Passive electrical properties of the neuron

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The following books were used for the lecture:

“From neuron to brain” Nicholls et al., 3rd Ed.

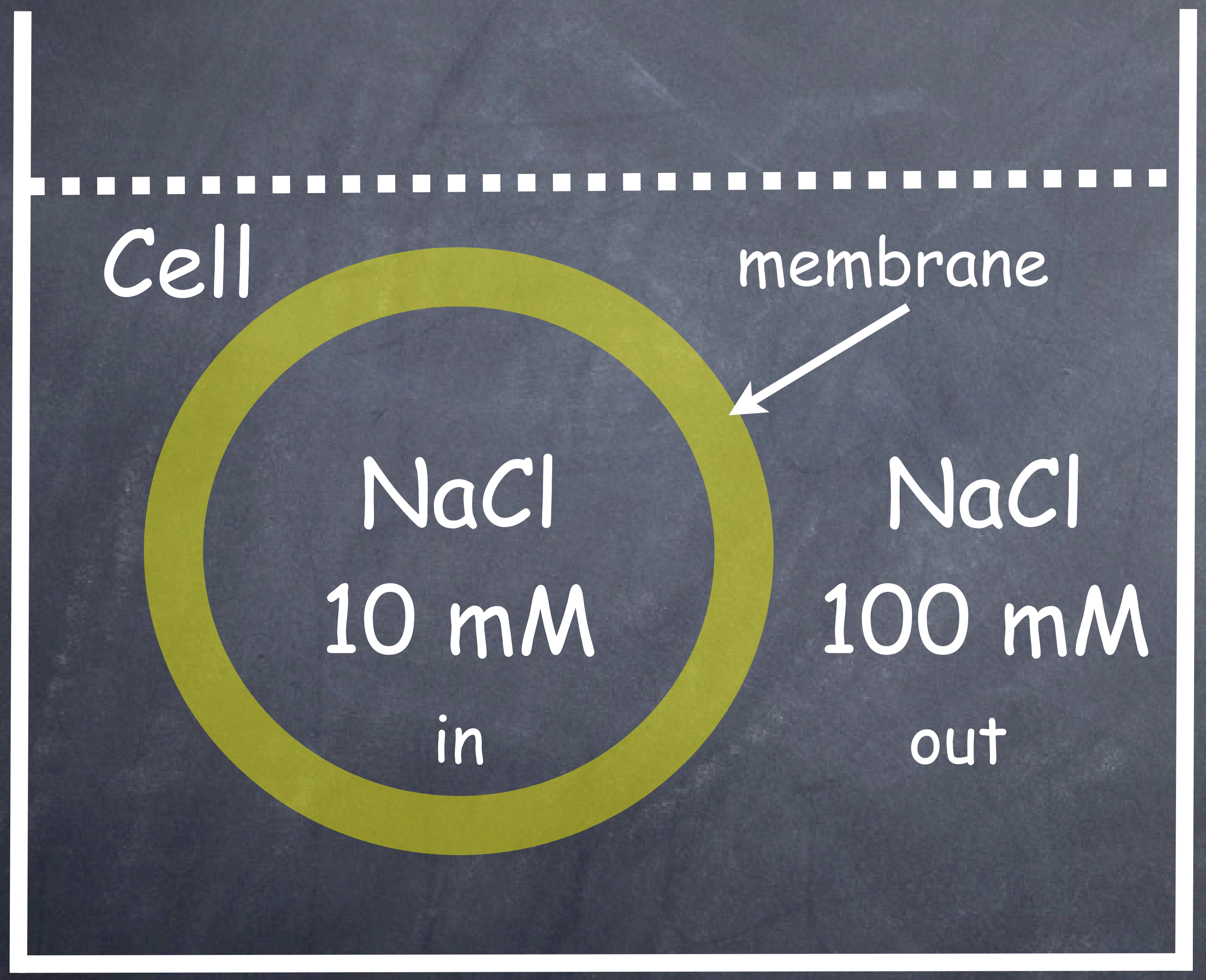
“Ion channels of excitable membranes” B. Hille, 3rd Ed.

“Fundamental neuroscience” Zigmond et al.

“Principles of neuroscience” Kandel et al., 4th Ed.

Review of basic concepts

Experiment 1



Cell

membrane

NaCl

10 mM

in

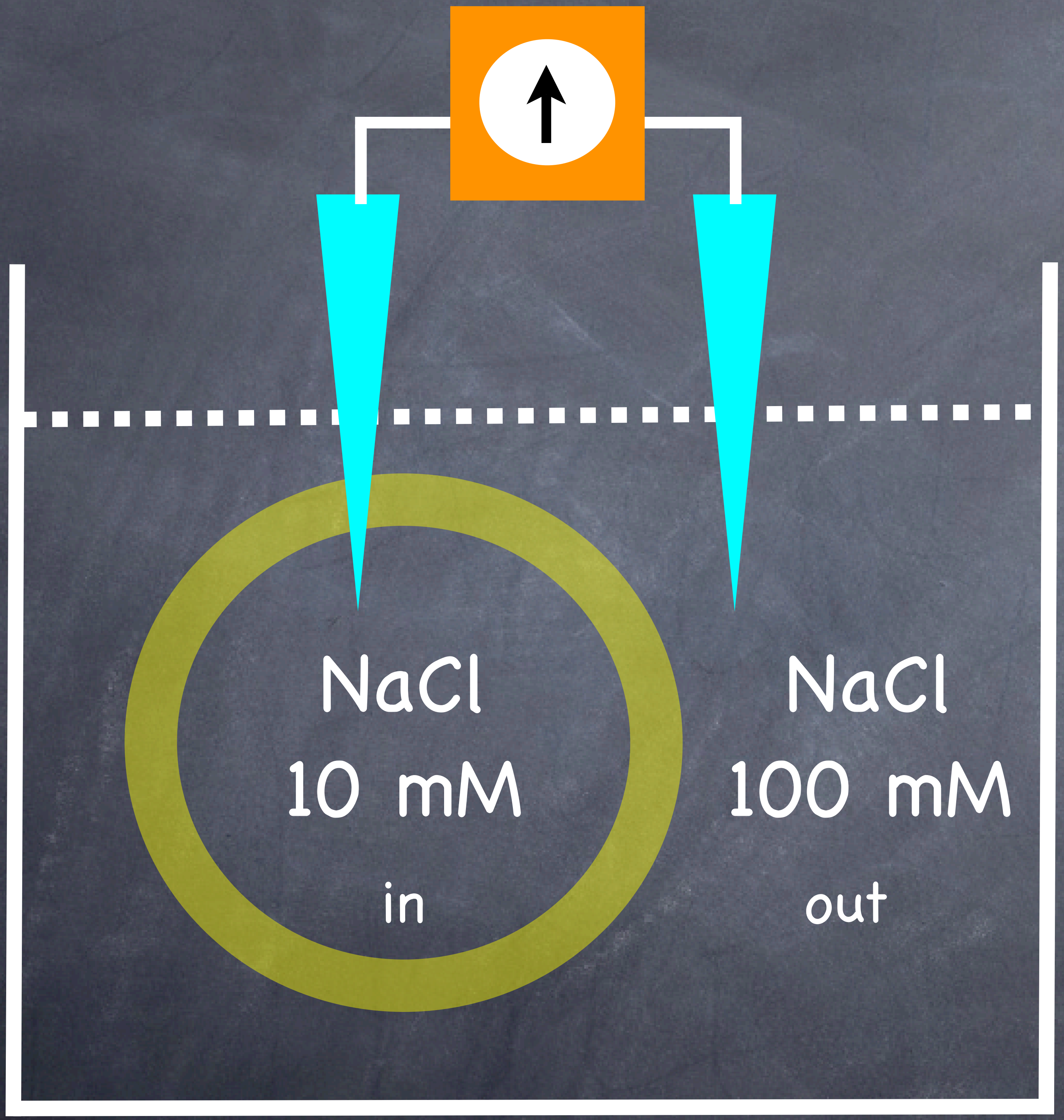
NaCl

100 mM

out

◆ Concentration Gradient

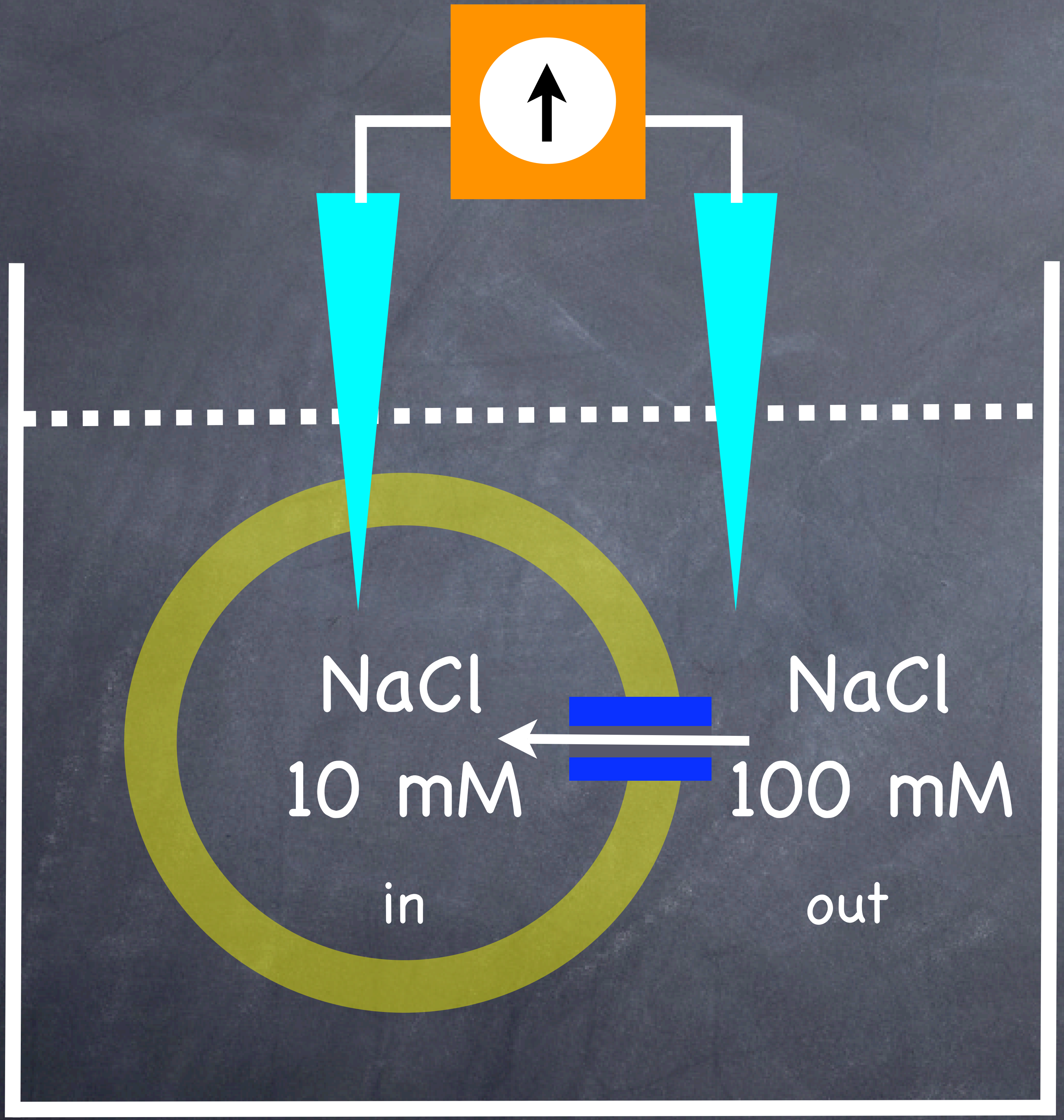
Measure membrane potential



- Concentration Gradient

V= ??

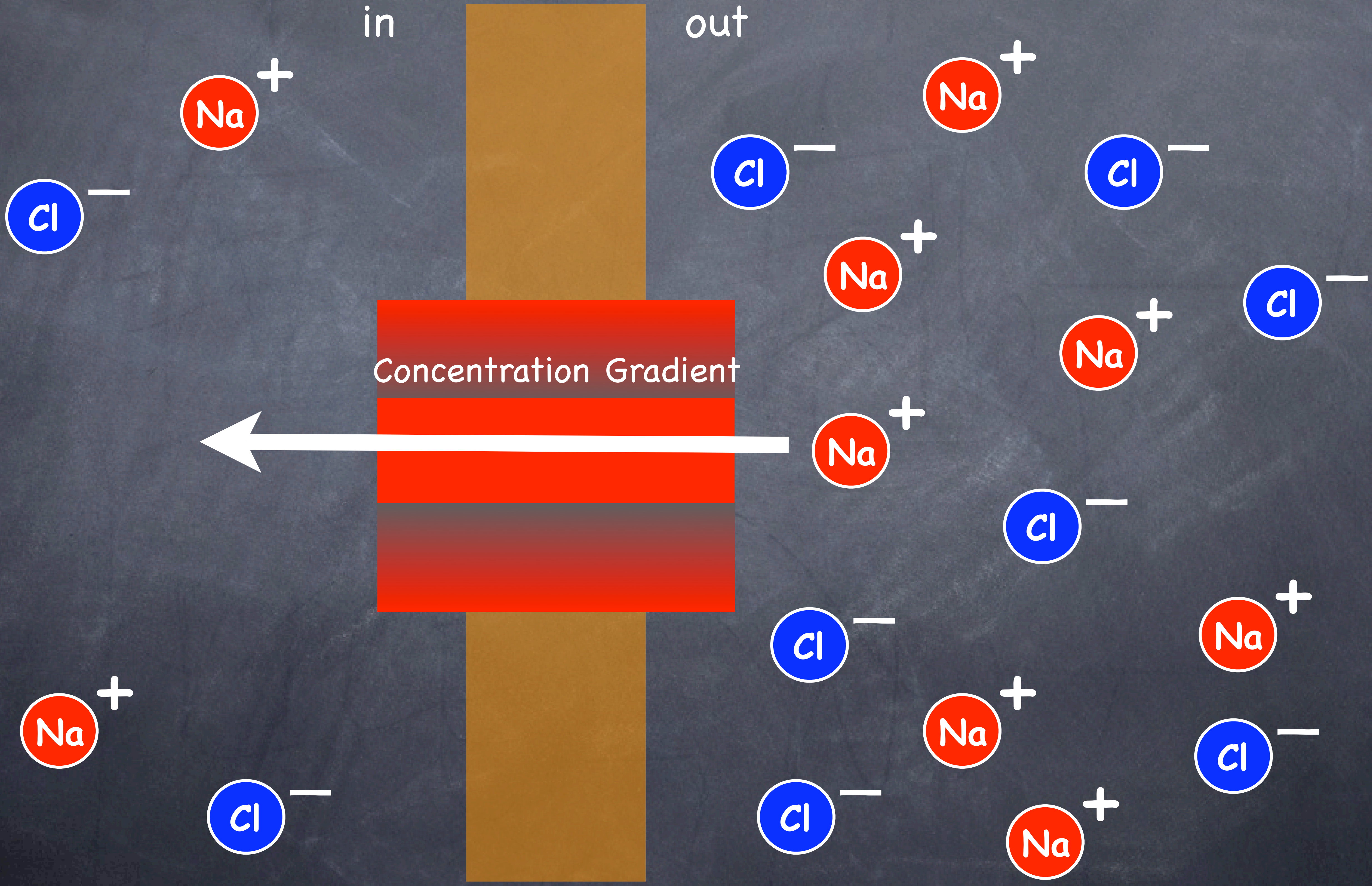
Measure membrane potential



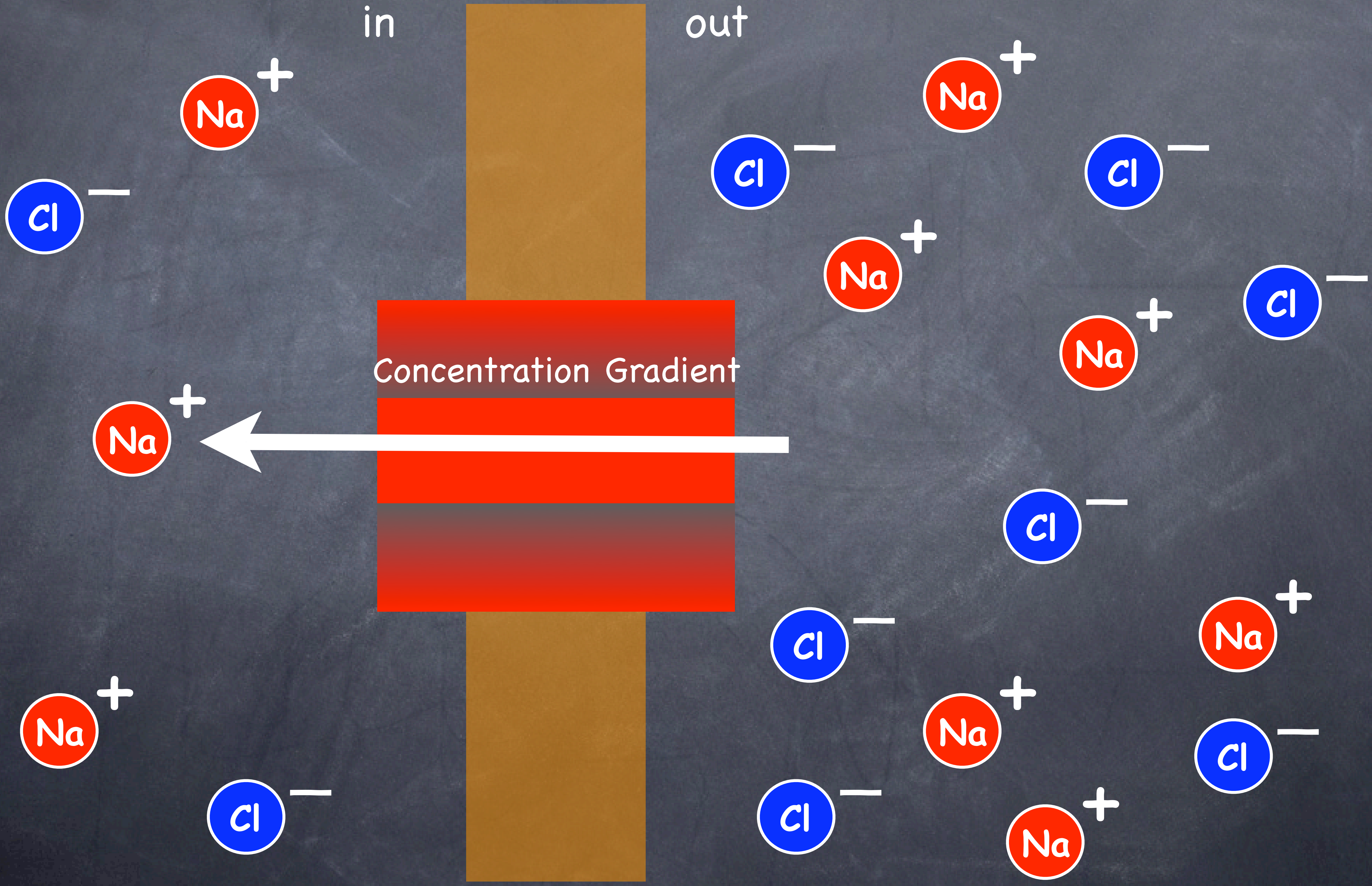
- ◆ Concentration Gradient
- ◆ membrane is equally permeable to both Na and Cl

V= ??

