Particle Physics of the early Universe

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Why do you need that?

Things I consider known

Beginning of the XXth century: attempts to explain atomic and nuclear physics, taking into account Quantum Mechanics and Relativity.

- The state of the system is described by the **complex wave function** $\psi(x)$
- $|\psi(x)|^2$ is interpreted as probability density and $\int d^3x|\psi(x)|^2 = \text{const}$ as the full probability
- $\vec{J} = \frac{i\hbar}{2m}(\psi^*\nabla\psi - \psi\nabla\psi^*)$ is interpreted as a **probability density current** so that

$$\frac{\partial|\psi|^2}{\partial t} + \text{div}\,\vec{J} = 0 \quad (1)$$

- The quantum system is described by Schrödinger equation:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \hat{\mathcal{H}}\psi(x,t) \quad (2)$$

where the operator $\hat{\mathcal{H}}$ is called **Hamiltonian**

- The operator Hamiltonian is built based on **correspondence principle**: **if** a classical system is described by the Hamiltonian $H(x,p)$ then the quantum system is described by $H(x,-i\hbar\partial_x)$. For example for a particle of mass m in external field we get

$$\text{Classical } H(x,p) = \frac{p^2}{2m} + V(x) \Rightarrow \text{Quantum } \hat{\mathcal{H}} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \quad (3)$$

- Uncertainty principle $\Delta x \Delta p \sim \hbar$

- Solution of the Schödinger equation for the hydrogen atom was predicting the energy spectrum

$$E_n = -\frac{m_e e^4}{2\hbar^2 n^2}, \quad n = 1, 2, \dots \quad (4)$$

for each n the degeneracy (the number of states) of the energy level is $\sum_{l=0}^{n-1}(2l+1) = n^2$.

- In non-relativistic quantum mechanics if Hamiltonian has the form $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}$ then the probability of transition between an initial state $\psi_i(x)$ and the final state $\psi_f(x)$ of **unperturbed** Hamiltonian $\hat{\mathcal{H}}_0$ is given by (Landau & Lifshitz, vol. 3, § 43):

$$dw_{if} = \frac{2\pi}{\hbar}|V_{if}|^2\delta(E_i - E_f)dn_f \tag{5}$$

  where $|V_{if}|$ is the matrix element between initial and final states; and dn_f is the number of final states with the energy E_f (degeneracy of the energy level).

- If the transition $i \to f$ is forbidden, the analog of Eq. (5) reads (c.f. Landau & Lifshitz, vol. 3, § 43):

$$dw_{if} = \frac{2\pi}{\hbar}\left|\int dn\,\frac{V_{in}V_{nf}}{E_i - E_n}\right|^2\delta(E_i - E_f)dn_f \tag{6}$$

where $\int dn$ is the integral over any basis of **intermediate states**.

Pauli Hamiltonian

In order to explain the emission spectra of elements Pauli introduced a spin (*"a two-valued quantum degree of freedom"*). Its interpretation was not known. Pauli postulated **Pauli (or Pauli-Schrödinger) equation**

$$\hat{\mathcal{H}}_{\text{Pauli}} = \frac{1}{2m}\Big[\vec{\sigma}\cdot(-i\hbar\vec{\nabla} - e\vec{A})\Big]^2 + eA_0\mathbb{1} \tag{7}$$

where **Pauli matrices** $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are defined as

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8}$$

and $\mathbb{1}$ is 2×2 identity matrix. Pauli Hamiltonian acts on the **2-component wave-function**

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \hat{\mathcal{H}}_{\text{Pauli}}\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} \tag{9}$$

Electromagnetic field as collection of oscillators [1]

- Consider the solution of wave equation for vector potential $\vec{A}$ (impose $A_0 = 0$ and $\mathrm{div}\,\vec{A} = 0$)

$$\frac{1}{c^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - \Delta \boldsymbol{A} = 0 \tag{10}$$

- Solution

$$\boldsymbol{A}(x,t) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\boldsymbol{a}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \boldsymbol{a}^*_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right] \tag{11}$$

where the complex functions $\boldsymbol{a}_{\boldsymbol{k}}(t)$ have the following time dependence

$$\boldsymbol{a}_{\boldsymbol{k}}(t) = \boldsymbol{a}_{\boldsymbol{k}}e^{-i\omega_{\boldsymbol{k}}t}, \qquad \omega_{\boldsymbol{k}} = |\boldsymbol{k}| \tag{12}$$

[0]See Landau & Lifshitz, Vol. 4, §2

Generalized coordinate and momentum

- $\boldsymbol{a_k}(t)$ and $\boldsymbol{a^*_k}(t)$ obey the following equations:

$$\dot{\boldsymbol{a}}_{\boldsymbol{k}}(t) = -i\omega_{\boldsymbol{k}}\boldsymbol{a}_{\boldsymbol{k}}(t) \quad , \quad \dot{\boldsymbol{a}}^*_{\boldsymbol{k}}(t) = i\omega_{\boldsymbol{k}}\boldsymbol{a}^*_{\boldsymbol{k}}(t) \tag{13}$$

- Notice that we re-wrote partial differential equation (11) second order in time into a set of ordinary differential equations for infinite set of functions $\boldsymbol{a_k}(t)$ and $\boldsymbol{a^*_k}(t)$. **Any solution of the free Maxwell's equation** is parametrized by the (infinite) set of complex number $\{\boldsymbol{a_k}, \boldsymbol{a^*_k}\}$

- To make the meaning of Eqs. (16) clear, let us introduce their imaginary and real parts.

$$\left.\begin{aligned} \boldsymbol{Q_k} &\equiv \boldsymbol{a_k} + \boldsymbol{a^*_k} \\ \boldsymbol{P_k} &\equiv -i\omega_{\boldsymbol{k}}\big(\boldsymbol{a_k} - \boldsymbol{a^*_k}\big) \end{aligned}\right\} \overset{\text{dynamics}}{\Longrightarrow} \left\{\begin{aligned} \dot{\boldsymbol{Q}}_{\boldsymbol{k}} &= \boldsymbol{P_k} \\ \dot{\boldsymbol{P}}_{\boldsymbol{k}} &= -\omega^2_{\boldsymbol{k}}\boldsymbol{Q_k} \end{aligned}\right. \tag{14}$$

Hamiltonian of electromagnetic field

- Hamiltonian (total energy) of electromagnetic field is given by

$$\mathcal{H} = \frac{1}{2}\int d^3\boldsymbol{x}\left[\boldsymbol{E}^2 + \boldsymbol{B}^2\right] = \frac{1}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\boldsymbol{E}_{\boldsymbol{k}}^2 + \boldsymbol{B}_{\boldsymbol{k}}^2\right] \qquad (15)$$

- Using mode expansion (11) and definition (14) we can write with frequencies $\omega_{\boldsymbol{k}}$

$$\mathcal{H}[\boldsymbol{Q}_{\boldsymbol{k}}, \boldsymbol{P}_{\boldsymbol{k}}] = \frac{1}{2}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\boldsymbol{P}_{\boldsymbol{k}}{}^2 + \omega_{\boldsymbol{k}}^2 \boldsymbol{Q}_{\boldsymbol{k}}{}^2\right] \qquad (16)$$

- Therefore dynamical equations (14) are nothing by the Hamiltonian equations

$$\dot{\boldsymbol{Q}}_{\boldsymbol{k}} = \frac{\partial \mathcal{H}}{\partial \boldsymbol{P}_{\boldsymbol{k}}} \quad , \quad \dot{\boldsymbol{P}}_{\boldsymbol{k}} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{Q}_{\boldsymbol{k}}} \qquad (17)$$

with Hamiltonian (16)

Hamiltonian of electromagnetic field

- Eqs. (16)–(17) describe Hamiltonian dynamics of a sum of independent oscillators with frequencies $\omega_{\boldsymbol{k}}$

Classical electromagnetic field can be considered as an infinite sum of oscillators with frequencies $\omega_{\boldsymbol{k}}$

- **Recall:** for quantum mechanical oscillator, described by the Hamiltonian

$$\hat{\mathcal{H}}_{\text{osc}} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2}{2}x^2 \qquad (18)$$

one can introduce **creation and annihilation operators**:

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + \hbar\partial_x) \quad ; \quad \hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x - \hbar\partial_x) \qquad (19)$$

- Commutation $[\hat{a}, \hat{a}^\dagger] = 1$

- Hamiltonian can be rewritten as $\hat{\mathcal{H}}_{\text{osc}} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$

Properties of creation/annihilation operators

- Commutation $[\hat{a}, \hat{a}^\dagger] = 1$

- If one defines a **vacuum** $|0\rangle$, such that $\hat{a}\,|0\rangle = 0$ (**Fock vacuum**) then a state $|n\rangle \equiv (\hat{a}^\dagger)^n\,|0\rangle$ is the eigenstate of the Hamiltonian (18) with $E_n = \hbar\omega(n + \frac{1}{2})$, $n = 0, 1, \ldots$

- Given $|n\rangle$, $n > 0$, $\hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$ and $\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle$

- Time evolution of the operators $\hat{a}, \hat{a}^\dagger$:

$$i\hbar\frac{\partial\hat{a}}{\partial t} = [\mathcal{H}_{\text{osc}}, \hat{a}] \tag{20}$$

and Hermitian conjugated for $\hat{a}^\dagger$

Birth of quantum field theory

- Dirac (1927) proposes to treat radiation as a collection of **quantum** oscillators

Paul A.M. Dirac ***Quantum theory of emission and absorption of radiation***

Proc.Roy.Soc.Lond. A114 (1927) 243

- Take the classical solution (11)

$$\boldsymbol{A}(x,t) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[\boldsymbol{a}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \boldsymbol{a}^*_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right]$$

- Introduce creation/annihilation operators $\hat{a}_{\boldsymbol{k}}$, $\hat{a}^\dagger_{\boldsymbol{k}}$

$$[\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{p}}] = \hbar\delta_{\boldsymbol{k},\boldsymbol{p}} \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{p}}] = 0 \qquad (21)$$

Birth of quantum field theory

- Replace Eq. (11) with a quantum operator

$$\hat{\boldsymbol{A}}(x,t) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left[\hat{\boldsymbol{a}}_{\boldsymbol{k}}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{\boldsymbol{a}}^{\dagger}_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right] \quad (22)$$

- Operator $\boxed{\hat{\boldsymbol{a}}^{\dagger}_{\boldsymbol{k}}}$ **creates photon with momentum k and frequency $\omega_{\boldsymbol{k}}$**

- Operator $\boxed{\hat{\boldsymbol{a}}_{\boldsymbol{k}}}$ **destroys photon with momentum k and frequency $\omega_{\boldsymbol{k}}$** (if exists in the initial state)

- State without photons $\leftrightarrow$ Fock vacuum:

$$\hat{\boldsymbol{a}}_{\boldsymbol{k}}\left|0\right\rangle = 0 \quad \forall \boldsymbol{k} \quad (23)$$

Birth of quantum field theory

- State with N photons with momenta $\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_N$:

$$|\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_N\rangle = \hat{\boldsymbol{a}}^\dagger_{\boldsymbol{k}_1} \hat{\boldsymbol{a}}^\dagger_{\boldsymbol{k}_2} \ldots \hat{\boldsymbol{a}}^\dagger_{\boldsymbol{k}_N} |0\rangle \tag{24}$$

Quantum electrodynamics (QED) – first quantum field theory has been created. Free fields with interaction treated **perturbatively** in *fine-structure constant:* $\alpha = \frac{e^2}{\hbar c}$

Quantum mechanics and relativity

Attempts to blend quantum mechanics and special relativity led to the problem of infinities.[2]

- Electromagnetic energy of uniformly charged ball of radius r_e should be smaller than the total mass of electron ($m_e c^2$)?

$$m_e c^2 > \frac{e^2}{r_e} \Longrightarrow r_e > \frac{e^2}{mc^2} = 3 \times 10^{-13}\,\mathrm{cm} \qquad (25)$$

  – larger than the size of atomic nucleus (Hydrogen $r_H \sim 1.75 \times 10^{-13}\,\mathrm{m}$)

- Pauli's idea of spin meant that electron possesses magnetic moment: $\mu_e = \frac{e\hbar}{2m_e c}$. The energy of magnetic sphere with radius r_e would be $\frac{\mu_e^2}{r_e^3}$ which would exceed $m_e c^2$ for $r \sim \frac{1}{m_e}\left(\frac{e\hbar}{c^4}\right)^{2/3}$ (even larger distances than (25)!

