Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 1 or 2: Pure Mathematics

Practice Paper C **Time: 2 hours**	**Paper Reference(s)** **9MA0/01 or 9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are xx questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1. $$\frac{18x^2-98x+78}{(x-4)^2(3x+1)}=\frac{A}{x-4}+\frac{B}{(x-4)^2}+\frac{C}{3x+1},\ x>4$$

Find the values of the constants A, B and C.

(6 marks)

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2. A curve C has equation $4^x = 2xy$ for $x > 0$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point C with coordinates (2, 4).

(5 marks)

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3. (a) Show that $\cos 7x + \cos 3x = 2\cos 5x \cos 2x$ by expanding $\cos(5x+2x)$ and $\cos(5x-2x)$ using the compound-angle formulae.

(3 marks)

(b) Hence find $\int(\cos 5x \cos 2x)\,\mathrm{d}x$.

(3 marks)

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4. The temperature of a mug of coffee at time t can be modelled by the equation

$$\mathrm{T}(t)=T_R+(90-T_R)\mathrm{e}^{-\frac{1}{20}t},$$

where $\mathrm{T}(t)$ is the temperature, in °C, of the coffee at time t minutes after the coffee was poured into the mug and T_R is the room temperature in °C.

Using the equation for this model,

(a) explain why the initial temperature of the coffee is independent of the initial room temperature.

(2 marks)

(b) Calculate the temperature of the coffee after 10 minutes if the room temperature is 20 °C.

(2 marks)

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5. Prove by contradiction that if n is odd, $n^3 + 1$ is even.

(5 marks)

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6. A curve C has parametric equations $x = \sec^2 t + 1$, $y = 2\sin t$, $-\frac{\pi}{4} \leqslant t \leqslant \frac{\pi}{4}$.

Show that a cartesian equation of C is $y = \sqrt{\frac{8-4x}{1-x}}$ for a suitable domain which should be stated.

(4 marks)

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7. An infinite geometric series has first four terms $1 - 4x + 16x^2 - 64x^3 + \ldots$ The series is convergent.

(a) Find the set of possible values of x for which the series converges.

(2 marks)

Given that $\sum_{r=1}^{\infty}(-4x)^{r-1} = 4$,

(b) calculate the value of x.

(3 marks)

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8. $f(x) = 2 - 3\sin^3 x - \cos x$, where x is in radians.

(a) Show that f(x) = 0 has a root α between x = 1.9 and x = 2.0.

(2 marks)

Using $x_0 = 1.95$ as a first approximation,

(b) apply the Newton–Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 decimal places.

(5 marks)

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9. Given that $(b-a)\mathbf{i} - 2abc\mathbf{j} + 2\mathbf{k} = 10\mathbf{i} - 96\mathbf{j} + (7a+5b)\mathbf{k}$, find the values of a, b and c.

(6 marks)

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10. Use proof by contradiction to show that there are no positive integer solutions to the statement $x^2 - y^2 = 1$.

(5 marks)

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11. The function g(x) is defined by $g(x) = x^2 - 8x + 7$, $x \in \mathbb{R}$, $x > 4$.

Find $g^{-1}(x)$ and state its domain and range.

(6 marks)

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12. $$\mathrm{f}(x) = \frac{4x^2 + x - 23}{(x-3)(4-x)(x+5)}, \quad x > 4.$$

Given that $\mathrm{f}(x)$ can be expressed in the form $\dfrac{A}{x-3} + \dfrac{B}{4-x} + \dfrac{C}{x+5}$, find the values of A, B and C.

(6 marks)

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13 The curve C has equation $y = x^3 + 6x^2 - 12x + 6$.

(a) Show that C is concave on the interval [–5, –3].

(3 marks)

(b) Find the coordinates of the point of inflection.

(3 marks)

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14 Find $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} \sin 4x(1 - \cos 4x)^3 \, \mathrm{d}x$.

(4 marks)

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15 $$\frac{4x^2 - 4x - 9}{(2x+1)(x-1)} \equiv A + \frac{B}{2x+1} + \frac{C}{x-1}.$$

(a) Find the values of the constants A, B and C.

(6 marks)

(b) Hence, or otherwise, expand $\dfrac{4x^2 - 4x - 9}{(2x+1)(x-1)}$ in ascending powers of x, as far as the x^2 term.

(6 marks)

(c) Explain why the expansion is not valid for $x = \dfrac{3}{4}$.

(1 mark)

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16 A large cylindrical tank has radius 40 m. Water flows into the cylinder from a pipe at a rate of $4000\pi \text{ m}^3 \text{ min}^{-1}$. At time t, the depth of water in the tank is h m. Water leaves the bottom of the tank through another pipe at a rate of $50\pi h \text{ m}^3 \text{ min}^{-1}$.

(a) Show that t minutes after water begins to flow out of the bottom of the cylinder, $160\dfrac{\mathrm{d}h}{\mathrm{d}t} = 400 - 5h$.

(6 marks)

When $t = 0$ min, $h = 50$ m.

(b) Find the exact value of t when $h = 60$ m.

(6 marks)

TOTAL FOR PAPER IS 97 MARKS

1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)\equiv 18x^2-98x+78$	**M1**	2.2a	7th Decompose algebraic fractions into partial fractions − repeated factors.
	Further states that: $A(3x^2-11x-4)+B(3x+1)+C(x^2-8x+16)\equiv 18x^2-98x+78$	**M1**	1.1b	
	Equates the various terms. Equating the coefficients of x^2: $3A+C=18$ Equating the coefficients of x: $-11A+3B-8C=-98$ Equating constant terms: $-4A+B+16C=78$	**M1**	2.2a	
	Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.	**M1**	1.1b	
	Finds the correct value of any one variable: either $A = 4$, $B = -2$ or $C = 6$	**A1**	1.1b	
	Finds the correct value of all three variables: $A = 4$, $B = -2$, $C = 6$	**A1**	1.1b	
				(6 marks)

Notes

Alternative method

Uses the substitution method, having first obtained this equation:

$A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)\equiv 18x^2-98x+78$

Substitutes $x = 4$ to obtain $13B = -26$

Substitutes $x=-\frac{1}{3}$ to obtain $\frac{169}{9}C=\frac{338}{3}\Rightarrow C=\frac{1014}{169}=6$

Equates the coefficients of x^2: $3A+C=18$

Substitutes the found value of C to obtain $3A = 12$

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Differentiates 4^x to obtain $4^x \ln 4$	**M1**	1.1b	7th Differentiate simple functions defined implicitly.
	Differentiates $2xy$ to obtain $2x\frac{dy}{dx}+2y$	**M1**	2.2a	
	Rearranges $4^x \ln 4 = 2x\frac{dy}{dx}+2y$ to obtain $\frac{dy}{dx}=\frac{4^x \ln 4 - 2y}{2x}$	**A1**	1.1b	
	Makes an attempt to substitute (2, 4)	**M1**	1.1b	
	States fully correct final answer: $4\ln 4 - 2$ Accept $\ln 256 - 2$	**A1**	1.1b	
				(5 marks)

Notes

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly states $\cos(5x+2x) \equiv \cos 5x \cos 2x - \sin 5x \sin 2x$	M1	1.1b	6th Integrate using trigonometric identities.
	Correctly states $\cos(5x-2x) \equiv \cos 5x \cos(-2x) - \sin 5x \sin(-2x)$ or states $\cos(5x-2x) \equiv \cos 5x \cos(2x) + \sin 5x \sin(2x)$	M1	1.1b	
	Adds the two above expressions and states $\cos 7x + \cos 3x \equiv 2\cos 5x \cos 2x$	A1	1.1b	
		(3)		
(b)	States that $\int(\cos 5x \cos 2x)\,\mathrm{d}x = \frac{1}{2}\int(\cos 7x + \cos 3x)\,\mathrm{d}x$	M1	2.2a	6th Integrate functions of the form f($ax + b$).
	Makes an attempt to integrate. Changing cos to sin constitutes an attempt.	M1	1.1b	
	Correctly states the final answer $\frac{1}{14}\sin 7x + \frac{1}{6}\sin 3x + C$ o.e.	A1	1.1b	
		(3)		
				(6 marks)

Notes

(b) Student does not need to state '+C' to be awarded the first method mark. Must be stated in the final answer.