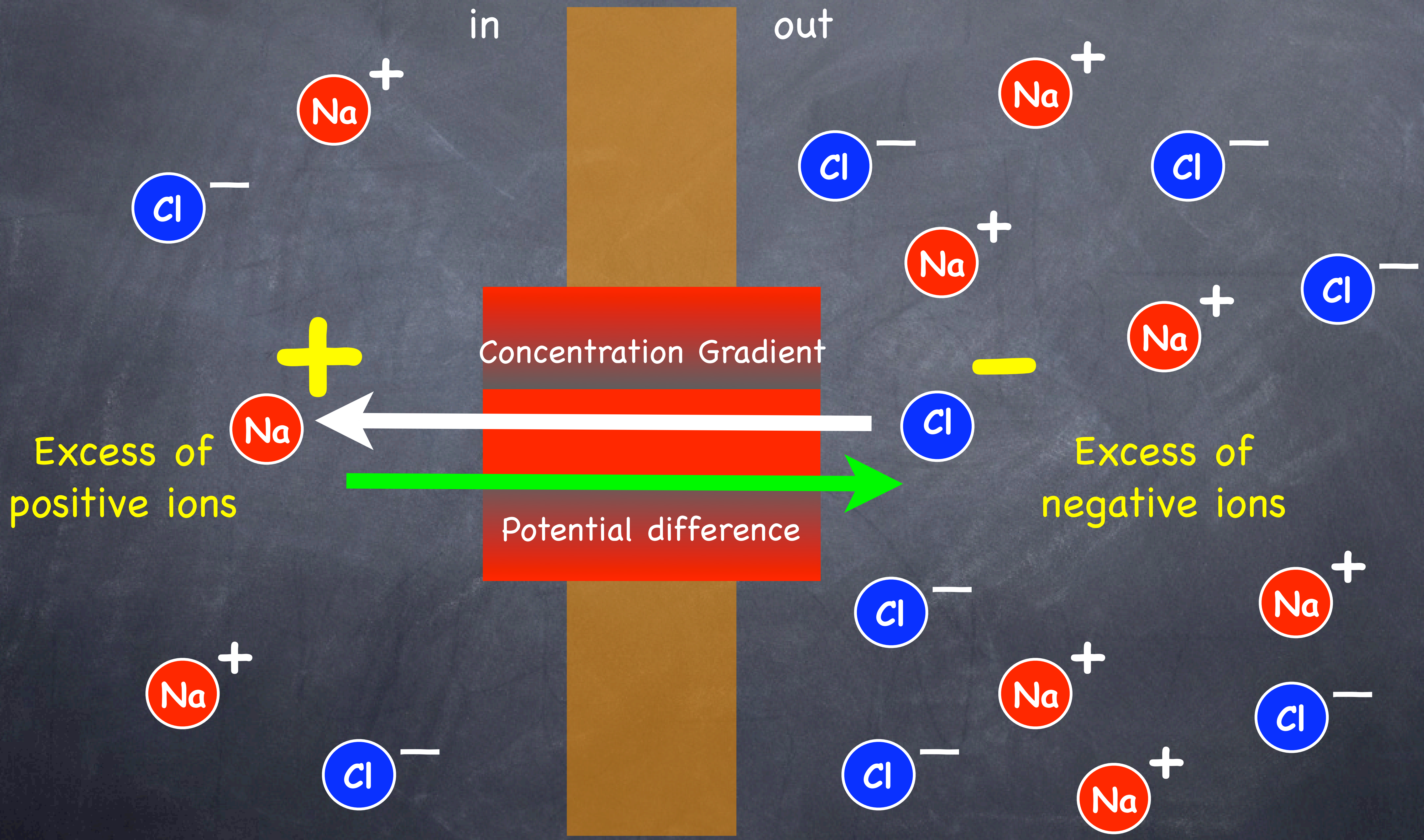
Membrane is selectively permeable to Sodium



Membrane is selectively permeable to Sodium



Membrane is selectively permeable to Sodium



in

out

Concentration Gradient

Potential difference

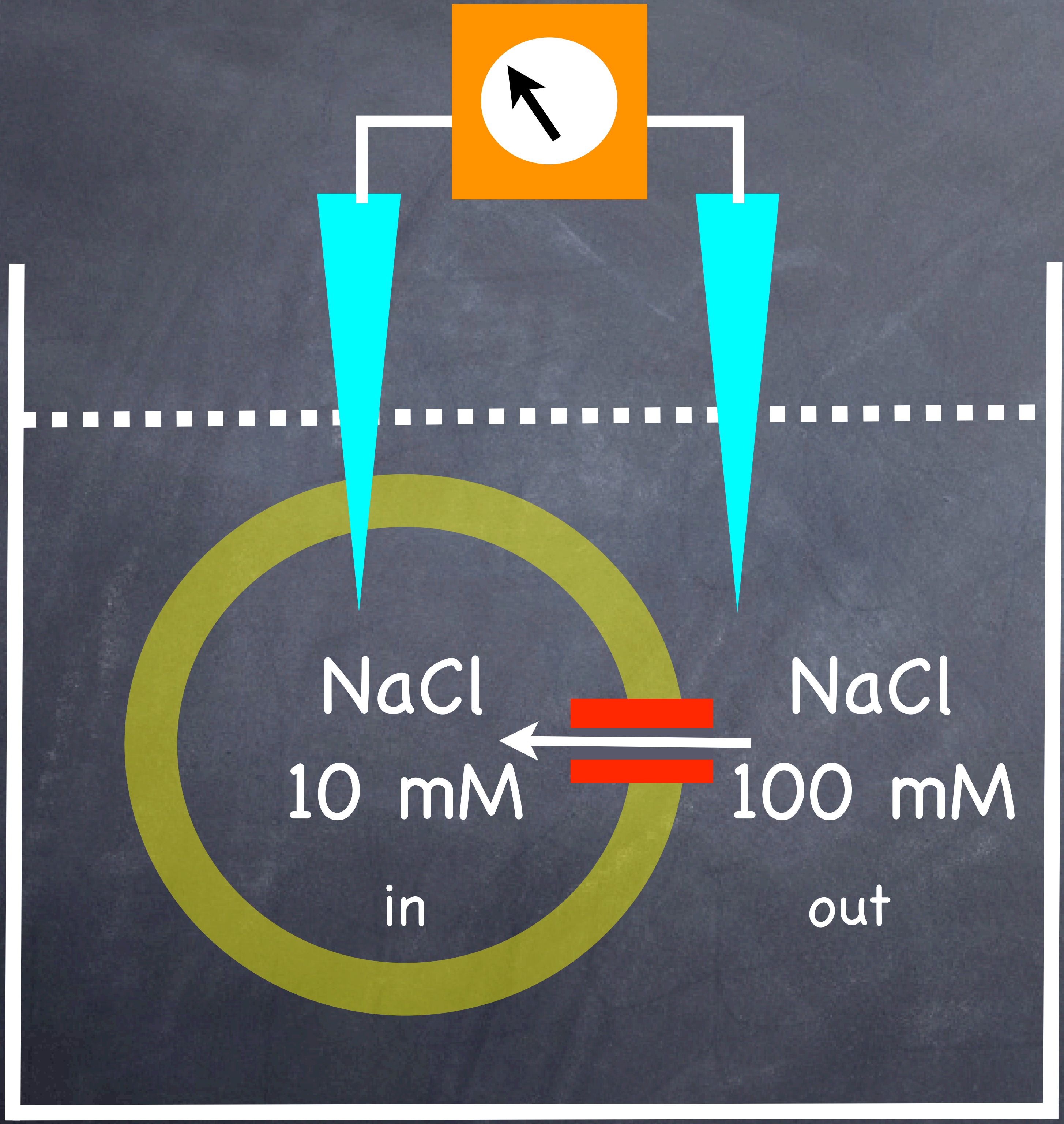
+

Excess of positive ions

−

Excess of negative ions

Membrane potential



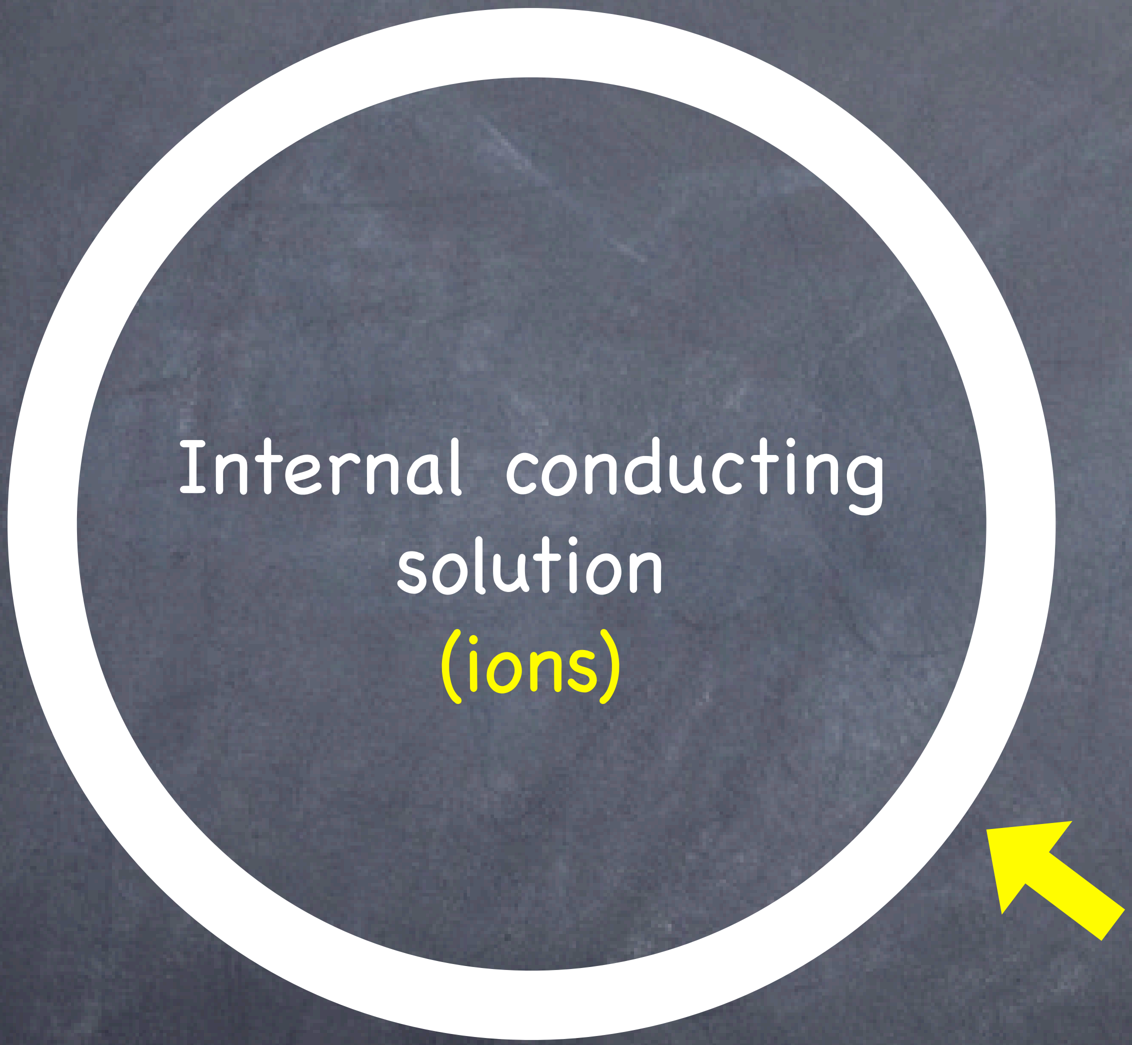
- Concentration Gradient across the membrane
- Membrane is selectively permeable to ions

$$V = \frac{RT}{F} \log \frac{[out]}{[in]}$$

$$V_{Na} = +58 \text{ mV}$$

$$V_{Cl} = -58 \text{ mV}$$

Membranes as capacitors

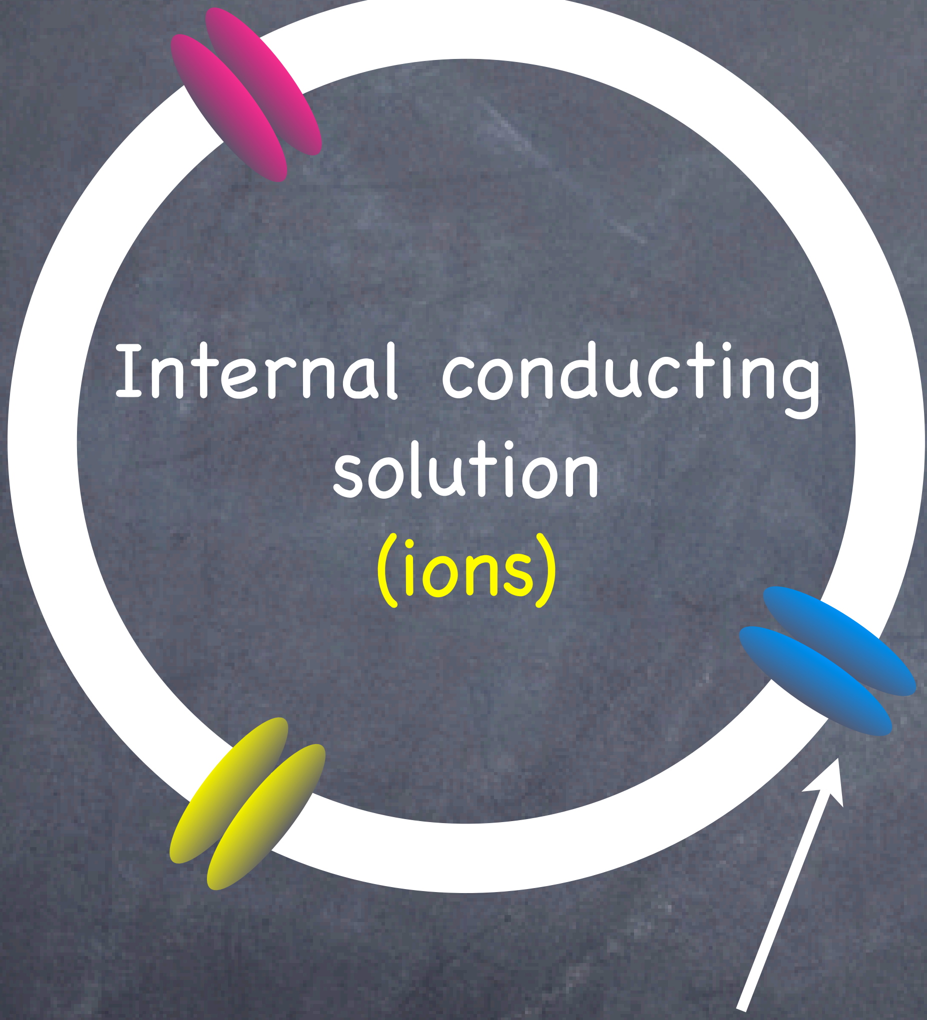


Internal conducting
solution
(ions)

External conducting
solution
(ions)

Thin insulating
layer
(membrane, 4nm)