[2]See an interesting exposition of the historical perspectives at http://people.bu.edu/gorelik/cGh_Bronstein_UFN-200510_Engl.htm, **Sec. 4**

Relativistic Schödinger equation [3]

- Free Schrödinger equation is **not Lorentz invariant** (indeed, it is obtained from $E = \frac{p^2}{2m}$ dispersion relation) $\Rightarrow$Replace r.h.s. by relativistic dispersion relation $E = \sqrt{p^2c^2 + m^2c^4}$

Relativistic Schrödinger equation ? $$i\hbar\frac{\partial\psi}{\partial t} = \boxed{\sqrt{-\hbar^2\vec{\nabla}^2 + m^2c^4}}\,\psi \quad (26)$$

- How to make sense of square root? Try to take square of Eq. (26)?

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = \left(-\hbar^2c^2\nabla^2 + m^2c^4\right)\psi \Leftrightarrow \left(\Box + \left(\frac{mc}{\hbar}\right)^2\right)\psi = 0 \quad (27)$$

This is **Klein-Gordon equation**.

- However, the Klein-Gordon equation does not admit probabilistic interpretation of ψ

[2]The presentation of this topic follows Bjorken & Drell, Chap. 1, Sec. 1.1–1.3

Dirac equation [4]

- Dirac (1928) had constructed linear in time evolution. Indeed, **if** $i\hbar\partial_t\psi = H\psi$ **then** the probability density is conserved **for any Hermitian Hamiltonian**

- Dirac proposed the form

$$i\hbar\frac{\partial\psi}{\partial t} = \underbrace{\left[-i\hbar c\left(\alpha_x\frac{\partial}{\partial x} + \alpha_y\frac{\partial}{\partial y} + \alpha_z\frac{\partial}{\partial z}\right) + \beta mc^2\right]}_{\text{Dirac Hamiltonian}}\psi \qquad (28)$$

where

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}; \qquad \beta = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \qquad (29)$$

- The Dirac equation

$$\left(i\frac{\partial}{\partial x^\mu}\left(\gamma^\mu\right)^\alpha_\beta - m\,\delta^\alpha_\beta\right)\psi^\beta = 0 \qquad (30)$$

[3] The presentation of this topic similar to that of Bjorken & Drell, Chap. 1, Sec. 1.1–1.3

Dirac equation [5]

involves 4 Dirac matrices γ^μ (index $\mu = 0, 1, 2, 3$), each matrix having size 4×4 (indices α, β run over their dimensions)

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}; \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{31}$$

Interaction with electromagnetic field [6]

- In the same paper [Proc. R. Soc. Lond. (1928) 610, 24] Dirac introduced coupling of spinors to electromagnetic field

- Recall in non-relativistic quantum mechanics coupling to the electromagnetic field was via "minimal coupling" (i.e. the momentum $\hat{p}_\mu \to \hat{P}_\mu = (\hat{p}_\mu - eA_\mu)$, P_μ is sometimes called "generalized momentum")

$$\frac{\hat{p}^2}{2m} \to \frac{1}{2m}\left(\hat{p} - e\vec{A}\right)^2 + eA_0(x) \tag{32}$$

- In Dirac equation, we make similar substitution

$$(\hat{p}_\mu\gamma^\mu - eA_\mu\gamma^\mu - m)\psi = 0 \tag{33}$$

- **Problem:** check that the Dirac equation (33) is invariant under the gauge transformation $A_\mu \to A_\mu - \partial_\mu f$, $\psi \to \psi e^{ief}$.

[5] Bjorken-Drell, Chap. 1, Sec. 1.4

Probability and current density

- Dirac has constructed an equation of the form $\boxed{(i\partial_t - H)\psi = 0}$
- Therefore the quantity

$$\int d^3x\,\psi^+\psi \tag{34}$$

  is conserved

- The quantity

$$\psi^\dagger\psi = |\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2 + |\psi_4|^2 \geq 0 \tag{35}$$

  can be interpreted as the **probability density**. Contrary to the Klein-Gordon case, it is non-negative by construction.

- The density $\psi^\dagger\psi$ is a part of the current vector

$$j^\mu = \psi^\dagger\gamma^0\gamma^\mu\psi \tag{36}$$

Probability and current density

- As a consequence of the Dirac equation, this current is conserved[7]

$$\partial_\mu j^\mu = 0 \tag{37}$$

therefore the spatial part of j^μ has the meaning of the current density.

[7] Check this

Dirac sea[9]

- As for any relativistic equation, Dirac equation has two branches of dispersion relation

$$E = \pm\sqrt{\boldsymbol{p}^2 + m^2} \tag{38}$$

- Now consider states with negative energy . The system is unstable, since it will try to choose the state with the lowest possible energy, but there is no natural lower bound on the value of the negative energy.

- The interpretation of Dirac: Since fermions obey the Pauli principle, the occupation number of each energy level cannot exceed 1[8]

- Assume that we start from the many-fermion system, where all the energy states with $E < 0$ are fully occupied (the Dirac sea).

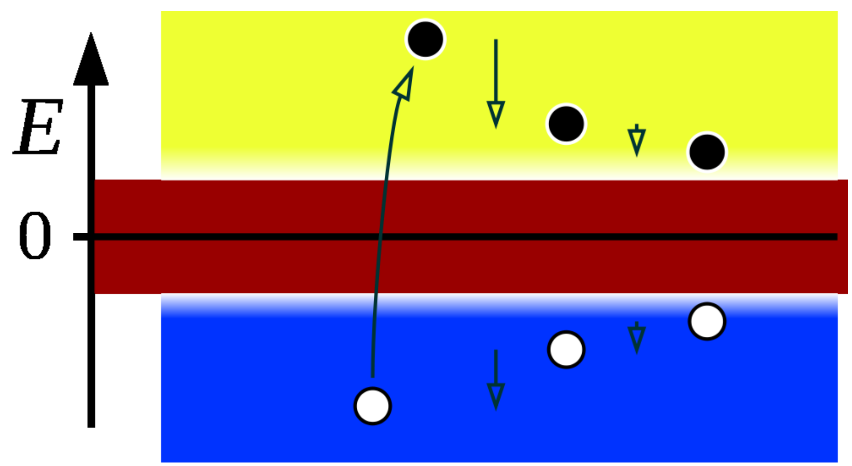
[8] Actually it cannot exceed 2, when we take into account two possible spin states

Dirac sea

- This fully-occupied state was interpreted as the vacuum
- Such vacuum is stable
- When we add an electron to the vacuum, it increases overall energy by $E > 0$. The dynamics of this additional particle is described by the positive-energy solutions of the Dirac equation.
- We may also **remove** one particle from the vacuum. The resulting state is called “hole”, and behaves like the absence of electron with $E = -\sqrt{\boldsymbol{p}^2 + m^2}$.

Therefore, the hole has[10]

- **positive** charge $e_{\text{hole}} = +|e|$
- **positive** energy $E_{hole} = +\sqrt{\boldsymbol{p}^2 + m^2}$
- opposite momentum $\boldsymbol{p}_{\text{hole}} = -\boldsymbol{p}$



[10] The analogy is an air bubble in water, compared to the drop of water in air: effectively, the bubble behaves like a drop with negative density.

Holes

The removal of fermion with negative energy **increases** the energy of the system. The vacuum has the lowest energy

Prediction of antiparticles

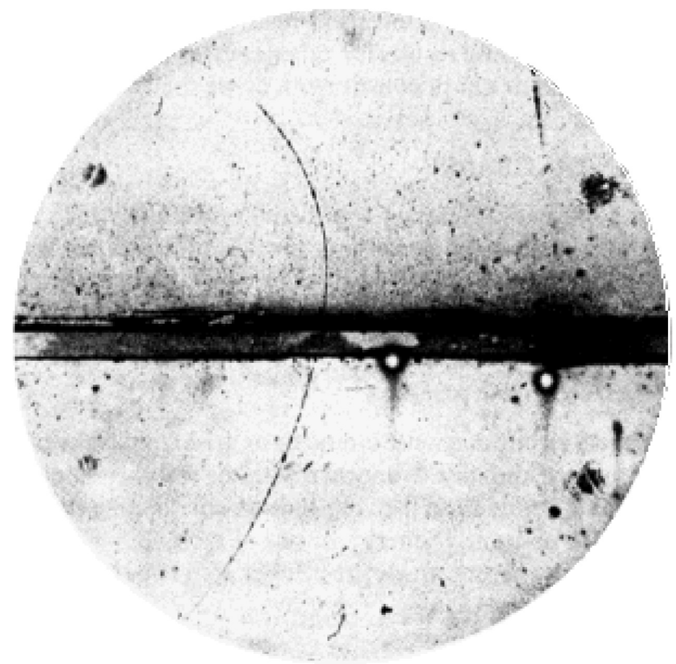
- The hole state is called the **antiparticle**
- For electron, there should exist a positively-charged antiparticle – **positron**. It was first predicted by Dirac.
- In 1932, positron was discovered by Anderson in cosmic rays
  Phys. Rev. 43, 491494 (1933) "*The positive electron*"
  `http://link.aps.org/doi/10.1103/PhysRev.43.491`

