#### PATHWAY LOGIC AND SYMBOLIC SYSTEMS BIOLOGY

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### PLAN

- Symbolic systems biology
- Executable Specification in RWL
- Biological Processes (What to model)
- Building a PL knowledge base
- Computing with PL models
  - Small KB
  - Egf Stimulation

# SYMBOLIC SYSTEMS BIOLOGY

# BIOLOGICAL SYSTEMS

- Biological processes are complex
  - genes, proteins, metabolites
  - cells, organs, organisms
- Dynamics that range over huge timescales
  - microseconds to years
- Spatial scales over 12 orders of magnitude
  - single protein to cell, cell to whole organism
- Oceans of experimental biological data generated
- Important intuitions captured in mental models that biologists build of biological processes

# SYMBOLIC SYSTEMS BIOLOGY

Symbolic -- represented in a logical framework

- Systems -- how things interact and work together, integration of multiple viewpoints and levels of abstraction
- Which biology? Causal networks of biomolecular interactions and reactions

• Goals:

- Develop formal models that are as close as possible to domain expert's mental models
- Compute with, analyze and reason about these complex networks
- New insights into / understanding of biological mechanisms

# LOGICAL FRAMEWORK

- Making description and reasoning precise
- Language
  - for describing things and/or properties
  - given by a signature and rules for generating expressions (terms, formulas)
- <u>Semantic model</u> -- mathematical structure (meaning)
  - interpretation of terms
  - satisfaction of formulas: M |= wff
- Reasoning -- rules for inferring valid formulae
- <u>Symbolic model</u> -- theory (axioms) used to answer questions

### EXECUTABLE SYMBOLIC MODELS

- Describe system states and rules for change
- Transition graph
  - nodes -- reachable states
  - edges -- rules connecting states
- Path -- sequence of nodes and edges in transition graph (computation / derivation)
- Execution strategy -- picks a path

# SYMBOLIC ANALYSIS I

- Static Analysis
  - how are elements organized -- sort hierarchy
  - control flow / dependencies
  - detection of incompleteness

Forward simulation from a given state
run model using a specific strategy
fast, first exploration of a model

# SYMBOLIC ANALYSIS 11

Forward search from a given state

- breadth first search of transition graph
- find ALL possible outcomes
- find only outcomes satisfying a given property

Backward search from a given state S

- run a model backwards from S
- find initial states leading to S
- find transitions that contribute to reaching S

# SYMBOLIC ANALYSIS III

#### Model checking

- determines if all pathways from a given state satisfy a given property, if not a counter example is returned
- example property:
  - molecule X is never produced before Y
- counter example:
  - pathway in which Y is produced after X

# SYMBOLIC ANALYSIS IV

- Constraint solving
  - Find values for a set of variables satisfying given constraints.
  - MaxSat deals with conflicts
    - weight constraints
    - find solutions that maximize the weight of satisfied constraints
  - Finding possible steady state flows of information or chemicals through a system can be formulated as a constraint problem.

## SYMBOLIC ANALYSIS V

- Meta analysis -- reasoning about the model itself
  - find transitions producing / consuming X
  - find all phosphorylation reactions
  - check that transitions satisfy some property such as stoichiometry
  - transform a model and property to another logic (for access to tools)

# A SAMPLING OF FORMALISMS

- Rule-based + Temporal logics
- Petri nets + Temporal logics
- Membrane calculi -- spatial process calculi / logics
- Statecharts + Live sequence charts
- Stochastic transitions systems and logics
- Hybrid Automata + Abstraction

# EXECUTABLE SPECIFICATION IN REWRITING LOGIC (MAUDE)

#### ABOUT REWRITING LOGIC

- Rewriting Logic is a logical formalism that is based on two simple ideas
  - states of a system are represented as elements of an algebraic data type
  - the behavior of a system is given by local transitions between states described by rewrite rules
- It is a logic for executable specification and analysis of software systems, that may be concurrent, distributed, or even mobile.
- It is also a (meta) logic for specifying and reasoning about formal systems, including itself (reflection!)