Membranes as Resistors

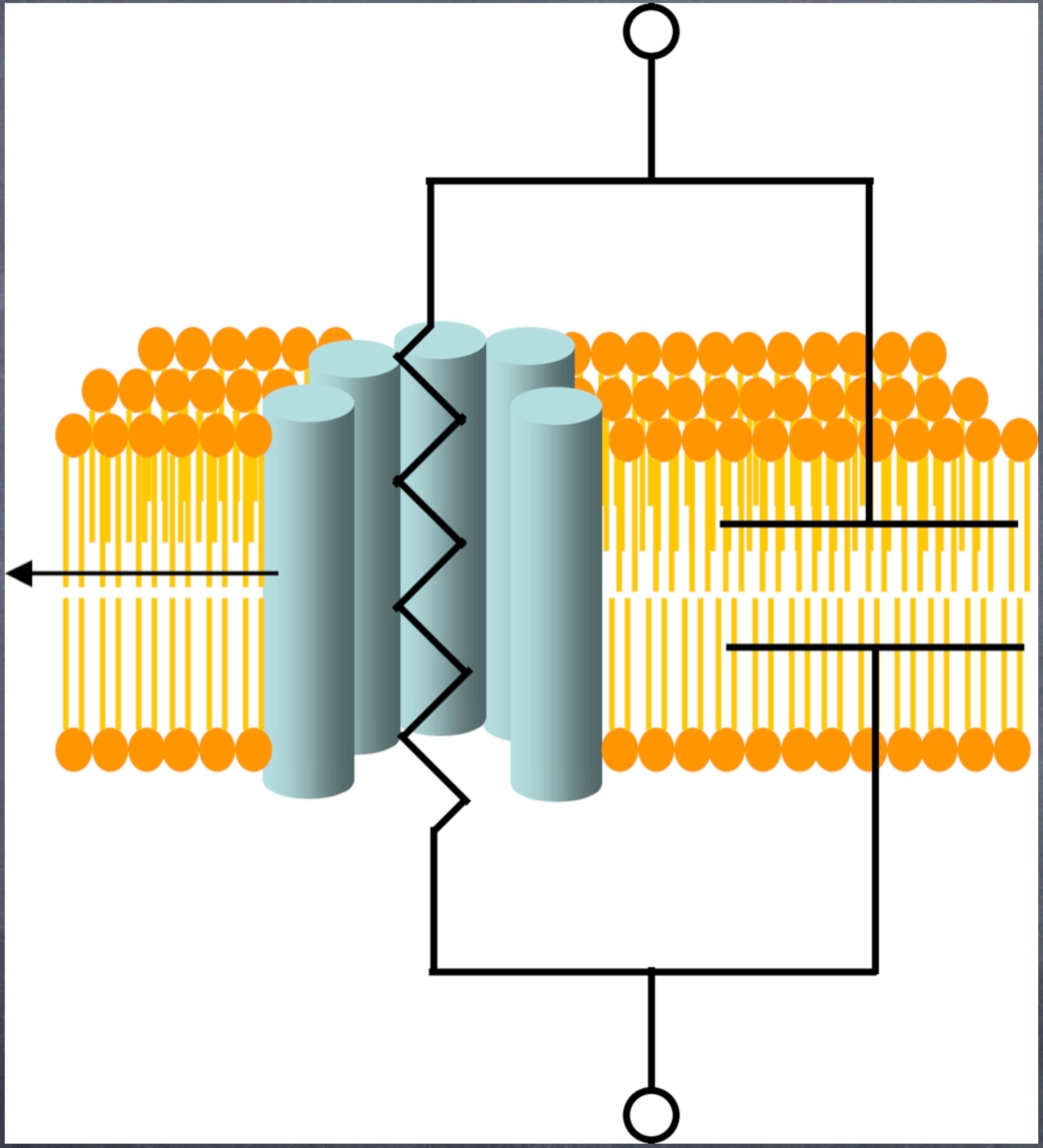


Internal conducting solution (ions)

External conducting solution (ions)

Ion **channels**
Voltage-gated, NT- gated etc.

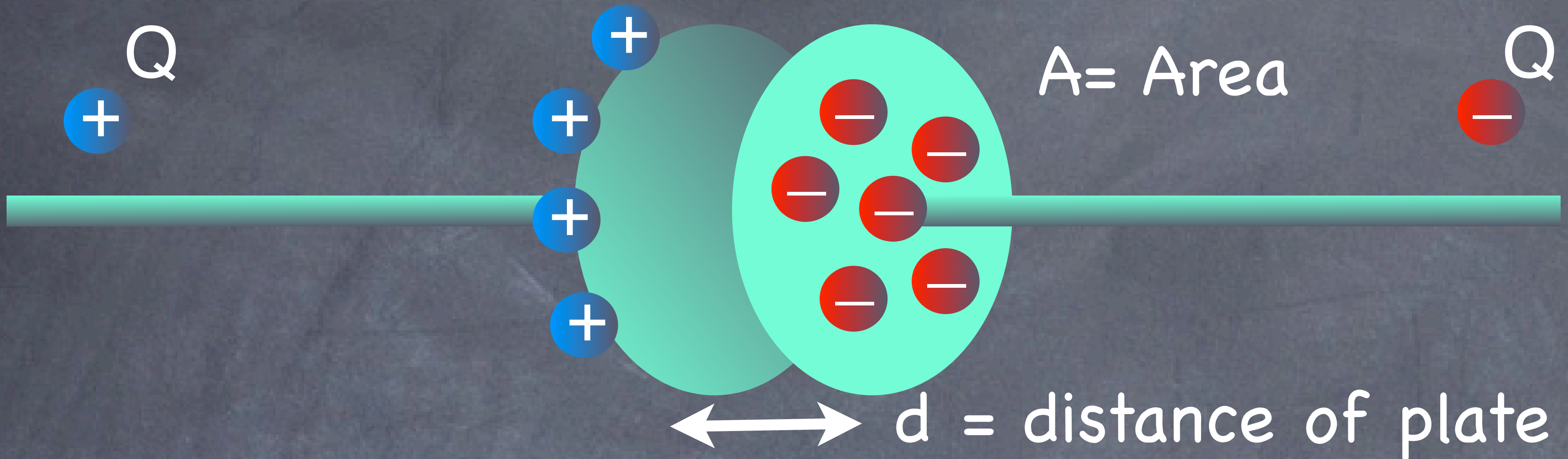
Electrical model of the cell membrane



Ion channel
RESISTOR

Membrane
CAPACITOR

Capacitor



A= Area

d = distance of plate separation

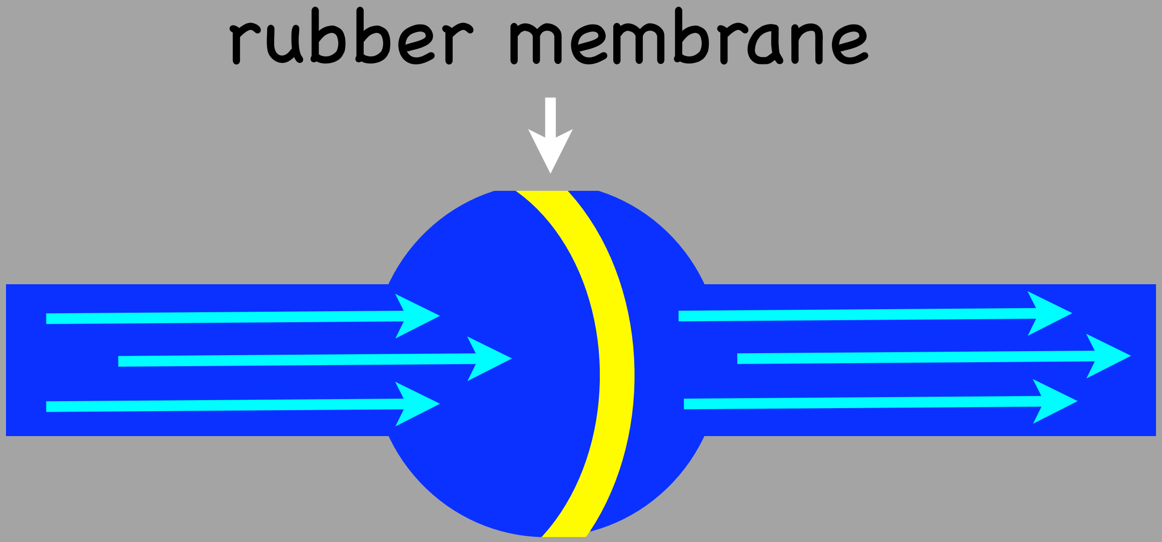
Capacitance

C = Q/V   Coulomb/Volt or Farads (F)

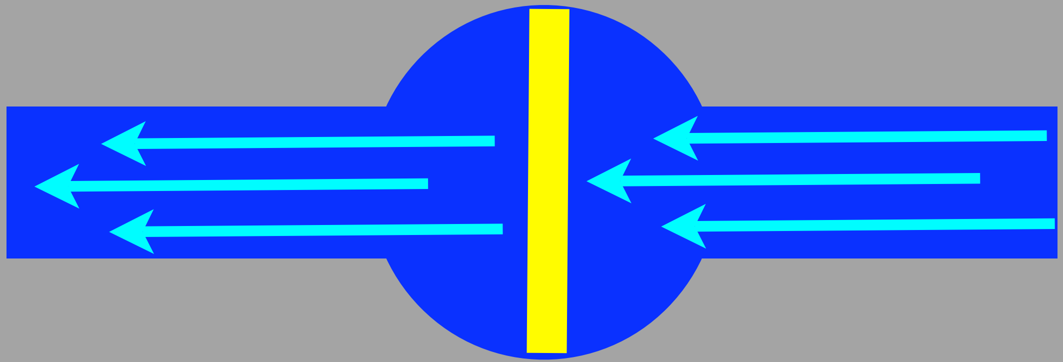
$C = \square_o\square A/d$ $\square_o$ electrostatic permittivity

↑A = ↑C ↑d = ↓C

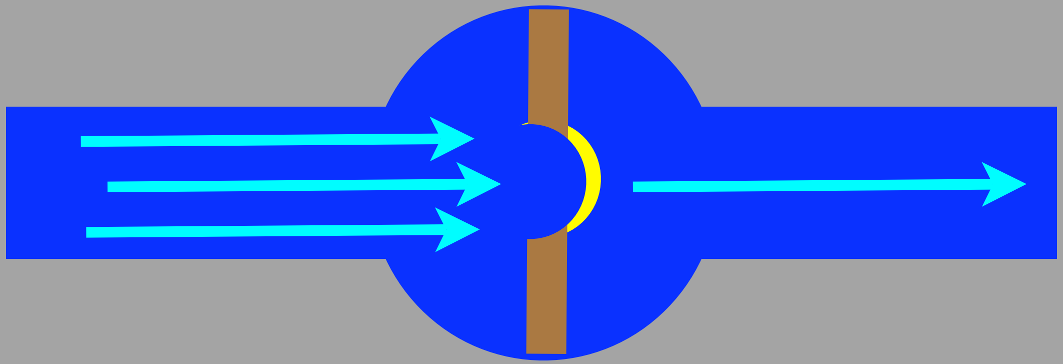
Capacitor



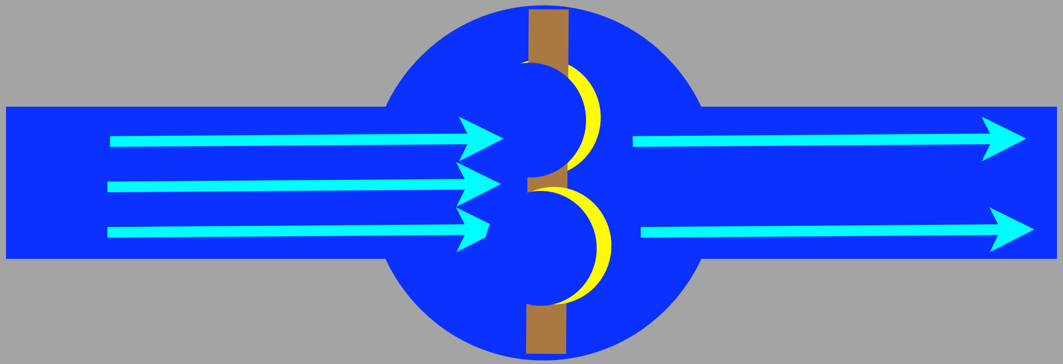
Water pressure



Release of pressure

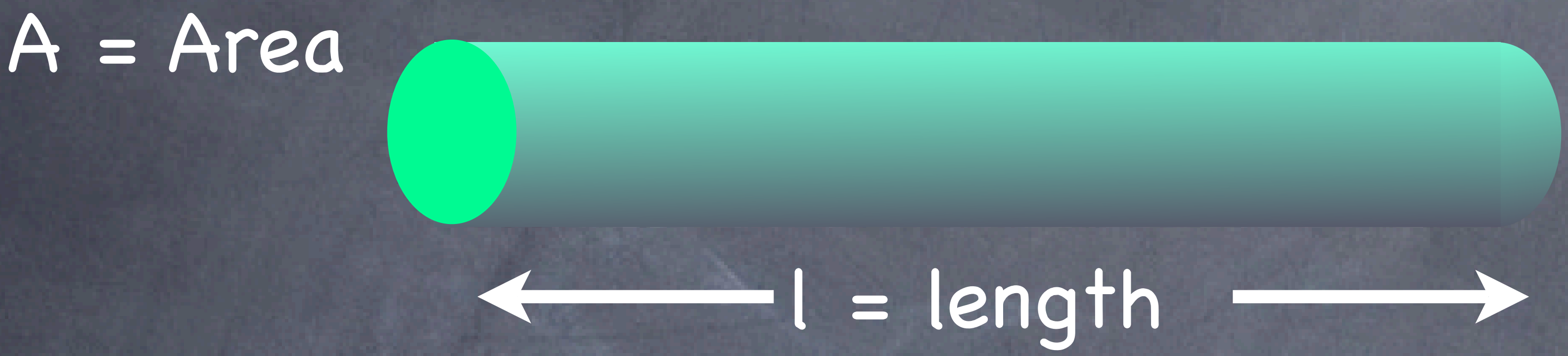


Small Capacitance



Capacitors in parallel add larger Capacitance

Resistance


A = Area

l = length

Ohm's law

$R = V/I$ Ohms (Ω)

$R = \rho l/A$

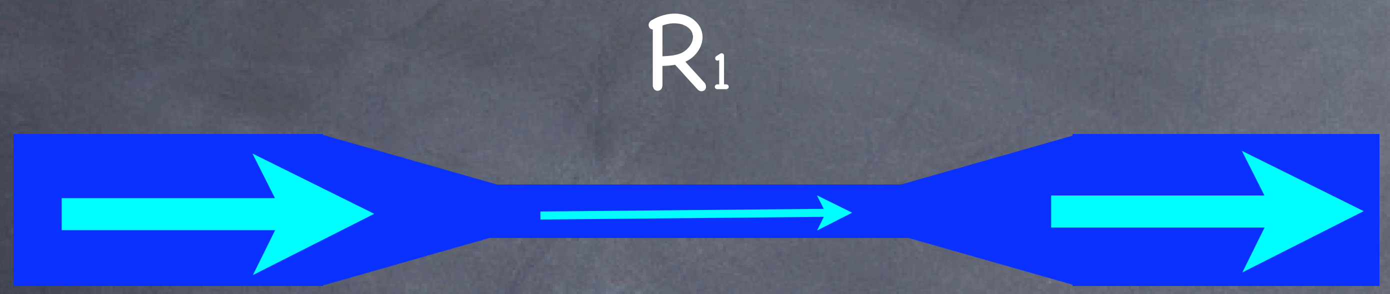
ρ resistivity

↑l = ↑R

↑A = ↓R

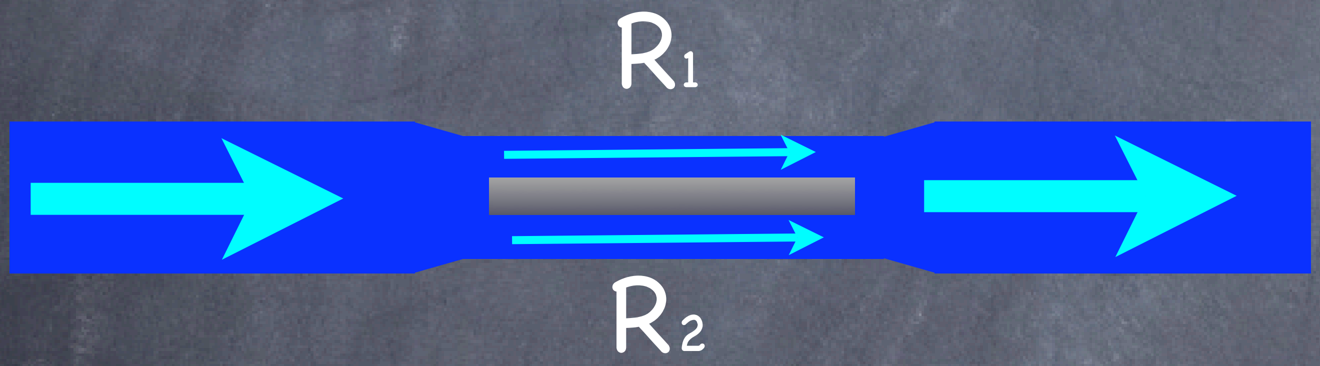
For the same current, a larger R produces larger V