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to substitute $t = 0$ into $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$. For example, $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^0$ or $\mathrm{T}(t) = T_R + (90 - T_R)$ is seen.	**M1**	3.1a	6th Set up and use exponential models of growth and decay.
	Concludes that the T_R terms will always cancel at $t = 0$, therefore the room temperature does not influence the initial coffee temperature.	**B1**	3.5a	
		(2)		
(b)	Makes an attempt to substitute $T_R = 20$ and $t = 10$ into $\mathrm{T}(t) = T_R + (90 - T_R)\mathrm{e}^{-\frac{1}{20}t}$. For example, $\mathrm{T}(10) = 20 + (90 - 20)\mathrm{e}^{-\frac{1}{20}(10)}$ is seen.	**M1**	1.1b	6th Set up and use exponential models of growth and decay.
	Finds $\mathrm{T}(10) = 62.457...°\mathrm{C}$. Accept awrt 62.5°.	**A1**	1.1b	
		(2)		
				(4 marks)

Notes

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. 'Assumption: there exists a number n such that n is odd and $n^3 + 1$ is also odd.'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Defines an odd number. 'Let $2k + 1$ be an odd number.'	**B1**	2.2a	
	Successfully calculates $(2k+1)^3+1$ $(2k+1)^3+1 \equiv \left(8k^3+12k^2+6k+1\right)+1 \equiv 8k^3+12k^2+6k+2$	**M1**	1.1b	
	Factors the expression and concludes that this number must be even. $8k^3+12k^2+6k+2 \equiv 2\left(4k^3+6k^2+3k+1\right)$ $2\left(4k^3+6k^2+3k+1\right)$ is even.	**M1**	1.1b	
	Makes a valid conclusion. This contradicts the assumption that there exists a number n such that n is odd and $n^3 + 1$ is also odd, so if n is odd, then $n^3 + 1$ is even.	**B1**	2.4	
				(5 marks)

Notes

Alternative method

Assume the opposite is true: there exists a number n such that n is odd and $n^3 + 1$ is also odd. (**B1**)

If $n^3 + 1$ is odd, then n^3 is even. (**B1**)

So 2 is a factor of n^3. (**M1**)

This implies 2 is a factor of n. (**M1**)

This contradicts the statement n is odd. (**B1**)

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises that the identity $\sin^2 t + \cos^2 t \equiv 1$ can be used to find the cartesian equation.	**M1**	2.2a	6th Convert between parametric equations and cartesian forms using trigonometry.
	States $\sin t = \frac{y}{2}$ or $\sin^2 t = \frac{y^2}{4}$ Also states $\cos^2 t = \frac{1}{x-1}$	**M1**	1.1b	
	Substitutes $\sin^2 t = \frac{y^2}{4}$ and $\cos^2 t = \frac{1}{x-1}$ into $\sin^2 t + \cos^2 t \equiv 1$ $\frac{y^2}{4} + \frac{1}{x-1} = 1 \Rightarrow \frac{y^2}{4} = \frac{x-2}{x-1}$	**M1**	1.1b	
	Solves to find $y = \sqrt{\frac{4x-8}{x-1}}$, accept $y = \sqrt{\frac{8-4x}{1-x}}$, $x < 1$ or $x \ldots 2$	**A1**	1.1b	
				(4 marks)

Notes

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Understands that for the series to be convergent $\lvert r\rvert<1$ or states $\lvert -4x\rvert<1$	M1	2.2a	6th Understand convergent geometric series and the sum to infinity.
	Correctly concludes that $\lvert x\rvert<\frac{1}{4}$. Accept $-\frac{1}{4}<x<\frac{1}{4}$	A1	1.1b	
		(2)		
(b)	Understands to use the sum to infinity formula. For example, states $\frac{1}{1+4x}=4$	M1	2.2a	5th Understand sigma notation.
	Makes an attempt to solve for x. For example, $4x=-\frac{3}{4}$ is seen.	M1	1.1b	
	States $x=-\frac{3}{16}$	A1	1.1b	
		(3)		
				(5 marks)

Notes

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\mathrm{f}(1.9) = -0.2188...$ and $\mathrm{f}(2.0) = (+)0.1606...$	M1	1.1b	5th Use a change of sign to locate roots.
	Change of sign and continuous function in the interval $\left[1.9, 2.0\right] \Rightarrow$ root	A1	2.4	
		(2)		
(b)	Makes an attempt to differentiate f(x)	M1	2.2a	6th Solve equations approximately using the Newton-Raphson method.
	Correctly finds $\mathrm{f}'(x) = -9\sin^2 x\cos x + \sin x$	A1	1.1b	
	Finds $\mathrm{f}(1.95) = -0.0348...$ and $\mathrm{f}'(1.95) = 3.8040...$	M1	1.1b	
	Attempts to find x_1 $x_1 = x_0 - \dfrac{\mathrm{f}(x_0)}{\mathrm{f}'(x_0)} \Rightarrow x_1 = 1.95 - \dfrac{-0.0348...}{3.8040...}$	M1	1.1b	
	Finds $x_1 = 1.959$	A1	1.1b	
		(5)		
				(7 marks)

Notes

(a) Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States $-a+b=10$ and $7a+5b=2$	**M1**	2.2a	6th Solve geometric problems using vectors in 3 dimensions
	Makes an attempt to solve the pair of simultaneous equations. Attempt could include making a substitution or multiplying the first equation by 5 or by 7.	**M1**	1.1b	
	Finds $a = -4$	**A1**	1.1b	
	Find $b = 6$	**A1**	1.1b	
	States $-2abc = -96$	**M1**	2.2a	
	Finds $c = -2$	**A1**	1.1b	
				(6 marks)

Notes

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. 'Assumption: there exist positive integer solutions to the statement $x^2 - y^2 = 1$'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Sets up the proof by factorising $x^2 - y^2$ and stating $(x-y)(x+y)=1$	**M1**	2.2a	
	States that there is only one way to multiply to make 1: $1 \times 1 = 1$ and concludes this means that: $x - y = 1$ $x + y = 1$	**M1**	1.1b	
	Solves this pair of simultaneous equations to find the values of x and y: $x = 1$ and $y = 0$	**M1**	1.1b	
	Makes a valid conclusion. $x = 1$, $y = 0$ are not both positive integers, which is a contradiction to the opening statement. Therefore there do not exist positive integers x and y such that $x^2 - y^2 = 1$	**B1**	2.4	
				(5 marks)

Notes

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Understands the need to complete the square, and makes an attempt to do this. For example, $(x-4)^2$ is seen.	**M1**	2.2a	6th Find the domain and range of inverse functions.
	Correctly writes $g(x)=(x-4)^2-9$	**A1**	1.1b	
	Demonstrates an understanding of the method for finding the inverse is to switch the *x* and *y*. For example, $x=(y-4)^2-9$ is seen.	**B1**	2.2a	
	Makes an attempt to rearrange to make *y* the subject. Attempt must include taking the square root.	**M1**	1.1b	
	Correctly states $g^{-1}(x)=\sqrt{x+9}+4$	**A1**	1.1b	
	Correctly states domain is $x > -9$ and range is $y > 4$	**B1**	3.2b	
				(6 marks)

Notes

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(4-x)(x+5)+B(x-3)(x+5)+C(x-3)(4-x)\equiv 4x^2+x-23$	**M1**	2.2a	6th Decompose algebraic fractions into partial fractions − three linear factors.
	Further states that: $A(-x^2-x+20)+B(x^2+2x-15)+C(-x^2+7x-12)\equiv 4x^2+x-23$	**M1**	1.1b	
	Equates the various terms. Equating the coefficients of x^2: $-A+B-C=4$ Equating the coefficients of x: $-A+2B+7C=1$ Equating constant terms: $20A-15B-12C=-23$	**M1***	2.2a	
	Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.	**M1***	1.1b	
	Finds the correct value of any one variable: either $A = 2$, $B = 5$ or $C = -1$	**A1***	1.1b	
	Finds the correct value of all three variables: $A = 2$, $B = 5$, $C = -1$	**A1**	1.1b	
				(6 marks)

Notes

Alternative method

Uses the substitution method, having first obtained this equation:

$A(4-x)(x+5)+B(x-3)(x+5)+C(x-3)(4-x)\equiv 4x^2+x-23$

Substitutes $x = 4$ to obtain $9B = 45$ (**M1**)

Substitutes $x = 3$ to obtain $8A = 16$ (**M1**)

Substitutes $x = -5$ to obtain $-72C = 72$ (**A1**)

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 12x - 12$	**M1**	1.1b	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x + 12$	**M1**	1.1b	
	States that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x + 12 \leq 0$ for all $-5 \leq x \leq -3$ and concludes this implies C is concave over the given interval.	**B1**	3.2a	
		(3)		
(b)	States or implies that a point of inflection occurs when $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$	**M1**	3.1a	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $x = -2$	**A1**	1.1b	
	Substitutes $x = -2$ into $y = x^3 + 6x^2 - 12x + 6$, obtaining $y = 46$	**A1**	1.1b	
		(3)		
				(6 marks)

Notes

14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to find $\int \sin 4x(1-\cos 4x)^3 \, dx$. Raising the power by 1 would constitute an attempt.	**M1**	2.2a	6th Integrate using the reverse chain rule.
	States a fully correct answer $\int \sin 4x(1-\cos 4x)^3 \, dx = \frac{1}{16}(1-\cos 4x)^4 + C$	**M1**	2.2a	
	Makes an attempt to substitute the limits $\frac{1}{16}\left[(1-0)^4 - \left(1-\frac{1}{2}\right)^4\right]$	**M1 ft**	1.1b	
	Correctly states answer is $\frac{15}{256}$	**A1 ft**	1.1b	
				(4 marks)

Notes

Student does not need to state '+C' to be awarded the second method mark.

