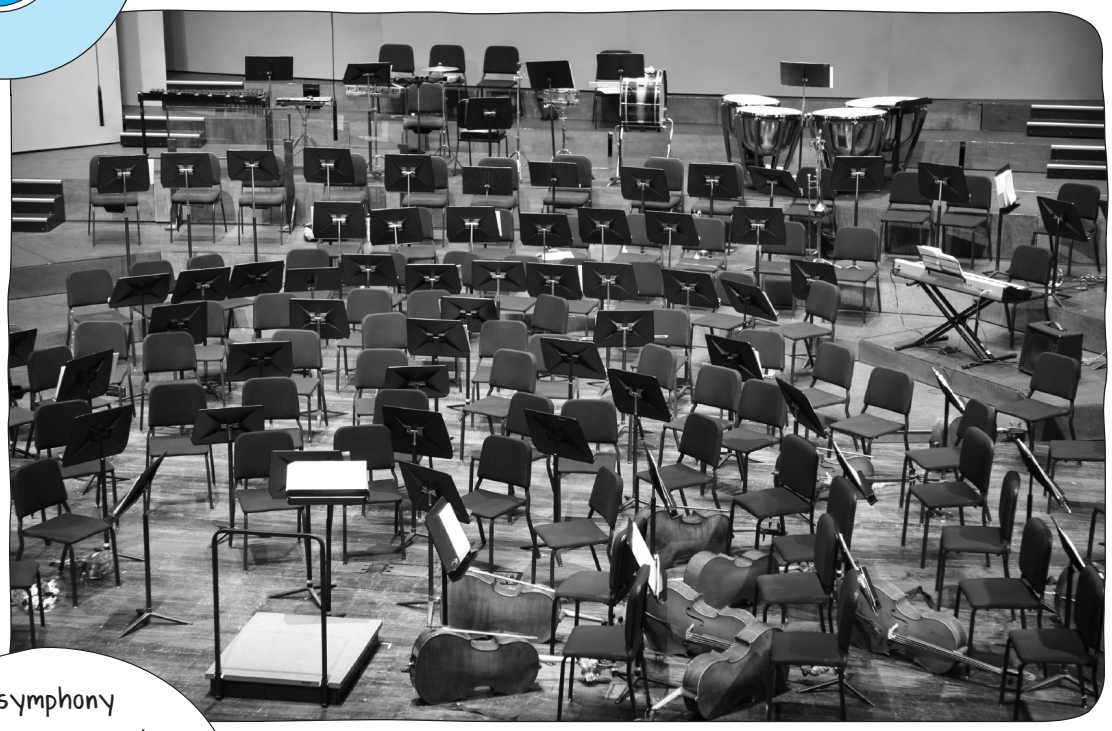
6 PERCENTS



A symphony orchestra can create a wide range of musical sounds. According to the Guinness Book of World Records, the largest orchestral work ever written is Havergal Brian's Symphony #1, which calls for 190 instruments (including a thunder machine and some chains) and a choir of 700 voices!

6.1 PERCENTS CAN MAKE OR BREAK YOU!

Introduction to Percents

Learning Goals

In this lesson, you will:

- Write fractions, decimals, and percents.
- Model percents on a hundredths grid.
- Explain the similarities and differences of percents, fractions, and decimals.

Key Term

- percent

What are the latest numbers in the polls? The word poll actually has several meanings. It can be used as another word for *survey*, but *poll* usually refers to the survey used to find out the opinions of voters during an election process. In almost any election, candidates and candidate advisors constantly monitor polls to see what the voters' opinions are about that candidate. What other types of polls have you seen? Where have you seen polls displayed?

Problem 1 They're All Part of the Same Family

What do these statements mean to you?

- There is an 80 percent chance of rain tomorrow.
- You earn 90 percent on a science test.
- Big Sale! 25 percent discount on all regular-priced items.
- Your bill at the Eat and Talk Restaurant is $40. Below the total, the restaurant adds a 20 percent tip.
- The star of the high school basketball team makes 80 percent of his free throws.
- Sales tax is 7 percent in Richmond County.
- Yuma, Arizona, has sunny days 90 percent of the time.

A **percent** is a fraction in which the denominator is 100. Percent is another name for hundredths. The percent symbol "%" means "out of 100." Therefore:

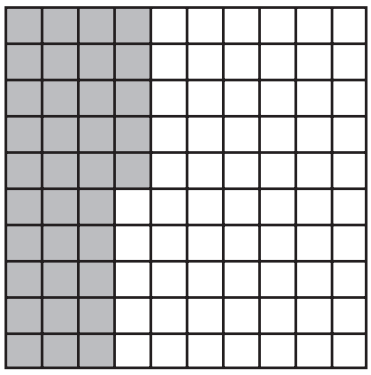
35% means 35 out of 100.

35% as a fraction is $\frac{35}{100}$.

35% as a decimal is 0.35.

35% as a ratio is 35 to 100, or 35 : 100.

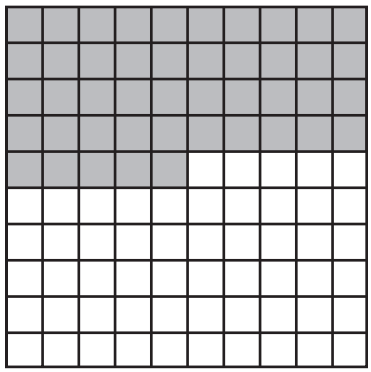
You can shade 35 of the 100 squares on the hundredths grid to represent 35%.



Percents, fractions, and decimals can be used interchangeably.

1. Each hundredths grid represents a whole. Write the shaded part as a fraction, decimal, and percent.

a.



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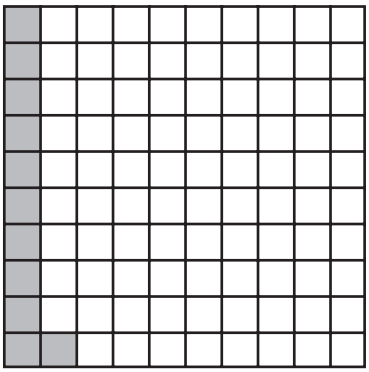

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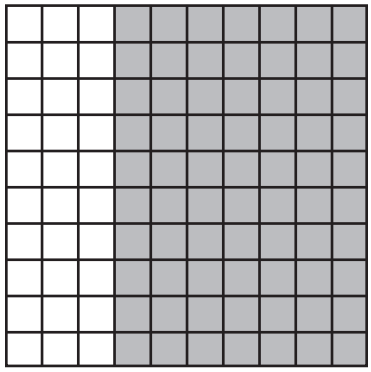

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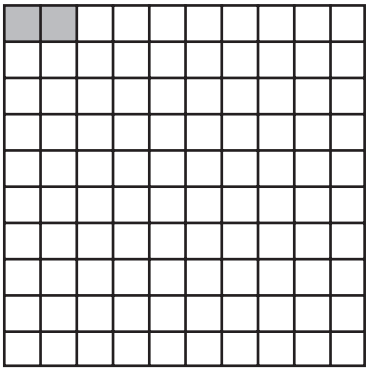

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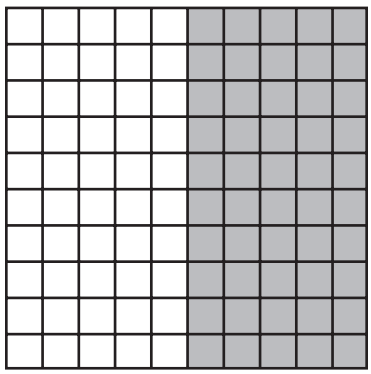

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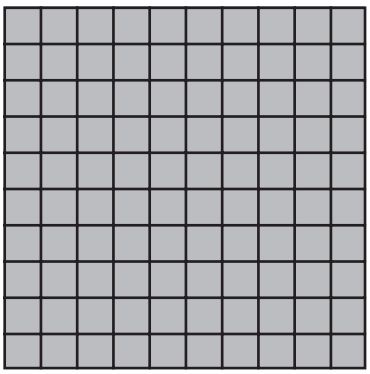
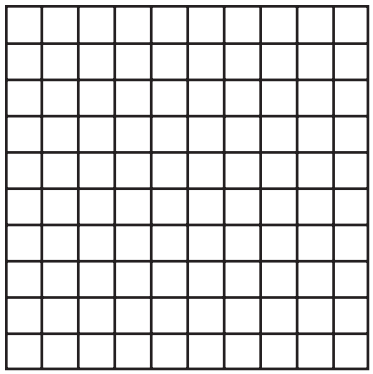
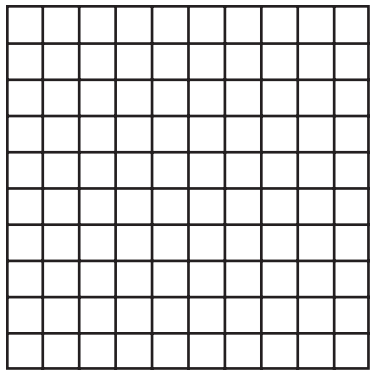


2. Shade the hundredths grids to represent each percent shown. Then, write the equivalent fraction and decimal for each percent.

