PDH Course E426 (3 PDH)

Voltage Drop Calculations

David A. Snyder, PE

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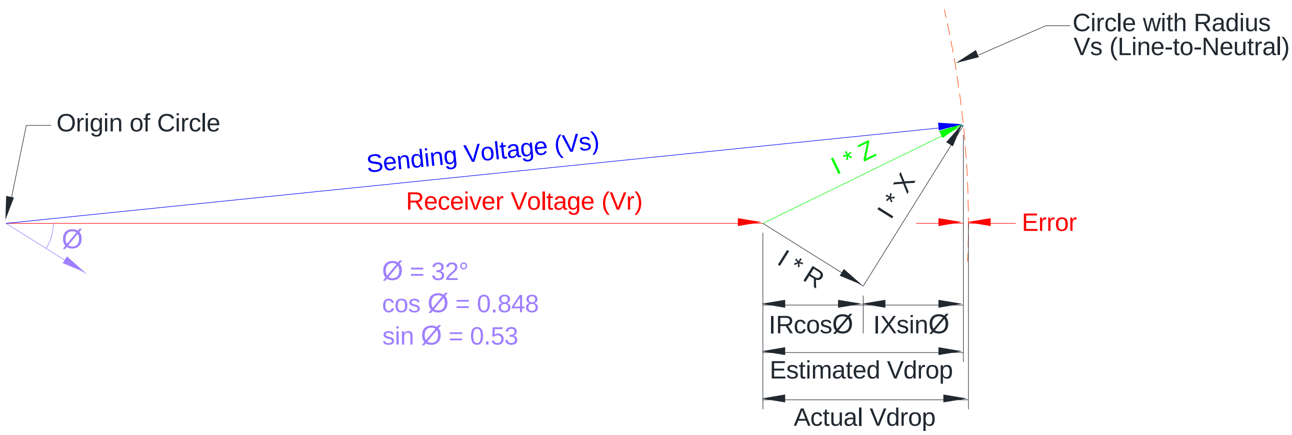

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Introduction

Voltage drop calculations are an everyday occurrence, but the method used and the level of detail can vary widely, based on the person performing the calculation and on the required accuracy.

A few caveats about this course and quiz:

- Voltages are RMS.
- All power factors are lagging.
- All conductors are stranded.
- Ambient temperatures for conductors are between 30 °C and 40 °C, unless stated otherwise. Termination temperatures are 75 °C, unless otherwise indicated.
- The term ‘resistance’ is used sometimes in this course to refer to impedance.
- The Greek letters ϕ (phi) and θ (theta) are used interchangeably in other technical publications to represent the angle between the current and voltage in alternating-current systems. In other words, some sources say the power factor PF = cos(ϕ) and others use PF = cos(θ).
- Three-phase voltages are counter-clockwise, rotating A, B, C in vector space.
- This course has several scaled drawings or figures. When printing a PDF with scaled drawings, choose “Actual Size” or a “Custom Scale” of 100% for accurate results. The Reader is encouraged to use a decimal scale or ruler (the decimal edge of a framing square will do, in a pinch) to measure the results illustrated in the scaled figures.

Topics that are not covered in this course:

- *Aluminum conductors, since the concepts are the same as for copper conductors.*
- *Direct-current (DC) systems.*
- *Leading power factor.*
- *Metric units.*
- *Voltage drop for fire pumps.*
- *Voltages greater than 600 V.*

We will start with well-known formulas for approximate voltage drop (Vdrop), then Estimated Vdrop, then derive the formula for exact or Actual Vdrop. As we shall see, some approximations of voltage drop are more approximate than others.

The first two types of formulas in single-phase sections and the three-phase sections are for approximate voltage drop calculations. This is differentiated from estimated voltage drop calculations in this course in order to point out that the Estimated Vdrop in the IEEE figures is not based on the approximate voltage drop calculations. The estimated voltage drop is the third type of formula.

The fourth type of voltage drop formula is the Actual Vdrop formula.

If you want to jump straight to the voltage drop formulas, here is a list of where they are located:

Type of Formula	Equation #	Page #	Accuracy
Single-Phase, Approximate, Using K	2	9	Least accurate for AC calc's
Equation 2 Rearranged to Find Minimum Circular Mils	19	48	
Single-Phase, Approximate, Using R	1	7	More accurate, depending on value chosen for R
Equation 1 Rearranged to Find Maximum Resistance	18	45	
Single-Phase, Estimated, Using Note 2 to NEC Table 9	8	29	Even more accurate
Single-Phase, Actual or Exact, Using IEEE Formula	15	36	Exactly Accurate
Three-Phase, Approximate, Using K	4	13	Least accurate for AC calc's
Equation 4 Rearranged to Find Minimum Circular Mils	21	50	
Three-Phase, Approximate, Using R	3	12	More accurate, depending on value chosen for R
Equation 3 Rearranged to Find Maximum Resistance	20	49	
Three-Phase, Estimated, Using Note 2 to NEC Table 9	9	29	Even more accurate
Three-Phase, Actual or Exact, Using IEEE Formula	16	36	Exactly Accurate

Table 1 – Quick Guide to Voltage Drop Formulas

As previously stated, we will start with formulas for approximate voltage drop calculations, then explore the formulas for estimated voltage drop, then derive the formula for exact or actual voltage drop calculations. The different types of formulas are listed in Table 1, with comments as to their relative accuracies. We will compare the results and accuracies of the various formulas numerous times.

Voltage Drop and the National Electrical Code

The following are National Electrical Code (NEC) references with regard to maximum voltage drop.

Branch Circuits

NEC 210.19(A)(1) Informational Note No. 4 limits the voltage drop at the furthest outlet of a load to 3% of the applied voltage. This allows 2% drop in the feeder. Alternatively, the maximum combined voltage drops on the feeder and branch circuits going to the furthest outlet of a load should be limited to 5%. This means the feeder could have 1% Vdrop if the branch had no more than 4%, or any other combination of feeder and branch voltage drops that did not add up to more than 5%. For example, if a panel board is located adjacent to the transformer feeding it, one might assert that there is nominally 0% voltage drop in the short feeder from the transformer secondary to the adjacent panel board, thus leaving the full 5% voltage drop for use on the branch circuits powered from that panel board.

Feeders

NEC 215.2(A)(4) Informational Note No. 2 also limits the voltage drop at the furthest outlet of a load to 3% of the applied voltage. This allows 2% drop in the feeder. Alternatively, the maximum combined voltage drops on the feeder and branch circuits going to the furthest outlet of a load should be limited to 5%. This is the same requirement stated above for branch circuits.

Sensitive Electronic Equipment

NEC 647.4(D) covers the use of separately-derived 120 V, single-phase, three-wire systems with 60 V between each of the two hot conductors and the neutral, similar to the 120/240V service to our houses, but at half the voltage. This NEC article limits the voltage drop on any branch circuit serving sensitive electronic equipment to 1.5% of the applied voltage. Alternatively, the maximum combined voltage drops on the feeder and branch circuits going to sensitive electronic equipment should be limited to 2.5%. These values are half of the values given for feeders and branch circuits.

These special electrical systems typically used for sensitive audio/video or similar types of electronic equipment and are only allowed in commercial or industrial occupancies and are not discussed further in this course.

Other Considerations

A 3% voltage drop on a single-phase, 120 VAC system would be 0.03 * 120 V = 3.6 V. Similarly, a 2% voltage drop on a three-phase, 480 VAC system would be 0.02 * 480 V = 9.6 V. The main concern of voltage drop calculations is to make sure the load has enough voltage to function properly. For example, a motor on a 480 VAC system is rated to operate at 460 VAC system, and there is even a tolerance around that lower voltage at which the motor will still start and function properly.

NEC 90.5(C) states that explanatory information provided in the Code, such as Informational Notes, are not enforceable as Code requirements, so voltage drop recommendations that appear as Informational Notes in the Code are not enforceable. They are usually required and expected, however, on most projects, for conductor run lengths that merit consideration: consider the Rule-of-Thumb section beginning on page 53. It is important to note that the voltage drop requirement for Sensitive Electronic Equipment in NEC 647.4(D) is a code-enforceable requirement, not an Informational Note.

Common Formulas for Voltage Drop Calculations

The Reader will probably see more than one formula for single-phase and three-phase voltage drop calculations, and many designers have a preference as to which formula they use.

Presented in this section are some common voltage drop formulas that provide approximate results.

Single-Phase Approximate Voltage Drop Formulas

Figure 1 below shows a circuit diagram for a single-phase, two-wire voltage drop calculation. Notice that the resistance R of each conductor appears in both the hot and the neutral conductors (R1 = R2 = R * L), since there is resistance to current going from the power source to the load and the same amount of resistance to that same current coming from the load back to the source. The neutral current returning from the load is the same as the line current going to the load in the single-phase circuit shown in this figure.

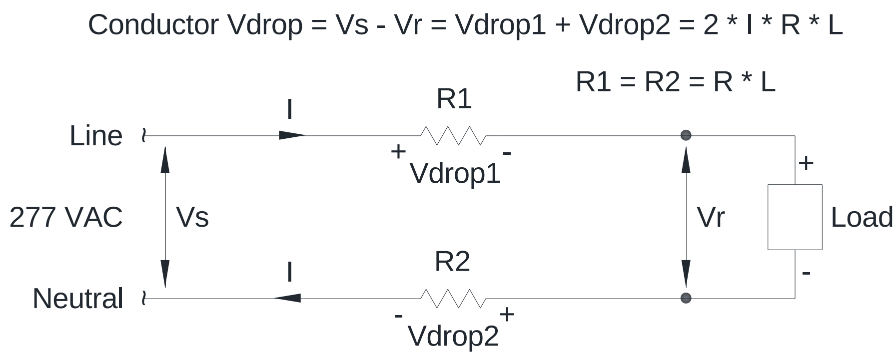

Figure 1 – Single-Phase, Two-Wire Voltage-Drop Circuit Diagram

Both conductors in Figure 1 are the same length and the same type of conductor, so they both have the same resistance. It is easy to see that the total voltage drop in the two conductors in this circuit is:

Vdrop = 2 * I * R * L round-trip Equation 1

where:

Vdrop = voltage drop, round-trip;

I = the current going to the load, which is also the current returning from the load;

R = the resistance (or impedance) per 1,000 feet of conductor;

L = the one-way length, in feet, of one conductor from the source to the load, or vice-versa, divided by 1,000.

The voltage drop in Equation 1 is described as 'round-trip' because the current goes out to the load and comes back on the neutral. A balanced, three-phase load, on the other hand, does not have any current coming back on the neutral.

The Reader will often see the variable R used in voltage drop calculations, but it is often an impedance, not a resistance, that the variable R represents. We will be pretty loose with the term 'resistance' in this course, until we discuss power factor, reactance, or phasor diagrams.

Why is the variable L present in Equation 1 and other voltage drop formulas? The resistance of a conductor is continuously distributed along its entire length. In other words, the longer the conductor, the higher the total resistance. Figure 2 below illustrates this concept using two conductors, similar to Figure 1 above.

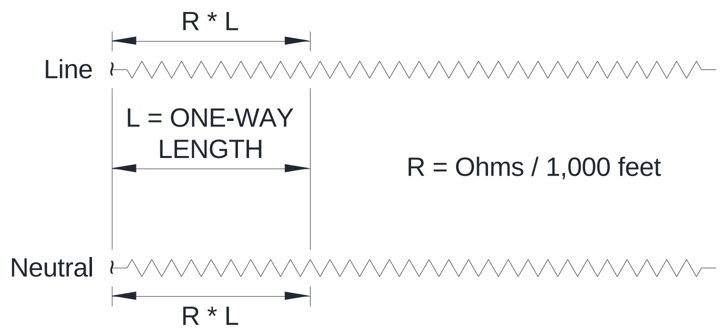

Figure 2 – Distributed Resistance in Conductors

A concept that will be repeated in this course is that the resistance, reactance, and impedance values found in the National Electrical Code (NEC) and other technical publications are the "line-to-neutral" or "ohms-to-neutral" resistance values, unless specifically stated otherwise.

Example 1: Using Equation 1 on page 7, what would be the voltage drop for a 12 A load powered by 120 VAC with 12 AWG conductors with a one-way length of 100 feet in PVC conduit? Let's assume a power factor of 0.85, which results in a resistance (impedance, really) value of 1.7 ohms per 1,000 feet for the 12 AWG conductors in PVC conduit (Effective Z at 0.85 PF column in NEC Table 9).

Vdrop = 2 * I * R * L

= 2 * 12 * 1.7 * (100 / 1,000)

= 4.08 V

This would be a voltage drop percentage of 4.08 V / 120 V = 3.4%. Alternatively, one could say that 120 V – 4.08 V = 115.92 V would be available at the receiver or load. All of the single-phase and three-phase voltages in this section are approximations, however. We will get to the exact or actual voltage drop calculations later on in this course, in the section called Calculating the Error Shown in the IEEE Phasor Diagram, beginning on page 34. Very often, approximate voltage drop calculations are acceptable, but is helpful to understand the use and inherent limitations of different types of voltage drop formulas.

End of Example

Another common single-phase approximate voltage drop formula is:

Vdrop = 2 * I * K * L / A round-trip Equation 2

where:

Vdrop = voltage drop, round-trip;

I = the current going to the load, which is also the current returning from the load;

K = ohms * circular mil per foot of conductor, which equals 12.9 for copper;

L = the one-way length, in feet, of one conductor from the source to the load;

A = the cross-sectional area of the conductor in circular mils (6,530 for 12 AWG from NEC Table 8).

The Author does not prefer Equation 2 for several reasons, one of which is because it uses the average value of DC resistance for all sizes of conductor. Also, Equation 2 has no provisions for the load's power factor – it says K = 12.9, regardless of the power factor of the load or the actual AC characteristics of the conductors. Equation 1, on the other hand, will allow you to substitute the NEC effective impedance (Effective Z) of the conductor, which incorporates the load's power factor, in place of R, thus turning the approximate voltage drop formula into an estimated voltage drop formula – a fine distinction that will be elaborated on later.

For more information on AWG sizes and circular mil areas, the Reader can refer to PDH Online course *E275 AWG and Circular Mils* listed in the Additional Reading section beginning on page 56.

Some sources quote different values for K, such as 11, 12, or 12.9. Where does this value of K originate? Consider the two-part Table 2 below.

Derivation of Value K for Copper Using NEC Table 8			
Wire Size (Stranded)	NEC Table 8 Circular Mils (C.M.)	NEC Table 8 DC Ohms per 1,000 ft	DC Ohms*C.M./ft
12	6,530	1.98	12.929
10	10,380	1.24	12.871
8	16,510	0.778	12.845
6	26,240	0.491	12.884
4	41,740	0.308	12.856
2	66,360	0.194	12.874
1	83,690	0.154	12.888
1/0	105,600	0.122	12.883
2/0	133,100	0.0967	12.871
250	250,000	0.0515	12.875
350	350,000	0.0367	12.845
500	500,000	0.0258	12.900
			12.877 Average Value

Comparison to NEC Table 9 in Steel Conduit					
Wire Size (Stranded)	NEC Table 8 Circular Mils (C.M.)	NEC Table 9 AC Ohms per 1,000 ft	AC Ohms*C.M./ft	NEC Table 9 Effective Z at 0.85 PF per 1,000 ft	Effective Z at 0.85 PF Ohms*C.M./ft
12	6,530	2.0	13.060	1.7	11.101
10	10,380	1.2	12.456	1.1	11.418
8	16,510	0.78	12.878	0.70	11.557
6	26,240	0.49	12.858	0.45	11.808
4	41,740	0.31	12.939	0.30	12.522
2	66,360	0.20	13.272	0.20	13.272
1	83,690	0.16	13.390	0.16	13.390
1/0	105,600	0.12	12.672	0.13	13.728
2/0	133,100	0.10	13.310	0.11	14.641
250	250,000	0.054	13.500	0.073	18.250
350	350,000	0.039	13.650	0.060	21.000
500	500,000	0.029	14.500	0.050	25.000

Table 2 – Derivation of the Value of K for Copper Conductors

As seen in Table 2 above, the value of K = 12.9 is based on the average value of the DC resistance of copper, as seen in the red columns. The average values of AC resistance (green columns) and Effective Z at 0.85 PF (purple columns) are not shown in the bottom portion of that table because it is not recommended to use the average value of an assortment of wire sizes when performing voltage drop calculations. Clearly, the use of the constant K can lead to inaccuracies in AC voltage drop calculations. Let's try Example 1 again, but with Equation 2.

Example 2:

Vdrop = 2 * I * K * L / A

= 2 * 12 * 12.9 * 100 / 6,530

= 4.74 V

This would be a voltage drop of 4.74 V / 120 V = 3.95%. Alternatively, one could say that 120 V - 4.74 V = 115.26 V would be available at the load.

End of Example

The results in Example 1 and Example 2 are not equal, but, as previously stated, the value of K used in Example 2 is based on the DC resistance of copper, whereas we used the Effective Z at 0.85 PF for the value of R in Example 1.

The single-phase, two-wire voltage drop calculation formulas we have seen in this section are also applicable to balanced single-phase, three-wire systems, such as the 120/240V services to our homes, since there will be no neutral current if the load is balanced. See Figure 3 below.

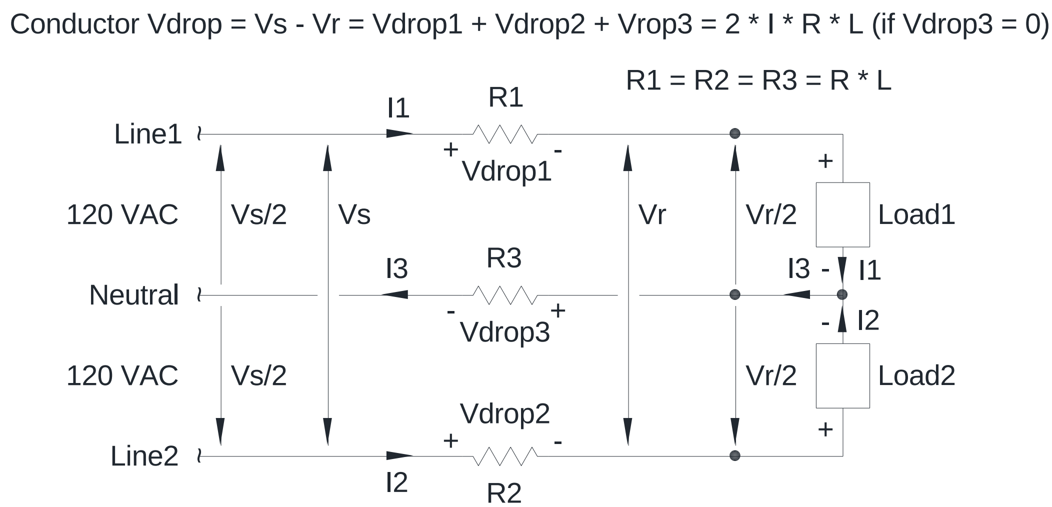

Figure 3 – Single-Phase, Three-Wire Voltage-Drop Circuit Diagram

To reiterate, the single-phase voltage drop calculations we have seen thus far (Equation 1 and Equation 2) will only work for Figure 3 above if the load is balanced, such that there is no neutral current. That means the Vdrop would be line-to-line, instead of round-trip, since there would be no current returning on the neutral conductor.

