-9

Perimeter, Circumference, and Area

What You'll Learn

- To find perimeters of rectangles and squares, and circumferences of circles
- To find areas of rectangles, squares, and circles
- ... And Why

To find the amount of fencing material needed to build a fence, as in Example 1

Vocabulary Tip

perimeter of a polygon as

the distance around it and

the area as the number of

square units it encloses.

You can think of the





3. |-2-6| 8

Simplify each absolute value. 1. |4 - 8| 4

Find the distance between the points to the nearest tenth.

2. |10 - (-5)| **15**

4. <i>A</i> (2, 3), <i>B</i> (5, 9)	6.7	

6. *W*(4, -7), *Z*(10, -2) **7.8 8.** *M*(-1, -10), *P*(-12, -3) **13.0** 5. K(-1, -3), L(0,0) 3.2
 7. C(-5,2), D(-7,6) 4.5
 9. O(-8, -4), R(-3, -10) 7.8

Finding Perimeter and Circumference

Hands-On Activity: Finding Perimeter and Area

Draw each figure on centimeter grid paper.

- a rectangle with length 5 cm and width 3 cm
- a rectangle with length 8 cm and height 2 cm
- a rectangle with each side 4 cm
- To find the perimeter of each rectangle, find the sum of the lengths of the sides. Record the perimeter of each rectangle. 1–2. See margin.
- **2.** To find the area of each rectangle, count the number of square centimeters in its interior. Record the area of each rectangle.
- 3. Do rectangles with equal perimeters have the same area? no
- 4. Do rectangles with the same area have the same perimeter? no
- 5. Use a piece of string and make a loop. Tie a slip knot. Adjust the loop and fix its total length at 36 cm. Use the loop to approximate different rectangles on your grid paper. Record their lengths, widths, perimeters, and areas. What do you notice? Check students' work.

active math

Inline

For: Perimeter/Area Activity Use: Interactive Textbook, 1-9 The perimeter P of a polygon is the sum of the lengths of its sides. The area A of a polygon is the number of square units it encloses. For special figures such as squares, rectangles, and circles, you can use formulas for perimeter (called circumference in circles) and area.

Some formulas for perimeter and area are given in the chart at the top of the next page. You will also find the chart on pages 764 and 765 to be useful at times.

Lesson 1-9 Perimeter, Circumference, and Area 61

learning style: verbal

Differentiated Instruction Solutions for All Lea	irners
Special Needs L1 In Example 3, encourage students to first estimate the perimeter. Ask: What is the size of a square unit on the coordinate grid? one square unit Students then shack that the colution is reasonable	Below Level L2 Review the difference between rational and irrational numbers before discussing why π is irrational.

1-9

1. Plan

Objectives

- 1 To find perimeters of rectangles and squares, and circumferences of circles
- 2 To find areas of rectangles, squares, and circles

Examples

- 1 Real-World Connection
- 2 Finding Circumference
- **3** Finding Perimeter in the Coordinate Plane
- 4 Finding Area of a Rectangle
- 5 Finding Area of a Circle
- 6 Finding Area of an Irregular Shape

Math Background

Strictly speaking, a polygon has no area because it is composed only of segments. A polygonal region is the union of a polygon and its interior. You can use Euclidean geometry to derive formulas for the areas of polygonal regions, but you need calculus to find the areas of some nonpolygonal regions.

More Math Background: p. 2D

Lesson Planning and Resources

See p. 2E for a list of the resources that support this lesson.



🧭 Check Skills You'll Need

For intervention, direct students to: Skills Handbook, p. 757

Finding Distance

Lesson 1–6: Example 1 Extra Skills, Word Problems, Proof Practice, Ch. 1

Activity

- 1. 5 cm by 3 cm \rightarrow 16 cm 8 cm by 2 cm \rightarrow 20 cm
- 4 cm by 4 cm \rightarrow 16 cm
- 2. 5 cm by 3 cm \rightarrow 15 cm² 8 cm by 2 cm \rightarrow 16 cm² 4 cm by 4 cm \rightarrow 61 16 cm²

learning style: visual

2. Teach

Guided Instruction

Hands-On Activity

Encourage students to use the term *counterexample* in Exercises 3 and 4.

1 EXAMPLE Error Prevention

Students may think they need to add 3 ft only once to each dimension. Discuss why 3 ft is added twice to each dimension. Have students examine a window frame to help clarify each new length and width.

2 EXAMPLE Teaching Tip

The calculator value for π is used for all the examples and exercises in this lesson.