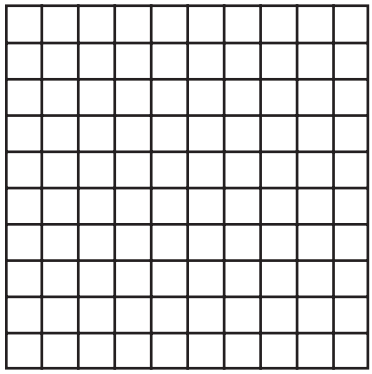
a. 44%



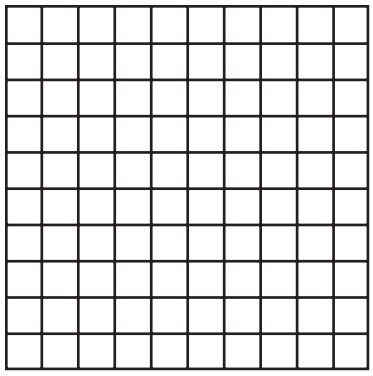
b. 16%

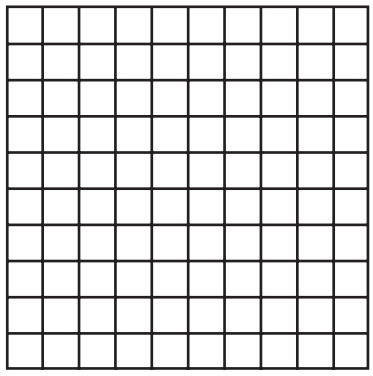


c. 97%



d. 117%





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3. Look at the percents and the decimals you wrote for Question 2 to determine a pattern. Use this pattern to describe how you can write any percent as a decimal.

4. Write each percent as a decimal.

a. 12%

b. 3%

c. 80%

d. 125%

5. Write each decimal as a percent.

a. 0.4

b. 0.37

c. 0.7381

d. 0.52

When the denominator is a factor of 100, scale up the fraction to write it as a percent. When the denominator is not a factor of 100, you can divide the numerator by the denominator to write the fraction as a decimal, which you can then write as a percent.

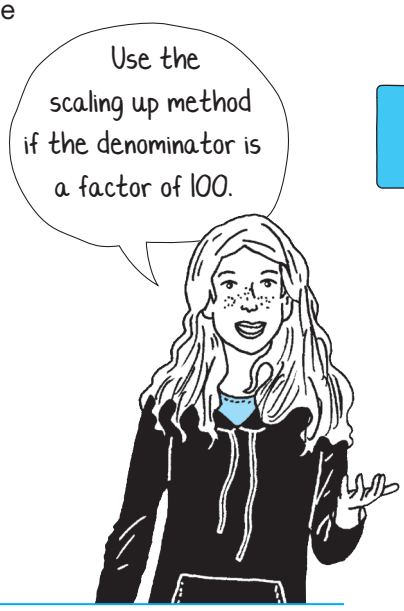
6. Write each fraction as a percent. Round your answer to the nearest tenth.

a. $\frac{4}{5}$

b. $\frac{3}{10}$

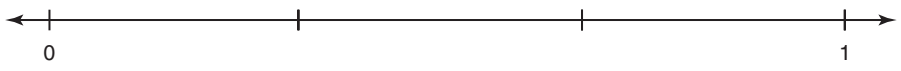
c. $\frac{3}{8}$

d. $\frac{1}{3}$



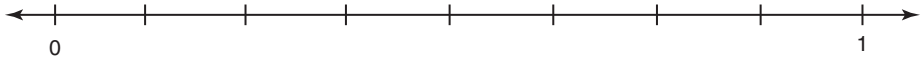
7. Label each mark on the number line with a fraction, decimal, and percent. Make sure your fractions are in simplest form.

a.



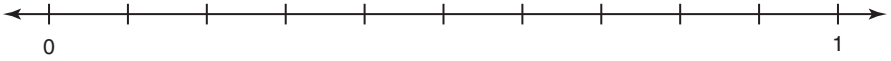
Fraction	0	$\frac{1}{3}$	____	1
Decimal	0.0	____	$0.6\overline{6}$	1.0
Percent	0%	____	____	100%

b.



Fraction	0	____	____	____	$\frac{1}{2}$	____	____	$\frac{7}{8}$	1
Decimal	0.0	0.125	____	____	____	0.625	0.75	____	1.0
Percent	0%	____	25%	37.5%	____	____	____	____	100%

c.



Fraction	0	$\frac{1}{10}$	____	____	$\frac{2}{5}$	____	____	$\frac{7}{10}$	____	____	1
Decimal	0.0	____	0.2	____	____	0.5	____	____	____	0.9	1.0
Percent	0%	____	____	30%	____	____	60%	____	80%	____	100%

Problem 2 Survey Says . . .

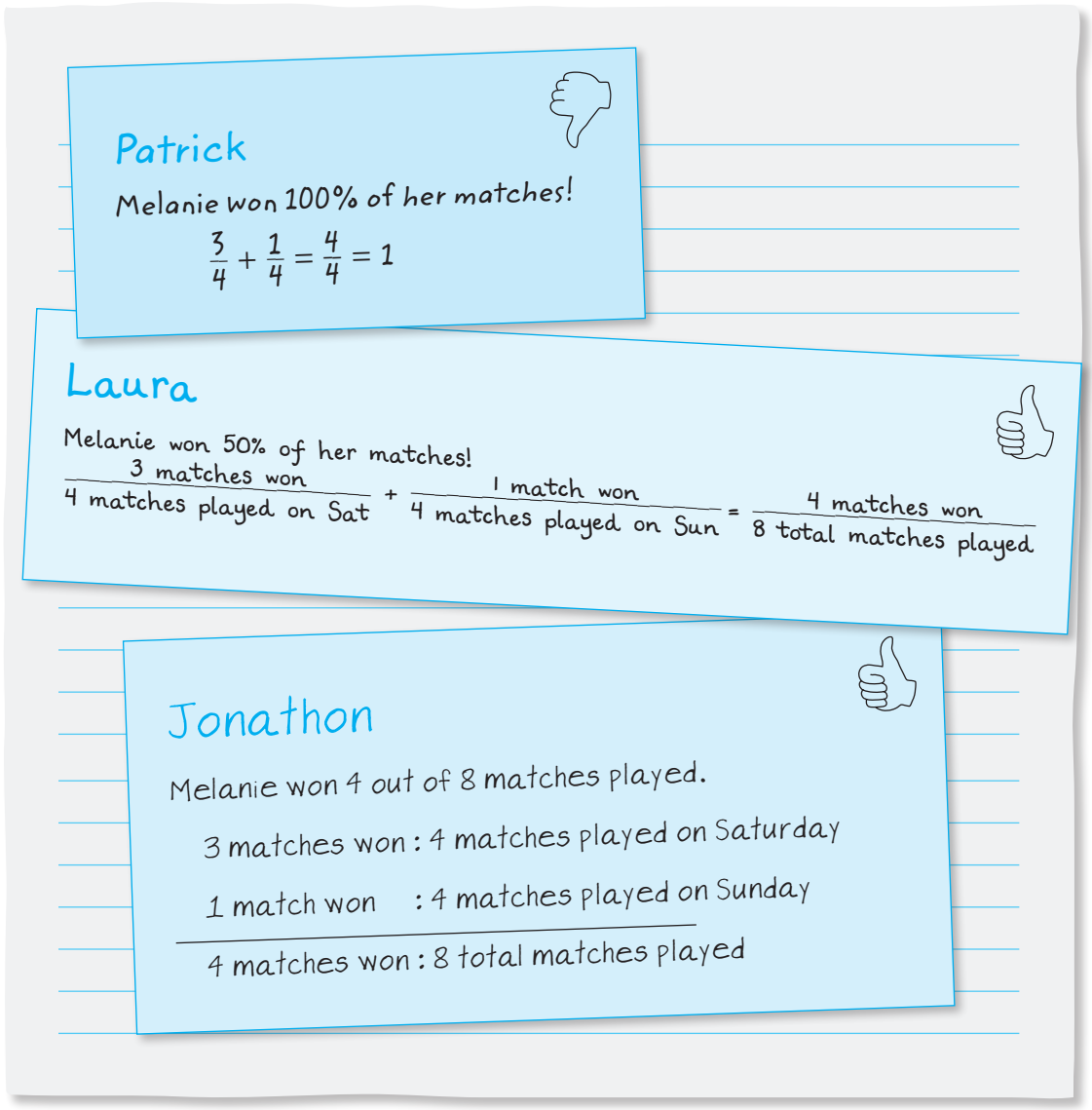
1. One hundred sixth-grade students took a survey that asked questions about an upcoming class trip.

   a. Complete the table to represent each survey result as a ratio using colon notation, as a fraction, as a decimal, and as a percent. Make sure your fractions and ratios are in simplest form.

	Ratio	Fraction	Decimal	Percent
How many days should we plan for the trip?				
Stay overnight two nights 60 out of 100 students				
Stay overnight one night 25 out of 100 students				
No overnight stay 15 out of 100 students				
Where should we go?				
Philadelphia 35 out of 100 students				
Washington, D.C. 22 out of 100 students				
New York City 30 out of 100 students				
Atlanta 13 out of 100 students				
How should we get there?				
Bus 25 out of 100 students				
Airplane 75 out of 100 students				
Are you planning on going on the trip?				
Yes 100 out of 100 students				

b. Write a summary of the results of the survey using percents.