Let's extend what we've seen for single-phase approximate voltage drop calculations into the sometimes inscrutable realm of three-phase systems.

Three-Phase Approximate Voltage Drop Formulas

Let's start off with two possibly controversial statements:

1.) All commonly-used voltage drop formulas for balanced three-phase loads are actually single-phase voltage drop formulas.

2.) Commonly-used three-phase voltage drop formulas are only valid for balanced three-phase loads.

The two single-phase approximate voltage drop formulas we just reviewed in the previous section are almost identical to the two three-phase approximate voltage drop formulas presented in this section, except that we replace the 2 in the single-phase formulas with √3 for three-phase voltage drop calculations. Consider Figure 4 below.

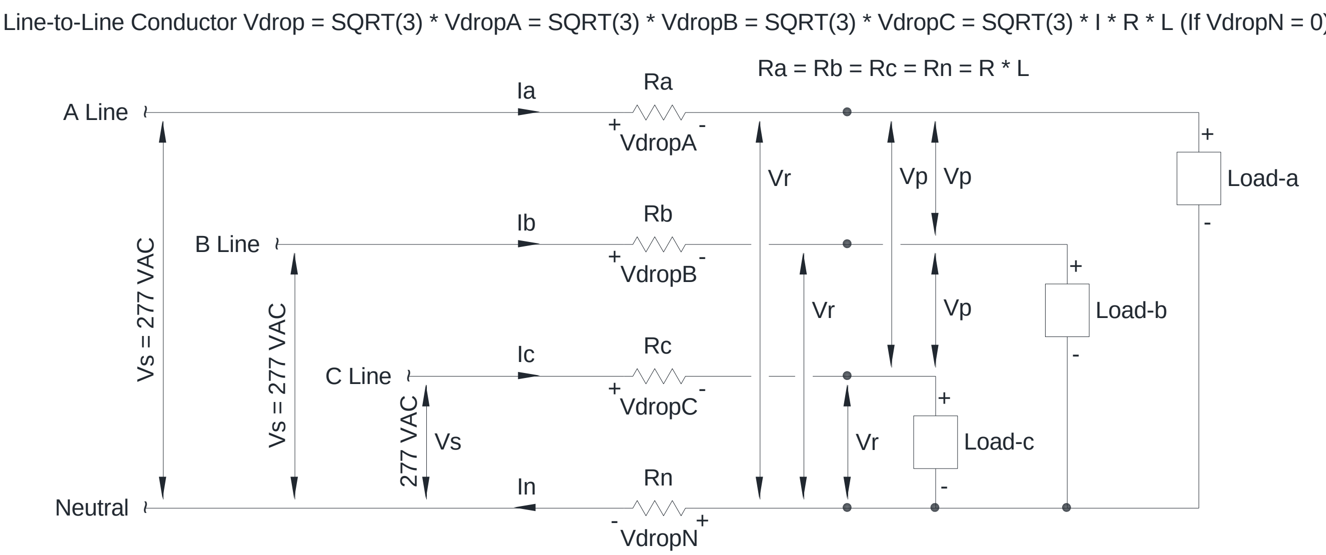

Figure 4 – Three-Phase Wye-Connected with Neutral Voltage-Drop Circuit Diagram

The value of Vs in Figure 4 above is the line-to-neutral source or supply voltage; Vdrop is the voltage drop across the resistance or impedance in the conductors, Vr is the line-to-neutral receiver or load voltage after the voltage drop, and Vp is the phase-to-phase voltage at the load. These values might be seen more easily in Figure 10 on page 19. Below is a common formula for balanced three-phase approximate voltage drop:

$V_{drop} = \sqrt{3} * R * I * L$ line-to-line Equation 3

where:

Vdrop = voltage drop, line-to-line;

R = ohms per 1,000 ft of conductor;

I = line current going to the load;

L = one-way conductor length, in feet, divided by 1,000.

Why is the square root of three (√3) included in this calculation? The square-root of three in voltage-drop calculations is not there for the return current on the neutral as the 2 is in single-phase calculations, the √3 is required to convert the line-to-neutral voltage drop to a line-to-line voltage drop, as described in Figure 10 on page 19. For commonly-used three-phase voltage drop equations to be valid, the three-phase load must be balanced between the three phases such that there is no current flow on the neutral conductor, if present, and therefore no voltage drop in the neutral conductor. It doesn't matter whether or not there is a neutral conductor, the load needs to be balanced for the equation to give accurate results.

Example 3: Using Equation 3, what would be the voltage drop for a 50 Hp motor at 480V/3Φ with 4 AWG conductors with a one-way length of 200 feet in steel conduit? Let's assume a power factor of 0.85, which results in a value of 0.3 ohms per 1,000 feet for R (Effective Z at 0.85 PF column in NEC Table 9). The full-load current for this motor is 65 A from NEC Table 430.250.

$$V_{drop} = \sqrt{3} * I * R * L$$

$$= \sqrt{3} * 65 * 0.3 * (200 / 1{,}000)$$

$$= 6.75 \text{ V}$$

This would be a voltage drop of 6.75 V / 480 V = 1.4%. Alternatively, one could say that 480 V – 6.75 V = 473.25 V line-to-line would be available at the load.

End of Example

Another three-phase approximate voltage drop formula is:

$$V_{drop} = \sqrt{3} * I * K * L / A \quad \text{line-to-line} \quad \text{Equation 4}$$

where:

Vdrop = voltage drop, line-to-line;

I = line current going to the load;

K = ohms * circular mil per foot of conductor, which equals 12.9 for copper;

L = the one-way length, in feet, of one conductor from the source to the load;

A = the cross-sectional area of the conductor in circular mils (41,740 for 4 AWG from NEC Table 8).

The Author does not prefer Equation 4 for the same reasons mentioned for Equation 2 on page 9.

Example 4: Using Equation 4, what would be the voltage drop for the installation described in Example 3?

$$V_{drop} = \sqrt{3} * I * K * L / A$$

$$= \sqrt{3} * 65 * 12.9 * 200 / 41{,}740$$

$$= 6.96 \text{ V}$$

This would be a voltage drop of 6.96 V / 480 V = 1.5%. Alternatively, one could say that 480 V – 6.96 V = 473.04 V would be available at the load. This result is only slightly different from the result of Example 3.

End of Example

Some designers substitute the effective impedance (Effective Z) value for the value of R in Equation 1 and Equation 3 by solving the formula in Note 2 of Table 9. This Effective Z formula is shown as Equation 5 on page 22 of this course and using it in place of R will result in a more accurate voltage drop, since it includes the conductors' reactance and AC resistance, as well as the power factor of the load. The formula for Effective Z will be explored in more detail in the section called Effective Z at Any Power Factor: Note 2 to Table 9 in the NEC, beginning on page 21. Further discussion on this topic is also in the section called Estimated Vdrop Derived from Impedance Phasor Diagrams, which starts on page 28.

The ultimate three-phase voltage drop calculation will be discussed later in this course, but it is not a common formula, so it is not listed in this section. It is the IEEE formula for Actual Vdrop, which is derived in the section called Calculating the Error Shown in the IEEE Phasor Diagram, beginning on page 34.

No Neutral Current in a Balanced Three-Phase System?

It might seem strange that we say there is no neutral current when there is a balanced three-phase load, whether or not a neutral conductor is present. How can we say there is no return current when there are three equal line currents going out to the balanced load? That is because the three equal currents are 120° apart from each other and therefore add up to zero. Consider the balanced three-phase load shown in Figure 5 below. The neutral current (In) is equal to the sum of the phase-currents, which is the same as the sum of the line currents (Ia + Ib + Ic). Since, in a balanced, three-phase circuit, all three of the line currents are of equal amplitude and 120° apart, they add up to zero, as illustrated in Figure 6.

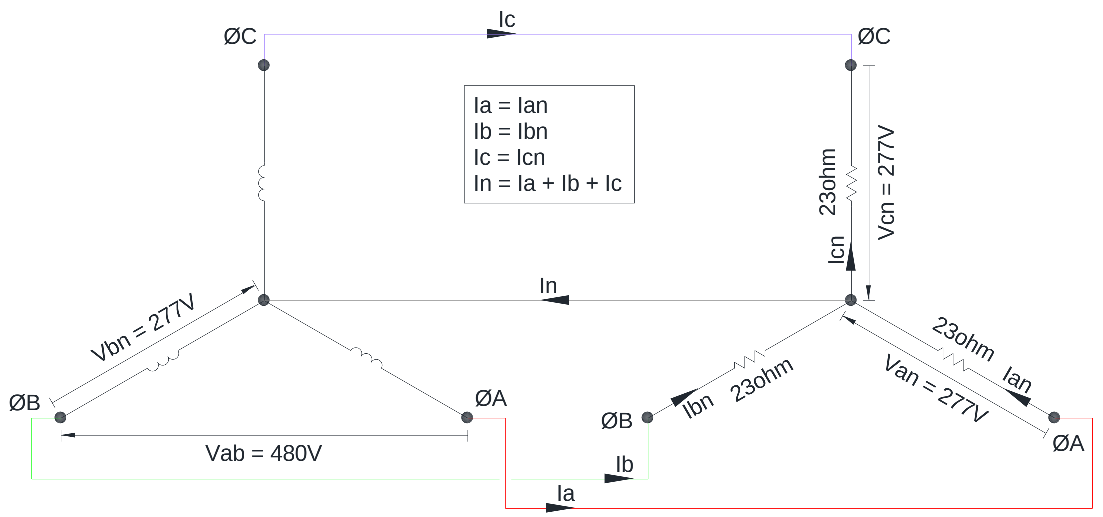

Figure 5 – Balanced, Wye-Connected, Three-Phase, Resistance Load, with Neutral Connection

It can be seen from the right-hand side of Figure 6 below that the neutral current (In) is equal to zero because it is the sum of the line currents (Ia + Ib + Ic), which add up vectorially to zero. In other words, starting at the beginning of vector Ian and working to the arrowhead of that vector, then adding vector Ibn, then adding vector Icn, brings us back to our starting point at the beginning of vector Ian, resulting in a total current of zero.

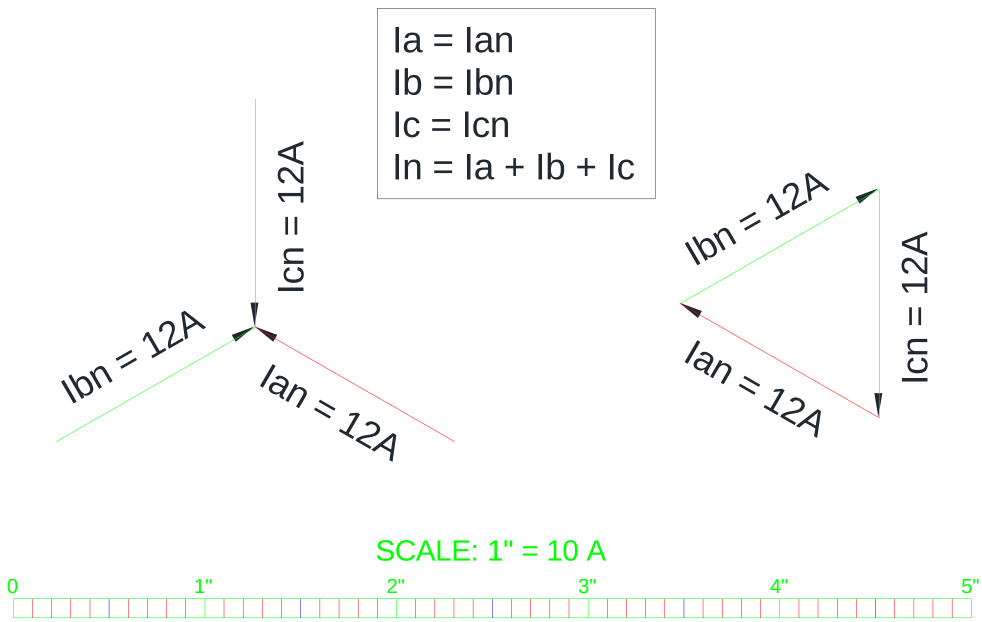

Figure 6 – Balanced, Wye-Connected, Three-Phase 10 KW Load at 480 V

Let's look at the ubiquitous square root of three (√3) and discuss why it is used in three-phase voltage drop calculations. Interested Readers can peruse PDH Online course *E431 The Square Root of Three (√3) in Electrical Calculations* listed in the Additional Reading section beginning on page 56 for further discussion of this topic.

√3 Relationship of Three-Phase Voltages

The square root of three is the ratio of the line-to-line or phase-to-phase voltage (480 V) to the line-to-neutral or phase-to-neutral voltage (277 V) in three-phase power systems. Figure xyz06 below illustrates that this relationship is based on simple geometry. This figure is drawn to scale and the Reader is encouraged to confirm the voltages by measuring them.

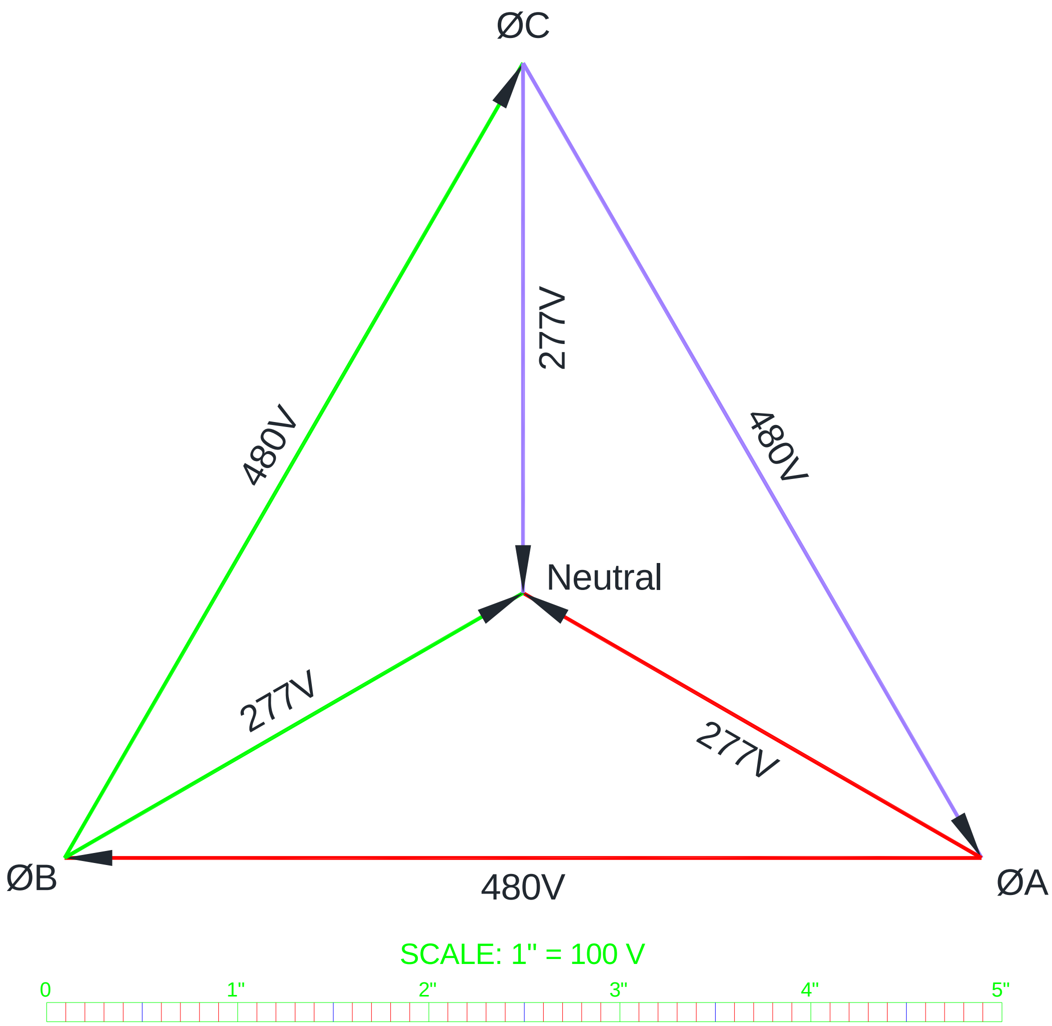

Figure 7 – 480Y/277V Wye-Delta Voltage Relationship

The voltage relationship between 277 V wye and 480 V delta from Figure 7 can be thought of as simple right-triangle geometry, where the hypotenuse is 277 V and the adjacent side to the 30° angle is half of 480 V, or 240 V. Figure 8 below is the bottom portion of Figure 7. The length of the short vertical side opposite the 30° angle is of no concern for this exercise.