Award ft marks for a correct answer using an incorrect initial answer.

15	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to set up a long division. For example, $2x^2 - x - 1\overline{)4x^2 - 4x - 9}$ is seen.	M1	2.2a	7th Expand rational functions using partial fraction decomposition.
	Long division completed so that a 2 is seen in the quotient and a remainder of $-2x - 7$ is also seen. $\begin{array}{r} 2 \\ 2x^2 - x - 1\overline{)4x^2 - 4x - 9} \\ \underline{4x^2 - 2x - 2} \\ -2x - 7 \end{array}$	M1	1.1b	
	States $B(x-1) + C(2x+1) \equiv -2x - 7$	M1	2.2a	
	Either equates variables or makes a substitution in an effort to find *B* or *C*.	M1	2.2a	
	Finds $B = 4$	A1	1.1b	
	Finds $C = -3$	A1	1.1b	
		(6)		
(b)	Correctly writes $4(2x+1)^{-1}$ or $4(1+2x)^{-1}$ as $4\left(1 + (-1)(2x) + \frac{(-1)(-2)(2)^2 x^2}{2} + \ldots\right)$	M1 ft	2.2a	6th Understand the binomial theorem for rational n.
	Simplifies to obtain $4 - 8x + 16x^2 + \ldots$	A1 ft	1.1b	
	Correctly writes $\frac{-3}{x-1}$ as $\frac{3}{1-x}$	M1 ft	2.2a	
	Correctly writes $3(1-x)^{-1}$ as $3\left(1 + (-1)(-x) + \frac{(-1)(-2)(-1)^2(-x)^2}{2} + \ldots\right)$	M1 ft	2.2a	
	Simplifies to obtain $3 + 3x + 3x^2 + \ldots$	A1 ft	1.1b	
	States the correct final answer: $9 - 5x + 19x^2$	A1 ft	1.1b	
		(6)		

(c)	The expansion is only valid for $\lvert x \rvert < \frac{1}{2}$	**B1**	3.2b	6th Understand the conditions for validity of the binomial theorem for rational n.
		(1)		
				(13 marks)

Notes

(a) Alternative method.

Writes the RHS as a single fraction.

Obtains $4x^2 - 4x - 9 = A(2x+1)(x-1) + B(x-1) + C(2x+1)$

Substitutes $x = 1$ to obtain $C = -3$

Substitutes $x = -\frac{1}{2}$ to obtain $B = 4$

Compares coefficients of x^2 to obtain $A = 2$

(b) Award all 6 marks for a correct answer using their incorrect values of *A*, *B* and/or *C* from part **a**.

16	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}h}{\mathrm{d}V}$	M1	3.1b	8th Solve differential equations in a range of contexts.
	Deduces that $V = \pi r^2 h = 1600\pi h$	M1	3.1b	
	Finds $\frac{\mathrm{d}V}{\mathrm{d}h} = 1600\pi$ and/or $\frac{\mathrm{d}h}{\mathrm{d}V} = \frac{1}{1600\pi}$	M1	1.1b	
	States $\frac{\mathrm{d}V}{\mathrm{d}t} = 4000\pi - 50\pi h$	M1	3.1b	
	Makes an attempt to find $\frac{\mathrm{d}h}{\mathrm{d}t} = \left(4000\pi - 50\pi h\right) \times \frac{1}{1600\pi}$	M1	1.1b	
	Shows a clear logical progression to state $160\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - 5h$	A1	1.1b	
		(6)		
(b)	Separates the variables $\int \left(\frac{1}{400-5h}\right) \mathrm{d}h = \int \frac{1}{160}\, \mathrm{d}t$	M1	2.2a	8th Solve differential equations in a range of contexts.
	Finds $-\frac{1}{5}\ln\left(400-5h\right) = \frac{t}{160} + C$	A1	1.1b	
	Uses the fact that $t = 0$ when $h = 50$ m to find C $C = -\frac{1}{5}\ln\left(150\right)$	M1	1.1b	
	Substitutes $h = 60$ into the equation $-\frac{1}{5}\ln\left(400-300\right) = \frac{t}{160} - \frac{1}{5}\ln\left(150\right)$	M1	3.1b	
	Uses law of logarithms to write $\frac{1}{5}\ln\left(150\right) - \frac{1}{5}\ln\left(100\right) = \frac{t}{160}$ $\Rightarrow \frac{1}{5}\ln\left(\frac{150}{100}\right) = \frac{t}{160}$	M1	2.2a	
	States correct final answer $t = 32\ln\left(\frac{3}{2}\right)$ minutes.	A1	1.1b	
		(6)		
				(12 marks)

Notes

Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 1 or 2: Pure Mathematics

Practice Paper D	**Paper Reference(s)**
Time: 2 hours	**9MA0/01 or 9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 13 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1. Given that

$$\frac{x^2-36}{x^2-11x+30}\times\frac{25-x^2}{Ax^2+Bx+C}\times\frac{6x^2+7x-3}{3x^2+17x-6}\equiv\frac{x+5}{6-x},$$

find the values of the constants A, B and C, where A, B and C are integers.

(5 marks)

---

2. (a) Use proof by contradiction to show that if n^2 is an even integer then n is also an even integer.

(4 marks)

(b) Prove that $\sqrt{2}$ is irrational.

(6 marks)

---

3. Given that in the expansion of $\dfrac{1}{(1+ax)^2}$ the coefficient of the x^2 term is 75, find

(a) the possible values of a,

(4 marks)

(b) the corresponding coefficients of the x^3 term.

(2 marks)

---

4. (a) Given that $\mathrm{f}(x) = \sin x$, show that

$$\mathrm{f}'(x)=\lim_{h\to 0}\left(\left(\frac{\cos h-1}{h}\right)\sin x+\frac{\sin h}{h}\cos x\right)$$

(4 marks)

(b) Hence prove that $\mathrm{f}'(x) = \cos x$.

(2 marks)

---

5. Given that $\displaystyle\int_a^4 (10-2x)^4\,\mathrm{d}x=\frac{211}{10}$, find the value of a.

(5 marks)

---

6. $$f(x) = x^4 - 8x^2 + 2$$

(a) Show that the equation f(x) = 0 can be written as $x = \sqrt{ax^4 + b}, x > 0$, where a and b are constants to be found.

(2 marks)

Let $x_0 = 1.5$.

(b) Use the iteration formula $x_{n+1} = \sqrt{ax_n^4 + b}$, together with your values of a and b from part (a), to find, to 4 decimal places, the values of x_1, x_2, x_3 and x_4.

(2 marks)

A root of f(x) = 0 is α. By choosing a suitable interval,

(c) prove that $\alpha = -2.782$ to 3 decimal places.

(3 marks)

---

7. The functions f and g are defined by $f(x) = e^{2x} + 4$, $x \in \mathbb{R}$ and $g(x) = \ln(x + 1)$, $x \in \mathbb{R}$, $x > -1$.

(a) Find fg(x) and state its range.

(4 marks)

(b) Solve fg(x) = 85

(3 marks)

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8. For an arithmetic sequence $a_4 = 98$ and $a_{11} = 56$.

(a) Find the value of the 20th term.

(4 marks)

Given that the sum of the first n terms is 78,

(b) find the value of n.

(4 marks)

---

9. Figure 1 shows the right-angled triangles ΔABC, ΔABD and ΔBDC, with $AB = 1$ and $\angle BAD = \theta$.

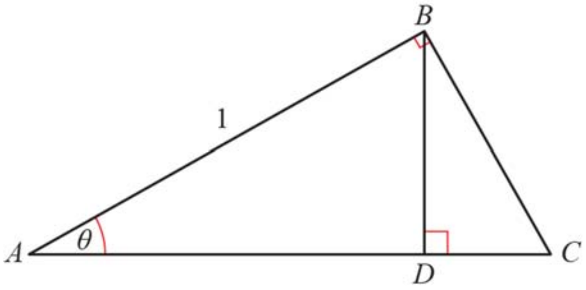

Figure 1

Prove that $1 + \tan^2\theta = \sec^2\theta$.

(8 marks)

---

10. A particle of mass 3 kg is acted on by three forces, $F_1 = \left(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\right)\text{N}$, $F_2 = \left(7\mathbf{i} + 8\mathbf{k}\right)\text{N}$ and $F_3 = \left(-3\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}\right)\text{N}$.

(a) Find the resultant force R acting on the particle.

(2 marks)

(b) Find the acceleration of the particle, giving your answer in the form $\left(p\mathbf{i} + q\mathbf{j} + r\mathbf{k}\right)\text{ms}^{-2}$

(2 marks)

(c) Find the magnitude of the acceleration.

(2 marks)

(d) Given that the particle starts at rest, find the exact distance travelled by the particle in the first 10 s.