2. On Saturday, Melanie won 3 out of 4 of her tennis matches at the Redstone Tournament. On Sunday, she won 1 out of 4 of her matches at the Mesa Tennis Tournament.



Patrick

Melanie won 100% of her matches!

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

Laura

Melanie won 50% of her matches!

$$\frac{3 \text{ matches won}}{4 \text{ matches played on Sat}} + \frac{1 \text{ match won}}{4 \text{ matches played on Sun}} = \frac{4 \text{ matches won}}{8 \text{ total matches played}}$$

Jonathon

Melanie won 4 out of 8 matches played.

3 matches won : 4 matches played on Saturday

1 match won : 4 matches played on Sunday

4 matches won : 8 total matches played

6

a. What is wrong with Patrick's reasoning?

b. How did Laura make her reasoning explicit?

c. What is the same about Laura's and Jonathon's reasoning? What is different?

d. What ratio of matches did Melanie win?

Problem 3 Do You Think You Know Your Percents?

It's time to play The Percentage Match Game. In this game, you will use your knowledge of percents, fractions, and decimals, and your memory.

Rules of the Game:

- Cut out the cards shown.
- Deal all the cards face down in an array.
- The first player chooses any card in the array. That player then turns over another card to see if it is an equivalent match. If the two cards are an equivalent match, then the two matched cards are put into the player's pile. The first player then picks again and repeats the process until a match is not found.
- If the first player does not have an equivalent match, it is the second player's turn. The same process for picking and matching cards described is now followed by the second player.
- The game continues until all the cards have been paired with an equivalent match.
- Both players then count the number of equivalent matches each player has, and receive 5 points for each equivalent match. The player with the most points wins!

$\frac{3}{5}$	$\frac{3}{10}$	$\frac{6}{10}$	30%
0.6	$\frac{1}{3}$	60%	33%
$\frac{1}{8}$	$\frac{2}{6}$	12.5%	$0.\overline{3}$
$\frac{1}{10}$	$\frac{1}{2}$	1%	50%
0.1	$\frac{2}{3}$	10%	$66.6\overline{6}\%$
$\frac{1}{5}$	$\frac{3}{4}$	$\frac{2}{10}$	$\frac{6}{8}$
$\frac{1}{4}$	0.75	$\frac{2}{8}$	75%

6

Talk the Talk

Percents, fractions, and decimals can be used interchangeably. The chart shows some common equivalent fractions, decimals, and percents.

Common Equivalent Fractions, Decimals, and Percents									
Fraction	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
Decimal	0.2	0.25	$0.33\overline{3}$	0.4	0.5	0.6	$0.66\overline{6}$	0.75	0.8
Percent	20%	25%	$33\frac{1}{3}\%$	40%	50%	60%	$66\frac{2}{3}\%$	75%	80%

1. How are percents similar to decimals? How are percents and decimals different?

2. How are percents similar to fractions? How are percents and fractions different?

3. How are percents similar to ratios? How are percents and ratios different?

Be prepared to share your solutions and methods.

6.2 WACKY WEATHER!

Estimating Percents

Learning Goals

In this lesson, you will:

- Estimate percents as fractions and decimals.
- Write fractions as percents.
- Identify equivalent forms of fractions, decimals, and percents.
- Order fractions, decimals, and percents.

Key Term

- benchmark percents

What do you think the following statement means?

"Tomorrow, there will be a 30 percent chance of rain."

Although this statement seems simple enough, a study showed that its meaning can vary dramatically from person to person. There appears to be three different interpretations of the statement: (1) It will rain 30 percent of the time during the day; (2) only 30 percent of the forecasted area will have rain, while the remaining areas will be dry; or (3) there is a 30 in 100, or 3 in 10, chance that it will actually rain. What is common with all of these interpretations is that they are all estimates, but that is where the similarities stop.

So, what do you think "30 percent chance of rain" means?

Problem 1 How Much Juice Does It Have?

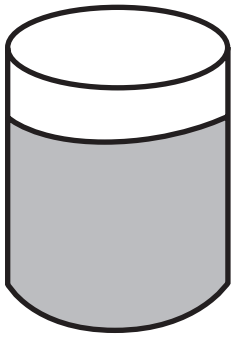
1. What does the saying, "I gave it 100 percent!" mean?

When you estimate with percents, it is easier to work with those you are familiar with. You know that 100% means one, or the whole, and 50% means half.

A laptop computer uses an icon of a battery on the toolbar to show how much power is left in the battery. When you glance at the icon, you can get a good estimate of how much battery life remains before you need to recharge the battery.

2. Estimate how much battery power remains by writing the percent under each battery icon.

a.



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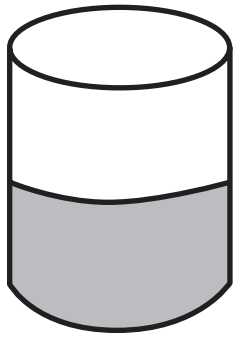

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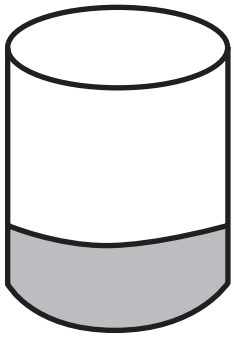

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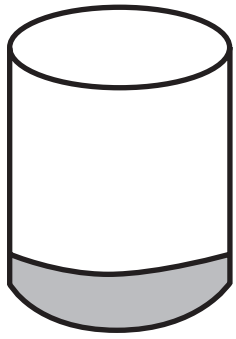

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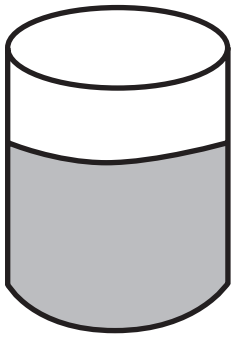

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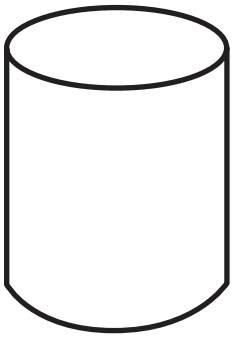
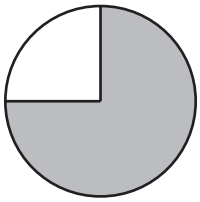


3. Estimate the shaded part of each circle shown, and write it as a percent.

a.

b.



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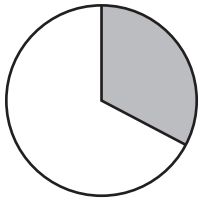

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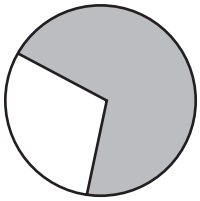

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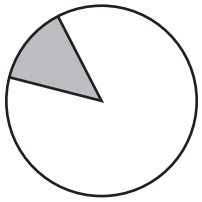

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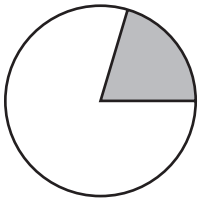
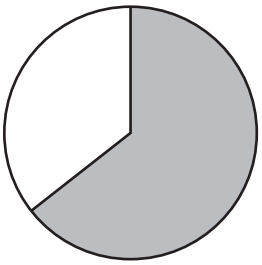
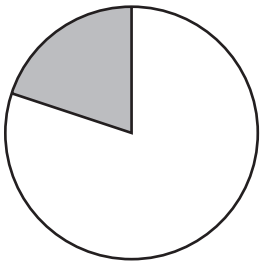


4. Estimate the shaded part of each model, and write it as a fraction, a decimal, and a percent. Make sure your fraction is in simplest form.

a.



b.



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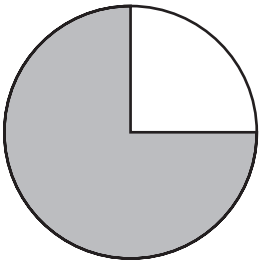

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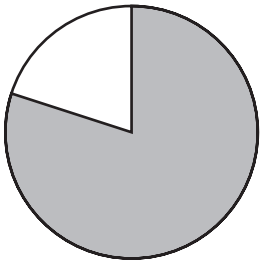

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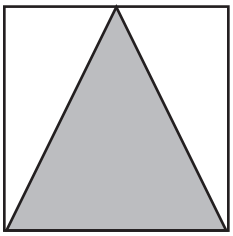

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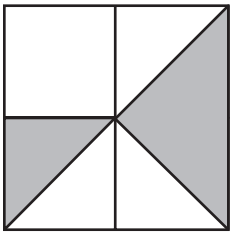


Problem 2 Benchmark Percents

1. Use your calculator to determine the percent of each number.

a. 1% of 28 =

b. 10% of 28 =

c. 1% of 234 =

d. 10% of 234 =

e. 1% of 0.85 =

f. 10% of 0.85 =

g. 1% of 5.86 =

h. 10% of 5.86 =

i. 1% of 98.72 =

j. 10% of 98.72 =

k. 1% of 1085.2 =

l. 10% of 1085.2 =

m. What patterns do you notice?

