

# Performance and Difficulties of Students in Formulating and Solving Quadratic Equations with One Unknown\*

Makbule Gozde Didis<sup>a</sup>

Gaziosmanpasa University

Ayhan Kursat Erbas<sup>b</sup>

Middle East Technical University

## Abstract

This study attempts to investigate the performance of tenth-grade students in solving quadratic equations with one unknown, using symbolic equation and word-problem representations. The participants were 217 tenth-grade students, from three different public high schools. Data was collected through an open-ended questionnaire comprising eight symbolic equations and four word problems; furthermore, semi-structured interviews were conducted with sixteen of the students. In the data analysis, the percentage of the students' correct, incorrect, blank, and incomplete responses was determined to obtain an overview of student performance in solving symbolic equations and word problems. In addition, the students' written responses and interview data were qualitatively analyzed to determine the nature of the students' difficulties in formulating and solving quadratic equations. The findings revealed that although students have difficulties in solving both symbolic quadratic equations and quadratic word problems, they performed better in the context of symbolic equations compared with word problems. Student difficulties in solving symbolic problems were mainly associated with arithmetic and algebraic manipulation errors. In the word problems, however, students had difficulties comprehending the context and were therefore unable to formulate the equation to be solved. Thus, it is concluded that the differences in the structural properties of the symbolic equations and word problem representations affected student performance in formulating and solving quadratic equations with one unknown.

**Keywords:** Mathematics education • Algebra • Quadratic equations • Quadratic word problems • Students' difficulties

---

\* An earlier version of this manuscript was presented at the 10th Congress of National Science and Mathematics Education held in Nigde University, Turkey on June 27-30, 2012.

Ayhan Kursat Erbas is supported by the Turkish Academy of Sciences through the Young Scientist Award Program (A.K.E./TÜBA-GEBİP/2012-11).

## a Corresponding author

Makbule Gozde Didis, Elementary Education Department, Faculty of Education, Gaziosmanpasa University, Tokat Turkey

Research areas: Mathematics teacher education; Mathematical modeling; Teaching and learning algebra

Email: gozde.didis@gop.edu.tr

b Ayhan Kursat Erbas, Secondary Science and Mathematics Education Department, Faculty of Education, Middle East Technical University, Ankara Turkey

Email: erbas@metu.edu.tr

Quadratic equations have been a fundamental topic, not only in secondary mathematics curricula around the world but also in the historical development of algebra. Various approaches for solving quadratic equations were used at different stages in this historical development, through representations including arithmetic or numerical, algebraic or symbolic, and visual or geometric (Katz & Barton, 2007). From a contemporary perspective, quadratic equations are considered important in school mathematics curricula because they serve as a bridge between mathematical topics such as linear equations, functions, and polynomials (Sağlam & Alacaci, 2012). Furthermore, like linear equations, quadratic equations are powerful representations used in other disciplines, such as physics, engineering, and design, due to their usefulness in solving many kinds of word problems and for modeling realistic or real-life situations.

### Student Performance in Solving Quadratic Equations

Various researchers (e.g., Vaiyavutjamai & Clements, 2006) have illustrated that very little attention has been paid to quadratic equations in mathematics education literature, and there is scarce research regarding the teaching and learning of quadratic equations. A limited number of research studies focusing on quadratic equations have documented the techniques students engage in while solving quadratic equations (Bossé & Nandakumar, 2005), geometric approaches used by students for solving quadratic equations (Allaire & Bradley, 2001), students' understanding of and difficulties with solving quadratic equations (Kotsopoulos, 2007; Lima, 2008; Tall, Lima, & Healy, 2014; Vaiyavutjamai, Ellerton, & Clements, 2005; Zakaria & Maat, 2010), the teaching and learning of quadratic equations in classrooms (Olteanu & Holmqvist, 2012; Vaiyavutjamai & Clements, 2006), comparing how quadratic equations are handled in mathematics textbooks in different countries (Sağlam & Alacaci, 2012), and the application of the history of quadratic equations in teacher preparation programs to highlight prospective teachers' knowledge (Clark, 2012).

In general, for most students, quadratic equations create challenges in various ways such as difficulties in algebraic procedures, (particularly in factoring quadratic equations), and an inability to apply meaning to the quadratics. Kotsopoulos (2007) suggests that recalling main multiplication facts directly influences a student's ability while engaged in factoring quadratics. Furthermore, since solving the quadratic equations by factorization

requires students to find factors rapidly, factoring simple quadratics becomes quite a challenge, while non-simple ones (i.e.,  $ax^2 + bx + c$  where  $a \neq 1$ ) become harder still. Factoring quadratics can be considerably complicated when the leading coefficient or the constant term has many pairs of factors (Bossé & Nandakumar, 2005). Lima (2008) and Tall et al. (2014) suggest that students' lack of understanding on the procedures of linear equations, and their understanding based on "procedural embodiments," affects students' work on quadratic equations. Students tend to allocate meaning to equations and solving methods, however, the given meaning is related to the movement of the symbols rather than the mathematical concept. They also documented that students perceive quadratic equations as mere calculations, without paying attention to the unknown as a fundamental characteristic of an equation. Students mostly focus on the symbolic world to perform operations with symbols. For example, students used procedural embodiment associated with the exponent of the unknown, and solved the equation by transforming it into  $m = \sqrt{9}$  to solve  $m^2 = 9$ . In this case, students' use of the procedural embodiments "switching power to roots" (p. 15) resulted in failing to recognize the other root (i.e.,  $m = -3$ ). Moreover, they reported that students attempted to transform quadratic equations into linear equations. For instance, while attempting to solve  $m^2 = 9$ , some students applied the exponent associated with the unknown as if it were the coefficient; that is,  $m^2$  equals to  $2m$ , and students showed a tendency to use the quadratic formula as the only valid method in solving every quadratic equation. Although it is expected that students would use factorization for solving equations such as  $t^2 - 2t = 0$  and  $3k^2 - k = 0$ , studies have shown that few students would do so. Vaiyavutjamai and Clements (2006) proposed that students' difficulties with quadratic equations stem from their lack of both instrumental understanding and relational understanding of the specific mathematics associated with solving quadratic equations. They suggest that while teacher-centered instruction with strong emphasis placed on the manipulation of symbols, rather than on the meaning of symbols, increases student performance regarding solving quadratic equations, their (relational) understanding would still be quite low, and they could develop misconceptions. For example, they found that many students had an inadequate understanding of the "null factor law". In solving  $(x - 3)(x - 5) = 0$ , although most students gave the correct answer;  $x = 3$  and  $x = 5$ , they considered two  $x$ s in the equation as

representatives of different variables, and thus, they must take different values. That is, when they were asked to check their solutions, they simultaneously substituted  $x = 3$  into  $(x - 3)$  and  $x = 5$  into  $(x - 5)$  and found that  $0.0 = 0$  and in doing so, decided that their solutions were correct. This misconception also appeared in students' solution of  $x^2 - x = 12$ .