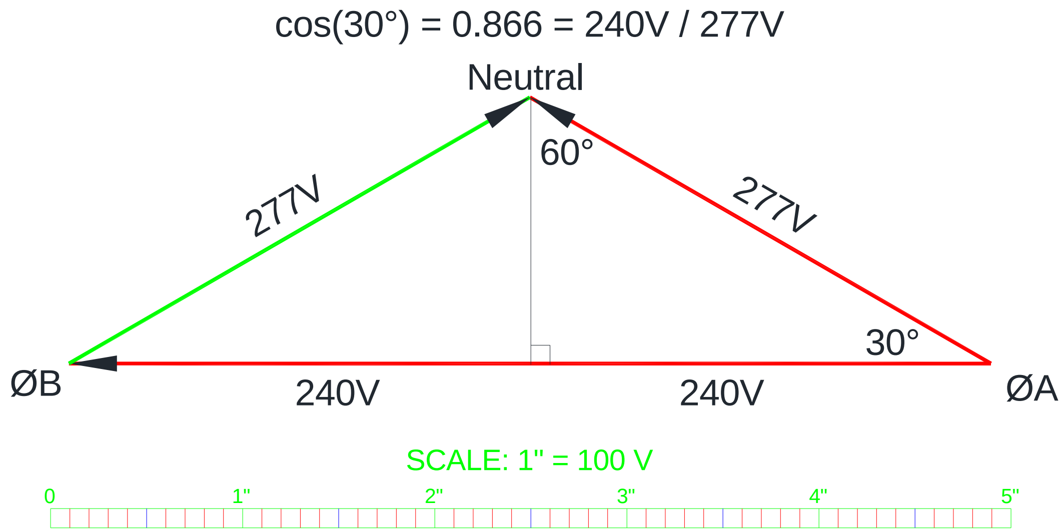

Figure 8 –Wye-Delta Voltage Relationship – Right Triangle Geometry

As shown in Figure 8, the length of the 240 V side of both right triangles is related to the length of the 277 V sides by the cosine of 30°. Alternatively, we could have used the 60° corner for reference in Figure 8 and stated that the relationship between 240 V and 277 V was defined by: sin(60°) = 0.866 = 240 V / 277 V to get the same result. The approximate value of 0.866 is actually $\sqrt{3} / 2$, as one might expect from the voltages shown in Figure 8.

The square root of three also comes into play for the voltage applied to a wye-start/delta-run motor. Interested Readers should see PDH Online Course *E413 Wye-Delta Motor Starters*, listed in the Additional Reading section beginning on page 56.

The same square root of three ($\sqrt{3}$) relationship between the line-to-line voltage and line-to-neutral voltage for 480Y/277V also holds true for 280Y/120V, as shown in Figure 9 below.

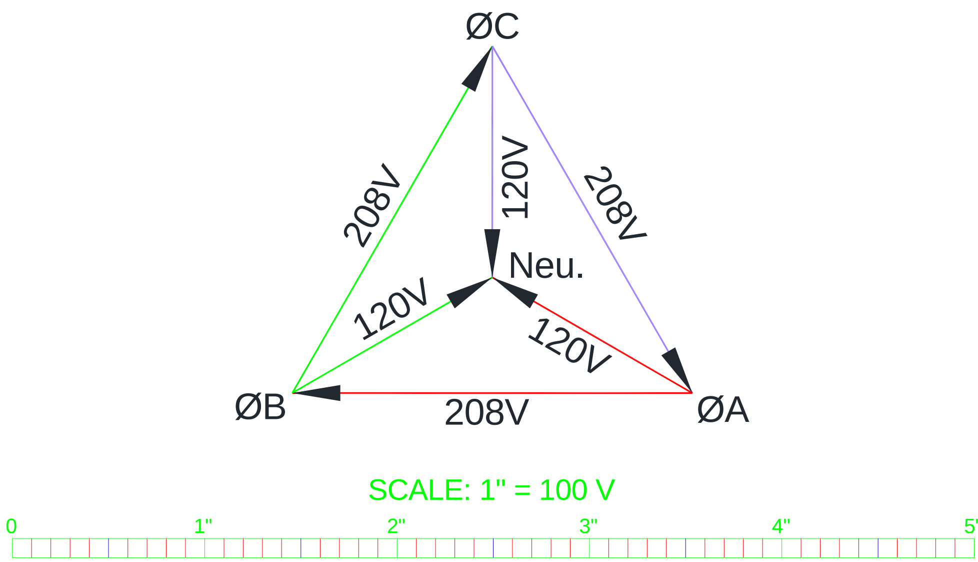

Figure 9 – 208Y/120V Wye-Delta Voltage Relationship

The square root of three (√3) relationship is an immutable characteristic for all three-phase power systems, including 400Y/230V and 600Y/347V. See PDH Online Course *E427 Standard AC System Voltages (600 V and Less)*, listed in the Additional Reading section beginning on page 56, for more information on various AC system voltages.

Having discussed the square root of three relationship of three-phase voltages, let's move on to our next topic, the use of the square root of three in balanced three-phase voltage drop calculations.

Why the √3 Is Used in Balanced, Three-Phase Voltage Drop Calculations

The square root of three is used in balanced, three-phase voltage drop calculations because it converts the line-to-neutral voltage drop into a line-to-line voltage drop. This is most easily demonstrated by confirming the measurements shown in Figure 10 below. The line-to-neutral supply voltage is 3 V, so the line-to-line supply voltage is √3 times that, or 5.196 V, as can be directly measured in that figure with reasonable accuracy.

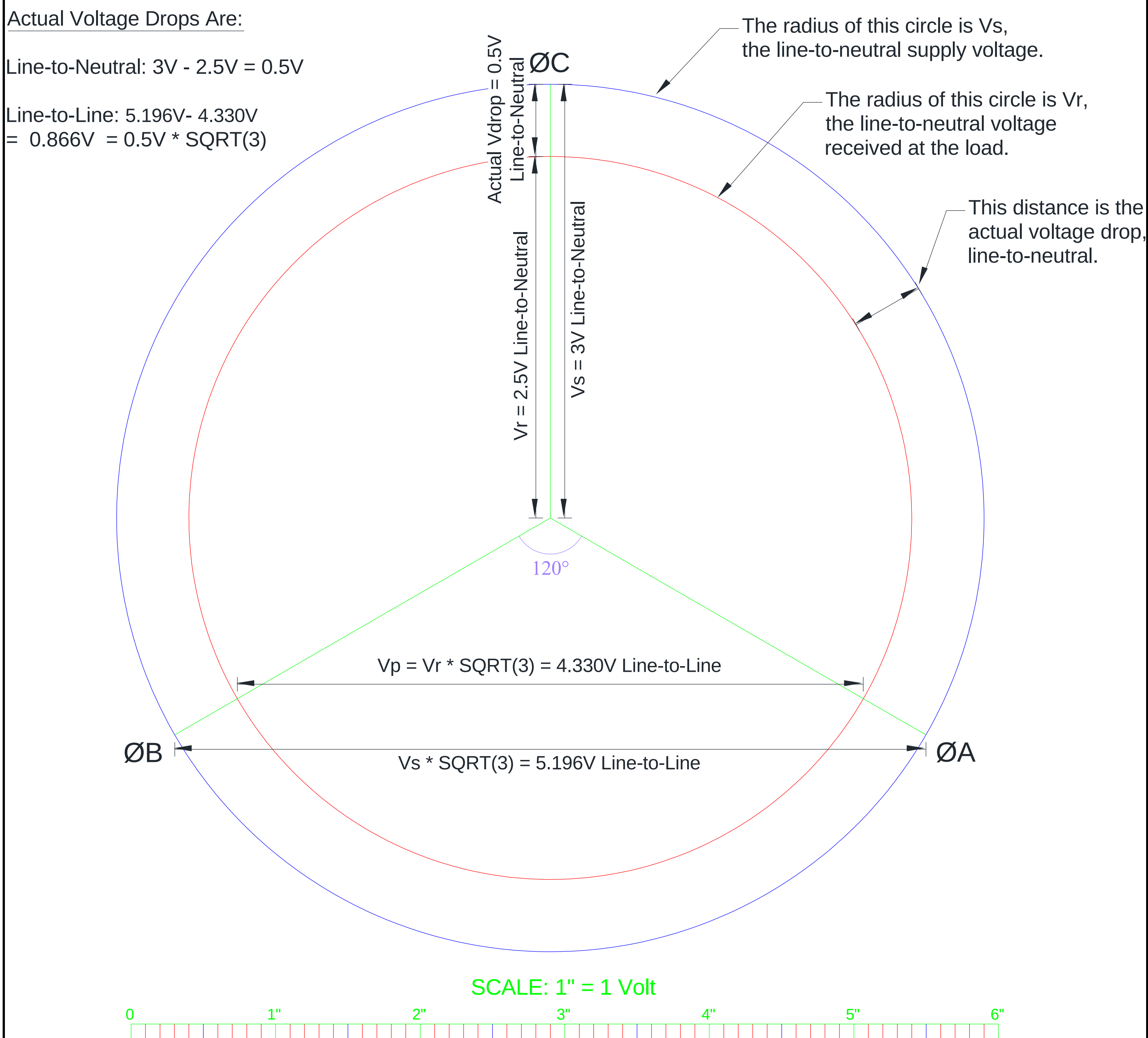

Figure 10 – The Square Root of Three in Three-Phase Voltage-Drop Calculations

As can be seen in Figure 10, there is nothing mystical or magical about using the square root of three in balanced three-phase voltage drop calculations – it is based on simple geometry. If the line-to-neutral voltage drop is 0.5 V, then the line-to-line voltage drop is $\sqrt{3}$ times that or 0.866 V. Confirm this by measuring the line-to-line voltages Vs (5.196 V) and Vr (4.330 V) to see that the line-to-line difference, the line-to-line voltage drop, is 0.866 V. Draw it in a CAD program for maximum accuracy to confirm the measurements.

Table 9 in the NEC

The NEC does not have any formulas for voltage drop calculations (although the NEC Handbook does), but it does have a formula for effective impedance, which is elaborated on in the section in this course called Effective Z at Any Power Factor: Note 2 to Table 9 in the NEC on page 21.

One of the resources that is used quite often in voltage drop calculations is Table 9 in the NEC. For AC applications, this table lists reactances, resistances, and impedances in units of 1) ohms-to-neutral per kilometer; or 2) ohms-to-neutral per 1,000 feet. The metric value of ohms per km is the top number and the other value of ohms per 1,000 feet is the bottom number in each cell. As mentioned previously, we will ignore the columns concerning aluminum conductors in this course, but the concepts are the same as for copper conductors.

Which Columns Are Applicable?

Some designers are uncertain as to which columns to use in Table 9 of the NEC for voltage drop calculations. The type of raceway (PVC, aluminum, or steel) is a straightforward issue, but which resistance value column is the most appropriate? A common decision is to use the worst-case value for the wire size in question, such as 0.05 Ω / 1,000 feet for 500 KCMIL copper conductors in steel conduit. What is the difference between all of these columns?

X_L (Reactance)

The first column, after the wire size, is called "X_L (Reactance) for all Wires". This is the value that would be used in place of X in the Effective Z formula that appears as Note 2 to Table 9, and is labeled Equation 5 on page 22 of this course.

Alternating-Current Resistance

The next column is called "Alternating-Current Resistance for Uncoated Copper Wires". This is the value that would be used in place of R in the Effective Z formula that appears as Note 2 to Table 9, and is labeled Equation 5 on page 22 of this course.

> *What are coated copper wires?*
>
> *Metal coatings, such as tin, silver, or nickel, are sometimes applied to the surface of the individual wire strands in copper conductors to reduce oxidation or to improve the electrical properties at high frequencies. The catalog cut-sheet of the conductor or cable should say whether or not the conductors are coated.*

Some designers might use the alternating-current resistance values in this column for the variable R in Equation 1 on page 7 and Equation 3 on page 12, but the values in this column do not account for the reactance of the conductors, nor the power factor of the load.

Effective Z at 0.85 PF

The Effective Z values for certain wire sizes at a power factor of 0.85 were shown previously in the purple column in Table 2 on page 10 so the Reader could compare those values to the DC resistances from which the constant K is derived. The Effective Z at 0.85 PF column in NEC Table 9 is the value of choice for many designers, since it is the worst-case for larger conductors, but also because it is applicable to or close enough for many everyday applications, including many types of motor loads. The values in this column are the result of using Reactance X_L from the first column and the Alternating-Current Resistance from the second column in the effective Z formula described in Note 2 to Table 9 of the NEC. Table 3 below shows some selected examples to prove this point:

Effective Z at 0.85 PF for Selected Wire Sizes in Steel Conduit						
Wire Size	X_L (Reactance)	Alternating-Current Resistance	Power Factor PF [cos(θ)]	Rcos(θ)	Xsin(θ)	Effective Z = Rcos(θ) + Xsin(θ)
12	0.068	2.0	0.85	1.700	0.036	1.7
10	0.063	1.2	0.85	1.020	0.033	1.1
4	0.060	0.31	0.85	0.264	0.032	0.30
2	0.057	0.20	0.85	0.170	0.030	0.20
1/0	0.055	0.12	0.85	0.102	0.029	0.13
250	0.052	0.054	0.85	0.046	0.027	0.073
500	0.048	0.029	0.85	0.025	0.025	0.050

Table 3 – Selected Effective Z Calculations at 0.85 PF (Ohms-to-Neutral per 1,000 feet)

Compare the calculated values in Table 3 to Table 9 of the NEC. The Effective Z values for 12 AWG, 250 KCMIL, and 500 KCMIL in Table 3 are shown graphically as phasor diagrams in Figure 11 on page 25, Figure 13 on page 26, and Figure 14 on page 27, respectively.

Notice that the reactance X_L is much lower than the resistance R in the smaller wire sizes, then is about equal at 250 KCMIL, then the reactance becomes greater than the resistance for larger conductor sizes. In other words, the resistance is the driving factor in the smaller wire sizes and the reactance becomes the significant player in the larger sizes. This can be seen by comparing the rows for different power factors in Table 4 below, where the Effective Z increases as power factor increases in smaller conductor sizes, but the Effective Z decreases in larger conductor sizes as the power factor increases. The former example can be easily seen in Figure 12 on page 26, and the latter example in Figure 15 on page 27.

Effective Z at Any Power Factor: Note 2 to Table 9 in the NEC

The formula in Note 2 to Table 9 in the NEC is often cited in technical documents. It is used to determine the effective impedance (Effective Z) of a conductor at any power factor. Table 4 below is a modified version of Table 3 above, with some of the additional power factor values included. The purple, last column in both tables is the result of the formula in Note 2 to Table 9 in the NEC, which is:

Effective Z = Rcosθ + Xsinθ Equation 5

where:

Effective Z = the effective impedance per 1,000 feet of conductor, ohms-to-neutral;

R = the resistance per 1,000 feet of conductor;

X = the reactance per 1,000 feet of conductor;

θ = the power factor angle of the load.

Effective Z at Selected PF for Selected Wire Sizes in Steel Conduit						
Wire Size	X_L (Reactance)	Alternating-Current Resistance	Power Factor PF [cos(θ)]	Rcos(θ)	Xsin(θ)	Effective Z = Rcos(θ) + Xsin(θ)
12	0.068	2.000	0.80	1.600	0.041	1.641
12	0.068	2.000	0.85	1.700	0.036	1.736
12	0.068	2.000	0.90	1.800	0.030	1.830
12	0.068	2.000	1.00	2.000	0.000	2.000
10	0.063	1.200	0.80	0.960	0.038	0.998
10	0.063	1.200	0.85	1.020	0.033	1.053
10	0.063	1.200	0.90	1.080	0.027	1.107
10	0.063	1.200	1.00	1.200	0.000	1.200
4	0.060	0.310	0.80	0.248	0.036	0.284
4	0.060	0.310	0.85	0.264	0.032	0.295
4	0.060	0.310	0.90	0.279	0.026	0.305
4	0.060	0.310	1.00	0.310	0.000	0.310
2	0.057	0.200	0.80	0.160	0.034	0.194
2	0.057	0.200	0.85	0.170	0.030	0.200
2	0.057	0.200	0.90	0.180	0.025	0.205
2	0.057	0.200	1.00	0.200	0.000	0.200
1/0	0.055	0.120	0.80	0.096	0.033	0.129
1/0	0.055	0.120	0.85	0.102	0.029	0.131
1/0	0.055	0.120	0.90	0.108	0.024	0.132
1/0	0.055	0.120	1.00	0.120	0.000	0.120
250	0.052	0.054	0.80	0.043	0.031	0.074
250	0.052	0.054	0.85	0.046	0.027	0.073
250	0.052	0.054	0.90	0.049	0.023	0.071
250	0.052	0.054	1.00	0.054	0.000	0.054
500	0.048	0.029	0.80	0.023	0.029	0.052
500	0.048	0.029	0.85	0.025	0.025	0.050
500	0.048	0.029	0.90	0.026	0.021	0.047
500	0.048	0.029	1.00	0.029	0.000	0.029

Table 4 – Effective Z Calculations for Selected Wire Sizes at Various Power Factors (Ohms-to-Neutral per 1,000 feet)

As mentioned earlier and as shown in Table 4 above, as the power factor improves (becomes larger) the effective impedance (Effective Z) becomes higher in the smaller wire sizes, but becomes lower in the larger wire sizes. Compare, again, Figure 12 on page 26 to Figure 15 on page 27, if desired.

Note 2 to Table 8 in the NEC

Table 8 in the NEC has DC resistance values for conductors. Note 2 under that table has a simple equation for the change in resistance at other than 75 °C conductor termination temperatures. At low voltages (600 V and less), most terminations are rated for 60 °C or 75 °C. See NEC 110.14(C) for more information on termination temperature ratings. If it is desired to use this formula underneath the DC resistance table, it is:

$$R2 = R1[1 + \alpha(T2 - 75)] \qquad \text{Equation 6}$$

where:

R2 = the new resistance value;

R1 = the original resistance value;

α = 0.00323 for copper;

T2 = the new temperature in °C.

Some technical sources might mention using Note 2 to Table 8 for correcting the resistance of the conductors for ambient temperatures in excess of 75 °C, possibly without considering that 75 °C is equivalent to 167 °F, a temperature that is rarely encountered outside of harsh industrial environments. The real intent for the formula in Note 2 to Table 8 is for conductor operation and termination temperatures other than 75 °C. Let's look at conductors operated at 60 °C and 90 °C in the next two examples.

Example 5: If the terminations, or equipment, or conductor insulation are only rated for 60 °C, then the conductors are only permitted to be operated at 60 °C, which would be a new resistance R2 of:

$$R2 = R1[1 + \alpha(T2 - 75)]$$

$$= R1[1 + (0.00323)(60 - 75)]$$

$$= R1[1 + (-0.04845)]$$

$$= R1[0.95155]$$

Using this formula, the resistance of conductors operated at 60 °C would be about 5% lower than the resistance when operated at 75 °C.

End of Example

Let's look at this formula for conductors operated at 90 °C.

Example 6: If the terminations, and equipment, and conductor insulation are rated for 90 °C, then the conductors are permitted to be operated at 90 °C, which would be a new resistance R2 of:

$$R2 = R1[1 + \alpha(T2 - 75)]$$

$$= R1[1 + (0.00323)(90 - 75)]$$

$$= R1[1 + (0.04845)]$$

$$= R1[1.04845]$$

Using this formula, the resistance of conductors operated at 90 °C would be about 5% higher than the resistance when operated at 75 °C.

End of Example

Now, let's turn to Table 9 in the NEC and use the information from that table to construct phasor diagrams of the resistance, reactance, and resulting impedance of various conductor sizes.

