



## PENETRATIVE TURBULENCE ASSOCIATED WITH MESOSCALE SURFACE HEAT FLUX VARIATIONS

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### ABSTRACT

Human modification of the earth's surface has a strong impact on penetrative turbulence in the atmospheric boundary layer, which increases uncertainty in the future climate projections. Such a surface induced turbulence is difficult to be parameterized in climate models, and requires further investigations. This article aims to investigate penetrative turbulence in the atmospheric boundary layer. Using a large eddy simulation approach, we study characteristics of the mixed layer with respect to surface heat flux variations in the range from  $231.48 \text{ W m}^{-2}$  to  $925.92 \text{ W m}^{-2}$ , and observe that the surface heterogeneity on a spatial scale of 20 km leads to downscale turbulent kinetic energy cascade. Coherent fluctuations of mesoscale horizontal wind is observed at 100 m above the ground. Such a surface induced temporal oscillations in the horizontal wind suggest a rapid jump in mesoscale wind forecasts. The present result with respect to a surface heterogeneity at a typical scale (20 km) of city characterizes the modifications needed for the subgrid scale parameterization schemes used in classical weather forecasting mesoscale models.

**Keywords:** large eddy simulation, atmospheric boundary layer, numerical method.

### INTRODUCTION

Earth's surface causes horizontal winds on various spatial scales, and is assumed to play a distinctive influence in weather and climate systems [15]. The surface influence generates horizontal convection as a result of differential heating along one horizontal boundary of the atmosphere (ABL) [8, 13]. Some studies investigated coherent mesoscale motions in the atmospheric boundary layer [14, 9]. In such studies, the effect of surface heterogeneity were investigated numerically on scales of few hundred kilometers, using a grid spacing of  $\Delta x \sim O(10 \text{ km})$ . Since the typical scale of a city is  $O(20 \text{ km})$ , a sufficiently high resolution is needed to investigate the differential heating due to urbanization. On the other hand, the geophysical fluid dynamics community debates on a controversial opinion on whether horizontal convection due to differential heating at the same level would lead to turbulent fluctuations, or would drive large scale overturning circulations [10, 13]. This opinion is based on a century old experimental result by Sandström (1908) [12], which implies that a sustained circulation cannot occur if the level of the heating source is the same as that of the cooling source. This hypothesis is supported theoretically by the anti-turbulence theorem of Paparella and Young (2002) [10]. Such a conclusion indicates that surface heterogeneity may not have a strong influence on the coherent mesoscale circulations. However, Scotti and White (2011) [13] employed a direct numerical simulation of horizontal convection at Rayleigh number 1010, and showed that a flow driven by the horizontal convection exhibits the characteristics of a true turbulent flow. Clearly, some fundamental aspects of solar heating at the earth's surface is not fully understood. It is necessary to investigate if differential heating at the scale of a typical city induces a sustained circulation that may have a strong impact on local weather. This article puts some contribution in this direction.

### Surface induced penetrative turbulence

Solar heating over a heterogeneous surface generates penetrative turbulence because an unstably stratified large body of fluid above the ground underlies a stably stratified fluid layer in the free atmosphere [6]. In this situation, turbulent eddies are driven vertically by the buoyancy force, and attempt to penetrate into the overlying stable fluid [15]. In the stable region, when an eddy reaches its level of buoyancy, it returns back downward into the unstable region. However, due to the gained momentum, eddies often overshoot their level of buoyancy, and thus, internal waves are excited from the interface between the overlying stably stratified fluid and the underlying unstable mixing layer [8, 15]. These waves transports kinetic energy to the upper atmosphere, and may initiate upper level clear air turbulence [16]. Deardorff *et al.* (1969) [6] investigated such penetrative turbulence experimentally. A brief review of observational work may shed further light in this direction.

