Patterns, Equations, and Graphs

Section 1-9

Goals

Goal

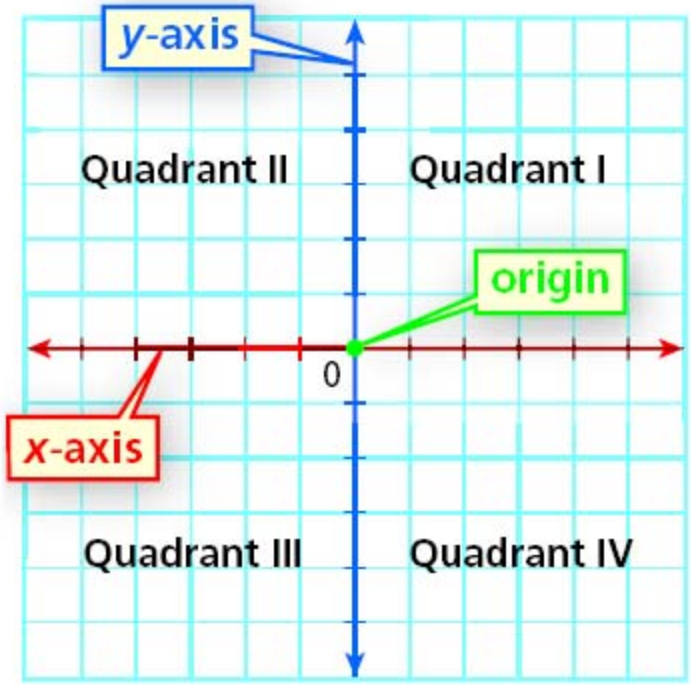
- To use tables, equations, and graphs to describe relationships.

Vocabulary

- Solution of an equation
- Inductive reasoning

Review: Graphing in the Coordinate Plane

The coordinate plane is formed by the intersection of two perpendicular number lines called ***axes***. The point of intersection, called the ***origin***, is at 0 on each number line. The horizontal number line is called the ***x-axis***, and the vertical number line is called the ***y-axis***.



Graphing in the Coordinate Plane

Points on the coordinate plane are described using ordered pairs. An ***ordered pair*** consists of an ***x-coordinate*** and a ***y-coordinate*** and is written (x, y). Points are often named by a capital letter.

Reading Math

The *x*-coordinate tells how many units to move left or right from the origin. The *y*-coordinate tells how many units to move up or down.

Example: Graphing in the Coordinate Plane

Graph each point.

A. *T*(–4, 4)

Start at the origin.

Move 4 units left and 4 units up.

B. *U*(0, –5)

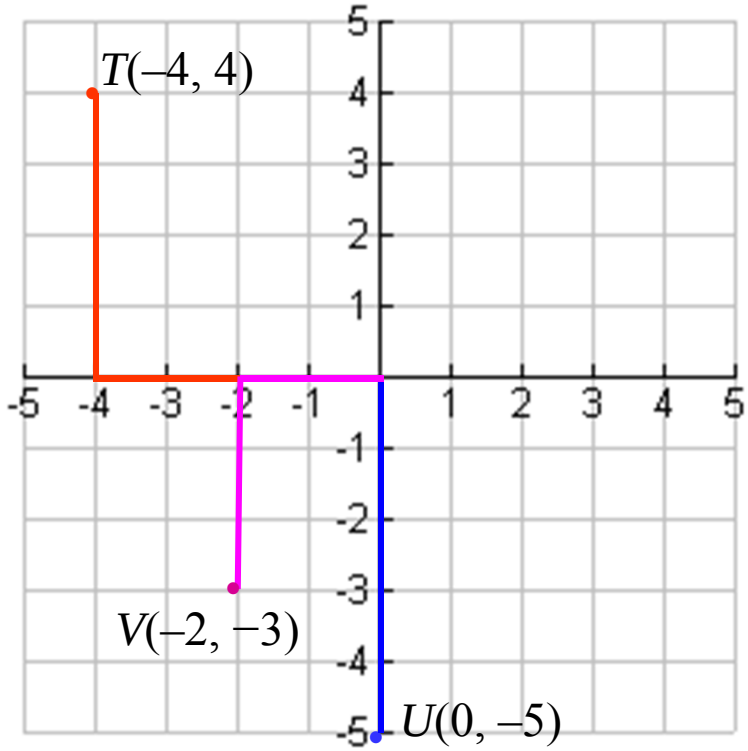
Start at the origin.

