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Fundamentals of Geometry

Perimeter and Area

1. The perimeter of an object in a plane is the length of its boundary.
    - A circle's perimeter is called its circumference.

2. The area of an object is the amount of surface that the object occupies.

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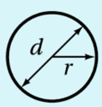
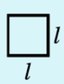
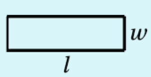
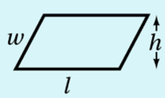
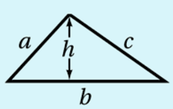
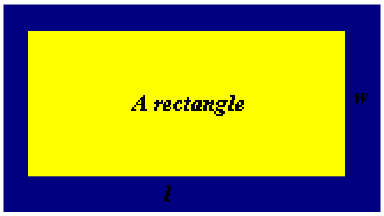
Perimeter and Area - Summary

TABLE 10.2 **Perimeter and Area Formulas for Familiar Two-Dimensional Objects**

OBJECT	PICTURE	PERIMETER	AREA
Circle		$2\pi r = \pi d$	πr^2
Square		$4l$	l^2
Rectangle		$2l + 2w$	lw
Parallelogram		$2l + 2w$	lh
Triangle		$a + b + c$	$\frac{1}{2}bh$

Perimeter and Area
Rectangles



<u>Perimeter</u>

$= l + w + l + w$

$= 2l + 2w$

<u>Area</u>

= length × width

$= l \times w$

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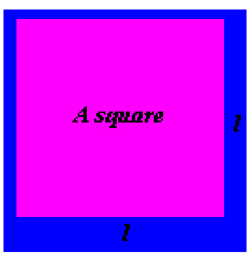
Perimeter and Area
Squares



<u>Perimeter</u>

$= l + l + l + l$

$= 4l$

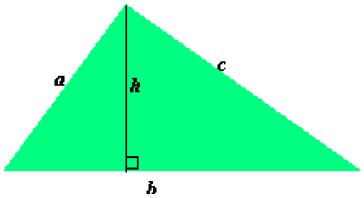
<u>Area</u>

= length × width

$= l \times l$

$= l^2$

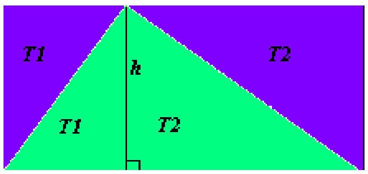
Perimeter and Area
Triangles



Perimeter

$= a + b + c$

Area



$= \frac{1}{2} \times b \times h$

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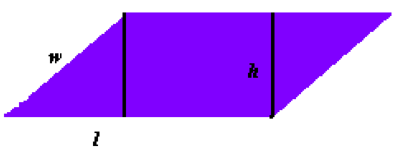
Perimeter and Area
Parallelograms

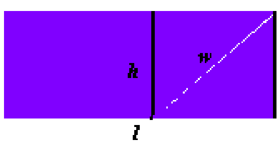
Perimeter

$= l + w + l + w$

$= 2l + 2w$



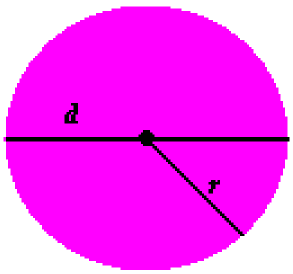
Area



= length × height

$= l \times h$

Perimeter and Area
Circles



Circumference(perimeter)

$= 2\pi r$

$= \pi d$

Area

$= \pi r^2$

$\pi \approx 3.14159\ldots$

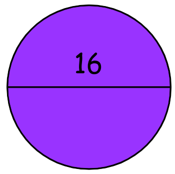
Practice with Area and Perimeter Formulas

Find the circumference/perimeter and area for each figure described:

43/617 A circle with diameter 16 centimeters

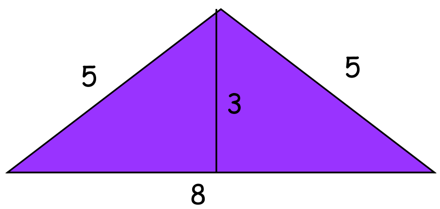
Circumference $= \pi d = \pi \times 16$ cm$=$ 16π cm

Area $= \pi r^2 = \pi \times (16/2 \text{ cm})^2 = 64\pi \text{ cm}^2$



Practice with Area and Perimeter Formulas

55/617 Find the perimeter and area of this triangle

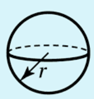
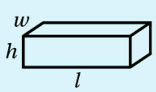
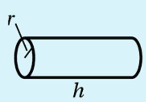


Perimeter = 5+5+8 = 18 units

Area = $\frac{1}{2}$ ×8×3 = 12 units^2

Surface Area and Volume

TABLE 10.3 **Surface Area and Volume Formulas for Familiar Three-Dimensional Objects**

OBJECT	PICTURE	SURFACE AREA	VOLUME
Sphere		$4\pi r^2$	$\frac{4}{3}\pi r^3$
Cube		$6l^2$	l^3
Rectangular prism (box)		$2(lw + lh + wh)$	lwh
Right circular cylinder		$2\pi r^2 + 2\pi rh$	$\pi r^2 h$

Ex. Consider a softball with a radius of 2 inches and a bowling ball with a radius of 6 inhes. Compute the surface area and volume for both balls.

