Photon-beam Dose Calculation Algorithms

Narayan Sahoo

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Main sources of the materials included in this lecture notes are:
(1) Radiation Oncology Physics: A Handbook for Teachers and Students Edited by E. B. Podgorsak
(2) The physics of Radiation Therapy by F. M. Khan
(3) Previous lecture notes for this course by Karl Prado, Ph.D.
(4) T. R. Mackie et. al, Chapter in 1996 AAPM Summer School Proceedings

Photon-Beam Dose Calculation Algorithms

- Photon calculation algorithms consist of the mathematical equations that are used to compute dose at any point within a volume irradiated by a photon beam
- Algorithms model the transport of radiation energy within the medium based on the physical interaction processes occurring during irradiation
  - It is necessary to understand their capabilities and recognize their limitations

Photon-Beam Dose Calculations

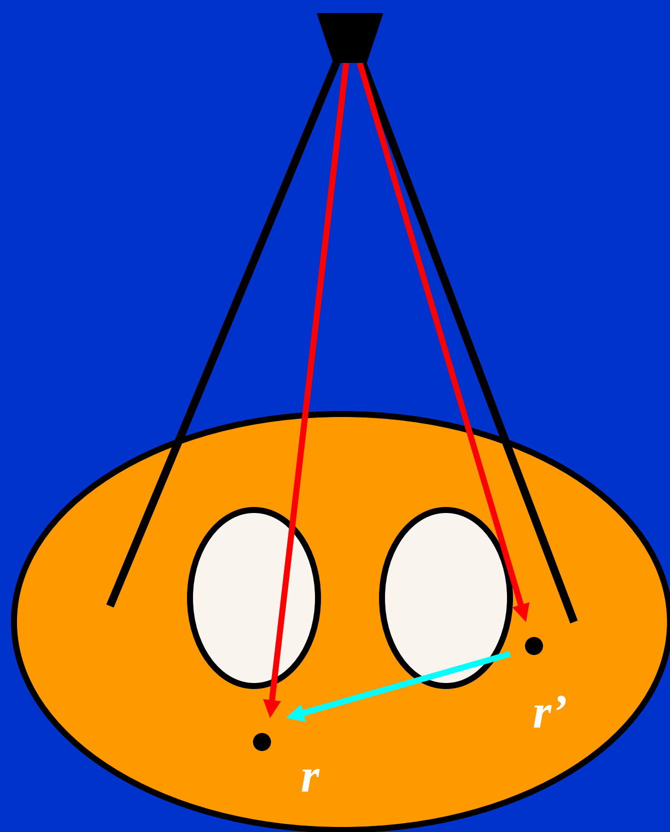
- Calculation Methods: Separation of Primary and Scatter
- Convolution Algorithm - General Form
- Scatter Integration Algorithm - General Form
  - Scatter Integration - Clarkson Integration

Primary and Scatter: Concepts

- Dose can be thought of as the energy deposited by electrons produced in photon interactions
- Dose from "Primary" Interactions
  - Dose produced in interactions by photons originating in the treatment unit itself
  - No (or minimal) scatter component
- Dose from "Scatter" Interactions
  - Dose from scattered photons produced at other points within the irradiated volume

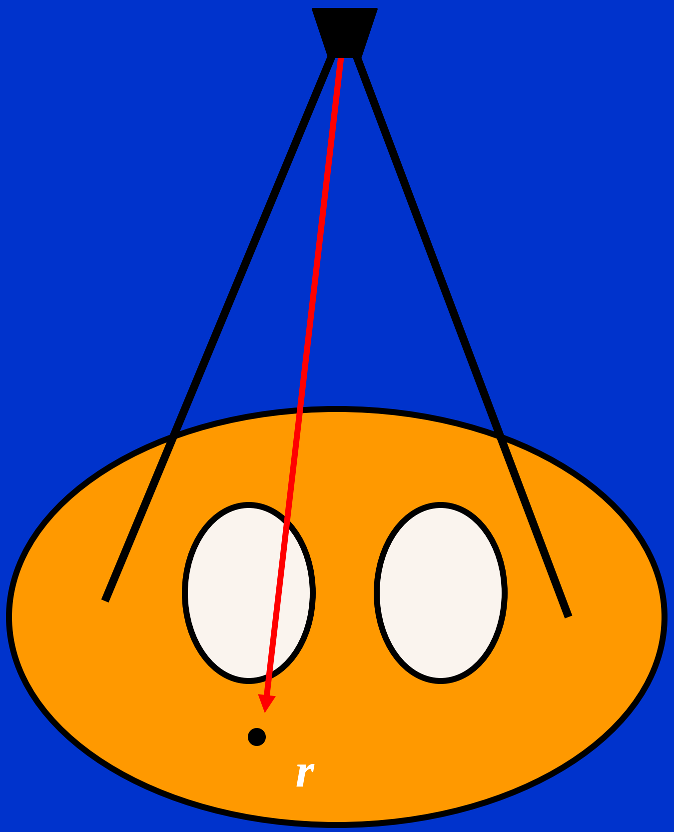
Primary and Scatter: Concepts

- The dose at point "*r*" can be thought of consisting of energy depositions from two classes of events:
  - Interactions from "primary" photons
  - Interactions from scatter photons



Dose from Primary Interactions

- Photons originating in the treatment unit interact in the medium producing electrons that deposit their energy locally (at ***r***):
  - Compute fluence
  - Correct for distance and attenuation
  - Compute energy transfer