Several researchers discussed the data obtained from aircraft measurements – some of which are reviewed here briefly. For example, lightning data over Houston, USA between years of 1989 and 2000 was analyzed, and the result indicates that highest flash densities occur over urban areas (e.g. Zhang *et al.* 2014 [17]). Analysis of the data for a 43 years period from 1958 to 2001 in the North Atlantic, USA, and European sectors concludes that clear-air turbulence increases in these regions by 40% – 90%, which is a result of the urban induced impact on vertical transport and mixing of penetrative ABL turbulence [16]. Note that the average horizontal flux of kinetic energy at a height  $\sim 100 \text{ m}$  from the ground is about. Such studies suggest that a destabilization of this huge energy flux due to differential heating on the surface may cause catastrophic impact on the turbulent atmosphere. Since penetrative turbulence in the atmosphere (see Stull, 1976 [15]) is primarily characterized by solar heating,



perturbations to boundary layer structures by human activities (e.g. urbanization) are sensitive to vertical mixing and transport [3, 8].

### Goal of the present research

The pioneering work of Deardorff (1970) [5] resulted in the state-of-the-art large eddy simulation (LES) methodology for atmospheric turbulence. This article investigates an LES based numerical model in order to characterize penetrative turbulence over a heterogeneous land surface. To resolve the energy containing near surface eddies, the present method filters a turbulent flow using a multiresolution approach, and employs a Smagorinsky type eddy viscosity model for the subfilter scale processes. Surface heating is directly incorporated into the LES model in the form of an elliptic equation for the background potential temperature. To avoid artificial numerical dissipation, we have used a sixth order weighted residual collocation method. The method is unconditionally stable, and its computational complexity is approximately, where  $N$  is the number of grid points. A details of the numerical approach used in this work is also given by Alam *et al.* (2014) [2]. Primary objectives of this short article includes the investigation of evidences whether horizontal convection transports large quantities of heat, as well as sustains a large amount of diapycnal mixing with a relatively small amount of dissipation. In this direction, the effect of surface heat flux on the downscale energy cascade has been investigated. We have found that differential surface heating on scales of a typical modern city cascades kinetic energy downscale, which – in the absence of a mechanical shear – is sufficient to initiate a turbulent flow. The overall methodology and numerical simulations have been summarized briefly in the following section.

## METHODOLOGY

### Equations

For a compressible atmospheric model, the temperature ( $T$ ) and the pressure ( $p$ ) are represented by the potential temperature ( $\theta$ ) and the Exner function ( $\pi'$ ) (e.g., [14]),

$$\theta = T \left( \frac{p_0}{p} \right)^{R_d/c_p}, \quad \pi' = \left( \frac{R_d}{p_0} \right)^{R_d/(c_p - R_d)},$$

respectively, where  $p_0$  is a reference pressure,  $R_d$  is the gas constant,  $c_p$  is the specific heat at constant pressure, and  $\rho$  is the density. The continuity equation is replaced with Equation (1). The notation  $(x_1, x_3) = (x, z)$  and  $(u_1, u_3) = (u, w)$  are adopted for simplicity. A spatial filter is applied to the momentum and energy equations, where  $u_i$  ( $\equiv \langle u_i \rangle$ ) and  $\theta$  ( $\equiv \langle \theta \rangle$ ) represent filtered velocity and temperature, respectively. More specifically,  $\tilde{u}_i = u_i + u'_i$  and  $\tilde{\theta} = \theta_0 + \bar{\theta}(z) + \theta(x, z, t) + \theta'$  (Deardorff, 1970, 1980 [4, 5]). Note the separation of the reference temperature  $\theta_0$  from the background temperature  $\bar{\theta}(z)$ , which is convenient for satisfying the

surface condition. The following equations are solved in the present work;

$$\frac{D\pi'}{Dt} = -\pi' \frac{\partial u_i}{\partial x_i}, \quad (1)$$

$$\frac{Du_i}{Dt} = -c_p \theta_0 \frac{\partial \pi'}{\partial x_i} + \frac{g\theta}{\theta_0} \delta_{i3} - \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

$$\frac{D\theta}{Dt} = -w \frac{\partial \bar{\theta}}{\partial z} - \frac{\partial \tau_{\theta j}}{\partial x_j} \quad (3)$$