Phasor Diagrams of Resistance, Reactance, and Impedance for Conductors

The phasor diagrams presented in this section are an easy, visual way to understand how the conductor resistance, reactance, and impedance interact with the load's power factor to produce the effective impedance (Effective Z) and the resulting Estimated Vdrop when current is applied to the conductor and load. It is important to note that the power factor of the load can have a significant effect on the Effective Z in these diagrams and on the Estimated Vdrop later, since we will also use these impedance phasor diagrams when we do voltage drop phasor diagrams.

The 12 AWG copper conductor at 0.85 PF in Table 3 on page 21 and Table 4 on page 22 is shown in a graphical fashion known as a phasor diagram in Figure 11 below. There is a right (90°) angle between the R and the X legs of the right triangle, there just isn't enough room to show it in this example. This relationship can be seen more readily in Figure 13 on page 26.

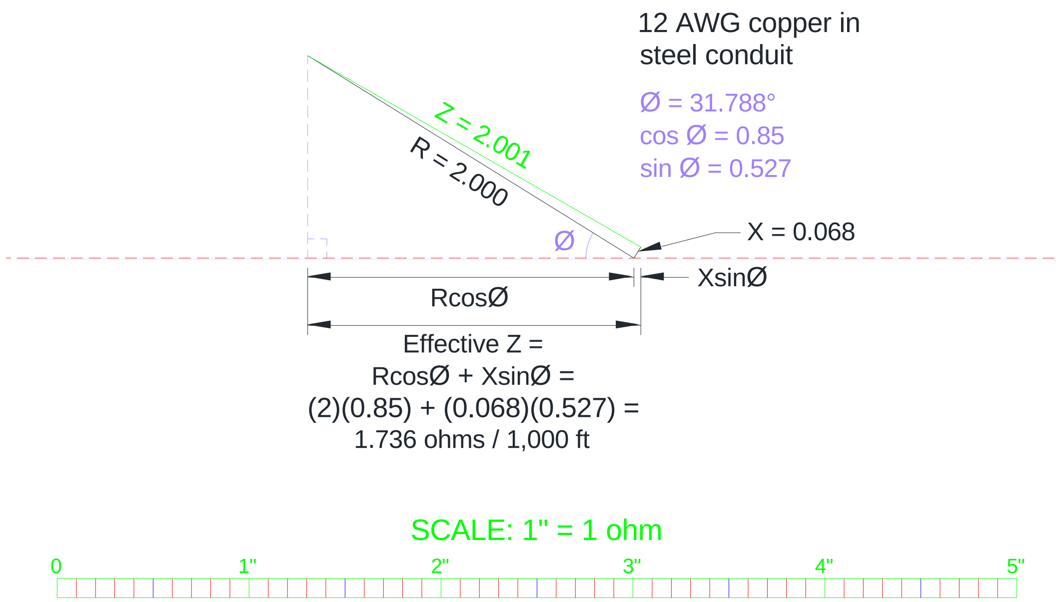

Figure 11 – Phasor Diagram of Resistance, Reactance, and Impedance for 12 AWG Copper Conductors in Steel Conduit at 0.85 PF

The effective impedance known as Effective Z is not the length of the Z vector in Figure 11 and similar figures in this course – it is only the horizontal component of this vector – the length of 1.736, not 2.001 in that particular figure. This horizontal component known as Effective Z will be used in voltage drop calculations later. The Effective Z in Figure 11 is 1.736 ohms / 1,000’, which matches the value for 12 AWG at 0.85 PF in Table 3 on page 21 and Table 4 on page 22. Let’s look at the other power factor values that were shown for 12 AWG copper conductors in steel conduit on Table 4 on page 22, as illustrated in Figure 12 below.

Notice that the resulting vector Z in Figure 11 is pointing in a downward direction. This is because the inductance X is so much smaller than the resistance R in this example. As we investigate larger wire sizes, the ratio of reactance to resistance increases and the vector for impedance Z starts pointing in a more upward direction. Compare Figure 11 to Figure 13 and Figure 14.

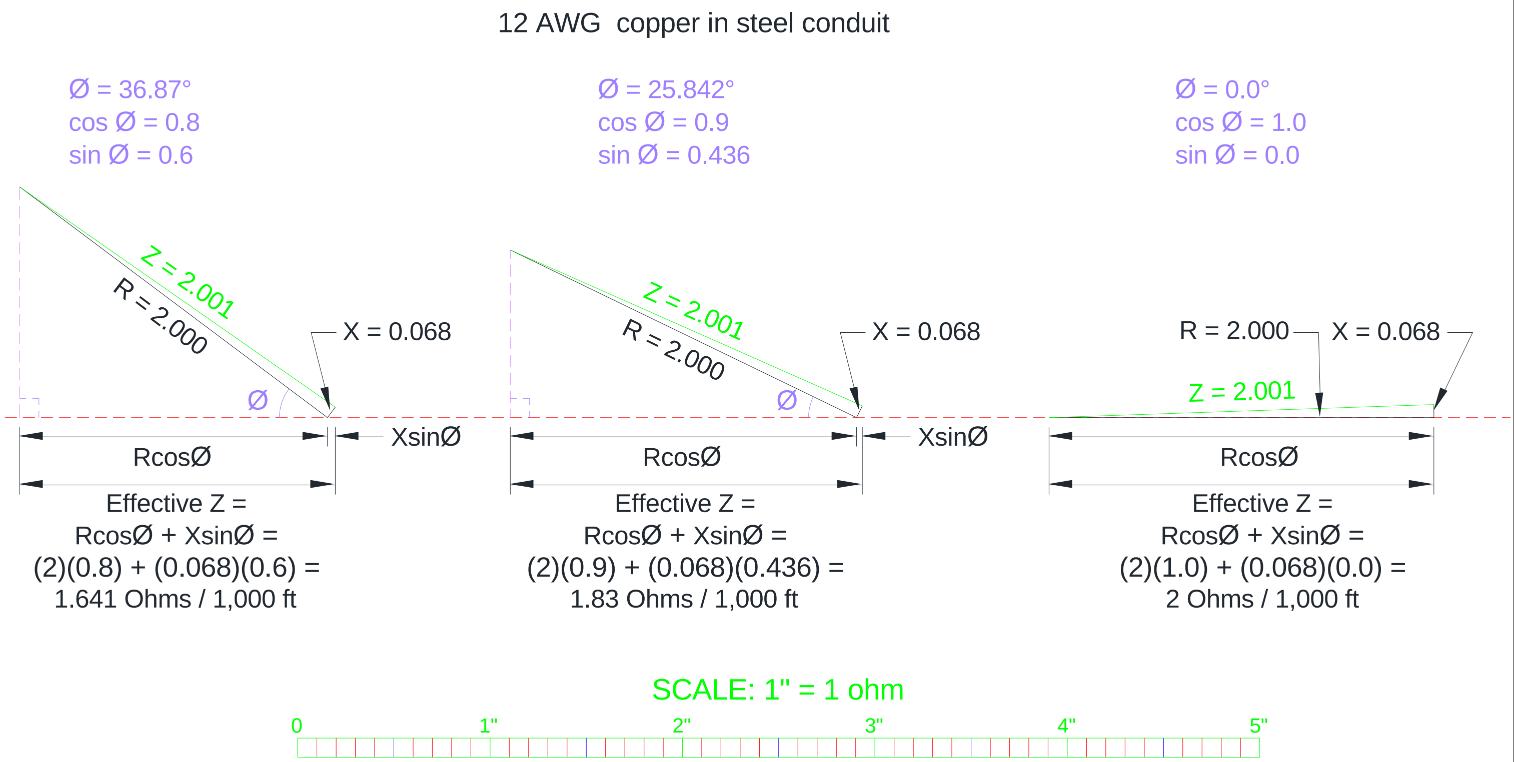

Figure 12 – Phasor Diagram of Resistance, Reactance, and Impedance for 12 AWG Copper Conductors in Steel Conduit at Selected Power Factor Values

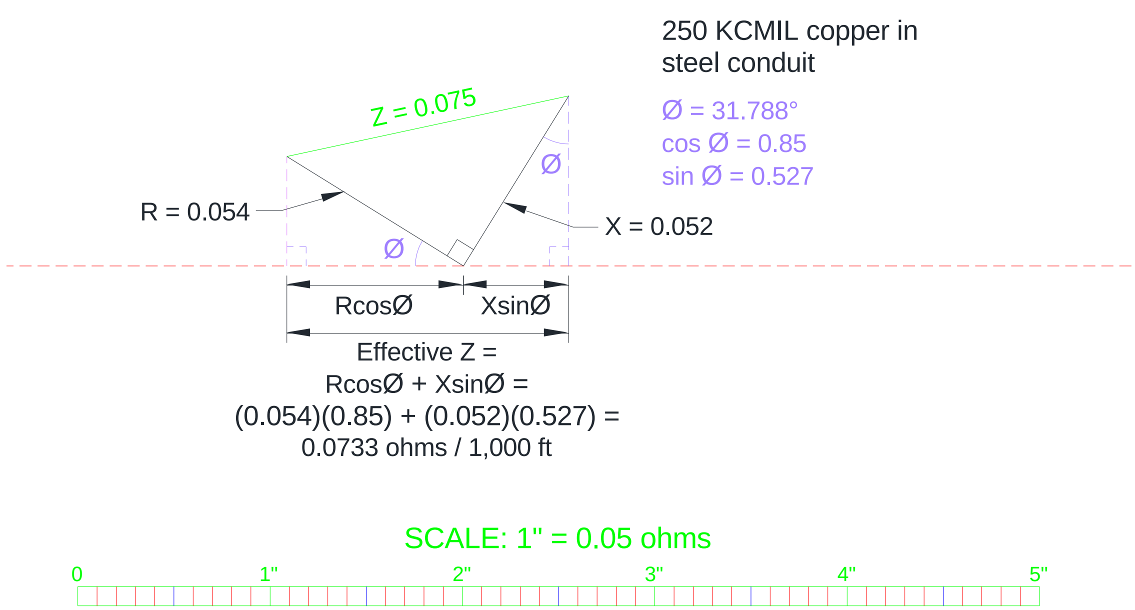

Figure 13 – Phasor Diagram of Resistance, Reactance, and Impedance for 250 KCMIL Copper Conductors in Steel Conduit at 0.85 PF

The Effective Z in Figure 13 can be rounded to 0.073, which matches the value for 250 KCMIL at 0.85 PF in Table 3 on page 21 and Table 4 on page 22. Note that the scale of inches-to-ohms in Figure 13 and similar figures is often much different than the scale in Figure 11 and Figure 12.

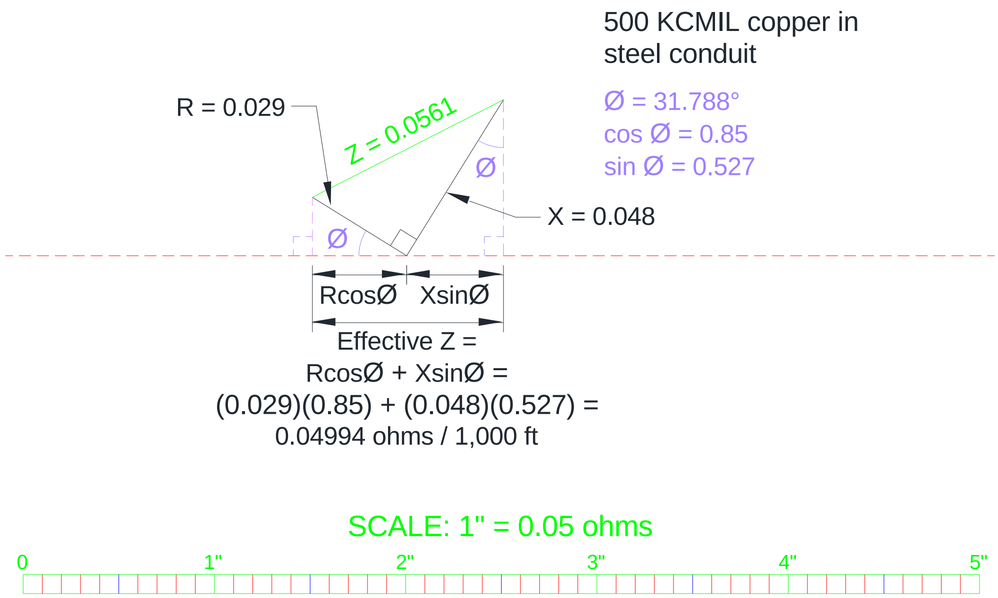

Figure 14 – Phasor Diagram of Resistance, Reactance, and Impedance for 500 KCMIL Copper Conductors in Steel Conduit at 0.85 PF

The Effective Z in Figure 14 can be rounded to 0.05 ohms / 1,000', which matches the value for 500 KCMIL at 0.85 PF in Table 3 on page 21 and Table 4 on page 22. Let's look at phasor diagrams for the other power factor values that were shown for 500 KCMIL copper conductors in steel conduit on Table 4 on page 22, as illustrated in Figure 15 below.

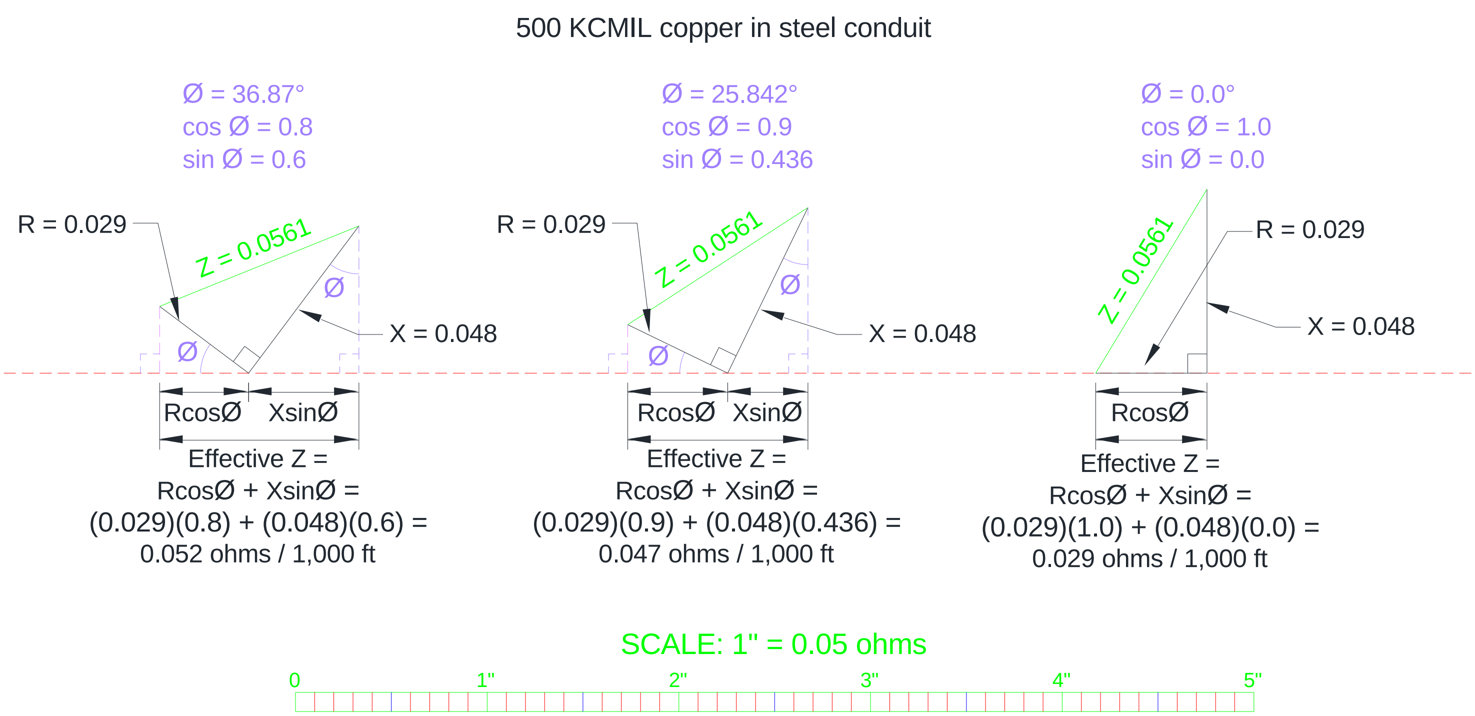

Figure 15 – Phasor Diagram of Resistance, Reactance, and Impedance for 500 KCMIL Copper Conductors in Steel Conduit at Selected Power Factor Values

As the power factor increases from 0.8 to 1.0 in Figure 15 above, the angle ϕ decreases, which lessens the effect that the reactance X has on the horizontal length known as Effective Z.

We have developed phasor diagrams for the resistance R and the reactance X at a certain power factor PF to calculate the Effective Z for various sizes of copper conductors. Let's apply 300 A of current to the phasor diagrams for the Effective Z for 500 KCMIL copper conductors in steel conduit in Figure 15 above, the results of which are shown in Figure 16 below.

Estimated Vdrop Derived from Impedance Phasor Diagrams

To get the Estimated Vdrop, we need to apply current to the impedance phasor diagrams that were generated in the previous section. Consider Figure 16 below, which is simply the 500 KCMIL copper conductor example in Figure 15 above, with 300 A of alternating current applied to each line conductor. Both the resistance R (0.029 Ω) and reactance X (0.048 Ω) are multiplied by the current I (300 A) to get the two legs of what is now a voltage phasor diagram, instead of a resistance, reactance, and impedance phasor diagram. When 300 A is applied to the conductors, the length of the resistance leg will be (300 A) * (0.029 Ω) = 8.7 V per 1,000 feet and the length of the reactance leg will be (300 A) * (0.048 Ω) = 14.4 V per 1,000 feet. The Estimated Vdrop in each of the three cases is determined by the power factor PF of the load.