Additional Examples

Margaret's garden is a square 12 ft on each side. She wants a 1-ft-wide path around the entire garden. What will the outside perimeter of the path be? 56 ft

2 \bigcirc G has a radius of 6.5 cm. Find the circumference of \bigcirc G in terms of π . Then find the circumference to the nearest tenth. **13** π ; about **40.8 cm**

3 Quadrilateral *ABCD* has vertices *A*(0, 0), *B*(9, 12), *C*(11, 12), and *D*(2, 0). Find the perimeter. **34**

SummaryPerimeter and Areasbdshdsddsddsddsddsddsddsddsdd<

The units of measurement for perimeter and circumference include inches, feet, yards, miles, centimeters, meters, and kilometers. When measuring area, use square units such as square inches (in.²), square centimeters (cm²), square meters (m²), and square miles (mi²).

EXAMPLE

Real-World 📢 Connection

Fencing Your pool is 15 ft wide and 20 ft long with a 3-ft wide deck surrounding it. You want to build a fence around the deck. How much fencing will you need?

To find the perimeter of the pool with the deck, first find the width and length of the pool with the deck.

Width of pool and deck = 15 + 3 + 3 = 21

Length of pool and deck = 20 + 3 + 3 = 26

Perimeter of a rectangle = 2b + 2h

20 ft 20 ft 20 ft 20 ft

3 ft

Use the formula for the perimeter of a rectangle.

P = 2(21) + 2(26) Substitute. P = 42 + 52 Simplify.

• You will need 94 ft of fencing.

Quick Check Duppose you want to frame a picture that is 6 in. by 7 in. with a $\frac{1}{2}$ -in. wide frame. a. Find the perimeter of the picture. 26 in.

b. Find the perimeter of the outside edge of the frame. **30 in.**

P = 94

Notice that the formulas for a circle involve π . Since the number π is irrational, $\pi = 3.1415926...,$

you cannot write it as a terminating decimal. For an approximate answer, you can use 3.14 or $\frac{22}{7} \left(3.14 \approx \frac{22}{7}\right)$ for π . You can also use the rounded decimal you get by pressing π on your calculator. For an exact answer leave the result in terms of π .

62 Chapter 1 Tools of Geometry

Differentiated Instruction Solutions for All Lea	rners	
Advanced Learners 4 After students find the perimeter in Example 1, have them find the area of the deck.	English Language Learners ELL Review the terms <i>radius, diameter,</i> and <i>circumference</i> Compare the radius of a bicycle wheel to its diamete Emphasize that <i>circumference</i> is the distance that the wheel rolls in one revolution.	
learning style: verbal	learning style: visual	

Vocabulary Tip

Key Concepts

For a rectangle, "length" and "width" are sometimes used in place of "base" and "height."



EXAMPLE Finding Perimeter in the Coordinate Plane



Algebra Find the perimeter of $\triangle ABC$.		
Find the length of each side. Add the lengths to find the perimeter.		
AB = 5 - (-1) = 6	Use the Ruler Postulate	
BC = 6 - (-2) = 8		
$AC = \sqrt{(5 - (-1))^2 + (6 - (-2))^2}$	Use the Distance Formula.	
$=\sqrt{6^2+8^2}=\sqrt{100}=10$		
AB + BC + AC = 6 + 8 + 10 = 24		
• The perimeter of $\triangle ABC$ is 24 units.		

Quick Check 3 Graph quadrilateral *KLMN* with vertices K(-3, -3), L(1, -3), M(1, 4), and N(-3, 1). Find the perimeter of *KLMN*. See margin.

Finding Area

To find area, you should use the same unit for both dimensions.

You are designing a rectangular banner for the front of

the museum. The banner will be 4 ft wide and 7 yd high.

7 yd = 21 ft Change yards to feet using 1 yd = 3 ft.

How much material do you need?

Finding Area of a Rectangle

EXAMPLE

Area = bh

A = 4(21)

A = 84



Video Tutor Help Visit: PHSchool.com Web Code: aue-0775



56.5 m

• The area of the banner is 84 square feet (ft^2). You need at least 84 ft^2 of material.

Use the formula for area of a rectangle.

Substitute 4 for b and 21 for h.

Quick Check 4 Find the area of the banner in Example 4 by first changing all units to yards. Compare your answer to the one in Example 4. How do they compare? 9¹/₃ yd²; 9¹/₃ is one-ninth of 84.