### Student Performance in Solving Algebra Word Problems

Application problems traditionally appear in the form of word problems (Verschaffel, Greer, & de Corte, 2000). Algebraic equations are considered beneficial for tracking students' comprehension processes and detecting differences among students' solutions of algebraic word problems (Hinsley, Hayes, & Simon, 1977). Studies have revealed that solving algebra word problems is challenging for the majority of students because the formal algebraic system creates a serious barrier to generating equations that represent the relationships within the problem (Kieran, 1992). However, students' challenges depend, not only on a formal algebraic system in the solution phase, but also on the linguistic form of the word problems in the comprehension phase (Koedinger & Nathan, 2004). Various researchers have proposed that solving algebra word problems consists of a "comprehension phase and [a] solution phase" (e.g., Cummins, Kintsch, Reusser, & Weimer, 1988; Mayer, 1982; Nathan, Kintsch, & Young, 1992). In the comprehension phase, a problem-solver comprehends and then forms the text base of the problem, utilizing words as an internal representation in his or her memory. In the solution phase, she or he expresses this internal representation externally and applies the rules of algebra to reach a conclusion (Koedinger & Nathan, 2004; Mayer, 1982).

A number of research studies have offered various evidence that word problems are difficult for students. For example, Clement (1982) indicated that students' difficulties in solving algebra word problems stem from the difficulties they have in symbolizing meaningful relationships within algebraic equations. Moreover, Briars and Larkin (1984) attributed the word problem-solving difficulty to the students' psychological processes. They also emphasized factors relating to the problem's features, such as the number of words in the problem, the presence of cue words and the size of the numbers. On the other hand, the text comprehension factor is the main issue for students in solving word problems (Cummins et al., 1988; Nathan et al., 1992).

According to Cummins et al. (1988), inadequate mapping of phrases causes student to fail at solving word problems. Similarly, Nathan et al. (1992) suggested that students make errors because they fail to see how the situational aspect of the problem is related to the formal expressions, in their attempts to produce the intended internal representations. Although the linguistic form of the problem's text conveys the significant factors that affect the comprehension process, Stacey and MacGregor (2000) claimed that a major reason for students' difficulties with word problems arises from not understanding the algebraic logic of a problem. They argue that because of their prior experiences with arithmetic word problems, students perceive the problem-solving process as a series of calculations and shift their thought process from algebraic thinking to arithmetic thinking when solving algebraic word problems.

The purpose of this study was to investigate the performance of 10<sup>th</sup> grade students and their difficulties in solving quadratic equations with symbolic and word-problem representations. The following questions have guided the study:

- How do 10<sup>th</sup> grade students perform in formulating and solving quadratic equations in one unknown?
- What types of difficulties do students have in solving quadratic equations that are represented symbolically?
- What types of difficulties do students have in solving quadratic equations that are represented as word problems?

This study aims to widen the existing body of knowledge regarding students' difficulties in solving quadratic equations with one unknown. Additionally it aims to contribute to a growing body of research concerning students' performances and understanding of symbolic-equation and word-problem representations when formulating and solving equations. In doing so, this study also attempts to address the need for research studies regarding the teaching and learning of quadratic equations in a Turkish context, as most of the existing research has focused on student difficulties in forming and solving equations with regards to linear equations (e.g., Erbaş, Çetinkaya, & Ersoy, 2009).

## Method

### Research Design

The research design utilized in this study can be described as mixed method. It draws on both qualitative and quantitative data to describe and analyze

students' performances and difficulties with quadratic equations in symbolic and word-problem contexts.

### Participants and the Context

The participants of this study were 217 tenth grade students from three public Anatolian high schools in Turkey; one is located in Ankara ( $n_1 = 84$ ) and the other two are in Çorum ( $n_2 = 78$  and  $n_3 = 55$ ). The participants' ages ranged from 14 to 16 ( $M = 15.95$  and  $SD = 0.46$ ). Furthermore, all of the participating students were in mathematics-natural sciences programs; emphasizing mathematics, physics, chemistry, and biology. Since this study aimed to gain insight into and describe the participating students' performances and difficulties, rather than to generalize participants' performance, a convenient sampling approach was considered appropriate. Also, as these schools were willing to open their doors to carry out the study, convenient sampling was the primary choice for the participant selection. However, the participants of the study cannot be considered representative of all tenth grade students in Turkey, as certain high schools accept students based on their scores in a centralized test which is carried out in 8th grade. The test comprises multiple-choice questions in certain domains, such as Turkish Literature, mathematics, and science. Therefore, the results of this study should be interpreted accordingly.

At the time of data collection, the participants had already studied quadratic equations in one unknown in about 15 lessons at the same grade level. They were introduced to factoring polynomials, including factoring quadratic polynomials (trinomials), before they began the unit on quadratic equations, inequalities, and functions. Teachers were expected to teach the subject through factorization, the quadratic formula, and completing square methods. In addition, although the curriculum emphasized teaching symbolic and tabular representations, it did not in any way address the teaching with regards to the graphical representations of quadratic equations with one unknown. Moreover, although the curriculum guidelines do not explicitly emphasize problem-solving with the use of quadratic equations, they suggest using word problems to find roots and solution sets of quadratic equations. Furthermore, the mathematics textbooks used in schools emphasize symbolic approaches regarding quadratic equations, such as factorization, the quadratic formula, and completing the square.

### Instrument

A test comprising 12 open-ended items presented in the form of either symbolic equations or word problems was developed to determine the students' performances and difficulties in quadratic equations. The items in the test were representative of those that can be found in most mathematics textbooks. The first eight items consisted of symbolic equations and were intended to measure students' procedural abilities in solving quadratic equations. These equations were presented in different symbolic structures with different structural properties, such as quadratic with rational coefficient, quadratic with leading coefficient having many pairs of factors, factorable and non-factorable quadratics and quadratics in standard as well as non-standard forms (e.g.,  $24x^2 + 7x - 6 = 0$ ;  $3x^2 = 6x$ ;  $x^2 + 2x - 1 = 0$ ;  $x^2 + \frac{2}{3}x - 1 = 0$ ). The remaining four items were presented as word problems, with each having a different problem context (e.g., rate and volume) as straightforward applications of quadratic equations with one unknown (see the Appendix). These items were intended to measure student performance in using symbolic algebra to identify and represent algebraic relationships in word problems, and to solve them. The items were aligned with higher-order thinking skills such as recognizing, exploring, and/or identifying algebraic relationships.