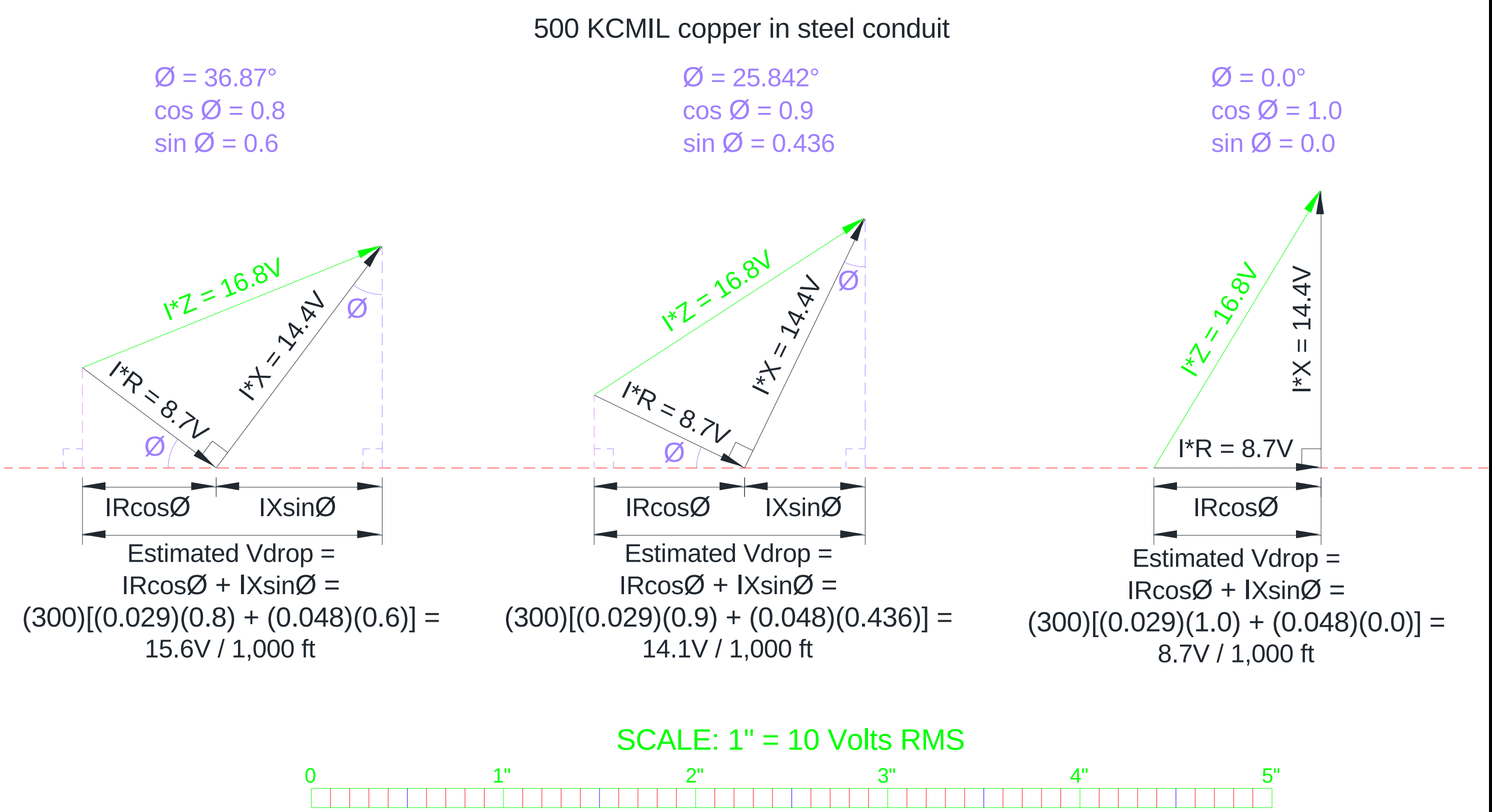

Figure 16 – Applying 300 Amps to the Impedance Phasor Diagram for 500 KCMIL Copper Conductors in Steel Conduit at Selected Power Factor Values

Now that we have applied a current to the resistance, reactance, and impedance phasors, we can apply directional arrowheads to the voltage vectors in Figure 16 above. Notice that the Estimated Vrop values are volts per 1,000 feet. As can be seen in that figure, as the power factor increases, the approximate voltage drop decreases. As we will see later in Figure 21 on page 33, however, as the power factor increases, the Error voltage associated with the Estimated Vdrop calculation in Equation 7 below also increases for this and other large conductor sizes.

In the middle of Figure 16 is an example where the line-to-neutral voltage drop is 14.1 V / 1,000 ft, so the line-to-line voltage drop would be √3 times that or 24.4 V / 1,000 ft. If the one-way conductor length is 200 feet, then the line-to-line voltage drop would be 24.4 V * 200 / 1000 = 4.88 V.

When we apply current to the NEC formula for Effective Z (Equation 5 on page 22), we get the portion of the voltage drop that is known as Estimated Vdrop. Equation 7 below is simply Equation 5 with the line current applied to it:

Estimated Vdrop = $I(R\cos\theta + X\sin\theta) = IR\cos\theta + IX\sin\theta$ line-to-neutral Equation 7

where:

Estimated Vdrop = voltage drop, line-to-neutral per 1,000 feet of conductor;

R = the resistance per 1,000 feet of conductor;

X = the reactance per 1,000 feet of conductor;

θ = the power factor angle of the load;

I = the line current going through the conductor to the load.

Estimated Vdrop, Single-Phase:

Equation 7 gives us the line-to-neutral Estimated Vdrop, but we need to multiply that value by 2 to get the round-trip Estimated Vdrop:

Estimated Vdrop = $2*\{IR\cos\theta + IX\sin\theta\}$ round-trip Equation 8

Estimated Vdrop, Three-Phase:

Equation 7 gives us the line-to-neutral Estimated Vdrop, but we need to multiply that value by √3, as illustrated in Figure 10 on page 19, to get the line-to-line Estimated Vdrop:

Estimated Vdrop = $\sqrt{3}*\{IR\cos\theta + IX\sin\theta\}$ line-to-line Equation 9

Notice that we have not included the value of the supply voltage in any of the voltage drop calculation formulas or diagrams we have looked at thus far. If voltage drop is a product (literally) of the current through a conductor and the conductor's resistance to that current, why would the voltage of the power source matter? As unlikely as it may seem, we need to know the line-to-neutral voltage of the power source in order to determine the rest of the voltage drop that occurs after the Estimated Vdrop. This 'rest of the voltage drop' is denoted as Error in Figure 17 in the next section.

Voltage Drop Phasor Diagram in IEEE Standard 141 (Red Book)

The voltage drop phasor diagram in Figure 17 below is based on "Figure 3-11 – Phasor Diagram of Voltage Relations for Voltage-Drop Calculations" of the 1993(R1999) edition of IEEE *Standard 141* (the Red Book). The same diagram from the Red Book also appears as "Figure 7 – Vector Diagram of Voltage Relations for Voltage-Drop Calculations" in the 1990 Edition of IEEE *Standard 241* (the Gray Book). See the Additional Reading section, beginning on page 56, for more information about these publications. The quantity known as Estimated Voltage Drop in Standard 141 is called Calculated Voltage Drop in Standard 241.

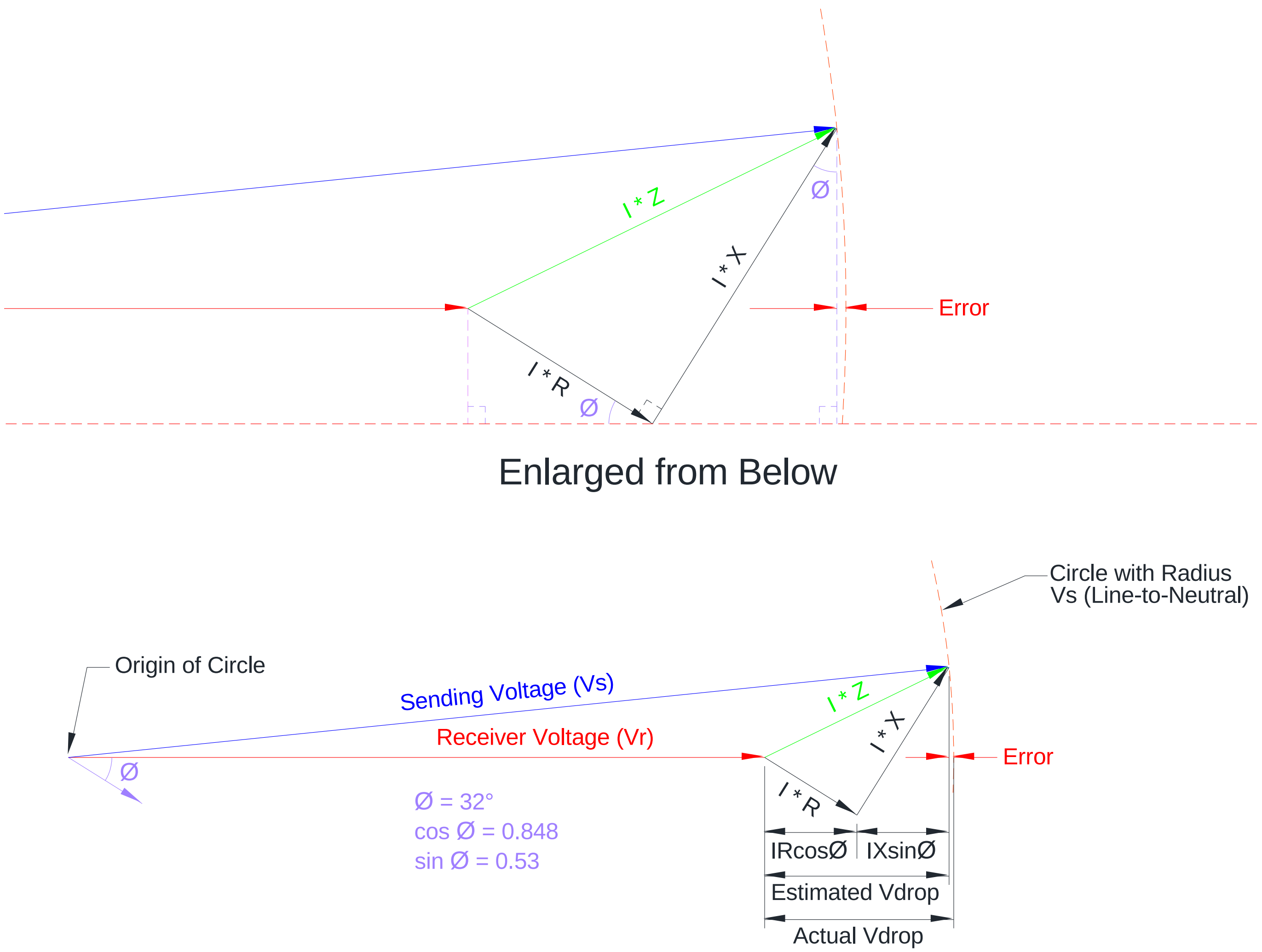

Figure 17 – IEEE Phasor Diagram of Voltage Drop

The phasor diagram in Figure 3-11 in IEEE Standard 141, upon which Figure 17 above is based, has been replicated and referenced in many technical documents and is a helpful representation of the concepts inherent to voltage drop calculations: both the Estimated Vdrop and the Actual Vdrop. In this phasor diagram, we can see the IRcosθ + IXsinθ formula that we have discussed as Equation 7 in previous sections. This phasor diagram also shows the mysterious element we have been alluding to, namely the Error between the Estimated Vdrop and the Actual Vdrop, which is the horizontal distance between IRcosθ + IXsinθ and the circumference of the circle that has a radius of the supply voltage or sending voltage Vs.

The phasor diagram that appears in the IEEE Standards is not forthcoming about how many places the power factor angle Φ or θ (depending on the Standard) appears in that diagram, but this can be inferred using simple geometric principles, as shown at the top of Figure 17 above. The horizontal component known as Estimated Vdrop is shown for triangle R-X-Z in that figure, but let's look at the vertical components of that triangle, since they are a main ingredient in calculating the Error and the resulting Actual Vdrop.

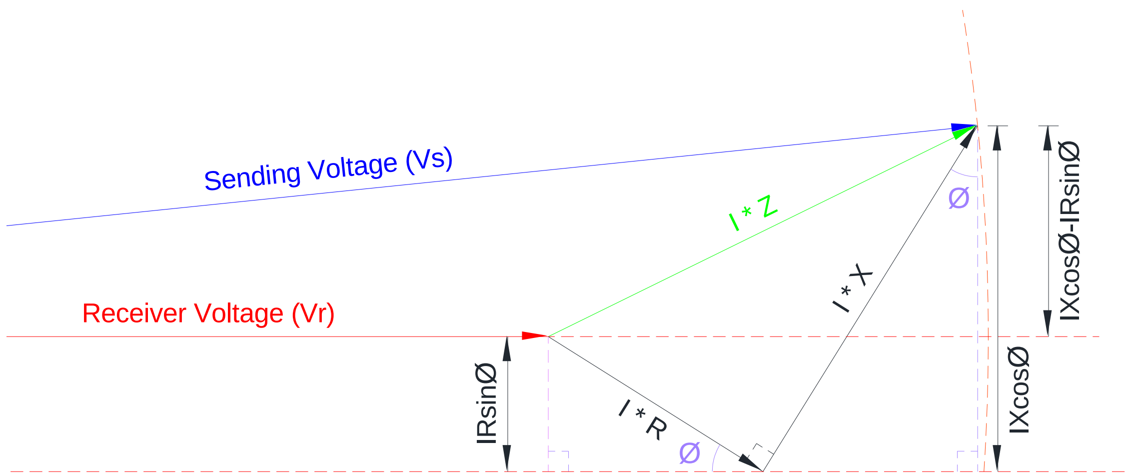

Figure 18 – Vertical Components of Phasor Diagram of Voltage Drop

The key piece of information we want from Figure 18 is the vertical distance or length denoted as IXcosΦ-IRsinΦ that rises above the horizontal line defined by the Receiver Voltage Vr. This is the vertical distance from the end of the Estimated Vdrop to the end of Vs. It is easy to see that this vertical length is formed by subtracting IRsinΦ from IXcosΦ. Let's use this vertical length in constructing a triangle with three sides that are well-known to us by now. See Figure 19.

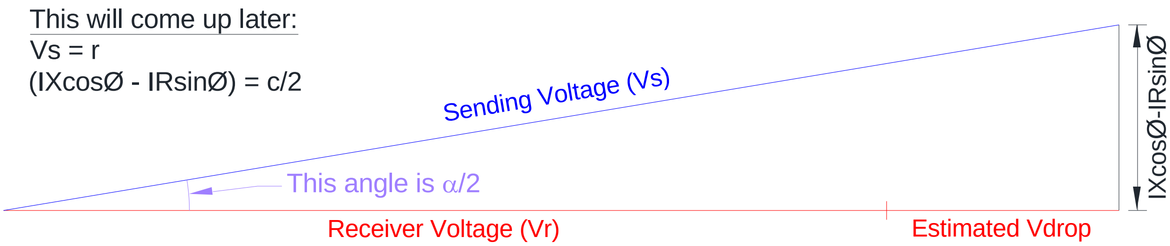

Figure 19 – Triangle Formed by Vs, Vr + Estimated Vdrop, and IXcosΦ-IRsinΦ

The triangle in Figure 19 above is a crucial step in solving the mystery of the Actual Vdrop. The two most important components that will come up later in the section called Calculating the Error Shown in the IEEE Phasor Diagram beginning on page 34 are the hypotenuse in blue and the vertical side in black.

It might seem convenient to say that the length or magnitude of vector I * Z in Figure 17 on page 30 is the Actual Vdrop, since it seems to line up nicely in the figure, but it is a little more complicated than that. The only instance in which the length or magnitude of vector I * Z would be equal to the Actual Vdrop would be when vector I * Z points at exactly 0° or the 3 o'clock position, which means the power factor of the load (cosθ = R / Z) would be exactly the ratio of the conductor's R to Z values, which is also when the sine of the power factor angle (sinθ = X / Z) would be exactly the ratio of the conductor's X to Z values. To put an even finer point on it, that is when tanθ = X / R. That means that if the power factor angle in Figure 20 is 58°, then the ratio of the conductor X to R would have to be equal to tan(58°) = 1.6.

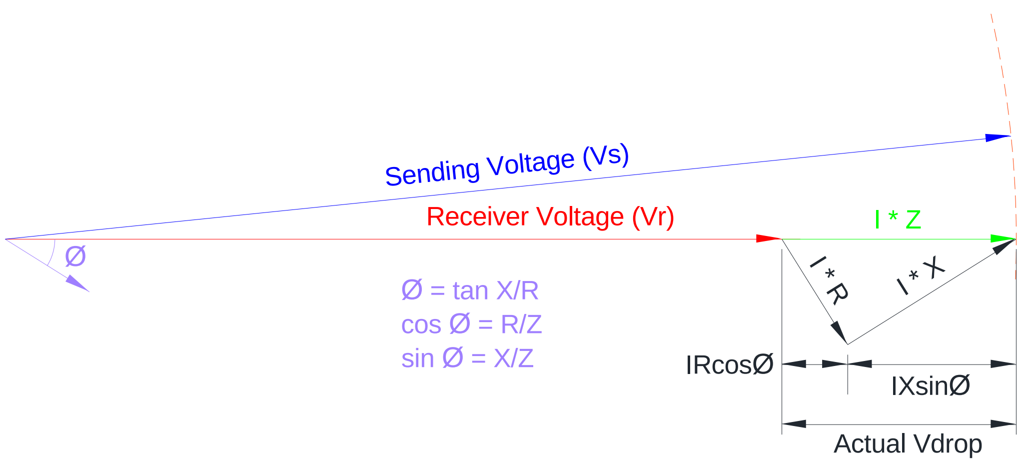

Figure 20 – When the Vector I * Z Really Is the Actual Vdrop

It would be a very strange circumstance, indeed, for the parameters of the conductor to be in exact accordance with the power factor of the load, so it would be a rare and singular occurrence for the length of vector I * Z to exactly equal to the Actual Vdrop.

Another item of interest that was mentioned previously, and one that might seem counter-intuitive, is that the Error in the voltage drop calculation increases as the power factor increases when X > R for the conductor chosen. See Figure 21 below for a graphical demonstration of this concept.