Guided Instruction

5 EXAMPLE Teaching Tip

Students may think that finding area in terms of π is less accurate than using an approximation for π , when the opposite is true. At this point, encourage students to find area both in terms of π and by using an approximation for π .



Use the figure from Example 6 to remind students that Postulate 1-10, *The area of a region is the sum of the areas of its nonoverlapping parts*, does not apply to perimeter.

Auditory Learners

Have students discuss ways to remember the formulas in this lesson. Encourage suggestions from the class.



To make a project, you need a rectangular piece of fabric 36 in. wide and 4 ft long. How many square feet of fabric do you need? 12 ft²

Quick Check



20 units





 $2.25\pi \,\mathrm{yd}^2$

6 Find the area of the figure below.



Resources

- Daily Notetaking Guide 1-9 13
- Daily Notetaking Guide 1-9-• Adapted Instruction L1

Closure

Find the area and perimeter of the square. Find the area and circumference of the circle in terms of π .



square: 64 cm²; 32 cm; circle: $32\pi \text{ cm}^2$; $8\sqrt{2\pi} \text{ cm}^2$



 $\frac{25}{4}\pi$ ft² **a.** Find the area in terms of π . **b.** Find the area to the nearest tenth. **19.6** ft²

The following postulates are useful in finding areas of figures with irregular shapes.

Key Concepts	Postulate 1-9
	If two figures are
	Postulate 1-10
	The area of a reg

Example 6 applies Postulate 1-10 by summing the areas of the parts of a figure.



64

6.

EXERCISES

For more exercises, see Extra Skill, Word Problem, and Proof Practice.



3. Practice

Assignment Guide

V A B 1	-19, 50, 55	
🛛 А В	20-49, 51-	54, 56-63
C Challenge	е	64-70
Test Prep		71-75
Mixed Revie	W	76-88

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 6, 37, 41, 46, 51.

Visual Learners

Exercises 6, 7 Encourage students to draw the rectangles, write the applicable formula next to each drawing, and label their drawings with the appropriate units.

Exercises 20-26 Use these exercises to highlight the importance of using the same units when working with measurements.

Differentiated Instruction Resources



- 21. 4320 in.² or $3\frac{1}{3}$ yd²
- 22. $1\frac{1}{6}$ ft² or 162 in.²
- 25. 120,000 cm² or 12 m²

Alternative Method

Exercises 37–38 Each figure can be separated in several ways. After students find the areas, have them share with a partner how they separated the figures.

Exercise 58 Once students understand the question, write $x \cdot \Box = (4x^2 - 2x)$ on the board and have them try to fill-in the box. Students should recognize that $x \cdot (4x - 2)$ and $4x^2 - 2x$ are equivalent.

Diversity

Exercise 60 Some students may be unfamiliar with weatherstripping. Invite a student to explain its use.

Exercise 64 If necessary, review the procedure for making tables on a graphing calculator.



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- Sample: For Exercise 44, vou use feet because the bulletin board is too big for inches.
 - 50. a. Yes; every square is a rectangle.
 - b. Answers may vary. Sample: No, not all rectangles are squares.