Move 5 units down.

C. *V* (–2, –3)

Start at the origin.

Move 2 units left and 3 units down.



Your Turn:

Graph each point.

A. *R*(2, –3)

Start at the origin.

Move 2 units right and 3 units down.

B. *S*(0, 2)

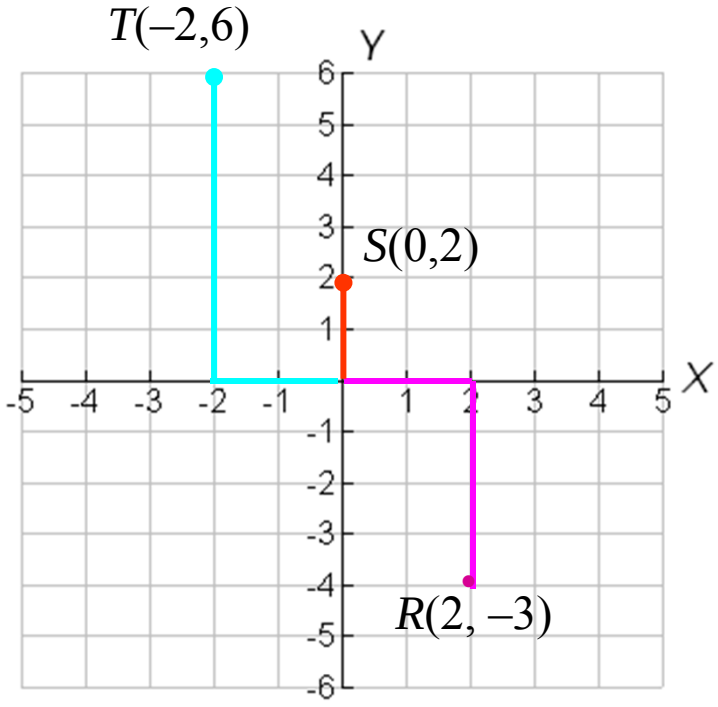
Start at the origin.

Move 2 units up.

C. *T*(–2, 6)

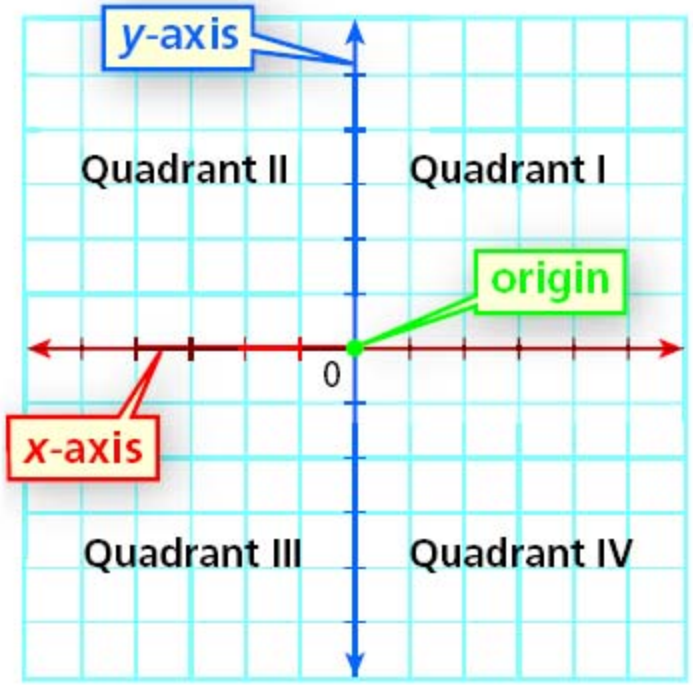
Start at the origin.