# REWRITING

- Rewrite theory: (Signature, Labels, Rules)
- Signature: (Sorts, Ops, Eqns) -- an equational theory
  - Describe data types, structure of system state
- Rules have the form label : t => t' if cond
- Rewriting operates modulo equations
  - rules apply locally
  - generates computations (pathways)

### MAUDE

Maude is a language and tool based on rewriting logic
See: <u>http://maude.cs.uiuc.edu</u>
Features: Executability -- position /rule/object fair rewriting High performance engine --- {ACI} matching Modularity and parameterization Builtins -- booleans, number hierarchy, strings Reflection -- using descent and ascent functions

Search and model-checking

#### MAUDE FUNCTIONAL MODULES THE EQUATIONAL PART

A Maude functional module describes an equational theory specifying data types, data constructors, and functions

fmod <modname> is
 <imports>
 <sorts>
 <opdecls>
 <eqns>
endfm

s> \*\*\* reuse, modularity \*\*\* data types and subtyping s> \*\*\* names/and arities of operations \*\*\* how to compute functions

The semantics of an fmod is an initial model
no junk---every data value is constructable
no confusion---two terms are equal only if forced by the equations

#### AN EQUATIONAL SPECIFICATION

Natural numbers are specified by

```
fmod NAT is
  sorts Zero NzNat Nat .
  subsort Zero NzNat < Nat .
  op 0 : -> Zero [ctor] .
  op s_ : Nat -> NzNat [ctor] .
    ....
endfm
```

Examples: 0, s s s 0 (also prints as 3)

#### AN EQUATIONAL SPECIFICATION III

Extend NATLIST with a function to sum the elements of a list

```
var n : Nat. var nl : NatList .
```

```
op sum : NatList -> Nat .
eq sum(nil) = 0 .
eq sum(n nl) = n + sum(nl) .
```

Equations are used to reduce terms to canonical form

```
sum(1 \ 2 \ 3) = 1 + sum(2 \ 3)
= 1 + 2 + sum(3)
= 1 + 2 + 3
= 6
```

#### MAUDE SYSTEM MODULES THE RULES PART

System dynamics are specified using rewrite rules

A system module defines a set of computations (derivations) over the data types specified by the functional part.

#### RULE DELCARATIONS

The <rules> part of a system module consists of rule declarations having one of the forms

```
rl[<id>]: <lhs> => <rhs> .
```

crl[<id>]: <lhs> => <rhs> if <cond> .

<lhs>, <rhs>, <cond> are terms possibly containing variables.
A rule applies to a term T if there is a substitution S (mapping
variables to terms) such that S(<lhs>) is a subterm of T
(<lhs> matches a subterm of T) and S(<cond>) rewrites to true.

In this case T can be rewritten by replacing the matched subterm by the matching instance of <rhs>, S(<rhs>).

# REWRITING RULES



closed under



congruence:



replacement:



#### RULE APPLICATION

Suppose we have the following rules (in a suitable module).

```
rl[fa2b]: f(a,x) => f(b,x) .
rl[hh2h]: h h(y) => h(y) .
```

then

```
g(c,f(a,d)) => g(c,f(b,d))
```

and

```
h h(g(c,f(a,d))) \Rightarrow h(g(c,f(b,d)))
```

also using replacement.

Before applying a rewrite rule, Maude reduces a term to canonical form using equations. The rule lhs is not reduced.

### ABOUT PETRI NETS

A Petri net is represented as a graph with two kinds of nodes:

- \* transitions/rules (reactions--squares)
- \* places/occurrences (reactants, products, modifiers--ovals)

A Petri net process has <u>tokens</u> on some of its places. A rule can fire if all of its inputs have tokens. Firing a rule moves tokens from input to output.



#### PETRI NET MODEL OF A VENDINGMACHINE



#### USING THE VENDING MACHINE

```
What is one way to use 3 $s?
  Maude> rew $ $ $ .
  result Marking: q a c c
How can I get 2 apples with 3 $s?
  Maude> search $ $ $ =>! a a M:Marking .
  Solution 1 (state 8)
  M:Marking --> q q c
  Solution 2 (state 9)
  M:Marking --> q q q a
  No more solutions.
  states: 10 rewrites: 12)
```

## MODEL CHECKING I

 Algorithm for determining if M |= P (M satisfies P) where M is a `model' and `P' is a property.

