PHY 132
ELECTRICITY, MAGNETISM AND MODERN PHYSICS

NATIONAL OPEN UNIVERSITY OF NIGERIA

PHY 132: ELECTRICITY, MAGNETISM AND MODERN PHYSICS

COURSE GUIDE

NATIONAL OPEN UNIVERSITY OF NIGERIA

1.0 Introduction

PHY132 electricity, magnetism and modern physics is a one semester 2 credits, foundation level course. It will be available to all students to take towards the core module of their B.Sc. Education, and other programmes B.Sc computer science, environmental studies and

The course comprises 20 study units (4 modules), which involve basic principles of Electricity, Magnetism and Modern Physics. The material ha been developed in such a way that students with at least a credit pass at the ordinary level of equivalent will follow quite easily.

There are no compulsory pre requisites for the course. However, you are strongly advised to have adequate grasp of Further Mathematics or Applied Mathematics.

This course guide tells you briefly what the course is about, what course materials you will be using and how to work your way through these materials. Is suggests some general guidelines for the TIME to complete it successfully. It also gives you some guidance on your tutor-marked assignments.

There are regular tutorial classes that are linked to the course. You are advised to attend these sessions regularly. Details of time and locations of tutorials will be given to you at the point of registration for the course.

2.0 What You Will Learn In This Course

The overall aim of PHY132 is to introduce the basic principle and application of Electrical Energy and its association with Magnetism. During the course you will learn that an electric field is always associated with a magnetic field and vice versa. You would see that this bond between Electricity and Magnetism is the basis of many scientific and technology developments during the last century.

Towards the end of the course you will be introduced into some aspects of Modern Physics where we have introduced some new concepts to explain sub-atomic phenomena. These include quantum theory, and energy levels in atoms.

3.0 Course Aims

The aim of this course can be summarized as follows: this course aims to give you an understanding of Electricity, Magnetism and Modern Physics and their applications in everyday life. This will be achieved by

- Introducing you to the fundamentals of Electricity, Magnetism and Modern Physics as subjects on their own right.
- Demonstrating how the various theories can be applied to real life situations.

- Explaining some fundamental concepts in Electricity, Magnetism and Modern Physics.
- Explaining the transistion from Newtonian Mechanics to Quantum Mechanics.
- Giving you some insight into possible future development in

these areas.

4.0 Course Objectives

The course sets overall objectives, to achieve the aims set out above.

In addition, each unit also has specific objectives. The unit objectives are always included at the beginning of a unit; you should read then before you start working through the unit. You may want to refer to them during your study of the Unit to check your progress. You should always look at the Unit objectives after completing a unit. In this way you can be sure that you have done what was required of you for the unit.

Set out below are the objectives of the Course as a whole. By meeting these objectives you should have achieved the aims of the Course as a whole.

On successful completion of the Course, you should be able to:

1. Describe the theory of electricity, magnetism and electromagnetic radiation
2. Explain the concepts of electric and magnetic fields.
3. Measure and compute electric current in d.c and a.c. circuit.
4. Illustrate the principles of electromagnetic induction as they apply to both d.c. and a.c. generators.
5. Demonstrate how circuit elements are connected.
6. Describe the principles of cathode ray oscilloscope, ammeters, voltmeters, x-ray tubes and dry cells as well as accumulators.
7. Identifying the advantages and disadvantages of x-rays
8. Describe the structure of the nuclear atom.
9. Distinguish between geographic and geomagnetic meridians.
10. Describe the terrestrial magnetic field.
11. Distinguish between nuclear fusion and nuclear fission.

12. Describe the generation and distribution of electric power.

5. 0 Working Through This Course

To complete this course you are required to read the study units, read set books and read other materials provided by NOUN. You will also need to do some practical exercise which will be arranged by your Course Tutor. Each unit contains self-assessment exercises, and at points in the course you are required to submit assignments for assessment purposes. At the end of the course, there is a final examination. The course shall take you about 45 weeks in total to complete . Below you will find listed all the components of the course, what you have to do and how you should allocate your time to each unit in order to complete the course successfully and on time.