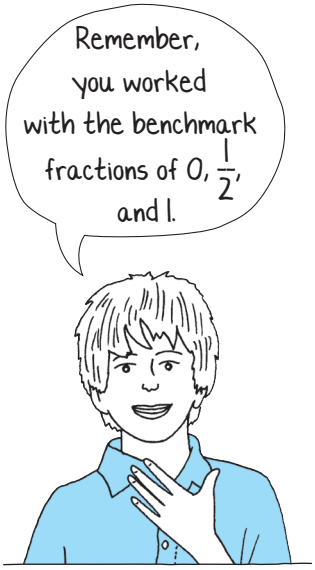
2. Write a rule to calculate 1% of any number.

3. Write a rule to calculate 10% of any number.

A **benchmark percent** is a percent that is commonly used, such as 1%, 5%, 10%, 25%, 50%, and 100%. You can use benchmark percents to calculate any whole percent of a number.

4. State each relationship.

a. How is 50% related to 100%?



b. How is 25% related to 100%? How is 25% related to 50%?

c. How is 10% related to 100%? How is 10% related to 50%?

d. How is 5% related to 10%?

e. How is 1% related to 10%? How is 1% related to 5%?

5. Try these percents mentally. Calculate the value of each using your knowledge of benchmark percents.

a. 100% of $300

b. 1% of $300

c. 50% of $300

d. 25% of $300

e. 10% of $300

f. 5% of $300

Akuro eats at the Eat and Talk Restaurant and decides to leave a 15% tip. Akuro says, "I can easily calculate 10% of any number, and then calculate half of that, which is equal to 5%. I can then add those two percent values together to get a sum of 15%."

6. Do you think Akuro's method is reasonable? How much should she leave for a tip of 15% on $16.00?

7. What is 15% of each restaurant check total? Explain how you calculated your answer. Round to the nearest hundredth if necessary.

a. $24.00

b. $32.56

c. $47.00

You can determine any whole percent of a number in your head by using 10%, 5%, and 1%.

8. How can you use 10%, 5%, and/or 1% to determine each percent given? Explain your reasoning.

a. How can you calculate 18% of a number?

b. How can you calculate 25% of a number?

c. How can you calculate 37% of a number?

6

9. Estimate each using 1%, 5%, and 10%.

a. 27% of 84

b. 43% of 116

c. 98% of 389

d. 77% of 1400

e. 12% of 1248

10. About 12% of the United States population is left-handed. Estimate how many left-handed students are in a class of:

a. 100 students.

b. 200 students.

c. 150 students.

Problem 3 Ordering Fractions, Decimals, and Percents

1. Order these numbers from greatest to least using what you have learned about fractions, decimals, and percents. Cut out the cards to help you order the numbers.

$33\frac{1}{3}\%$	$\frac{1}{4}$	$\frac{13}{50}$	78%
0.0666…	0.1%	$\frac{3}{4}$	$\frac{50}{75}$
0.098	0.51	$\frac{3}{5}$	80%
0.98	1.0	27%	$\frac{198}{200}$

Talk the Talk

The rules for ways to calculate common equivalent fractions, decimals, and percents are shown.

	Fraction	Decimal	Percent
Percent	Write the percent as a fraction with a denominator of 100. Simplify. $28\% = \frac{28}{100} = \frac{7}{25}$	Write the percent as a fraction with a denominator of 100. Write the fraction as a decimal. $42\% = \frac{42}{100} = 0.42$ OR Move the decimal point two places to the left and remove the percent sign. $38\% = 0.38$	
Fraction		Write the fraction as an equivalent fraction with a denominator of 10, 100, 1000 . . . Then, write it as a decimal. $\frac{7}{20} = \frac{35}{100} = 0.35$ OR Divide the numerator by the denominator. $\frac{2}{9} = 2 \div 9 = 0.2\overline{2}\ldots$	Write an equivalent fraction with a denominator of 100. Write the fraction as a percent. $\frac{3}{5} = \frac{60}{100} = 60\%$ OR Use division to write the fraction as a decimal, and then a percent. $\frac{5}{8} = 5 \div 8 = 0.625 = 62.5\%$
Decimal	Write the decimal as a fraction with a denominator of 10, 100, 1000 . . . Simplify. $0.28 = \frac{28}{100} = \frac{7}{25}$		Write the decimal as a fraction. Then, write the fraction as a percent. $0.08 = \frac{8}{100} = 8\%$ OR Write the decimal as a fraction. Then, write the equivalent fraction with a denominator of 100. Then, write the fraction as a percent. $0.4 = \frac{4}{10} = \frac{40}{100} = 40\%$ OR Move the decimal point two places to the right and add the % sign. $0.08 = 8\%$

1. Complete the table. Write each as a fraction, decimal, and percent.

Fraction	Decimal	Percent
		3%
	1.5	
$\frac{13}{20}$		
$\frac{2}{3}$		

Be prepared to share your solutions and methods.

6.3 IT'S ALL IN THE FOLLOW-THROUGH

Determining the Percent of a Number

Learning Goals

In this lesson, you will:

- Determine the percent of a number.
- Use double number lines.

Did you know that a professional basketball player can shoot thousands of shots over his or her career? Think about it! Basketball players routinely shoot thousands upon thousands of shots during practices, scrimmages, drills, and the actual games. In fact, one basketball player shot 28,307 shots during career games, making 15,837 of them. This doesn't even include the shots made during practices! How would you calculate this player's shots-made percentage?

Problem 1 There's More than One Way . . .

Mr. Goodwin, the sixth grade math teacher, asked the class to determine 25% of 44. Five different student responses are shown.

Kendra

"I used the multiplication by a fraction method.

$25\% = \frac{25}{100} = \frac{1}{4}$.

Then, I multiply $\frac{1}{4} \cdot 44 = 11$."

Hank

"I like decimals much better than fractions."

$25\% = 0.25$

$0.25 \cdot 44 = 11$

Ryan

"25% is easy to do in my head.

50% of 44 is 22.

25% is $\frac{1}{2}$ of 50%, so 25% of 44 is $\frac{1}{2}$ of 22, which is equal to 11."

Simon

"Since 25% is the same as $\frac{1}{4}$, I just divided by four."

$44 \div 4 = 11$

Pamela

"I prefer to use the benchmarks of 10% and 5%.

10% of 44 = 4.4.

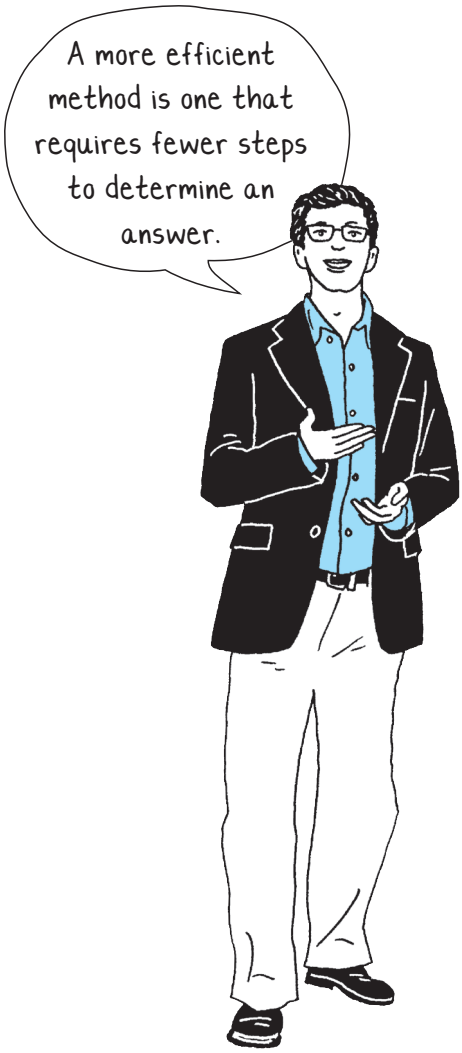
20% is $2 \cdot 10\% = 2 \cdot 4.4 = 8.8$.

5% is half of 10% = 2.2.

Therefore, $20\% + 5\% = 8.8 + 2.2 = 11.0$"

1. Discuss each student method used.

a. When is Kendra's method most efficient to use?



b. When is Ryan's method most efficient to use?

c. When is Simon's method most efficient to use?

d. When is Pamela's method most efficient to use?

2. Which method(s) can be used in any situation?

Ellen said, "All the methods are correct, and everyone got the correct answer, but what if Mr. Goodwin gave us the problem 32% of 732?"