Primary Interactions

- Dose from primary source (treatment machine)
  - Fluence (photons per unit area X photon's energy)
    - Effect of beam modifiers (collimation, attenuators, etc.) is taken into consideration:

$$\Psi_0(E) = \int_E \phi_0(E) \times E \times d(E)$$

    - Ray-Trace Attenuation

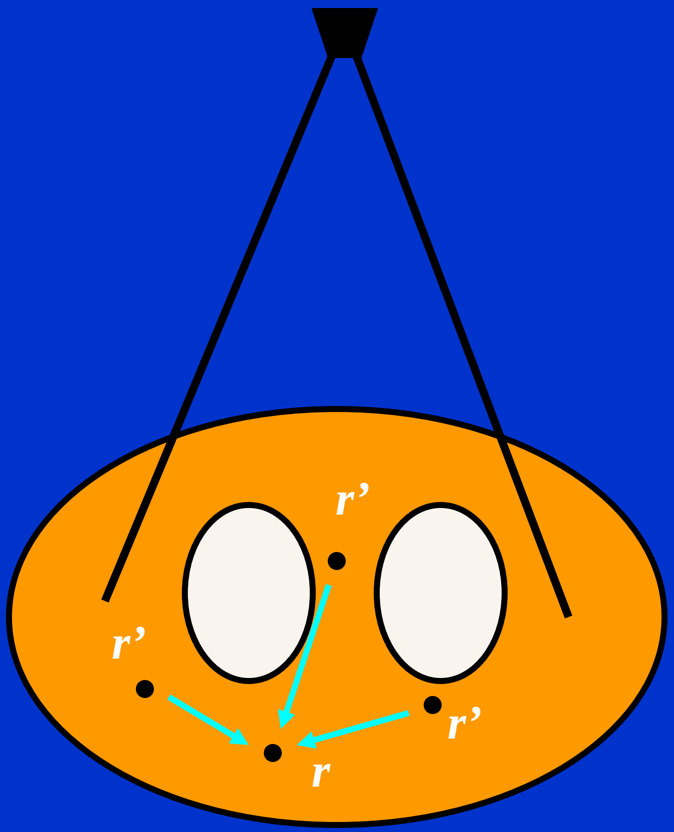
$$\Psi_d(E) = \Psi_0(E) \times e^{-\mu(d-dm)}$$

  - Primary dose (no scatter contribution):

$$D_d = \Psi_d(E) \times \left(\mu_{en}/\rho\right)(E)$$

Dose from Scatter Interactions

- Scattered radiation (secondary) that is produced at other sites (*r'*) in the irradiated volume deposit energy at *r*:
  - Compute the fraction of the energy deposited at some point *r'* that is made available to *r*
  - Repeat for all points *r'* and sum



Scatter Interactions

- Dose from Secondary (Scatter) Radiation:
  - The magnitude of the dose from scattered radiation at some given point can be quantified in a few ways:
    - Convolution Kernels

$$K(r) = \int_{r'} K(r' \rightarrow r)$$

    - Scatter-Air or Scatter-Maximum Ratios (SARs, SMRs)

$$TAR(r,d) = TAR(0,d) + \left(\frac{1}{n}\right)\sum_{i=1}^{n} SAR(r_i,d)$$

  - The dose from scatter is then added to the dose from primary to obtain the total

Convolution Algorithm - General Form

$$D(r) = \int_{r'} \frac{\mu}{\rho}(r') \times \Psi(r') \times \mathrm{K}(r' \rightarrow r)$$

- The incident fluence, Ψ ***(r')***, is projected onto the CT representation, μ / ρ ***(r')***, and is attenuated using a ray-tracing technique
- The available energy is spatially distributed in accordance with the applicable energy deposition kernel ***K (r' $\rightarrow$ r)***

Primary Fluence - $\Psi(r')$

- The primary fluence $\Psi(r')$ is a description of the number and energy of primary photons that exist at the point r'
  - It is the in-air photon fluence that exits the head of the treatment unit, is moderated by beam modifiers, and is subsequently attenuated by the patient
  - It contains all primary radiation output, collimation, inverse-square, off-axis, and beam modifier corrections

Attenuation Coefficient - $\mu / \rho\ (r')$

- Energy is removed from the available primary fluence existing at ***r'*** in proportion to the mass-energy attenuation coefficient, μ_{en}/ρ, at ***r'***
  - The attenuation coefficient, μ_{en}/ρ, is a function of the electron density corresponding to the CT number) of the voxel at ***r'***

TERMA - *T(r')*

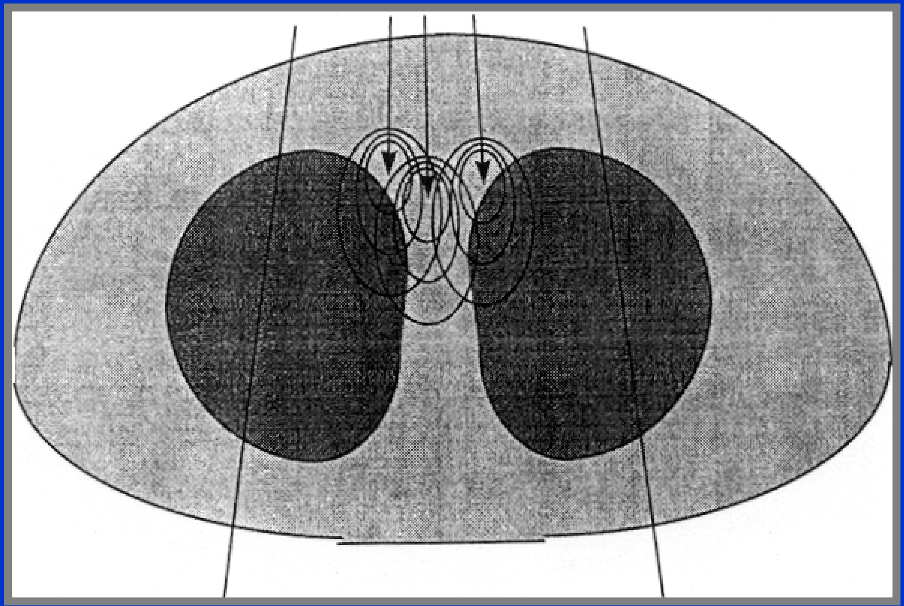
- The product of Ψ ***(r')*** and μ / ρ ***(r')*** is a quantity that represents the total radiation energy released per mass at the point ***r'*** (the "TERMA" at ***r'***)