Prediction of antiparticles

From **Phys. Rev. 43, 491494 (1933) "*The positive electron*"**

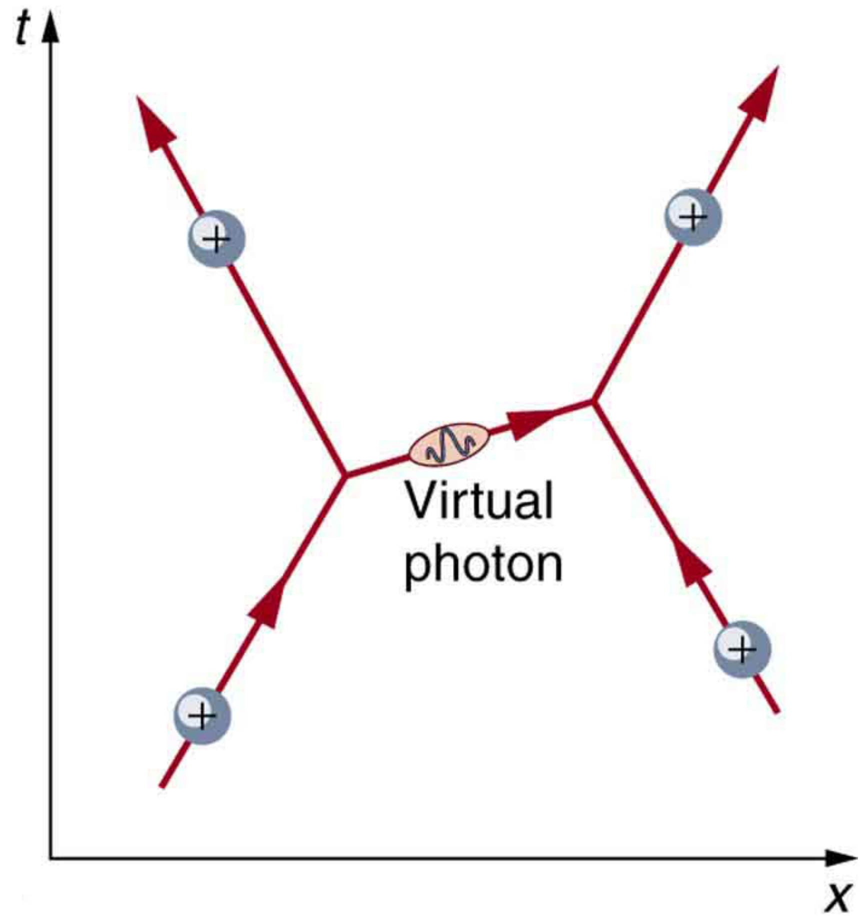
http://link.aps.org/doi/10.1103/PhysRev.43.491

Abstract. Out of a group of 1300 photographs of cosmic-ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than twice, and is probably exactly equal to, that of the proton. If these particles carry unit positive charge the curvatures and ionizations produced require the mass to be less than twenty times the electron mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.



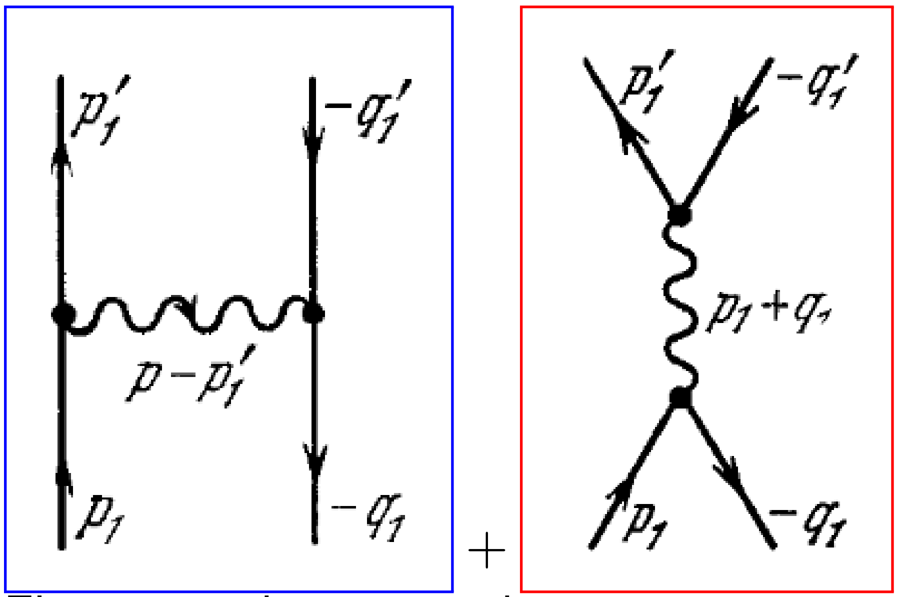
Interaction of light with the Dirac sea

Interaction of charged particles goes via exchange of virtual photon quanta



Electron-positron interaction

- Consider now electron-positron scattering.[11]



Electron-positron scattering

- Recall, that the processes the process $e^- + e^+ \to \gamma$ and $e^- \to e^- + \gamma$ are forbidden for real particles if both energy **and** momentum are conserved.

[11] See Bjoren & Drell, Sec. 7.9

Second order perturbation theory

- The Hamiltonian $\mathcal{H}_0$ is perturbed by $\hat{V}$ given by
$$\hat{V} = \int d^3\boldsymbol{x}\,\left(\bar{\psi}(x)\gamma^\mu\psi(x)\right)\hat{A}_\mu(x)$$

- Let us start with the blue diagram. A full system of **intermediate states** $|n\rangle$ contains both electron-positron **and** photon. That is
  – the initial state $|i\rangle = |e^-(p_1), e^+(q_1)\rangle \otimes |0\rangle$
  – the final state $|f\rangle = |e^-(p_1'), e^+(q_1')\rangle \otimes |0\rangle$
  – and the intermediate state $|n\rangle = |e^-(p_1'), e^+(q_1)\rangle \otimes |\boldsymbol{k}\rangle$
  (momentum of electron in the intermediate state is equal to its final momentum!)

- The state $|\boldsymbol{k}\rangle$ is the free photon eigen state, such that its x representation is given by

$$\langle x\,|\boldsymbol{k}\rangle = \frac{\epsilon_\mu}{\sqrt{2\omega_{\boldsymbol{k}}}}e^{-i\omega_{\boldsymbol{k}}t+i\boldsymbol{k}\cdot\boldsymbol{x}} \quad \text{or} \quad \langle x\,|\boldsymbol{k}\rangle = \frac{\epsilon_\mu}{\sqrt{2\omega_{\boldsymbol{k}}}}e^{+i\omega_{\boldsymbol{k}}t+i\boldsymbol{k}\cdot\boldsymbol{x}} \qquad (39)$$

Computation of the red diagram

- In this case we have different intermediate state: $|n\rangle = |0\rangle \otimes |\boldsymbol{k}\rangle$. The energy of intermediate state: $E_n = \pm\omega_k$

- Matrix element

$$V_{in} \equiv \langle e^+e^- | \hat{V} | \boldsymbol{k}\rangle = \bar{v}(q_1)\gamma^\mu u(p_1)\frac{\epsilon_\mu}{\sqrt{2\omega_{\boldsymbol{k}}}}\int d^3\boldsymbol{x}\, e^{i(\boldsymbol{p}_1+\boldsymbol{q}_1-\boldsymbol{k})\cdot\boldsymbol{x}} e^{i(E_i-\omega_{\boldsymbol{k}})t}$$
$$=\bar{v}(q_1)\gamma^\mu u(p_1)\frac{\epsilon_\mu}{\sqrt{2\omega_{\boldsymbol{k}}}}\delta^{(3)}(\boldsymbol{p}_1+\boldsymbol{q}_1-\boldsymbol{k})e^{i(E_i-\omega_{\boldsymbol{k}})t} + \{\omega_{\boldsymbol{k}} \leftrightarrow -\omega_{\boldsymbol{k}}\} \tag{40}$$

where the initial energy $E_i = E(\boldsymbol{p}_1) + E(\boldsymbol{q}_1)$.

- Similarly, for the V_{nf} element we get

$$V_{nf} = \bar{u}(p_1')\gamma^\mu v(q_1')\frac{\epsilon_\mu}{\sqrt{2\omega_{\boldsymbol{k}}}}\delta^{(3)}(\boldsymbol{p}_1'+\boldsymbol{q}_1'-\boldsymbol{k})e^{i(\omega_{\boldsymbol{k}}-E_f)t} \tag{41}$$

Computation of the red diagram

- As a result, we get:

$$M_{if} = \int dn \frac{V_{in}V_{nf}}{E_i - E_n}$$
$$= e^{i(E_i - E_f)t}\delta^{(3)}(\boldsymbol{p}_1' + \boldsymbol{q}_1' - \boldsymbol{p}_1 - \boldsymbol{q}_1)\frac{(\bar{v}(q_1)\gamma^\mu u(p_1))(\bar{u}(p_1')\gamma^\mu v(q_1'))}{E_i^2 - \omega_{\boldsymbol{k}}^2}\Big|_{\boldsymbol{k}=\boldsymbol{p}_1+\boldsymbol{q}_1} \quad (42)$$

Total matrix element

- Amplitudes of two processes (blue and red on the Figure) should be **added together** before $|\dots|^2$ is taken. That is the probability of the process is proportional to $|\mathcal{M}_1+\mathcal{M}_2|^2$ rather than $|\mathcal{M}_1|^2+|\mathcal{M}_2|^2$ (interference terms are present)

$$|\mathcal{M}|^2 = (\dots)\left|\underbrace{i\frac{\big(\bar u(p_1')\gamma_\mu u(p_1)\big)\big(\bar v(q_1)\gamma^\mu v(q_1')\big)}{(p_1-p_1')^2}}_{\text{blue diagram}} - \underbrace{i\frac{\big(\bar u(p_1')\gamma_\mu v(q_1')\big)\big(\bar v(q_1)\gamma^\mu u(p_1)\big)}{(p_1+q_1)^2}}_{\text{red diagram}}\right|^2 \quad (43)$$

where $\dots$ is a prefactor, depending on energies/masses of particles.