The model region is a vertical plane ( $x, z$ ) that extends 100 km horizontally ( $-50 \leq x \leq 50$ ) and 2 km vertically ( $0 \leq z \leq 2$ ). A city of scale 20 km exists for  $-10 \leq x \leq 10$ , which is surrounded by rural areas. Recent literature indicates that the most appropriate model for the near surface penetrative turbulence is not fully understood [17]. As a compromise, as discussed by Pope (2000) [11], we have adopted a resolution that is finer than what is used by many LES models of the ABL. The finest resolution uses  $\Delta x = 97.65$  m and  $\Delta z = 3.9$  m. Thus, a significant fraction of the energy containing large eddies is resolved. Here, some advantages of the costly three-dimensional simulation are sacrificed for a high resolution two-dimensional idealization [8].

### Large eddy simulation

The subgrid scale turbulent stress is estimated by the popular Smagorinsky (1963) model,  $\tau_{ij} = -2(C_s \Delta)^2 |S| S_{ij}$ , where  $|S| = 2S_{ij}S_{ij}$  and  $S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ . We define the filter width by  $\Delta = \sqrt{\Delta x \cdot \Delta z}$ , and take  $C_s = 0.18$  for the Smagorinsky constant.

According to Deardorff (1980) [4], the subgrid scale eddy coefficients is computed by  $K_h = (1 + 2l/\Delta)K_m$  and  $K_m = 2(C_s \Delta)^2 |S|$ , where  $l \leq \Delta$  is a subgrid scale mixing length. The length scale,  $l$ , is related to the subgrid scale turbulence energy  $e'$  and the buoyancy frequency,  $N^2 = \frac{g}{\theta_0} \frac{\partial \theta}{\partial z}$ , i.e.  $l = 0.76 \sqrt{e'}/N$ . When the scale of resolved eddies is  $\Delta = 20$ m, then a turbulent Prandtl number  $Pr = 0.71$  gives  $l \approx 4$  m. The SGS flux for  $\tau_{\theta i} = -K_h \frac{\partial \theta}{\partial x_i}$  (Deardorff, 1980).

### Surface heat-flux variation

We consider a time independent profile for the surface heat flux variation  $H_{sfc}(x) = \langle H_{sfc} \rangle + H_0$  that is given by

$$H_{sfc(x)} = \langle H_{sfc} \rangle + A[\tanh \xi(x + \lambda/2) - \tanh \xi(x - \lambda/2)] \quad (4)$$

Here,  $\langle H_{sfc} \rangle = 0.03 \text{ K m s}^{-1}$  (about  $37 \text{ W m}^{-2}$ ) represents a domain average heat flux for all  $x$  in the range from  $-50$  to  $50$  km, where there is no city induced heat flux  $H_0$ .  $\lambda$  is the characteristic wavelength for the



surface heat flux variation  $H_0$ , where the heating region (city) of the surface is from  $-10$  to  $10$  km, i.e.  $\lambda = 20$  km in the present simulation. This choice for  $\lambda$  represents the typical resolved scale for mesoscale numerical weather prediction models.  $\xi = 100$  is a dimensionless number that leads to a continuous sharp interface between the central heating region and other part of the surface. Thus, the surface heat flux  $H_{sfc}(x)$  takes approximately the form of a square wave without sharp corners, and models the effect of urban-rural heat flux variation. This article summarizes the sensitivity of surface heat flux on penetrative turbulence for 6 values of  $H_0$ . A purpose of these simulations is to understand the sensitivity of high-amplitude surface heat-flux heterogeneity for temporal oscillation in mesoscale atmospheric circulations.