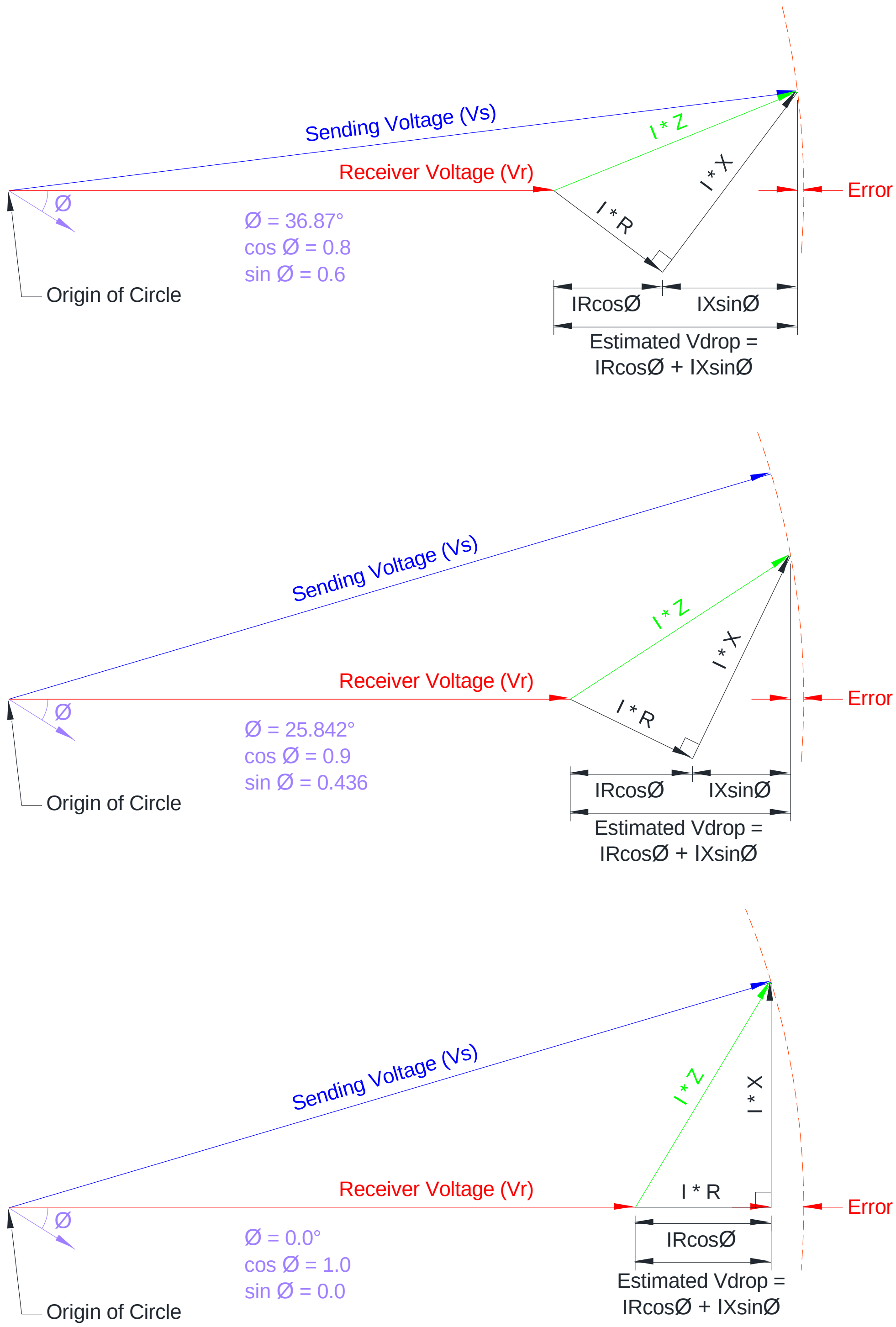

Figure 21 – If Conductor X > R, the Error Increases as Power Factor Increases

We have already seen at Figure 15 on page 27 that the voltage drop in larger conductors decreases as the power factor increases, since X > R for conductors larger than 250KCMIL, as can be seen in Table 9 of the NEC and Table 4 on page 22. Figure 21 above reinforces the concept that the voltage drop decreases as power factor increases when X > R, but also illustrates that the Error value increases as the longer X leg of the triangle pushes the top corner of the triangle more and more to the left as the triangle rotates in a counter-clockwise direction as the power factor increases.

Let's derive the formula for Actual Vdrop, which is the sum of the Estimated Vdrop plus the Error.

Calculating the Error Shown in the IEEE Phasor Diagram

A horizontal distance called Error is shown in Figure 17 on page 30. This is the difference between 1) the Estimated Vdrop that we have been calculating by applying current to the NEC formula to get the horizontal distance IRcosθ + IXsinθ and 2) the Actual Vdrop. In other words, it is the rest of the line-to-neutral voltage drop, which cannot be reached by Equation 7 on page 29.

We can calculate this Error value by finding the height h of the circular segment shown in Figure 22 below. Refer back to Figure 19 on page 32, then notice its similarity to the upper triangle shown in Figure 22 below, with sides d, c/2, and r. The height h of the circular segment is shown on the right hand side of the circle in the figure below, at the 3-o'clock position.

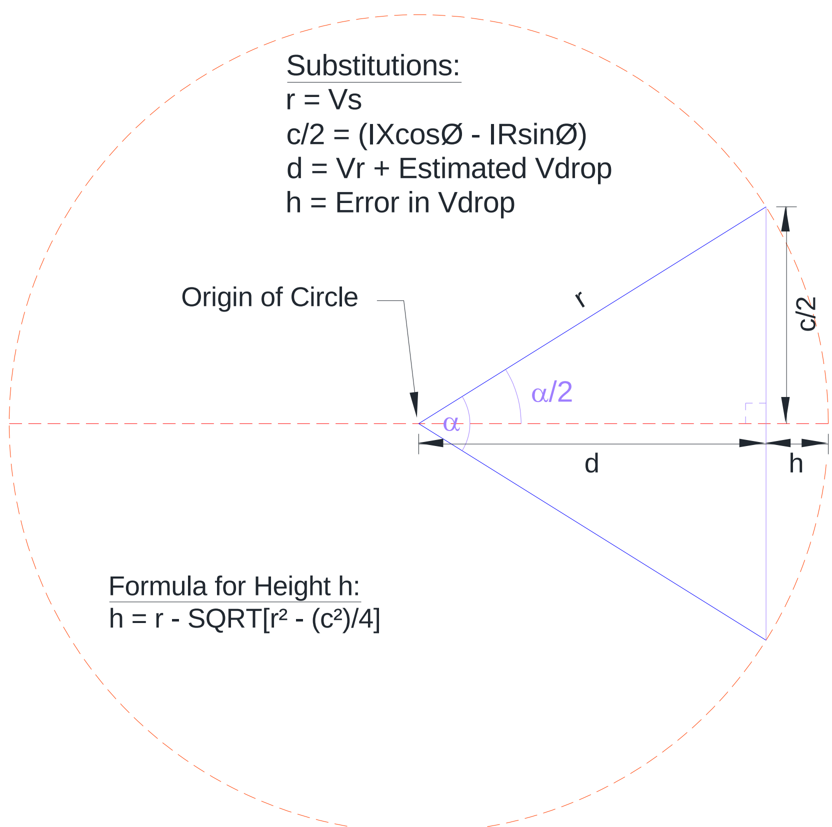

Figure 22 – Finding the Height h of a Circular Segment

The h in Figure 22 represents the Error shown in Figure 17 on page 30. We can calculate the height h by using this formula for circular segments:

$$h = r - \sqrt{r^2 - \frac{c^2}{4}}$$ Equation 10

For more information on the geometry of a circular segment, one source is listed in the Additional Reading section, beginning on page 56. Using Equation 10, and substituting the values shown in Figure 22, we have:

$$h = r - \sqrt{r^2 - \frac{c^2}{4}}$$

where:

h = the height of the circular segment, which is the Error;

r = the radius of the circle, which is the sending or supply voltage Vs, line-to-neutral;

c/2 = half of the cord of the circular segment, which is equal to IXcosΦ-IRsinΦ.

Since the $(c^2)/4$ portion of the equation is equivalent to $(c/2)^2$, we now have, simply by direct substitution:

$$Error = Vs - \sqrt{Vs^2 - (IXcos\Phi - IRsin\Phi)^2}$$ line-to-neutral Equation 11

When we combine the Estimated Vdrop formula (Equation 7 on page 29) with the Error formula (Equation 11) we get the exact or Actual Vdrop:

Actual Vdrop = Estimated Vdrop + Error

$$= \mathrm{IRcos}\theta + \mathrm{IXsin}\theta + \mathrm{Vs} - \sqrt{\mathrm{Vs}^2 - (\mathrm{IXcos}\Phi - \mathrm{IRsin}\Phi)^2}$$ line-to-neutral

Rearranging terms to look like the formula in IEEE Standards 141 and 241:

$$Actual\ Vdrop = Vs + IRcos\Phi + IXsin\Phi - \sqrt{Vs^2 - (IXcos\Phi - IRsin\Phi)^2}$$ line-to-neutral
Equation 12

Remember that the values in Equation 11 and Equation 12 are the voltage drop line-to-neutral values, so they all have to be multiplied by 2 for single-phase or √3 for three-phase in order to calculate the round-trip and line-to-line voltage drops, respectively, which is done in the equations below. This is also what we had to do for Estimated Vdrop Equation 7 on page 29 in order to get Equation 8 and Equation 9, also respectively.

Single-Phase Formulas for Error and Actual Vdrop:

Error = $2*\{ Vs - \sqrt{Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2} \}$ round-trip Equation 13

Actual Vdrop = $2*\{ Vs + IR\cos\Phi + IX\sin\Phi - \sqrt{Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2} \}$ round-trip Equation 14

Three-Phase Formulas for Error and Actual Vdrop:

Error = $\sqrt{3}*\{ Vs - \sqrt{Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2} \}$ line-to-line Equation 15

Actual Vdrop = $\sqrt{3}*\{ Vs + IR\cos\Phi + IX\sin\Phi - \sqrt{Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2} \}$ line-to-line Equation 16

If, as many technical publications do, we want to express the Actual Vdrop in terms of the voltage received at the load (Vr), we can say:

Vr = Vs – Actual Vdrop

$\mathrm{Vr} = \mathrm{Vs} - \{\mathrm{Vs} + \mathrm{IR}\cos\Phi + \mathrm{IX}\sin\Phi - \sqrt{\mathrm{Vs}^2 - (\mathrm{IX}\cos\Phi - \mathrm{IR}\sin\Phi)^2} \}$

$\mathrm{Vr} = -\mathrm{IR}\cos\Phi - \mathrm{IX}\sin\Phi + \sqrt{\mathrm{Vs}^2 - (\mathrm{IX}\cos\Phi - \mathrm{IR}\sin\Phi)^2}$

Putting the Estimated Vdrop in parentheses and moving it to the end, so the formula doesn't start with a negative sign, we have the same formula that appears in IEEE Standard 241:

$\mathrm{Vr} = \sqrt{\mathrm{Vs}^2 - (\mathrm{IX}\cos\Phi - \mathrm{IR}\sin\Phi)^2} - (\mathrm{IR}\cos\Phi + \mathrm{IX}\sin\Phi)$ line-to-neutral Equation 17

In the 1990 edition of IEEE Standard 241, the formula is not numbered, but it appears at the top of page 72, just after (Eq 2) at the bottom of page 71 in that publication. It also has a typo: the = sign is missing.

It is important to keep track of whether the voltage you have calculated is line-to-neutral, round-trip, or line-to-line.

Let's do a little bit of foreshadowing by calculating the Error voltage for the real-world examples that will be discussed in the next section.

Vdrop Error for 10 Hp at 480V/3Φ, 0.85 PF											
Wire Size	**Table 8 Cir. Mils**	**Table 8 DC Ohms**	**Table 9 X_L (React) Ohms/k-ft**	**Table 9 AC Ohms/k-ft**	**Power Factor PF [cos(θ)]**	**Rcos(θ) per k-ft**	**Xsin(θ) per k-ft**	**NEC Effective Z at 0.85 PF**	**Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft**	**Full-Load Current**	**One-Way Wire Length (ft)**
12	6,530	1.98	0.068	2.000	0.85	1.700	0.036	1.700	1.736	14.0	200.0
Equation 11:	Error = $\{Vs - \sqrt{[Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2]}\}$							Result:	0.0140 V Error line-to-neutral		
Equation 15:	Error = $\sqrt{3}*\{Vs - \sqrt{[Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2]}\}$							Result:	0.0243 V Error line-to-line		

Vdrop Error for 15 KW at 480V/3Φ, 1.0 PF											
Wire Size	**Table 8 Cir. Mils**	**Table 8 DC Ohms**	**Table 9 X_L (React) Ohms/k-ft**	**Table 9 AC Ohms/k-ft**	**Power Factor PF [cos(θ)]**	**Rcos(θ) per k-ft**	**Xsin(θ) per k-ft**	**NEC Effective Z at 0.85 PF**	**Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft**	**Full-Load Current**	**One-Way Wire Length (ft)**
10	10,380	1.24	0.063	1.200	1.00	1.200	0.000	1.100	1.200	18.0	200.0
Equation 11:	Error = $\{Vs - \sqrt{[Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2]}\}$							Result:	0.0001 V Error line-to-neutral		
Equation 15:	Error = $\sqrt{3}*\{Vs - \sqrt{[Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2]}\}$							Result:	0.0002 V Error line-to-line		

Vdrop Error for 250 Hp at 480V/3Φ, 0.9 PF											
Wire Size	**Table 8 Cir. Mils**	**Table 8 DC Ohms**	**Table 9 X_L (React) Ohms/k-ft**	**Table 9 AC Ohms/k-ft**	**Power Factor PF [cos(θ)]**	**Rcos(θ) per k-ft**	**Xsin(θ) per k-ft**	**NEC Effective Z at 0.85 PF**	**Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft**	**Full-Load Current**	**One-Way Wire Length (ft)**
500	500,000	.0258	0.048	0.029	0.90	0.026	0.021	0.050	0.047	302.0	200.0
Equation 11:	Error = $\{Vs - \sqrt{[Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2]}\}$							Result:	0.006 V Error line-to-neutral		
Equation 15:	Error = $\sqrt{3}*\{Vs - \sqrt{[Vs^2 - (IX\cos\Phi - IR\sin\Phi)^2]}\}$							Result:	0.011 V Error line-to-line		

Table 5 – Error Voltage Drop Calculations for Real-World Examples in Next Section

The Error voltages in Table 5 above are explored in more detail in the real-world examples listed in Table 6 below. The line-to-neutral Error voltages in Table 5 agree with the Error voltages shown in the applicable figures and tables for the real-world examples, but the line-to-line Error voltages are not an exact match. This is due only to the fact that we are using the rounded-off value of 277 V for 480 V / √3.

One might wonder, why go through all of this trouble to include the miniscule Error in order to arrive at the Actual voltage drop? One response to that might be, why try to balance your checkbook to the penny?

Now that we have derived the Actual voltage drop formulas shown in IEEE Standards 141 and 241, let's apply them to some everyday installations.

Real-World Examples

In this section, we will look at some typical examples that might occur in the day-to-day functions of an electrical design. The following table can serve as a quick guide in finding the various examples:

Type of Load for Three-Phase Line Current Examples	Starts on Page #
10 Hp Motor at 480V/3Φ with 12 AWG Conductors	38
15 KW Heater at 480V/3Φ with 10 AWG Conductors	41
250 Hp Motor at 480V/3Φ with 500 KCMIL Conductors	43

Table 6 – Real-World Examples

For each of the examples in the table above, we will assume a one-way copper conductor length of 200 feet in steel conduit and an ambient temperature between 30 °C and 40 °C. A maximum voltage drop of 2% is also assumed, which would be equivalent to 9.6 V line-to-line.

10 Hp Motor at 480V/3Φ with 12 AWG Conductors

A 10 Hp motor at 480V/3Φ has an NEC full-load current rating of 14 A. It is typical to specify 12 AWG conductors for this load. Assume a power factor of 0.85. Figure 23 below is a graphical representation of the voltage drop and Table 7 compares the four different types of three-phase voltage drop calculations we have already discussed. If you notice any miniscule discrepancies to the right of the decimal points line-to-line voltages for Vr and Actual Vdrop in Figure 23, it is due to rounding off the value of 480 V / √3 to exactly 277 V, as mentioned previously.

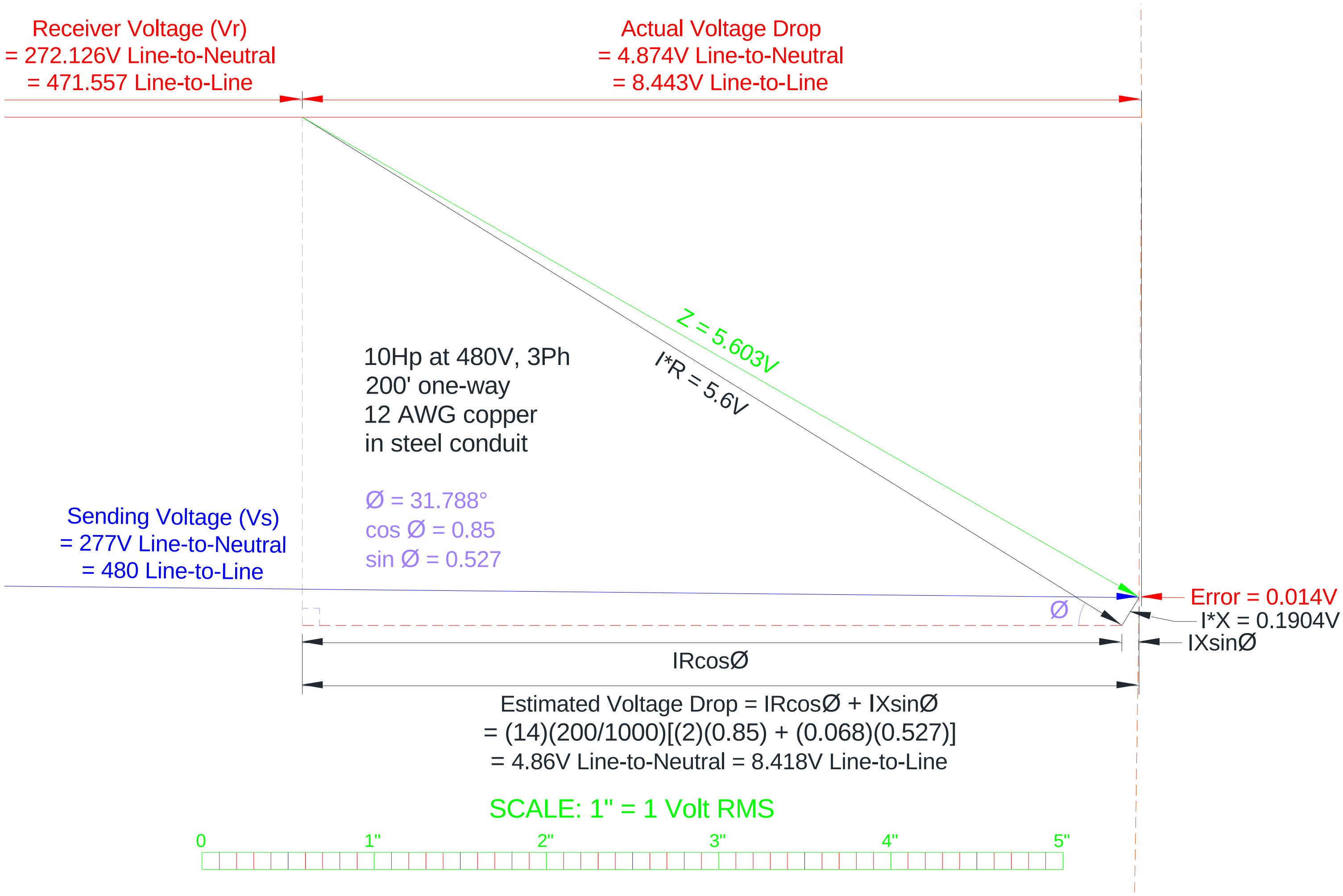

Figure 23 – Line-to-Neutral Voltage Drop for 10 Hp Motor at 480V/3Φ with 200' of 12 AWG Conductors

Remember that the resistance, reactance, and impedance values, and consequently the voltage values, shown in Table 9 in the NEC are for line-to-neutral resistance or ohms-to-neutral resistance. This means that the circle defined by Vs needs to be the line-to-neutral or phase-to neutral voltage, which would be 277 V in a 480 V system.