Prior to the main study, an initial version of the test comprised seven items in the form of symbolic equations. In order to check the suitability of the questions and the time that students would need to complete the test, a pilot study was conducted with 46 10th-grade students from a different Anatolian high school. Students were given 20 minutes to complete the test. After the pilot study, two questions were eliminated, as their structural properties were similar to other questions in the test, and three symbolic equations having different symbolic structures were added. This allowed for better structural diversity. Moreover, in accordance with the objective of the study, the researchers decided to add four word problems which required forming and solving quadratic equations. Thus, the final form of the test consisted of 12 items. Furthermore, the pilot study confirmed that a class hour was enough time for students to complete the test.

In order to establish content validity of the test, all items in the final version were discussed with two-experienced high school mathematics teachers in terms of mathematical correctness and pedagogical alignment. That is, the suitability of the items

for the students and their learning opportunities presented in the class or in the textbooks used at this level. Furthermore, the Cronbach's alpha value from the current sample was .75.

### Data Collection and Analysis

All participants in this study were administered the open-ended test by their mathematics teachers during regular class time and were given 45 minutes to complete it. Furthermore, semi-structured interviews were conducted with 16 volunteer students from the participating schools to obtain insights into student reasoning and thought processes underlying their written solutions, and to learn about the factors affecting their difficulties in solving the items. The interviews were conducted one day after the test was administered to avoid retention loss. In the interviews, students were asked to explain the solutions they had given in the test, especially the incorrect ones. They were also asked about which of the items were most difficult for them and why. Each interview was audio recorded and lasted about 10 minutes.

For data analyses, each student's written responses to each question were initially coded as correct, incorrect, blank or incomplete. To obtain a general view of performance, the percentages of the coding were calculated. In the symbolic equation context, all responses those were not totally completed, including those that followed a mathematically correct process, were categorized as incomplete. In the word-problem context, responses where students set up the correct equation to solve the problem, but did not solve it, were also deemed incomplete. The aim of this process was to have an overall descriptive picture of the students' solutions. Afterwards, qualitative analyses of the students' responses in the written test were conducted to identify common reasons for mistakes in both contexts that had caused students to display low performance. For both contexts, the analysis of students' written responses was drawn from students' errors, and the difficulties they faced. Given this factor, students' incorrect and incomplete answers for all questions in both contexts were analyzed item-by-item. In terms of symbolic equations, before the data was coded, by considering the students' difficulties and errors in solving and formulating quadratic equations in one unknown revealed by prior research, a code list was created. In addition, some codes also evolved from the data itself when the participants' difficulties and errors were not compatible with those present in prior research. As a result of cod-

ing students' incorrect answers, we decided to categorize student performance in solving quadratic equations with respect to their use of methods (i.e., quadratic formula, factoring method, completing square method, and square root property) to solve quadratic equations. In terms of word problems, the codes and categories came from conceptual frameworks and prior research relating to solving arithmetic and algebraic word problems (Cummins et al., 1988; Koedinger & Nathan, 2004). Students' incorrect and incomplete answers were coded and categorized with respect to their comprehension of the problem statement, setting up the correct relationship and formulating quadratic equations in one unknown, and solving the formulated quadratic equation. For both contexts, the authors discussed the codes, and any emerging problems concerning the codes were resolved. While the data were being coded, the researchers agreed on how to group the most common reasons for incorrect solutions and the interpretations to be drawn from them. For the analysis of the interview data, the students' responses were open coded to reveal information about their level of understanding, difficulties, and affecting factors. Difficulties, the methods and reasoning used, and the reasons for incorrect, incomplete, and blank solutions were some major codes that evolved from the data.

## Results

### Student Performance in Solving Symbolic Equations

The quantitative analysis of the data revealed that only about ten percent of the students ( $N = 217$ ) solved all of the symbolic equation questions correctly. The data shows that the percentage of correct solutions ranged from 25.8% to 80.6% (see Table 1). The highest and the lowest number of correct solutions were for Q1 and Q4 respectively. Almost one-third of the students did not attempt to solve Q4 and left it blank. Moreover, the number of students who correctly solved Q3 was also quite low. For questions 2, 6, 7, and 8, more than half of the students solved the given quadratic equation correctly. The qualitative analyses of the students' incorrect and incomplete responses for all symbolic equation questions are illustrated in the following categories.

Table 1  
Distribution of Students' Correct, Incorrect, Blank, and Incomplete Answers for the Symbolic Equation Questions (N = 217)

Symbolic Equations	Correct f (%)	Incorrect f (%)	Blank f (%)	Incomplete f (%)
Q1. $x^2 - 5x + 6 = 0$	175 (80.6)	35 (16.1)	1 (0.5)	6 (2.8)
Q2. $24x^2 + 7x - 6 = 0$	121 (55.8)	64 (29.5)	26 (11.9)	6 (2.8)
Q3. $x^2 + 2x - 1 = 0$	76 (35.0)	112 (51.6)	21 (9.7)	8 (3.7)
Q4. $x^2 + \frac{2}{3}x - 1 = 0$	56 (25.8)	77 (35.5)	64 (29.5)	20 (9.2)
Q5. $10x^2 - 9x = 7$	146 (67.3)	48 (22.1)	14 (6.5)	9 (4.1)
Q6. $\frac{5}{4}x^2 - \frac{1}{4} = x$	109 (50.2)	50 (23.1)	54 (24.9)	4 (1.8)
Q7. $9x^2 - 25 = 0$	126 (58.1)	63 (29)	19 (6.8)	9 (4.1)
Q8. $3x^2 = 6x$	143 (65.9)	60 (27.6)	11 (5.1)	3 (1.4)

**Students' Difficulties in Using Factoring Method:** We observed that students employed different approaches to factorization depending on the kind or structure of the quadratic equation to be solved, and thus experienced difficulties in different stages of the process. These are explained in the following sections:

**False Guess of Factors While Using the Cross-multiplication Method:** Certain errors made by the students revealed they had factorized the quadratic equation into two linear factors incorrectly, and determined the roots incorrectly because they had made false guesses while using the cross-multiplication method. This kind of error occurred mostly in Q1 and Q5, where many students used cross-multiplication as a factoring technique to find the roots of the quadratic equation. The most common error emerged when students used a cross-multiplication method, based on a kind of "guess-and-check" approach, while factorizing the quadratic equations. For example, students guessed the factors of the constant term 6 in Q1 incorrectly as -6 and 1 instead of -2 and -3, respectively. Similarly for Q5, although students guessed the factors of the coefficient of  $x^2$  correctly, they were not able to determine the factors of the constant correctly (i.e., as -7 and 1 instead of 7 and -1).