### Validation

The total heat flux is composed of the turbulent  $(\overline{w'\theta'})$  and the viscous  $(\alpha \frac{\partial \theta}{\partial z})$  components, where the turbulent component vanishes on a flat surface. The dimensionless surface heat flux takes the form  $\frac{1}{\sqrt{RaPr}} \frac{\partial \theta}{\partial z} \Big|_{z=0}$ , where the potential temperature is equal to the surface temperature. A scale analysis helps to compare the present model with that presented by Dubois and Touzani (2009) [7]. With a fully developed turbulence in the convective boundary layer, the vertical temperature profile satisfies  $\frac{\partial \theta}{\partial z} = 0$  in the mixing region ( $z > 0$ ) [6]. In this case, natural convection heat transfer is characterized by the Rayleigh number  $Ra = \frac{gH^3}{\theta_0 \nu \kappa} \frac{H_0 H}{\alpha}$ , where  $H$  is a vertical length scale,  $1/\theta_0$  is the coefficient of thermal expansion (1/K),  $\nu$  is the kinematic viscosity ( $m^2/s$ ),  $\kappa$  is thermal conductivity (W/m·K),  $g$  is the acceleration due to gravity ( $m/s^2$ ), and  $\alpha$  is the thermal diffusivity ( $m^2/s$ ). In an LES,  $\nu$  and  $\alpha$  can be replaced with  $K_m$  and  $K_h$ , respectively. So, we set to match the adiabatic lapse rate ( $10^\circ K/km$ ) of  $\frac{H_0 H}{\alpha} = 10^\circ K$  the atmosphere so that  $\frac{\partial \theta}{\partial z} = 0$ . As a result, an increase of  $H_0$  by a factor of 2 increases  $Ra$  by a factor of 10. Thus, the values of  $H_0 = 57.87 Wm^{-2}$  and  $115.74 Wm^{-2}$  represents  $Ra = 10^4$  and  $10^5$ , respectively. The comparison is summarized in Table 1, minimum potential temperature ( $\theta_{min}$ ), maximum horizontal velocity ( $u_{max}$ ), maximum vertical velocity ( $w_{max}$ ), and the Nusselt number has been reported. The discrepancy between the present result and that of Dubois and Touzani (2009) [7] is reasonable because our equations and numerical method both are different than that of Dubois and Touzani (2009) [7]. In addition, we have compared the vertical profile of mean temperature with that obtained from the Wangara day 33 experiment, which shows a good qualitative agreement on satisfying  $\frac{\partial \theta}{\partial z} = 0$  in the mixed layer (the plot is not shown to save space).

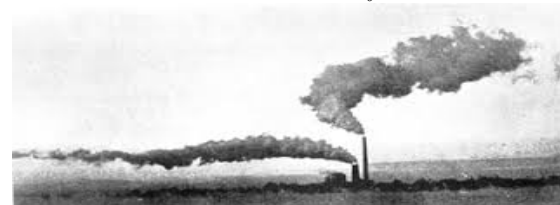
**Table-1.** Comparison of dimensionless extreme values with that from [7].

$H_0$	$57.87 Wm^{-2}$		$115.74 Wm^{-2}$	
	Present	D & T	Present	D & T
$\theta_{min}$	-0.0644	-0.0712	-0.1672	-0.1663
$u_{max}$	0.1761	0.1748	0.1796	0.1790
$w_{max}$	0.2275	0.2282	0.3294	0.3224
$Nu$	0.3262	0.2951	0.6892	0.6435

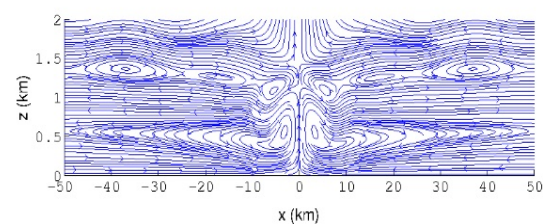
## RESULT

### Surface induced impact of air pollution

Figure-1(a) demonstrates an example of surface induced impact on air pollution. The turbulent plume from the lower chimney (75 m tall) moves toward the region of warmer surface (city area), and that from the higher chimney (150 m tall) moves toward the region of cooler surface (rural area). The atmosphere over the warmer surface has a decreased thermal stability, which causes a low level converging flow. Thus, the air parcels move horizontally toward the center of the heated region, where they rise upward. The observed behavior of plume dispersion in an urban-rural region – as shown in Figure-1(a) has been simulated with the present model – as shown in Figure-1(b), where streamlines are employed to demonstrate the flow pattern. The comparison in Figure-1 explains the method we have developed to model penetrative turbulence in the context of differential heating on the surface. The streamline plot in Figure 1(b) shows that air moves to the left in the lower portion of the boundary layer above  $x > 0$ , and the air in the upper portion of the boundary layer above  $x > 0$  moves to the right. The circulation is symmetric with respect to  $x=0$ . This flow pattern is computed at  $t = 6.5$  h from the simulation with a surface heat flux  $H_0 = 231.48 W m^{-2}$ .