Resistors



For ion channels is better to think in terms of conductance

$$R_1 = 1/g_1$$

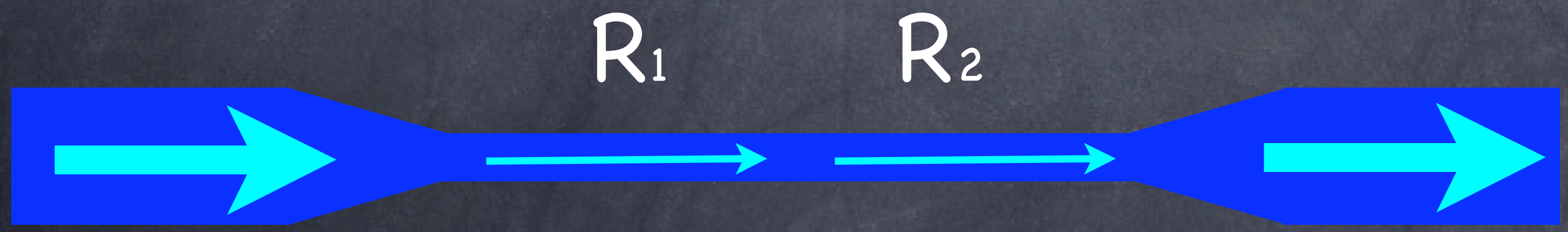


As the # of Rs in parallel increases RT decreases!

$$1/R_T = 1/R_1 + 1/R_2$$

More (open) channels in the membrane more conductance

$$g_T = g_1 + g_2$$



$$R_T = R_1 + R_2$$

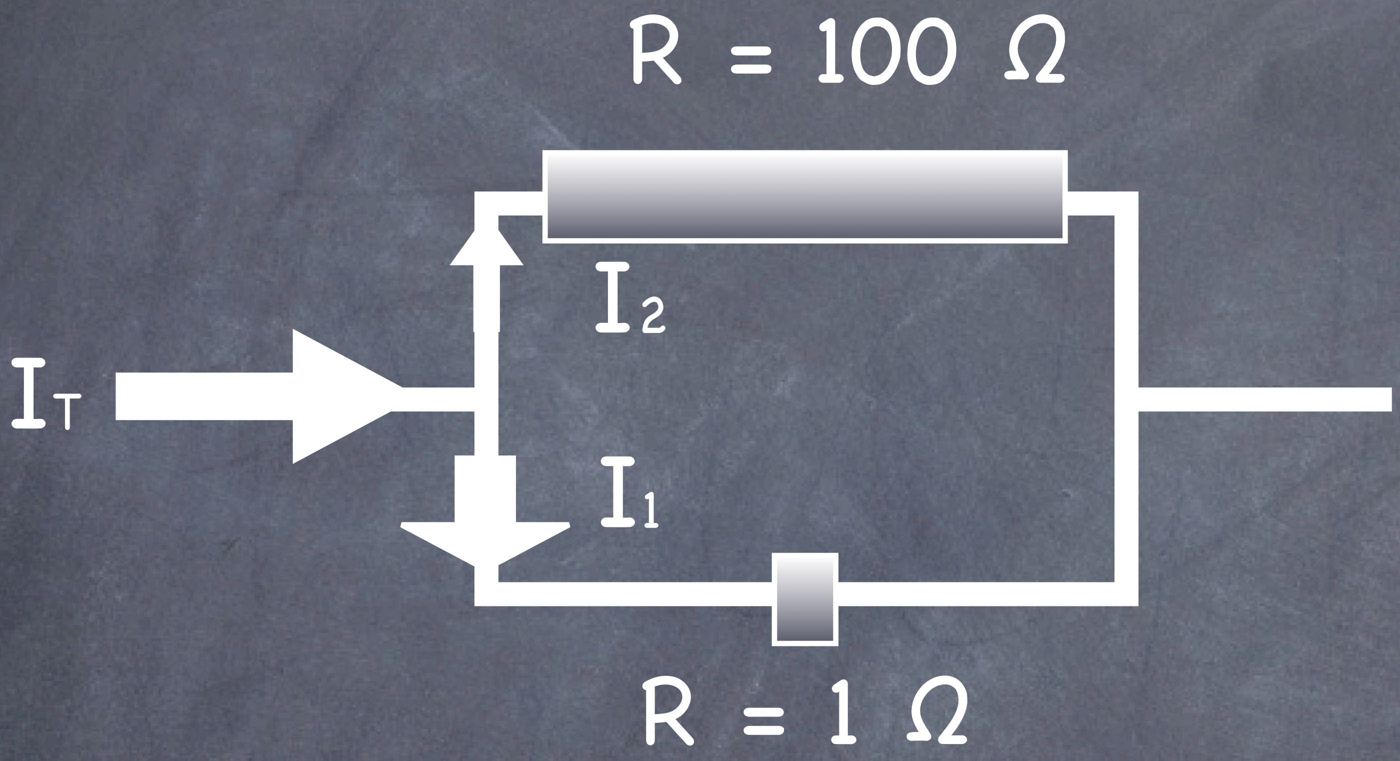
Long, thin parts of a neuron have large resistance!

Some useful equations

Current	I = Coulombs/second Amperes (A)
Ohm's law	$V = IR$
Capacitance	$C = Q/V$ Coulombs/Volts (F)
Voltage across capacitor	$V = Q/C$
Changing the voltage in a capacitor	$\Delta V = \Delta Q/C$
We change the charge by passing current	$Ic = \Delta Q/\Delta t$
The change in V depends on the duration of Ic	$\Delta V = Ic \square \Delta t/C$

Also remember...

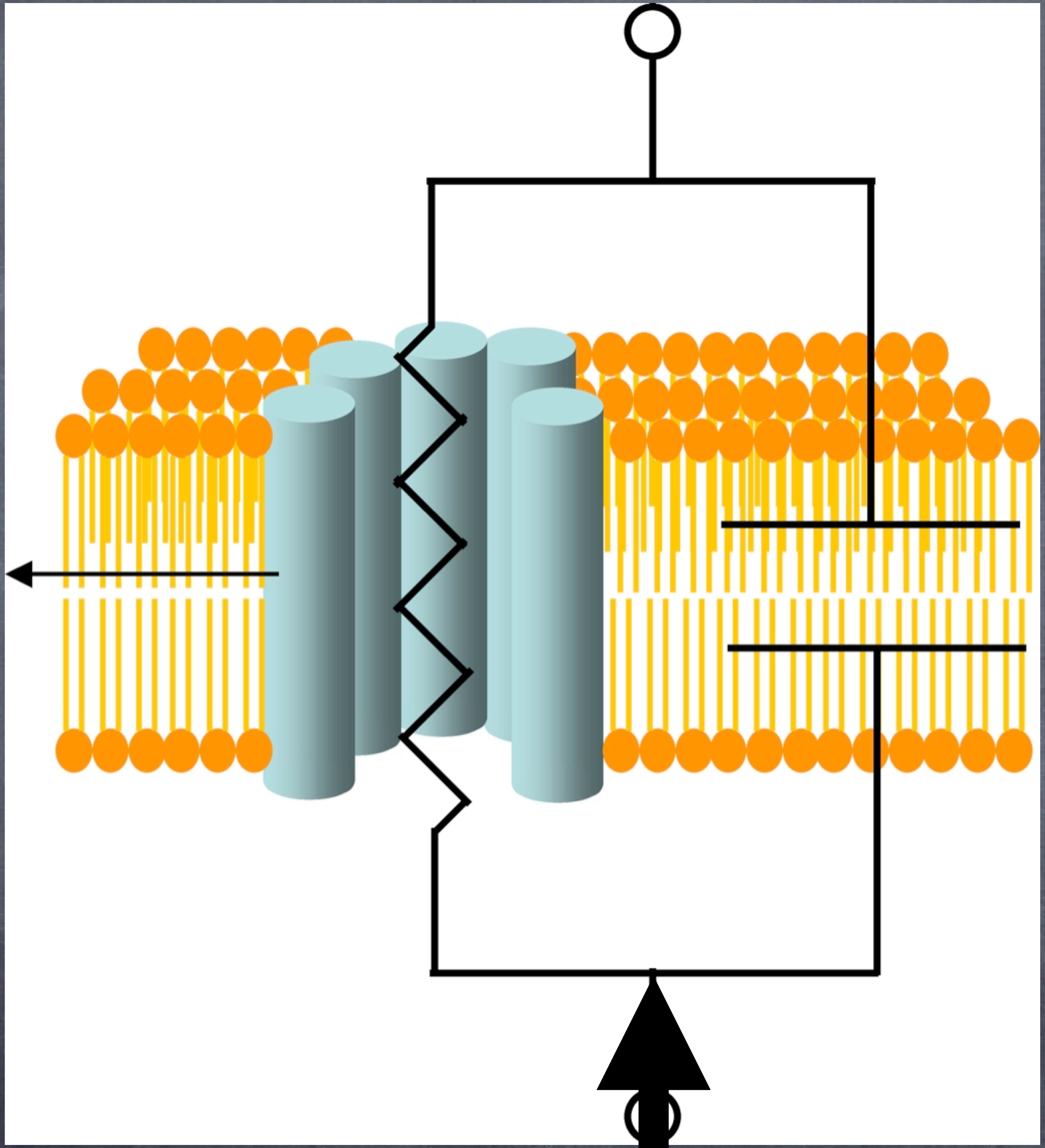
Current likes to flow through the path with less resistance



And

$$I_T = I_1 + I_2$$

Electrical model of the cell membrane



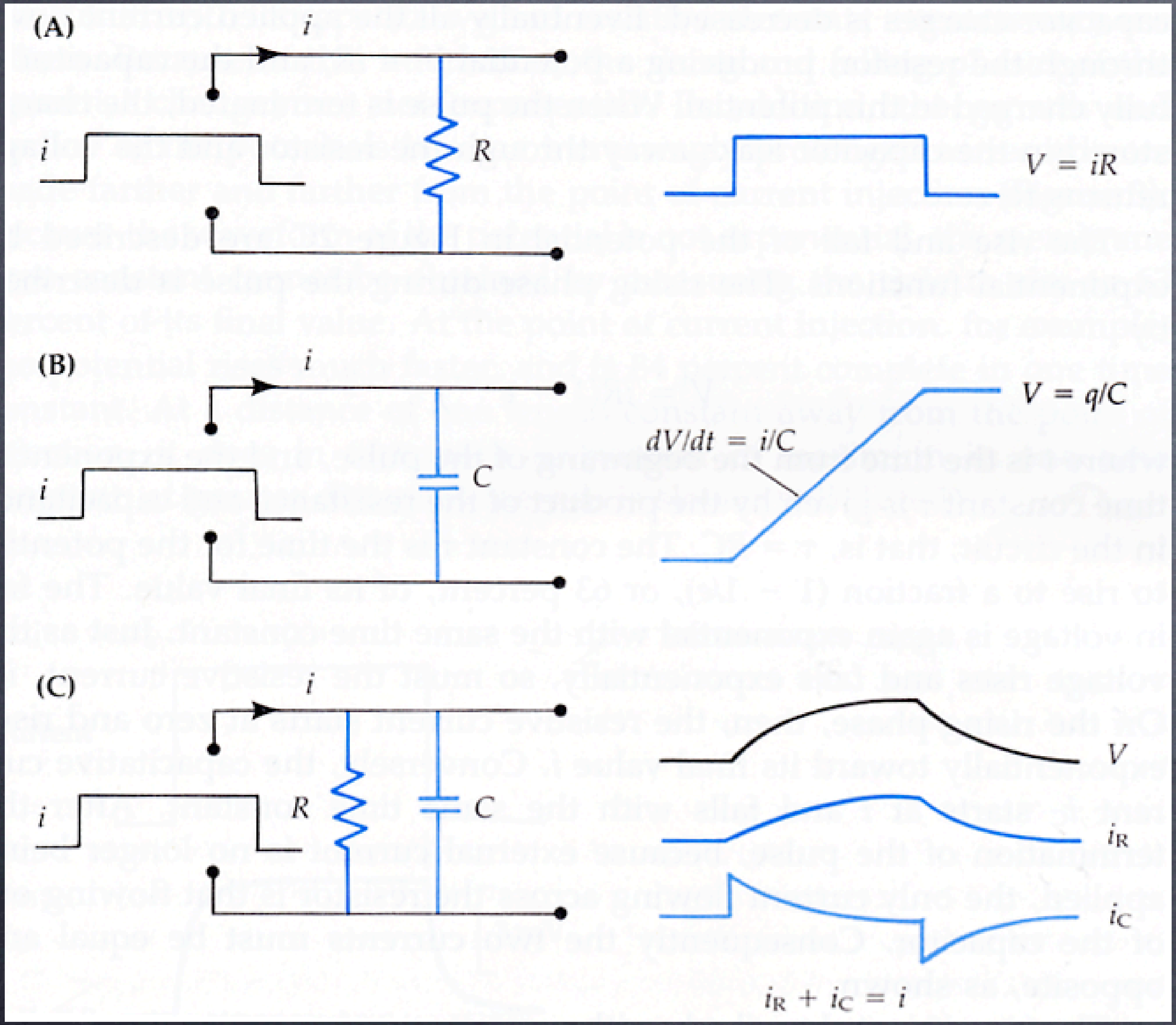
Ionic membrane current I_i

Capacitive membrane current I_c

Membrane current I_m

$$I_m = I_i + I_c$$

Effects of passing current on circuits containing R and C

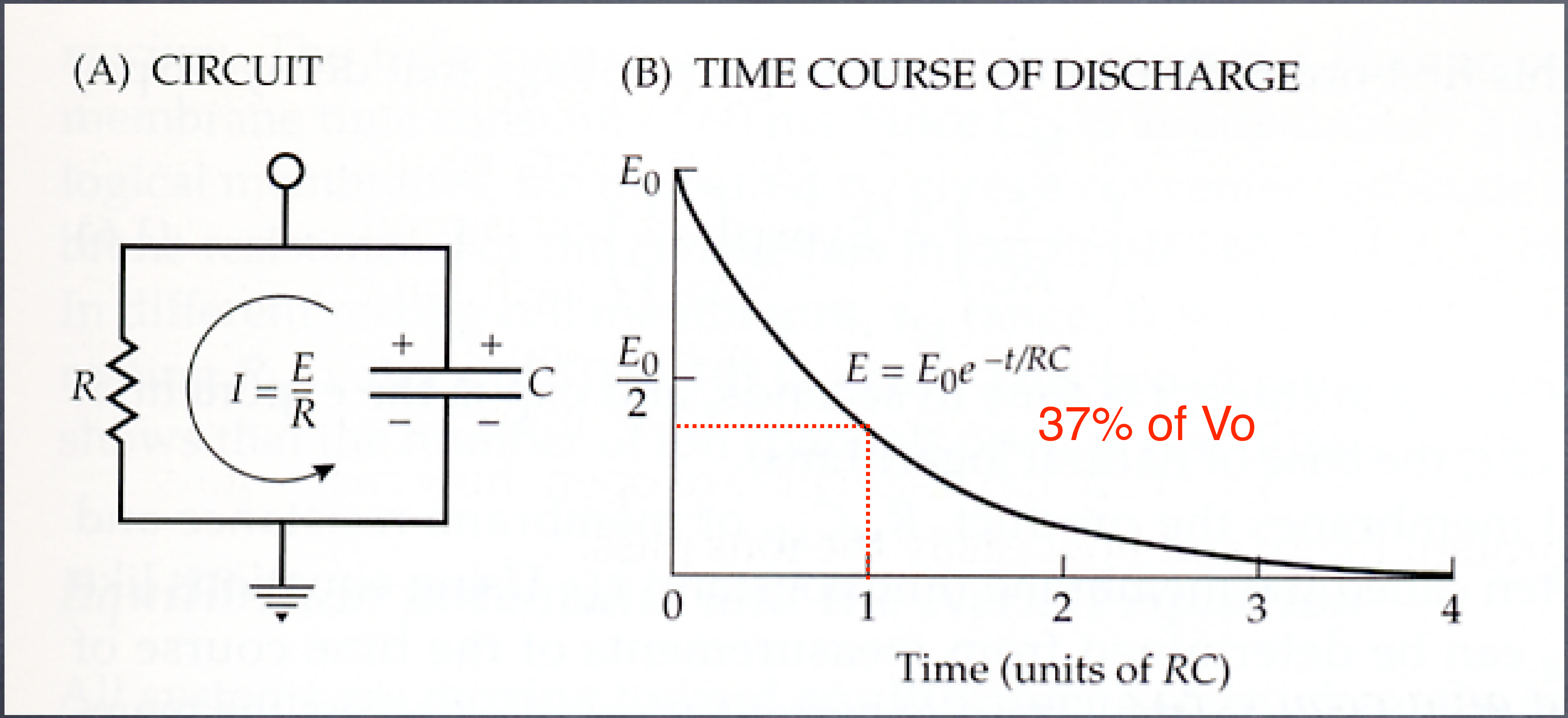


V changes instantaneously with I

V changes linearly in time with I

V changes exponentially with a time constant = RC

RC circuits



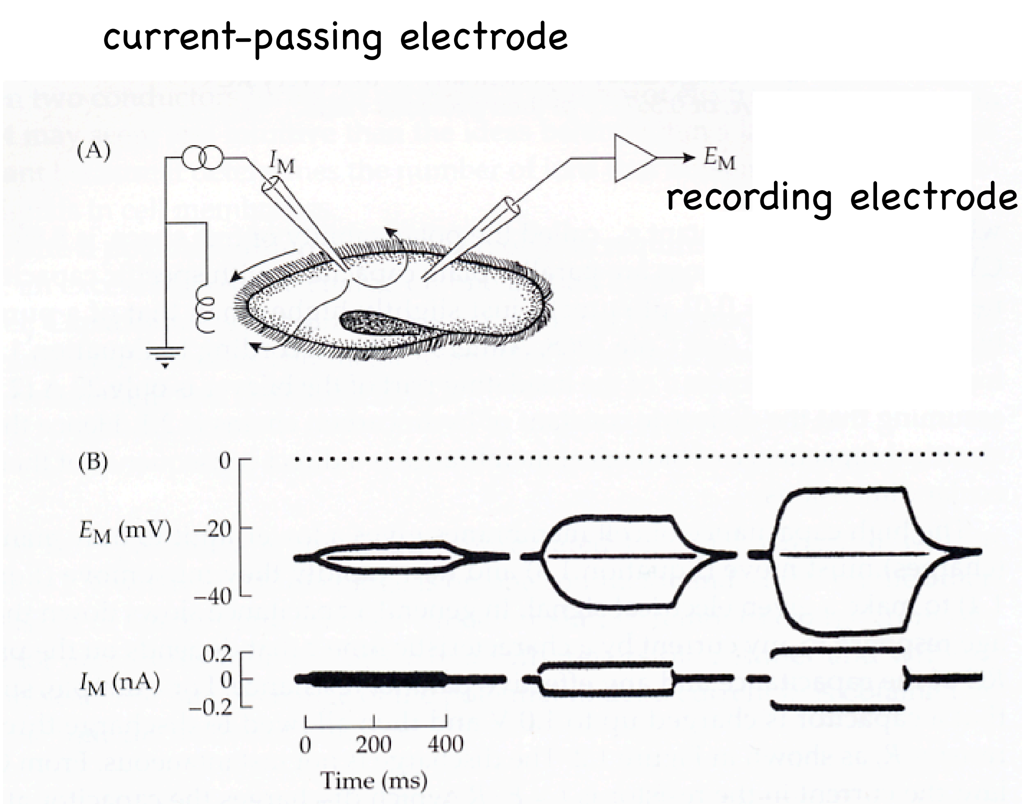
(E and V are the same)

