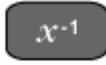# Performing Matrix Operations on the TI-83/84

While the layout of most TI-83/84 models are basically the same, of the things that can be different, one of those is the location of the Matrix key.

For most models, the Matrix menu is found by clicking on [2nd] and [$x^{-1}$], but on some models **Matrix** is its own key. For the rest of this handout we will just say "access the Matrix menu" rather than refer to specific keystrokes. We'll do the same thing for other common operations.

The Matrix menu opens to a screen that looks like this:

```
NAMES MATH EDIT
1:[A]
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```
Names is the menu you use when calling up a matrix to the main screen; Math is for calculation operations, and Edit is for entering the contents of the matrix.

Scroll over to the Edit menu (it looks just like this one, but with the sizes of previously entered matrices listed; if none have been entered, no dimensions are listed). To choose the matrix you want to edit, click on the number of the matrix 1-9 and 0 or scroll down to the desired matrix and press **Enter**.

```
MATRIX[A] 1 ×1
[ 0        ]
```
Enter the size of the matrix you want to work with. For starters, let's work with an augmented matrix for a two variable system of equations:

$$\begin{cases} 2x + 3y = 16 \\ -x + 4y = 10 \end{cases}$$

This matrix will have two rows and three columns. Press 2 Enter and 3 Enter to input the dimensions.

```
MATRIX[A] 2 ×3
[0    0    0   ]
[0    0    0   ]
```
Then enter the values of the matrix entries. By typing the value and then pressing Enter to move to the next entry position. The cursor will naturally move along rows and then return to the next line. You can also use the cursor to move between entries.

```
1,1=0
```

To perform any calculations on this matrix, we'll need to return to the main screen. To **Quit** back to the home screen, press [2nd] [MODE] .

```
MATRIX[A] 2 ×3
[ 2    3    16  ]
[ -1   4    10  ]

2,3=10
```

To change a matrix, just type over the old entries.

Let's try some simple operations.

**Example 1**.  Using the A matrix we've entered, find $A^T$.

```
NAMES MATH EDIT
1:det(
2:T
3:dim(
4:Fill(
5:identity(
6:randM(
7↓augment(
```

To perform this operation, go to the matrix menu and then call up the **[A]** matrix to the main screen by pressing 1 or Enter.  Then return to the Matrix menu and scroll over to Math menu.  The transpose operation is 2 on this list.  Press 2 or scroll down and press Enter to call the operation up onto the main screen.  The screen will appear like this.

$[A]^T$

Now press Enter to execute the operation.

$[A]^T$

```
     [[2  -1]
      [3   4]
      [16 10]]
```

The transpose operation just transforms columns into rows and rows into columns.  This operation is used in calculating inner products and some least-squares problems.

Most of the operations in the Math menu we won't use, but two that we will use regularly are the row echelon form command (ref) and the reduced row echelon form (rref).  We can use these commands to solve systems of equations like the one we started with.  Both these commands will work on matrices that are either square or with more columns than rows.  You will receive an error if you try to reduce ones with fewer columns than rows.

**Example 2**.  Solve the system of equations by row-reducing the matrix **[A]**.

From the main screen, access the matrix menu and scroll over to Math.  The REF command is letter A on the list or you can scroll to find it (it's faster to scroll up) and press Enter to put it on the main screen.  Then, call up matrix A (go to the matrix menu and press enter).

```
NAMES MATH EDIT
0↑cumSum(
A:ref(
B:rref(
C:rowSwap(
D:row+(
E:*row(
F:*row+(
```

ref([A])

Then press Enter.  The calculator will perform a series of row operations.  The calculator does this very algorithmically, and will not work to avoid fractions.  The row echelon form of the matrix is NOT unique, so do not expect it to look like the one you solve by hand.  Only the last row and any pivot positions can be expected to be identical.

```
ref([A])
[[1 1.5 8      …
 [0 1   3.27272…
Ans▶Frac
 [[1 3/2 8     ]
  [0 1   36/11]]
```

The TI's in this model range will typically display decimals as the default, but by pressing [MATH] Enter (▶Frac) to convert the matrix entries into the fractions that are normally desirable.

To run the RREF command, to fully reduce the matrix to a form that *is* unique, the commands are similar to the ones for REF.  Go to the matrix menu and Math.  Press B or scroll to put your

cursor on the command and press Enter.  Then call up the matrix [A] to the screen and press Enter to execute the command.

```
rref([A])
[[1 0 3.0909090…
 [0 1 3.2727272…
Ans▶Frac
  [[1 0 34/11]
   [0 1 36/11]]
```
The matrix is now fully reduced and as before, we can convert decimals to fractions using the convert to fraction command from the Math menu.

Some more recent models have a feature that will allow you to set the output to fractions all the time.  Check the  MODE  menu for this option.

The other important matrix commands we need to know how to do need to be performed only on a square matrix: inverses and determinants.

**Example 3**. Calculate the determinant of the coefficient matrix for the system above.

The coefficient matrix for the system was $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$.  Go into the matrix menu and change the size of the matrix to 2x2.  Because we used the augmented matrix before, the augment should just drop off and leave the values we need.  But, you can reenter the values if need be.