(a)



(b)

**Figure-1.** (a) Movement of plumes from two chimneys of unequal height as a result of differential surface heating.

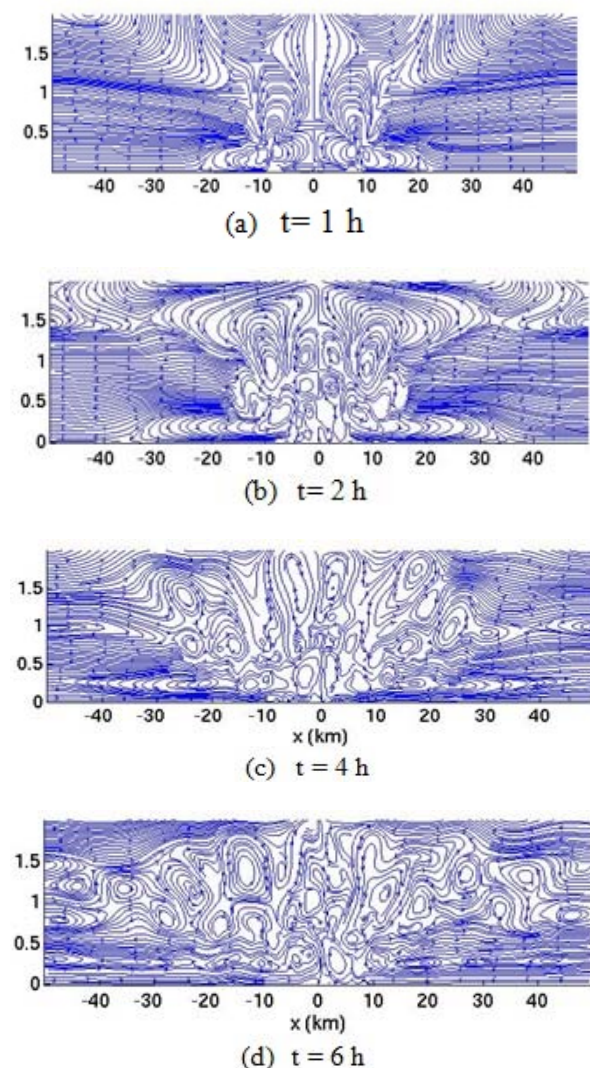
The figure is adapted from Google image database, and represents a pattern of expected circulation. (b) Low level converging flow and high level diverging flow is seen from this streamline at  $t = 6.5$  h for  $H_0 = 115.74 W m^{-2}$ .





### Turbulent mixed layer formation

To understand the mixing and turbulent transport in the absence of shear or mechanically driven turbulence, we have initialized the flow with a stable stratification and no background wind everywhere except an unstable stratification is introduced near the surface. The surface Richardson number  $Ri_0 = \left( \frac{g}{\theta_0} \frac{\partial \theta}{\partial z} \Big|_{z=0} \right) H^2 / U^2$  controls the initial strength of unstable stratification. The flow is initialized with  $Ri_0 = -1$ , and allowed to evolve naturally with time. Thus, the resulting circulation characterizes penetrative turbulent convection [6]. To demonstrate the span-wise vorticity pattern, the time evolution of the streamlines in the entire domain is shown in Figure-2.



**Figure-2.** The time evolution of span-wise vorticity pattern is demonstrated using the streamline at  $t = 1$  h,  $t = 2$  h,  $t = 4$  h, and  $t = 6$  h for the simulation with  $H_0 = 925.92 \text{ W m}^{-2}$ .

