

PART 1

**MICROFOUNDATIONS OF
MACROECONOMICS**

1 Overview of the book 3

Thus the history of the individual firm cannot be made in the history of an industry any more than the history of an individual man can be made into the history of mankind. And yet the history of mankind is the outcome of the history of individuals and the aggregate production for a general market is the outcome of the motives which induce individual producers to expand or contract their production. It is just here that our device of a representative firm comes to aid. (Alfred Marshall, *Principles of Economics*, 8th Edition pp. 380–381) ‘Thus the representative firm is in a sense an average firm.’ (Ibid., p. 265).

CHAPTER 1

Overview of the book

1.1 Introduction

This chapter provides an overview of the book within different dimensions. First it motivates how it is an extension of the microfoundation approach of using general equilibrium theory for macroeconomics. It highlights new elements of the text in terms of providing aggregate demand and supply analysis in the dynamic context, and in explaining business cycles with basic comparative statics of goods and time endowment changes.

The role of human capital in the text is explained and then the text is outlined in terms of fundamental margins of economic activity. Concepts of permanent income and wealth are traced across the chapters of the book, in a section providing a summary mathematical reference section. The single agent and heterogeneous agent approach are outlined. Finally, a brief summary of uses of the text in teaching are presented.

1.1.1 Learning objective

The aim of the chapter is to provide the intuition for the approach of using only microeconomics to derive all of macroeconomics. Then the specifics are presented so that a complete overview of the text can be glimpsed. This chapter is meant as a reference to pull together all of the separate chapters of the book. It provides a basis to which the student might return at later points so as to get additional intuition into the subsequent Chapters 2–20.

1.1.2 Who made it happen?

Paul A. Samuelson's PhD thesis, published as *Foundations of Economic Analysis* (1947), is often cited as the beginning of modern economic analysis. For example it formulated the modern mathematical analysis of changes in equilibrium, known as comparative statics. Samuelson (Nobel Laureate 1970) developed static macroeconomic analysis in multiple editions of his textbook, *Economics: An Introductory Analysis*, starting in 1948, and finishing with a 19th edition in 2009 co-authored with William Nordhaus. In this he developed the aggregate supply and demand analysis that became known as the Keynesian Cross, and which morphed into the *IS-LM* analysis that remains the dominant paradigm of introductory macroeconomic analysis.

Robert E. Lucas, Jr. writes in his 2001 'Professional Memoir': 'Samuelson was the Julia Child (American television kitchen Chef) of economics, somehow teaching you the basics and giving you the feeling of becoming an insider in a complex culture all at the same time. I loved the *Foundations*. Like so many others in my cohort, I internalised its view that

if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing. I came to the position that mathematical analysis is not one of many ways of doing economic theory: It is the only way. Economic theory is mathematical analysis. Everything else is just pictures and talk.'

R.D.G. Allen provided a summary of the state of mathematical macroeconomic analysis in his 1968 *Macro-Economic Theory*, but utility maximisation was not a part of this. Jump forward to the current statement of dynamic macroeconomic theory such as in Stokey and Lucas, with Prescott, in their 1989 *Recursive Methods in Economic Dynamics*. This expounded the two-period approach to capturing the full infinite horizon of the dynamic consumer problem. This text follows that approach in its simplest form, while attempting the contextual approach of aggregate supply and demand that Samuelson motivates. In certain ways it also formalises and extends the general equilibrium approach seen in Robert Barro's Macroeconomic textbook.

Note that subsections called 'Who made it happen?' give some of the background of how the ideas presented in the particular chapter evolved over time. Those economists named here are not an exclusive group but rather a partially representative group that inevitably omits others that are equally deserving in credit for the development of certain theories. The idea is to give historical flavour that presents the concepts as an evolving part of economics rather than a fixed law of a textbook that never will be amended. People make economics as a science develop and it can be thought-provoking to view it that way.

1.2 The microfoundations approach

Macroeconomics has had a history of divorcing itself from the need to derive its equilibrium conditions, which is the basis of microeconomics. Sets of equilibrium conditions have been presented and analysed, but not derived from the consumer and firm maximisation frameworks. Microeconomics is the study of how the consumer and firm do maximise subject to constraints, thereby giving the supply and demand of goods and the factor markets of labour and capital. Modern macroeconomic practice has returned to the microeconomic foundation of deriving the equilibrium conditions through optimisation problems using microeconomic principles. This has given name to the microfoundations approach to macroeconomics.

Deriving the equilibrium conditions from the agent's optimisation problem is the central part of the microfoundations to macroeconomics. This approach also has come to mean avoiding the addition of features into the model that are not arguably technological features of the aggregate economy or particular industries. The pure microfoundations approach avoids so-called 'ad hoc' elements, since this is the characteristic feature of macroeconomic models not derived from optimisation.

A microfoundations approach is extremely popular in modern macroeconomics, but many ad hoc elements are nonetheless found in most work. This evolution of macro shows that while the microfoundations approach is the goal, it is still subject to compromise in order to better explain certain aggregate events. But now the compromise more often takes place by adding in the ad hoc features into the optimisation problem itself. This way all of the equilibrium conditions are still derived. There are important exceptions though, with equilibrium conditions simply added onto the model after optimisation, and such instances are further from a pure microfoundations approach.

A text for modern macroeconomics at the intermediate undergraduate level can well justify presenting only the pure microfoundations approach to macroeconomics and so leave detailed ad hoc elements for higher graduate level study. And this is what this text does. Yet it is astonishing how much of the subject of macroeconomics can be covered in this way. Arguably all of the elements of macroeconomics are covered in this way, and this

text lays out an extensive set of these elements. In its pure microfoundations approach, only the utility and production technology plus goods and time constraints account for all of the equilibrium conditions of the text. And only standard utility and technology forms are used.

1.2.1 Building macro from micro: new elements

The modern macroeconomics here implies equilibrium supply and demand conditions in all of its extensions such that a modern definition of aggregate supply and demand emerges and permeates the text. The text's basic novelty is that the equilibrium conditions and implied aggregate markets are derived strictly by using standard microeconomics. This makes the transition from microeconomics to macroeconomics a direct extension, rather than a new learning of the science.

Many previous books have integrated increasing amounts of microeconomically founded macroeconomics into more traditional macroeconomic approaches that are not strictly based on microeconomics. In a sense, this text departs from this model by starting with a clean slate and building the macroeconomics without any recourse to the usual approaches except those consistent with pure microeconomics. This is both the goal and the methodology of the text: to write what we think of as modern macroeconomics solely in terms of pure microeconomics. This gives a methodologically consistent structure from the beginning to the end, with each chapter building upon the last.

The full approach of the book is made accessible to the student in that every model solution is an explicit analytic solution. Simulations are not used, but rather only explicit functional forms that give the solutions of the variables. For the baseline dynamic model, all variables including the state variable are found with explicit analytic solutions, with all variables being solved as an exact function of the economy's exogenous parameters. In extensions including human capital, the solution involves an analytic equation in just one variable, with the equilibrium solution being the solution of a quadratic equation.

Every model in the book thus can be solved and graphed using their exact functional forms. And the graphs are those we are familiar with: aggregate supply and demand. The aggregate supply and demand equations are also used to derive the analytic solution to the economy. And these graphs are even extended to show a complete graphical exposition of the general equilibrium solution that includes graphical derivation of the capital stock in the dynamic model.

1.2.2 Microeconomic foundations of macroeconomics

Deriving aggregate markets in general equilibrium follows directly from decentralising the representative agent general equilibrium problem into consumer and firm parts. Starting with the centralised economy, the consumer maximises utility subject to the technology of goods production, thereby acting as both consumer and firm simultaneously. Here prices are not explicit or necessary for the optimisation problem to be solved.