 In our case a model is a Maude specification of a system together with a staAlgorithm for determining if M |= P (M satisfies P) where M is a `model' and `P' is a property.

 In our case a model is a Maude specification of a system together with a state of interest.

### MODEL CHECKING 11

- Temporal formulas are built from propositions about states, using boolean connectives, and temporal operators
   [] (always) and <> (eventually).
- Satisfaction for M |= A is axiomatized by equations
  - M |= P if C |= P for every computation C of M
  - C |= A if the first state of C satisfies A
  - C |= [] P if every suffix of C satisfies P
  - C |= <>P if some suffix of C satisfies P

#### MODEL CHECKING DEFINING PROPERTIES

```
mod MC-VENDING-MACHINE is
inc VENDING-MACHINE . inc MODEL-CHECKER . inc NAT .
```

```
op vm : Marking -> State [ctor] .
op countPlace : Marking Place -> Nat .
op value : Marking -> Nat . *** in units of quarters
....
```

```
ops fiveQ lte4Q : -> Prop .
ops nCakes nApples val : Nat -> Prop .
```

```
eq vm(M) |= fiveQ = countPlace(M,q) == 5 .
eq vm(M) |= lte4Q = countPlace(M,q) <= 4 .
eq vm(M) |= nApples(n) = countPlace(M,a) == n .
eq vm(M) |= val(n) = value(M) == n .
endm</pre>
```

### MODEL CHECKING THE VENDING MACHINEI

Starting with 5 \$s, can we get 6 apples without accumulating more than 4 quarters? Model check the claim that we can't.

#### Maude>

```
red modelCheck(vm($ $ $ $),[]~(lte4Q U nApples(6))) .
result ModelCheckResult: counterexample(
```

```
{vm($ $ $ $ $ $),'buy-a}
{vm($ $ $ $ $ q a),'buy-a}
{vm($ $ $ $ q q a a),'buy-a}
{vm($ $ q q q a a a),'buy-a}
{vm($ q q q q a a a a),'buy-a}
{vm($ $ a a a a),'buy-a}
{vm($ $ a a a a a),'buy-a},
{vm($ q a a a a a a), 'buy-a},
```

A counterexample to a formula is a pair of transition lists representing an infinite compution which fails to satisfy the formula. A transition is a state and a rule identifier. The second list (deadlock) represents a loop.

### MODEL CHECKING THE VENDING MACHINE II

Starting with 5 \$s, can we get 5 quarters then 6 apples?

```
Maude>
red modelCheck(vm($ $ $ $ $),
      []~(<>fiveQ /\ (fiveQ |-> nApples(6))) .
result ModelCheckResult: counterexample(...)
```

Is value conserved?

Maude> red modelCheck(vm(\$ \$ \$ \$),[]val(20) .
result Bool: true

# BIOLOGICAL PROCESSES ----WHAT TO MODEL

## CELLULAR SIGNALING

- Cells respond to changes in their environment through biochemical pathways that detect, transduce, and transmit information to effector molecules within different cellular compartments.
- Most signaling pathways involve hierarchical assembly in space and time of multi-protein complexes that regulate the flow of information according to logical rules.
- Biological subnetworks interact to produce high levels of physiological organization (e.g., circadian clock subnetworks are integrated with metabolic, survival, and growth subnetworks).

### SIGNALING PATHWAYS

- Signaling pathways involve the modification and/or assembly of proteins and other molecules within cellular compartments into complexes that coordinate and regulate the flow of information.
- Signaling pathways are distributed in networks having stimulatory (positive) and inhibitory (negative) feedback loops, and other concurrent interactions to ensure that signals are propagated and interpreted appropriately in a particular cell or tissue.
- Signaling networks are robust and adaptive, in part because of combinatorial complex formation (several building blocks for forming the same type of complex), redundant pathways, and feedback loops.