- Perturbative series in $\hat V = e\int d^3\boldsymbol{x}(j^\mu A_\mu)$

$$M_{if} = M_{if}^{(1)} + M_{if}^{(2)} + \dots$$

- We saw that $M_{if}^{(1)}$ is equal to zero (Eq. (5)). If non-zero, $M_{if}^{(1)} \propto e$ (charge in $\hat V$)

Total matrix element

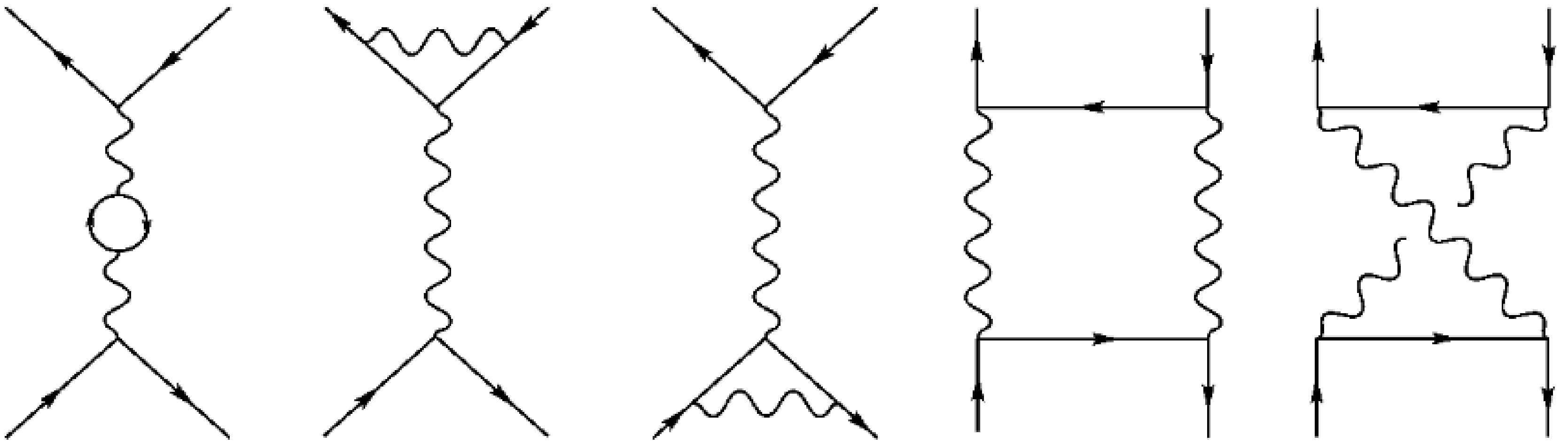
- We found $M_{if}^{(2)}$ (2nd order perturbation theory in $\hat{V}$)

- This expression $M_{if}^{(2)}$ is proportional to e^2

- This parameter e is known experimentally to be small. The expansion parameter of electron-photon interaction is known as the **fine structure constant** $\alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137}$

- Indeed, consider next order in expansion. It includes more intermediate states – higher order in e

Next order



- There are e^+e^- scattering processes that go through **several** intermediate states:

$$M_{if}^{(3)} = \int dn_1 \int dn_2 \frac{V_{in_1}V_{n_1n_2}V_{n_2f}}{(E_i - E_{n_1})(E_{n_1} - E_{n_2})}$$

$$M_{if}^{(4)} = \int dn_1 \int dn_2 \int dn_3 \frac{V_{in_1}V_{n_1n_2}V_{n_2n_3}V_{n_3f}}{(E_i - E_{n_1})(E_{n_1} - E_{n_2})(E_{n_2} - E_{n_3})}$$

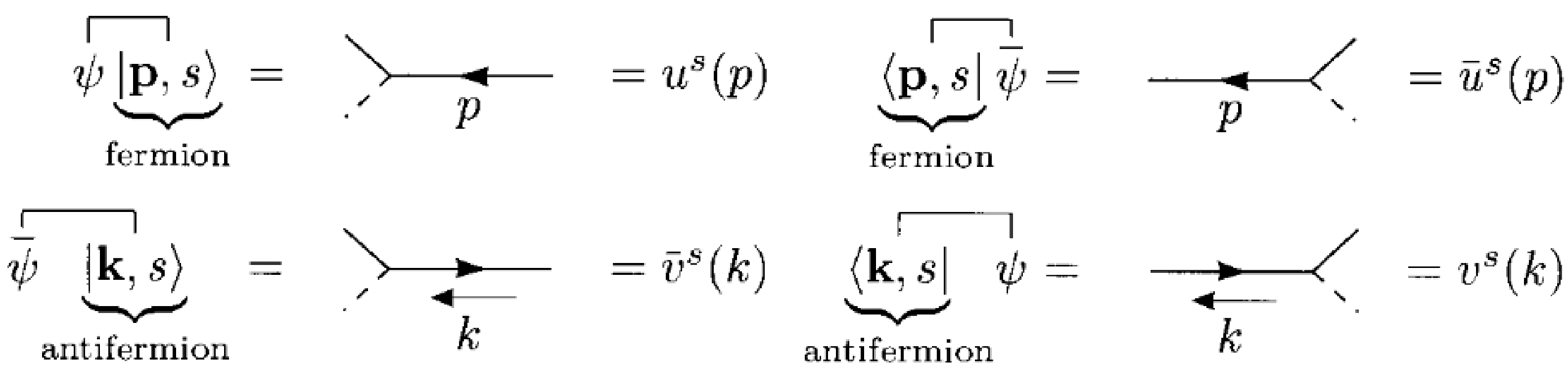
Virtual particles

- In the above computations we have explicitly separate space and time.
- The intermediate states were "physical" (energy and momentum were related via $E^2 = \boldsymbol{p}^2 + m^2$), but only 3-momentum conservation was imposed in every vertex. The total (initial - final) energy was conserved, but for intermediate states it was not
- It is possible to construct explicitly Lorentz-invariant technique for computation of such matrix elements
- This is called **Feynman technique**. Its rules are presented in

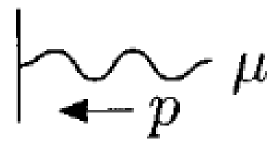
Feynman rules

- There are three types of objects in constructing Feynman graphs:
  - external lines (real particles)
  - internal lines (virtual particles)
  - vertices (interaction points)

- To each **external fermion line** one associates a spinor u, v , etc. according to the following rule:



- To each **external photon line** one associates a polarization vector

External photon lines: $\overbrace{A_\mu \, |\mathbf{p}\rangle} =$  $= \epsilon_\mu(p)$

$\overbrace{\langle\mathbf{p}| \, A_\mu} =$  $= \epsilon^*_\mu(p)$

- Each virtual line adds a propagator:
  - Virtual fermion:

$$S_F(p) = \frac{i(\not p + m)}{p^2 - m^2 + i\epsilon}$$

  - Virtual photon:

$$D_{\mu\nu}(p) = \frac{-i\eta_{\mu\nu}}{p^2 + i\epsilon}$$

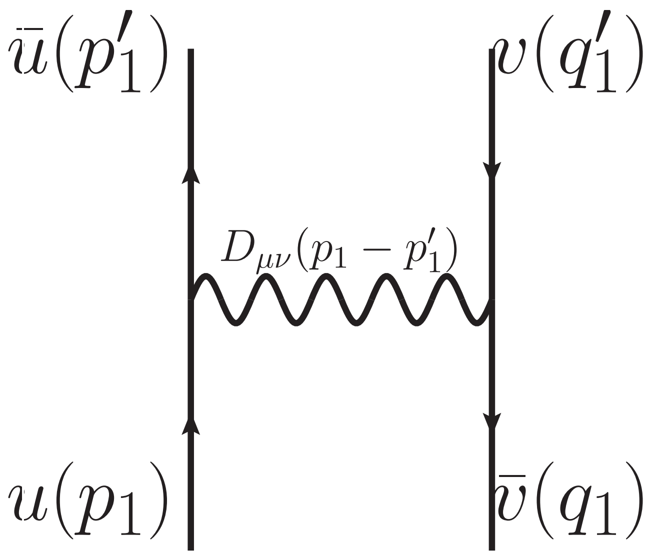
- Each vertex (two fermion lines plus one photon line) receives a factor $-ie\gamma_\mu$

Feynman rules

- **Energy-momentum conservation** is imposed at every vertex

Electron-positron scattering

- Let us repeat the computation of electron-positron scattering

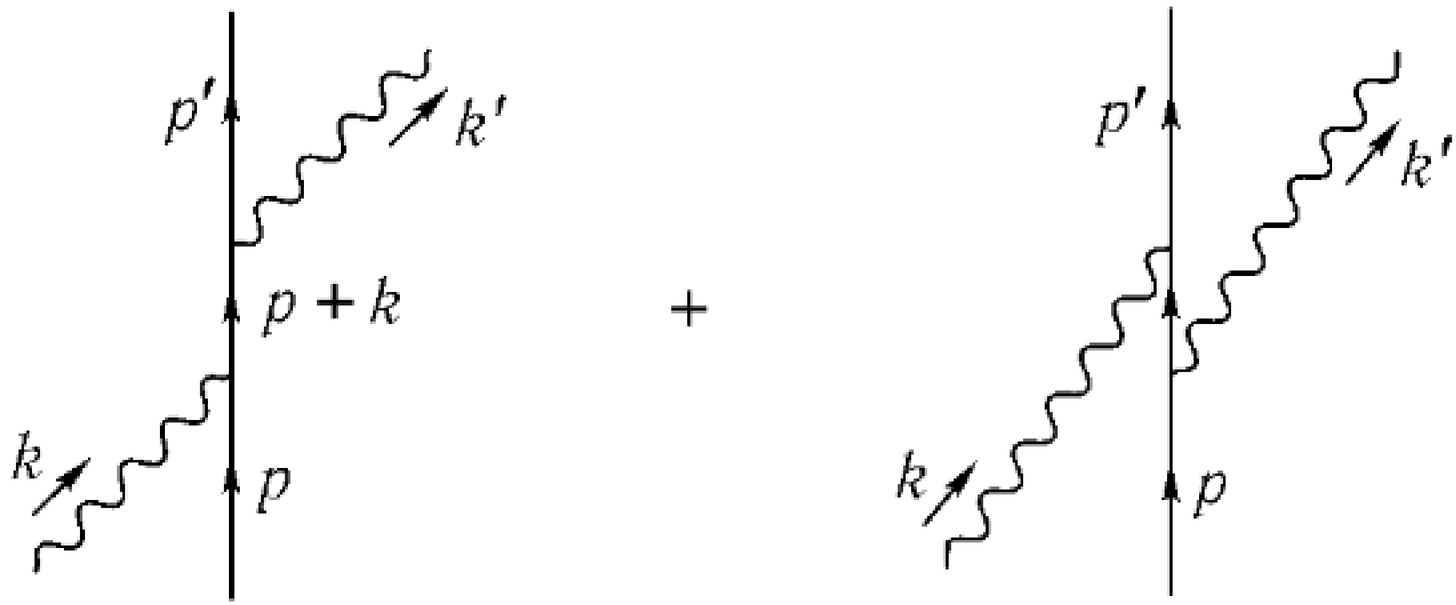


$$\mathcal{M} = \frac{\left(\bar{u}(p_1')(-ie\gamma_\mu)u(p_1)\right)\left(\bar{v}(q_1)(-ie\gamma^\mu)v(q_1')\right)}{(p_1 - p_1')^2}$$

- Similarly

$$\mathcal{M} = \frac{\left(\bar{u}(p_1')(-ie\gamma_\mu)v(q_1')\right)\left(\bar{v}(q_1)(-ie\gamma^\mu)u(p_1)\right)}{(p_1 + q_1)^2}$$

Compton scattering



- Write general form of matrix elements for Compton scattering

- Derive differential cross-section in non-relativistic case (photon energy $\omega \ll m_e$) and in ultra-relativistic case ($\omega \gg m_e$)