**Attempting to Factorize Non-factorable Quadratic Equations:** The data revealed that some of the students did not correctly judge whether the quadratic equation to be solved was factorable over some domain, such as rational numbers. For example, some students attempted to factor the equations;

" $x^2 + 2x - 1 = 0$ " and " $x^2 + \frac{2}{3}x - 1 = 0$ ," although they are not factorable, over the rational numbers. For the quadratic equation  $x^2 + 2x - 1 = 0$ , some students tried to factorize it as  $a(x^2 - 1)^2, (x + 1)^2$  or  $(x - 1)(x + 1)$ . For this quadratic equation, all students who had successfully found the roots used

the quadratic formula. On the other hand, a small number of students who tried to apply completing the square, failed. Similarly, a few students attempted to factorize  $x^2 + \frac{2}{3}x - 1 = 0$  as  $(x + \frac{1}{2})(x + 1)$ .

In addition, some of the students tried to factorize it after multiplying both sides of the equation by the denominator of the rational coefficient, in order to make all coefficients integers, and they rewrote the equation as  $3x^2 + 2x - 3 = 0$  (Figure 1).

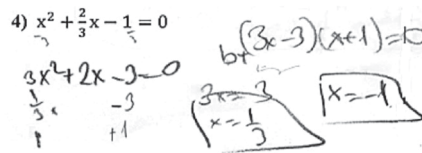


Figure 1: An example of a student's attempt to factorize a non-factorable quadratic equation.

**Incorrect Use of either Difference of Two Squares or the Greatest Common Factor Techniques:** Some students could not correctly apply the algebraic identity  $a^2 - b^2 = (a - b)(a + b)$  to factorize quadratics. For example, students factorized the polynomial  $9x^2 - 25$  incorrectly, as  $(9x - 5)(9x + 5)$  or  $(3x - 5)^2$  and calculated the roots as  $\left\{\frac{5}{9}, -\frac{5}{9}\right\}$  or just as a single root  $\left\{\frac{5}{3}\right\}$ . On the other hand, the data revealed that some students factorized the quadratic equation  $3x^2 - 6x$  incorrectly as  $3x(x - 3), 3x(x - 6)$  or  $3x(x - 2)$ .

As shown in Figure 2, some students initially moved the term  $6x$  to the left side of the equation. They then identified the greatest common factor of the polynomial;  $3x$ , and rewrote the polynomial using the factored terms. However, although students put the common term in front of the parentheses correctly, they put the resulting expression inside



the parentheses incorrectly. Therefore, when they equaled the factors to zero, they ended up obtaining one of the roots of the quadratic equation incorrectly.

$$\begin{aligned}
 &8) \quad 3x^2 = 6x \\
 &\quad 3x^2 - 6x = 0 \\
 &3x \cdot (3x - 2) = 0 \quad G = \left\{ 0, \frac{2}{3} \right\} \\
 &3x = 0 \quad 3x = 2 \\
 &x = 0 \quad x = \frac{2}{3}
 \end{aligned}$$

Figure 2: An example of a student's errors in the use of the greatest common factor technique.

**Students' Difficulties in Using the Quadratic Formula:**

The data showed that students encountered the following challenges while applying the quadratic formula to find the roots of the quadratic equations: (i) students either computed the discriminant incorrectly because of calculation errors, or could not compute it at all; (ii) students computed the discriminant correctly, but applied the quadratic formula incorrectly, since they had misremembered it; (iii) students computed the discriminant incorrectly but they applied the quadratic formula correctly. Particularly, for questions 2, 3, 4, and 6, where the students mostly used the quadratic formula to find the roots, students' incorrect solutions were mainly based on either the incorrect calculation of the discriminant or incorrect use of the quadratic formula.

Most of the students were not able to solve the Q2 quadratic equation correctly, because they made calculation errors while they were finding the discriminant of the quadratic equation. On the other hand, most of students calculated the discriminant correctly in questions 3, 4, and 6, but they did not use the correct form of the quadratic formula. For example, many of the students misremembered the quadratic

formula and applied the following forms to solve the equations:  $\frac{\pm b - \sqrt{\Delta}}{2a}$ ,  $\frac{b^2 \pm \sqrt{\Delta}}{2ac}$ ,  $\frac{b \pm \sqrt{\Delta}}{2a}$ ,  $\frac{-b \pm \sqrt{\Delta}}{c}$ , or  $\Delta = \frac{b^2 - 4ac}{2}$  (Figure 3).

$$\begin{aligned}
 &x^2 + 2x - 1 = 0 \\
 &\Delta = b^2 - 4ac \\
 &\Delta = 4 - 4 \cdot -1 \\
 &\Delta = 8 \\
 &x_1 = \frac{-b + \sqrt{\Delta}}{c} \\
 &x_1 = \frac{-2 + 2\sqrt{2}}{-1} \\
 &x_1 = 2 - 2\sqrt{2} \\
 &x_2 = \frac{-b - \sqrt{\Delta}}{c} \\
 &x_2 = \frac{-2 - 2\sqrt{2}}{-1} \\
 &x_2 = 2 + 2\sqrt{2}
 \end{aligned}$$

Figure 3: An example of incorrect use of the quadratic formula.

Furthermore, the students' explanations in the interviews support the possibility that students either misremembered the quadratic formula or totally forgot the correct form of the formula leading to an inability to solve the quadratic equation, especially when the equation was not factorable into binomials with rational coefficients. This determination is illustrated in the following excerpt:

**Interviewer [I]:** For instance, when we look at question three...

**Student 11 [S11]:** I omitted... I tried to use the discriminant and I even found here that there were two positive roots, but then... there is a formula. Umm... minus b... I think I remembered it wrong; so I couldn't go any further. I tried but...

Similarly, the following dialogue illustrates one student's difficulty remembering the correct form of the quadratic formula and his tendency to forget it after some time had passed after initially memorizing it.

$$\begin{aligned}
 &x_1 = \frac{b - \sqrt{\Delta}}{2ac} \quad \Delta = 4 - 16 = -8 \\
 &x_1 = \frac{4 - \sqrt{-8}}{2 \cdot -1} = \frac{4 - 2\sqrt{2}}{2} = 2 - \sqrt{2}
 \end{aligned}$$

**I:** Let us look at question three.

**S7:** My solution is wrong (referring to the solution above)

The formula I remembered was  $x_1$  (one of the roots) is  $\frac{b^2 - \sqrt{\Delta}}{2ac}$

**I:** Well, your solution is wrong since you misremembered the formula, is it not?

**S7:** Umm... how can I say... we learned the topic [quadratic equation] a few months ago. To be honest, I forgot it because I did not repeat it at all after I learnt it.