6.0 Course Materials

1. Course guide
2. Study units
3. Assignment file
4. Presentation schedule

7.0 Study Units

There are 20 Study Units in this Course, as follows:

Unit 1 Electric charge, Force and Field

Unit 2 Gauss's Law

Unit 3 Electric Potential

Unit 4 Potential for Continuous Charge Distribution And Energy

Unit 5 Dielectrics and Capacitors

Unit 6 Electric Current

Unit 7 Direct-Current Circuits and Instruments

Unit 8 The Magnetic Field

Unit 9 Motion of Charge Particles in Electric and Magnetic Field

Unit 10 Electrolysis and Cells

Unit 11 Thermal Effects of Electric Currents And Electric Power

Unit 12 Magnetic Properties of Matter

Unit 13 Terrestrial Magnetism

Unit 14 Electromagnetic Induction I

Unit 15 Electromagnetic Induction 11

Unit 16 Alternating Current Theory 1

Unit 17 Alternating Current Theory 11

Unit 18 Thermoelectric, Photoelectric Thermionic Effects

Unit 19 Modern Physics 1

Unit 20 Modern Physics 11

Each study unit consists of two to three weeks' work, and includes specific objectives. Each unit contains a number of self-tests. In general, these self-tests, question you on the material you have just covered or require you to apply it in some way and, thereby, help you to gauge your progress and reinforce your understanding of the material. Together with tutor-marked assignments, these exercises will assist you in achieving the stated learning objectives of the individual units and of the course.

8.0 Set Textbooks

Duncan Tom (1982) Physics. A Textbook for Advanced Level Students John Murray (Publishers) Ltd. London.

S.M. Geddes (1981) Advanced Physics. Macmillan Education Ltd. London McKenzie A.E.E (1973) A Second Course of Electricity. The University Press, Cambridge

9.0 Assignment File

The assignment file will be supplied by NOUN. In this file you will find all the details of the work you must submit to your tutor for marking. The marks you obtain for these assignments will count towards the final mark you obtain for this course. Further information on assignments will be found in the assignment file itself and later in this course guide in the section on assessment.

10.0 Presentation Schedule

The presentation schedule included in your course materials may show the important dates for the completion of tutor-marked assignments. Remember, you are required to submit all your assignments by the due date as dictated by your facilitator. You should guide against falling behind in your work.

11.0 Assessment

There are two aspects to the assessment of the course. First are the tutor-marked assignment; second, there is a written examination.

In doing the assignment, you are expected to apply information, knowledge and techniques gathered during the course. The assignments must be submitted to your tutor for formal assessment in accordance with the dead-lines stated in the presentation schedule and the assignment file. The work you submit to your tutor for assessment will count for 40% of your total course work.

At the end of the course you will need to sit for a final written examination of three hours 'duration'. This examination will also count for 60% of your course mark.

12.0 Tutor-Marked Assignments (TMA)

The TMAs are listed as item 6.0 in each unit. Generally, you will be able to complete your assignments from the information and martial contained in the study units, set books and other reading. However, it is desirable in all degree level education to demonstrate that you have read and researched more widely than the required minimum. Using other references will give you a broader viewpoint and may provide a deeper understanding of the subject.

When you have completed each assignment, send it, together with a TMA form, to your tutor. Make sure that each assignment reaches your tutor on or before the deadline given in the presentation schedule and assignment file. If, for any reason you cannot complete your work on time contact your tutor before the assignment is due to discuss the possibility of an extension. Extensions will not be granted after the due date unless there are exceptional circumstances.

13.0 Final Examination and Grading

The final examination for PHY 132 will be of three hours duration and have a value of 60% of the total course grade. The examination will consist of quantities which reflect the types of self-testing practice exercises and tutor-marked problems you have previously encountered. All areas of the course will be assessed.

You are advised to use the time between finishing the last unit and sitting the examination to revise the entire course. You might find it useful to review your self-tests, tutor-marked assignments and comments on them before the examination.

14.0 Course Marking Scheme

The following table shows how the actual course marking is broken down.

Assessment	Marks
Assignments	40% of course marks
Final examination	60% of overall course marks
Total	100% of course marks

Table 1 course marking scheme

PHY 132: ELECTRICITY, MAGNETISM AND MODERN PHYSICS

COURSE DEVELOPMENT

Course Developer
Fred Ebunu

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NATIONAL OPEN UNIVERSITY OF NIGERIA

UNIT 1

ELECTRIC CHARGE, FORCE AND FIELD

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1.0 Introduction

Lightning and thunder are two common phenomena in our hot and humid atmosphere in Nigeria. Have you ever given a thought to what is responsible for the occurrence of the phenomena?

A physicist, Benjamin Franklin demonstrated as long ago as 1752 that thunder clouds are charged with electricity. These charged clouds, when discharged in the atmosphere, give rise to a great spark, which is referred to as lightening. It will interest you to know that the amount of electric current during the discharge is about 20KA.