- Kendra said, "My fraction method is not as easy this time."

$$\frac{\cancel{32}^{8}}{\cancel{100}_{25}} \cdot \frac{732}{1} = \frac{5856}{25} = 234.24$$

- Hank said,
  "32% = 0.32
  0.32 · 732 = 234.24
  My method is not that different."
- Ryan said, "I can still estimate, but my answer will be close, not exact.
  32% is close to $\frac{1}{3}$ and $\frac{1}{3}$ of 732 is 244."
- Simon said, "I don't have an easy fraction to use for 32%, so my method only works for certain percents."
- Pamela said, "I can still use my method."
  32% = 10% + 10% + 10% + 1% + 1%
  10% of 732 = 73.2
  73.2 · 3 = 219.6
  1% of 732 = 7.32
  7.32 · 2 = 14.64
  219.6 + 14.64 = 234.24

3. Which method do you prefer with this problem? Discuss with your partner and explain your thinking.

6

4. Determine the percent for each by using your preferred method.

a. 7% of 80

b. 15% of 55

c. 12% of 320

d. 8% of 300

e. 75% of 240

f. 37% of 120

g. 60% of 232

h. 150% of 27

i. 12.5% of 64

5. You are the coach of the Gators' basketball team. The Gators are playing the Crocs. The game is tied 64 to 64 with time running out. Just before the buzzer sounds, the Crocs' coach illegally steps out onto the floor while play is taking place. This results in a penalty—and a free throw shot for the Gators! You have to pick a Gators' player to shoot the free throw. If the player makes the shot, the Gators win!

You can choose one of four players to shoot the free throws. During the past few games,

- Natalie made 17 out of 25 free throws.
- Angela made 15 out of 20 free throws.
- Casey made 7 out of 10 free throws.
- Erin made 37 out of 50 free throws.

You want to select the player who is most likely to make the shot. Which player should you choose? Use mathematical reasoning and the data on the players' performance in the last few games to explain your choice.

Problem 2 Giving Back to the Community

This school year, students are asked to volunteer for a service project to help in the community. The results of the questionnaire that was given to the sixth-grade students are shown in the table. Three hundred students returned the questionnaire, with each student choosing one service project.

Working at the . . .	Boys	Girls
Food Bank	42	48
Soup Kitchen	40	48
Senior Center	28	94

1. Explain each statement mathematically using your knowledge of fractions, decimals, and percents.

a. 30% of the students prefer working at the Food Bank.

b. About 30% of the students prefer working at the Soup Kitchen.

c. About 40% of the students prefer working at the Senior Center.

d. About 50% of the girls prefer working at the Senior Center.

e. About 25% of the boys prefer working at the Senior Center, and about 25% of the girls prefer working at the Soup Kitchen.

f. The boys prefer working at the Food Bank over the girls.

g. The boys prefer working at the Soup Kitchen over the girls.

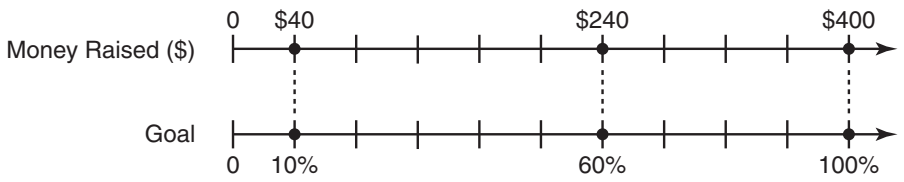
h. The ratio of boys who prefer *not* working at the Soup Kitchen to total boys is 7 to 11.

Problem 3 Percent Problems

Karla is in charge of designing a way to keep a running total of the money raised by her homeroom for the Food Bank project. As of today, her homeroom has raised $240, which is 60% of their goal.

Karla decided to use a double number line to record the money raised and the percent of the goal raised.

A double number line is a graph that has two number lines with the same intervals indicated with tick marks. Each number line represents different data value. However, the data values are related to each other.



In the graph shown, the bottom number line represents the percent of the homeroom goal. The top number line represents the amount of money raised.

Karla's homeroom has raised $240, which is 60% of the goal.

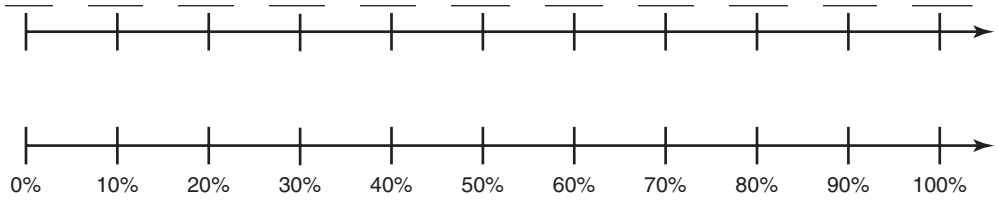
1. Which way of reporting is more informative: the amount of money raised, or the percent of money raised? Explain your thinking.

6

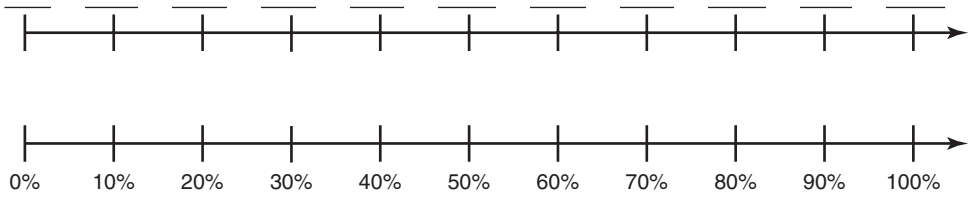
2. Create double number lines to represent the goals of the other sixth-grade homerooms using the information from the table. Write the equivalent dollar amount for each percent on the double numbers lines.

Homeroom	Goal (dollars)
6B	240
6C	360
6D	480
6E	120
6F	280

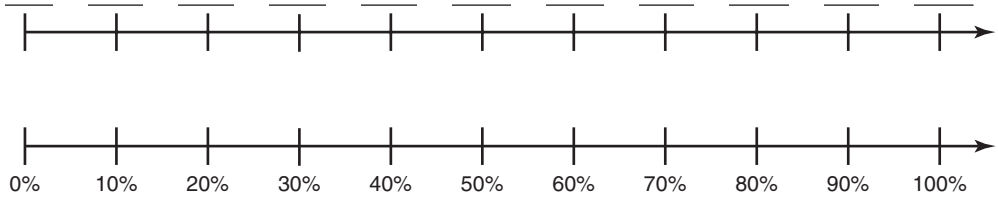
a. Homeroom 6B



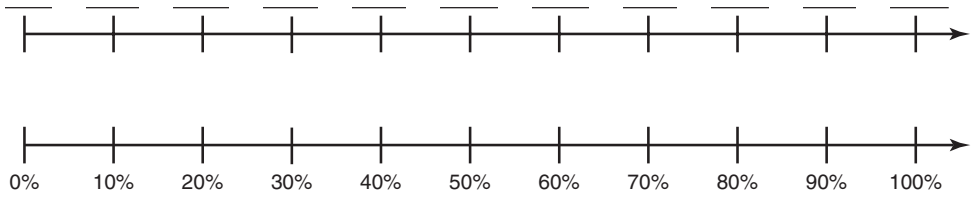
b. Homeroom 6C



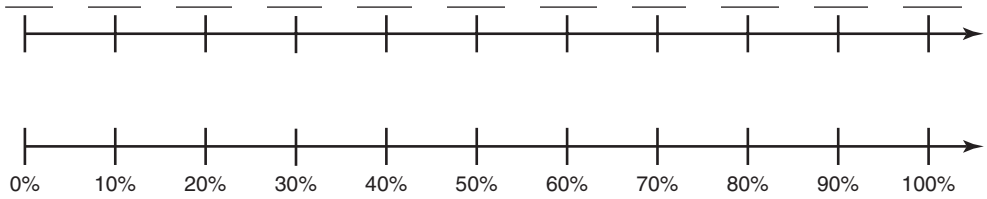
c. Homeroom 6D



d. Homeroom 6E



e. Homeroom 6F



Problem 4 Wow! What a Bargain!

Everyone loves a bargain. When you have a lot of shopping to do, every discount or sale helps reduce the amount of money spent. Sometimes, this is called bargain shopping.

Back to School Sale		
Item	**Regular Price**	**Now**
Notebook	$2.95	25% off
Pencils	$1.20/dozen	15% off
Pens	$0.99	12% off
Markers	$4.95/dozen	30% off
Erasers	$1.50/dozen	20% off
Scissors	$2.25	10% off
Yearly Planners	$7.92	30% off

1. The school supplies you need for the upcoming school year include:

- 2 notebooks
- 2 pens
- 1 dozen pencils
- 1 yearly planner

a. How much is your total, *not* including the discounts?

b. How much is your total, including the discounts?

c. Tax in your county is 7%. How much tax will you pay on the discounted total?

d. How much will you spend, including tax?

2. Your brother's school supplies for the upcoming school year include:

- 2 dozen pencils
- 1 dozen markers
- 1 dozen erasers
- 1 pair of scissors

a. How much is his total, including the discounts?

b. How much tax will he pay on his total?

c. What is his total cost?

The newspaper has a coupon for 30% off the price of planners. Kendra has two strategies for calculating the discounted price. She said that to determine the price of a planner, she could calculate 30% of $7.92 and then subtract that amount from the original price—a two-step process.