(3 marks)

---

11. Find the values of the constants A, B, C, D and E in the following identity:

$$5x^4 - 4x^3 + 17x^2 - 5x + 7 \equiv (Ax^2 + Bx + C)(x^2 + 2) + Dx + E$$

(5 marks)

12.

$$\text{f}(x) = \frac{21 - 14x}{\left(1 - 4x\right)\left(2x + 3\right)}, x \neq \frac{1}{4}, x \neq -\frac{3}{2}$$

(a) Given that $\text{f}(x) = \frac{A}{1 - 4x} + \frac{B}{2x + 3}$, find the values of the constants A and B.

(5 marks)

(b) Find the exact value of $\int_{-1}^{0} \text{f}(x)\,\text{d}x$

(5 marks)

---

13. Figure 2 shows the curve C with parametric equations $x = t + 2$, $y = \dfrac{t-1}{t+2}$, $t \neq -2$. The curve passes through the x-axis at P.

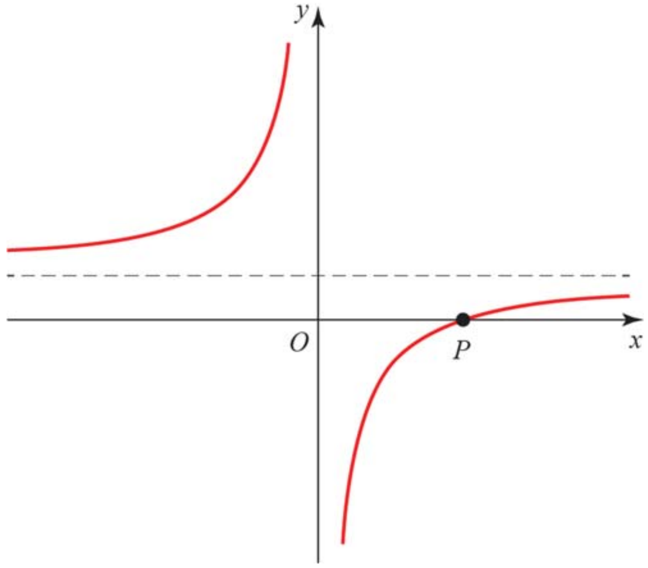

Figure 2

(a) Find the coordinate of P.

(2 marks)

(b) Find the cartesian equation of the curve.

(2 marks)

(c) Find the equation of the normal to the curve at the point $t = -1$. Give your answer in the form $ax + by + c = 0$.

(6 marks)

(d) Find the coordinates of the point where the normal meets C.

(4 marks)

TOTAL FOR PAPER IS 100 MARKS

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1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to factor all the quadratics on the left-hand side of the identity.	**M1**	2.2a	5th Simplify algebraic fractions.
	Correctly factors each expression on the left-hand side of the identity: $\frac{(x-6)(x+6)}{(x-5)(x-6)} \times \frac{(5-x)(5+x)}{Ax^2+Bx+C} \times \frac{(3x-1)(2x+3)}{(3x-1)(x+6)}$	**A1**	2.2a	
	Successfully cancels common factors: $\frac{(-1)(5+x)(2x+3)}{Ax^2+Bx+C} \equiv \frac{x+5}{(-1)(x-6)}$	**M1**	1.1b	
	States that $Ax^2+Bx+C \equiv (2x+3)(x-6)$	**M1**	1.1b	
	States or implies that $A = 2$, $B = -9$ and $C = -18$	**A1**	1.1b	
				(5 marks)

Notes

Alternative method

Makes an attempt to substitute $x = 0$ (**M1**)

Finds $C = -18$ (**A1**)

Substitutes $x = 1$ to give $A + B = -7$ (**M1**)

Substitutes $x = -1$ to give $A - B = 11$ (**M1**)

Solves to get $A = 2$, $B = -9$ and $C = -18$ (**A1**)

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Begins the proof by assuming the opposite is true. 'Assumption: there exists a number *n* such that n^2 is even and *n* is odd.'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Defines an odd number (choice of variable is not important) and successfully calculates n^2 Let $2k + 1$ be an odd number. $n^2 = (2k+1)^2 = 4k^2 + 4k + 1$	**M1**	2.2a	
	Factors the expression and concludes that this number must be odd. $4k^2 + 4k + 1 = 2\left(2k^2 + 2k\right) + 1$, so n^2 is odd.	**M1**	1.1b	
	Makes a valid conclusion. This contradicts the assumption n^2 is even. Therefore if n^2 is even, *n* must be even.	**B1**	2.4	
		(4)		

(b)	Begins the proof by assuming the opposite is true. 'Assumption: $\sqrt{2}$ is a rational number.'	**B1**	3.1	7th Complete proofs using proof by contradiction.
	Defines the rational number: $\sqrt{2} = \frac{a}{b}$ for some integers a and b, where a and b have no common factors.	**M1**	2.2a	
	Squares both sides and concludes that a is even: $\sqrt{2} = \frac{a}{b} \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$ From part **a**: a^2 is even implies that a is even.	**M1**	1.1b	
	Further states that if a is even, then $a = 2c$. Choice of variable is not important.	**M1**	1.1b	
	Makes a substitution and works through to find $b^2 = 2c^2$, concluding that b is also even. $a^2 = 2b^2 \Rightarrow (2c)^2 = 2b^2 \Rightarrow 4c^2 = 2b^2 \Rightarrow b^2 = 2c^2$ From part **a**: b^2 is even implies that b is even.	**M1**	1.1b	
	Makes a valid conclusion. If a and b are even, then they have a common factor of 2, which contradicts the statement that a and b have no common factors. Therefore $\sqrt{2}$ is an irrational number.	**B1**	2.4	
		(6)		
				(10 marks)

Notes

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly states that $(1+ax)^{-2} = 1 + (-2)(ax) + \frac{(-2)(-3)(ax)^2}{2} + \frac{(-2)(-3)(-4)(ax)^3}{6} + ...$	**M1**	2.2a	6th Understand the binomial theorem for rational n.
	Simplifies to obtain $(1+ax)^{-2} = 1 - 2ax + 3a^2x^2 - 4a^3x^3 \ldots$	**M1**	1.1b	
	Deduces that $3a^2 = 75$	**M1**	2.2a	
	Solves to find $a = \pm 5$	**A1**	1.1b	
		(4)		
(b)	$a = 5 \Rightarrow -4(125)x^3 = -500x^3$. Award mark for –500 seen.	**A1**	1.1b	6th Understand the binomial theorem for rational n.
	$a = -5 \Rightarrow -4(-125)x^3 = 500x^3$. Award mark for 500 seen.	**A1**	1.1b	
		(2)		
				(6 marks)

Notes

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x+h-x}$	**M1**	3.1b	5th Differentiate simple trigonometric functions.
	Makes correct substitutions: $f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$	**M1**	1.1b	
	Uses the appropriate trigonometric addition formula to write $f'(x) = \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	**M1**	2.2a	
	Groups the terms appropriately $f'(x) = \lim_{h \to 0} \left(\left(\frac{\cos h - 1}{h} \right) \sin x + \left(\frac{\sin h}{h} \right) \cos x \right)$	**A1**	2.2a	
		(4)		
(b)	Explains that as $h \to 0$, $\frac{\cos h - 1}{h} \to 0$ and $\frac{\sin h}{h} \to 1$	**M1**	3.2b	5th Differentiate simple trigonometric functions.
	Concludes that this leaves $0 \times \sin x + 1 \times \cos x$ So if $f(x) = \sin x$, $f'(x) = \cos x$	**A1**	3.2b	
		(2)		
				(6 marks)

Notes

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to find $\int (10-2x)^4 \, dx$. Raising the power by 1 would constitute an attempt.	**M1**	1.1b	6^{th} Integrate using the reverse chain rule.
	Correctly states $\int (10-2x)^4 \, dx = -\frac{1}{10}(10-2x)^5$	**A1**	2.2a	
	States $-\frac{1}{10}(2)^5 + \frac{1}{10}(10-2a)^5 = \frac{211}{10}$	**M1 ft**	1.1b	
	Makes an attempt to solve this equation. For example, $\frac{1}{10}(10-2a)^5 = \frac{243}{10}$ or $(10-2a)^5 = 243$ is seen.	**M1 ft**	1.1b	
	Solves to find $a = \frac{7}{2}$	**A1 ft**	1.1b	
				(5 marks)

Notes

Student does not need to state '+C' in an answer unless it is the final answer to an indefinite integral.

Award ft marks for a correct answer using an incorrect initial answer.