$$T(r') = \frac{\mu}{\rho}(r')\,\Psi(r')$$

- It represents the total amount of radiation energy available at ***r'*** for deposition

Convolution Kernel *K (r′→ r)*

- The dose-spread kernel, ***K (r′ → r)***, represents the energy distribution from the primary interaction site throughout the volume
  - Simply, ***K (r′ → r)*** is the ratio of the energy deposited at ***r*** to the total energy released at ***r′***

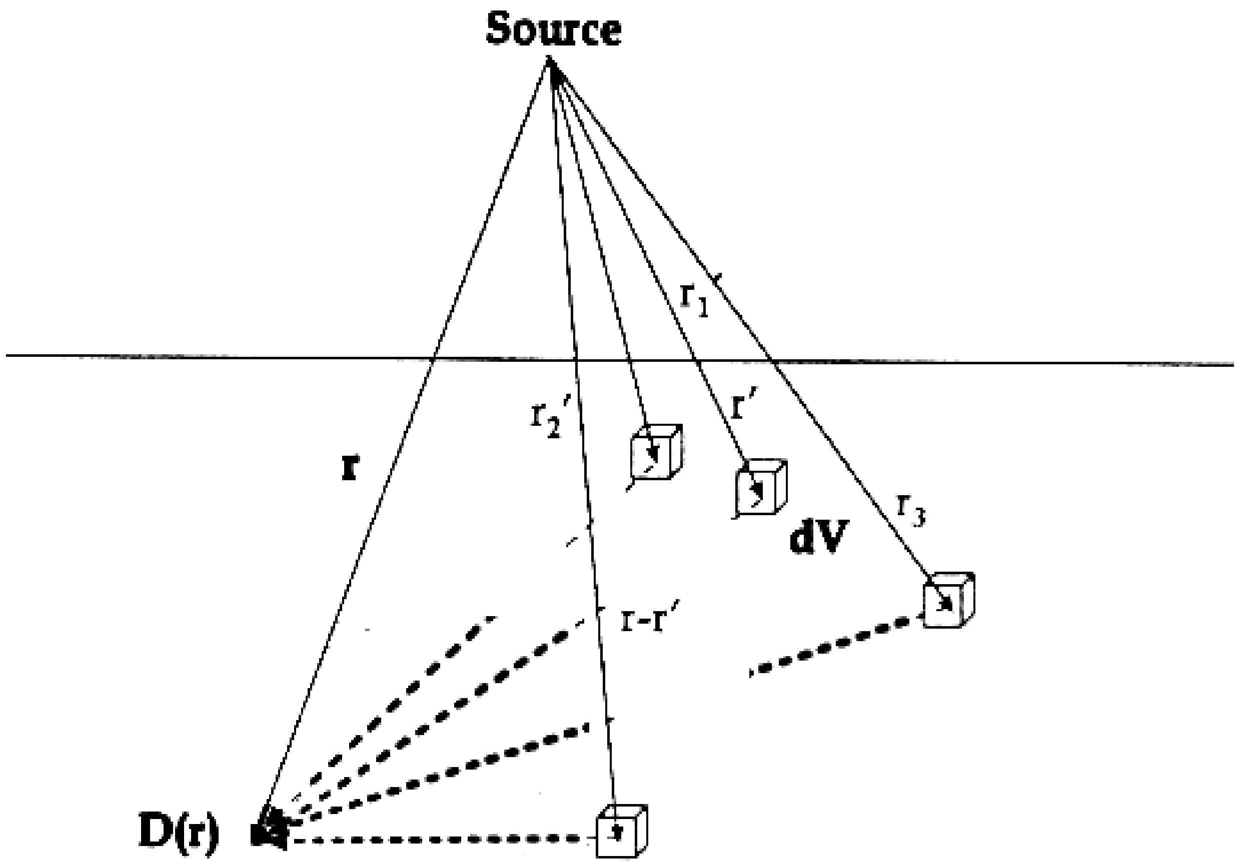


Convolution - Summation

$$D(r) = \int_{r'} \frac{\mu}{\rho}(r') \times \Psi(r') \times \mathrm{K}(r' \rightarrow r)$$

- The TERMA, ***Ψ (r') ´ μ / ρ (r')***, available at all points ***r'*** is deposited at all points ***r*** as given by the energy deposition kernel ***K (r' → r)***
- The total dose at r is then the sum (∫) of the interactions occurring at all points ***r'***
- This process is shown schematically as follows:

Convolution Geometry



$$D(\vec{r}) = \sum \left\{ \frac{\mu}{\rho}(\vec{r}_1')\Psi(\vec{r}_1')K(\vec{r}-\vec{r}_1') + \frac{\mu}{\rho}(\vec{r}_2')\Psi(\vec{r}_2')K(\vec{r}-\vec{r}_2') + \ldots \right\}$$