In addition to students' difficulties in applying the correct form of the quadratic formula, many students made calculation errors even if they applied the correct quadratic formula.

**Students' Difficulties in Completing the Square:**

The data showed that, among all questions, students applied the completing square method only for solving  $x^2 + 2x - 1 = 0$ . Indeed, only eleven students attempted to use completing the square method to solve the quadratic equation, while the majority of the students applied the quadratic formula. Students' incorrect responses revealed that each student who attempted to solve the quadratic equation by completing the square method encoun-

tered a different challenge that led them to failure. For example, some of the students had difficulties adding the numbers correctly on both sides of the equations to balance it and converting the left hand side of the equation to its squared form. On the other hand, several students did not complete their solutions although they correctly converted the left hand side of the equation to its squared form.

On the other hand, the interview data showed that some students found the use of the completing square method challenging, and as such, they did not attempt to use it to solve the quadratic equations. The following excerpt demonstrates one reason why the student preferred not to use the completing the square method.

I: Why did you omit question 3?

S12: I tried to factor it but it did not work because there was 1. I tried to solve it by using the discriminant, however, I could not reach a solution.

I: Well, did you consider using completing the square method?

S12: Umm ... I thought about it but it is challenging. It is difficult to think which number has to be added. I prefer not to use this method.

**Students' Difficulties in Using the Square Root Property:** The data also revealed that, in solving the two-term equation  $9x^2 - 25 = 0$ , some students used neither factorization nor the quadratic formula. Rather, they tried to use the square root method. In this case, in order to isolate the squared variable, students initially moved 25 to the right side of the equation, and then put it underneath, to get  $\frac{25}{9}$  and took the square root. That is, the exponent passed to the other side as a square root. However, the majority of the students who followed this procedure found only one correct root, and they neglected one of the roots of the quadratic equations, particularly the negative root (Figure 4).

Figure 4: An example of a student's errors relating to finding a single root as the solution.

**Other Difficulties; "Student-Generated Methods":**

In solving the quadratic equation  $3x^2 = 6x$ , some students preferred to apply procedural methods rather than use formal methods. In this case, the procedural process the students followed was erroneous and led them to failure. As seen in Figure 5, when  $x$  was simultaneously canceled from both sides, the equation became linear, and students solved it as if it were linear. As such, those students neglected to recognize 0 as one of the roots when  $x$  was simultaneously canceled from both sides; they found only one of the roots.

Figure 5: An example of a student-generated method related to finding a single root as the solution.

**Student Performance in Using Quadratic Equations to Formulate and Solve Word Problems**

Only one student, who was also successful in solving all the symbolic equation questions, was successful in solving all the questions in the word-problem context. While the students were mostly successful in solving word problem 2 (46.1%), they were least successful in solving the word problem 4 (3.7%) (see Table 2).

Table 2  
Distribution of Students' Correct, Incorrect, Blank, and Incomplete Answers for the Word Problems (N = 217)

Word Problems	Correct f(%)	Incorrect f(%)	Blank f(%)	Incomplete f(%)
1	18 (8.3)	80 (36.9)	105 (48.4)	14 (6.4)
2	100 (46.1)	50 (23.0)	27 (12.5)	40 (18.4)
3	37 (17.1)	109 (50.2)	63 (29.0)	8 (3.7)
4	8 (3.7)	68 (31.3)	137 (63.2)	4 (1.8)

The students' solutions indicated that their performance differed depending on the structural characteristics of the word problems. The large majority of the students, except for when tackling the second word problem, either did not formulate the quadratic equations to represent the information given in the problem, or did not solve it correctly. Furthermore, the rate of incomplete solutions was highest in the second problem, where the students were usually able to set up the correct quadratic equation but then did or could not solve it.

On the other hand, the students who provided the correct answer for the word problems did not necessarily solve the problem using quadratic equa-



tions (see Table 3). In particular, while some students solved the first problem with a guess-and-test strategy, some students attempted to solve the third problem by initially making a drawing and then examining it from a different point of view, without formulating a quadratic equation to represent the relationship. For the first problem, students attempted to use guess-and-test strategy in order to find the number of days that they initially planned to get the order ready. For the third problem, the students initially drew a triangle to show the data and to see what was going on. Then, these students recognized that the triangle was familiar with the 3:4:5 triangle, and concluded that the dimensions of the right triangle in the diagram must be a 30-40-50 triangle.

Table 3  
Distribution of Algebraic or Symbolic Methods and Other Methods Used by the Students in Determining Correct Answers for the Word Problems

Word Problems	Algebraic or Symbolic Methods f(%)	Other Methods f(%)	Total Correct Answers f(%)
1	6 (33.3)	12 (66.7)	18 (100)
2	99 (99)	1 (1)	100 (100)
3	7 (18.08)	30 (81.92)	37(100)
4	7 (87.5)	1 (12.5)	8 (100)

Analysis of students' incorrect solutions and the interview data revealed that the reasons for low performance in forming quadratic equations stated as word problems are threefold: (i) students did not fully comprehend the problem; (ii) students understood the problem, however, they did not know how to represent the information as a quadratic equation (or approach the problem differently); (iii) students understood the problem and represented the information as a quadratic equation, however, they had difficulty solving the problem and thus, also with interpreting it.

**Comprehension of the Word Problems:** The students' solutions revealed that, since a large proportion of students either did not comprehend or miscomprehended the text in the word problems, particularly in the fourth problem, they could not formulate the related equation. Figure 6 presents an example where the student miscomprehended the

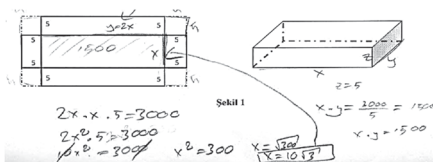


Figure 6: An example of an incorrect solution due to the miscomprehension of the problem statement.

dimension of the rectangular paperboard. Although the length of the paperboard is twice its width, before cutting and making it into an open box (i.e., a rectangular prism), the student symbolizes the dimensions of the open box as  $x$  for the width and  $2x$  for the length, and then forms the equation by way of a volume formula for a rectangular prism. Here the student correctly symbolizes the relationships and formulates the quadratic equation, however, his misinterpretation leads to an incorrect solution.

The interview data supported the factor that forming quadratic equations was quite challenging for students due to their difficulty comprehending the problem statement. During the interviews, several students expressed that they did not comprehend the problem statement (especially regarding the fourth problem), nor the information presented within.

I: What was it that caused you difficulty in the fourth question?