Two-step method:

30% of $7.92

$0.30 \cdot \$7.92 = \$2.38$

$\$7.92 - \$2.38 = \$5.54$

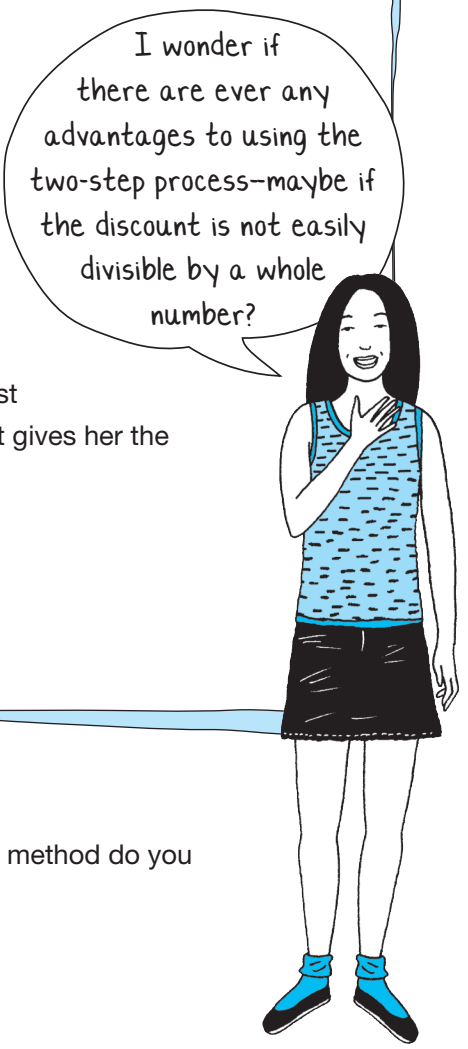
Kendra can also calculate the new price in one step.

Kendra can determine 70% of $7.92, since she is receiving a 30% discount. With the discount, she is actually paying 70% of the cost of the item. Kendra can do one calculation that gives her the final discounted price.

One-step method:

70% of $7.92

$0.70 \cdot \$7.92 = \$5.54$



3. What do you think about Kendra's methods? Which method do you prefer? Explain your reasoning.

4. If you receive a 10% discount, what percent of the original price will you pay?

5. If you receive a 25% discount, what percent of the original price will you pay?

6. If you receive a 48% discount, what percent of the original price will you pay?

Problem 5 Who's Correct?

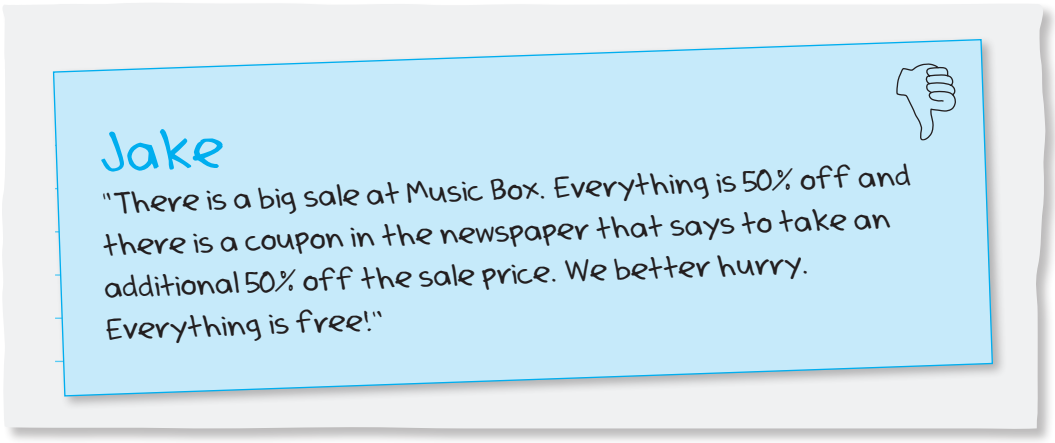
1. An in-store flyer has a coupon for an extra 10% off the sale price of yearly planners.

Extra **10% off** the sale price of planners!

- Patricia says, "The price will be $4.75, which is 40% off, because the price is currently 30% off, and I will get an additional 10% off with the coupon. 40% off of $7.92 is $4.75."
- Karla says, "The price is $4.99 because the 30% discount is calculated first, which makes the price $5.54. Then, the coupon will give you an extra 10% off the sale price, which will make the price $4.99."

Who is correct? Explain your reasoning.

2.



You doubt that Music Box could be giving everything away for free. Explain to Jake what is wrong with his reasoning. Be sure he understands exactly what percent he will be paying.

3. Allen thought that if Kendra's method worked for calculating a discount, it could probably work for calculating the tax and other percentages that get added onto a base price. If an item costs $10.50 plus 7% tax, what is the total cost of the item.

He calculated the total cost using a two-step method first.

Two-step method:

7% of $10.50

$0.07 \cdot \$10.50 = \$0.74$ tax

$\$10.50 + \$0.74 = \$11.24$

Allen thought that if he was paying the base price plus 7% tax, it is like paying 100% + 7% or 107%.

One-step method:

107% of $10.50

$1.07 \cdot \$10.50 = \$11.24$

What do you think about Allen's one-step method? Explain your reasoning.

4. What percent can you use to represent paying an 8% tax?

5. What percent can you use to represent paying a 25% surcharge?

6. What percent can you use to represent adding a 15% tip?

7. What is more helpful to know, $2.00 off or $20% off?

Problem 6 Target Heart Rate

It is important to maintain the proper pacing during exercise. The maximum heart rate for a person is calculated by subtracting a person's age (in years) from 220, which represents the *maximum* number of heart beats per minute for a person. A heart rate should stay within the 50% to 85% range of the maximum number of heart beats per minute.

1. Complete the chart shown using the information about healthy heart rates.

Age (years)	Maximum Heart Rate (beats per minute)	Target Heart Rate Zone (beats per minute)
20	200	100–170
25	195	98–166
30	190	95–162
35		
40		
45		
50		
55		
60		
65		
70		

Be prepared to share your solutions and methods.

6.4 MI MI MI MI MI MI MI!

Determining the Part, Whole, or Percent of Percent Problems

Learning Goals

In this lesson, you will:

- Determine the percent given the part and the whole.
- Determine the whole given a part and the percent.
- Determine the part given the whole and the percent.

Have you ever seen a choir sing? Most choirs have four voice parts: basses, tenors, altos, and sopranos. Women generally sing the alto and soprano parts because those parts are higher in pitch. Men generally sing the tenor and bass parts because those parts are lower in pitch.

What do you think the choir might sound like if there were more sopranos than tenors? What do you think the choir might sound like if there were more basses than altos? Do you think most choirs strive to have an even amount of people sing various parts?

Problem 1 What Percent Is It?

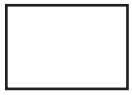
Previously, you have learned how to determine the percent of a number. You can use your knowledge of percents to determine the whole of a group or the percent of the whole that a certain number represents.

1. This picture shows triangles.

a. If the picture shown is 30% of the triangles, draw 100% of the triangles.

b. What percent is 1 triangle?

2. Analyze the rectangle shown.



If the rectangle shown is 25% of another rectangle:

a. Draw 50% of the other rectangle.

b. Draw 75% of the other rectangle.

c. Draw 100% of the other rectangle.

6

3. The figure shown represents 75% of the whole figure.

a. Draw 25% of the figure.

b. Draw 100% of the figure.

4. The rectangle shown represents 150%.

a. On the rectangle, shade 50% of the rectangle.

b. What is the percent of the rectangle that is not shaded?

5. The figure shown represents 700%.

Draw 100%.

6. The figure shown represents 160%.

x x x x
x x x x
x x x x
x x x x

a. Draw 10%

b. Draw 100%

Problem 2 Determining Parts, Wholes, and Percents

Percent problems involve three quantities: the part, the whole, and the percent. If you know two of the quantities, you can determine the third.

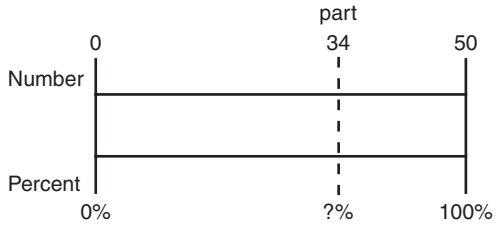
Previously, you calculated the price of a notebook after the 25% discount. The notebook cost $2.95 and the discount was 25%. You calculated either 25% of $2.95 and subtracted the difference from the total, or you calculated 75% of $2.95.

The whole usually comes after the word "of" in percent problems. The percent is generally indicated in percent problems.

Let's investigate a situation that involves determining the percent if you know the part and the whole.

In the middle school chorus, 34 of the 50 students are girls. What percent of the chorus is girls?

Holly used a double number line to think about this problem.



Then, she wrote equivalent fractions.

$\frac{\text{part}}{\text{whole}}$

$\frac{34}{50} = \frac{?}{100}$ (x 2)

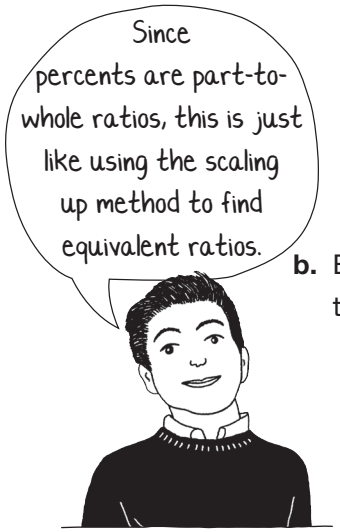
$= \frac{68}{100} = 68\%$ (x 2)

Holly determined that by doubling 50, she could calculate the percent. Since $50 \times 2 = 100$, she calculated the correct denominator for a percent. Then, she multiplied the numerator by the same factor, 2, which is $34 \times 2 = 68$.