For a rising exponential

$$V = V_0(1-e^{-t/RC})$$

Experiment 2

Passing current and recording the membrane potential from a paramecium

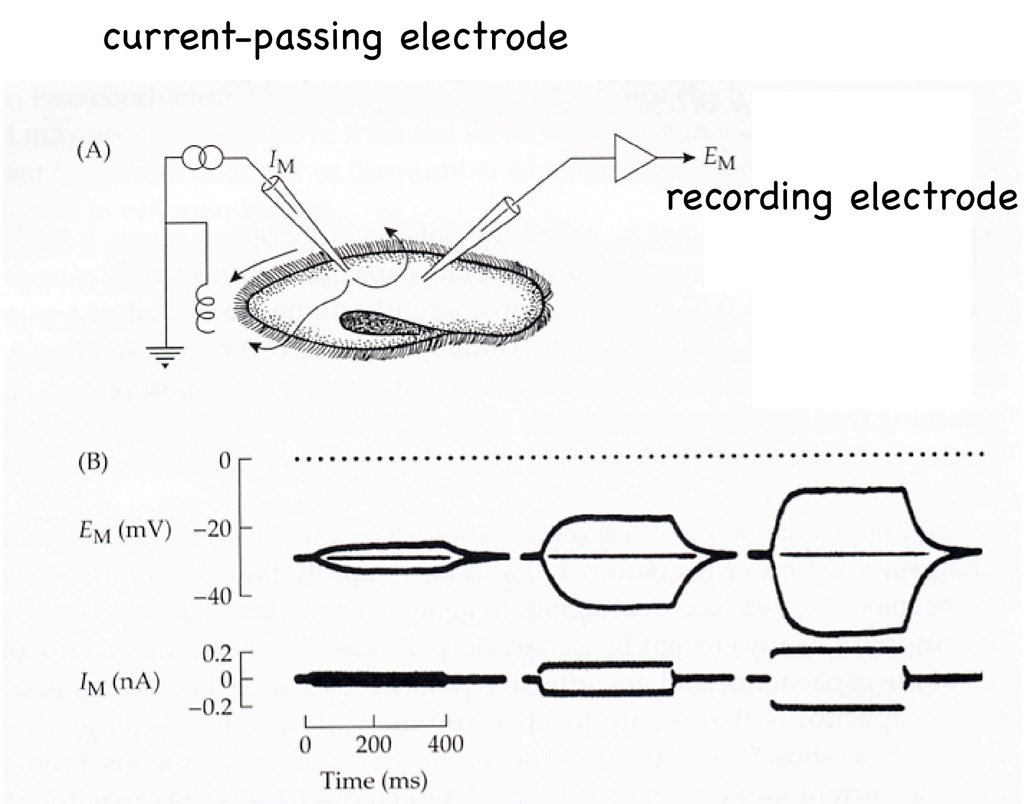


“electrotonic potential”

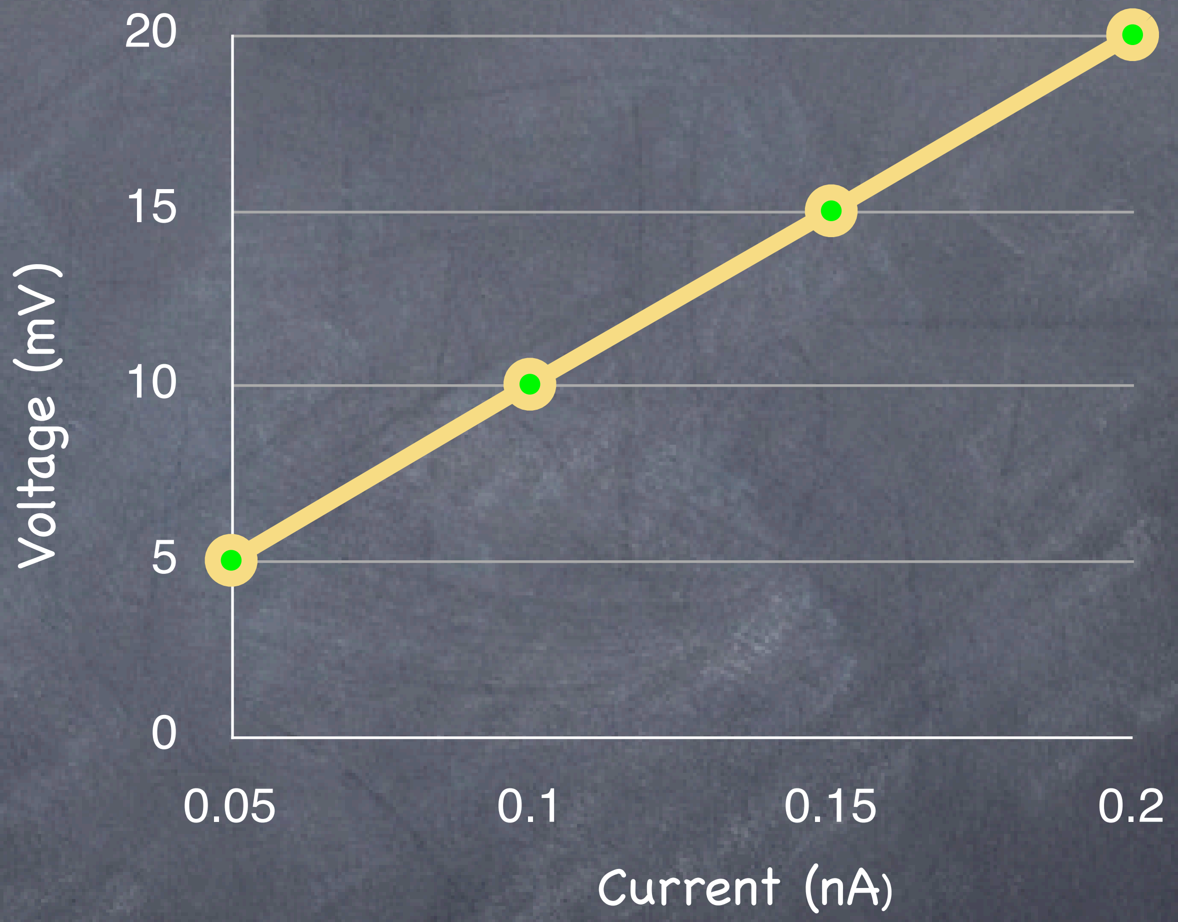
Negative current makes the membrane potential more negative
hyperpolarization

Positive current makes the membrane potential more positive
depolarization

Linear relationship between current and voltage



“electrotonic potential”



$$V = I \square R_{in}$$

Input Resistance $R_{in} = 100\ \mathrm{M\Omega}$

Specific membrane resistance

cross section of a cell

To compare cell with different sizes

The specific membrane resistance

(resistance per area)

$$R_M = \Omega \cdot cm^2$$

depends on the # of channels per cm^2

More channels make R_M smaller

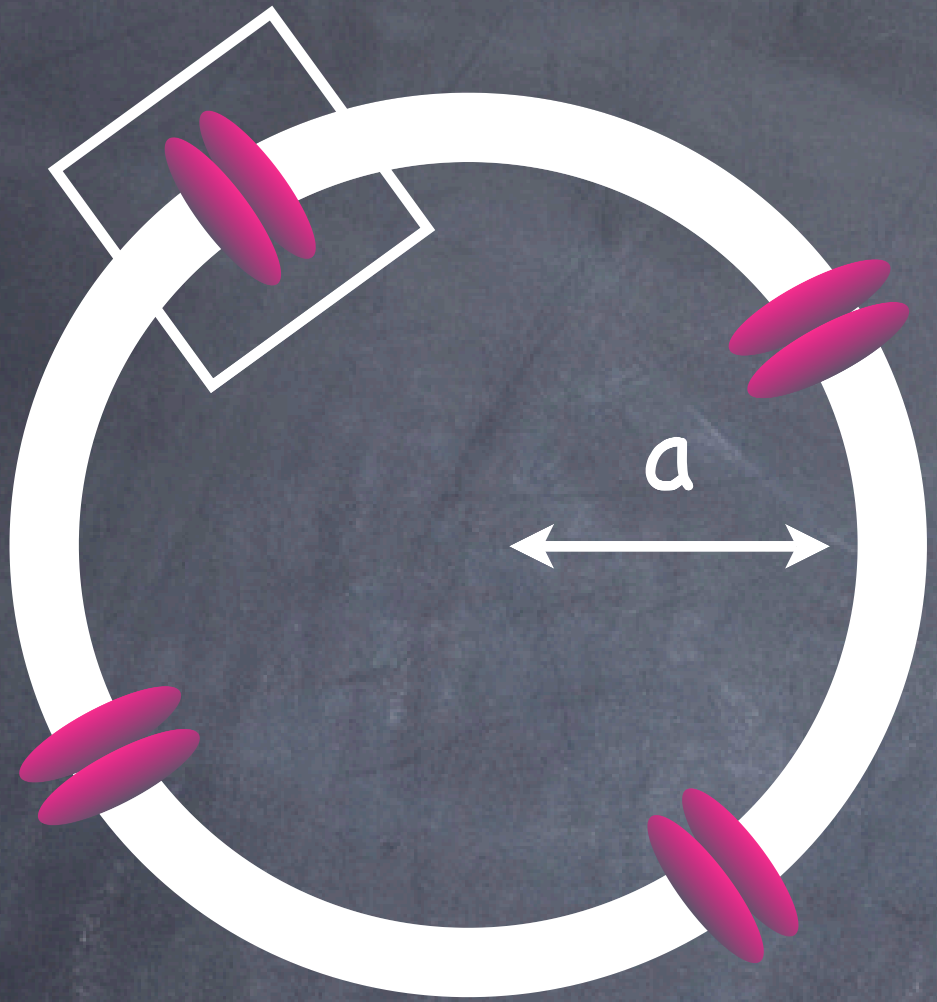
For a spherical cell

$$R_{in} = R_M / 4\pi a^2 \qquad a = \text{radius}$$

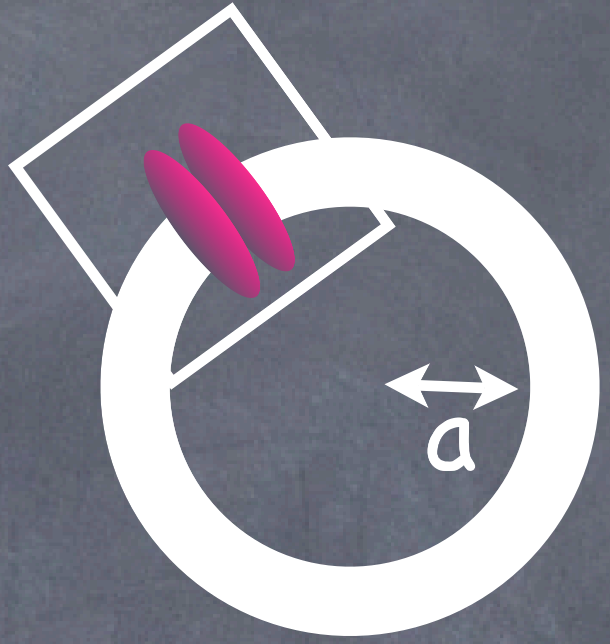
R_{in} determines how much the cell depolarizes in response to a steady current

Example

same $R_M = 2000\ \Omega\square\text{cm}^2$



Cell diameter is 50 m



Cell diameter is 5 m

$a = 25\ \ \text{m} = 25\square10^{-4}\text{cm}$

$R_{in} = R_M/4\pi a^2$

$R_{in} = 2000\ \Omega\square\cancel{\text{cm}^2}/4\pi(25\square10^{-4}\cancel{\text{cm})^2}$

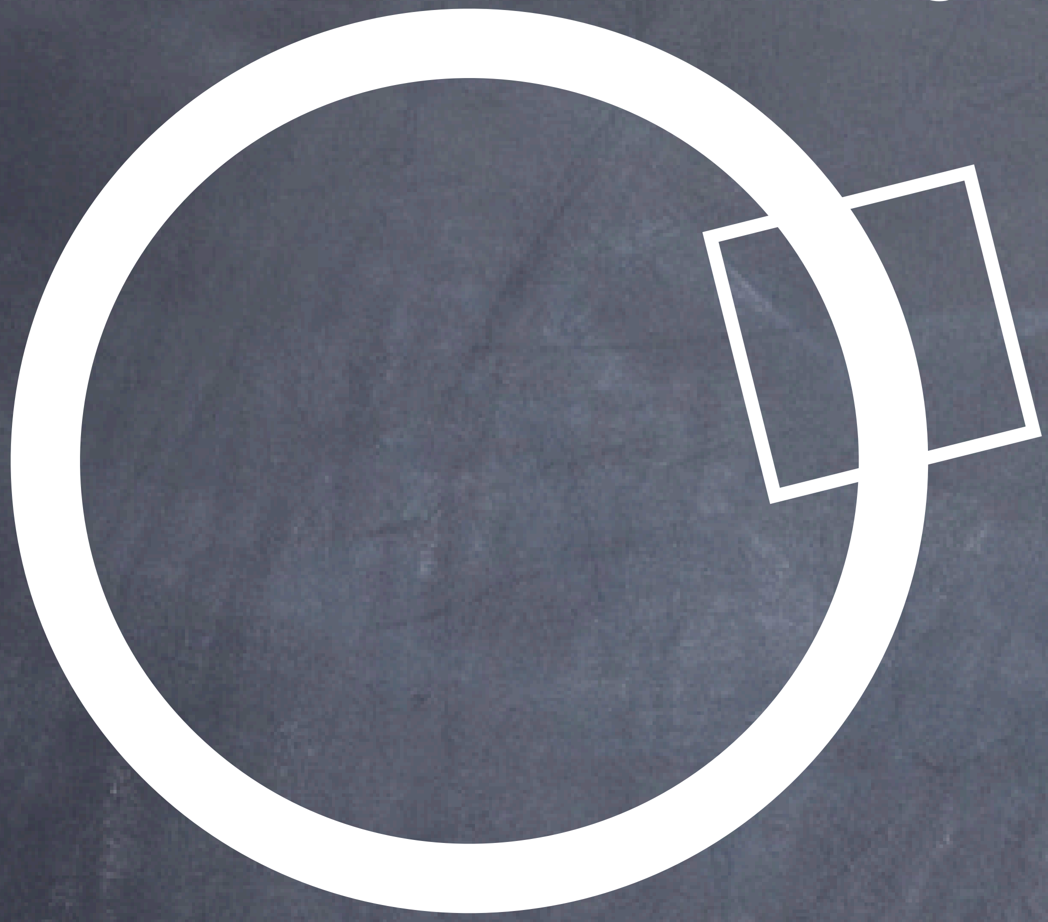
$R_{in} = 25\ \text{M}\Omega$

your numbers here...