S9: Obviously, I read the first part; "The length of a rectangular shaped paperboard is twice its width. An open box is constructed by first cutting out squares with 5 cm side lengths from each corner..." then I read it again, and I still did not understand it.

I: Was the problem too long? Or you did not understand what you read?

S9: Both; due to the length of the problem and the reading comprehension.

**Difficulty in Setting up the Correct Relationship:**

The data revealed that certain students understood the problem statement, however, they did not know how to represent the given information as a quadratic equation. For example, in the third word problem, most of the students provided a common incorrect solution (Figure 7). It seems that these students comprehended the problem, but misinterpreted the mathematical situation and could not set up the correct relationship, such as  $(2x)^2 + (2x + 10)^2 = 50^2 \Rightarrow x^2 + 5x - 300 = 0$ , which represents the relationship among the distances.

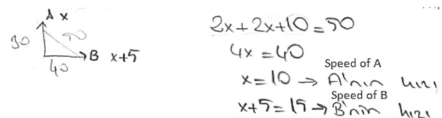


Figure 7: An example of a student's difficulties in setting up the correct relationship.

As shown in Figure 7, the student assigned values  $x$  and  $x + 5$  for the speeds of bus A and bus B, respec-

tively, and represented the situation as a diagram. The distance between buses A and B after two hours; 50, represents the length of the hypotenuse. It seems that because the student is familiar with 3:4:5 triangles, the student determined that the dimensions of the right triangle in the diagram must be 30-40-50. Thus, instead of applying Pythagorean theorem and getting a quadratic equation that represents the relationship among the distances in the situation, the student set up an incorrect relationship;  $2x + (2x + 10) = 50$ , as if the sum of the distances traveled after two hours was 50 km. It is safe to say that the problem can be solved easily;  $2x = 30$  or  $2(x + 5) = 40$ , upon recognizing the triangle as 30-40-50. However, the student failed to apply the Pythagorean Theorem to set up the correct linear relationships.

Similarly, in the second word problem, some students did not formulate the correct algebraic relationship between the side length of the square and the width of the rectangle. In the interviews, 12 students claimed that, although they did comprehend the problem statement, they could not interpret the information presented, in order to form the quadratic equation, or could not set up the quadratic equation in the correct form. This is exemplified in the following exchange:

- I: What did you think about question 4?  
**S10:** I actually read and tried to interpret;  $2x \dots$  From there, I tried to formulate the equation but I do not know what happened and I could not do it. Then, I thought that I can do this if I set up the equation because the volume of the box is  $3000\text{cm}^3$  after folding up the edges. But I could not set up the equation. Actually, I initially set [it] up, but then I [got] confused.

**Difficulty in Solving the Quadratic Equation:** The data revealed that although some students set up the quadratic equation correctly in the word problems, they made mistakes while solving it. The most common error for the second word problem was that the students constructed the algebraic relationships and formulated the quadratic equation correctly as;  $2a^2 + 6a - 176 = 0$  however, they made calculation errors while using the cross-multiplication method and zero product property, as illustrated in Figure 8.

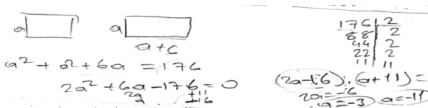


Figure 8: An example of a student's errors in solving a formulated quadratic equation.

On the other hand, some students set up the quadratic equation correctly but they could not solve the equation and as such, their solutions were incomplete. Although this issue was observed in students' attempts at solving each of the word problems, it was mainly encountered in the second word problem (Table 2). Figure 9 shows an example of how a student formulated the quadratic equation correctly by using the Pythagorean Theorem, but did not solve the equation in any way conclusively.

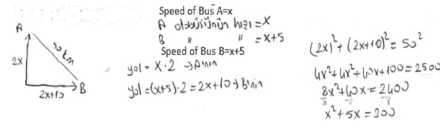


Figure 9: An example of a student's incomplete solution.

### Discussion and Conclusion

The first major result of the study was the determination that although student performance regarding solving quadratic equations stated in symbolic equation was not high, their performance depended on the structural properties of the symbolic form of the quadratic equation. Their performance also depended on how effectively they used factorization, completing to the square, and quadratic formula for solving quadratic equations. As shown in Table 1, the structural properties of the quadratic equation in Q1 (the question with the highest percentage of correct solutions) and Q4 (the one with the lowest percentage of correct solutions) differ from each other, although both are in the standard form (i.e.,  $ax^2 + bx + c = 0$  where  $a \neq 0$ ). Although the quadratic equation in Q1 is factorable, has integer coefficients, and its roots are all rational numbers, the quadratic equation in Q4 is not factorable; its second coefficient is a rational number, and its roots are irrational numbers. Similarly, the quadratic equation in Q3, which got the second lowest percentage of correct solutions, is also non-factorable, and its roots are irrational numbers.

Our data suggest that students displayed high performance in solving the quadratic equation in Q1 because it was easy to factor, has rational roots, and students have had more practice and greater procedural abilities in solving these types of equations. However, when it came to the non-factorable structure of the quadratic equation in Q4, students were unsuccessful, due to their limited procedural algebraic and arithmetic abilities. This question required more algebraic symbol manipulations and arithmetic operations, with rational and radical numbers, while applying either a quadratic formula or complete square.

The data also revealed that the method chosen to solve each quadratic equation affected the students' performance. The analysis of students' responses initially showed that the majority of the students attempted to use either factorization or quadratic formula methods. Moreover, some of the students attempted to use a square root method for Q8 in the form of  $x^2 - a = 0$ . On the other hand, the number of students who applied the completing the square method was quite low. In fact, although the nature of the quadratic equation in Q3 explicitly allowed students to use the completing the square method, few students attempted to use it, and none solved the quadratic equation correctly with this method. One of the fundamental reasons for student preference to use the quadratic formula and factorization may be explained by the fact that the students in this study lacked sufficient algebraic and arithmetic abilities to complete operations with the necessary fractional and radical expressions to efficiently use the complete the square technique, as indicated by Bossé and Nandakumar (2005). In addition, students might have felt safer using a quadratic formula and factorization methods.