Egf stimulation of the Mitogen Activated Protein Kinase (MAPK) pathway.

 $\mathsf{Egf} \to \mathsf{EgfR} \to \mathsf{Grb2} \to \mathsf{Sos1} \to \mathsf{Ras} \to \mathsf{Raf1} \to \mathsf{Mek} \to \mathsf{Erk}$ 

- Egf (EGF) binds to the Egf receptor (EgfR) and stimulates its protein tyrosine kinase activity to cause autophosphorylation, thus activating EgfR.
- The adaptor protein Grb2 (GRB2) and the guanine nucleotide exchange factor Sos1 (SOS) are recruited to the membrane, binding to EgfR.
- The EgfR complex activates a Ras family GTPase
- Activated Ras activates Raf1, a member of the RAF serine/threonine protein kinase family.
- Raf1 activates the protein kinase Mek (MEK), which then activates Erk (MAPK)

### FEATURESI

#### Naming

- different biologists use different names for the same protein Egf vs EGF, Erk vs MAPK, EgfR vs ErbB1 vs HerbB1
- link name to `standard' source: SwissProt, KEGG, HUGO ...
- Activity / state -- a protein may need to be is a specific state (active) to carry out its function
- Location -- what compartment, where in the compartment
  - media -- outside a cell
  - Cell compartment
    - Membrane -- integral, surface, interior
    - Cytoplasm

# FEATURES 11

- Roles / functions
  - Ligand -- Egf
  - Receptor -- EgfR -- binds ligand
  - Scaffold / adaptor -- complex formation
  - Kinase -- Raf1, Mek, Erk
- Processes
  - recruiting
  - postranslational modification
    - phosphorylation (by kinase)
    - ubiquitination

# PATHWAY LOGIC (PL) REPRESENTATION OF SIGNALING

http://pl.csl.sri.com/

### ABOUT PATHWAY LOGIC

Pathway Logic (PL) is an approach to modeling biological processes as executable formal specifications (in Maude) The resulting models can be queried

- using formal methods tools: given an initial state
  - execute --- find some pathway
  - search --- find all reachable states satisfying a given property
  - model-check --- find a pathway satisfying a temporal formula
     using reflection
    - find all rules that use / produce X (for example, activated Rac)
    - find rules down stream of a given rule or component

### PATHWAY LOGIC ORGANIZATION

A Pathway Logic (PL) system has four parts

- Theops --- sorts and operations
- Components --- specific proteins, chemicals ...
- Rules --- signal transduction reactions
- Dishes --- candidate initial states

Knowledge base: Theops + Components + Rules Equational part: Theops + Components A PL cell signaling model is generated from

- a knowledge base
- a dish

# THEOPS

Specifies sorts and operations (data types) used to represent cells:

- Proteins and other compounds
- Complexes
- Soup --- mixtures / solutions / supernatant ...
- Post-translational modifications
- Locations --- cellular compartments refined
- Cells --- collection of locations
- Dishes --- for experiments, think Petri dish

### SAMPLE FROM COMPONENTS

```
sort ErbB1L . subsort ErbB1L < Protein . *** ErbB1 Ligand</pre>
op Eqf : -> ErbB1L [metadata "(\
  (spname EGF HUMAN) \
  (spnumber P01133)
  (hugosym EGF) \
  (category Ligand) \
  (synonyms \"Pro-epidermal growth factor precursor, EGF\" \
            \"Contains: Epidermal growth factor, Urogastrone \"))"].
op EqfR : -> Protein [metadata "(\
  (spname EGFR HUMAN) \
  (spnumber P00533)\
  (hugosym EGFR) \
  (category Receptor) \
  (synonyms \"Epidermal growth factor receptor precursor\" \
            \"Receptor tyrosine-protein kinase ErbB-1, ERBB1 \"))"].
op PIP2 : -> Chemical [metadata "(\
   (category Chemical)
   (keggcpd C04569)\
   (synonyms \"Phosphatidylinositol-4,5P \" ))"].
```