The decentralisation allows the prices and profit to become explicit, in contrast to the centralised general equilibrium. Decentralisation allows the consumer to only consume, while the firm does the production. However, the consumer still owns the firm and receives back any profit of the firm as part of the budget constraint. The consumer then maximises utility subject to the budget constraint. The firm maximises the same budget constraint, which now is in the form of the firm's profit, subject to the production technology. Thus the budget constraint becomes the means of separating the centralised economy without prices and profit into the decentralised economy with consumer and firm problems.

The equilibrium of the consumer and firm problems then imply the supply and demand for quantities of goods, labour and capital as functions of the relative prices.

The decentralisation into the consumer and firm problems, which are the mainstays of microeconomics, thereby allows for the markets of goods, labour and capital to be made explicit. The aggregate demand and supply of goods, labour and capital can all be derived in functional forms and this is done throughout the text.

1.3 Static versus dynamic general equilibrium

While the derivation of the static aggregate demand and supply is commonplace, the derivation of the dynamic aggregate supply and demand of goods, labour and capital markets is not so easy to find. And this difference is what makes fully microeconomic founded macroeconomics different from microeconomics. Microeconomics considers capital a fixed factor and does not solve the problem dynamically. Micro-founded macroeconomics solves the dynamic problem with the equilibrium capital as an integral part of the solution.

Apparently the exact derivation of the aggregate supply (AS) and demand (AD) of the dynamic 'neoclassical' model, instead of the $AS-AD$ that we write down loosely in introductory textbooks, is a new approach of this textbook. The $AS-AD$ is right there in the modern macroeconomics model, waiting to be made clear to students, but up until now it has remained buried. This focus of the book brings the modern dynamic method down to terms of AS and AD , with the advantage of greatly simplifying the results in terms of common graphs. Of course this is why we have always liked to use supply and demand in economics: to make intuitively obvious the nature of the results.

The text brings out the dynamic model exactly as it is, and really how it must be. There are many ways to derive the $AS-AD$ model in non-microfounded macroeconomic approaches, with each textbook seeming to offer a different way to do such a derivation of our fundamental concept of supply and demand in the aggregate context. Here it could be said is just one more approach to $AS-AD$. But it may emerge over time that there is one standard way to derive $AS-AD$ in a pure microfoundations approach within the standard dynamic neoclassical model and this approach is presented here.

Such $AS-AD$ analysis is useful by making exact how changes to the equilibrium in aggregate markets occur. And these changes are the foundation of the dynamic growth and business cycle theory of modern macroeconomics. Showing how these changes can be formulated within $AS-AD$ and applied is a central task of the text.

To make the modern macroeconomics of $AS-AD$ as clear as possible, the first part of the text previews the two marginal rates of substitutions that are the bedrock of the $AS-AD$ by deriving them in a static framework. The static warm-up takes these margins one dimension at a time. Then when they are derived in the equivalent form in the dynamic framework they are already old friends to whom students can compare stories of equilibrium. This makes the dynamic framework immediately approachable since the equilibrium conditions are nearly the same as those that have been derived statically.

Similarly by deriving the static $AS-AD$ in these static frameworks, the derivation of $AS-AD$ in the dynamic framework likewise is a reasonable next step from what has already been learned. Students can then focus on seeing how this aggregate framework is used in all of its extensions.

Changes in the equilibrium known as comparative statics are also developed in a similar fashion, first in the static frameworks and then again in the dynamic framework with the static framework available for comparison. In particular the change in sectoral productivity is the main comparative static exercise that is found throughout the book, first in the static context and then dynamic. And this lays the foundation for understanding the neoclassical growth and business cycle models, as an extension of the same productivity shift studied from the very beginning of the text. Dynamic models can be described in terms of their essential comparative static exercise, even if this occurs over time.

1.3.1 Dynamic AS–AD

Some tricks are necessary to accomplish the expiation of the modern AS–AD and its standard extensions. It must be analysed as a stationary equilibrium concept as its first presentation. There are no ‘transitional dynamics’ discussed in the text; rather only the movement from one stationary equilibrium state to another. The study of stationary equilibrium is the first step upon which transition dynamics can then be studied at advanced levels.

The use of stationary equilibrium also suggests how to present the analysis. Dynamic analysis requires that an output growth rate of the economy needs to be assumed, or derived endogenously. Therefore, along with the concept of an output growth rate, must be the concept of a balanced growth path equilibrium. Then the dynamic AS–AD can be derived, which is done starting in Part 4.

To derive the dynamic AS–AD along the balanced growth path stationary equilibrium still requires thought about what makes the modern dynamic analysis in fact dynamic. The dynamic part is the accumulation of capital. This creates a fundamental ‘state’ variable that must be consistent with the AS–AD equilibrium; the AS–AD is written as a function of the state variable, and it must be the correct value of the state variable. Therefore the third trick, after assuming a growth rate and realising that the equilibrium must be along the balanced growth path, is that the state variable must be computed accurately for the equilibrium to be correct.

To see the complexity that arises with the need for computation of the equilibrium state variable, consider that for any comparative static change in the parameters of the model, the AS–AD functions that have been derived as a function of the state variable must be recomputed with the new equilibrium state variable substituted in. This is really the only hard part of the computation of the AS–AD: the computation of the state variable, and how it changes with comparative static exercises. Only then can the analysis show the change from one balanced growth path equilibrium to another balanced growth equilibrium.

The computation of the state variable is shown in the text and at the same time the importance of the state variable in the equilibrium also influences the modern approach of presenting the optimisation problem. The ‘recursive’ framework of writing down the optimisation problem as a function of the state variable provides a concise, helpful way of deriving the equilibrium conditions. It avoids the more cumbersome infinite horizon approach that speaks almost immediately to the graduate level but not below. In place it puts a simple structure of two time periods that once understood and accepted makes it possible for the student to accept the dynamic model and so derive its equilibrium conditions. And it helps that the text shows that using the recursive approach gives the same two, now-familiar, margins already learned in the preceding static problems of Parts 2 and 3. The only difference is that now both are derived at the same time.

In summary, Parts 2 and 3 build the static elements of AS–AD analysis that are consistent with the dynamic development of AS–AD. Part 4 then presents the dynamic AS–AD analysis. And then this dynamic analysis is applied in the remaining parts of the book, with a set of fundamental extensions. A full graphical analysis of the dynamic general equilibrium AS–AD that includes the derivation of the capital stock is provided in Chapter 10.

1.3.2 General equilibrium output and input dimensions

A complimentary feature to showing the supply and demand in goods and labour markets, with the goods market being called the AS–AD analysis, is to show the general equilibrium output in terms of both goods and leisure. In addition we have the general equilibrium inputs in terms of labour and capital, the input equilibrium.

In the output dimensions, this is done using what is typically called a production possibility curve and a utility level curve. Here this same graph is constructed for the equilibria throughout the book using the exact production functions and exact utility level curves. For the dynamic analysis that starts in Part 4 and continues to the end of the book, the input space is in terms of labour and capital. And these dimensions are also graphed for examples using the exact functional forms throughout Parts 4–7 of the text.

1.3.3 Consumption smoothing

A central theme of the economics of the text is that consumption is always guided by the natural desire by the consumer to ‘smooth’ consumption of goods across different dimensions of the economic problem. This smoothing is the result of optimisation of a normally shaped utility function subject to the resource constraints. For goods and leisure in Part 2, this means that the consumer chooses a balance of goods consumption versus leisure, in the ‘intra-temporal’ decision that allocates resources during the current time period. For goods consumption across time in Part 3, this means that the consumer chooses a balance of consumption today versus consumption tomorrow, in the ‘inter-temporal’ decision that allocates resources across time periods. For intra-temporal and inter-temporal consumption in Part 4, this means that the consumer chooses a balance of consumption simultaneously intra-temporally and inter-temporally.