Move 2 units left and 6 units up.



Graphing in the Coordinate Plane

The axes divide the coordinate plane into four ***quadrants***. Points that lie on an axis are not in any quadrant.



Example: Locating Points

Name the quadrant in which each point lies.

A. *E*

Quadrant ll

B. *F*

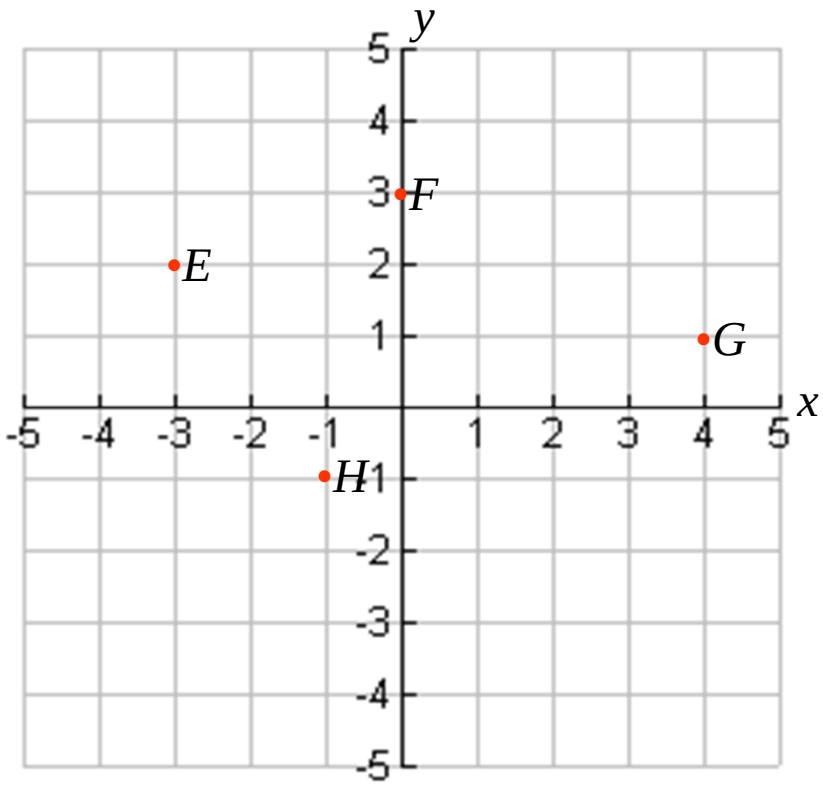
no quadrant (*y*-axis)

C. *G*

Quadrant l

D. *H*

Quadrant lll



Your Turn:

Name the quadrant in which each point lies.

A. *T*

no quadrant (*y*-axis)