Peskin & Schroeder, Sec. 5.5

Pair creation

- Compute cross-section of $\gamma + \gamma \to e^+ + e^-$ (pair production)
- Derive differential cross-section in ultra-relativistic case ($\omega \gg m_e$)

Peskin & Schroeder, Sec. 5.5

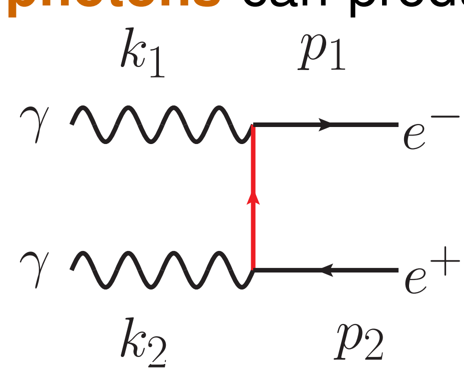
Onther consequences of Dirac theory of positrons

- Photons are **bosons** (particles of spin = 1). Electrons/positrons are **fermions** particles of spin = 1/2. Therefore, angular momentum conservation means that **photon couples to electron + positron**

- Photons could produce electron-positron pairs. **However**, the process $\boxed{\gamma \to e^+e^-}$ is not possible if all particle are "real" (i.e. photon obeys $E = cp$, electron/positron $E = \sqrt{p^2c^2 + m_e^2c^4}$ – "**on-shell conditions**")

- Instead, a **pair of photons** can produce electron-positron pair via

$\boxed{\gamma + \gamma \to e^+e^-}$:



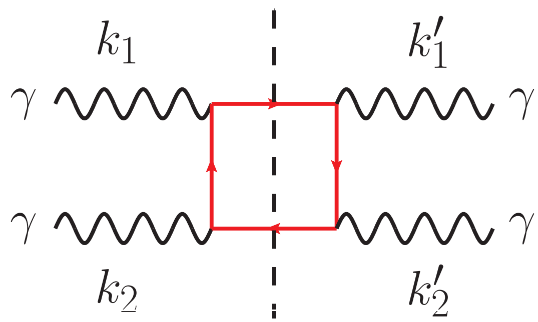
where $(k_1 + k_2)^2 \geq 4m_e^2$

- Similarly, electron-positron pair can **annihilate** into a pair of

Onther consequences of Dirac theory of positrons

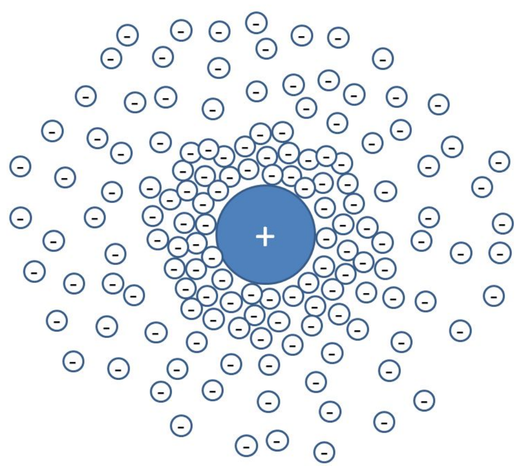
photons

- Kinematically, the red electron is **virtual** (i.e. for it $E \neq \sqrt{p^2c^2 + m^2c^4}$ – **check this**)

- 

If energies of incoming photons are smaller than twice the electron mass (i.e. $(k_1 + k_2)^2 < 4m_e^2$) photons produce only virtual electron-positron pair which can then "annihilate" into another pair of photons – **light-on-light scattering**

- Charge screening:



Systems with many particles

- Presence of the negative-energy levels means that you can create particle-antiparticle pairs out of "nowhere"

- Particles in the pair can be real, but they can be also virtual (i.e. $E^2 - \boldsymbol{p}^2 \neq m^2$)

- According to the Heisenberg uncertainty relation $\Delta E\ \Delta t\ \gtrsim\ 1$, if one measures the state of system two times, separated by a short period $\Delta t \ll 1/m$, one will find a state with $1, 2, 3, ...$ additional pairs.

- It means that we no longer work with definite number of particles: number of particles may change! (Contrary to non-relativistic quantum mechanics)

- We need an approach that naturally takes into account states with different number of particles (we will return to this point in this Lecture)

Birth of quantum field theory

- Quantum electrodynamics (QED) – first quantum field theory. Free fields with interaction treated **perturbatively** in *fine-structure constant:* $\alpha = \frac{e^2}{\hbar c}$
- Divergencies? Many answers beyond tree-level (1st order) perturbation theory were infinite, because one had to sum up contributions from infinite number of **virtual particles** with growing energies $E_p = \pm\sqrt{m^2 + p^2}$

New particles

- Nuclear physics, two types of nuclear physics phenomena: α-**decay** and β**-decay**
- Cosmic rays, first accelerators produced many new particles (positrons, muon, pions, ...)
- These phenomena did not find their explanation in the framework of QED

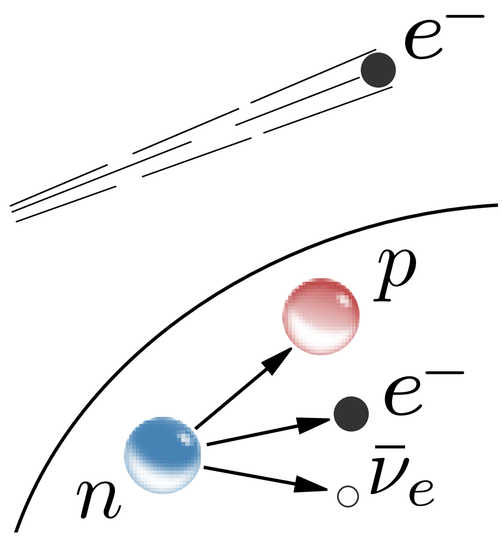
From α–decay to strong interactions:

- Discovery of proton (1919)
- Discovery of neutron (1932)
- Lots of new particles (mesons, baryons)
- Classified according to representation of SU(2) and SU(3) group
- Existence of quarks (1964)
- Color. Confinement
- Quantum Chromodynamics (QCD): theory of "strong interactions" (1973)

See timeline here: hep-ph/0001283

From β-decay to weak interactions

- Prediction of neutrino (1930)
- Discovery of positron (1932)
- Fermi theory (1934)
- Discovery of muon (1940)
- Parity violation (1955-1957) QCD
- V-A currents
- Theoretical problems with Fermi theory.
- Vector boson
- Electroweak theory (1964 – 1968)



- Neutron decay $n \to p + e^- + \bar{\nu}_e$
- Two papers by E. Fermi:

  An attempt of a theory of beta radiation. 1. (In German) Z.Phys. 88 (1934) 161-177 DOI: 10.1007/BF01351864

  Trends to a Theory of beta Radiation. (In Italian) Nuovo Cim. 11 (1934) 1-19 DOI: 10.1007/BF02959820

Continuum spectrum of electrons (**1927**)
Prediction of neutrino (**1930, 1934**)
Fermi theory (**1934**)
Universality of Fermi interactions (**1949**)

- Fermi 4-fermion theory:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] \qquad (44)$$

- New phenomenological constant, G_F, **Fermi constant**.

[11] History of β-decay (see **[hep-ph/0001283]**, Sec. 1,1); Cheng & Li, Chap. 11, Sec. 11.1)

Fermi theory of β-decay [14]

- Dimension of the Fermi constant? .
  $[\mathcal{L}] = [G_F][(\bar\psi\psi)^2]$. Dimension of Lagrangian – $[\mathcal{L}] = M^4$.[12] Mass operator $M\bar\psi\psi \Rightarrow [\bar\psi\psi] = M^3 \Longrightarrow [G_F] = M^{-2}$

- Value determined experimentally to be $G_F \approx 10^{-5}$ GeV^{-2} (More precisely $\frac{G_F}{(\hbar c)^3} = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$.)

- Fermi Lagrangian includes leptonic and hardronic terms:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\left[J^\dagger_{\text{lepton}}(x) + J^\dagger_{\text{hadron}}(x)\right]\cdot\left[J_{\text{lepton}}(x) + J_{\text{hadron}}(x)\right] \quad (45)$$

- Lagrangian (45) predicts universality of weak interactions: all processes in Fermi theory can be described in terms of only one constant, G_F

[12] Dimension $[A_\mu] = [\partial_\mu] = M$. Therefore $[\mathcal{L}] = [F^2_{\mu\nu}] = M^4$.

Neutrino-electron scattering

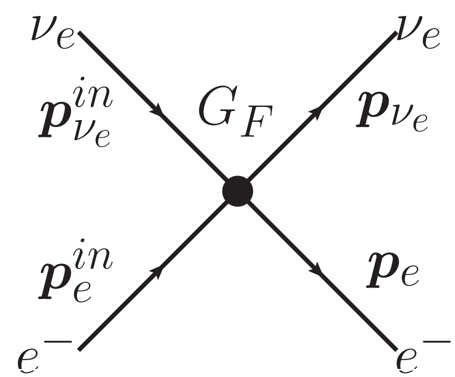
- Fermi theory predicts **lepton-only** weak interactions, such as $e + \nu_e \to e + \nu_e$ scattering

$$\mathcal{L}_{\nu e} = \frac{4G_F}{\sqrt{2}}(\bar{e}\gamma_\lambda \nu_e)(\bar{\nu}_e \gamma^\lambda e) \qquad (46)$$

- Matrix element for $e + \nu_e \to e + \nu_e$ scattering $|\mathcal{M}|^2 = \left|\langle e^-, \nu_e| \mathcal{L}_{\text{Fermi}} |e^-, \nu_e\rangle\right|^2$ (summed over spins of outgoing particles and averaged over spins of initial particles)

$$\sum_{\text{spins}} |\mathcal{M}|^2 \propto \left|G_F \bar{u}(p_e)\gamma_\lambda u(p_e^{in})\bar{u}(p_\nu)\gamma^\lambda u(p_\nu^{in})\right|^2$$
$$\propto G_F^2 (p_e^{in}\cdot p_\nu^{in})(p_\nu \cdot p_e) \propto \boxed{G_F^2 E_{c.m.}^4} \qquad (47)$$

Neutrino-electron scattering



- center-of-mass energy of the system is $E_{c.m.}^2 = \frac{1}{4}(p_e^{in} + p_\nu^{in})^2 \approx (p_e^{in} \cdot p_\nu^{in})$ for $E_{c.m.} \gg m_e$
- momentum of one of the incoming particles $\boldsymbol{p}_{c.m.} \approx E_{c.m.}$ for $E_{c.m.} \gg m_e$
- Cross-section:

$$d\sigma = \frac{1}{4|\boldsymbol{p}_{c.m.}|E_{c.m.}} \frac{d^3\boldsymbol{p}_e}{(2E_e)} \frac{d^3\boldsymbol{p}_\nu}{(2E_\nu)} |\mathcal{M}|^2 \delta(E_{c.m.} - E_e - E_\nu)\delta^3(\boldsymbol{p}_e + \boldsymbol{p}_\nu) \quad (48)$$