Even if a quadratic formula and factorization seemed safer and easier for students to use, our data showed that students also had difficulties using these methods. Bossé and Nandakumar (2005) outlined that factoring quadratic equations can be complicated if the leading coefficient and/or constant includes many pairs, since a lot of factoring attempts would need to be tested in order to find the correct pairs of factors. The results of the current study revealed that even if both the leading coefficient and/or constant do not include many pairs of factors, as in Q1 and Q5, students still could not guess and/or choose the factors correctly, which led them to failure. As such, this result can be explained by the students' lack of computational competency (or procedural knowledge) which is required to complete basic multiplication and addition when using factoring. Moreover, unlike findings reported in previous research, the results of this study revealed that many students had difficulty remembering the correct forms of the quadratic formula, and some students had difficulty with accurately applying the cross-multiplication method, difference of two squares method, and the greatest common factor method, to factor quadratic equations. Furthermore, students were challenged when solving non-factorable quadratic equations. Particularly in the case of quadratic equation  $x^2 + 2x - 1 = 0$ , students' incorrect responses provided evidence that they had confused this quadratic equation with

either  $x^2 - 1 = 0$ ,  $x^2 - 2x + 1 = 0$  or  $x^2 + 2x + 1 = 0$ . Essentially, these results indicate that students may have blindly memorized the rules and applied them mechanically without enough thought.

Furthermore, as Tall, Lima, and Healy (2014) emphasized that "met-before" can affect students' current learning, in this study, students' previous experiences in arithmetic and algebra influenced their interpretation of a new context in negative ways. One such case appeared in students' incorrect responses emerging from solving  $3x^2 = 6x$  and  $9x^2 - 25 = 0$ . In solving the former, some students developed their own ways, and they simultaneously canceled  $x$  from both sides of the equation without thinking that they are working with a variable. Similarly, when students attempted to solve the quadratic equation in the form of  $x^2 - a = 0$  (where  $a > 0$ ), they obtained only one root and neglected the other, usually the negative one. Similar findings have also been reported by others (Lima, 2008; Tall et al., 2014; Vaiyavutjamai et al., 2005). A reason for this result may be that those students do not realize what finding the roots of a quadratic equation means, how many roots a quadratic equation should have, or under which conditions a quadratic equation has two real solutions (Vaiyavutjamai et al., 2005). Furthermore, students might have scant knowledge about the relationship between square roots and absolute values (i.e.,  $\sqrt{a^2} = |a|$ ,  $a \in R$ ). Thus, students' previous knowledge regarding square root might also cause problems confusions when they are working with variables to solve an (quadratic) equation (Tall et al., 2014).

In general, results of this study indicated that students' difficulties in symbolic-equation context stem mainly from errors in algebraic and arithmetic manipulations, as the data supports that the students knew some rules (or procedures) such as finding the discriminant, quadratic formula or factorization techniques related to solving quadratics. Nevertheless, the results indicate that students mainly focus on the symbols to find the roots of the equations and perceive the quadratic equation as a calculation without thinking about its meaning (Lima, 2008; Tall et al., 2014). Therefore, since students memorize the rules, formulas, and algebraic procedures to solve quadratic equations without understanding the meaning, they could not transfer these rules, formulas, and procedures to solve the quadratic equations with non-standard structured properties. They also have a tendency to forget the formula after some time has passed since they learnt it. In addition, students usually do not think

about alternative techniques for solving quadratic equations in terms of their effectiveness and usefulness in a context and thus choosing one over other possible methods (Bossé & Nandakumar, 2005).

Similar to other findings regarding word problems in linear equations (Clement, 1982; Cummins et al., 1988; Stacey & MacGregor, 2000), the second major observation made from this study is that the quadratic word problems are quite difficult for students. Consistent with Cummins et al. (1988), we found comprehension of the problem statement to be the central reason for students' difficulties with the word problems, rather than cognitive challenges in the solution phase of the symbolic equations. Our data offers evidence that students' difficulties in the comprehension phase were influenced by the structural properties of the word problems. Students in this study experienced difficulties comprehending and interpreting the problem, as well as in representing the relationships symbolically due to complex syntactic structures. Moreover, even though some of the students did comprehend the problem statement fully, they experienced failure because they did not know which mathematical procedures to conduct or how to formulate the correct relationships. Some students' incorrect solutions indicated that they knew which quantity (or quantities) in the problems could be symbolized as the unknown, however, they could not construct certain meaningful relationships in terms of a quadratic equation. This could be explained by the fact that students were required to recognize the underlying structure of relationships between quantities, in order to use algebraic methods (MacGregor & Stacey, 1998). On the other hand, findings regarding the students' incomplete solutions in the word problems presented evidence that some students could not solve the quadratic equations even if they did formulate them correctly. This may well be directly related to student performance in solving quadratic equations through symbolic computation or manipulation. For example, students' incomplete responses commonly displayed that the coefficient and constant of the quadratic equations, which students formulated for solving word problems, included many pairs of factors (e.g., for the third word problem,  $8x^2 + 40x = 2400$  or  $x^2 + 5x = 300$  when simplified). Therefore, factoring such equations might have become challenging as well as time-consuming for the students, when the constant of the quadratic had many pairs of factors (Bossé & Nandakumar, 2005).

In conclusion, the findings of this study have revealed that although students performed better in a symbolic context than in a word-problem context, they had considerable difficulty in solving and/or using quadrat-

ic equations in both contexts. Students performing better in the symbolic equation context compared to the word-problem context can be explained with the reasoning that all students participating in this study had studied the topic through experience (or learning opportunities), emphasizing symbolic or algebraic methods. After all, the instructions they received were mainly based on the symbolic computations involved with quadratic equations in their mathematics courses. However, considering the student difficulties and mistakes reported in previous literature, it becomes obvious that high school students in various countries potentially experience common difficulties and make common mistakes regarding the quadratics. Thus, students' difficulties and mistakes regarding quadratic equations may not solely be attributed to how quadratics is taught. It seems that regardless of the form of instruction, the cognitive processes involved in forming and solving quadratic equations may also affect students' difficulties and mistakes. Thus, the cognitive processes involved in forming and solving quadratic equations in different contexts could be of interest for further research. On the other hand, students' difficulties with quadratics demonstrated in this study, whether in the form of symbolical equations or word problems, could also be explained by students' reliance on rote thinking and reasoning in mathematics (Lithner, 2008). As Lithner (2000) explained, global strategy choices made by students are guided by reasoning based on established experiences, which also tends to dominate students' plausible reasoning at the local level, however, "this causes problems when the familiar routines do not work for different reasons" (p. 187). Thus, it is important to design teaching and learning environments that can actively promote *creative mathematically founded reasoning over rote learning and imitative reasoning* (i.e., *memorized and algorithmic reasoning*) (Lithner, 2008).

The results of the study not only temper the conclusions reported in previous research regarding students' difficulties with quadratic equations, but also extend on previously obtained results by providing a systematic and comprehensive classification of student difficulties in both symbolic and word-problem contexts. Some of students' difficulties reported in this study have not been previously mentioned in the existing literature. As such, the findings of this study add further support to the limited body of research concerning students' difficulties with quadratic equations and to the large body of research noting the comparative difficulty students have with contextualized word problems versus symbolic equations, particularly with linear equations.