$R_{in} = 637\ \text{M}\Omega$

R_{in} is larger in a smaller cell

Specific membrane capacitance of biological membranes



$C_M = 1 \quad F/cm^2$

For a cell at -80 mV how many ions is this?

$$C_M = Q/V$$

$$Q = 10^{-6}\ C/V \times 0.08\ V$$

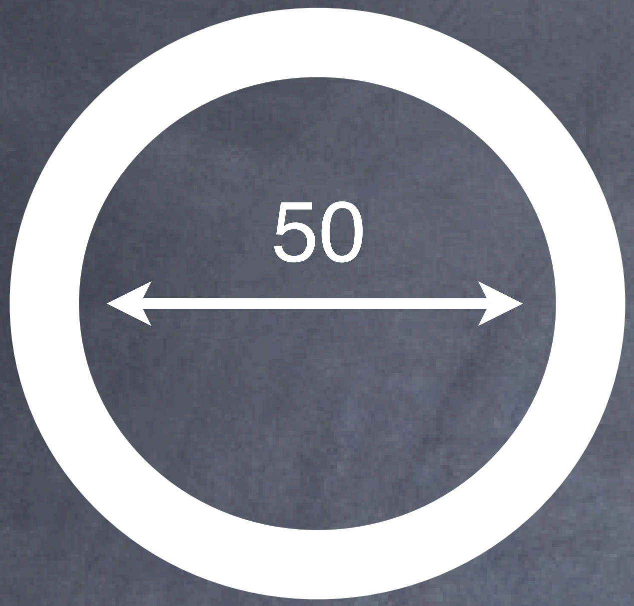
$$Q = 8 \times 10^{-8}\ C/cm^2$$

Faraday constant ≈ 10^{5} Coulombs/mole

Avogadro's number = $6.02 \times 10^{23}\ mole^{-1}$

Then this is 4.8×10^{11} ions/cm^{2} Is this a lot???

Let's assume the cell is 50 m in diameter



$a = 25 \;\; m = 25\square 10^{-4}\,cm$

<u>surface</u> of sphere $A = 4\pi a^2$

$A = 7.85\square 10^{-5}\,cm^2$

$4.8\square 10^{11}$ ions in 1 cm^2 So total is $4\square 10^{7}$ ions

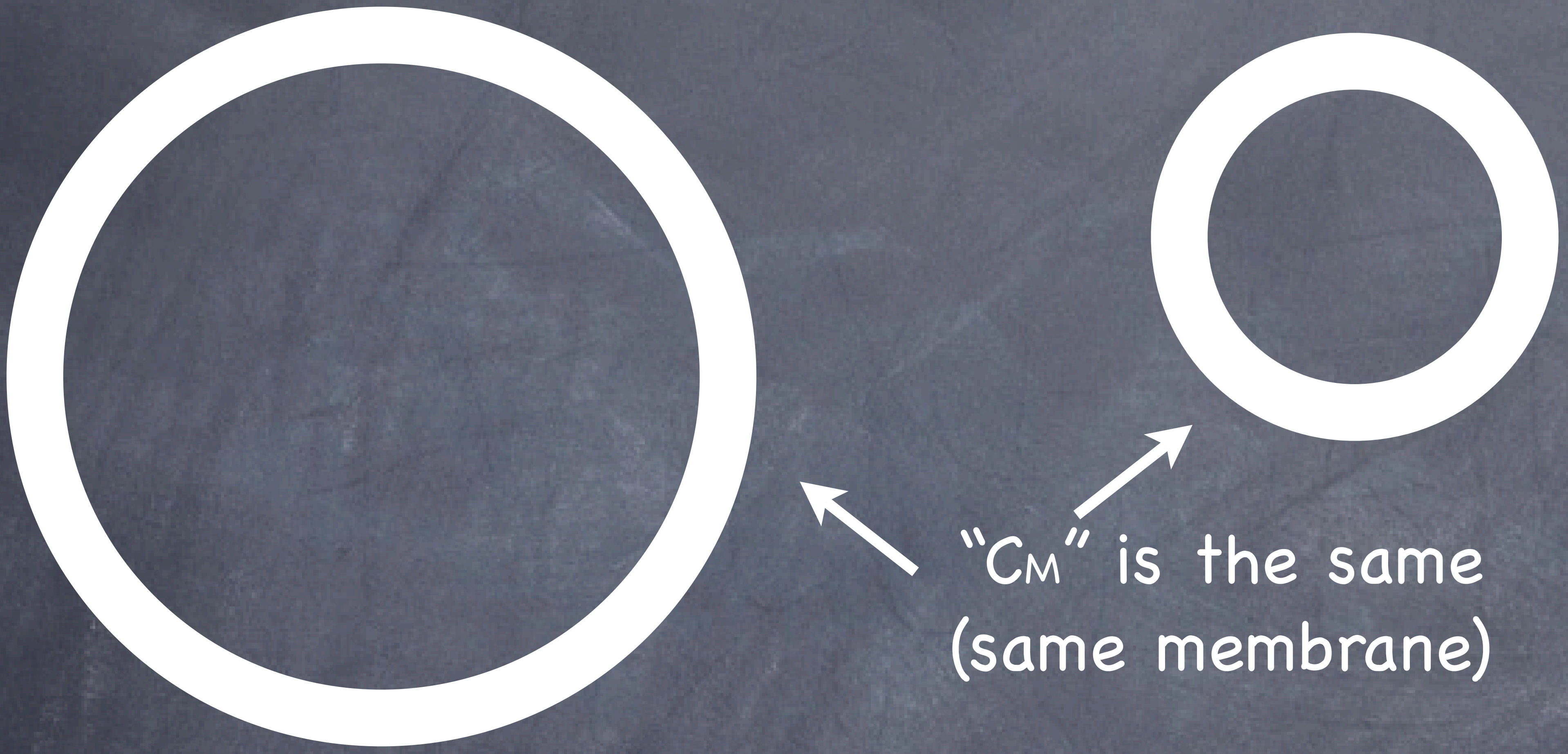
The <u>volume</u> of this cell is $6.55\square 10^{-8}\,cm^3$

Then this number of ions is $\sim 10^{-6}$ M

If KCl inside is 120 mM this means that only ~1/120,000 ions is in excess!

Large cell
large capacitance

Small cell
small capacitance



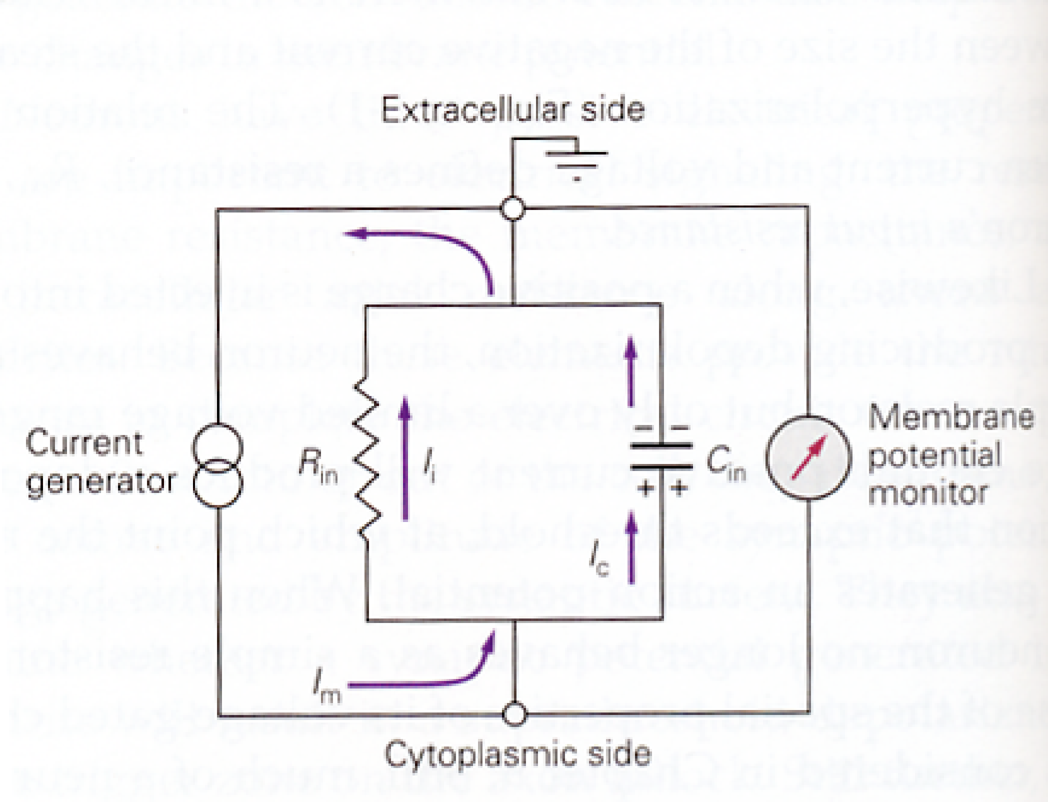
"C_M" is the same
(same membrane)

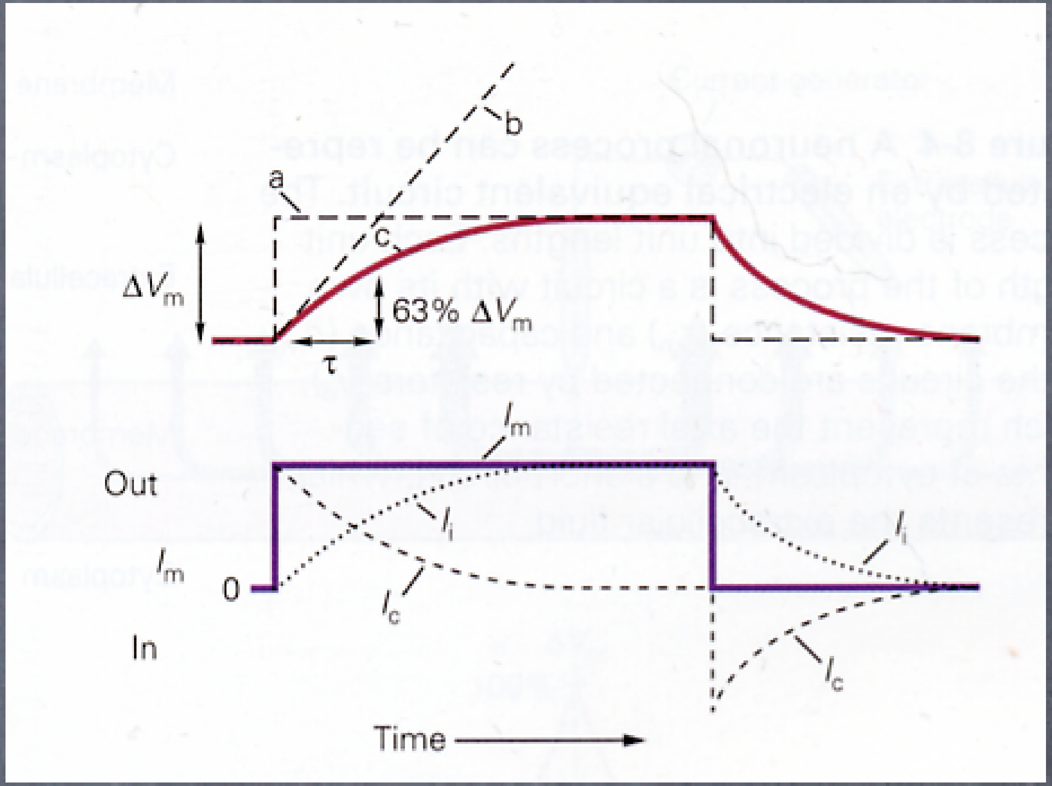
For a spherical cell, the input capacitance

$$C_{in} = C_M \cdot 4\pi a^2 \quad a = \text{radius}$$

More charge (current) is required to change the voltage across a large cell

In summary





$R_{in} = R_M / 4\pi a^2$

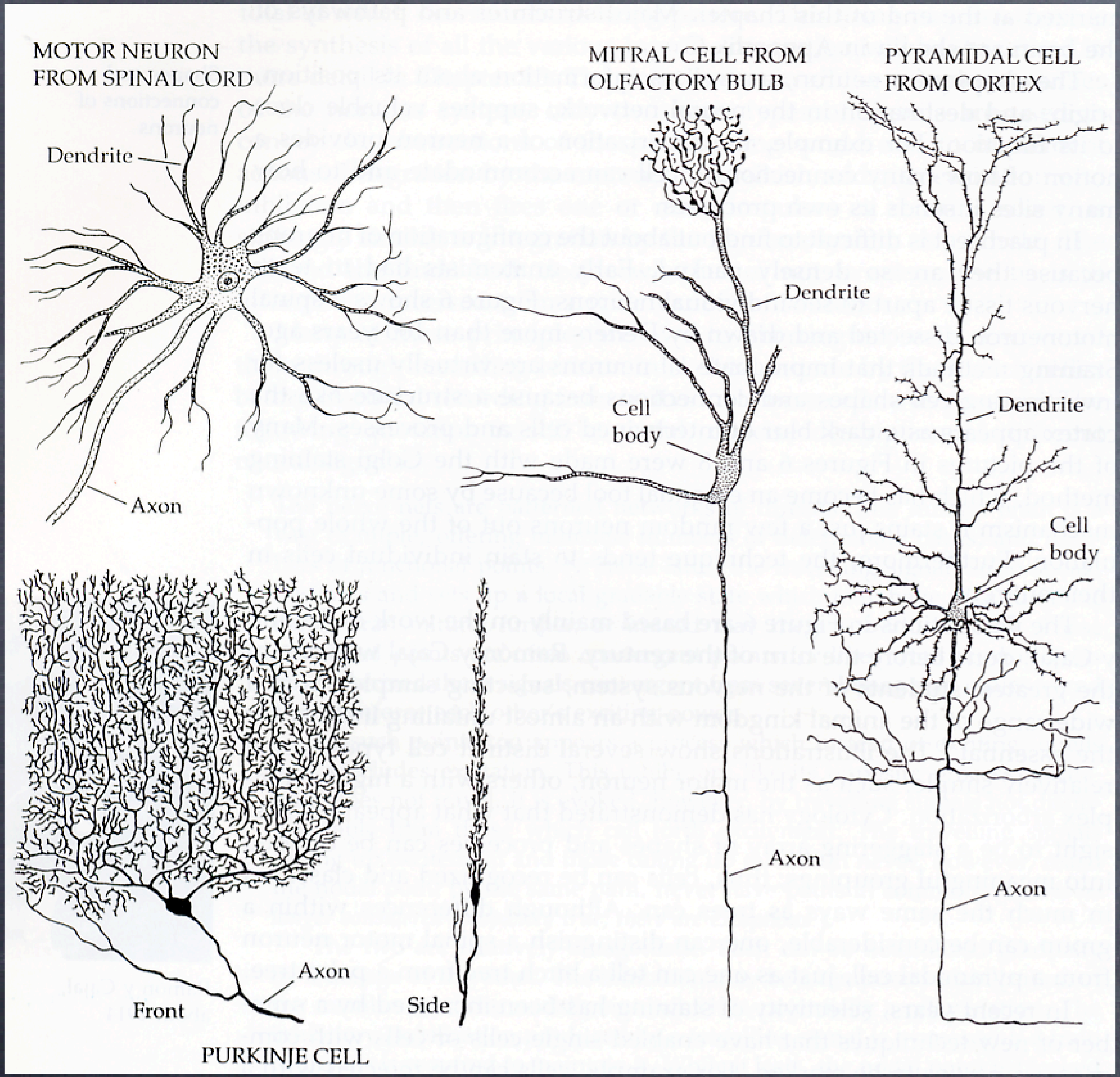
$C_{in} = C_M \square 4\pi a^2$

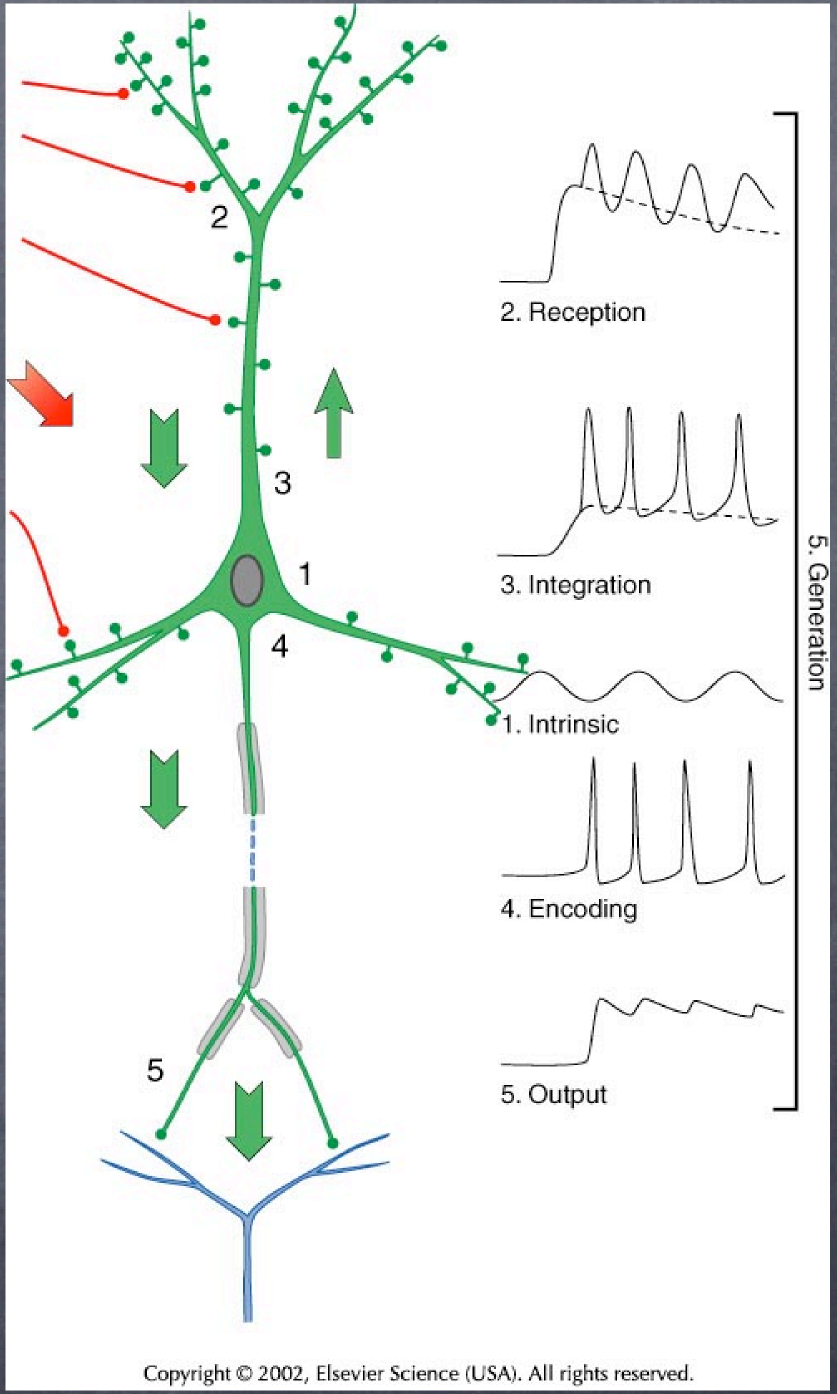
$\tau = R_{in} \square C_M$

The product of input Capacitance and Resistance (τ) determines the time it takes change the potential