The heart of Part 4 is the dynamic baseline model in which the consumer’s consumption demand is derived (Chapter 8). And consumption is shown to depend upon permanent income. This is an important concept in macroeconomic theory that is expostulated here in terms of the elements from the labour–leisure intra-temporal choice and the current–future consumption inter-temporal choice. The consumption demand is shown to be a simple fraction of permanent income, just as in the permanent income hypothesis of consumption that was first specified as the modern dynamic baseline model was being developed.

By consuming a fraction of permanent income, the consumer smooths the consumption across time and during the period. To raise permanent income, the consumer needs to invest in capital. A steady amount of investment is necessary for sustained economic growth, a subject of Part 5 (Chapter 11). Permanent income is also affected by how much the consumer invests in other forms of capital. The investment into human capital raises the permanent income and consumption stream, while determining the output growth rate. Such endogenous growth is also the subject of Part 5 (Chapter 12). How the consumption stream is affected by variations in income around the permanent income level comprises the ‘business cycle’ study of the economy also in Part 5 (Chapter 13). Trade also affects consumption in the dynamic model with endogenous growth in the last part of Part 5 (Chapter 14).

Investment by the consumer in other forms of transferring capital across time and space, and the subsequent smoothing of consumption across time and space, is the subject of the next Part 6. This shows a general problem of smoothing consumption across uncertain states of nature, such as good and bad states that arise with certain probabilities. The analysis involves an extension of the so-called Arrow-Debreu theory of uncertain state consumption smoothing. Here costs are assumed to be involved in such consumption smoothing, giving rise to an extension of the theory, but in a very intuitive way. With costs introduced, consumption smoothing is not perfect, but instead is done as best as resources allow. Banks are also introduced in Part 6 as the means of making such transference of consumption across uncertain states, time and space (Chapter 15). And then the consumer’s investment in banks as financial intermediaries to do this consumption smoothing is introduced (Chapter 16). Consumer investment directly in asset markets and asset pricing is also introduced, in the last part of Part 6 (Chapter 17).

Part 7 shows how government intervenes to try to implement better consumption smoothing by the representative consumer. The government does this using its budget constraint (Chapter 18), and the corresponding fiscal (Chapter 19) and monetary policy (Chapter 20). Taxes necessary to raise funds for such government efforts in themselves create distortions that decrease the ability of the consumer to smooth consumption, and this is a problem. But the government can try to balance this loss of consumption smoothing from taxation against a greater ability to smooth consumption through government programmes involving ‘social insurance’ as it appears in its many manifestations.

1.3.4 Taxes, regulations and inefficiencies

The text provides full descriptions and general equilibrium examples of how taxes on goods, labour and capital affect the economy, and how this decreases the consumption smoothing. So-called ‘wedges’ get driven between the consumer and producer so that there is less work and less consumption during the period, with goods and labour taxes, and/or less investment and consumption over time with capital taxes. Using human capital, goods, labour and capital taxes are all shown to decrease the growth rate of the economy. So all taxes cause less current and future consumption through their distortions on the consumer’s incentives to work, save, and invest compared to when there are no taxes. Of course taxes are necessary if there is government spending. And it is possible that the effect of government spending is so positive as to more than offset such negative incentive effects.

Regulations likewise act like taxes. Government regulations are analysed in a similar way to government taxes (Chapter 2). And the analysis also allows for privately-induced regulations to result in effect upon the economy, such as through unions or corruption. Analyzing taxes and regulations in general equilibrium in the different dimensions of intratemporal work–leisure, and intertemporal savings–investment provides a introduction to the effect of such distortions on the economy.

Chapters 3 and 6 have labour, goods and capital taxes in a static environment. Chapter 9 shows the effect of a labour tax in the dynamic baseline model. Chapter 19 focuses within the dynamic model on the effect of labour, capital and goods taxes on both output levels and on the economy’s growth rate. And Chapter 20 shows the distorting effect of the inflation tax to be similar to a labour tax.

1.4 Comparative statics and business cycles

Comparative statics are done by changing one exogenous parameter of the economy and finding the new equilibrium, and comparing it to the initial equilibrium before the parameter was changed. Such analysis is conducted throughout the book, being a comparison of new balanced growth path equilibria in the dynamic models of Parts 4–7. And these are used to show how a typical business cycle might be explained.

1.4.1 A focus on productivity changes

Productivity changes are key to studying modern research-based macroeconomics. Productivity increases in the aggregate goods production sector are the way that economic growth is formulated. And fluctuations in such productivity is how business cycles are explained. Therefore productivity changes are examined throughout the text.

Parts 2 and 3 introduce the comparative static experiment of increasing productivity and seeing how this affects equilibrium in the economy. Part 4 shows how such productivity changes affect the dynamic $AS-AD$ framework, including the subsequent change in

the capital stock. Part 5 shows that continuous productivity increases over time, within the dynamic model, provides a modern theory of economic growth; and changes to such productivity likewise can introduce the modern theory of business cycles. Productivity increases in the human capital sector allow for changes in the growth rate, in Part 5. And human capital productivity increases that occur along with the general aggregate goods sector productivity changes allow for resolution of certain puzzles of the baseline dynamic model.

Productivity increases also allow for increased ability to smooth consumption across uncertain states of nature in Part 6. Similarly, banking productivity changes can cause changes in capital. And unexpected productivity changes in the banking sector can introduce so-called aggregate risk into such uncertain state consumption smoothing, also in Part 6.

1.4.2 Basic puzzle of too much smoothing

The basic dynamic model of Part 4 can explain major parts of the economy. However productivity changes show that the model cannot explain changes in labour employment very well. And so in a sense the model can be said to yield employment that is too smooth relative to the evidence.

A productivity increase leads to no change in labour employed in the static model in Part 2, and also in the dynamic model in Part 4. Yet business cycles are formulated as being a result of changes in aggregate goods sector productivity. This presents a problem in that labour employment does not change when a productivity change occurs and the business cycle occurs. It is a problem because a rise and fall in the employment rate of labour is a key feature of business cycles and one that needs to be a part of a good business cycle explanation. Thus labour employment is too smooth in the baseline static and dynamic model.

One approach to the smoothing puzzles is to look at market distortions that keep markets from working properly. In effect what is assumed is that there is some 'ad hoc' reason for the market not to be working as the general equilibrium economy demands. The text presents the simplest case for this type of ad hoc approach. Labour, goods and capital markets are shown to be non-clearing by assuming that the input prices of labour and capital, these being the wage rate and the capital rental rate, are for some reason fixed at the current level. The inability to lower the wage rate and/or the interest rate when there is an aggregate productivity decrease causes 'unemployment' of labour and/or capital (Chapters 3 and 5).

The assumption of fixed relative prices is outside of a full general equilibrium analysis or, more accurately, in violation of the assumptions of general equilibrium. As part of the microeconomic foundations to macroeconomics is to use proper general equilibrium analysis, the text does not favour this approach as a full understanding of the resolution of the smoothing puzzles. It is a segmented, temporary solution that allows the model to have unemployed resources. But fixed relative prices violates the notion of equilibrium and so is hard to accept as a full solution of the problem.

Instead, the text shows a basic approach towards ameliorating the smoothing puzzle, under conditions of a complete general equilibrium analysis, without assumed violations to the equilibrium. It does this through the introduction of a second comparative static experiment: a change in the time allowed for goods and leisure. This is done by changing the time endowment in Parts 2–4. And then in Part 5, a change in the time for goods and leisure is made endogenous. This is done by allowing for changes in the productivity of human capital investment. And this leads to changes in the time devoted to human capital investment. The result is that the time devoted to goods and leisure is then endogenously changed as the productivity of the human capital investment sector is changed. And this

human capital approach also solves a related major complication of the baseline model: it cannot explain endogenously the growth rate, but human capital accumulation does explain growth.

