

## Scale

What happens when all the dimensions are doubled?


## In 3-D what happens when all dimensions are doubled?


$2 x$

Surface Area $=6 * x^{2}$
Volume $=x^{3}$
Area: multiplied by $4=2^{2}$
Volume: multiplied by $8=2^{3}$


$$
\text { Surface Area }=6^{*}(2 x)^{2}=24 x^{2}
$$

$$
\text { Volume }=(2 x)^{3}=8 x^{3}
$$

$2 x$

When all linear dimensions are multiplied by 2

| Perimeter | Multiplied by 2 |
| :--- | :--- |
| Area | Multiplied by 4 |
| Surface Area | Multiplied by 4 |
| Volume | Multiplied by 8 |



Table of scale factors
Scale factor of $s$

| scale <br> factor | 2 | 3 | s |
| :--- | :---: | :---: | :---: |
|  | multiplied <br> by | multiplied <br> by | multiplied <br> by |
| Lengths | 2 | 3 | s |
| Areas | 4 | 9 | $\mathrm{~s}^{2}$ |
| Volumes | 8 | 27 | $\mathrm{~s}^{3}$ |

8 Foot Model of DaVinci's Horse



|  | Leonardo's Horse scale factor = 3 |  |
| :---: | :---: | :---: |
| height | 8 feet | 24 feet |
| Length of mane | 4.5 feet |  |
| Size of hoofprint | 48 sq in |  |
| Width of horseshoe |  | 16 inches |
| Weight (volume) |  | 15 tons |

more photos of horse are at: http://www.math.msu.edu/~winter/horse/

## Leonardo's Horse

scale factor $=3$

| height | 8 foot | 24 foot |
| :--- | :--- | :--- |
| Length of mane | 4.5 feet | $3^{*} 4.5=13.5$ |
| Size of hoofprint | 48 sq in | $9^{*} 48=432$ |
| Width of <br> horseshoe | $16 / 3=5.333$ | 16 inches |
| Weight (volume) | $15 / 27=.5555$ | 15 tons |

## Area is multiplied by 9




Suppose both horses are hollow. Fill the 15 -inch model with water, and pour it into the 24 -foot horse. How many "fills" are needed?

15 inches $=1.25$ feet. Scale factor is $s=24 / 1.25=19.2$

This is a volume problem. Answer is $\mathrm{s}^{3}=7077.888$

More Typical Problem - Two similar bricks

| Surface area | 58 sq in | 362.5 sq in |
| :--- | :--- | :--- |
| length | x | 5 |
| width | 28 cu in | y |
| volume | s |  |
| Scale factor <br> Small to large |  |  |

$$
\begin{array}{ll}
\text { Two bricks } & 58 \mathrm{~s}^{2}=362.5 \\
\mathrm{~s}=2.5
\end{array}
$$

| Surface area | 58 sq in | 362.5 sq in <br> $=\mathrm{s}^{2} 58$ |
| :--- | :--- | :--- |
| length | x | $5=\mathrm{s}^{*} \mathrm{x}$ |
| width | 3.5 | y <br> $=\mathrm{s}^{*} 3.5$ |
| volume | 28 cu in | z <br> $=\mathrm{s}^{3 *} 28$ |
| Scale factor <br> Small to large |  |  |

Two bricks

| Surface area | 58 sq in | 362.5 sq in |
| :--- | :--- | :--- |
| length | $\mathrm{x}=5 / 2.5=2$ | 5 |
| width | 3.5 | $\mathrm{y}=3.5^{*} 2.5$ <br> $=8.75$ |
| volume | 28 cu in | $\mathrm{z}=28^{*} 2.5^{3}$ <br> $=437.5$ |
| Scale factor <br> Small to large | $58 \mathrm{~s}^{2}=362.5$ <br> $\mathrm{~s}=2.5$ |  |

## Spheres, Cylinders



Surface area $=4 \pi r^{2}$
Volume $=\frac{4}{3} \pi r^{3}$


Volume =
(Area of base)height
Surface area $=$ $2^{*} A+P^{*} h$ where $P$ is the perimeter of the base and $h$ is the height

## Volume of a Cone



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Cones, Pyramids
(base is circle or polygon)


$$
\mathrm{V}=\frac{1}{3} \mathrm{Ah}
$$

Remember: triangles with same base and height have same area; same is true for pyramids.


Pyramid; Base is a polygon
Can be tipped.
$\mathrm{h}=$ perpendicular height above the base = perpendicular distance E to base ABCD



## Problem

Tennis ball can

Which is greater:
the height of three stacked balls
or
the circumference of the can holding them?
height $=3 \mathrm{~d}$
circumference of can $=$ circumference of ball $=\pi^{\star} d$, which is larger

Tennis ball can

Which is greater:
the height of three stacked balls
or the circumference of the can holding them?