Find the area of each circle to the nearest tenth. **33.** r = 7 ft **35.** d = 24 cm **34.** d = 8.3 m**36.** *r* = 12 in. 153.9 ft² 54.1 m² 452.4 in.² 452.4 cm² Find the area of the shaded region. All angles are right angles. Example 6 (page 64) 37. 310 m² 38. 80 in.² 20 m 4 in. 18 m 4 in. 8 in 5 m 10 m 5 m 12 in. **Apply Your Skills 39. a.** What is the area of a square whose sides are 12 in. long? **144 in.² b.** What is the area of a square whose sides are 1 ft long? **1 ft²** 39c. There are 144 square c. Reasoning How many square inches are in a square foot? Explain. See left. inches in one square 40. a. Count squares to find the area of the 1 in. foot. A square whose polygon outlined in blue. 30 squares sides are 12 in. long **b.** Use a formula to find the area of each square outlined in red. 16; 9; 4; 1 **c.** How does the sum of your results in part (b) compare to your result in part (a)? Which postulate does this support? They are =. Post. 1-10 **41. Estimation** On a postcard from Mexico, Ky sketched the "footprint" of the pyramid known as El Castillo in the ancient Mayan city Chichen Itza. He said he estimated the three different lengths on each side to be 22 m, 6 m, and 11 m. Use those estimates to estimate the area of El Castillo's footprint. 3289 m² 42-45. Answers may vary. Check students' work. Samples are given. Estimation Estimate the perimeter and area of each object. **39 in.; 93.5 in.² 43.** the front cover of your notebook 42. the front cover of this book $\frac{38 \text{ in.;}}{90 \text{ in.}^2}$ 44. a classroom bulletin board **45.** the top of your desk 12 ft; 8 ft² 8 ft; 3.75 ft² 46. Writing Choose one exercise from Exercises 42–45 and explain why you chose your unit of length. See margin. **47.** The area of an 11-cm wide rectangle is 176 cm². What is its length? **16 cm 48.** The perimeter of a rectangle is 40 cm and the base is 12 cm. What is its area? **49.** A square and a rectangle have equal area. The rectangle is 64 cm by 81 cm. What is the perimeter of the square? 288 cm 50. a. Critical Thinking Can you use the formula for the perimeter of a rectangle to find the perimeter of any square? Explain. See margin. b. Can you use the formula for the perimeter of a square to find the perimeter of any rectangle? Explain. See margin. c. Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter. $A = {\frac{P}{4}}^2$ or $A = {\frac{P^2}{16}}$ **51.** The surface area of a three-dimensional figure is the GPS sum of the areas of all of its surfaces. You can find the surface area by finding the area of a net for the figure. 6 in **a.** Draw a net for the solid shown. Label 8 in. the dimensions. See back of book. **b.** What is the area of the net? What is the surface area of the solid? 208 in.², 208 in.² Chapter 1 Tools of Geometry



Real-World Connection Four 6 in.-by-6 in. tiles will cover 1 ft².







52. Tiling The students in the Art Club are tiling a wall that is 8 ft by 16 ft at the entrance to the community center. They are using tiles that are 6 in. by 6 in. to create a multi-colored design. How many tiles do the students need? **512 tiles**

 x^2 Algebra Draw each rectangle in the coordinate plane. Find its perimeter and area.

53. *A*(−3, 2), *B*(−2, 2), *C*(−2, −2), *D*(−3, −2) **53–54.** See back of book. **54.** *A*(−2, −6), *B*(−2, −3), *C*(3, −3), *D*(3, −6)

Coordinate Geometry On graph paper, draw polygon *ABCDEFGH* with vertices A(1, 1), B(10, 1), C(10, 8), D(7, 8), E(7, 5), F(4, 5), G(4, 8), and H(1, 8).

55. Find the perimeter of the polygon. 38 units

56. Divide the polygon into rectangles. Find the area of the polygon. 54 units²

- **57. Biology** In the Pacific Northwest, a red fox has a circular home range with a radius of about 718 meters. To the nearest thousand square meters, what is the area of the home range of a red fox? **1,620,000 m²**
 - **58.** Multiple Choice A rectangle has a base of x units. The area is $(4x^2 2x)$ square units. What is the height of the rectangle in terms of x? **D** (A) (4 - x) units (B) $(4x^3 - 2x^2)$ units
 - \bigcirc (x-2) units \bigcirc (4x-2) units

Home Maintenance To determine how much of each item to buy, tell whether you need to know area or perimeter. Explain your choice. 59–62. See margin.

- **59.** wallpaper for a bedroom**60.** weatherstripping for a door**61.** fence for a garden**62.** paint for a basement floor
- **63.** Coordinate Geometry The endpoints of a diameter of a circle are A(2, 1) and B(5, 5). Find the area of the circle in terms of π . **6.25** π units²
- **64. Graphing Calculator** You want to build a rectangular corral by using the side of a barn for one side and 100 ft of fencing for the other three sides.
 - **a.** Make a table on your graphing calculator listing integer values for the base and the corresponding values of the height and area.
 - **b.** Make a graph using your table values. Graph the base on the horizontal axis and area on the vertical axis. **a-b. See back of book.**
 - c. What are the dimensions of the corral with the greatest area? 25 ft by 50 ft



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are X(-6, 2), Y(8, 2), and Z(3, 14). $4x^3 - 2x^2$) units 5. Find the area of the figure

 Find the area of the figure below. All angles are right angles. 256 in.²



Alternative Assessment

Have students draw and label a rectangle and a circle, each having an area between 20 and 25 in.² They should include with each drawing a written explanation of how each area can be verified.

- **65.** How many circles with the given radius are needed for the sum of their areas to equal the area of a circle with the second given radius?
 - a. 1 in., 3 in. 9
 b. 2 in., 6 in. 9
 c. 3 in., 9 in. 9
 d. Make a Conjecture How many circles with a radius of *n* in. are needed for the sum of their areas to equal the area of a circle with a radius of 3*n* in.? 9

x^2 Algebra Find the area of each figure.