Nikos Papanikolaou, Ph.D

Scatter Integration Algorithm

- Group conventional dosimetry quantities into "primary" and "scatter" categories:

Phantom-Scatter Dependent	**Phantom-Scatter Independent**	**Possibly Dependent**
PDD / TMR	Inverse-Square	Transmission Factors
Phantom Scatter	Collimator Scatter	Off-Axis Factors

- Phantom-scatter dependent quantities are those exhibiting a relatively strong field-size (in phantom) dependence:
  - ***PDD / TAR / TMR***
  - Phantom Scatter (***S_p*** or ***NPSF***)

Scatter Integration Algorithm - General Form

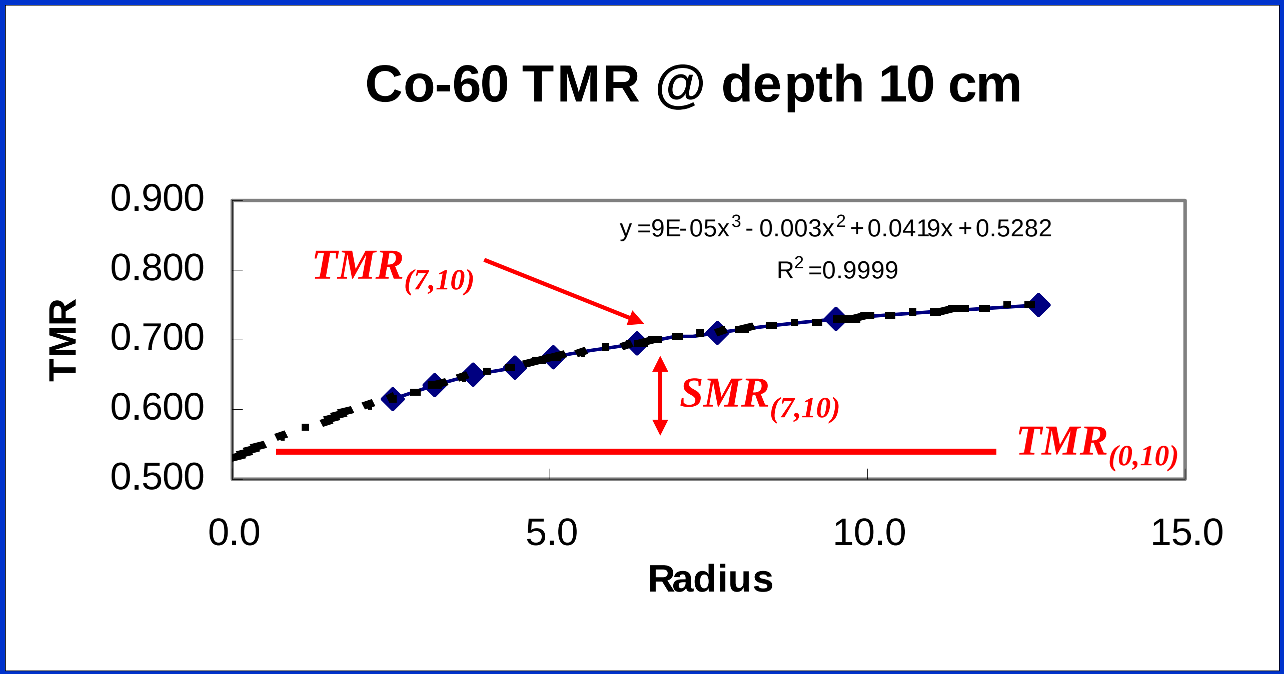
$$D(f,r,d,x) = D_{ref} \times OF_{pri}(r) \times ISF(f+d) \times OAF(x,d) \times T(r) \times OF_{scat}(r) \times [TPR(0,d) + SPR_{avg}(r,d)]$$

Scatter contribution

- Primary and scatter quantities are evaluated and computed separately, and then summed
- The quantities $OF_{scat}(r)$ and $SPR_{avg}(r,d)$ represent scatter in a fashion analogous to the dose-spread kernel $K(r)$

Separation of Primary and Scatter: The "0X0" Field

- Determine "primary" beam effects from full-field data
  - Extrapolation from field-size dependent data:



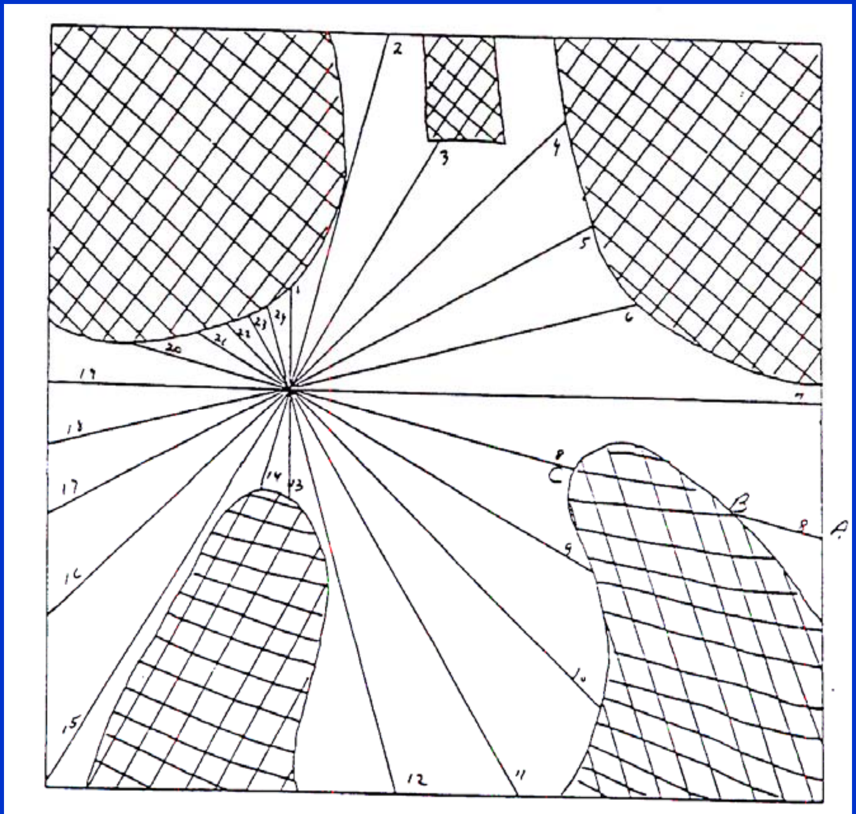
$$r = S \Big/ \sqrt{\pi}$$

Clarkson Integration

- The Clarkson Integration is an application of Scatter Integration concepts
- It has been used, traditionally, to estimate the amount of scatter at any point in a field of irregular shape:
  - The field is divided into a series of "sectors" surrounding the point, each sector represented by a radius r_i
  - The scatter from each sector is determined and a total is obtained by summation

Clarkson Integration

- Field sectors are represented by equally-spaced radii from the calculation point to the edge of the field (either to the collimator jaw or to a block edge)



Clarkson Integration - Example

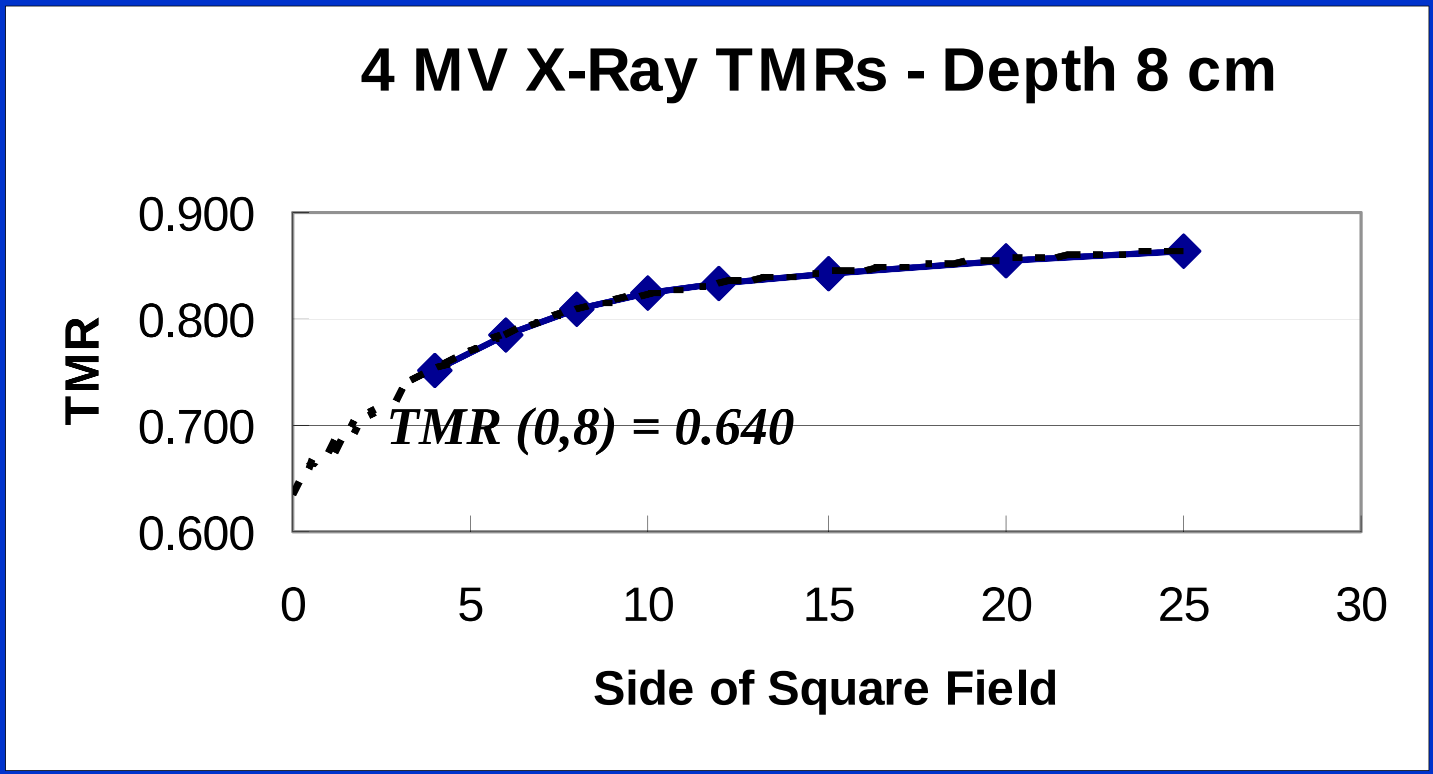
- Create a 4 MV X-Ray Depth 8 cm TMR Table

4 MV X-Ray TMRs – Depth 8 cm

Square Field	TMR
4×4	0.753
6×6	0.785
8×8	0.809
10×10	0.823
12×12	0.834
15×15	0.843
20×20	0.856
25×25	0.863