- **Reproduce** this calculation using the Lagrangian (46). Keep in mind that neutrino is a purely massless two-component spinor, i.e.

$$\gamma^\mu p_\mu (1 + \gamma_5)\nu(p) = 0$$

where $\frac{1}{2}(1+\gamma_5)$ is the projector that selects only two (left) component of any 4-component spinor

- Show that any solution of the Dirac equation for neutrino depending only on t and one spatial coordinate (z) is "left-moving" (i.e. the wave-function has the general form $\psi(t, z) = f(t + z)$)

Low energy weak processes

- Muon (μ^-) discovered in cosmic rays (charge of the electron, mass $\approx 100\,\mathrm{MeV}$) was first confused with the meson (predicted by Yukawa theory). Originally called μ-meson

- Later π mesons and other mesons were discovered. They were all similar, but μ^- was not interacting with nuclear forces as any meson

- So muon turned out to be just a "heavy electron"

- Fermi Lagrangian (45) was extended so that leptonic current J_{lepton} would include muon:

$$J^\lambda_{\text{lepton}} = \bar{\nu}_e \gamma^\mu e + \bar{\nu}_\mu \gamma^\lambda \mu$$

- …describing for example, the muon decay

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

implies the existence of **muon neutrino**

ν_μ discovered in 1962

- Muon mass $m_\mu \approx 105\,\mathrm{MeV}$. $m_\mu \gg m_e$. In the rest frame of muon e^-, ν_e, ν_μ can be taken as approximately massless.[15]

- The answer should be constructed as a product of two dimensionful quantities. Can construct the only object with dimension of time, proportional to G_F^{-2} .[16]

$$\text{Decay width } \Gamma_\mu \propto \frac{G_F^2 m_\mu^5}{\text{some powers of } \pi \ldots}$$

- **Find** the exact answer: $\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$ using the analog of Lagrangian (46) where one $\bar{e}\gamma_\lambda\nu_e$ term is substituted with $\bar{\mu}\gamma_\lambda\nu_\mu$ term

[15] There can be a caveat here! Will see it when analyzing pion decay

[16] Actually, we will talk about **decay width** with the dimension sec^{-1} or equivalently, GeV. Recall that decay width of $1\,\mathrm{GeV}$ corresponds to the lifetime $6.6 \times 10^{-25}\,\mathrm{sec}$!

Low energy weak processes

- Notice, that similar argument would not work on the neutron's decay:

$$n \to p + e^- + \bar{\nu}_e$$

as this time $m_n \approx m_p$ and therefore one cannot simply write $\Gamma_n \propto G_F^2 m_n^5$. Rather it should be some combination of $m_n = 939.5\,\mathrm{MeV}$ and $(m_n - m_p) \approx 1.3\,\mathrm{MeV}$.

Parity transformation

- **Parity transformation** is a discrete space-time symmetry, such that all spatial coordinates flip $\vec{x} \to -\vec{x}$ and time does not change $t \to t$

- **Show** that:

P-even	P-odd
time	position $\vec{x}$
angular momentum	momentum $\vec{p}$
mass density	Force
electric charge	electric current
magnetic field	electric field

- For fermions parity is related to the notion of **chirality**

- Dirac equation without mass can be split into two non-interacting parts

$$(i\gamma^\mu \partial_\mu)\psi = \begin{pmatrix} 0 & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

Parity transformation

- **Show** that if particle moves only in one direction $\boldsymbol{p} = (p_x, 0, 0)$, then these two components $\psi_{L,R}$ are **left-moving** and **right-moving** along x-direction.

- One can define the $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. In the above basis (Peskin & Schroeder conventions)':

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

  and

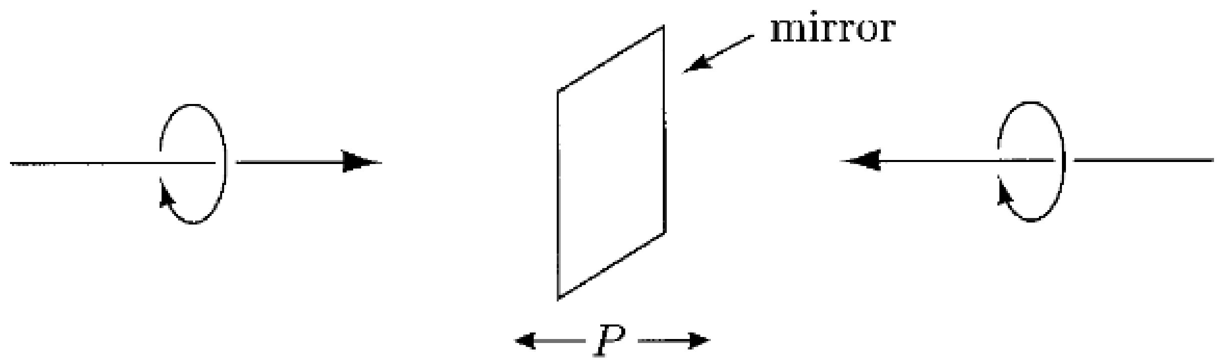
$$\gamma_5\psi_{R,L} = \pm\psi_{R,L}$$

- **Show** that for massless fermions one can define a conserved quantity **helicity**: projection of spin onto momentum:

$$h \equiv \frac{(\boldsymbol{p}\cdot\boldsymbol{s})}{|\boldsymbol{p}|}$$

  Show that left/right chiral particles have definite helicity ± 1.

Parity transformation



- For fermions: Left-Handed $\rightleftarrows$ Right-Handed. $|\psi_L(\vec{p})\rangle \xrightarrow{P} |\psi_R(-\vec{p})\rangle$.

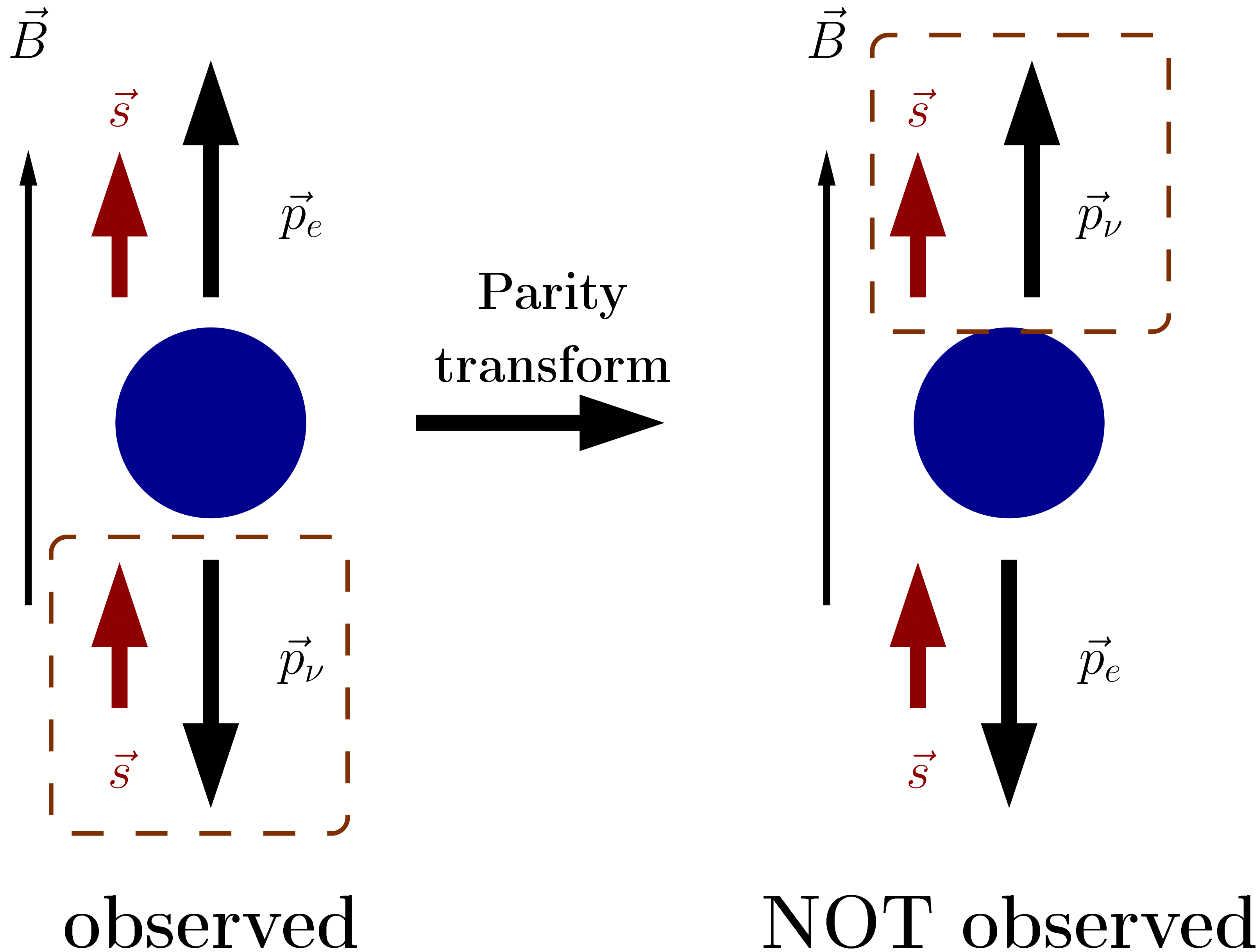
- The neutrino being massless particle would have two states:
  - Left neutrino: spin **anti-parallel** to the momentum $\boldsymbol{p}$
  - Right neutrino: spin **parallel** to the momentum $\boldsymbol{p}$

- This has been tested in the experiment by Wu et al. in 1957

Parity violation in weak interactions

- Put nuclei into the strong magnetic field to align their spins
- Cool the system down (to reduce fluctuations, flipping spin of the Cobalt nucleus)
- The transition from ^{60}Co to ^{60}Ni has momentum difference $\Delta J = 1$ (spins of nuclei were known)
- Spins of electron and neutrino are parallel to each other
- Electron and neutrino fly in the opposite directions
- **Parity** flips momentum but does not flip angular momentum/spin/magnetic field