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Rearranges $x^4 - 8x^2 + 2 = 0$ to find $x^2 = \frac{x^4 + 2}{8}$	**M1**	1.1b	5th Understand the concept of roots of equations.
	States $x = \sqrt{\frac{x^4+2}{8}}$ and therefore $a = \frac{1}{8}$ and $b = \frac{1}{4}$ or states $x = \sqrt{\frac{1}{8}x^4 + \frac{1}{4}}$	**A1**	1.1b	
		(2)		
(b)	Attempts to use iterative procedure to find subsequent values.	**M1**	1.1b	6th Solve equations approximately using the method of iteration.
	Correctly finds: $x_1 = 0.9396$ $x_2 = 0.5894$ $x_3 = 0.5149$ $x_4 = 0.5087$	**A1**	1.1b	
		(2)		
(c)	Demonstrates an understanding that the two values of f(x) to be calculated are for x = –2.7815 and x = –2.7825.	**M1***	2.2a	5th Use a change of sign to locate roots.
	Finds $\text{f}(-2.7815) = -0.0367...$ and $\text{f}(-2.7825) = (+)0.00485...$	**M1**	1.1b	
	Change of sign and continuous function in the interval $\left[-2.7825, -2.7815\right] \Rightarrow$ root	**A1**	2.4	
		(3)		
				(7 marks)

Notes

(b) Award M1 if finds at least one correct answer.

(c) Any two numbers that produce a change of sign, where one is greater than –2.782 and one is less than –2.782, and both numbers round to –2.782 to 3 decimal places, are acceptable. Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to find $\mathrm{fg}(x)$. For example, writing $\mathrm{fg}(x) = \mathrm{e}^{2\ln(x+1)} + 4$	**M1**	2.2a	5th Find composite functions.
	Uses the law of logarithms to write $\mathrm{fg}(x) = \mathrm{e}^{\ln(x+1)^2} + 4$	**M1**	1.1b	
	States that $\mathrm{fg}(x) = (x+1)^2 + 4$	**A1**	1.1b	
	States that the range is $y > 4$ or $\mathrm{fg}(x) > 4$	**B1**	3.2b	
		(4)		
(b)	States that $(x+1)^2 + 4 = 85$	**M1**	1.1b	5th Find the domain and range of composite functions.
	Makes an attempt to solve for *x*, including attempting to take the square root of both sides of the equation. For example, $x + 1 = \pm 9$	**M1**	1.1b	
	States that $x = 8$. Does not need to state that $x \neq -10$, but do not award the mark if $x = -10$ is stated.	**A1**	3.2b	
		(3)		
				(7 marks)

Notes

8	**Scheme**	**Marks**	**AOs**	**Pearson Progression Step and Progress descriptor**
(a)	Forms a pair of simultaneous equations, using the given values $a+3d=98$ $a+10d=56$	**M1**	2.2a	4th Understand simple arithmetic sequences.
	Correctly solves to find $d=-6$	**A1**	1.1b	
	Finds $a=116$	**A1**	1.1b	
	Uses $a_n=a+(n-1)d$ to find $a_{20}=116+19\times(-6)=2$	**A1**	1.1b	
		(4)		
(b)	Uses the sum of an arithmetic series to form the equation $\frac{n}{2}\left[232+(n-1)(-6)\right]=78$	**M1 ft**	2.2a	5th Understand simple arithmetic series.
	Successfully multiplies out the brackets and simplifies. Fully simplified quadratic of $3n^2-119n+78=0$ is seen or $6n^2-238n+156=0$ is seen.	**M1 ft**	1.1b	
	Correctly factorises: $(3n-2)(n-39)=0$	**M1 ft**	1.1b	
	States that $n=39$ is the correct answer.	**A1**	1.1b	
		(4)		
				(8 marks)

Notes

(a) Can use elimination or substitution to solve the simultaneous equations.

(b) Award method marks for a correct attempt to solve the equation using their incorrect values from part **a**.

9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that $\sin\theta = \frac{BD}{1}$ and concludes that $BD = \sin\theta$	**M1**	3.1	6th Prove $\sec^2 x = 1 + \tan^2 x$ and $\text{cosec}^2 x = 1 + \cot^2 x$.
	States that $\cos\theta = \frac{AD}{1}$ and concludes that $AD = \cos\theta$	**M1**	3.1	
	States that $\angle DBC = \theta$	**M1**	2.2a	
	States that $\tan\theta = \frac{DC}{\sin\theta}$ and concludes that $DC = \frac{\sin^2\theta}{\cos\theta}$ oe.	**M1**	3.1	
	States that $\cos\theta = \frac{\sin\theta}{BC}$ and concludes that $BC = \tan\theta$ oe.	**M1**	3.1	
	Recognises the need to use Pythagoras' theorem. For example, $AB^2 + BC^2 = AC^2$	**M1**	2.2a	
	Makes substitutions and begins to manipulate the equation: $1+\tan^2\theta = \left(\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta}\right)^2$ $1+\tan^2\theta = \left(\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right)^2$	**M1**	1.1b	
	Uses a clear algebraic progression to arrive at the final answer: $1+\tan^2\theta = \left(\frac{1}{\cos\theta}\right)^2$ $1+\tan^2\theta = \sec^2\theta$	**A1**	1.1b	
				(8 marks)

Notes

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to find the resultant force by adding the three force vectors together.	**M1**	3.1a	6th Solve contextualised problems in mechanics using 3D vectors.
	Finds $R = (6\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$ N	**A1**	1.1b	
		(2)		
(b)	States $F = ma$ or writes $(6\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}) = 3(a)$	**M1**	3.1a	6th Solve contextualised problems in mechanics using 3D vectors.
	Finds $a = (2\mathbf{i} + \mathbf{j} + \mathbf{k})\text{ms}^{-2}$	**A1**	1.1b	
		(2)		
(c)	Demonstrates an attempt to find $\|a\|$ For example, $\|a\| = \sqrt{(2)^2 + (1)^2 + (1)^2}$	**M1**	3.1a	6th Solve contextualised problems in mechanics using 3D vectors.
	Finds $\|a\| = \sqrt{6}$ m s^{-2}	**A1**	1.1b	
		(2)		
(d)	States $s = ut + \frac{1}{2}at^2$	**M1**	3.1a	6th Solve contextualised problems in mechanics using 3D vectors.
	Makes an attempt to substitute values into the equation. $s = (0)(10) + \frac{1}{2}(\sqrt{6})(10)^2$	**M1 ft**	1.1b	
	Finds $s = 50\sqrt{6}$ m	**A1 ft**	1.1b	
		(3)		
				(9 marks)

Notes

(d) Award ft marks for a correct answer to part **d** using their incorrect answer from part **c**.

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Equating the coefficients of x^4: $A = 5$	**A1**	2.2a	6th Solve problems using the remainder theorem linked to improper algebraic fractions.
	Equating the coefficients of x^3: $B = -4$	**A1**	1.1b	
	Equating the coefficients of x^2: $2A + C = 17$, $C = 7$	**A1**	1.1b	
	Equating the coefficients of x: $2B + D = -5$, $D = 3$	**A1**	1.1b	
	Equating constant terms: $2C + E = 7$, $E = -7$	**A1**	1.1b	
				(5 marks)

Notes

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States that $A(2x+3)+B(1-4x)\equiv 21-14x$	M1	1.1b	6th Decompose algebraic fractions into partial fractions − linear factors.
	Equates the various terms. Equating xs $2A-4B=-14$ Equating numbers $3A+B=21$	M1	1.1b	
	Multiplies or or both of the equations in an effort to equate one of the two variables.	M1	1.1b	
	Finds $A = 5$	A1	1.1b	
	Find $B = 6$	A1	1.1b	
		(5)		
(b)	Writes $\int_{-1}^{0}\left(\frac{5}{1-4x}+\frac{6}{2x+3}\right)\mathrm{d}x$ as $\int_{-1}^{0}\left(5(1-4x)^{-1}+6(2x+3)^{-1}\right)\mathrm{d}x$	M1 ft	2.2a	6th Integrate functions using the reverse chain rule.
	Makes an attempt to integrate the expression. Attempt would constitute the use of logarithms.	M1 ft	2.2a	
	Integrates the expression to find $\left[-\frac{5}{4}\ln(1-4x)+3\ln(2x+3)\right]_{-1}^{0}$	A1 ft	1.1b	
	Makes an attempt to substitute the limits $\left(-\frac{5}{4}\ln(1-4(0))+3\ln(2(0)+3)\right)$ $-\left(-\frac{5}{4}\ln(1-4(-1))+3\ln(2(-1)+3)\right)$	M1 ft	1.1b	
	Simplifies to find $\ln 27+\frac{5}{4}\ln 5$ o.e.	A1 ft	1.1b	
		(5)		
				(10 marks)

Notes

Award ft marks for a correct answer to part **b** using incorrect values from part **a**.