```
det([A])
■
```
                    11

Access the determinant (DET) function from the Matrix Math menu.  Then call up the [A] matrix.  The screen will display the value.

Assuming that the determinant is not zero (which it isn't here), we can also find the inverse of the matrix.

**Example 4**. Find the inverse of the coefficient matrix.

```
[A]⁻¹
[[.3636363636 -…
 [.0909090909 .…
Ans▶Frac
  [[4/11 -3/11]
   [1/11 2/11 ]]
```
This operation is not in the Matrix Math menu like the others.  Instead we use the $x^{-1}$ key.  First call up the matrix from the Matrix menu and then press the inverse key.

As before, use the convert to fraction feature to get rid of the decimals.

```
ERR:SINGULAR MAT
1▪Quit
2:Goto

ERR:INVALID DIM
1▪Quit
2:Goto
```
Suppose I replace the top row with zeros: $\begin{bmatrix} 0 & 0 \\ -1 & 4 \end{bmatrix}$.  You'll get an error.  You'll get this error any time you try to take the inverse of a matrix whose determinant is zero.

If you try to take an inverse of a non-square matrix, you'll also receive an error.  You will also get this dimension error when you try to take

the determinant of a non-square matrix, or multiply matrices with non-matching dimensions.

Another potential pitfall with the inverse function is in taking the inverse of a product of matrices. We'll handle this in a separate example.

**Example 5**. Perform matrix operations on the indicated matrices.

Let $A = \begin{bmatrix} 2 & 3 & 16 \\ -1 & 4 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 9 \\ 1 & 2 \\ -3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 5 & -4 \end{bmatrix}$, $D = \begin{bmatrix} 7 & 3 \\ -2 & 8 \end{bmatrix}$

   a. Calculate AB.

```
[A]*[B]
     [[-45 104]
      [-26  49 ]]
■
```

Enter the two matrices into the calculator as matrix [A] and matrix [B].

Remember that the interior dimensions must match or you'll get the dimension error mentioned in Example 4. Here our dimensions are 2x**3** and **3**x2.

   b. Calculate C+3D.

```
[C]+3[D]
      [[22  9 ]
       [-1 20]]
■
```

Enter the values for matrices [C] and [D] in the matrix menu. Then call up the matrices on the screen in the form of the matrix equation we want to solve. Press Enter. The calculator will perform the operation.

   c. Calculate $C^2$.

```
[C]^2
     [[1   0 ]
      [-15 16]]
```

We can also raise square matrices to powers. Non-square matrices will give a dimension error.

**Example 6**. Combining operations.

Sometimes we can combine multiple operations and sometimes we can't. The insertion of extra sets of parentheses can help.

```
([A]ᵀ[A])⁻¹[A]ᵀ
```

For instance, to solve a least-squares problem we use the equation $A^T A\vec{x} = A^T \vec{b}$, or if the columns of A are linearly independent we can use inverse methods to get $(A^T A)^{-1} A^T \vec{b} = \vec{x}$. However, sometimes entering this equation into the calculator as is can create errors. The calculator sometimes operates ruthlessly left-to-right. If the dimensions of A don't match the dimensions of $(A^T A)^{-1}$, then the operation won't work. Entering an extra set of parentheses around $A^T$ can help, but sometimes you just have to perform the entries one at a time. The calculator can sometimes be a little

quirky. I've gotten this formula to work with some matrices and not others for the same format, so just try not to get frustrated if it doesn't work like you'd expect the first time.

The last example I want to do is from the Matrix Math menu, but it creates a matrix rather than performs operations on a matrix.

**Example 7**. Create a 4x4 identity matrix.

```
identity(4)
      [[1 0 0 0]
       [0 1 0 0]
       [0 0 1 0]
       [0 0 0 1]]
■
```

From the Matrix Math menu select command 5 (or scroll down and choose enter). In parentheses, put the size of the matrix you want. Here, want a 4x4. Since the identity must be square, we only need to enter 4.

Creating an identity matrix of a certain size is useful in calculating certain types of problems, such as the steady state vector of Markov chains.

The TI-83/84 can perform additional functions such as row operations, generating random matrices and other things, but these are these operations aren't used on a regular basis, so check your operator's manual for more details.

**How is this different if I have a TI-89?**

The most of these commands exist in the TI-89. The main difference will be in entering matrices.

Matrices in the TI-89/92 aren't normally named; you can work with unnamed vectors. To enter the matrix from the augmented system we had above, you can enter the matrix in square brackets. Separate the entries by commas and use a semi-colon to break to the next row.

$$[2,3,16; -1,4,10]$$

Beyond that, you'll need to check your operator's manual for more information.