1.4.3 Human capital

The text extends the mainstream dynamic model by including the long tradition in human capital study. It does this for the fundamental reason of providing solutions to the problems of the baseline model. Sufficient changes in the labour employment rate during the business cycle is accomplished first by changing the time endowment exogenously. And then with human capital this is accomplished by having a second production sector, besides the aggregate goods sector. This second sector is the production of human capital investment.

The text shows that the output of the goods sector expands as the human capital sector moves in the other direction. Labour flows from the 'non-market' human capital sector to the goods sector, and this gives the central result. The employment rate rises in the goods sector. And in a decline of goods sector productivity, employment shifts towards the human capital sector, and the employment rate falls in the goods sector, this being a key fact of a business cycle contraction.

The human capital sector allows the growth rate to be endogenous rather than being assumed as exogenous, and so leaves it unexplained as in the baseline dynamic model. Goods sector productivity changes have no effect on the growth rate. But changes in the human capital sector productivity do affect the growth rate of output. This gives a fuller explanation of economic growth and, in particular, how tax distortions affect the output growth rate.

1.4.4 Banking in general equilibrium

Alongside the introduction of human capital is the introduction of banking in general equilibrium. Human capital allows the economic choice of the capitalisation rate of the consumer's time value, over time. The banking sector allows the economic choice of the transfer of goods, rather than time, over space, time and states of nature under certain conditions. These conditions are that it is no longer a 'frictionless' world in which such transfers can take place without any cost. When such transfers are costly, and involve entire large industries to manifest the transformation of goods across various dimensions, then it is desirable to analyse how this costly transformation is performed in an optimal way. And this is where banking is introduced, in order to model how the economy does this transformation of goods, or capital.

Banking is modelled as the way in which goods are transferred across uncertain states of nature, in the case when such transformation involves cost. This can be thought of generally as insurance being provided by the insurance industry, which itself can be considered more broadly as part of the financial intermediation, or banking, industry. Banking in this way can also be viewed as the conduit for providing pensions, and even health insurance. This approach of modelling financial intermediation in general equilibrium results in an extension of the Arrow-Debreu theory of how goods are transferred across uncertain states of nature under the assumption of no cost in manifesting the goods transfer. Banking is a way to model these costs. This provides a window into the view of how a drop in banking productivity can cause such disruption to the economy, and be a cause of so-called unexpected aggregate risk (Chapter 15).

Banking also allows the economy to intermediate the savings of the consumer into the investment of the firm, when such intermediation cannot be done by the consumer directly. Describing the production of this intermediation service is the way in which

banking is brought into the general equilibrium. The banking extension then again focuses on productivity changes, this time to the productivity of producing the intermediation service in the banking industry. When for example there is a downwards drop in productivity, as in a banking crisis, then less savings is turned into investment, and consumption is not smoothed as well as otherwise (Chapter 16).

1.4.5 Private and public finance

The text tries to minimise the extensions to the baseline model while still covering the elements of modern macroeconomics, including private and public finance. Initially the consumer can directly invest in physical capital, which the consumer then rents to the firm. Then the consumer is assumed to no longer be able to invest directly in physical capital, and instead can either use financial intermediation through banks (Chapter 16), or invest in ownership shares of the firm, getting back profit in the form of dividends. And the firm itself then invests in capital directly, as occurs in actual economies (Chapter 17).

The private finance of firms by the consumer is then extended also to allow the public finance of the government by the consumer. Here the consumer is allowed to invest directly in government bonds, making another form of investment without directly investing in physical capital (Chapter 17).

When the consumer invests in government bonds, either directly or through the private bank, this is a way in which the consumer takes part in government finance (Chapter 18). The nature of government finance can affect the economy's growth rate (Chapter 19). The consumer's holding of money printed by the government's central bank is also a part of government finance and this also affects the economy's growth rate (Chapter 20).

1.5 Explaining business cycles and growth

The text shows how the neoclassical model succeeds in explaining business cycles in simple elemental terms of comparative static exercises. This is contrary to conventional wisdom of standard textbooks. Conventional wisdom says that a productivity change in the output sector is the basis of the business cycle model. Yet comparative statics show the sense in which the productivity change causes insufficient employment change. In other words, for the labour market, the worker has no external margin for labour, about whether to enter the labour force or not, but only an internal one on how many hours to work. And the internal margin does not change because income and substitution effects of the productivity change result in little or no employment change.

This problem is overcome by considering not only changes in the productivity of the output sector, but also changes in the endowment of time. The goods productivity increase, given the production technology, amounts to an increase in the endowment of goods in the economy. And this is just one major side of the economic equation of directing goods and time efficiently. The endowment of time also is the other major side of the balancing of resources. Sickness affects this; age affects this; and perhaps most important for basic analysis, education time affects the time endowment leftover for everything else. Thus both the goods and time endowment can be affected by productivity changes, either in goods production, or in human capital production.

Allowing the endowment of time to increase or decrease in a comparative static fashion is intuitively similar to allowing changes in the labour force participation rate, or the external margin. This causes a shift out and back of the labour supply, with a rise and fall in the time endowment; and the result is a rise and fall of the equilibrium employment rate. But in Part 2 it also makes the wage rate go down when time endowment increases and up

when time endowment decreases. Allowing the time endowment to go up in a business cycle expansion allows labour employment to rise, but counterfactually the wage rate falls. However, combining this with the goods productivity change solves the problem.

A comparative static explanation of business cycles that extends the standard approach is to allow both the goods productivity and the time endowment to rise in an expansion. In Chapter 3 this is done with an equal percentage rise in the expansion and decrease in a contraction. The result is that in an expansion, labour employment rises, as labour supply shifts out, but so also does the wage rate rise, as productivity rises and offsets the fall in the wage from the shift out in the labour supply. In a contraction, the employment and wage rate fall. This explains normal expansions and contractions.

A fixed wage rate is additionally allowed to show an occasional extreme decrease in employment, in both static and dynamic economies (Chapters 3 and 9). This is done by having the productivity decrease and the time decrease occur at the same time that the wage is fixed. Then employment drops severely as in a depression, in a way to illustrate Keynes's original ideas.

Such goods and time comparative statics marks an approach that allows for a more cohesive story to emerge. The same comparative statics that are performed in the model without dynamic capital accumulation in Part 2 are performed with the baseline dynamic model in Part 4. The same results occur, so that a business cycle is explained with the same combination of comparative static changes in goods and time endowment, even within the dynamic model. Further, in the dynamic model the fixed wage is added onto such productivity changes to again show a depression-like decrease in employment as in Part 2.

In Part 5, the model is extended with human capital and endogenous growth to again examine the business cycle. When the productivity of the human capital sector rises, more time is spent producing human capital. This causes a decrease in the time left for labour and leisure. This decrease in time left over is analogous to a decrease in the time endowment for labour and leisure that is performed in the exogenous growth standard dynamic model. Changing human capital productivity thereby endogenises the exogenous change in time endowment seen in earlier chapters of the text. And it does this in a way consistent with explaining the business cycle.

When human capital productivity shifts up, labour in the goods sector decreases as in a business cycle contraction, and time spent in the human capital sector increases as has been interpreted to occur when labour shifts from the market to the non-market sector in a contraction. When human capital productivity falls, labour shifts back to the goods production sector, as the time endowment for labour and leisure is endogenously increased by the fall in human capital productivity. This gives the movement of labour as seen in the business cycle.