Parity violation in weak interactions



Parity violation in weak interactions

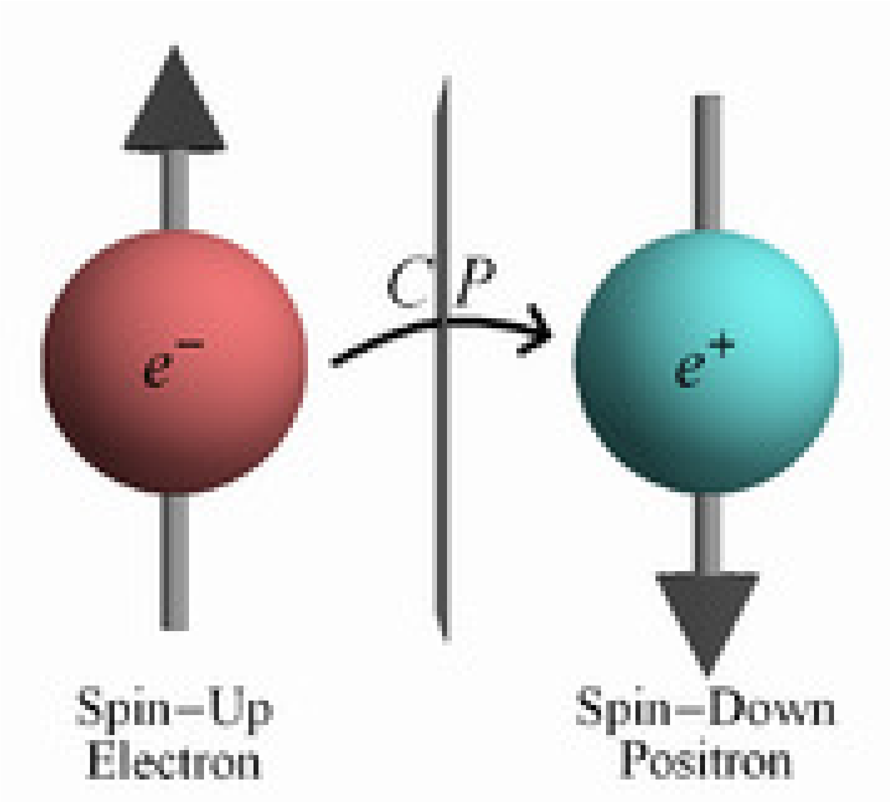
- This result means that neutrino always has spin anti-parallel to its momentum (**left-chiral particle**)

- Parity exchanges left and right chiralities. As neutrino is always left-polarized this means that

Particle looked in the mirror and did not see itself ???

- How can this be?

- Another symmetry, **charge conjugation** comes to rescue. Charge conjugation exchanges particle and anti-particle. Combine it with parity (**CP-symmetry**):



  – P: $|\nu_L\rangle \to |\nu_R\rangle$ – **impossible**

  – CP: $|\nu_L\rangle \to |\bar{\nu}_R\rangle$ – **possible**. Anti-neutrino exists and is always right-polarized

  – Life turned out to be more complicated. CP-symmetry is also broken

Unitarity bound (1957)

- Recall: in QED the coupling constant α is dimensionless. Therefore, electron-photon interaction (Compton scattering) has cross-section, decreasing at high energies:

$$\sigma_{\rm EM} \propto \frac{\alpha^2}{E^2}$$

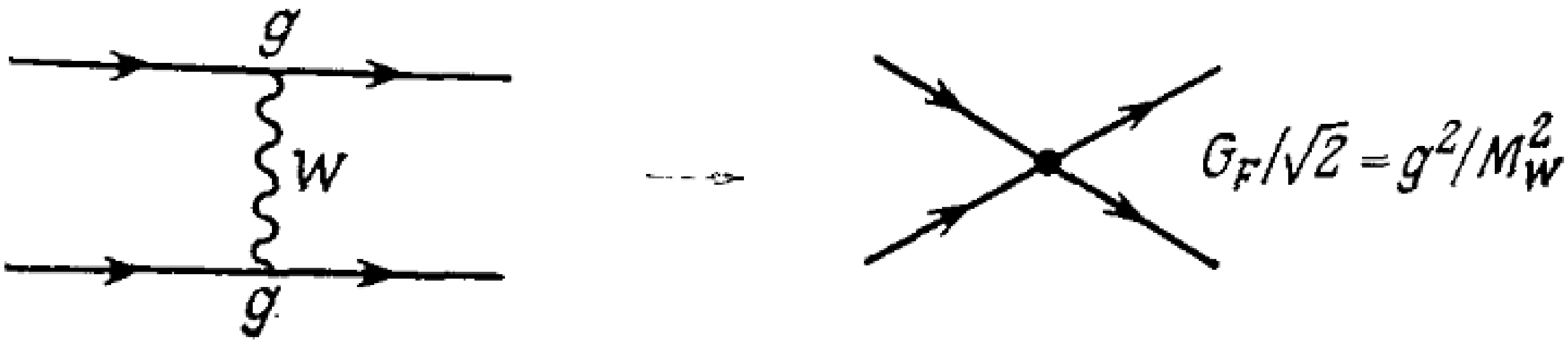
- In Fermi theory coupling constant is **dimensionful** ($[G_F] =$ GeV^{-2}). The cross-section

$$\sigma_{\rm weak} \propto G_F^2 E^2$$

  – grows with energy and violates **unitarity bound** at energies $E \sim 300$ GeV

- The theory should be modified? Make G_F dependent on energy?

Vector boson



- Promote point-like 4-fermion Fermi interaction to interaction, mediated by a **vector boson** : $\mathcal{L}_{\text{Fermi}} \to \mathcal{L}_W = g(W^\mu J_\mu + h.c.)$ 1957

Theory of massive vector bosons?

- Theory of interacting vector boson can only have sense if they are gauge bosons of some gauge group.

- Easy to understand even for photon: the propagator

$$\langle A_\mu(p) A_\nu(-p)\rangle = -i\frac{\eta_{\mu\nu}}{p^2}$$

  means that **norm of the state $|A_0\rangle = A_0|0\rangle$ is negative** – consequence of Lorentz invariance for spin 1 particles

- Gauge symmetry allows to remove these unphysical states

Theory of massive vector bosons?

- How can gauge boson be massive?

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m^2}{2}A_\mu^2$$

$A_\mu \to A_\mu + \partial_\mu \lambda(x)$? – Lagrangian is not gauge invariant!!

- If mass breaks gauge symmetry – we are immediately back to the problem with the negative norm states

- The only possibility is **spontaneous symmetry breaking**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m^2}{2}\Big(\partial_\mu \theta(x) + A_\mu\Big)^2$$

Gauge transformation shifts $\theta(x)$: $\theta(x) \to \theta(x) - \lambda(x)$.

- **Fix the gauge:** $\theta(x) = 0$ and **obtain massive gauge boson**!

Vector boson vs. photon

- Propagator of the massive vector boson:

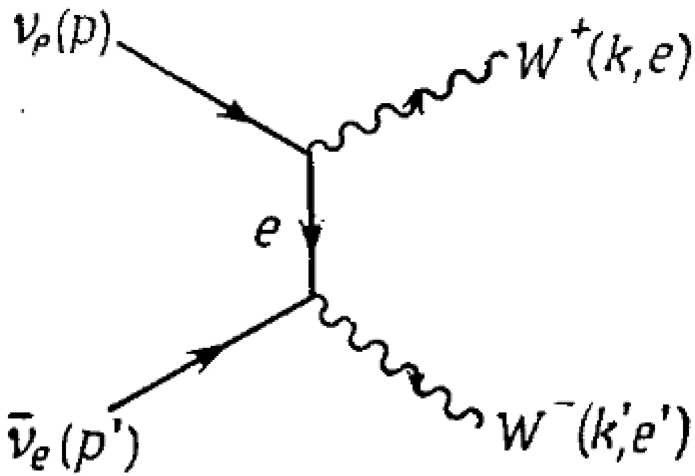
$$\langle W_\mu(p) W_\nu(-p)\rangle = -i\frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2}$$

- Recall: photon propagator: $\langle A_\mu(p) A_\nu(-p)\rangle = -i\frac{\eta_{\mu\nu}}{p^2}$

- Try to put $M_W \to 0$. Will you recover photon-like Lagrangian? **No!**
  Trouble with the term in the numerator

- Violation of tree-level unitarity reappeared in case of WW scattering

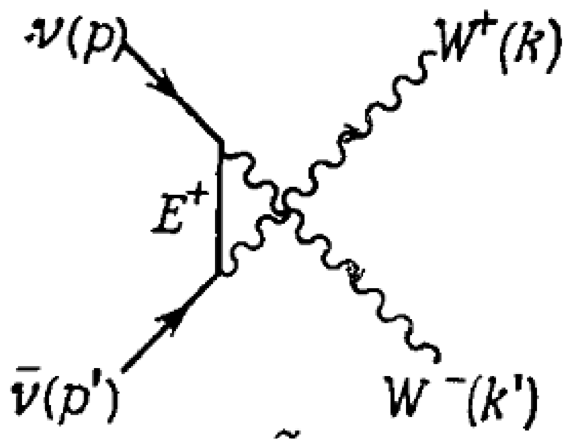
$\nu\bar{\nu} \to W^+W^-$ and neutral currents

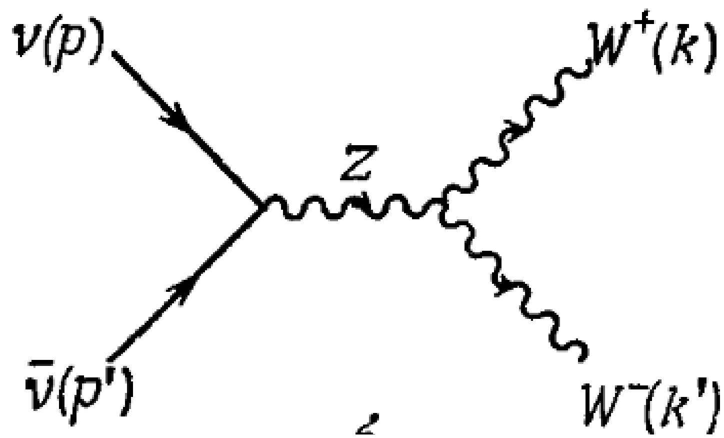


Roughly the cross-section is as in QED times an *enhancement factor* $\left(\frac{E}{M_W}\right)^4$ for *longitudinal polarizations* at high energies

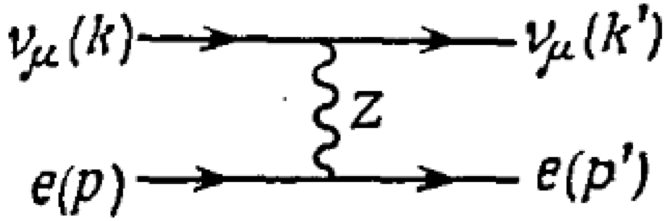
$$\sigma_{\nu\bar{\nu}\to WW} \propto \frac{g^2}{E^2}\left(\frac{E}{M_W}\right)^4$$

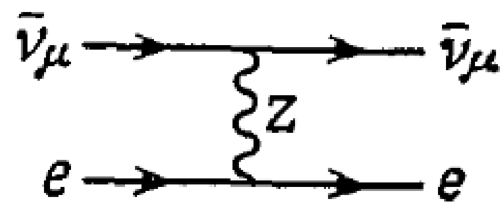
Need to introduce **new particles** to cancel this growing divergence. Two possibilities were considered: new charged fermion (E^+ on the left panel) and new neutral vector boson (Z on the right panel)





- Adding a neutral boson (Z^0 boson) meant that there should be additional processes, such as neutrino scattering:





- Discovered in CERN in 1974 (In 1973 the huge Gargamelle bubble chamber in CERN photographed the tracks of a few electrons suddenly starting to move, seemingly of their own accord. This is interpreted as a neutrino interacting with the electron by the exchange of an unseen Z boson. The neutrino is otherwise undetectable, so the only observable effect is the momentum imparted to the electron by the interaction)

- Successful prediction of the theory!