Notice that (τ) is not affected by "a"

Real Neurons





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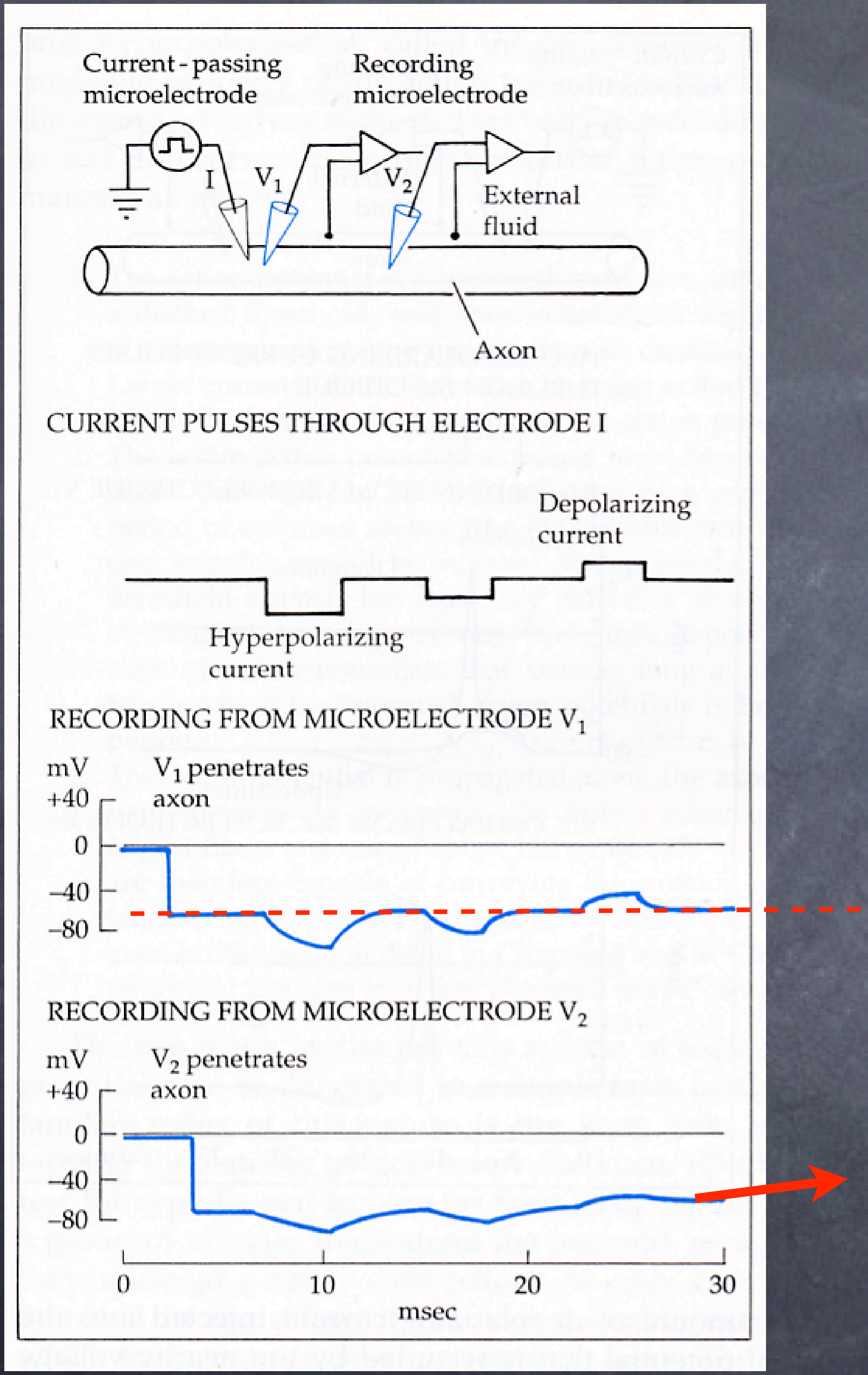
How are signals affected by the passive properties of the membrane ?

Experiment 3

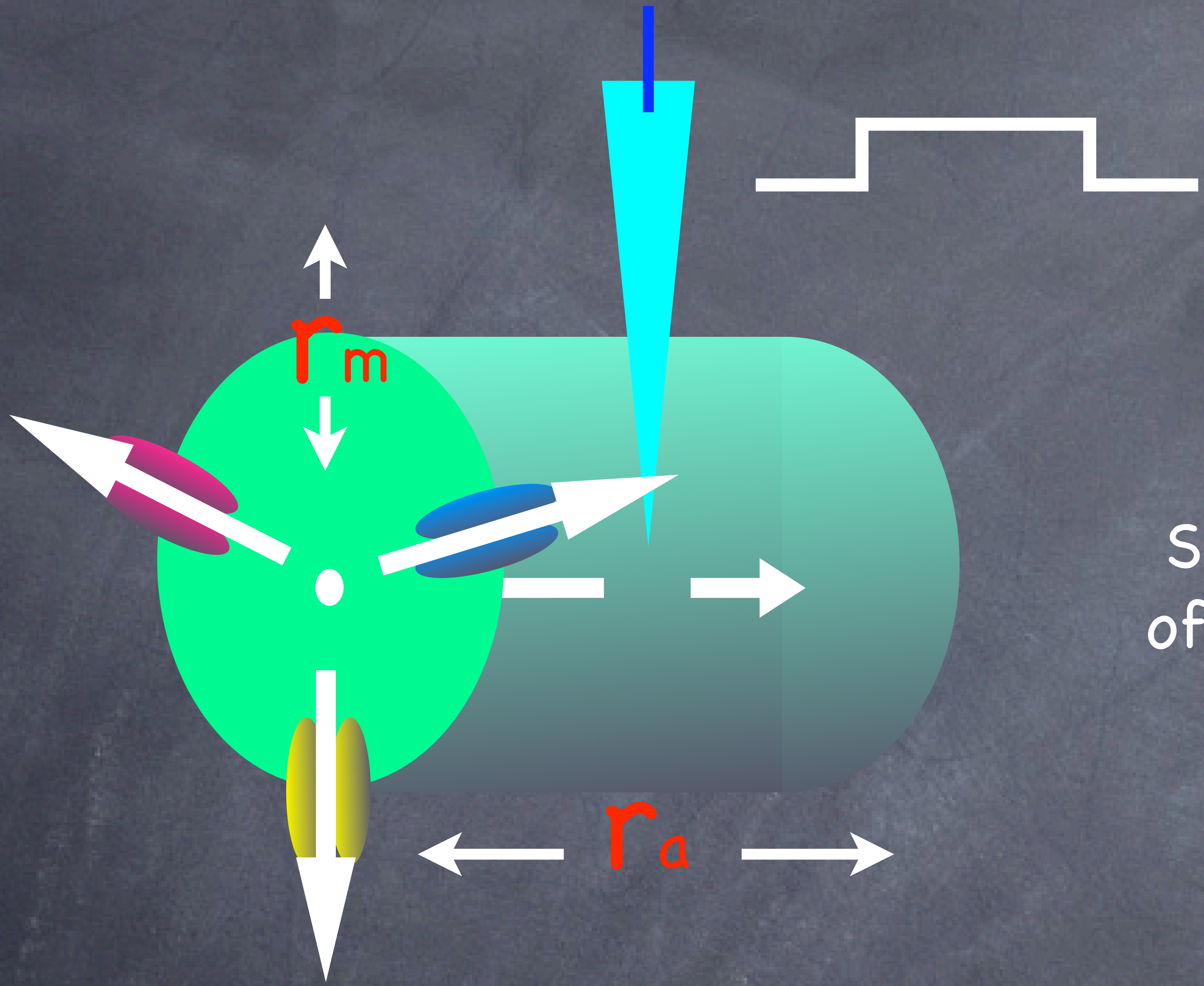


Local potentials are graded

resting potential

Almost no potential change is observed, why?

Current in axons and dendrites



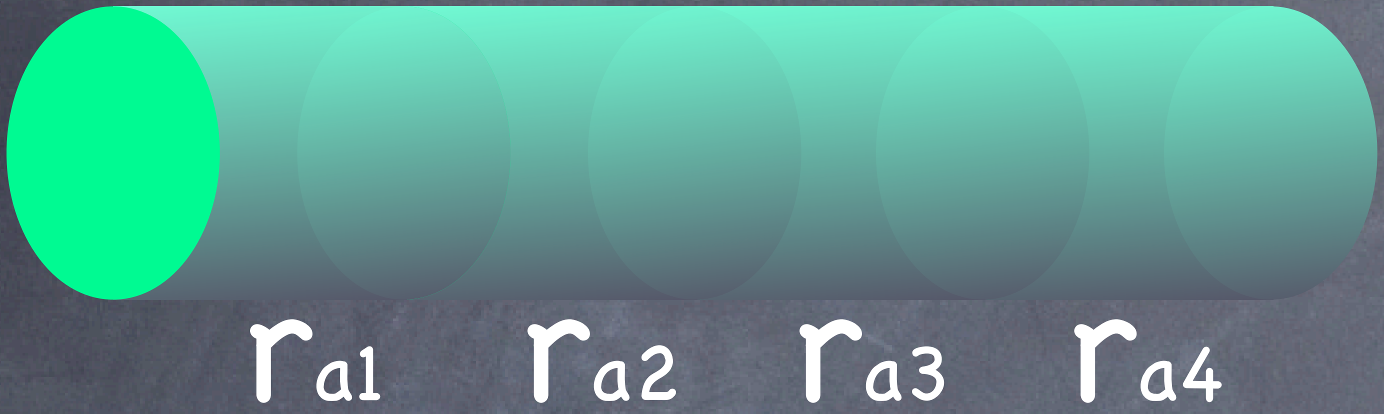
Current pulse

Section of axon or dendrite of determined length (x). In this case 1 cm.

r_a axial resistance (Ω/cm)

r_m membrane resistance ($\Omega \cdot$cm)

Axial resistance increases with distance (x)



Total axial resistance

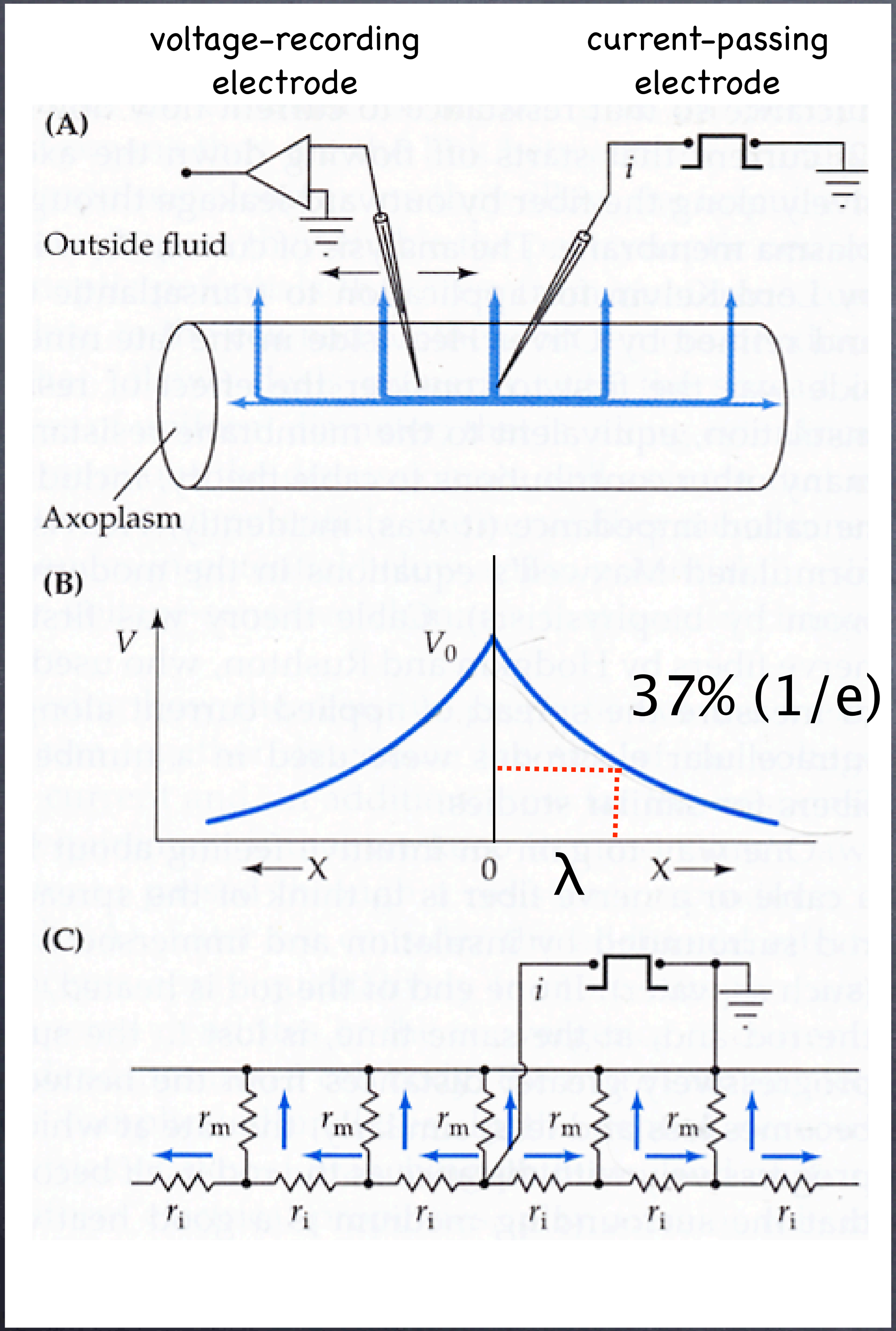
$$r_x = r_a \square x \quad (x = 4)$$

(remember RT = R1 + R2)

Near the site of injection, the current flows through r_m (less resistance)

Then $V_o = I_m \square r_m$

What is the value of V at increasing distances from the site of current injection?



$$V = V_0\,e^{-x/\lambda}$$

λ is the length constant

$$\lambda = \sqrt{(r_m/r_a)} \quad (\text{cm})$$

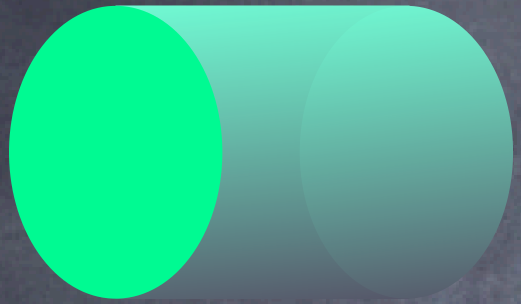
Increasing r_m increases λ

Decreasing r_a increases λ

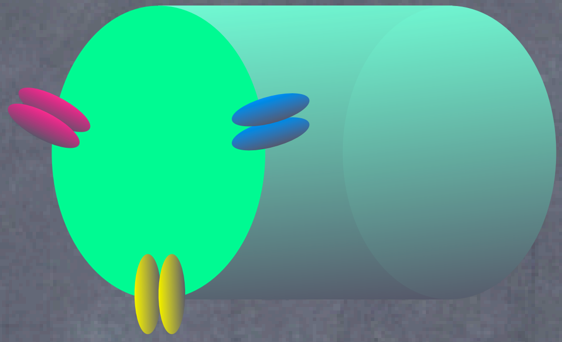
i.e. V is closer to V_0

For 1 cm of cytoplasm (dendrites or axon)