Combining the fall in human capital productivity with a rise in goods sector productivity, the wage rate indeed rises in an expansion. In a contraction, a decrease in goods sector productivity combined with a rise in human capital sector productivity produces a decrease in the wage rate and in employment in the goods sector. Therefore human capital theory endogenises the previously exogenous change in the external margin, being the amount of time endowed for labour and leisure.

The occasional use of fixed real wages to generate a depression on top of a normal contraction is ad hoc. This feature is eliminated by making the occasional depression endogenous to the model, through banking. In Part 6, a banking sector is specified that intermediates savings into investment. Using the standard approach of changing productivity, a third productivity comparative static exercise is introduced. When the banking sector productivity is decreased sharply, as occurs occasionally such as in the 1930s and 2007–09, the model shows how a normal recession can become a depression, but now without fixed wages.

The story that results is that normal business cycles can be explained by changing productivity in the goods and human capital sector. And a further depression can be explained by decreasing productivity in the banking sector. It demonstrates a surprising vigour of the standard model to explain what is often our main focus: business cycles. And it does this without any ad hoc elements that are outside of the model other than model parameters. In particular only productivity parameters end up being changed.

The human capital feature that is crucial to this elemental business cycle explanation also explains important long term phenomena: the long secular fall in the labour work-week, the long secular increase in the time spent in education, and the long gradual increase in the worldwide growth rate. These are all a direct result within the model of the human capital sector, productivity increasing very slightly but steadily over time.

Human capital therefore allows for long term growth puzzles to be explained, as well as business cycles, all within the *AS-AD* analysis, and using only comparative static changes in productivity parameters. Banking adds the further productivity event that plausibly adds to the model's repertoire the explanation of the Great Depression, again within *AS-AD* analysis of shifting aggregate supply and demand. This gives a slowly expanded baseline model, with each step consistent with earlier elements, but with added fundamentalism so that only comparative static changes in sectoral productivities are used in the end to explain a broad set of phenomena. The elemental expansion slowly builds from Chapter 2 and continues until the end of the text.

1.6 Content by margins

A theme of the book is that just two basic types of margins can explain the behaviour of the representative agent. Economic margins are simply the equilibrium conditions that describe the simultaneous balancing of the costs and benefits of alternatives during the period, across time, and across states of nature. The two primary margins are the marginal rate of substitution between goods and time (leisure) during the period and the marginal rate of substitution between goods, or time, over time or across states of nature.

The goods-leisure margin links goods to time during the period. The intertemporal-interstate margin links the transfer of goods or time across time or across states of nature to the investment set aside for such transfers during the current period. Such investment is reaped only in a future time or a different state of nature. While the investment of goods or time for transfer across states of nature might be thought of as a separate, additional, margin to the intertemporal transfer of goods or time across time, both are forms of investment of goods or time now for a return from the investment in another time or state-space. In this sense, no other margins are introduced in the book.

Part 2 first develops the goods-leisure margin for the single representative agent, 'closed economy' (Chapter 2). The marginal disincentive effect of a tax on goods or labour income is presented (Chapter 3). And the text presents the same margin but for two representative agents identical in utility but different in labour productivity. This creates comparative advantage between the two-agents, one being better at goods production and the other being better at leisure consumption, and equilibrium requires trade between the agents (Chapter 4).

Within the goods-leisure margin, the real wage focuses the consumer's decision of whether to consume goods or instead to consume leisure. As the relative price of leisure versus goods, the real wage w plays both a substitution and an income role in this dimension of consumption theory. The substitution effect, as the wage rises, is that the consumer chooses less leisure, more work, and more goods consumption. The income effect causes, as the real wage rises, every hour of work to yield more real income, and this induces

higher goods consumption. And from the firm side, this real wage is the marginal product of labour in producing goods.

Part 3 develops the intertemporal margin for the representative agent that explains how the consumer decides how much to save and invest today in order to get a certain amount of goods at a future point in time (Chapter 5). Business cycles and taxes are investigated on the basis of this margin (Chapter 6), and the analysis is extended to two representative agents again with identical utility but different productivity of capital (Chapter 7).

Physical capital theory focuses on investment of goods to create even more future period goods, thereby balancing the consumption of goods across time. The interest rate, like the real wage, is a relative price; it gives the price of goods consumed today or in the current period, relative to goods consumed tomorrow or in the next period. Consider that goods consumed today could be instead invested with a net rate of return equal to r so that tomorrow the abstaining consumer would have $1 + r$ multiplied by the goods invested today. Thus r , the net interest rate, or $1 + r$, the gross interest rate, is the 'cost' of the current relative to the future consumption. And from the supply or producer side in equilibrium the interest rate r equals the marginal product of capital.

The substitution effect of an increase in the interest rate is less current consumption in favour of more future consumption. The income effect of a change in the interest rate depends on whether the consumer is a borrower or lender (Chapter 7). On balance, the representative consumer has zero net borrowing since this represents the net borrowing of the whole closed economy, which must be zero. This zero net borrowing means the lack of any income effect on the representative consumer. But with two-agents, one can borrow from the other (Chapter 7).

Part 4 develops the baseline model which simultaneously derives the two main margins of intratemporal consumption, being goods versus leisure, and of intertemporal consumption, about savings and investment (Chapters 8, 9, 10). Part 5 extends both margins. Growth in the baseline model depends on exogenous changes in labour and capital productivity (Chapter 11). Human capital requires time to accumulate and it enables higher wage earnings in the future, which makes it a form of savings/investment. Here time is in effect invested today in order to yield more time in the future. Thus the balance is again current versus future consumption, but of time rather than goods.

Time is increased in effect by increasing the human capital stock so that the effective time, which is the raw time multiplied by the human capital stock, is higher. And the higher the human capital, the higher is the effective wage for the human capital augmented time. Because time is the ultimately scarce resource that cannot be reproduced, the decision on how much human capital to accumulate thereby determines how fast the economy can grow (Chapter 12).

In Part 5, changing the productivity of production alters how the consumer decides across both intratemporal and intertemporal margins. And the consumer response to the changes then gives rise to an explanation of the business cycle (Chapter 13). Allowing the productivity of human capital production to be different across agents, and then allowing for trade between agents, allows for a simultaneous combination of the wage and interest rate bases of labour trade in Chapter 4 and capital trade in Chapter 7. With different wage rates and interest rates under autarky, the trade equilibrium establishes international factor price equalisation, and the trade flows between nations required to establish this, a key theorem of international trade (Chapter 14).

Part 6 allows for the intertemporal margin to be extended to allow for investment across uncertain states (Chapter 15), for investment across time through banking (Chapter 16), and for investment directly into asset markets as a conduit of the private finance of the firm's capital investment (Chapter 17). Part 7 considers the public finance dimensions intertemporally (Chapter 18). This introduces tax distortions to the goods-leisure intratemporal margin, and to the return on intertemporal capital investment (Chapter 19). And

with money comes the inflation tax distortion to the goods–leisure margin that lowers the human capital investment return, and lowers the growth rate of output (Chapter 20).

1.7 Consumption, permanent income and wealth

Understanding the mathematical structure of the text’s macroeconomics requires study of the consumer problem, from its simplest form to a continually extended form. A mathematical overview of consumption and output demand can be provided in order to outline some of the mathematical content. The presentation of certain equations thereby provides an overview that can be returned to as a reference while deriving these progressively throughout the text.

The mathematics is made as simple as is possible. The log-utility function is used throughout the text, on the consumer side. And the standard Cobb-Douglas production function is also used throughout, on the firm side. These standard functions are described in Chapter 2. The optimisation problems are reduced to the task of taking the partial derivative with respect to just one variable in many of the models. This requires use of the chain rule of derivation in order to get the ‘first-order’ equilibrium condition or conditions of the optimisation problem. And the other challenge is solving the full set of equilibrium conditions for the unknown variables of the model.