Clarkson Integration - Example

- Extrapolate $TMR_{d=8\ cm}$ data to obtain ***TMR (0,8)***

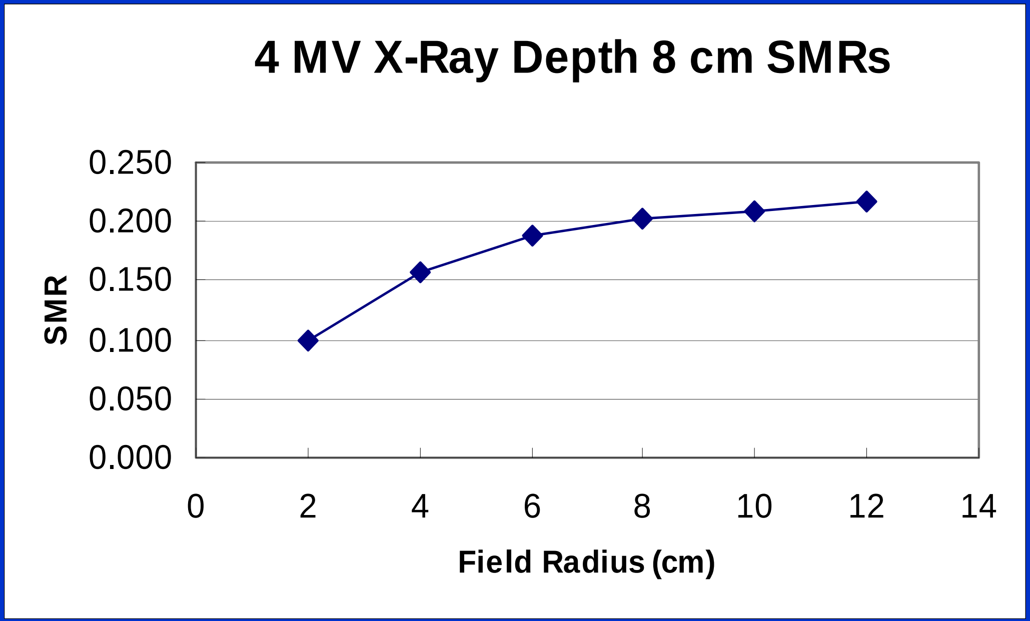


Clarkson Integration - Example

- Calculate and Tabulate $SMR_{d=8\ cm}$:

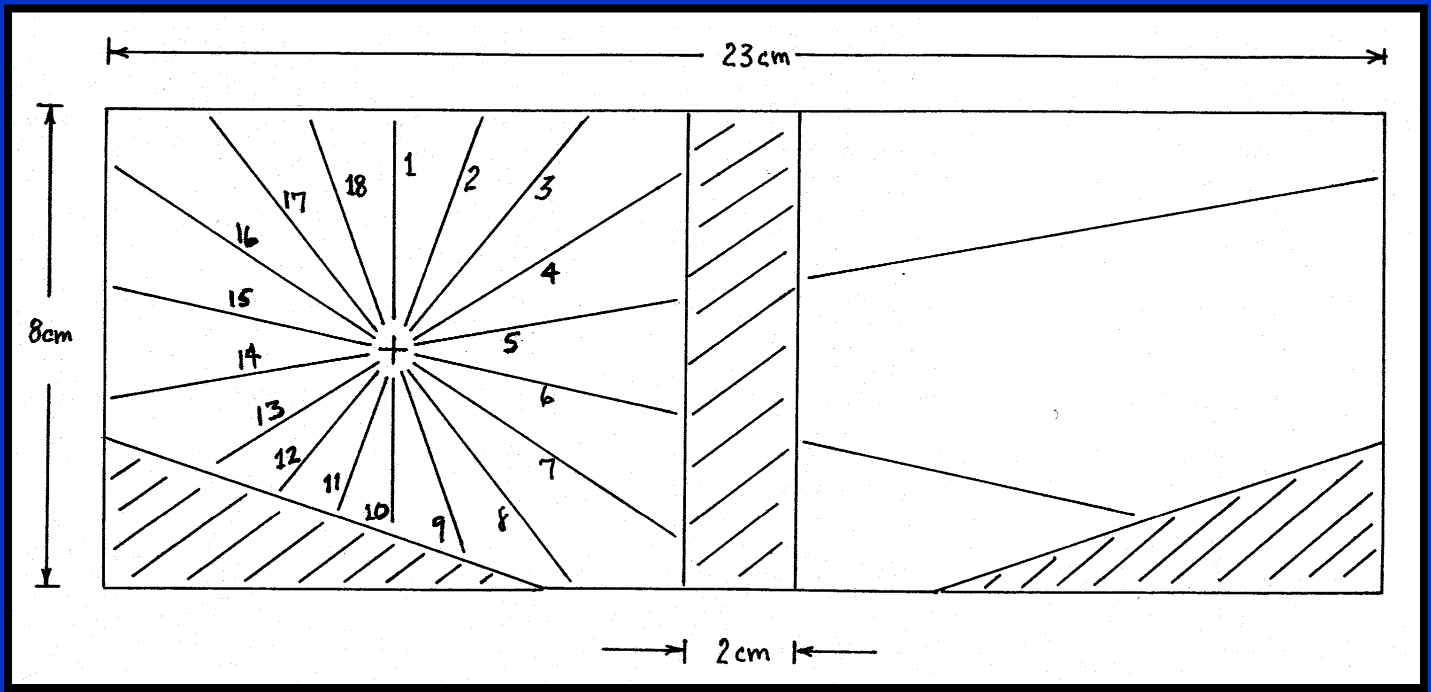
Radius	SMR (r,8)
2 cm	0.098
4 cm	0.157
6 cm	0.189
8 cm	0.203
10 cm	0.209