B. *U*

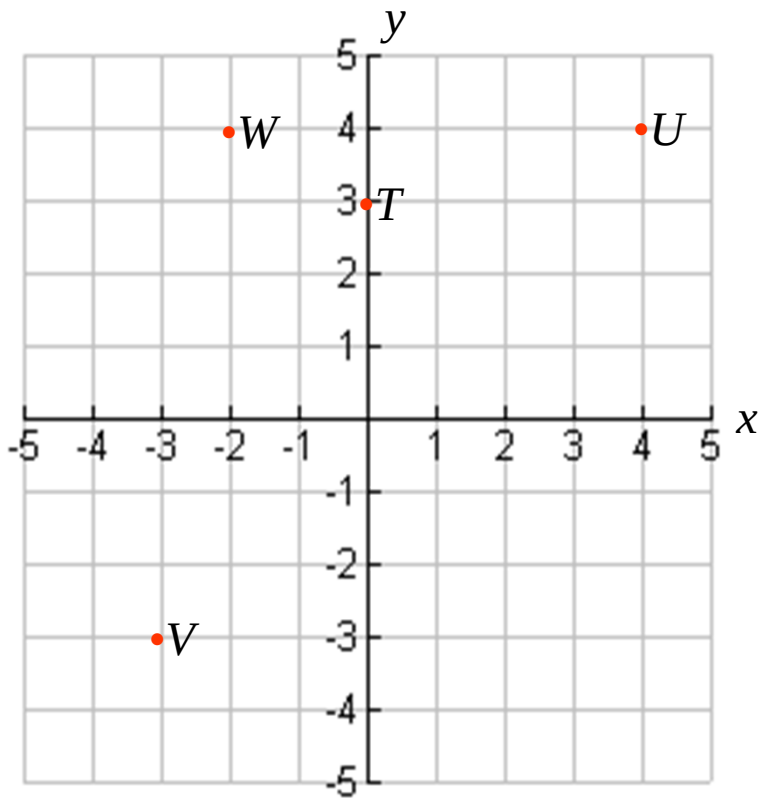
Quadrant I

C. *V*

Quadrant III

D. *W*

Quadrant II



The Rectangular Coordinate System

SUMMARY: The Rectangular Coordinate System

- Composed of two real number lines – one horizontal (the *x*-axis) and one vertical (the *y*-axis). The *x*- and *y*-axes intersect at the origin.
- Also called the Cartesian plane or *xy*-plane.
- Points in the rectangular coordinate system are denoted (x, y) and are called the coordinates of the point. We call the *x* the *x*-coordinate and the *y* the *y*-coordinate.
- If both *x* and *y* are positive, the point lies in quadrant I; if *x* is negative, but *y* is positive, the point lies in quadrant II; if *x* is negative and *y* is negative, the point lies in quadrant III; if *x* is positive and *y* is negative, the point lies in quadrant IV.
- Points on the *x*-axis have a *y*-coordinate of 0; points on the *y*-axis have an *x*-coordinate of 0.

Equation in Two Variables

An **equation in two variables**, x and y, is a statement in which the algebraic expressions involving x and y are equal. The expressions are called **sides** of the equation.

$x + y = 15$ $\qquad$ $x^2 - 2y^2 = 4$ $\qquad$ $y = 1 + 4x$

Any values of the variables that make the equation a true statement are said to be **solutions** of the equation.

$$x + y = 15$$

The ordered pair (5, 10) is a *solution* of the equation.

$$5 + 10 = 15$$
$$15 = 15$$

Solutions to Equations

Example:
Determine if the following ordered pairs satisfy the equation $2x + y = 5$.

a.) $(2, 1)$

$$2x + y = 5$$
$$2(2) + (1) = 5$$
$$4 + 1 = 5$$
$$5 = 5 \quad \text{True}$$

$(2, 1)$ is a solution.

b.) $(3, -4)$

$$2x + y = 5$$
$$2(3) + (-4) = 5$$
$$6 + (-4) = 5$$
$$2 = 5 \quad \text{False}$$

$(3, -4)$ is not a solution.

Equation in Two Variables

An equation that contains two variables can be used as a rule to generate ordered pairs. When you substitute a value for *x*, you generate a value for *y*. The value substituted for *x* is called the ***input***, and the value generated for *y* is called the ***output***.

Output **Input**

$$y = 10x + 5$$

Table of Values

Use the equation $y = 6x + 5$ to complete the table and list the ordered pairs that are solutions to the equation.

x	y	(x, y)
-2		$(-2, -7)$
0		$(0, 5)$
2		$(2, 17)$

$x = -2$

$y = 6x + 5$

$y = 6(-2) + 5$

$y = -12 + 5$

$y = -7$

$x = 0$

$y = 6x + 5$

$y = 6(0) + 5$

$y = 0 + 5$

$y = 5$

$x = 2$

$y = 6x + 5$

$y = 6(2) + 5$

$y = 12 + 5$

$y = 17$

Example: Application

An engraver charges a setup fee of $10 plus $2 for every word engraved. Write a rule for the engraver's fee. Write ordered pairs for the engraver's fee when there are 5, 10, 15, and 20 words engraved.

Let y represent the engraver's fee and x represent the number of words engraved.