All of the following equations concerning consumption are derived and worked out at length in the text. The first mathematical current consumption function, derived in Part 2, is that consumption is a fixed fraction of permanent income from wages and profit from the firm that produces goods. Here c^d denotes the consumer’s demand for consumption goods, w the real wage rate, l^s the fraction of time supplied as labour, Π the profit from firm production of goods that goes to the consumer, x the fraction of time spent as leisure, and α a positive parameter indicating degree of preference for leisure. The budget constraint is that consumption purchases are made with labour income wl^s and profit from goods production Π ,

$$c^d = wl^s + \Pi.$$

The exogenous time allocation of T is assumed to be divided between work and leisure, so that $T = l^s + x$ and so consumption is also written as

$$c^d = w(T - x) + \Pi.$$

It will be shown in detail that the marginal rate of substitution between goods and leisure with log utility tells us that

$$x = \frac{\alpha c^d}{w}.$$

And substituting in for x , and solving for c^d , the result is that consumption is a constant fraction, $\frac{1}{1+\alpha}$, of the flow value of time, wT , plus the profit, that is $wT + \Pi$, or

$$c^d = \frac{1}{1+\alpha} (wT + \Pi). \quad (1.1)$$

Since consumption equals total output here, this is also the aggregate demand for output as a function of the real wage.

Alternatively, with permanent income defined as

$$y_P \equiv wT + \Pi,$$

then consumption is the fraction $\frac{1}{1+\alpha}$ of permanent income:

$$c^d = \frac{y^P}{1+\alpha}. \quad (1.2)$$

Inversely, to get an aggregate demand, or AD , curve that can be easily graphed as a downward sloping function of the relative price, the relative price needs to be solved for as a function of the consumption, rather than solving for the consumption as a function of the relative price as in equation (1.1). Solving for the price of goods relative to leisure, which is the goods price of 1, divided by the leisure shadow price of w , then $\frac{1}{w}$ is the relative price and the AD function is also given by

$$\frac{1}{w} = \frac{T_t}{c^d(1+\alpha) - \Pi}.$$

This gives a downsloping demand for goods as a function of the relative price $\frac{1}{w}$. Given the parameters T , α , and given Π (for now, although this is found to be a function of w that needs to be substituted in), then as c^d goes up, the price goes down: a downward sloping demand function.

In Part 3, the two-period model is very useful in establishing the idea of how intertemporal consumption relates to the interest rate and time preference. The static consumption demand of equation (1.1) is modified when the dynamic model is specified. Dynamics means that there is an accumulation of capital over time. As a step towards the dynamic aspect of capital accumulation, Part 3 develops at length how savings of capital and investment in capital are optimally chosen over just two-periods. With log utility, this captures that main intertemporal margin mathematically. With c_0 and c_1 denoting consumption in time period 0 and 1, the growth in consumption over the two-periods depends on whether the interest rate \hat{r} exceeds the consumer's rate of time discount, a positive parameter denoted by ρ :

$$\frac{c_1}{c_0} = \frac{1+\hat{r}}{1+\rho}. \quad (1.3)$$

Note that here in the two-period model, full depreciation of capital is assumed, so that $\frac{c_1}{c_0} = \frac{r}{1+\rho}$, using the standard definition of r . But with $1+\hat{r} \equiv r$, the intuition can be developed that carries over to the full dynamic model (see Appendix A5).

In the dynamic model over the complete time horizon rather than just two-periods, Part 4 shows how the consumption function of Part 2 is modified. This is done by using the additional intertemporal equilibrium concept of Part 3. Consumption depends on wages as before and now, instead of profit, on the interest income from the capital stock at time t , with this stock denoted by k_t . This means that the profit of Part 3, which is similar to capital income in a model in which capital is not yet explicitly introduced, is replaced by the explicit capital income in Part 4. This capital income forms part of the consumer's permanent income, as the interest flow on capital that adds to the wage flow from labour. Also depreciation of capital is introduced, with the depreciation rate denoted by δ_k .

Using time t subscripts, the Part 4 dynamic model is the simplest next step in developing the consumption function. The model is developed to show that the modified consumption demand is

$$c_t^d = \frac{1}{1+\alpha} [w_t T_t + (r - \delta_k) k_t]. \quad (1.4)$$

Optimal dynamic capital accumulation also implies that

$$\frac{c_t}{c_{t-1}} = \frac{1 + r_t - \delta_k}{1 + \rho}, \quad (1.5)$$

which is similar to equation (1.3) except for the time subscripts and the depreciation rate.

The interest flow on capital is then found in the equilibrium with zero growth. Zero growth of the economy implies that $c_t = c_{t-1}$, and so that the interest rate net of depreciation is equal to the rate of time preference: $r_t - \delta_k = \rho$. The consumption demand becomes simply

$$c_t^d = \frac{1}{1 + \alpha} (w_t T_t + \rho k_t). \quad (1.6)$$

Again consumption is a simple fraction, $\frac{1}{1 + \alpha}$, of permanent income, which in turn is the flow value of time $w_t T_t$ and the interest flow from capital, or $w_t T_t + \rho k_t$. The profit in the previous definition of permanent income is made more explicit in this extension with dynamic capital accumulation. Now the interest flow on capital ρk_t replaces profit Π in the permanent income of Part 3, and now this interest flow is added to the flow value of time $w_t T_t$.

The consumer wealth can also be specified. The concept of wealth is that it is the interest flow of permanent income. Given the consumer's permanent income of

$$y_{Pt} = w_t T_t + \rho k_t,$$

wealth W_t can be defined as the present value of the infinite income stream of $w_t + \rho k_t$, discounted with the interest rate $r_t - \delta_k = \rho$. Then wealth is

$$W_t = \frac{w_t T_t + (r - \delta_k) k_t}{r - \delta_k} = \frac{w_t T_t}{\rho} + k_t.$$

This means that wealth is the sum of both the discounted flow value of time, or 'human capital', and the physical capital.

To get the aggregate demand AD for the output y_t , with capital accumulation and zero growth, Part 4 shows that the investment demand is added to the consumption demand of equation (1.6). With zero growth, investment need only cover the depreciated capital, and so it equals $\delta_k k_t$. The AD function adds this $\delta_k k_t$ investment to the consumer demand for goods, to get that

$$y_t^d = c_t^d + \delta_k k_t = \frac{1}{1 + \alpha} (w_t T_t + [\rho + \delta_k (1 + \alpha)] k_t). \quad (1.7)$$

Output is a fraction of permanent income, made up of the time value flow $w_t T_t$ and the capital income flow plus capital depreciation of $[\rho + \delta_k (1 + \alpha)] k_t$. This can be written inversely by solving for the relative price of goods, $\frac{1}{w}$, so as to graph the AD function in price quantity dimensions, as

$$\frac{1}{w_t} = \frac{T_t}{y_t^d (1 + \alpha) - [\rho + \delta_k (1 + \alpha)] k_t}.$$

This gives a downward sloping aggregate demand function.