When calculating percents, you may want to avoid simplifying fractions. Instead, determine an equivalent fraction with a denominator of 100.

1. The middle school band has 25 students.

a. Twelve students play brass instruments. What percent of band members play brass instruments?



b. Eight students play the drums. What percent of band members play the drums?

c. Five students play woodwinds. What percent of band members play woodwinds?

You can write an equivalent fraction with a denominator of 100 by doubling, tripling, or quadrupling the original denominator. However, some denominators are not a factor of 100. When this occurs, you can simply divide the numerator by the denominator to calculate the decimal equivalent of the fraction. Then, you can multiply the decimal by 100 to determine the percent.

6

2. Jasmine is a piano and organ teacher. Of the 36 students she teaches, 19 take piano lessons. What percent of her students take piano lessons?

3. The seventh-grade marching band has 40 members. First, calculate the percent for each type of instrument (brass, woodwinds, percussion). Then, calculate the percent of each instrument in the band.

Band Section	Number of Students	Percent
Brass Instruments	**23**	
Cornet	8	
Trumpet	9	
Tuba	6	
Woodwinds	**10**	
Clarinet	7	
Flute	2	
Saxophone	1	
Percussion	**7**	
Bass Drum	4	
Cymbals	3	
Total	**40**	**100**

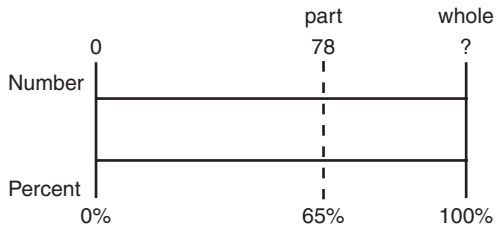
The sixth-grade class was asked the question, "How do you spend most of your time on computers?"

The results of this survey are shown:

Activity Done While Spending Time on Computers	Number of Students
Email – Instant Messaging	88
Homework	70
Games	30
Movies/DVDs/TV	24
Other Internet Activities	38

4. What percent of the sixth-grade class preferred each activity? Round your answers to the nearest whole percent.

Carlos is told that 65% of the students, or 78 students, prefer pizza for lunch according to a recent survey. He wants to know how many students were surveyed. He drew the double number line shown to visualize the problem.



He then wrote the equivalent fractions and determined that 120 students were surveyed.

$\frac{\text{part}}{\text{whole}}$ $\quad \frac{78}{?} = \frac{65}{100}$

$\frac{65}{100} \overset{+5}{=} \frac{13}{20} \overset{\times 6}{=} \frac{78}{?}$

$\frac{78}{120} = \frac{13}{20}$

5. How did Carlos determine the total number? Explain Carlos' calculations.

6. Determine the whole in each situation. Explain your reasoning.

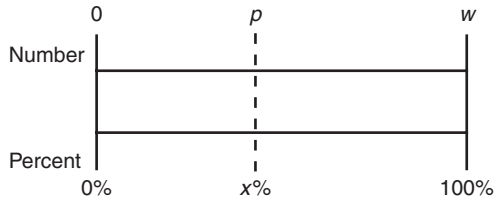
a. Your friends ate at a restaurant and left a $2.40 tip. They left a 15% tip. What was the cost of their bill?

b. The best player on your school basketball team makes 60% of her free throws. If she scored 90 points with free throw shots, which are worth one point each, how many free throws did she attempt?

c. Sandy made a 30% deposit on a computer. She gave the clerk $168. What is the price of the computer?

d. You got a quiz back and your teacher wrote +16, and 80% at the top. How many points was the quiz worth?

Talk the Talk



Given the percent equation, p represents the part, w represents the whole, and $\frac{x}{100}$ represents the percent.

1. Calculate a part given the percent and the whole.
   25% of 48 is what number?

2. Calculate a percent given the part and the whole.
   12 is what percent of 48?

3. Calculate a whole given the percent and the part.
   25% of what number is 12?

Be prepared to share your solutions and methods.

6

6.5 PRACTICAL PERCENTS PRACTICE!

Using Percents in Real-World Situations

Learning Goals

In this lesson, you will:

- Calculate the percent increase and decrease.
- Calculate the discount of a base price.
- Calculate additional discount on sales price.
- Calculate gratuity on a bill.
- Calculate sales tax.

Key Terms

- commission
- gratuity

Many people who eat at restaurants leave a tip for the waiter at the end of their meal. This is a little extra money—usually 15% or 20% of the cost of the meal—that people give directly to the server for waiting on them.

In 2007, however, a waitress at a pizza place in Angola, Indiana, got much much more than a 20% tip. The family that she waited on every Friday—who ordered the same thing on every visit—left their 20-year-old waitress, Jessica Osborne, a tip of $10,000!

Can you estimate what percent *that* tip amounted to? Do you think the $10,000 tip is greater or less than 100% of the bill?

Problem 1 Using Percents

1. Determine the value for each using your knowledge of percents. Round your answer to the nearest tenth, if necessary.

a. What is 25% of 60?

b. 15 is what percent of 50?

c. 45 is what percent of 60?

d. What number is 20% of 80?

e. 36 is 40% of what number?

f. What is 15% of 40?

g. 27 is what percent of 90?

Problem 2 How Much are You Going to Save?

Kala and Keisha are shopping for new basketball shoes. They notice a flyer that reads:

> Take an additional
> **40% off**
> the sales-tag price
> of all merchandise!

Kala finds a pair of shoes she likes and in her size, but the additional sale price tag has been removed. The original price is $120, and the original sale price is 25% off. Kala says, "I think I can determine the final cost of the shoes. Since the shoes were already reduced 25% and are being reduced again by 40%, then the shoes must be 65% off!"

Keisha disagrees. She says, "You cannot just add the discounts together. You have to read the flyer more closely. First, you must calculate the sale price after the 25% discount. Then, once you know the sale price, then you can reduce the sale price by 40%. That's the way you can determine the price after the additional 40% off."

1. What would the sale price be if you used Kala's method?

2. What would the sale price be if you used Keisha's method?

3. Who is correct in their reasoning? Explain how you determined your answer.

4. Kala's brother said that the shoes would have been cheaper if the store had taken 40% off first, and then taken an additional 25% off the sale price. Is he correct? Explain your reasoning.

5. You need a graphing calculator for math class. You saw the same graphing calculator at 3 different stores. On the day before school starts, you decide to do some comparative shopping for the calculator. All three stores are having a Back-to-School Sale, and as luck has it, all three stores have the graphing calculator you want on sale.

All Things Math has the price of the graphing calculator down 30%, but if you show your student ID card, you receive an additional 25% off the original price.

Rational Numbers for Rational Math has the price of the graphing calculator marked down 25%, but if you come between 10:00 am and 1:00 pm, you can get an additional 30% off the sale price.

Pi, Protractors, and Percents has the price of the graphing calculator marked down 50%.

Which store has the best price of the calculator? Explain how you determined your answer.

Problem 3 Is This for Here? Or to Go?

A year ago, Isosceles Triangle Restaurants (ITR) decided to offer curbside dining as another option for potential diners to eat. Now, restaurant executives are determining whether they should continue the curbside option. In the past year, ITR had 2,650,000 diners. The diners are listed in the following fashion:

- 636,000 ate in the restaurant
- 1,590,000 used the drive-thru window
- 424,000 used curbside dining

1. What percent of customers used the drive-thru windows?

2. What percent of customers ate in the restaurant?

3. What percent used curbside dining?

Out of the 2,650,000 customers, 75% ordered beef, and the other 25% ordered chicken.

4. Determine the number of customers that ordered beef using the information given. Explain how you determined your answer.

5. Determine the number of customers that ordered chicken. Explain your reasoning.

Problem 4 I Got the Sale! Now, What's My Cut?

Sales **commission** is an amount or percent of an item that is paid to employees or companies that sell merchandise in stores, or by calling customers. The commission is meant to motivate sales people to sell more. A commission may be paid in addition to a salary, or in place of a salary. Commissions are typically paid in the business of real estate, marketing, automobile sales—and even in textbook sales!

1. An automobile saleswoman earns 12% on all of her sales. Last month, she sold 3 cars for a total sales amount of $28,950. What is her commission?

2. A real estate agent earns 6% of the selling price of each house he sells. If he sells a home for $250,000, how much of a commission will he make?

3. If a car salesman made $2450 last month from a 12% commission, what is the total sales amount of all the cars that he sold?

4. A real estate agent made $7500 on a $150,000 home sale. What was the percent of her commission?

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Problem 5 When Black Equals Green!

Many experts have determined that the day after Thanksgiving is the busiest shopping day of the year in the United States. Nicknamed Black Friday, this day is seen by many stores and retail outlets as a huge opportunity to make a lot of money on the sales of merchandise. To lure customers to the big sales, some stores open at midnight on Black Friday and sometimes offer discounts of over 50% off the original price.