Engraver's fee	is	$10	plus	$2	for each	word
y	=	10	+	2	·	x

$$y = 10 + 2x$$

Writing Math

The engraver's fee is determined by the number of words in the engraving. So the number of words is the input and the engraver's fee is the output.

Example: Solution

Number of Words Engraved	Rule	Charges	Ordered Pair
x (input)	$y = 10 + 2x$	y (output)	(x, y)
5	$y = 10 + 2(5)$	20	(5, 20)
10	$y = 10 + 2(10)$	30	(10, 30)
15	$y = 10 + 2(15)$	40	(15, 40)
20	$y = 10 + 2(20)$	50	(20, 50)

Your Turn:

What if…? **The caricature artist increased his fees. He now charges a $10 set up fee plus $20 for each person in the picture. Write a rule for the artist's new fee. Find the artist's fee when there are 1, 2, 3 and 4 people in the picture.**

Let y represent the artist's fee and x represent the number of people in the picture.

Artist's fee		is	$10	plus	$20	for each	person
y		=	10	+	20	·	x

$$y = 10 + 20x$$

Solution:

Number of People in Picture	Rule	Charges	Ordered Pair
x (input)	$y = 10 + 20x$	y (output)	(x, y)
1	$y = 10 + 20(1)$	30	(1, 30)
2	$y = 10 + 20(2)$	50	(2, 50)
3	$y = 10 + 20(3)$	70	(3, 70)
4	$y = 10 + 20(4)$	90	(4, 90)

Graphing Ordered Pairs

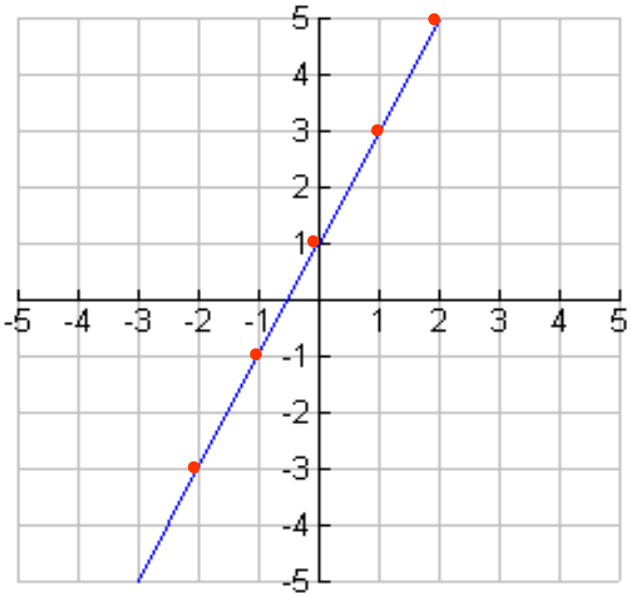
When you graph ordered pairs generated by a function, they may create a pattern.

Example: Graphing Ordered Pairs

Generate ordered pairs for the function using the given values for *x*. Graph the ordered pairs and describe the pattern.

$y = 2x + 1$; $x = -2, -1, 0, 1, 2$

Input	Output	Ordered Pair
x	y	(x, y)
-2	$2(-2) + 1 = -3$	$(-2, -3)$
-1	$2(-1) + 1 = -1$	$(-1, -1)$
0	$2(0) + 1 = 1$	$(0, 1)$
1	$2(1) + 1 = 3$	$(1, 3)$
2	$2(2) + 1 = 5$	$(2, 5)$