The sending or supply voltage Vs is shown underneath the load or receiver voltage Vr in Figure 23 because of the downward slope of the Effective Z vector for smaller conductor sizes, as discussed previously.

Let's see what the various formulas discussed in this course would yield for the line-to-line voltage drop shown in Figure 23. Consider Table 7 below.

Comparing the Results of Different Line-to-Line Vdrop Formulas for 10 Hp at 480V/3Φ, 0.85 PF											
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
12	6,530	1.98	0.068	2.000	0.85	1.700	0.036	1.700	1.736	14.0	200.0
Equation 4:	Vdrop = √3 * I * K * R * L / A			See Note below.		K = 12.9		Result:	9.581	V Approx.	1.995%
Equation 3:	Vdrop = √3 * I * R * L					Using DC Ohms Only		Result:	9.602	V Approx.	2%
Equation 3:	Vdrop = √3 * I * R * L					Using AC Ohms Only		Result:	9.699	V Approx.	2.02%
Equation 3:	Vdrop = √3 * I * R * L					Using Effective Z at 0.85 PF		Result:	8.245	V Approx.	1.717%
Equation 9:	Vdrop = √3 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	8.418	V Estimated	1.753%
Equation 16:	Vdrop = √3 * {Vs + IRLcosθ + IXLsinθ - √[Vs² - (IXLcosΦ-IRLsinΦ)²]}					Vs = 277V line-to-neutral		Result:	8.443	V Actual	1.758%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 7 – Line-to-Line Voltage Drop for 10 Hp Motor at 480V/3Φ with 200' of 12 AWG Conductors

As one can see from Table 7 above, the results of the different types of voltage drop calculations can vary significantly. In each formula in that example, however, 12 AWG conductors are acceptable for a maximum of 2% voltage drop, in the Author's opinion.

15 KW Heater at 480V/3Φ with 10 AWG Conductors

A 15KW heater at 480V/3Φ has a full-load current of 18 A. It is typical to specify 10 AWG conductors for this load. Assume a power factor of 1.0. Figure 24 below is a graphical representation of the voltage drop and Table 8 compares the four different types of three-phase voltage drop formulas.

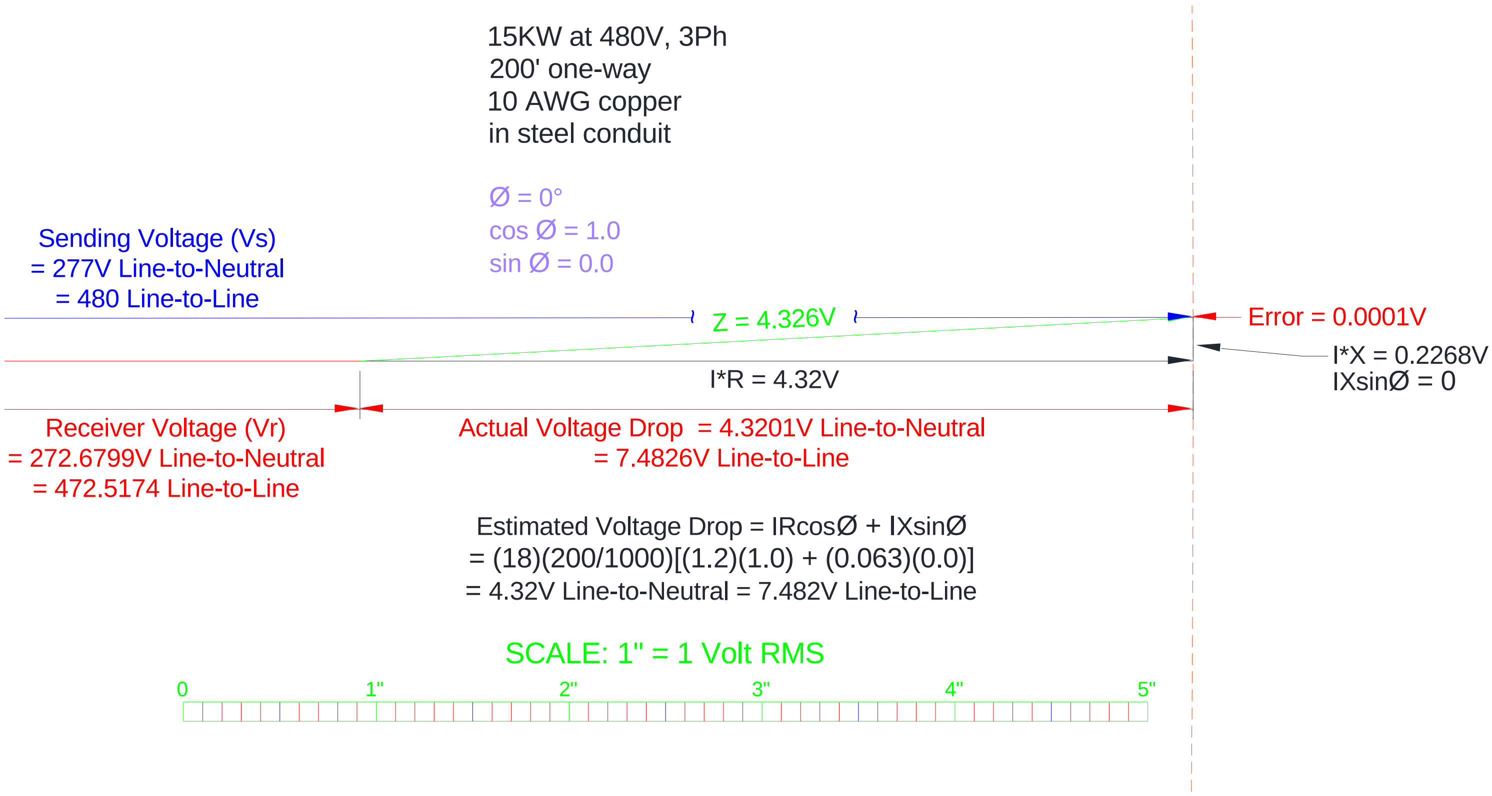

Figure 24 – Line-to-Neutral Voltage Drop for 15 KW Load at 480V/3Φ with 200' of 10 AWG Conductors

Comparing the Results of Different Line-to-Line Vdrop Formulas for 15 KW at 480V/3Φ, 1.0 PF											
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
10	10,380	1.24	0.063	1.200	1.00	1.200	0.000	1.100	1.200	18.0	200.0
Equation 4:	Vdrop = √3 * I * K * R * L / A			See Note below.		K = 12.9		Result:	7.749 V Approx.		1.614%
Equation 3:	Vdrop = √3 * I * R * L					Using DC Ohms Only		Result:	7.732 V Approx.		1.61%
Equation 3:	Vdrop = √3 * I * R * L					Using AC Ohms Only		Result:	7.482 V Approx.		1.558%
Equation 3:	Vdrop = √3 * I * R * L					Using Effective Z at 0.85 PF		Result:	6.859 V Approx.		1.428%
Equation 9:	Vdrop = √3 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	7.482 V Estimated		1.558%
Equation 16:	Vdrop = √3 * {Vs + IRLcosθ + IXLsinθ - √[Vs² - (IXLcosΦ-IRLsinΦ)²]}					Vs = 277V line-to-neutral		Result:	7.4826 V Actual		1.558%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 8 – Line-to-Line Voltage Drop for 15 KW Load at 480V/3Φ with 200' of 10 AWG Conductors

Since the power factor PF = 1.0 in Figure 24 and Table 8, the reactance is not part of the Estimated Vdrop, but it does contribute 0.0006 V of Error voltage to get to the Actual Vdrop. Also, since PF = 1.0, Equation 3 using only the AC ohms gives the same result as Equation 9 using the calculated Effective Z: 7.482 V.

As can be seen from Table 8 above, we would have gotten the least-accurate result if we had used the Effective Z at 0.85 PF column for this 1.0 PF load.

250 Hp Motor at 480V/3Φ with 500 KCMIL Conductors

A 250 Hp motor at 480V/3Φ has an NEC full-load current rating of 302 A. It is typical to specify 500 KCMIL conductors for this load. Assume a power factor of 0.9. Figure 25 below is a graphical representation of the voltage drop and Table 9 compares the four different types of three-phase voltage drop formulas. As mentioned for Figure 23 on page 39, if you notice any miniscule discrepancies to the right of the decimal points for line-to-line voltages Vr and Actual Vdrop in Figure 25, it is due to rounding off the value of 480 V / √3 to exactly 277 V.

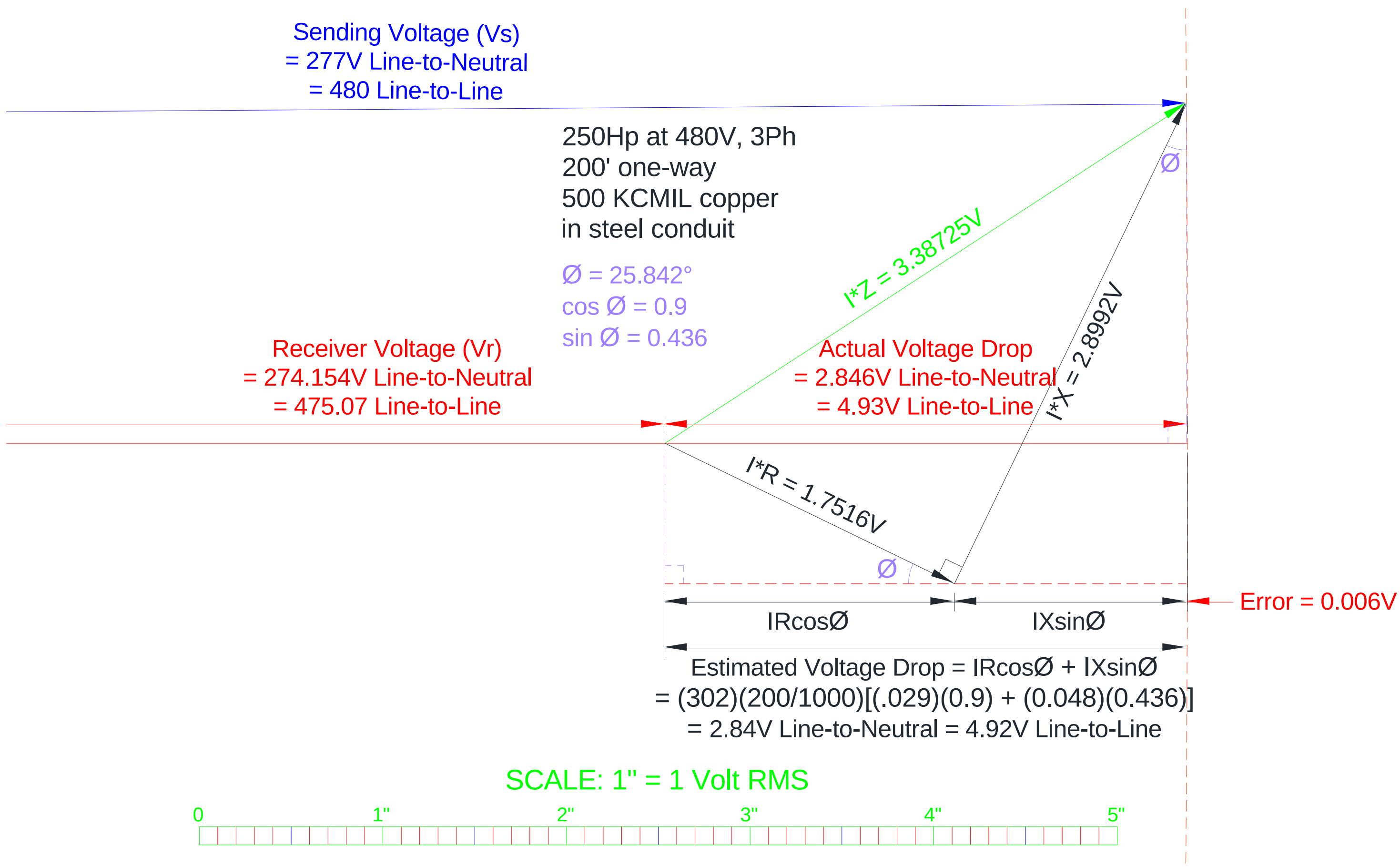

Figure 25 – Line-to-Neutral Voltage Drop for 250 Hp Motor at 480V/3Φ with 200' of 500 KCMIL Conductors

Comparing the Results of Different Line-to-Line Vdrop Formulas for 250 Hp at 480V/3Φ, 0.9 PF											
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
500	500,000	.0258	0.048	0.029	0.90	0.026	0.021	0.050	0.047	302.0	200.0
Equation 4:	Vdrop = √3 * I * K * R * L / A			See Note below.		K = 12.9		Result:	2.699 V Approx.		0.562%
Equation 3:	Vdrop = √3 * I * R * L					Using DC Ohms Only		Result:	2.699 V Approx.		0.562%
Equation 3:	Vdrop = √3 * I * R * L					Using AC Ohms Only		Result:	3.034 V Approx.		0.632%
Equation 3:	Vdrop = √3 * I * R * L					Using Effective Z at 0.85 PF		Result:	5.231 V Approx.		1.089%
Equation 9:	Vdrop = √3 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	4.919 V Estimated		1.024%
Equation 16:	Vdrop = √3 * {Vs + IRLcosθ + IXLsinθ - √[Vs² - (IXLcosΦ-IRLsinΦ)²]}					Vs = 277V line-to-neutral		Result:	4.930 V Actual		1.027%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 9 – Line-to-Line Voltage Drop for 250 Hp Motor at 480V/3Φ with 200' of 500 KCMIL Conductors

Notice how inaccurate the first three equations in Table 9 are, which use K = 12.9, only DC ohms, and only AC ohms. This underscores the significance that the reactance of larger conductors has in voltage drop calculations. In this example, the Estimated voltage drop at 0.9 PF (4.919 V) is lower than the approximate voltage drop at 0.85 PF (5.231 V).

Rearranging the Formulas Used for Approximate Vdrop

Many people prefer to rearrange the formulas for approximate Vdrop to put the unknown value on the left side of the = sign and the known values on the right side. This type of rearrangement makes sense for the two approximate voltage drop formulas, since there is only one unknown, either the resistance R or the circular-mil area A. This is not as simple for the more accurate formulas for Estimated Vdrop and Actual Vdrop, since those have two unknowns – reactance X and resistance R. One possibility is to use a rearranged approximate Vdrop formula to pick out the conductor size, then use the Estimated Vdrop or Actual Vdrop formula to double-check the suitability of the conductor size, as will be done in Example 7 below.

Single-Phase Voltage Drop Formulas – Rearranged

Equation 1 on page 7 could be rewritten as:

R = (Vdrop) / (2 * I * L) round-trip Equation 18

where:

R = the maximum allowable resistance (or impedance) per 1,000 feet of conductor;

Vdrop = the maximum allowable round-trip voltage drop;

I = the current going to the load, which is also the current returning from the load;

L = the one-way length, in feet, of one conductor from the source to the load, divided by 1,000.

Let's try an example in which we will select the proper wire size for an unnamed load with a current draw of 10 A and PF = 0.9 at 120V/1Φ with a 100-ft one-way length in steel conduit. We will use 2% for the maximum allowable voltage drop on this branch circuit, which means the maximum Vdrop would be 120 V * 0.02 = 2.4 V.

Example 7: What would be the maximum allowable value of R to meet this maximum voltage drop limit? Using Equation 18:

R ≤ (Vdrop) / (2 * I * L)

R ≤ 2.4 / (2 *10 * (100 / 1,000))

R ≤ 2.4 / 2

R ≤ 1.2 Ω per 1,000 feet

Starting with the 0.85 PF column of Table 9 in the NEC, the Author would consider 10 AWG as the interim solution to this maximum resistance limit, but would perform the following Estimated Vdrop calculation to ensure that this selection is still valid at PF = 0.9 (θ = 25.842°), since there is no column in NEC Table 9 for this power factor:

From Equation 8 on page 29 for Estimated round-trip voltage drop at a certain power factor:

Estimated Vdrop = 2 * (IRcosθ + IXsinθ)

Vdrop = 2* {[(10) * (1.2 * 100 / 1,000) * (0.9)] + [(10) * (0.063 * 100 / 1,000) * (0.436)]}

Vdrop = 2 * {[1.08] + [0.0275]}

Vdrop = 2.215 V round-trip

This result is less than the maximum allowable voltage drop, but not so small that we need to consider a smaller conductor size. Table 10 below is a spreadsheet for this example.

Line-to-Line Vdrop for 10 A, 0.9 PF with 100' of 10 AWG Conductors									120 V/1Φ		
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
10	10,380	1.24	0.063	1.200	0.90	1.080	0.027	1.100	1.107	10.0	100.0
Equation 4:	Vdrop = 2 * I * K * R * L / A					K = 12.9		Result:	2.486 V Approx.		2.071%
Equation 3:	Vdrop = 2 * I * R * L					Using DC Ohms Only		Result:	2.480 V Approx.		2.066%
Equation 3:	Vdrop = 2 * I * R * L					Using AC Ohms Only		Result:	2.400 V Approx.		2%
Equation 3:	Vdrop = 2 * I * R * L					Using Effective Z at 0.85 PF		Result:	2.200 V Approx.		1.833%
Equation 9:	Vdrop = 2 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	2.215 V Estimated		1.845%
Equation 16:	Vdrop = 2 * {Vs + IRLcosθ + IXLsinθ - √[Vs² - (IXLcosΦ-IRLsinΦ)²]}					Vs = 277V line-to-neutral		Result:	2.216 V Actual		1.846%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 10 – Voltage Drop for 10 A Load at 120V/1Φ with 100' of 10 AWG Conductors

The Estimated Vdrop is very close to the Actual Vdrop in this example.