66. a rectangle with side lengths of $\frac{2a}{5b}$ units and $\frac{3b}{8}$ units $\frac{3a}{20}$ units²

67. a square with perimeter 10n units $\frac{25n^2}{4}$ units²

68. a square with side lengths of (3m - 4n) units $(9m^2 - 24mn + 16n^2)$ units²

Lesson 1-9 Perimeter, Circumference, and Area

nline lesson quiz, PHSchool.com, Web Code: aua-0109

- 59. Area; the wall is a surface.
- 60. Perimeter; weatherstripping must fit the edges of the door.
- 61. Perimeter; the fence must fit the perimeter of the garden.
- 62. Area; the floor is a surface.

4. Assess & Reteach

A rectangle is 9 ft long and 40 in.

1. Find the perimeter in inches.

2. Find the area in square feet.

is 18 cm. Find the area in terms

3. The diameter of a circle

 Find the perimeter of a triangle whose vertices

of π . 81 π cm²



wide.

296 in.

30 ft²

Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 75
- Test-Taking Strategies, p.70
- Test-Taking Strategies with Transparencies

69. Answers may vary. Sample: one 8 in.-by-8 in. square + one 5 in.-by-5 in. square + two 4 in.-by-4 in. squares **69. Open-Ended** The area of a 5 in.-by-5 in. square is the same as the sum of the areas of a 3 in.-by-3 in. square and a 4 in.-by-4 in. square. Find two or more squares whose total area is the same as the area of an 11 in.-by-11 in. square. **See left.**

70. Track An athletic field is a rectangle, 100 yards by 40 yards, with a semicircle at each of the short sides. A running track 10 yards wide surrounds the field. Find the perimeter of the outside of the running track to the nearest tenth of a yard. 388.5 yd



Test Prep

Gridded Response

For Exercises 71 and 72, a rectangular garden has a rectangular walkway around it. The width of the walkway is 8 ft.

- **71.** How many feet greater than the perimeter of the garden is the outside perimeter of the walkway? **64**
- 72. If the garden is a square with a perimeter of 260 ft, what is the area of the walkway in square feet? 2336
- **73.** You need to tile a 12 ft-by-15 ft floor. The color you want allows you the choices found in the table at the right. How many dollars would it cost to tile the floor with 12 in.-by-12 in. tiles? **540**

Size of Tiles	Cost
$12'' \times 12''$	\$3/ft ²
$11'' \times 11''$	\$3/ft ²
$10'' \times 12''$	\$4/ft ²
6'' imes 8''	\$4.50/ft ²

74. How many tiles would cover the 12 ft-by-15 ft floor if you choose the 10 in.-by-12 in. tiles? **216**

75. How many dollars would it cost to cover the 12 ft-by-15 ft floor with the tiles that are 6 in. by 8 in.? **810**

🔵 Mixe	ed Review				
GO for Help	Lesson 1-8	76. The midpoint of \overline{CD} has coordinates (5, 6). Point <i>C</i> has coordinates (-5, -1). Find the coordinates of point <i>D</i> . (15, 13)			
		Find (a) AB to the nearest tenth and (b) the coordinates of the midpoint of \overline{AB} .			
	Lesson 1-7	77. $A(4, 1), B(7, 9)$ 8.5 units; (5.5, 5) 80. $A(0, 1), B(-4, 6)$ 6.4 units; (-2, 3.5) BG is the perpendicular b 83. What is $m \angle BIR$? 90 85. \overline{WR} has length 124. Wh	78. A(0, 3), B(3, 8) 5.8 units; (1.5, 5.5) 81. A(4, 10), B(-2, 3) 9.2 units; (1, 6.5) isector of WR at point I. 84. Name two nat is the length of \overline{IR} ? 62 u	79. $A(9,2), B(-3,9)$ 13.9 units; (3, 5.5) 82. $A(-1,1), B(-4,-5)$ 6.7 units; (-2.5, -2) WI $\cong \overline{RI}$ wo congruent segments. nits	
	Lesson 1-5	For the given coordinates, find PQ.			
		86. <i>P</i> : 12, <i>Q</i> : −6 18 units	87. <i>P</i> : 3, <i>Q</i> : 9 6 units	88. <i>P</i> : -23, <i>Q</i> : 10 33 units	
68 Chapte	er 1 Tools of Ge	eometry			

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