The findings of this study imply that it is important for students to make clear connections between symbolic representations and word-problem representations relating to quadratic equations. In order to help students achieve this, teachers need to allocate time in class to practice both symbolic and word-problem representations of quadratic equations. Then, they can introduce students to the strengths and weaknesses of the various approaches to solving quadratic equations and encourage students to use these methods with the awareness of when and why one should be chosen over another (Allaire & Bradley, 2001; Bossé & Nandakumar, 2005). Furthermore, teachers should be aware that solving quadratic equations is not based only on procedures and rules and attempt to seek out alternative ways of teaching quadratic equations. Moreover, students should also be given opportunities to explore a range of situations in which they are required to construct, interpret, and then solve quadratic equations.

The classification of students' difficulties and mistakes that stem from various sources may be of interest to mathematics teacher educators, in-service and prospective mathematics teachers. It is important for teachers to both recognize and understand the fundamental reasons behind students' difficulties, misconceptions and mistakes in algebra. This knowledge would enable teachers to design their lessons in accordance with students' needs; that is, they may develop or select appropriate tasks for students and help them effectively during instruction (Smith, 2001). Mathematics teacher educators can use this classification in pre-service teacher preparation courses, and professional development programs for in-service teachers, in order to increase their knowledge of students' difficulties regarding quadratic equations in both symbolic equations and word-problem contexts.

## References

- Allaire, P., & Bradley, E. R. (2001). Geometric approaches to quadratic equations from other times and places. *Mathematics Teacher*, *94*(4), 308–313.
- Bossé, M. J., & Nandakumar, N. R. (2005). The factorability of quadratics: Motivation for more techniques. *Teaching Mathematics and its Applications*, *24*(4), 143–153.
- Briars, D. J., & Larkin, J. H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction*, *1*(3), 245–296.
- Clark, M. K. (2012). History of mathematics: Illuminating understanding of school mathematics concepts for prospective mathematics teachers. *Educational Studies in Mathematics*, *81*, 67–84.
- Clement, J. (1982). Algebra word problem solutions thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, *13*(1), 16–30.
- Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology*, *20*, 405–438.
- Erbaş, A. K., Çetinkaya, B., & Ersoy, Y. (2009). Öğrencilerin basit doğrusal denklemlerin çözümünde karşılaştıkları güçlükler ve kavram yanılgıları [Student difficulties and misconceptions in solving simple linear equations]. *Education and Science*, *34*(152), 44–59.
- Hinsley, D. A., Hayes, J. R., & Simon, H. A. (1977). From words to equations: Meaning and representation in algebra word problems. In P. A. Carpenter, & M. A. Just (Eds.), *Cognitive processes in comprehension* (pp. 89–10). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Katz, V., & Barton, B. (2007). Stages in the history of algebra with implications for teaching. *Educational Studies in Mathematics*, *66*, 185–201.
- Kieran, C. (1992). Learning and teaching algebra at the middle school through college levels. In D. A. Grouws (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 390–419). New York, NY: Macmillan.
- Koedinger, K. R., & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *The Journal of the Learning Sciences*, *13*(2), 129–164.
- Kotsopoulos, D. (2007). Unravelling student challenges with quadratics: A cognitive approach. *Australian Mathematics Teacher*, *63*(2), 19–24.
- Lima, R. N. (2008, July). *Procedural embodiment and quadratic equations*. Paper presented at the 11<sup>th</sup> International Congress on Mathematical Education (ICME-11), Monterrey, México. Retrieved from <http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2010x-lima-quadratics-draft.pdf>
- Lithner, J. (2000). Mathematical reasoning in school tasks. *Educational studies in Mathematics*, *41*(2), 165–190.
- Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics*, *67*(3), 255–276.
- MacGregor, M., & Stacey, K. (1998). Cognitive models underlying algebraic and non-algebraic solutions to unequal partition problems. *Mathematics Education Research Journal*, *10*(2), 46–60.
- Mayer, E. R. (1982). Memory for algebra story problems. *Journal of Educational Psychology*, *74*(2), 199–216.
- Nathan, J. N., Kintsch, W., & Young, E. (1992). A theory of algebra word problem comprehension and its implications for design of learning environments. *Cognition and Instruction*, *9*(4), 329–389.

Olteanu, C., & Holmqvist, M. (2012). Differences in success in solving second-degree equations due to the differences in classroom instruction. *International Journal of Mathematical Education in Science and Technology*, 43(5), 575-587.

Sağlam, R., & Alacacı, C. (2012). A comparative analysis of quadratics unit in Singaporean, Turkish and IMDP mathematics textbooks. *Turkish Journal of Computer and Mathematics Education*, 3(3), 131-147.

Smith, M. S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: National Council of Teachers of Mathematics

Stacey, K., & MacGregor, M. (2000). Learning the algebraic method of solving problems. *Journal of Mathematical Behavior*, 18(2), 149-167.

Tall, D., de Lima, R. N., & Healy, L. (2014). Evolving a three-world framework for solving algebraic equations in the light of what a student has met before. *The Journal of Mathematical Behavior*, 34, 1-13.

Vaiyavutjamai, P., & Clements, M. A. (2006). Effects of classroom instruction on students' understanding of quadratic equations. *Mathematics Education Research Journal*, 18(1), 47-77.

Vaiyavutjamai, P., Ellerton, N. F., & Clements, M. A. (2005). *Students' attempts to solve two elementary quadratic equations: A study in three nations*. Retrieved from www.merga.net.au/documents/RP852005.pdf.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets & Zeitlinger.

Zakaria, E., & Maat, M. S. (2010). Analysis of students' error in learning of quadratic equations. *International Education Studies*, 3(3), 105-110.

Appendix

Quadratic Word Problems Used in the Study

1. A sewing workshop plans to get an order of 360 (identical) shirts ready in a certain number of days by making a certain number of shirts each day. By sewing 4 more shirts per day than planned, they completed the order a day before it was initially planned. What is the number of days that they initially planned to get the order ready?
2. Side length of a square is equal to the width of a rectangle whose length is 6 meters longer than its width. If the sum of the areas of the square and the rectangle is 176 m<sup>2</sup>, what is the side length of the square?

3. The buses A and B leave a bus station at the same time, one traveling to the north and the other travels to the east. The speed of the bus B is 5 km/h is more than that of the bus A. Determine the (average) speeds of the buses if they are 50 km apart after 2 hours they left the station.
4. The length of a rectangular shaped paperboard is twice its width. An open box is constructed by first cutting out the squares from each corner with side length 5 cm and then folding up the edges as shown in the figure below. If the volume of the box is 3000 cm<sup>3</sup>, find the width of the paperboard before cutting out the squares from each corner.