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Shows or implies that if $y = 0$, $t = 1$	M1	1.1b	7th Solve coordinate geometry problems involving parametric equations.
	Finds the coordinates of P. $t=1 \Rightarrow x=3$ $P(3,0)$	A1	1.1b	
		(2)		
(b)	Attempts to find a cartesian equation of the curve. For example, $t = x - 2$ is substituted into $y=\frac{t-1}{t+2}$	M1	2.2a	7th Solve coordinate geometry problems involving parametric equations.
	Correctly finds the cartesian equation of the curve $y=\frac{x-3}{x}$ Accept any equivalent answer. For example, $y=1-\frac{3}{x}$	A1	1.1b	
		(2)		
(c)	Finds $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^{-2}=\frac{3}{x^2}$	M1	2.2a	7th Solve coordinate geometry problems involving parametric equations.
	Substitutes $t = -1$ to find $x = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{(1)^2}=3$	M1	1.1b	
	Finds the gradient of the normal $m_N=-\frac{1}{3}$	M1	1.1b	
	Substitutes $t = -1$ to find $x = 1$ and $y = -2$	A1	1.1b	
	Makes an attempt to find the equation of the normal. For example, $y+2=-\frac{1}{3}(x-1)$ is seen.	M1	1.1b	
	States fully correct answer $x+3y+5=0$	A1	1.1b	
		(6)		

(d)	Substitutes $x = t + 2$ and $y = \dfrac{t-1}{t+2}$ into $x + 3y + 5 = 0$ obtaining $t + 2 + 3\left(\dfrac{t-1}{t+2}\right) + 5 = 0$	**M1 ft**	2.2a	7th Solve coordinate geometry problems involving parametric equations.
	Manipulates and simplifies this equation to obtain $t^2 + 12t + 11 = 0$	**M1 ft**	1.1b	
	Factorises and solves to find $t = -1$ or $t = -11$	**M1 ft**	1.1b	
	Substitutes $t = -11$ to find $x = -9$ and $y = \dfrac{4}{3}$, i.e. $\left(-9, \dfrac{4}{3}\right)$	**A1 ft**	1.1b	
		(4)		
				(14 marks)

Notes

(c) Award ft marks for correct answer using incorrect values from part **b**.

Pearson Edexcel Level 3	
GCE Mathematics **Advanced Level** **Paper 3: Statistics & Mechanics**	
Practice Paper H **Time: 2 hours**	**Paper Reference(s)** **9MA0/03**
You must have: **Mathematical Formulae and Statistical Tables, calculator**	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this paper. The total is 100.
- The for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

SECTION A: STATISTICS

Answer ALL questions.

1. The distributions for the heights for a sample of females and males at a UK university can be modelled using normal distributions with mean 165 cm, standard deviation 9 cm and mean 178 cm, standard deviation 10 cm respectively.

A female's height of 177 cm and a male's height of 190 cm are both 12 cm above their means.

By calculating z-values, or otherwise, explain which is relatively taller.

(Total 4 marks)

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2. The table shows some data collected on the temperature, in °C, of a cup of coffee, c, and the time, t in minutes, after which it was made.

t	0	2	4	5	7	11	13	17	25
c	81.9	75.9	70.1	65.1	60.9	51.9	50.8	45.1	39.2

The data is coded using the changes of variable $x = t$ and $y = \log_{10} c$.

The regression line of y on x is found to be $y = 1.89 - 0.0131x$.

(a) Given that the data can be modelled by an equation of the form $c = ab^t$ where a and b are constants, find the values of a and b.

(3)

(b) Give an interpretation of the constant b in this equation.

(1)

(c) Explain why this model is not reliable for estimating the temperature of the coffee after an hour.

(1)

(Total 5 marks)

---

3. The table below shows the number of gold, silver and bronze medals won by two teams in an athletics competition.

	Gold	**Silver**	**Bronze**
Team *A*	29	17	18
Team *C*	21	23	17

The events G, S and B are that a medal is gold, silver or bronze respectively. Let A be the event that team A won a medal and C team C won a medal. A medal winner is selected at random. Find

(a) P(G),

(2)

(b) P($[A \cap S]'$).

(2)

(c) Explain, showing your working, whether or not events S and A are statistically independent. Give reasons for your answer.

(2)

(d) Determine whether or not events B and C are mutually exclusive. Give a reason for your answer.

(2)

(e) Given that 30% of the gold medal winners are female, 60% of the silver medal winners are female and 40% of the bronze medal winners are female, find the probability that a randomly selected medal winner is female.

(2)

(Total 10 marks)

---

4. A certain type of cabbage has a mass M which is normally distributed with mean 900 g and standard deviation 100 g.

(a) Find P(M < 850).

(1)

10% of the cabbages are too light and 10% are too heavy to be packaged and sold at a fixed price.

(b) Find the minimum and maximum weights of the cabbages that are packaged.

(3)

(Total 4 marks)

---

5. The data and scatter diagram (Figure 1) show the weight of chickens, x kilograms, and the average weight, y grams, of eggs laid by a random sample of 10 chickens.

Weight of chickens (kg)	2.9	1.9	1.6	2.7	3.1	2.2	2.7	1.9	1.7	2.6
Average weight of eggs (g)	58	56	55	66	47	63	49	56	53	53

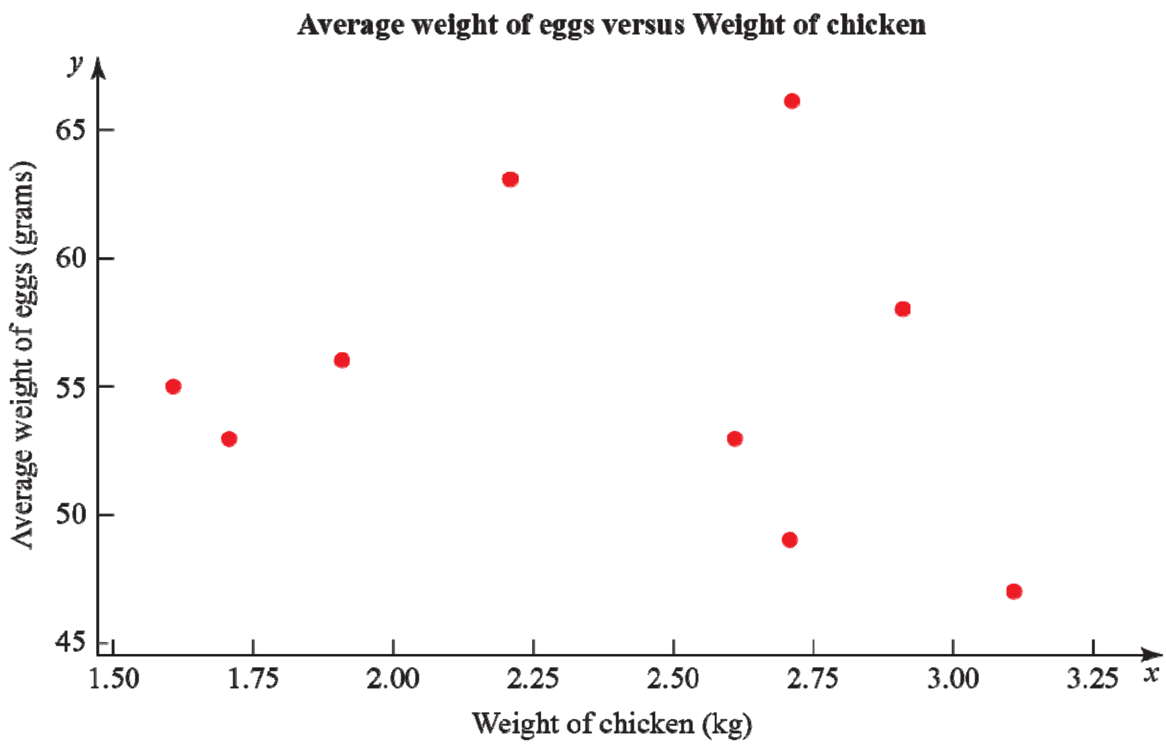

Figure 1

The product moment correlation coefficient for the average weight of eggs and weight of chickens is −0.136.

(a) Test for evidence of a negative population product moment correlation coefficient at the 2.5% significance level. Interpret this result in context.

(3)

(b) Explain why even if the population product moment correlation coefficient between two variables is close to zero there may still be a relationship between them.

(2)

(Total 5 marks)

6. (a) State the conditions under which the normal distribution may be used as an appoximation to the binomial distribution $X \sim B(n, p)$.

(2)

(b) Write down the mean and variance of the normal approximation to X in terms of n and p.

(2)

A manufacturer claims that more than 55% of its batteries last for at least 15 hours of continuous use.

(c) Write down a reason why the manufacturer should not justify their claim by testing all the batteries they produce.

(1)

To test the manufacturer's claim, a random sample of 300 batteries were tested.

(d) State the hypotheses for a one-tailed test of the manufacturer's claim.

(1)

(e) Given that 184 of the 300 batteries lasted for at least 15 hours of continuous use a normal approximation to test, at the 5% level of significance, whether or not the manufacturer's claim is justified.

(7)

(Total 13 marks)

---

7. The mean body temperature for women is normally distributed with mean 36.73 °C with variance 0.1482 $(°C)^2$. Kay has a temperature of 38.1 °C.

(a) Calculate the probability of a woman having a temperature greater than 38.1 °C.

(2)

(b) Advise whether should Kay get medical advice. Give a reason for your advice.