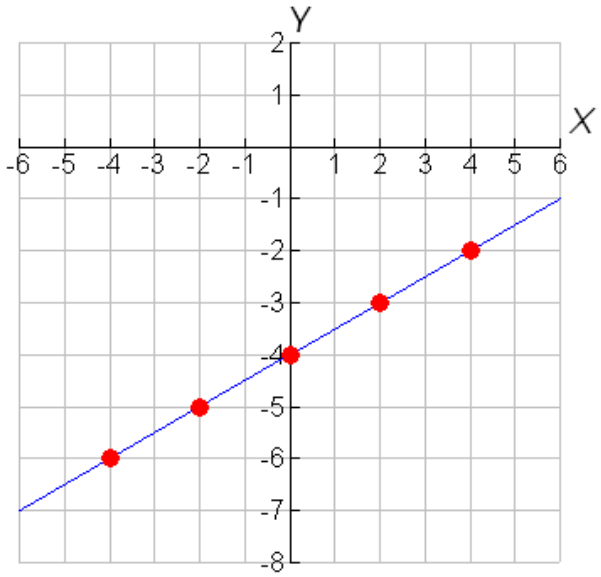


The points form a line.

Your Turn: Generate ordered pairs for the function using the given values for x. Graph the ordered pairs and describe the pattern.

$$y = \frac{1}{2}x - 4;\ x = -4, -2, 0, 2, 4$$

Input	Output	Ordered Pair
x	y	(x, y)
-4	$-2 - 4 = -6$	$(-4, -6)$
-2	$-1 - 4 = -5$	$(-2, -5)$
0	$0 - 4 = -4$	$(0, -4)$
2	$1 - 4 = -3$	$(2, -3)$
4	$2 - 4 = -2$	$(4, -2)$



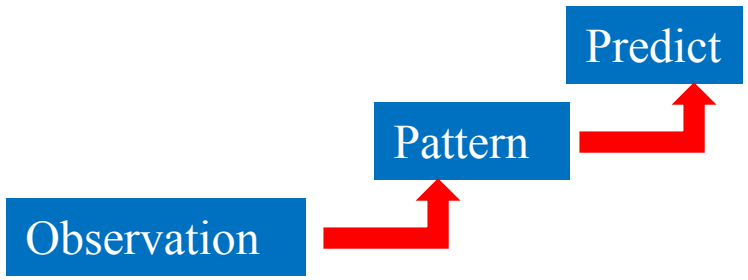
The points form a line.

Definition

- *Inductive Reasoning* – is the process of reaching a conclusion based on an observed pattern.
  - Can be used to predict values based on a pattern.

Inductive Reasoning

- Moves from specific observations to broader generalizations or predictions from a pattern.
- Steps:
  1. Observing data.
  2. Detect and recognizing patterns.
  3. Make generalizations or predictions from those patterns.

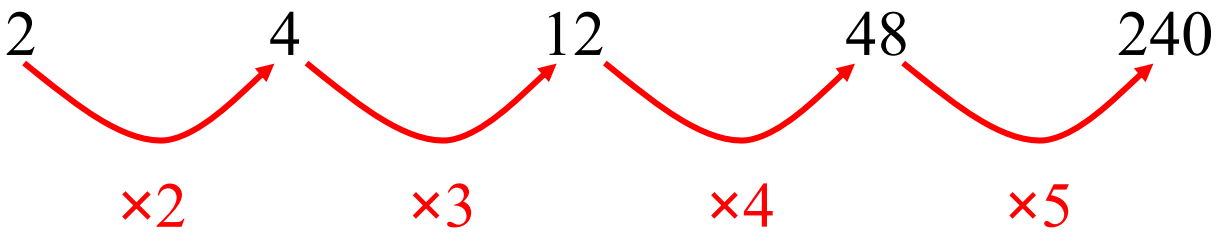


Example: Inductive Reasoning

Make a prediction about the next number based on the pattern.

2, 4, 12, 48, 240

Find a pattern:



The numbers are multiplied by 2, 3, 4, and 5.

Prediction: The next number will be multiplied by 6. So, it will be (6)(240) or 1440.

Answer: 1440

Your Turn:

Make a prediction about the next number based on the pattern.

$$1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$$

Answer: The next number will be $\frac{1}{6^2}$ or $\frac{1}{36}$

Joke Time

- What's a fresh vegetable?
- One that insults a farmer.

- What did one knife say to the other?
- Look sharp!

- What did one strawberry say to the other?
- "Look at the jam you've gotten us into!"