When there is a positive but exogenous growth rate of g in the equilibrium, the aggregate demand is modified accordingly. Part 5 develops the growth model and shows that greater growth changes the capital income to $k_t (r_t - \delta_k - g)$, instead of $k_t (r_t - \delta_k)$ when there is zero growth. With this change, and using equation (1.5) to define the growth rate as the consumption growth rate,

$$1 + g_t \equiv \frac{c_t}{c_{t-1}} = \frac{1 + r_t - \delta_k}{1 + \rho},$$

this implies that $r_t - \delta_k - g$ can be rewritten as $\rho(1+g)$. This makes the modified consumption function a simple fraction of permanent income, with this income now being $w_t + \rho(1+g)k_t$:

$$c_t^d = \frac{1}{1+\alpha} [w_t + \rho(1+g)k_t]. \quad (1.8)$$

With a growth rate of g , the permanent income is

$$y_{Pt} = w_t T_t + \rho(1+g)k_t,$$

while wealth is again given by the discounted infinite stream of the permanent income. With the discount rate $r_t - \delta_k - g$,

$$W_t = \frac{w_t T_t + (r_t - \delta_k - g)k_t}{r_t - \delta_k - g} = \frac{w_t T_t}{\rho(1+g)} + k_t.$$

Moving back to the aggregate demand for goods, investment is higher than just the depreciated capital when there is positive growth. The investment is shown to become $\delta_k(1+g)k_t$, so that adding this to the consumer's goods demand gives an aggregate demand AD of

$$y_t^d = \frac{1}{1+\alpha} (w_t T_t + k_t [\rho(1+g) + (\delta_k + g)(1+\alpha)]). \quad (1.9)$$

Inversely in terms of the relative price $\frac{1}{w}$, this AD is

$$\frac{1}{w_t} = \frac{T_t}{y_t^d (1+\alpha) - k_t [\rho(1+g) + (\delta_k + g)(1+\alpha)]}.$$

Part 5 goes the next step by making the growth rate endogenous. To do this, time is invested in human capital, and the growth rate is higher the higher is this human capital investment. But investing time in this way reduces time available for work. This makes the current wage flow lower, yet it ultimately makes the human capital stock and wage flow higher.

Let the time invested in human capital at time t be denoted by l_{Ht} . Then Part 5 develops the consumption function and shows it is given by

$$c_t^d = \frac{1}{1+\alpha} [w_t (1 - l_{Ht}) h_t + \rho(1+g)k_t]. \quad (1.10)$$

The time in human capital is solved within the model and the result is that it depends on the productivity of human capital investment, denoted by A_H , and the depreciation rate of human capital, denoted by δ_h , along with the growth rate g . Using the actual solution that $l_{Ht} = \frac{g+\delta_h}{A_H}$, the consumption function becomes

$$c_t^d = \frac{1}{1+\alpha} \left[w_t \left(1 - \frac{g+\delta_h}{A_H} \right) h_t + \rho(1+g)k_t \right]. \quad (1.11)$$

The consumption demand can also be written by defining the sum of the labour and leisure time as T_t , as in Parts 2, 3 and 4. Then $1 - l_{Ht} = T_t$, and consumption is

$$c_t^d = \frac{1}{1+\alpha} [w_t T_t h_t + \rho(1+g)k_t], \quad (1.12)$$

as in Part 4.

The permanent income stream is again

$$y_{Pt} = w_t T_t h_t + \rho(1+g)k_t,$$

and with $r_t - \delta_k - g = \rho(1 + g)$, wealth is again

$$W_t = \frac{w_t T_t h_t + (r_t - \delta_k - g) k_t}{r_t - \delta_k - g} = \frac{w_t T_t h_t}{\rho(1 + g)} + k_t.$$

Using that $T_t = 1 - \frac{g + \delta_h}{A_H}$, aggregate output AD becomes

$$y_t^d = \frac{1}{1 + \alpha} \left[w_t \left(1 - \frac{g + \delta_h}{A_H} \right) h_t + k_t [\rho(1 + g) + (\delta_k + g)(1 + \alpha)] \right],$$

or inversely,

$$\frac{1}{w_t} = \frac{\left(1 - \frac{g + \delta_h}{A_H} \right) h_t}{y_t^d (1 + \alpha) - k_t [\rho(1 + g) + (g + \delta_k)(1 + \alpha)]}. \quad (1.13)$$

Normalising $h_t = 1$, and using $1 - \frac{g + \delta_h}{A_H} = T_t$, this AD function is the same as in Part 4:

$$\frac{1}{w_t} = \frac{T_t}{y_t^d (1 + \alpha) - k_t [\rho(1 + g) + (g + \delta_k)(1 + \alpha)]}. \quad (1.14)$$

Endogenising the growth rate through human capital accumulation helps solve certain problems in the exogenous growth model and provides a better explanation of a variety of long term and even business cycle issues. This does complicate the model, and so it is focused upon primarily in Part 5. Endogenous growth appears again in Part 7 in order to show the effect of taxes on growth, which requires an endogenous growth model.

Part 6 of the text has the same consumption function as the baseline dynamic model of Part 4 with exogenous growth, but now the capital financing is done in a variety of ways. Capital savings from the consumer is intermediated through the banking sector to the firm for investment. With a banking crisis that lowers the productivity of the banking sector, the cost of capital rises and the capital stock falls. And this lowers the k_t that enters the standard AD function of equation (1.6).

Part 6 also shows how capital is raised by the firm through asset markets. And capital is raised by the government through taxation in Part 7. The basic AD is affected only if there are costs of raising the capital, either through asset markets or through the government. Taxes on labour, goods, capital and on money use all cause the basic AD function to be affected by changes in the after tax real wage and real interest rate.

1.8 A methodological outline

Starting with a representative agent framework, there is a certain methodology available to build the fundamentals first for the closed and then for the open economy. This is done by having a single agent for the closed economy and having multiple agents for the open economy who differ in some fashion. The difference in agents gives rise to the name of 'heterogenous' agents, meaning that they are not homogenous with all of the same features but rather differ in some way. The text assumes a simple case of the heterogenous agent economy: only two different types of agents exist. These two-agents then differ in only one way: different marginal productivities. In Chapter 4, the type A and B agents have different marginal productivities of labour. In Chapter 7, the type A and B agents have a different marginal productivity of capital. In Chapter 14, the two-agents, 1 and 2, have different human capital productivities, which result in different equilibrium marginal products of both labour and capital when they exist in isolation without trade. Trade makes these marginal products the same in both countries.

1.8.1 Closed economy methodology

The closed economy is derived under two equivalent views of the representative agent. There are no externalities introduced in any part of the text. Therefore there is what can be called a 'centralised' version of the economy that is equivalent to what is called the 'decentralised' version of the economy. The difference is that prices and profit, and supply and demand functions within markets, are explicit only once the economy is decentralised. Then relative prices reflect all conditions that affect the equilibrium when there are no externalities and so the centralised and decentralised economies are equivalent.

If there are factors external to the markets, then the relative prices do not reflect all of these factors and so the centralised and decentralised economies would be not the same. With externalities, a set of relative prices cannot 'support' the same equilibrium found in the centralised economy. This complication gives rise to the concept of a 'central planner' that makes allocations in the centralised economy. But all of these complications are avoided in the text by not introducing any externalities, and there are no central planners that might internalise any externalities.

Instead the text allows the representative agent to act jointly as consumer/producer, in the centralised economy, or to be split into being each a separate consumer and a separate firm, in the decentralised economy, with the relative prices separating the consumer from the firm.

Centralised economy

The agent that acts jointly as consumer and firm simply maximises utility subject to production technology, like Robinson Crusoe living alone on an island. Relative prices are not needed to find the general equilibrium. The aggregate demand and supply are equal at the equilibrium point. The relative prices and profit are implicit and not explicitly solved. And this lack of explicit prices does not allow for supply and demand functions to be derived. But this provides a simple and valid characterisation of the equilibrium. It is analogous to a closed world economy with the agent as the average agent.

Decentralised economy

Alternatively the agent acts in separate problems each as consumer and as firm. This puts a market line, also called either a budget line or a budget constraint, into the problem. This enables the derivation of aggregate demand and supply functions, and for the solutions of the equilibrium relative prices and profits. The consumer maximises utility subject to a budget constraint, and the firm maximises profit subject to production technology. The trick here is that the consumer budget constraint is the same equation in equilibrium as the firm profit function.