Ex. Which holds more soup -- a can with diameter of 3 inches and height of 4 inches, or a can with diameter of 4 inches and a height of 3 inches?

Ex. An empty water tank is in the shape of a cylinder with a diameter of 15 yards and height 25 yards. Water flows into the tank at a rate of 2500 cubic feet per minute. How many hours will it take to fill up the tank?

Scaling Laws

A **scale factor** is a number which scales, or multiplies, some quantity. In the equation $y=Cx$, C is the scale factor for x. For example, doubling distances corresponds to a scale factor of 2 for distance, while cutting a cake in half results in pieces with a scale factor of ½.

- *Lengths* always scale with the scale factor
- *Areas* always scale with the *square* of scale factor.
- *Volumes* always scale with the *cube* of scale factor.

If your size (height, width, and depth) doubled:

a. By what size has your waist increased?
b. How much more material will be required for your clothes?
c. By what factor has your weight changed?

If a shape doubles, the scale factor is 2; if the shape triples in size, the scale factor is 3, and so on.

If the scale factor is ½ what would be the area of the smaller object, assuming the area of the bigger one is known?

The ***volume rule*** is to multiply the volume of the known times the scale factor^3. Remember you are increasing the length, width, and height of a shape, thus cube ***- ing***.

In example above, what would be the volume of the smaller object, assuming the volume of the bigger one is known?

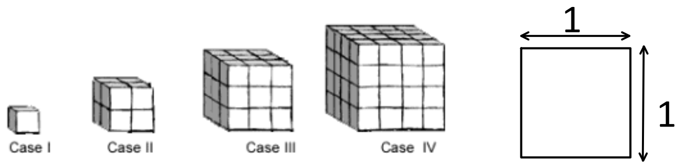


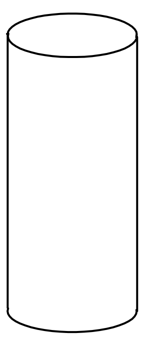
Figure 1: Cubes of varying sizes. The side of the first cube is 1 unit, the second cube is 2 units, the third cube is 3 units and the fourth cube is 4 units.

Parameters	Case I	Case II	Case III	Case IV
Length (L)	**1**	**2**	**3**	**4**
Face Area (L^2)	**1**	**4**	**9**	**16**
Volume (L^3)	**1**	**8**	**27**	**64**
Surface Area (L^2 x 6 faces)	**6**	**24**	**54**	**96**
Area/Volume ratio	**6**	**3**	**2**	**1.5**

Table 1: Cubes with increasing length (L). Comparing the first four cubes: as each unit of length (L) increases from 1, to 2, 3, and 4 for each new set of cubes, their volumes (L^3), increase from 1 to 8, to 27 and then 64. Each time the length doubles, the volume increases by eight-fold. Look at cubes 1 and 3. Each edge on the third cube is three times as long as the edge on the first cube. The third cube has nine times the surface area (L^2 x 6 sides), but 27 times the volume of the first cube. As the volume increases with length (L), mass increases at the same rate.

Ex. If this model is scaled up so that the new height is 17 m, find the surface area and the volume of the new tank.

Model:

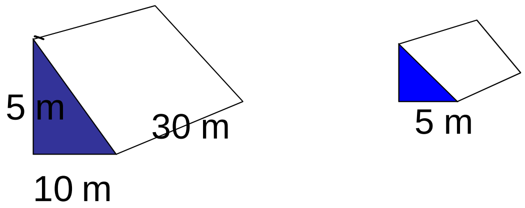


height = 2 m

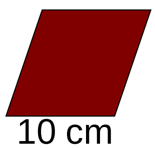
surface area = 82.07 m²

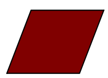
volume = 47.52 m³

If we scale it down so that the height is 0.5 m?



What is the volume of the smaller?





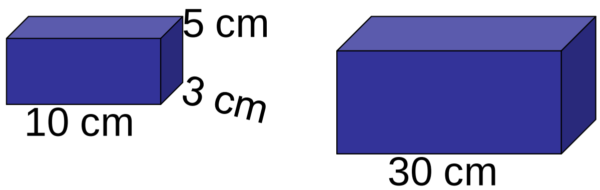
If the scale factor is ½ what would be the area of the smaller?

remember the rule

Area of the first times the scale factor^2

$10 * 10 * (1/2)^2 =$

$100 * ¼ =$ $25\ cm^2$



The scale factor is 3, because the size has tripled. What is the volume of the larger prism?

Hint: volume of small x scale factor^3

$150 \times 3^3 =$ $150 \times 27 =$ $4050\ cm^3$