π–mp3 player	**18%**	$195.00
π–Box 314	**21%**	$299.99
π–Game Console	**12%**	$259.98

1. Analyze the Black Friday Sales Flyer shown.

   a. Determine the sale price of the π-mp3 player. How much will a customer save from the discount?

   b. Determine the sale price of the π-Box 314.

   c. Determine the sale price of the π-Game Console.

2. During a Black Friday sale, Alberto paid $158.00 for new smart cell phone that was reduced by 30%. What was the original price of the smart phone?

3. Kalisha paid $243.19 for a flat-screen television that was originally $319.99. What percent discount did she receive?

While stores offer sale prices on items, they also tend to *mark up* the products they sell. Typically, stores buy items from a wholesaler or distributor, and then they increase the price when they sell the items to consumers. The increase in price provides money for the operation of the store and the salaries of people who work in the store.

A store may have a rule that the price of a certain type of item needs to be increased by a certain percentage to determine how much to sell it for. This percentage is called the markup.

4. Oh! Shiny! Jewelry Store marks up all of their jewelry by a percent that allows for a profit even if they have to offer a discount during the holiday season. Analyze the table shown and determine the appropriate values to complete the table.

Jewelry Item	Markup Percent	Original Cost (dollars)	Customer Price (dollars)
Necklace	100	119	
Earrings	200		234
Bracelet		324	810
Watch	125	85	
Cuff links		63	252
Ring	250		938

Problem 6 No Taxation Without Calculation

1. The table shown lists 20 states and the different sales tax added to every dollar of a purchase as of February 2010. Complete the table by determining the tax amount added to each purchase amount. If necessary, round your answer to the nearest penny.

State	Sales Tax (percent)	Tax on $10.00 (dollars)	Tax on $100.00 (dollars)	Tax on $1000.00 (dollars)
Indiana	7			
California	8.25			
Florida	6			
Arizona	5.6			
Kansas	5.3			
Alabama	4			
Missouri	4.225			
Minnesota	6.875			
Oklahoma	4.5			
Utah	4.7			
Colorado	2.9			
Maine	5			
Illinois	5.25			
North Carolina	5.75			
Nebraska	5.5			

Problem 7 Thanks for the Service!

In many service-oriented occupations, employees work for an hourly wage, plus a tip that is left by a customer to show appreciation for good service. A tip, also called a **gratuity**, for wait staff in a restaurant is generally a percent of the total amount of the bill. In the United States, the typical gratuity for good service ranges between 15% to 20%. Sometimes, if a party is more than six people, the restaurant will automatically add a gratuity charge to the bill.

1. Darlene works at a restaurant during the dinner shift. At her restaurant, the gratuity is automatically added to the bill. The gratuity is 18% of the total bill. If Darlene is hoping to earn $125.00 in tips, what will the total cost of dinners need to be?

2. Rajan is a dog groomer at a local pet store. He receives a 15% tip of the total from the dog owners. If he hopes to make $54 this weekend, how much money must be spent on dog baths? Show how you determined your answer.

3. If each dog bath and grooming costs $12.00, how many dog baths and grooming must he do to receive $54 in tips?

Be prepared to share your solutions and methods.

Chapter 6 Summary

Key Terms

- percent (6.1)
- benchmark percents (6.2)
- commission (6.5)
- gratuity (6.5)

6.1 Introducing Percents

A percent is a fraction with a denominator of 100. It means the same thing as hundredths. The percent symbol, "%," means "out of 100." Percents, fractions, and decimals are often used interchangeably.

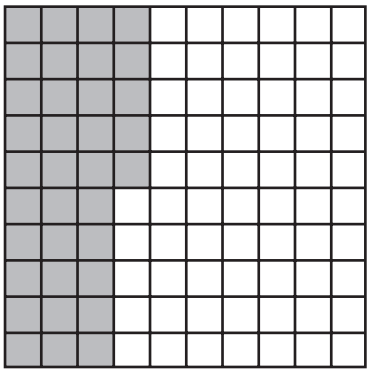
Example

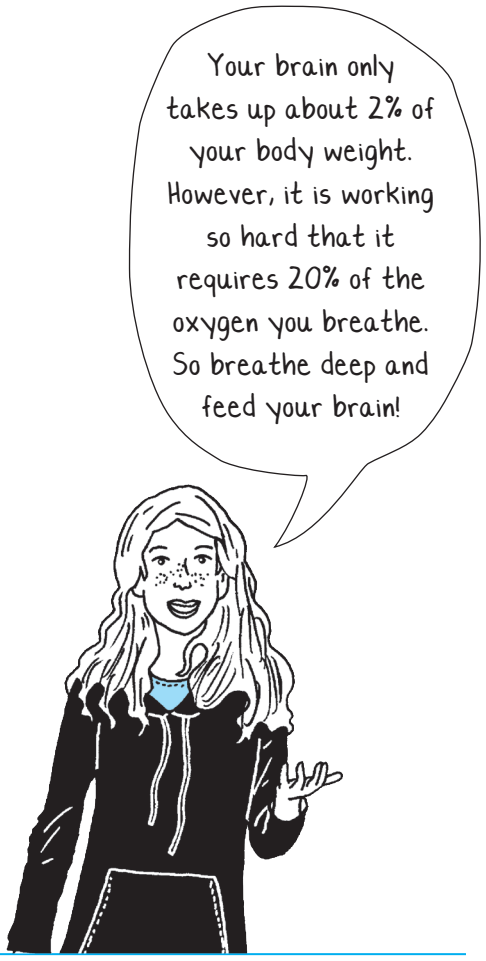
The shaded part of the hundredths grid is shown as a fraction, decimal, and percent.

Fraction: $\frac{35}{100}$

Decimal: 0.35

Percent: 35%



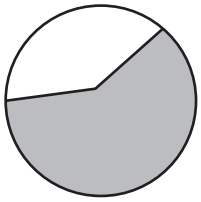


6.2 Estimating Percents with Models

A whole is the same as 100%.

Example

The estimate of the shaded part of the circle is 60%.



6.2 Estimating Percents Using Benchmarks

A benchmark percent is a commonly used percent, such as 1%, 5%, 10%, 25%, 50%, and 100%. Benchmark percents can help calculate the percent of any number.

Example

Benchmark percents are used to calculate 35% of 6500 as shown.

10% of 6500 is 650.
30% of 6500 is 3(650) = 1950.
Half of 10% is 5%, so 5% of 6500 is half of 650, or 325.
35% of 6500 is 1950 + 325 = 2275.

So, 35% of 6500 is 2275.

6.3 Determining the Percent of a Number

To determine the percent of a number, convert the percent to a decimal, and then multiply.

Example

What is 38% of 1200?

$38\% \text{ of } 1200 = 0.38 \times 1200$

$= 456$

So, 456 is 38% of 1200.

6.4 Determining Parts, Wholes, and Percents

A percent problem involves three quantities: the part, the whole, and the percent. If you know two of the quantities, you can determine the third. Given the percent equation, $\frac{x}{100} = \frac{p}{w}$, p represents the part, w represents the whole, and $\frac{x}{100}$ represents the percent.

Examples

a. 16 is what percent of 25?

$$\frac{x}{100} = \frac{p}{w}$$

$$\frac{x}{100} \overset{\times 4}{\underset{\times 4}{=}} \frac{16}{25}$$

$$x = 64$$

So, 16 is 64% of 25.

b. 72% of what number is 54?

$$\frac{x}{100} = \frac{p}{w}$$

$$\frac{72}{100} = \frac{54}{w}$$

$$\frac{18}{25} \overset{\times 3}{\underset{\times 3}{=}} \frac{54}{w}$$

$$75 = w$$

So, 72% of 75 is 54.

6.5 Calculating the Sale Price

To calculate a sale price, you can multiply the percent off by the base price to determine the discount. Then, subtract the discount from the base price.

Example

Felipe bought a pair of jeans on sale. The jeans were originally priced at $33 but were on sale for 40% off.

base price: $33
percent off: 0.40

discount: $33 × 0.40 = $13.20

sale price: $33 − $13.20 = $19.80

Felipe paid $19.80 for the jeans on sale.

6.5 Calculating Commission

Sales commission is a percent of a total sale paid to the salesperson. To determine a commission, multiply the percent by the total sale.

Example

A makeup salesperson had total sales of $1400 in one month. The salesperson earns 5% commission.

$1400 \times 0.05 = \$70$

So, the salesperson earned $70 in commission.

6.5 Calculating Sales Tax and Tips

A sales tax is an amount added to each dollar of a purchase. A tip, also called a gratuity, is a percent of the total amount of a bill that is often given to a waiter. You can use percents to calculate sales tax and tip amounts.

Mr. Clark's restaurant bill, without tax, was $55.79. He must pay an 8% sales tax.

$\$55.79 \times 1.08 = \$60.25$

His total bill, after sales tax, is $60.25

Mr. Clark leaves a 20% tip on top of the total bill.

$\$60.25 \times 1.20 = \$72.30$

So he pays $72.30 altogether.

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