End of Example

What would Equation 2 on page 9 have told us about Example 7? Like Equation 1 was rearranged into Equation 18, Equation 2 could be rearranged as:

A = (2 * I * K * L) / (Vdrop) round-trip Equation 19

where:

A = the minimum cross-sectional area of the conductor in circular mils;

I = the current going to the load, which is also the current returning from the load;

K = ohms * circular mil per foot of conductor, which equals 12.9 for copper;

L = the one-way length, in feet, of one conductor from the source to the load;

Vdrop = the maximum allowable round-trip voltage drop.

Example 8: Using the parameters of Example 7, let's try Equation 19:

A ≥ (2 * I * K * L) / (Vdrop)

A ≥ (2 * 10 * 12.9 * 100) / (2.4)

A ≥ (25,800) / (2.4)

A ≥ 10,750 circular mils

The conductor size that meets the 10,750 circular mil minimum requirement would be 8 AWG, with a cross-sectional conductor area of 16,510 circular mils, which is unnecessarily larger than the selection of 10 AWG using Equation 18.

Let's check that result by using the Estimated Vdrop formula. From Equation 8 on page 29 for estimated round-trip voltage drop at a certain power factor, using 8AWG conductors:

Estimated Vdrop = 2 * (IRcosθ + IXsinθ)

Vdrop = 2* {[(10) * (0.78 * 100 / 1,000) * (0.9)] + [(10) * (0.066 * 100 / 1,000) * (0.436)]}

Vdrop = 2 * {[0.702] + [0.02834]}

Vdrop = 1.46 V round-trip or a voltage drop percentage of 1.217%.

This result is less than the maximum allowable voltage drop, but it seems like it might be too small, so we might need to consider a smaller conductor size, such as 10 AWG. This was already done at the end of Example 7 above

End of Example

Three-Phase Voltage Drop Formulas – Rearranged

In keeping with the discussions directly above for rearranged single-phase voltage drop formulas, Equation 3 on page 12 could be rearranged as:

R = (Vdrop) / (√3 * I * L) line-to-line Equation 20

where:

R = the maximum allowable resistance (or impedance) per 1,000 feet of conductor;

Vdrop = the maximum allowable line-to-line voltage drop;

I = line current going to the load;

L = the one-way length, in feet, of one conductor from the source to the load, divided by 1,000.

Let's try an example in which we will select the proper wire size for a 10 Hp motor at 480V/3Φ at 0.85 PF with a 200-ft one-way length in steel conduit.

Example 9: We will use the typical value of 3% as the maximum voltage drop on this run. This means the maximum Vdrop would be 480 V * 0.03 = 14.4 V. What would be the maximum allowable value of R to meet this maximum voltage drop limit? Using Equation 20:

R = (Vdrop) / (√3 * I * L)

= 14.4 / (√3 * 14 * (200 / 1,000))

= 14.4 / 4.8497

= 2.887 Ω per 1,000 feet

The Author would choose 12 AWG from NEC Table 9, Effective Z at 0.85 PF column, even though 14 AWG would also meet the maximum resistance requirements, The Author's choice would be 12 AWG because 12 AWG is a typical minimum size for many projects. The voltage drop calculations for this motor and conductor combination have already been performed in Table 7 on page 40, and every one of those calculations gives a voltage drop of less than 14.4 V. See, also, Figure 23 on page 39 for a graphical representation of the Estimated Vdrop and Actual Vdrop for this example.

End of Example

As was done for three-phase approximate voltage drop Equation 3 to get Equation 20, Equation 4 on page 13 could be rearranged as:

$A = (\sqrt{3} * I * K * L) / (Vdrop)$ line-to-line Equation 21

where:

A = the minimum cross-sectional area of the conductor in circular mils;

I = line current going to the load;

K = ohms * circular mil per foot of conductor, which equals 12.9 for copper;

L = the one-way length, in feet, of one conductor from the source to the load;

Vdrop = the maximum allowable line-to-line voltage drop.

Let's use Equation 21 to re-work Example 9.

Example 10:

$A \geq (\sqrt{3} * I * K * L) / (Vdrop)$

$A \geq (\sqrt{3} * 14 * 12.9 * 200) / (14.4)$

$A \geq (62{,}561) / (14.4)$

$A \geq 4{,}345$ circular mils

The conductor size that meets the 4,345 circular mil minimum requirement would be 12 AWG, which has a cross-sectional conductor area of 6,530 circular mils. It is also interesting to note that 14 AWG would be too small for the result of this formula.

End of Example

Having thoroughly reviewed the derivation and use of various voltage drop formulas and calculations, it is time to consider a few other topics before concluding this course.

Other Considerations

When power or ungrounded or 'hot' conductors are increased in size to accommodate voltage drop, there are several other topics that need to be considered as a result. The following are actions that might need to be taken after increasing the size of power conductors due to voltage drop calculations.

Increase Equipment Grounding Conductor Size

NEC 250.122(B) requires the equipment grounding conductor (EGC) to be increased in size when the ungrounded (hot) conductors are increased in size to accommodate voltage drop. The increase in size of the EGC is required to be based on the same ratio as the hot conductors were increased. Let's look at an example.

Example 11: Consider a 150 A feeder protected by a 200 A circuit breaker at 480V/3Φ with a 500-foot one-way run of copper conductors in steel conduit, with 75 °C-rated terminations at each end. Let's do the voltage drop calculation then evaluate the EGC required size. Assume a maximum voltage drop of 2.5%, a power factor of 0.8 (θ = 36.87°), and an ambient temperature between 30 °C and 40 °C. Assume, also, that this is a continuous 150 A load.

What is a 150A Feeder Circuit?

Is a 150 A feeder circuit a circuit that has a 150 A circuit breaker and conductors sized for the 150 A circuit breaker? That is one interpretation, but for low-voltage (600 V and less) breakers with trip ratings less than 800 A or thereabouts, it is typical to only use 80% of the breaker trip rating, which would be 120 A for a 150 A breaker. In the Author's opinion, a 150 A feeder circuit will be able to provide the full 150 A, which would typically require a 200 A breaker.

Since this is a continuous 150A load, the conductors need to be sized for 125% of 150 A or 187.5 A. The 75 °C column of NEC Table 310.15(B)(16) tells us that 3/0 AWG ungrounded or hot conductors are appropriate for this continuous 150 A load. NEC Table 250.122 tells us that 6 AWG is the proper EGC size (before any adjustments) for this circuit, based on the 200 A breaker. The maximum allowable voltage drop for this installation is 480 * 0.025 = 12 V. Let's do the Estimated voltage drop calculation using Equation 9 on page 29.

$$\text{Estimated Vdrop} = \sqrt{3}*\{IR\cos\theta + IX\sin\theta\} \qquad \text{line-to-line}$$

$$= \sqrt{3}*\{[(150)*(0.079 * 500 / 1{,}000)*(0.8)] + [(150)*(0.052 * 500 / 1{,}000)*(0.6)]\}$$

$$= \sqrt{3}*\{[4.74] + [2.34]\}$$

$$= \sqrt{3}*\{7.08\}$$

$$= 12.26 \text{ V}$$

This voltage drop slightly exceeds the maximum of 12 V, so let's bump the conductor size up to 4/0 AWG and re-check the voltage drop:

Estimated Vdrop = $\sqrt{3}$*{IRcosθ + IXsinθ} line-to-line

= $\sqrt{3}$*{[(150)*(0.063 * 500 / 1,000)*(0.8)] + [(150)*(0.051 * 500 / 1,000)*(0.6)]}

= $\sqrt{3}$*{[3.78] + [2.295]}

= $\sqrt{3}$*{6.075}

= 10.52 V

The voltage drop with 4/0 AWG conductors is acceptable, so how much do we need to increase the size of the EGC? The circular mils of 4/0 AWG and 3/0 AWG are 211,600 and 167,800, respectively, so we increased the hot conductors by a ratio of 133,100 / 105,600 = 1.26. The circular mil area of 6 AWG is 26,240, so we multiply that by the ratio we just calculated to get 26,240 * 1.26 = 33,073. The conductor size that meets this minimum requirement is 4 AWG, with 41,740 circular mils. So, the conductors for this circuit would be (3) 4/0 AWG with a 4 AWG EGC. This is available as a standard three-conductor-with-ground cable, but sometimes the result of this type of calculation requires a conductor combination that is not available as a manufacturer's standard multiconductor cable offering.

Increase Grounded (Neutral) Conductor Size

NEC 240.23 permits, but does not require, a change in the size of the neutral conductor when the size of the ungrounded (hot) conductors is increased. When the hot conductors are increased in size for voltage drop, the neutral conductor is permitted to be increased in size by the same ratio as the hot conductors were increased.

Verify Wire Size and Quantity Capacity of Terminations at Both Ends

If you have to increase a 350 KCMIL set of conductors to 600 KCMIL to allow for voltage drop, it is a good idea to confirm that the terminations at both ends of these conductors will accept 600 KCMIL conductors. Similarly, if the circuit is changed from one set of 500 KCMIL to three sets of 250 KCMIL due to voltage drop, one should make sure that the terminations at both ends will accept three sets of conductors per phase, of that particular size.

Verify Conduit Size

This might go without saying, but be careful when using tables that give standard conductor and conduit sizes for pre-defined loads. If the conductor sizes are increased for voltage drop, be sure to confirm the conduit size and not rely on the conduit size listed for standard applications in a table. See PDH Online Course E276 Conduit Fill Calculations, listed in the Additional Reading section beginning on page 56 for more information on this topic.

Rule-of-Thumb

A common rule-of-thumb that many people use to determine whether or not a voltage drop calculation is required is to compare the one-way cable length in feet to the applied voltage. If the one-way cable length in feet is almost or more than the value of the applied voltage, then a voltage drop calculation should probably be considered. Consider a 20 A circuit breaker loaded at 50%, that is, 10 A, and various power supply voltages of 120V/1Φ, 208V/3Φ, 277V/1Φ, and 480/3Φ. Table 11 through Table 14 below show the voltage drop calculations for one-way cable lengths in feet that are equal to the applied voltages, all at a power factor of 0.9. The examples in these four tables lend credence to the rule-of-thumb stated in this section. Table 11 is very similar to Table 10 on page 47.

Converting Formulas from Single-Phase to Three-Phase

Some technical publications present only the single-phase, round-trip Vdrop formula, such as Equation 1 on page 7 and Equation 2 on page 9. The Reader is then instructed to multiply the result of the single-phase calculation by 0.866 in order to get the three-phase voltage drop. This is merely to replace the 2 in the formula with the $\sqrt{3}$, since $0.866 = (\sqrt{3}) / 2$.

Line-to-Line Vdrop for 10 A, 0.9 PF at One-Way Cable Length in Feet = Applied Voltage									120 V/1Φ		
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
12	6,530	1.98	0.068	2.000	0.90	1.800	0.030	1.700	1.830	10.0	120.0
Equation 4:	Vdrop = 2 * I * K * R * L / A					K = 12.9		Result:	4.741 V Approx.		3.95%
Equation 3:	Vdrop = 2 * I * R * L					Using DC Ohms Only		Result:	4.752 V Approx.		3.96%
Equation 3:	Vdrop = 2 * I * R * L					Using AC Ohms Only		Result:	4.800 V Approx.		4%
Equation 3:	Vdrop = 2 * I * R * L					Using Effective Z at 0.85 PF		Result:	4.080 V Approx.		3.4%
Equation 9:	Vdrop = 2 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	4.391 V Estimated		3.659%
Equation 16:	Vdrop = 2 * {Vs + IRLcosθ + IXLsinθ - √[Vs^2 - (IXLcosΦ-IRLsinΦ)^2]}					Vs = 120V line-to-neutral		Result:	4.399 V Actual		3.665%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 11 – Voltage Drop for 10 A Load at 120V/1Φ with 120' of 12 AWG Conductors

Line-to-Line Vdrop for 10 A, 0.9 PF at One-Way Cable Length in Feet = Applied Voltage									208 V/3Φ		
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
12	6,530	1.98	0.068	2.000	0.90	1.800	0.030	1.700	1.830	10.0	208.0
Equation 4:	Vdrop = √3 * I * K * R * L / A					K = 12.9		Result:	7.117 V Approx.		3.421%
Equation 3:	Vdrop = √3 * I * R * L					Using DC Ohms Only		Result:	7.133 V Approx.		3.429%
Equation 3:	Vdrop = √3 * I * R * L					Using AC Ohms Only		Result:	7.205 V Approx.		3.464%
Equation 3:	Vdrop = √3 * I * R * L					Using Effective Z at 0.85 PF		Result:	6.125 V Approx.		2.944%
Equation 9:	Vdrop = √3 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	6.592 V Approx.		3.169%
Equation 16:	Vdrop = √3 * {Vs + IRLcosθ + IXLsinθ - √[Vs^2 - (IXLcosΦ-IRLsinΦ)^2]}					Vs = 120V line-to-neutral		Result:	6.612 V (Actual)		3.178%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 12 – Voltage Drop for 10 A Load at 208V/3Φ with 208' of 12 AWG Conductors

Line-to-Line Vdrop for 10 A, 0.9 PF at One-Way Cable Length in Feet = Applied Voltage									277 V/1Φ		
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
12	6,530	1.98	0.068	2.000	0.90	1.800	0.030	1.700	1.830	10.0	277.0
Equation 4:	Vdrop = 2 * I * K * R * L / A					K = 12.9		Result:	10.944 V Approx.		3.95%
Equation 3:	Vdrop = 2 * I * R * L					Using DC Ohms Only		Result:	10.969 V Approx.		3.96%
Equation 3:	Vdrop = 2 * I * R * L					Using AC Ohms Only		Result:	11.080 V Approx.		4%
Equation 3:	Vdrop = 2 * I * R * L					Using Effective Z at 0.85 PF		Result:	9.418 V Approx.		3.4%
Equation 9:	Vdrop = 2 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	10.136 V Estimated		3.659%
Equation 16:	Vdrop = 2 * {Vs + IRLcosθ + IXLsinθ - √[Vs² - (IXLcosΦ-IRLsinΦ)²]}					Vs = 277V line-to-neutral		Result:	10.154 V Actual		3.665%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 13 – Voltage Drop for 10 A Load at 277V/1Φ with 277' of 12 AWG Conductors

Line-to-Line Vdrop for 10 A, 0.9 PF at One-Way Cable Length in Feet = Applied Voltage									480 V/3Φ		
Wire Size	Table 8 Cir. Mils	Table 8 DC Ohms	Table 9 X_L (React) Ohms/k-ft	Table 9 AC Ohms/k-ft	Power Factor PF [cos(θ)]	Rcos(θ) per k-ft	Xsin(θ) per k-ft	NEC Effective Z at 0.85 PF	Effective Z = Rcos(θ) + Xsin(θ) Ohms/k-ft	Full-Load Current	One-Way Wire Length (ft)
12	6,530	1.98	0.068	2.000	0.90	1.800	0.030	1.700	1.830	10.0	480.0
Equation 4:	Vdrop = √3 * I * K * R * L / A					K = 12.9		Result:	16.424 V Approx.		3.421%
Equation 3:	Vdrop = √3 * I * R * L					Using DC Ohms Only		Result:	16.461 V Approx.		3.429%
Equation 3:	Vdrop = √3 * I * R * L					Using AC Ohms Only		Result:	16.628 V Approx.		3.464%
Equation 3:	Vdrop = √3 * I * R * L					Using Effective Z at 0.85 PF		Result:	14.134 V Approx.		2.944%
Equation 9:	Vdrop = √3 * (IRcosθ + Ixsinθ) * L					Calculated Effective Z		Result:	15.211 V Approx.		3.169%
Equation 16:	Vdrop = √3 * {Vs + IRLcosθ + IXLsinθ - √[Vs² - (IXLcosΦ-IRLsinΦ)²]}					Vs = 277V line-to-neutral		Result:	15.259 V (Actual)		3.178%

Note: Equation 4 above is the only equation in which the one-way wire length is not divided by 1,000, since the units of K are (Ohms*C.M.) / ft.

Table 14 – Voltage Drop for 10 A Load at 480V/3Φ with 480' of 12 AWG Conductors

In Closing

Voltage drop calculations can be as simple or as intricate as the Reader chooses to make them. Likewise, they can be as accurate or approximate as the Reader decides is necessary. It is hoped that the information presented in this course will assist the Reader in choosing the most appropriate formula or method for performing voltage drop calculations, and in providing justification for that decision.

Abbreviations

A	Amp or Amps
AC	Alternating Current
AWG	American Wire Gage
CAD	Computer-Aided Drafting
EGC	Equipment Grounding Conductor
FLC	Full-Load Current, typical value for motors running at speeds usual for belted motors and motors with normal torque characteristics.
Hz	Hertz or cycles-per-second
I	Current in amperes
L	Length
N.A.	Not Applicable
NEC	National Electrical Code, 2014 Edition
PF	Power Factor
R	Resistance
RMS	Root-Mean Squared
V	Volt or Volts
VAC	Volts Alternating-Current
Vdrop	Voltage Drop
X	Reactance
Z	Impedance

Additional Reading

Circular Segment at http://en.wikipedia.org/wiki/Circular_segment

IEEE Standard 141 – 1993(R1999) IEEE Recommended Practice for Electric Power Distribution for Industrial Plants (the Red Book) at www.ieee.org

IEEE Standard 241 – 1990 IEEE Recommended Practice for Electric Power Systems in Commercial Buildings (the Gray Book) at www.ieee.org

NFPA 70 National Electrical Code, (NEC) 2014 Edition at www.nfpa.org

PDH Online course E275 AWG and Circular Mils at www.pdhonline.org

PDH Online course E276 Conduit Fill Calculations at www.pdhonline.org

PDH Online course E413 Wye-Delta Motor Starters at www.pdhonline.org

PDH Online course E427 Standard AC System Voltages (600 V and Less) at www.pdhonline.org

PDH Online course E431 The Square Root of Three (√3) in Electrical Calculations at www.pdhonline.org

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