### THEOPS: CELLS & DISHES

```
mod CELL is inc LOCATION .
  sorts Cell CellType .
  subsort Cell < Soup .</pre>
  op [ ] : CellType Soup -> Cell .
  op Cell : -> CellType .
  . . .
endm
Example cell:
    [Cell | {CLm | EqfR PIP2}{CLi | [Hras - GDP] Src}
            {CLc | Gab1 Grb2 Pi3k Plcq Sos1}]
mod DISH is inc CELL .
  sort Dish .
  op PD : Soup -> Dish .
endm
Example rasDish:
PD(Egf [Cell | {CLm | EgfR PIP2}{CLi | [Hras - GDP] Src}
                {CLc | Gab1 Grb2 Pi3k Plcg Sos1}])
```

### THEOPS: SOUP

```
fmod THING is inc PROTEIN . *** Basic building blocks
  sort Thing Family Composite Complex Chemical Signature .
  subsorts Protein Family Composite Complex Chemical Signature
         < Thing .
  op (:): Thing Thing -> Complex [assoc comm].
  ... endfm
Example complex: Eqf : [EqfR - act]
                            *** mixtures
fmod SOUP is inc THING .
                             *** a multiset of Thing
  sort Soup .
  subsort Thing < Soup .
 op empty : -> Soup .
  op : Soup Soup -> Soup [assoc comm id: empty ] .
  op has : Soup Thing -> Bool .
  eq (T1:Thing S:Soup ) has T2:Thing =
    if T1:Thing == T2:Thing
    then true else S:Soup has T2:Thing fi .
  eq (S:Soup has T:Thing) = false [owise] .
  ... endfm
Example soups:
      Gab1 Grb2 Pi3k Plcq Sos1
       [Hras - GDP] Src
```

#### THEOPS: MODIFICATIONS & LOCATIONS

```
fmod MODIFICATION is pr SOUP .
   sorts Modification ModSet . *** multisets of modifications
   subsort Modification < ModSet .
   op none : -> ModSet .
   op _____: ModSet ModSet -> ModSet [assoc comm id: none] .
   op [___] : Protein ModSet -> Protein [right id: none] .
   op act : -> Modification .
   op phos : ___> Modification .
   op Yphos : -> Modification .
   ops GTP GDP : -> Modification .
    *** used for small GTPases
endfm
```

Examples: [EgfR - act] [Hras - GTP]

```
fmod LOCATION is inc MODIFICATION .
sort Location LocName .
subsort Location < Soup .
op {_|_} : LocName Soup -> Location [format (n d d t d d)] .
ops CLo CLm CLi CLc : -> LocName . *** Cell - out,mem,in,cytosol
ops NUo NUm NUi NUc : -> LocName . *** Nucleus - out,mem,in,cytosol
...
endfm
```

# PL RULES

 A PL rule specifies the change in a cell due to an enabled reaction. The rule label gives a hint as to what happens.

In addition rules must be annotated with evidence
 literature citations
 pubmed id (type: review, data) brief description
 curator notes

### RULE 1: RECEPTOR BINDING

If a dish contains an EgfR ligand (?ErbB1L:ErbB1L) outside a cell with EgfR in the cell membrane then the ligand binds to exterior part of the receptor and the receptor is activated.

```
rl[1.EgfR.act]:
?ErbB1L:ErbB1L [CellType:CellType | ct {CLm | clm EgfR}]
=>
[CellType:CellType | ct {CLm | clm ([EgfR - act] : ?ErbB1L:ErbB1L)}].
```

Rule 1 applies to rasDish with the match

```
?ErbB1L:ErbB1L := Egf
clm := PIP2
ct := {CLi | [Hras - GDP] Src} {CLc | Gab1 Grb2 Pi3k Plcg Sos1}
```

and the resulting dish (rasDish1) is

```
PD([Cell |
   {CLm | ([EgfR - act] : Egf) PIP2}
   {CLi | [Hras - GDP] Src}
   {CLc | Gab1 Grb2 Pi3k Plcg Sos1}]).
```