Acting each as consumer and as firm/producer, the agent has separately identifiable supply and demand functions for outputs and for factor inputs. This is still a closed economy, with a particular known equilibrium set of quantities demanded and supplied. But now rather than just the quantities of supply and demand being known, the functions of supply and demand, which say how quantities depend on the relative prices, are known. This allows for examination of how the equilibrium changes when prices change. And this decentralisation is useful for tax analysis and for the open economy model.

Mathematically, the equilibrium price is determined from the market clearing condition. For example, for the consumption good as a function of the price, $c\left(\frac{1}{w}\right)$, setting the demand function for consumption, $c^d\left(\frac{1}{w}\right)$, equal to the supply function of consumption, $c^s\left(\frac{1}{w}\right)$, allows for determination of the equilibrium price $\frac{1}{w}$:

$$c^d\left(\frac{1}{w}\right) = c^s\left(\frac{1}{w}\right).$$

1.8.2 Open economy methodology

By adding just one more type of agent, the model can represent the open general equilibrium economy with trade. With two representative agents that differ only in productive capacity, each of the agents act separately as both consumer and firm. Different supply and demand functions can then be derived for each agent, and trade patterns established as functions of relative prices. Now the consumption demand is the sum of both agents A and B , as is consumption supply:

$$\begin{aligned}c^d &= c_A^d + c_B^d; \\c^s &= c_A^s + c_B^s.\end{aligned}$$

And the equilibrium relative price is determined by setting the total supply equal to the total demand of both agents. This gives the expanded market clear condition:

$$c^d = c_A^d + c_B^d = c_A^s + c_B^s = c^s.$$

Different productivities induce comparative advantage and open up gains from trade. Trade ‘triangles’ result when moving from autarky (which means no trade) to trade. This approach assimilates part of the microeconomic ‘pure theory of international trade’. Different cultures with different technologies can trade with each other.

The representative agent approach to the open economy, with two types of agents, is easily extended to any number of agents. Say there are 1000 agents in all, 400 of A and 600 of B type. The first-order equilibrium conditions for each type of agent are completely unaffected by the number of each type of agent. Only when the market clearing condition is applied to find the equilibrium price does the number of agents affect the computation of the equilibrium solution. In particular, now to get total consumption goods demand equal to consumption goods supply, and so to find the equilibrium relative price, the market clearing condition is instead

$$c^d = 400c_A^d + 600c_B^d = 400c_A^s + 600c_B^s = c^s.$$

And with consumption a function of $\frac{1}{w}$, this altered market clearing condition determines the new equilibrium $\frac{1}{w}$.

A two-agent general equilibrium, with the agents differing only in productivity, is presented in Chapters 4, 7 and 14. These chapters cover the labour-only dimension, the capital-only dimension, and the fully dynamic model with both labour and capital. Factor price equalisation occurs in each chapter. The wage equalises in Chapter 4, the interest rate equalises in Chapter 7 and both equalise in Chapter 14. However Chapter 14 requires endogenous growth so that the interest rate is endogenous and both interest rates and wage rates can as a result be equalised. And while Chapters 4 and 7 assume the agents differ only in their goods productivities, in Chapter 14 the agents differ only in their human capital investment productivities.

1.9 Suggested ways to use the text

The book can be read by students focusing on the intuition of the margins, learning to derive the margins presented, and working through the examples that are presented. It allows conceptual thinking about how each margin is first formulated and then how a margin is being affected by each extension.

Familiarity of the margins of the baseline recursive model then allows for the extensions to be made as just one more step beyond well-known compounds. Parts 2, 3 and 4 are designed to be a one semester course in modern macroeconomics. The elements of

modern macroeconomics are derived along the two static margins in Parts 2 and 3 and then brought together simultaneously in the baseline dynamic model in Part 4.

The Part 5–7 extensions of the model beyond the baseline formulation make up what is designed to be the second semester course in undergraduate modern macroeconomics. These extensions are along four dimensions: Part 5, growth and business cycles; Part 6, risk, banking and asset prices; Part 7 fiscal and monetary policy.

1.9.1 Possible course usage of text

The text is intended to bring to undergraduates the main elements of macroeconomics used in research. This involves additional mathematical rigour. And yet, while the mathematics is involved, it uses only derivatives and solving systems of equations. The solution of a quadratic equation is also used at times, although this is not essential as the solution is also shown graphically. And the text allows a graphical representation to be emphasised over the mathematics if that is desired. All of the graphs show the exact mathematical form of the equations of the particular example, and so the graphs are exact representations of the economy.

Undergraduates can be taught the text as a single intermediate macroeconomics one-semester course, as a two-semester course in intermediate macroeconomics, or as an advanced macroeconomics one-semester course. For example, for a full two semesters in undergraduate macroeconomics, the book's 20 chapters can be split in half. Masters level courses are typically as in undergraduate intermediate courses or as in advanced undergraduate courses, and so a Masters level course could select across the entire book. More advanced graduate courses could use the book as an introduction.

1.9.2 A summary

1. Two semester undergraduate intermediate Macroeconomics sequence:
 - (a) Semester one, first 10 chapters, selectively.
 - (b) Semester two, second 10 chapters, selectively.
2. One semester undergraduate intermediate Macroeconomics course:
 - (a) Emphasise the first half of the book, in Parts 1–4.
 - (b) Select across the first half of the book, in Parts 1–4 and add in other sections such as growth theory and banking in Part 5, and fiscal and monetary policy in Part 6.
 - (c) Emphasise only the static analysis of Parts 1–3, if you do not want to do the full dynamic analysis.
3. Undergraduate Advanced Macroeconomics Course: one semester, selected chapters across all seven Parts of the book.
4. Masters Level Macroeconomics Course: one semester, the entire book, selectively.

1.10 Questions

1. The microfoundations approach to macroeconomics stresses
 - (a) the representative agent analysis;
 - (b) relative price analysis;
 - (c) $AD-AS$ analysis;
 - (d) more than one of the above.
2. From the general equilibrium economy, where there are two outputs, x and y , and a representative consumer/producer, we can derive
 - (a) the amount of goods 'traded' (the difference between consumption and production, if any);
 - (b) the quantity supplied of x ;

- (c) the demand schedule for y ;
 - (d) all of the above.
3. If the representative agent consumes goods c , according to the consumption function $c = \left(\frac{1}{1+\alpha}\right) y_p$, where y_p is permanent income, and α a constant parameter, then
- (a) an increase in permanent income raises consumption;
 - (b) an increase in the interest rate raises wealth;
 - (c) an increase in the real wage rate lowers consumption;
 - (d) all of the above.
4. Suppose the consumption function is

$$c = \left(\frac{1}{1+\alpha}\right) y_p$$

where y_p is permanent income and α a constant parameter. Focusing on substitution and income effects, indicate how to use this function to explain each of the following:

- (a) business cycles;
 - (b) economic growth;
 - (c) old age pension insurance;
 - (d) the distribution of income.
5. What are the main margins of economic analysis that make up the microfoundations approach to macroeconomic analysis?
6. What role does the capital stock play in forming the aggregate demand and supply analysis in the dynamic model?
7. Define permanent income and explain how it affects the consumer's demand for goods.
8. Explain how the consumer's demand for goods, or consumption function, changes as the economy goes from a static one to a dynamic one.
9. Explain how aggregate demand is formed from the consumer's demand for goods, in terms of capital investment?
10. What role can comparative statics play in explaining business cycles and severe recessions?
11. What economic facts can be explained by the inclusion of human capital?

1.11 References

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