(1)

(Total 3 marks)

---

8. To investigate if there is a correlation between daily mean temperature (°C) and daily mean pressure (hPa) the location Hurn 2015 was randomly selected from:

Camborne 2015	Camborne 1987
Hurn 2015	Hurn 1987
Leuchars 2015	Leuchars 1987
Leeming 2015	Leeming 1987
Heathrow 2015	Heathrow 1987

(Source: Pearson Edexcel GCE AS and A Level Mathematics data set.)

(a) State the definition of a test statistic.

(1)

(b) The product moment correlation coefficient between daily mean temperature and daily mean pressure for these data is −0.258 with a p-value of 0.001. Use a 5% significance level to test whether or not there is evidence of a correlation between the daily mean temperature and daily mean pressure.

(3)

(c) The scatter diagram in Figure 2 shows daily mean temperature versus daily mean pressure, by season, for Hurn 2015. Give two interpretations on the split of the data between summer and autumn.

(2)

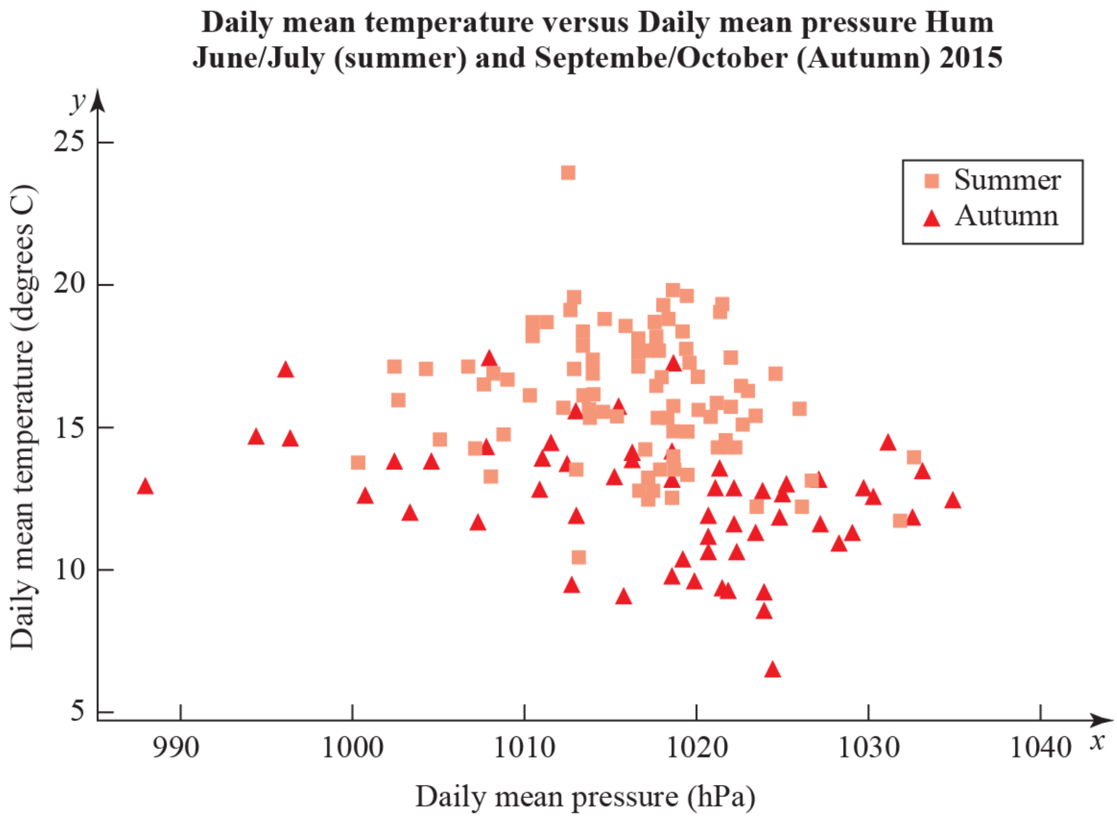

Figure 2

(Total 6 marks)

SECTION B: MECHANICS

Answer ALL questions.

9. At time t seconds, a 2 kg particle experiences a force $\mathbf{F}$ N, where $\mathbf{F} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} t + \begin{pmatrix} 6 \\ -12 \end{pmatrix} t^2$

(a) Find the acceleration of the particle at time t seconds.

(3)

The particle is initially at rest at the origin.

(b) Find the position of the particle at time t seconds.

(6)

(c) Find the particle's velocity when $t = 1$.

(3)

(Total 12 marks)

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10. An archer shoots an arrow at 10 m s^{-1} from the origin and hits a target at $(10, -5)$ m. The initial velocity of the arrow is at an angle θ above the horizontal. The arrow is modelled as a particle moving freely under gravity.

(In this question, take $g = 10 \text{ m s}^{-2}$.)

(a) Show that $(\tan\theta - 1)^2 = 1$.

(11)

(b) Find the possible values of θ .

(3)

(Total 14 marks)

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11. Figure 3 shows a 5000 kg bus hanging 12 m over the edge of a cliff with 1000 kg of gold at the front. The gold sits on a wheeled cart. A group of n people, each weighing 70 kg, stands at the other end. The bus is 20 m long.

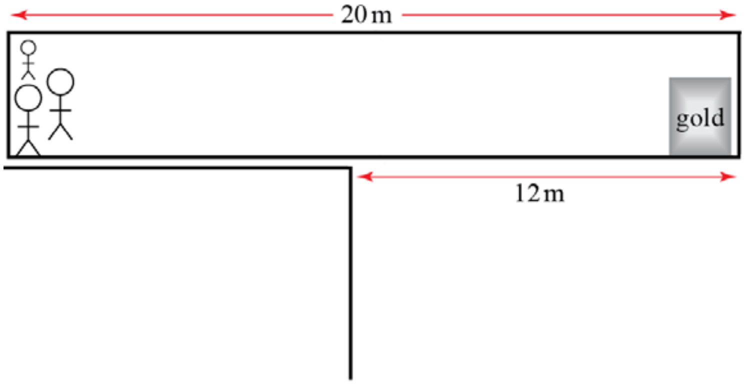

Figure 3

(a) Write down the total clockwise moment about the cliff edge in terms of n.

(7)

(b) Find the smallest number of people needed to stop the bus falling over the cliff.

(2)

(c) One person needs to walk to the end of the bus to retrieve the gold. Find the smallest number of people needed to stop the bus falling over the cliff in this situation, including the one retrieving the gold.

(4)

(Total 13 marks)

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12. A car travels along a long, straight road for one hour, starting from rest. After t hours, its acceleration is a km h^{-2}, where $a = 180 - 360t$.

(a) Find the speed of the car, in km h^{-1} in terms of t.

(2)

The speed limit is 40 km h^{-1}.

(b) Find the range of times during which the car is breaking the speed limit. Give your answer in minutes.

(4)

(c) Find the average speed of the car over the whole journey.

(5)

(Total 11 marks)

TOTAL FOR PAPER IS 100

H1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	$X \sim$ females $X \sim N(165, 9^2)$, $Y \sim$ males $Y \sim N(178, 10^2)$	**M1**	3.3	5th Calculate probabilities for the standard normal distribution using a calculator.
	$P(X > 177) = P(Z > 1.33)$ (or = 0.0912)	**M1**	1.1b	
	$P(Y > 190) = P(Z > 1.20)$ (or = 0.1151)	**A1**	1.1b	
	Therefore the females are relatively taller.	**A1**	2.2a	
				(4 marks)

H2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\log_{10} c = 1.89 - 0.0131t$ $c = 10^{1.89-0.0131t}$ $c = 77.6 \times 0.970^t$ (3 s.f.)	**M1** **M1** **A1**	1.1a 1.1b 1.1b	6th Understand exponential models in bivariate data.
		(3)		
b	b is the proportional rate at which the temperature changes per minute.	**A1**	3.2a	6th Understand exponential models in bivariate data.
		(1)		
c	Extrapolation/out of the range of the data.	**A1**	2.4	4th Understand the concepts of interpolation and extrapolation.
		(1)		
				(5 marks)
Notes				

H3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$\frac{29+21}{29+21+17+23+18+17}=\frac{50}{125}$	**M1**	1.1b	2nd Calculate probabilities from relative frequency tables and real data.
	$= 0.4$	**A1**	1.1b	
		(2)		
b	$\frac{125-17}{125}=\frac{108}{125}$	**M1**	3.1a	4th Understand set notation.
	$= 0.864$	**A1**	1.1b	
		(2)		
c	$\text{P}(S \cap A) = \frac{17}{125} = 0.136 \neq \text{P}(S) \times \text{P}(A) = \frac{40}{125} \times \frac{64}{125} = 0.163\ldots$	**M1**	2.1	4th Understand and use the definition of independence in probability calculations.
	So, S and A are not statistically independent.	**A1**	2.4	
		(2)		
d	B and C are not mutally exclusive	**B1**	2.2a	3rd Understand and use the definition of mutually exclusive in probability calculations.
	Being in team C does not exclude the possibility of winning a bronze medal	**B1**	2.4	
		(2)		
e	$\frac{15+24+14}{125}=\frac{53}{125}$	**M1**	3.1b	5th Calculate conditional probabilities using formulae.
	$= 0.424$	**A1**	1.1b	
		(2)		
				(10 marks)