- Z and $W^{\pm}$ bosons were seen at LEP in 1980s

- introduce an SU(2) **doublet** $L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ and the SU(2) gauge field (triplet of fields $\vec{A}_\mu = (A^1_\mu, A^2_\mu, A^3_\mu)$) then we can write the Lagrangian:

$$\mathcal{L} = \bar{L}\Big(i\gamma^\mu\partial_\mu + g\vec{\tau}\cdot\vec{A}_\mu\Big)L$$

- If one introduces $W^\pm_\mu \equiv A^1_\mu \pm iA^2_\mu$ then the gauge interaction becomes exactly the **charge currents** predicted from parity-violating nature of weak interactions:

$$J^+_\mu = \bar{\nu}_e\gamma^\mu(1-\gamma^5)e$$

and coupled to **intermediate vector boson** W^- as $W^+_\mu J^-_\mu + h.c$

- The SU(2) Lagrangian with the doublet becomes the Lagrangian of lepton weak interactions.

Electroweak interactions

- Kinetic term for the non-Abelian $\vec{A}_\mu$ field:

$$\mathcal{L}_{SU(2)} = -\frac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu)^2$$

- Model explains charged current interactions, but does not provides neutral current explanations

- Add an additional U(1) gauge boson (**hypercharge field Y_μ**). The full Lagrangian becomes:
  - Three massive vector bosons $W^\pm$ and Z^0 together with photon γ are unified into the same gauge group $SU(2) \times U(1)$.
  - There is a scalar field – **Higgs field** – that obtains **vacuum expectation** . The field is SU(2) doublet and is charged with respect to an additional U(1) field
  - The photon is the only unbroken configuration of the $SU(2)\times U(1)$ gauge group: $A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(-g' A^3_\mu + gY_\mu)$

Quarks

All these particles are bound states of 3 **quarks** with fractional electric charge, fractional baryon number, spin $\frac{1}{2}$

	Q	T	T_3	Y	S	B
u	$2/3$	$1/2$	$1/2$	$1/3$	0	$1/3$
d	$-1/3$	$1/2$	$-1/2$	$1/3$	0	$1/3$
s	$-1/3$	0	0	$-2/3$	-1	$1/3$

Mesons:

$$\pi^+ \sim \bar{d}u, \quad \pi^0 \sim (\bar{u}u - \bar{d}d)/\sqrt{2}, \quad \pi^- \sim \bar{u}d,$$
$$K^+ \sim \bar{s}u, \quad K^0 \sim \bar{s}d, \quad \overline{K}^0 \sim \bar{d}s, \quad K^- \sim \bar{u}s,$$
$$\eta^0 \sim (\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}.$$

Baryons:

$$p \sim udu, \quad n \sim udd,$$
$$\Sigma^+ \sim suu, \quad \Sigma^0 \sim s\,(ud + du)/\sqrt{2}, \quad \Sigma^- \sim sdd,$$
$$\Xi^0 \sim ssu, \quad \Xi^- \sim ssd,$$
$$\Lambda^0 \sim s\,(uu + dd - 2ud)/\sqrt{2},$$

Quark color

- **Puzzle** of $\Delta^{++} = |u\uparrow\ u\uparrow\ u\uparrow\rangle$ resonance: state of 3 u-quarks with the total spin $\frac{3}{2}$ – totally symmetric wave-function of 3 fermions **?!**

- **Resolution:** introduce new quantum number: **color**

- Quarks form fundamental representation of SU(3) color symmetry (**not to be confused with the SU(3) symmetry of the Eightfold Way!!!**)

- All observed states are singles under this symmetry.

- **Quantum chromodynamics:** SU(3) gauge symmetry with non-Abelian gauge group. Interaction mediated by **gluons**, forming adjoint representation of SU(3). There are $3^2 - 1 = 8$ gluons

- Perturbation theories does not work! Quantum corrections become small at low energy.

Standard Model: great success of particle physics

Standard Model of

FUNDAMENTAL PARTICLES AND INTERACTIONS

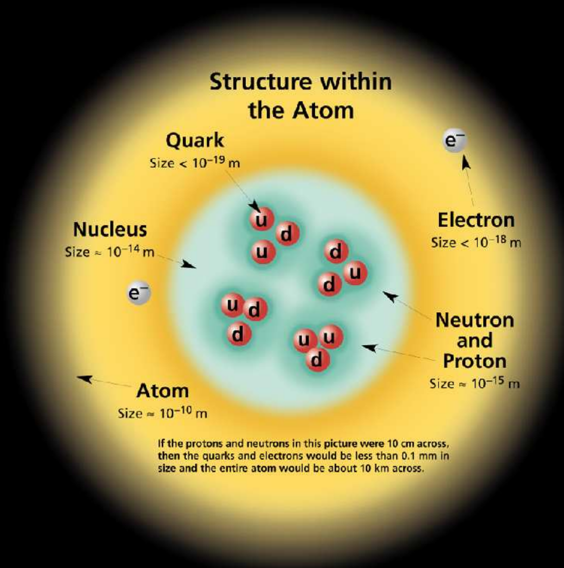
FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3



BOSONS force carriers spin = 0, 1, 2, ...

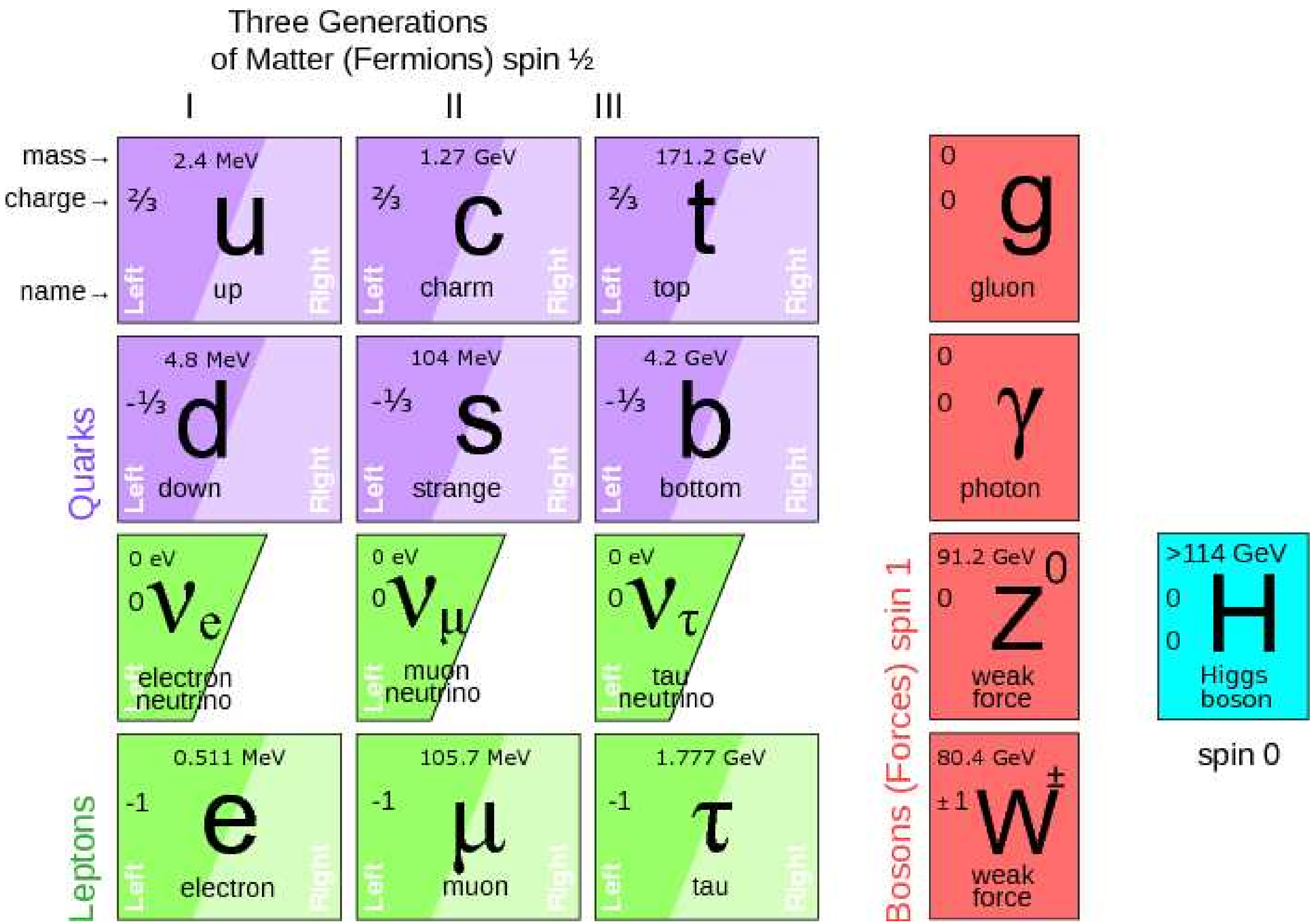
Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

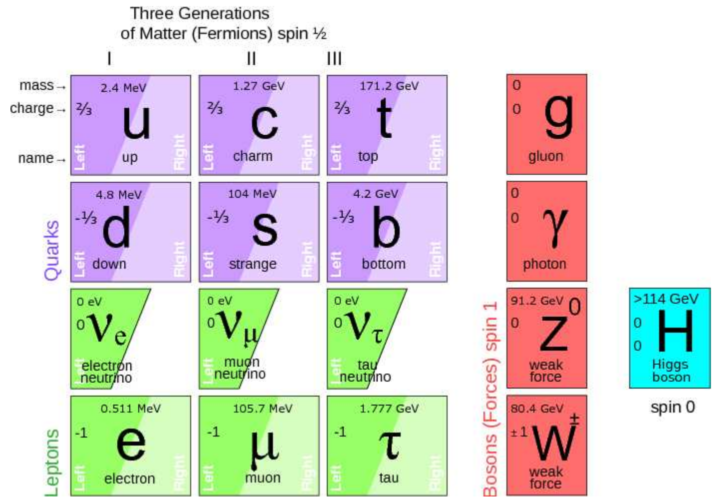
Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Standard Model



Standard Model



- The Standard Model describes all the confirmed data obtained using particle accelerators and has enabled many successful theoretical predictions.

- It has been tested with amazing accuracy, and its calculable quantum corrections play an essential role. Its only missing feature is a particle, called the Higgs boson, whose coupling to the other particles is believed to generate their masses.