### RULE 5: RECRUITMENT

Activated EgfR recruits Grb2 to the inside of the cell membrane

```
rl[5.Grb2.reloc]:
  {CLm | clm [EgfR - act] }
  {CLi | cli }
  {CLc | clc Grb2 }
  =>
  {CLm | clm [EgfR - act] }
  {CLi | cli [Grb2 - reloc] }
  {CLc | clc }
}
```

Rewriting rasDish1 with rule 5 results in

```
PD([Cell |
    {CLm | ([EgfR - act] : Egf) PIP2}
    {CLi | [Hras - GDP] Src [Grb2 - reloc]}
    {CLc | Gab1 Pi3k Plcg Sos1}]).
```



# A SMALL KB VERY EARLY EGF STIMULATION



#### WHERE DID THE GRAPH COME FROM?

A rule can be represented as a rule node with lhs/rhs represented as incoming/ outgoing `occurrence' nodes.

A occurrence label represents a components modifications and location. EgfR-CLm represents EgfR in the CLm location.

EgfR-act-CLm represents activated EgfR ([EgfR - act]) in the CLm location.



### RULES WITH MODIFIERS

A rule element that is the same on the left and right hand sides is represented using one occurrence node connected to the rule with a dashed arrow.



#### THE PATHWAY LOGIC ASSISTANT (PLA)

- Provides a means to interact with a PL model
- Manages multiple representations
  - Maude module (logical representation)
  - PetriNet (process representation for efficient query)
  - Graph (for interactive visualization)
- Exports Representations to other tools
  - Lola (and SAL model checkers)
  - Dot -- graph layout
  - JLambda (interactive visualization, Java side)
  - SBML (xml based standard for model exchange)



#### A SIMPLE QUERY LANGUAGE

- Given a Petri net with transitions P and initial marking O (for occurrences) there are two types of query
  - subnet
  - findPath a computation / unfolding
- For each type there are three parameters
  - G: a goal set---occurrences required to be present at the end of a path
  - A: an avoid set---occurrences that must not appear in any transition fired
  - H: as list of identifiers of transitions that must not be fired
- findPath returns a pathway (transition list) generating a computation satisfying the requiremments.
- subnet returns a subnet containing all (minimal) such pathways.





# FULL MODEL OF EGF STIMULATION

#### THE ERBB NETWORK (CARTOON FORM)



# PLEGFMODEL

Events that could occur in response to Egf



### CANONICAL EGF-ERK PATHWAY

Egf  $\rightarrow$  EgfR  $\rightarrow$  Grb2  $\rightarrow$  Sos1  $\rightarrow$  a Ras family member  $\rightarrow$  Raf1  $\rightarrow$  MEK1/2  $\rightarrow$  ERK1/2

- Egf binds to the EGF receptor (EgfR), stimulates its protein tyrosine kinase activity, causing autophosphorylation (and activation).
- a complex containing the adaptor protein Grb2 and the guanine nucleotide exchange factor Sos1 docks to the activated EgfR.
- the Sos1-containing EgfR complex activates a Ras family GTPase,
- the activated Ras protein activates Raf1, a member of the RAF serine/threonine protein kinase family.
- Raf1 then activates the dual-specificity protein kinases Mek1 and/or Mek2 (MEK1/2),
- MEK1/2 then activates Erk1 and/or Erk2 (ERK1/2).

#### SUBNET RELEVANT TO ERK ACTIVATION

Subnet containing all pathways leading to activation of Erk.

Obtained by backwards followed by forwards collection





#### CONSTRAINING THE EGF MAP

The idea is to go from all possible pathways to a plausible set, given the context.

a list of 85 protein state changes demonstrated experimentally to occur in response to a short stimulus with Egf were set as goals and a set of concurrent paths were produced by PLA. This subnet ensures that the paths used to reach chosen goals are mutually compatible.
(reachability of all of the goals is also a test of the model)
Egf Rules, with requirements specific to Egf signaling, were given preference over Common Rules



### ACTIVATION OF ERK IRT EGF

The path leading to activation of Erk1/2 in the constrained Egf network.

This path exists in the context of all the other experimental observations,