Notes

H4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$P(M < 850) = 0.3085$ (using calculator)	**B1**	1.1b	5th Calculate probabilities for the standard normal distribution using a calculator.
		(1)		
b	$P(M < a) = 0.1$ and $P(M < b) = 0.9$	**M1**	3.1b	5th Calculate probabilities for the standard normal distribution using a calculator.
	(using calculator) $a = 772$ g	**A1**	1.1b	
	$b = 1028$ g	**A1**	1.1b	
		(3)		
				(4 marks)

Notes

H5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$H_0 : \rho = 0$, $H_1 : \rho < 0$	**B1**	2.5	6th
	Critical value = −0.6319	**M1**	1.1a	Carry out a hypothesis test for zero correlation.
	$-0.6319 < -0.136$ no evidence to reject H_0 (test statistic not in critical region)			
	There is insufficient evidence to suggest that the weight of chickens and average weight of eggs are negatively correlated.	**A1**	2.2b	
		(3)		
b	Sensible explanation. For example, correlation shows there is no (or extremely weak) linear realtionship between the two variables.	**B1**	1.2	7th Interpret the results of a hypothesis test for zero correlation.
	For example, there could be a non-linear relationship between the two variables.	**B1**	3.5b	
		(2)		
				(5 marks)

Notes

H6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	n is large	B1	1.2	5th Understand the binomial distribution (and its notation) and its use as a model.
	p is close to 0.5	B1	1.2	
		(2)		
b	Mean = np	B1	1.2	5th Understand the binomial distribution (and its notation) and its use as a model.
	Variance = $np(1-p)$	B1	1.2	
		(2)		
c	There would be no batteries left.	B1	2.4	5th Select and critique a sampling technique in a given context.
		(1)		
d	H_0: $p = 0.55$ H_1: $p > 0.55$	B1	2.5	5th Carry out 1-tail tests for the binomial distribution.
		(1)		
e	$X \sim N(165, 74.25)$ $P(X \geqslant 183.5)$ $= P\left(Z \geqslant \frac{183.5-165}{\sqrt{74.25}}\right)$ $= P(Z \geqslant 2.146...)$ $=1 - 0.9838$ $= 0.0159$ Reject H_0, it is in the critical region. There is evidence to support the manufacturer's claim.	B1 M1 M1 A1 A1 M1 A1	3.3 3.4 1.1b 1.1b 1.1b 1.1b 2.2b	7th Interpret the results of a hypothesis test for the mean of a normal distribution.
		(7)		
				(13 marks)

Notes

H7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	$X \sim$ women's body temperature $X \sim \text{N}(36.73, 0.1482)$	M1	3.3	5th
	$\text{P}(X > 38.1) = 0.000186$	B1	1.1b	Calculate probabilities for the standard normal distribution using a calculator.
		(2)		
b	Sensible reason. For example, Call the doctor as very unlikely the temperature would be so high.	B1	2.2a	8th Solve real-life problems in context using probability distributions.
		(1)		
				(3 marks)

Notes

H8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	A statistic that is calculated from sample data in order to test a hypothesis about a population.	**B1**	1.2	5th Understand the language of hypothesis testing.
		(1)		
b	$H_0 : \rho = 0$, $H_1 : \rho \neq 0$	**B1**	2.5	6th Carry out a hypothesis test for zero correlation.
	p-value < 0.05	**M1**		
	There is evidence to reject H_0		1.1b	
	There is evidence (at 5% level) of a correlation between the daily mean temperature and daily mean pressure.	**A1**	2.2b	
		(3)		
c	Two sensible interpretations or observations. For example, Two distinct distributions Similar gradients of regression line. Similar correlations for each season. Lower temperaure in autumn. More spread for the daily mean pressure in autumn.	**B2**	3.2a	4th Use the principles of bivariate data analysis in the context of the large data set.
		(2)		
				(6 marks)

Notes

H9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Use of Newton's second law.	M1	3.1b	8th Understand general kinematics problems with vectors.
	$\mathbf{a} = \frac{\mathbf{F}}{2}$	M1	1.1b	
	$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} t + \begin{pmatrix} 3 \\ -6 \end{pmatrix} t^2 \ (\text{m s}^{-2})$	A1	1.1b	
		(3)		
b	Integrate **a**	M1	1.1a	8th Solve general kinematics problems using calculus of vectors.
	$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} t^2 + \begin{pmatrix} 1 \\ -2 \end{pmatrix} t^3 + \mathbf{c} \ (\text{m s}^{-1})$	A1	1.1b	
	$\mathbf{c} = 0$ because initially at rest.	A1	2.4	
	Integrate **v**	M1	1.1a	
	$\mathbf{r} = \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} t^3 + \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{2} \end{pmatrix} t^4 + \mathbf{c} \ (\text{m})$	A1	1.1b	
	$\mathbf{c} = 0$ because initially at origin.	A1	2.4	
		(6)		
c	Subsititute $t = 1$	M1	1.1a	6th Understand general kinematics problems with vectors.
	$\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$	M1	1.1b	
	$= \begin{pmatrix} 3 \\ -1 \end{pmatrix} (\text{m s}^{-1})$	A1	1.1b	
		(3)		
				(12 marks)

Notes

H11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Moment from bus = $5000 \times 2 \times g$	**M1**	3.1a	5th Find resultant moments by considering direction.
	$= 10\,000g$ (N m)	**A1**	1.1b	
	Moment from gold = $1000 \times 12 \times g$	**M1**	3.1b	
	$= 12\,000g$ (N m)	**A1**	1.1b	
	Moment from people = $70 \times 8 \times n \times g$	**M1**	3.1a	
	$= 560ng$ (N m)	**A1**	1.1b	
	Total moment = $(22\,000 - 560n)g$ (N m)	**A1**	1.1b	
		(7)		
b	Forming an equation or inequality for n and solving to find ($n = 39.28\ldots$)	**M1**	1.1b	5th Solve equilibrium problems involving horizontal bars.
	Need 40 people.	**A1**	3.2a	
		(2)		
c	New moment from gold and extra person is $1070 \times 12 \times g$ (N)	**M1**	3.1a	5th Solve equilibrium problems involving horizontal bars.
	New total moment = $(22840 - 560n)g$ (N m)	**M1**	1.1b	
	$n = 40.78\ldots$	**A1**	3.2a	
	42 people (including the extra)	**A1**	2.4	
		(4)		
				(13 marks)

H10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Use of suvat equations	M1	1.1a	8th Derive formulae for projectile motion.
	$x = 10t\cos\theta$	A1	1.1b	
	$y = 10t\sin\theta - \frac{1}{2}gt^2$	M1	1.1b	
	$= 10t\sin\theta - 5t^2$	A1	1.1b	
	Substitute $x = 10$ and $y = -5$	M1	1.1a	
	Solve x equation for t	M1	1.1b	
	$t = \frac{1}{\cos\theta}$	A1	1.1b	
	Substitute into y equation	M1	1.1a	
	$-5 = 10\tan\theta - 5\sec^2\theta$	A1	2.1	
	Use of $\sec^2\theta = 1 + \tan^2\theta$	M1	2.1	
	$(\tan\theta - 1)^2 = 1$ legitimately obtained	A1	2.1	
		(11)		
b	Solve for $\tan\theta$	M1	1.1a	8th Solve problems in unfamiliar contexts using the concepts of friction and motion.
	$\tan\theta = 0$ or $\tan\theta = 2$	A1	1.1b	
	$\theta = 0$ or 63.43…(°) (accept awrt 63)	A1	1.1b	
		(3)		
				(14 marks)

Notes

H12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
a	Integrate a w.r.t. t	M1	1.1a	5th Use integration to determine functions for velocity and/or displacement.
	$a = 180t - 180t^2$	A1	1.1b	
		(2)		
b	$180t - 180t^2 > 40$	M1	3.1a	7th Solve general kinematics problems in less familiar contexts.
	$20(3t-2)(3t-1) < 0$	A1	1.1b	
	$\frac{1}{3} < t < \frac{2}{3}$	A1	2.4	
	Breaking the speed limit between 20 and 40 minutes.	A1	3.2a	
		(4)		
c	Integrate v w.r.t. t	M1	1.1a	5th Use integration to determine functions for velocity and/or displacement.
	$x = 90t^2 - 60t^3 \ (+C)$	A1	1.1b	
	When $t = 1, x = 30$	A1	3.1b	
	Average speed $= \frac{\text{distance}}{\text{time}}$	M1	1.1b	
	30 km h^{-1}	A1	1.1b	
		(5)		
				(11 marks)

Notes