SMR (r,8) = (TMR (r,8)*S_p(r)/S_p(0))- TMR (0,8)



Clarkson Integration - Example

- Determine the radial distances from the point of calculation to field edges at fixed angular intervals (10 - 20°) and determine *SMR* (r,d_8)



Clarkson Integration - Results:

Sector #	Radius 1	SMR 1	Radius 2	SMR 2	Radius 3	SMR 3	Net SMR
1	4.0	0.157					0.157
2	4.3	0.163					0.163
3	5.4	0.181					0.181
4	8.2	0.203					0.203
5	18.0	0.220	7.4	0.200	5.4	0.181	0.202
6	13.8	0.220	7.5	0.200	5.4	0.181	0.201
7	7.7	0.201					0.201
8	5.3	0.180					0.180
9	3.8	0.153					0.153
10	3.1	0.134					0.134
11	2.9	0.129					0.129
12	3.1	0.134					0.134
13	3.8	0.153					0.153
14	5.3	0.180					0.180
15	5.3	0.180					0.180
16	6.1	0.190					0.190
17	5.4	0.181					0.181
18	4.3	0.163					0.163
	Avg SMR:	**0.174**				**Avg SMR:**	**0.171**
	TMR (0,8):	**0.640**				**TMR (0,8):**	**0.640**
	TMR (r,8):	**0.814**				**TMR (r,8):**	**0.811**

Subtract

Corrections for dose from Clarkson Integration

Beam Modifier Correction

$$WF(d, W) = \frac{\text{Dose with Wedge } (d,W)}{\text{Dose without Wedge } (d,W)}$$

$$\mu'(d, W) = -\frac{1}{x} \ln\left[WF(d, W)\right] \qquad \Phi = \Phi_0 \, e^{-\mu' x}$$

Similar attenuation to account for block attenuation

Contour Corrections

Effective attenuation method

Tissue-air or TPR or TMR ratio method

Isodose shift method

Contour Corrections

Effective attenuation method

CF = exp (-μx), where x is depth of missing tissue above the calculation point

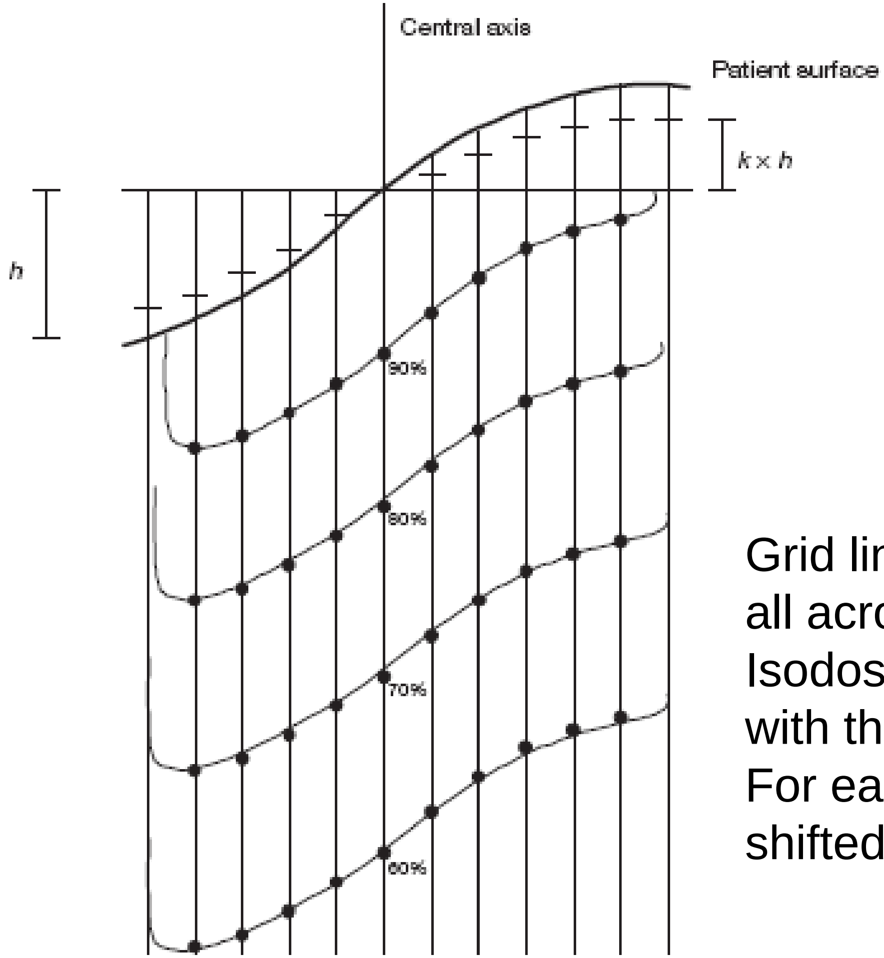
μ is lienar attenuation coefficient of tissue for a given energy

Tissue-air or TPR or TMR ratio method

$$\text{CF} = \frac{\text{TAR}(z-h, A_Q)}{\text{TAR}(z, A_Q)}$$

TPR or TMR can be used in place of TAR

Contour Corrections



Photon energy (MV)	k (approximate)
<1	0.8
^{60}Co–5	0.7
5–15	0.6
15–30	0.5
>30	0.4

Grid lines are drawn parallel to the beam CAX all across the field
Isodose lines for a flat phantom is aligned with the central axis on the patient contour
For each grid line, the overlaid isodose lines are shifted up or down by an amount of k x h