For the city area  $-10 \leq x \leq 10$ , let us consider the relative surface heat flux  $H_0 = 925.92 \text{ W m}^{-2}$  ( $\sim 0.75 \text{ K m}$

$\text{s}^{-1}$ ), where the background potential temperature  $\bar{\theta}(z)$  in the stable region has a gradient  $10^\circ/1 \text{ km}$ , and the buoyancy frequency is  $N \approx 10^{-2} \text{ s}^{-1}$ . As seen from Figure-2, when the sun heats the surface, eddies begin to form and rise upward. However, they return back downward due to stable stratification in the upper atmosphere. This generates an unstable mixing layer that is adjacent to the surface. This turbulent mixed layer underlies a stable region aloft. Based on the mean vertical profile of potential temperature (plot is not shown to save space), the depth of the mixed layer for this simulation is approximately 800 m. This value is consistent with typical mixed layer height [17].

When an eddy loses buoyancy, it rises upward, and a negative horizontal buoyancy gradient occurs near its left edge, which generates a cyclonic circulation; similarly, anticyclonic circulation is formed on the other edge of the eddy [1, 8]. The pattern of such a circulation and the time evolution of the associated span-wise vorticity, during a penetrative turbulent convection, is realized from the streamlines in Figure 2. This vortical pattern play its role as a heat transfer agent, thereby causing an imbalance between the buoyancy force and gravitational force. An important question of meteorological interest is whether the process leads to horizontal turbulent fluctuation, and if such fluctuation affects local weather pattern. At  $t = 1$  h, cyclonic/anticyclonic eddies have been formed near the outer edges of the heating region, and have reached a height of about 500 m. Later, horizontal convection is observed; i.e. turbulent eddies move horizontally, where turbulent eddies reach a maximum vertical height of about 800 m. Entrainment/detrainment associated with such movements of energy containing eddies is a primary mechanism for turbulent energy cascade [11].

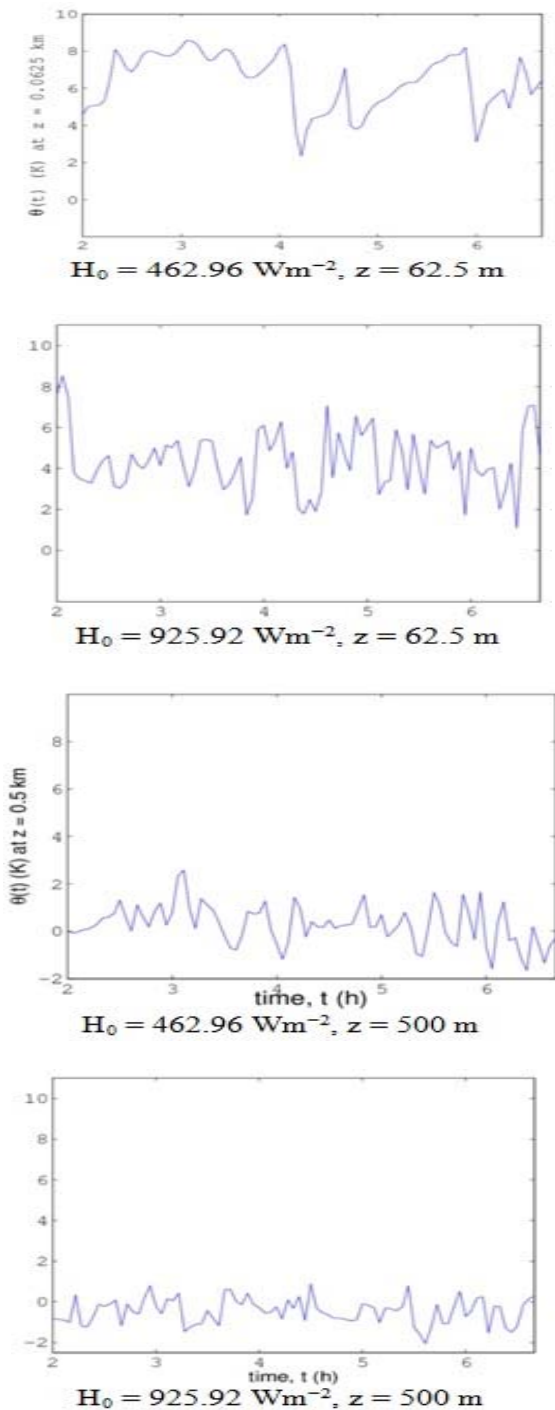
### Temporal oscillation and downscale energy cascade

