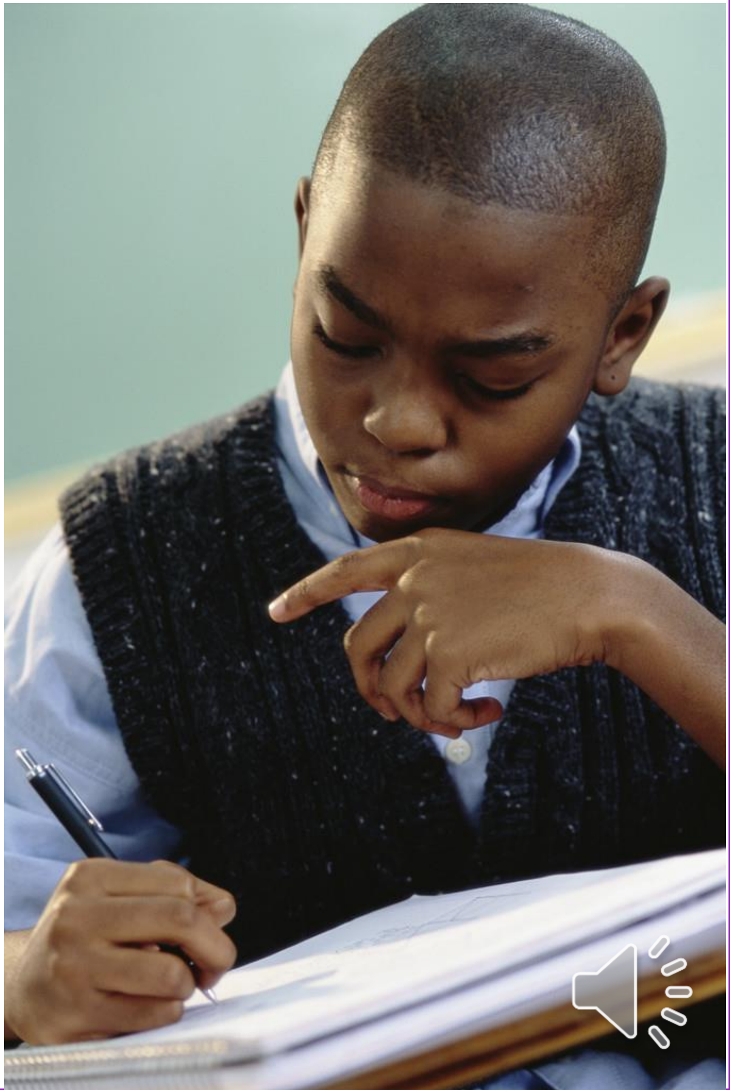
- **High Complexity** response modes require students to construct extended written responses that may also incorporate the use of online tools such as an equation editor, graphing tool, or other online feature that is essential to responding.

5. Processing Demand

Reading load and linguistic demands in item stems, instructions for responding to an item, and response options contribute to the cognitive complexity of items.

PARCC Content Specific Performance Level Descriptors (PLDs)

- The PARCC PLD writing panels consisted of educators from across the PARCC States.
- The PARCC PLD writing panels were focused on staying true to the CCSS.
- The foundation of the PARCC PLDs are the PARCC Evidence Statements and the PARCC Cognitive Complexity Framework.



Capturing What Students Can Do

PARCC PLDs

- capture how **all** students perform
- show understandings and skill development across the spectrum of standards and complexity levels assessed

Looking at the PLDs

Gives the Conceptual Concept the PLD is based on

Gives the Sub-Claim that the PLD is written for (A-Major Content)

Gives the PLD by performance level ranging from 2-5. Level 1 indicates a range from no work shown to Minimal command

	Grade 4 Math : Sub-Claim A The student solves problems involving the Major Content for the grade/course with connections to the Standards for Mathematical Practice.			
	Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**
Fractions and Decimals 4.NF.1-2 4.NF.2-1 4.NF.A.Int.1 4.NF.5 4.NF.6 4.NF.7	Compares decimals to hundredths; uses decimal notations for fractions (tenths and hundredths); compares fractions, with like or unlike numerators and denominators, by creating equivalent fractions with common denominators, comparing to a benchmark fraction and generating equivalent fractions. Recognizes that decimals and fractions must refer to the same whole in order to compare. Shows results using symbols. Demonstrates the use of conceptual understanding of fractional equivalence and ordering when solving simple word problems requiring fraction	Compares decimals to hundredths; uses decimal notations for fractions (tenths and hundredths); compares fractions, with like or unlike numerators and denominators, by creating equivalent fractions with common denominators, comparing to a benchmark fraction and generating equivalent fractions. Recognizes that decimals and fractions must refer to the same whole in order to compare. Shows results using symbols. **Demonstrates the use of conceptual understanding of fractional equivalence and ordering when solving** simple word problems requiring fraction	Given a visual model and/or manipulatives, compares decimals to hundredths; uses decimal notations for fractions (tenths and hundredths); compares fractions, with like or unlike numerators and denominators, by creating equivalent fractions with common denominators and comparing to a benchmark fraction. Recognizes that decimals and fractions must refer to the same whole in order to compare. Shows results using symbols. **Solves simple word problems requiring fraction comparison.**	Given a visual model and/or manipulatives, compares decimals to hundredths; uses decimal notations for fractions (tenths and hundredths); compares fractions, with like or unlike numerators and denominators by comparing to a benchmark fraction. Recognizes that decimals and fractions must refer to the same whole in order to compare. Shows results using symbols. Solves simple word problems requiring fraction comparison with scaffolding.

What's Next for PARCC Mathematics?

- Begin Phase 2 of item development this fall (last 50% of item bank)
- Begin the forms construction and forms review process
- Develop and release additional sample items this fall
- Conduct Field Testing in Spring 2014
- Data Review in Summer 2014

PARCC
Partnership for Assessment of Readiness for College and Careers

Discussion

- Implications for higher education
- Changes that might take place as the standards are implemented

With the new knowledge of the students, how might this affect:

- Course Offerings
- Syllabi
- Others…

Resources

- Any publicly released assessment policies, sample items and prototypes, Model Content Frameworks, PLDs can be found at www.PARCConline.org

- Additional item prototypes can be found at http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html

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