□ (Ω□cm) resistive property of 1 cm^3 of cytoplasm (dendrites or axon)



$r_a = \square/(\square a^2)$



$r_m = R_m/(2\square a)$

← 1 cm ←

$$\lambda = \sqrt{(r_m/r_a)}$$

$$\lambda = \sqrt{\frac{R_m \square a}{2\square}}$$

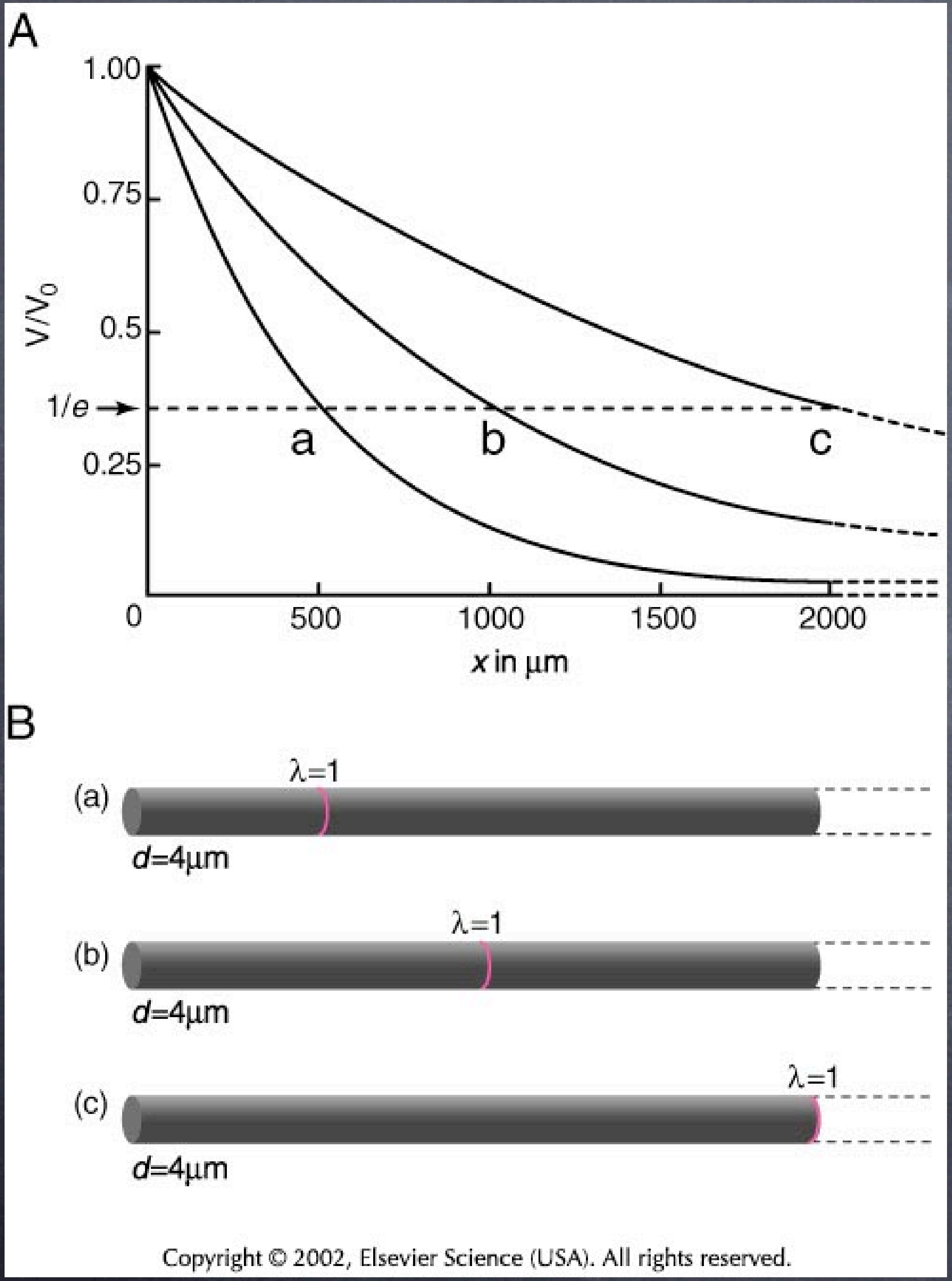
If R_m and □ are constant

$$\lambda = \sqrt{K\,a}$$

The length constant is proportional to the square root of the radius of the process

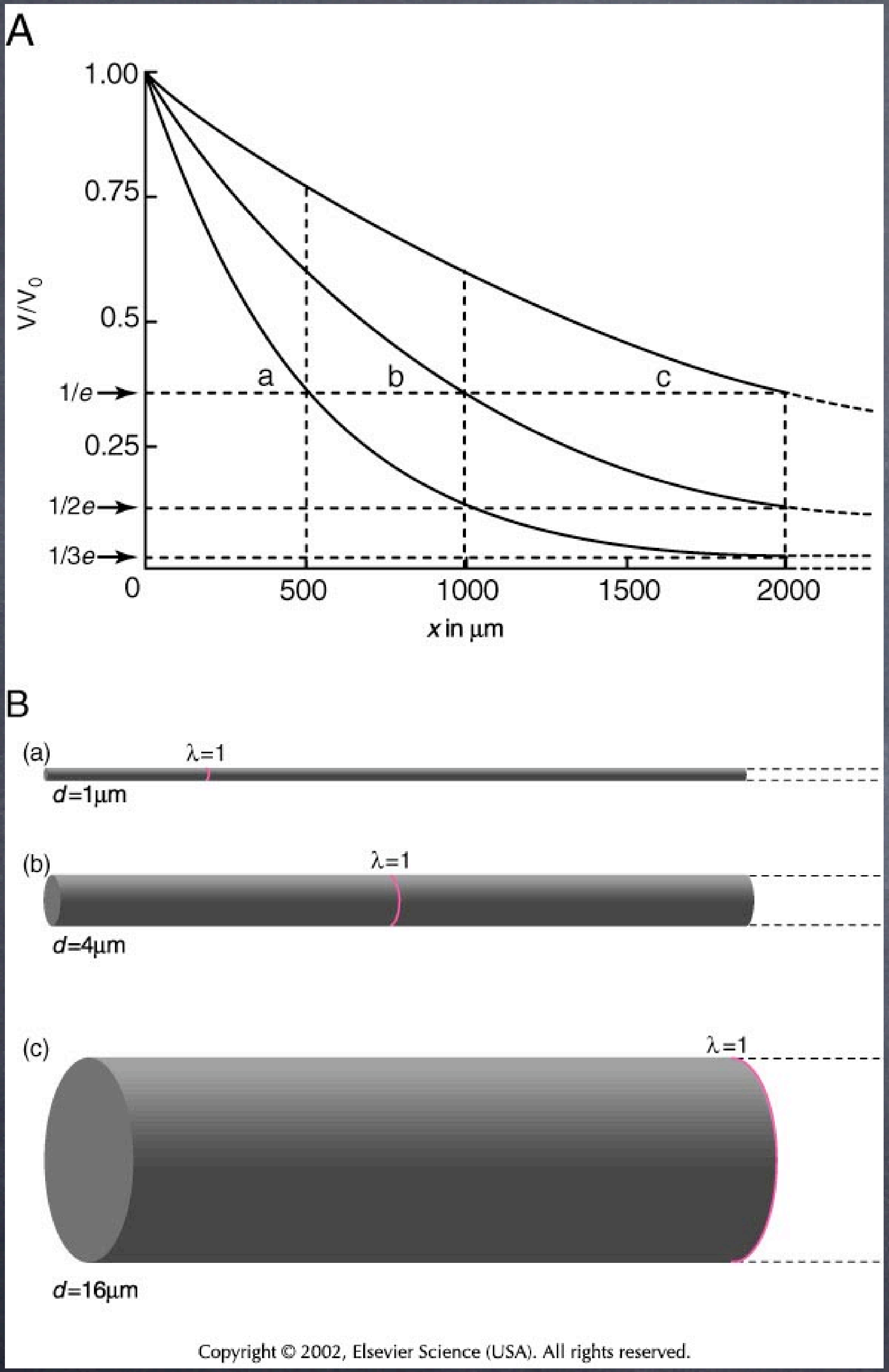
For neurons is usually 0.1 to 1 mm

Effect of length constant



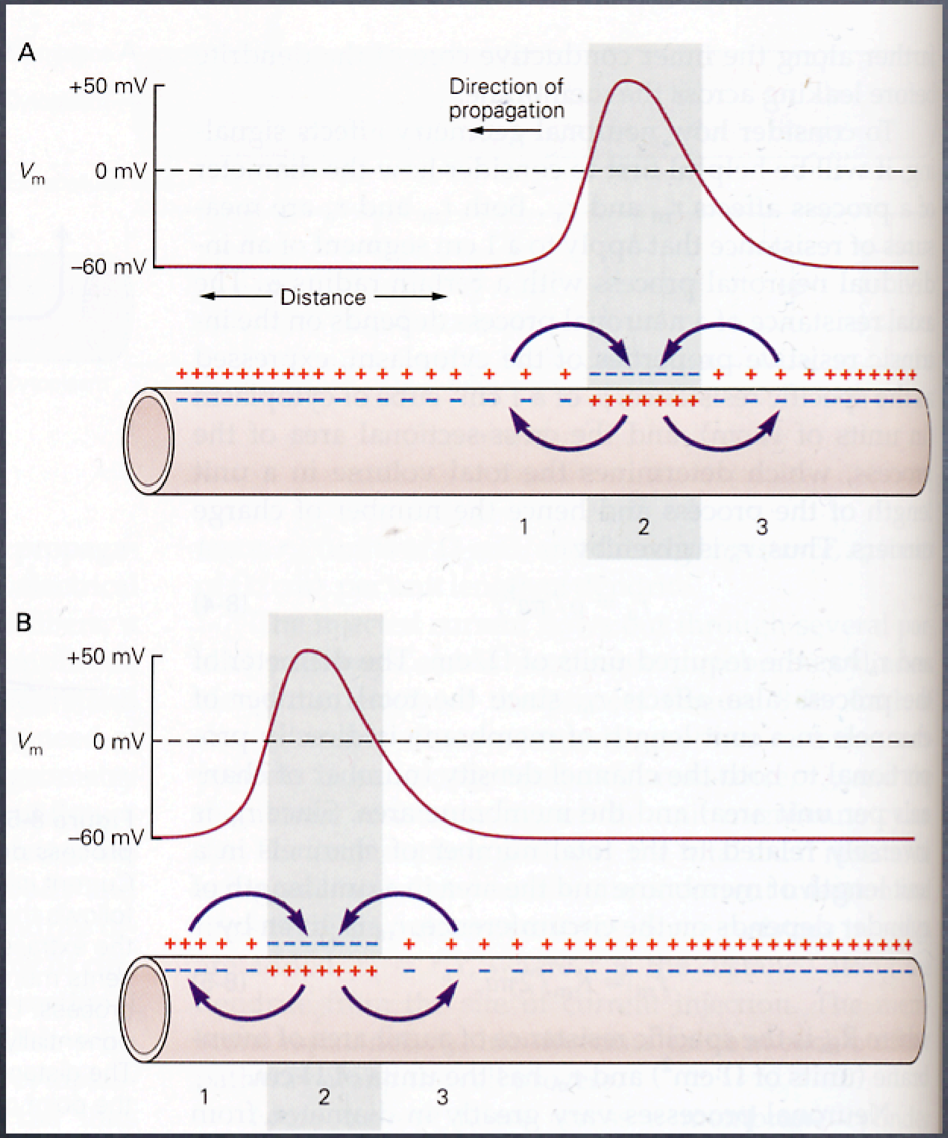
For a dendrite or axon with the same diameter as the <u>length constant increases the potential decreases less with distance</u>.

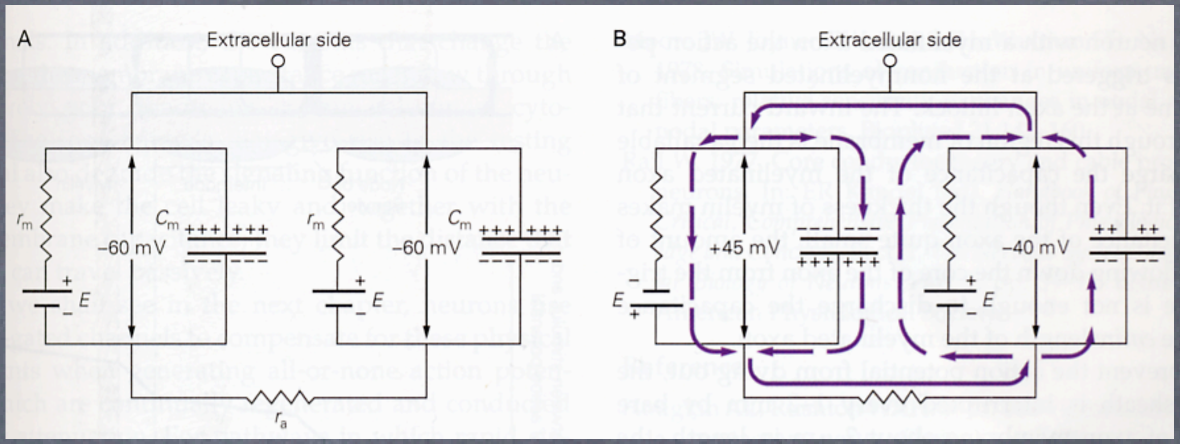
Effect of diameter



For a dendrite or axon with increasing diameters the length constant increases and the potential decreases less with distance.

The passive properties of membranes and axon diameter affect the speed of conduction of action potentials



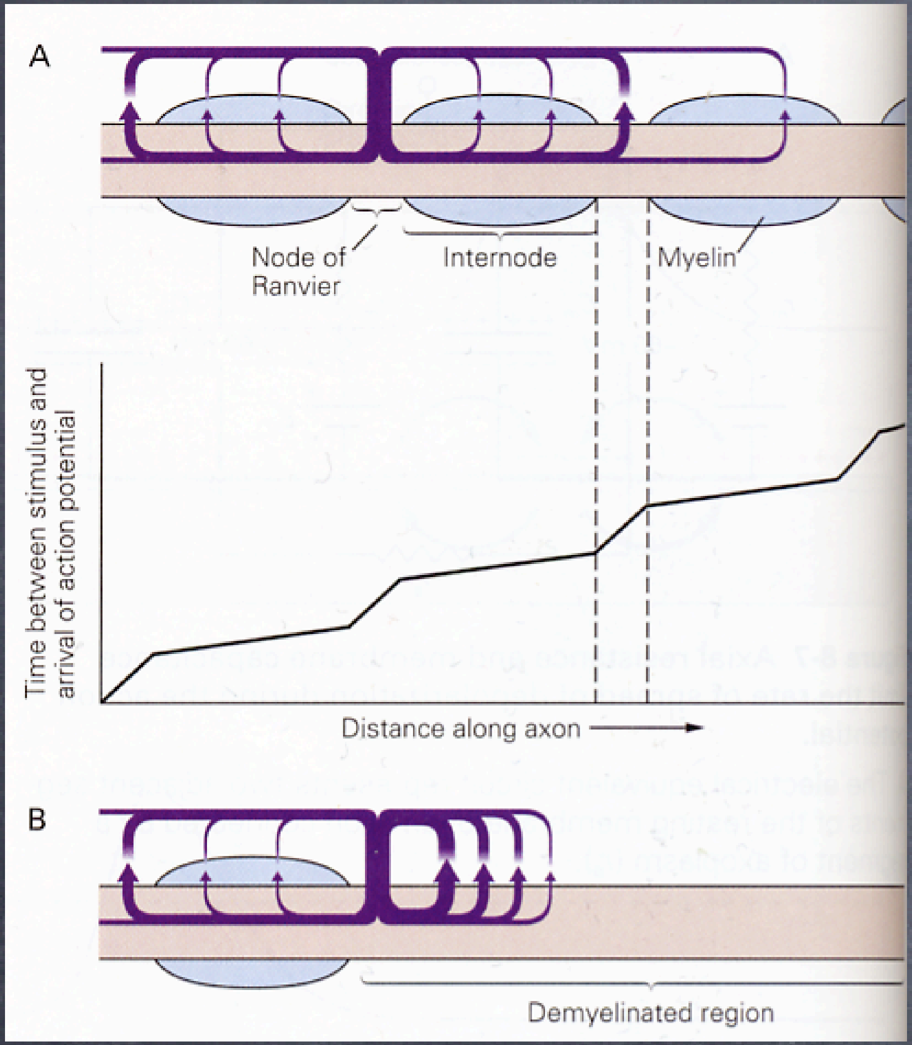


speed of conduction of action potentials is inversely related to r_a□c_m

speed of conduction is increased by increasing the diameter of the axon which decreases r_a

The giant axon of the squid 1 mm !

Myelination, the alternative to increasing the diameter of the axon.



Glial cell wrap around axons many times (20-160 times) this like adding 320 membranes (in series).

This increases Rm and decreases Cm