According to the Taylor's hypothesis (see [11]), if turbulent statistics is approximately stationary and homogeneous, then the turbulent field is advected over the time scales of interest. Under this hypothesis, time series of potential temperature and horizontal velocity, as shown in Figure-3, indicate that surface heterogeneity on a scale  $O(20 \text{ km})$  contributes toward downscale energy cascade. The growth of the horizontal potential temperature gradient develops a horizontal pressure gradient, which in turn generates horizontal wind. The time series of  $\theta$  and  $u$  at several vertical locations have been analyzed to characterize penetrative turbulence.

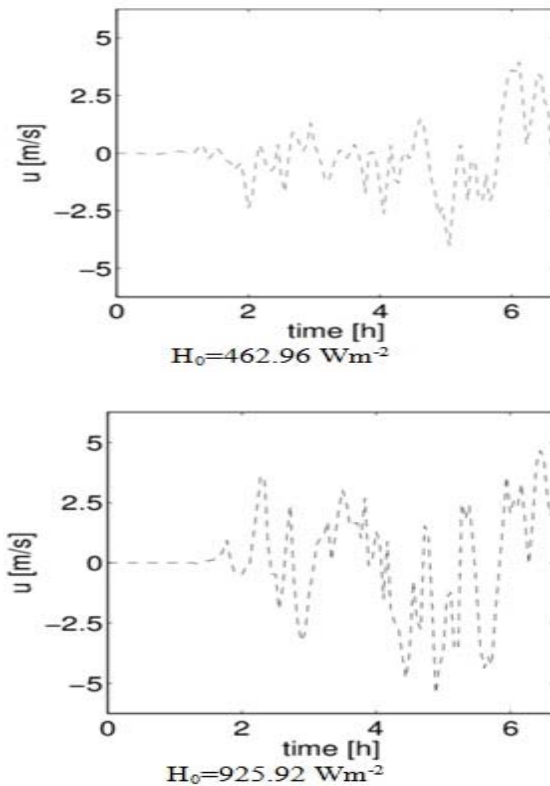
The sensitivity of surface heat flux on the temporal fluctuation of  $\theta$  is clear from Figure-3. At  $z = 62.5 \text{ m}$ , frequency of oscillation increased at  $H_0 = 925.92 \text{ W m}^{-2}$  compared to  $H_0 = 462.96 \text{ W m}^{-2}$ . However, at  $z = 500 \text{ m}$ , turbulence is seen fully developed in both cases. This indicates that turbulent kinetic energy cascades downscale as the energy is transported by internal waves. To verify that air flow over the simulated city is characterized by a horizontal flow of numerous rotating eddies, we present the horizontal velocity  $u$ , which is the



wind component that is parallel to the direction of the surface heat flux variation. The onset of turbulent fluctuations is compared between two surface fluxes in Figure-4.



**Figure-3.** Time series plots of the potential temperature  $\theta(0, z, t)$ ; top row – near the surface ( $z = 62.5$  m) and bottom row – near the top of the mixing layer ( $z = 500$  m).



**Figure-4.** The onset of turbulent fluctuation is shown through time series of horizontal velocity at  $z = 100$  m from the surface.

### CONCLUDING REMARKS

In this article, we report some aspects of penetrative turbulence in an idealized daytime boundary layer over a city that is surrounded by rural areas. As internal waves excite from the interface between the mixed layer and stable layer, kinetic energy of boundary layer turbulence is transferred to upper label free atmosphere. A complete understanding of this mechanism remains a challenging future research topic. However, in the present article, we have shown that energy cascades downscale as turbulence penetrates upward. Our simulations demonstrate that characteristics of the induced horizontal flow may be significantly different depending on the surface heterogeneity from the perspective of the turbulence that is generated.