Corrections for tissue inhomegeneities

Presence of inhomogeneities in patients leads to
1. Changes in the absorption of primary beam
2. Changes in scatter photon distribution
3. Changes in secondary electron fluence

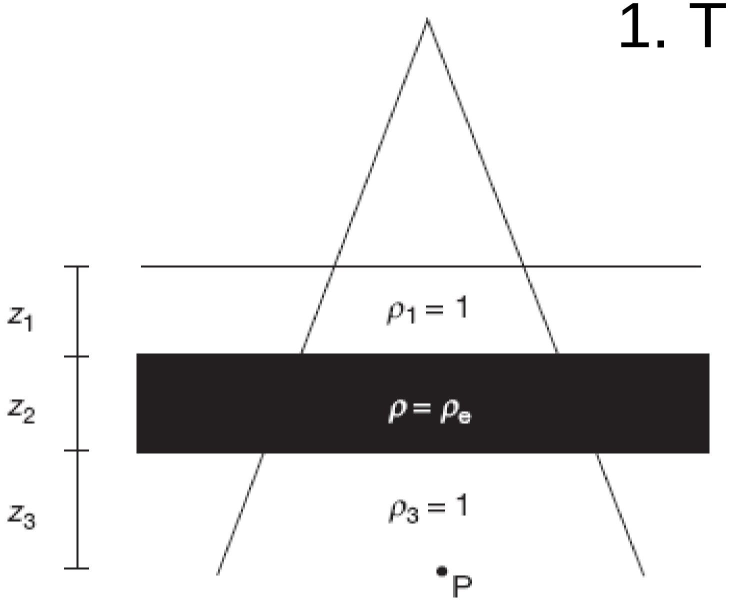
Changes depend upon the location of the point of interest relative to the inhomogeneity

In MVX beam Compton interaction dominates and the interaction cross-section depends on electron density of the media

Low energy x-rays, photoelectric effect can lead to higher dose to high Z-material like bone

1. An effective depth can be used for attenuation of the primary fluence
2. Near the interface, there may be loss of CPE

Corrections for tissue inhomegeneities



1. Tissue-air ratio method

$$CF = \frac{TAR(z', r_d)}{TAR(z, r_d)}$$

Where

$$z' = z_1 + \rho_e z_2 + z_3 \qquad z = z_1 + z_2 + z_3$$

Does not account the position relative to inhomegeneity
Assumes infinity lateral extension

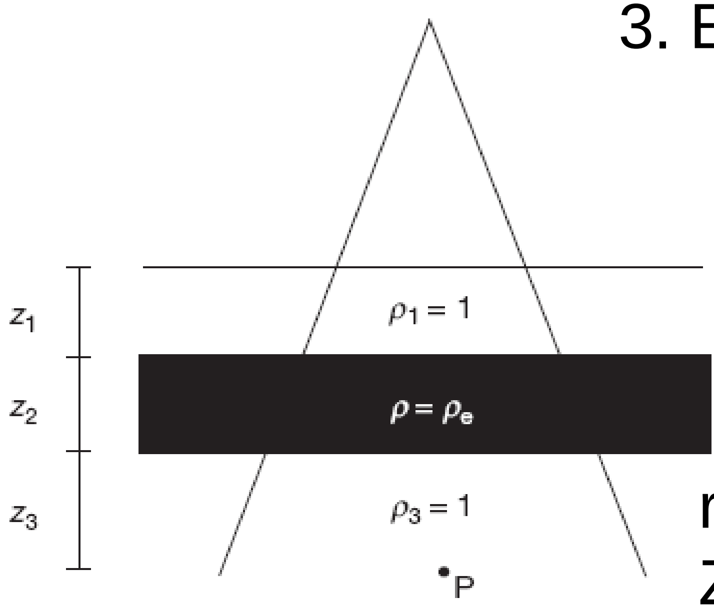
2. Batho Power law method

$$CF = \frac{TAR(z_3, r_d)^{\rho_3 - \rho_2}}{TAR(z, r_d)^{1-\rho_2}}$$

$$z = z_1 + z_2 + z_3$$

Takes into account the position relative to inhomegeneity
Assumes infinity lateral extension

Corrections for tissue inhomegeneities



3. Equivalent Tissue-air ratio method

$$CF = \frac{TAR(z', r_d')}{TAR(z, r_d)}$$

Where

$$z' = z_1 + \rho_e z_2 + z_3 \qquad z = z_1 + z_2 + z_3$$

r_d is beam dimension at actual depth Z

$r_d' = r_d \times \tilde{\rho}$ where $\tilde{\rho} = \frac{\sum\sum\sum \rho_{ijk} \times W_{ijk}}{\sum\sum\sum W_{ijk}}$

ρ_{ijk} is the relative electron density of the scattering element

W_{ijk} is the weight factor assigned to these element and is a function of the distance and angle relative to the calculation point, and its photon fluence

i, j, k corresponds to the x, y, z coordinate of the voxel. These are calculated using Compton scatter cross-sections and integration over entire volume

Corrections for tissue inhomegeneities

4. Isodose shift method
Isodose curves beyond the inhomogeneity are moved by an amount given by "n" times the thickness of the inhomgeneity
Towards the skin for high density material
Away from the skin for low density material

For 4 MV x-ray:
n = -0.6 for air cavity
n = -0.4 for lung
n = 0.5 / 0.25 for hard / spongy bone

Photon Algorithms: Summary

- Calculation algorithms are the mathematical equations used to compute dose within an irradiated volume
- Algorithms <u>model</u> radiation transport
- Algorithms differ in their calculation methods though their underlying principles are essentially the same
- An understanding of their capabilities is necessary in order to recognize their limitations