The present simulation is idealized. However, we have studied characteristics of horizontal flow with surface heat-flux variation on the scale of 20 km, which is the typical grid spacing in numerical weather prediction models. One of our objectives is to understand if thermally induced mesoscale perturbation has a connection to penetrative turbulence. In other words, we show that differential heating on the same label leads to a sustained turbulent flow. Note that we have compromised the three-dimensional simulation with a two-dimensional one in order to capture the energy containing large eddies. These results suggest that surface heterogeneity induced differential heating cause sub-grid scale turbulent



fluctuations, and the impact of surface heterogeneity could be more substantial on meso-scale atmospheric flows. In future studies, we plan to continue fully three-dimensional LES of penetrative turbulence. We plan to investigate how temporal oscillations characterize turbulent energy cascade, particularly with background wind conditions and rough surfaces.

## REFERENCES

- [1] Alam J. 2011. Towards a multi-scale approach for computational atmospheric modelling. *Monthly Weather Review*. 139(12).
- [2] Alam J. M., Walsh R. P., Alamgir Hossain M., and Rose A. M. 2014. A computational methodology for two-dimensional fluid flows. *International Journal for Numerical Methods in Fluids*. 75(12):835–859.
- [3] Bryan G. H. and Fritsch J. M. 2002. A benchmark simulation for moist nonhydrostatic numerical model. *Mon. Wea. Rev.* 130.
- [4] Deardorff J. 1980. Stratocumulus-capped mixed layers derived from a three-dimensional model. *Boundary-Layer Meteorology*. 18(4):495–527.
- [5] Deardorff J. W. 1970. A three-dimensional numerical investigation of idealized planetary boundary layer. *Geophys. Fluid Dyn.* 1:377–410.
- [6] Deardorff J. W., Willis G. E. and Lilly D. K. (1969). Laboratory investigation of non-steady penetrative convection. *Journal of Fluid Mechanics*. 35.
- [7] Dubois T. and Touzani R. 2009. A numerical study of heat island flows: Stationary solutions. *International Journal for Numerical Methods in Fluids*. 59(6):631–655.
- [8] Lane T. P. 2008. The vortical response to penetrative convection and the associated gravity-wave generation. *Atmos. Sci. Lett.* 9:103–110.
- [9] Lin Y. L. 2007. *Mesoscale Dynamics*. Cambridge University Press.
- [10] Paparella F. and Young W. R. 2002. Horizontal convection is non-turbulent. *Journal of Fluid Mechanics*. 466:205–214.
- [11] Pope S. B. 2000. *Turbulent Flows*. Cambridge University Press.
- [12] Sandström J. W. 1908. Dynamische versuche mit meerwasser. *Annalen der Hydrographie und Maritimen Meteorologie*. 36:6–23.
- [13] Scotti A. and White B. 2011. Is horizontal convection really non-turbulent. *Geophysical Research Letters*. 38(21).
- [14] Skamarock W. C. and Klemp J. B. 2008. A time-split nonhydrostatic atmospheric model for weather research and forecasting applications. *J. Comput. Phys.* 227(7):3465–3485.
- [15] Smolarkiewicz P. K., Kühnlein C. and Wedi N. P. 2014. A consistent framework for discrete integrations of soundproof and compressible pdes of atmospheric dynamics. *J. Comput. Phys.* 263:185–205.
- [16] Stull R. B. 1976. Internal gravity waves generated by penetrative convection. *J. of the Atmospheric Sciences*. pp. 1279–1286.
- [17] Williams P. D. and Joshi M. M. 2013. Intensification of winter transatlantic aviation turbulence in response to climate change. *Nature Climate Change*. 3:644–648.
- [18] Zhang N., Wang X. and Peng Z. 2014. Large eddy simulation of mesoscale circulations forced by inhomogeneous Urban Heat Island. *Boundary Layer Meteorology*. 151: 179-194.