The electric discharge which gives rise to lightning also produces a great amount of heat. In a fraction of a second, temperature rises to about 15000^0C. The lightening develops in a small area which is about 20cm in width. However, as a result of the heat amount of great produced in that small area the air molecules move fast and cause the intense sound which we call thunder. When the sound is reflected by clouds, hill or any other obstacle, we hear the roaring of clouds.

A very important thing about electric charges is that the forces between them are very large. The force is known as electrostatic force (or electric force) and is responsible for holding electrons to nuclei to form atoms and for holding the groups of the atoms together to form molecules, solids and liquids.

The study of these static charges is known as electrostatic. Indeed, electrostatic was the first branch of electricity to be investigated and, for some time, it was regarded as a subject which had no practical value. However, it is now known to have practical industrial applications. For example, we shall see later in this course that a knowledge of electrostatic is important in the design of cathode ray tubes for television, in electrical prospecting for minerals. Electrostatic loudspeakers and microphone are in common use as well as electrostatic photocopying machines.

2.0 Objectives

After studying this unit, you will be able to:

* Distinguish between the two types of electric charge
* Show that the total electric charge in an isolated system is conserved
* State Coulomb's law and use it to find the electrostatic force between two charges
* State the superposition principle
* Calculate the vector sum of the electric field strength due to a number of point charges.
* Sketch the field lines for some simple distribution of charge.

3.1 Electric charge

3.1.1 Types of charges

The ancient Greeks discovered that amber when rubbed with silk acquired the property of attracting light objects such as pieces of chaff . William Gilbert discovered that other substances exhibit the same effect, and that the magnitude of the effect is roughly proportional to the area of the surface rubbed. He was then led to the idea of a charge of electricity.

Du Fay (1745) discovered that there are two kinds of electricity. Two ebonite rods when rubbed with fur exert a force of repulsion on each other. Two glass rod rubbed with silk also repel one another. However, an ebonite rod which has been rubbed with fur attracts a glass rod which has been rubbed with silk. Any substance rubbed with a different substance acquires a charge of electricity, and is found either to repel charged ebonite and attract charged glass, or vice versa. Since the two kinds of electricity can neutralize each others effect, one is called positive and the other negative. Note that the choice as to which is positive was purely arbitrary. Glass rubbed with silk is said to have a positive charge and ebonite rubbed with fur a negative charge.

The origin of the +ve and –ve charge of proton and electron

The law of force between charges may be stated as follows: like charges repel, unlike charges attract

Let us now understand clearly the origin of the two types of charges; we remember that an atom consists of a positively charged nucleus with negatively charged electrons around it. The nucleus is made up of proton and electron . The neutron is neutral (no charge) which the proton and electron have equal but opposite charges (positive and negative).

The proton and neutron in the nucleus are held together very tightly by a nuclear force. So strong is the nuclear force that the protons are un able to move away from the nucleus. On the other hand, the force holding electron to the atomic nucleus is much weaker than the nuclear force. Hence the electrons may move away from the atom.

When two different materials are rubbed together, electrons get transferred fairly easily from one material to the other. Since some materials tend to hold their electrons more strongly than others, the direction of transfer of electrons depends on the materials. For example, when a plastic ruler is rubbed with a woolen cloth, electrons flow from wool to plastic, so that it carries net negative charges whereas the wool, with a deficit of electrons, carries a positive charge of equal magnitude.

This process of charging the bodies by means of rubbing them together is called charging by friction. In any case, we should note that friction actually has nothing to do with the charging process. It would appear that friction is only borrowed to describe the rubbing process.

Question

There are two charged bodies, x and y which attract each other. X repels a third charged body Z. Do you think z will attract or repel Y?

3.1.2 Unit of Charge

In the System International (SI), electric charge is measured in coulombs (C), which is defined in terms of ampere. A coulomb is the quantity of charge flowing per second through a conductor in which there is a steady current of 1A.

Note:
The definition of ampere involves force between currents. We shall see this is Modules 3.

A Coulombs is the amount of charge that flow through a cross-section of a wire in one second if there is a steady current of one ampere (IA) in the wire. In symbols,

$$q = It \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

Where q is in coulombs, if I is in ampere a.d t is in seconds. The main reason for defining the coulomb in terms of ampere is that it is easy to maintain, control and measure a current through a conductor rather than the amount of charge.

3.1.3 Conservation of Charge

In the method of charging by friction (rubbing) which is discussed in section 3.1, no new charges are created. The algebraic sum of the individual charges, that is the net charge, always remains constant. Let us see how this is the case. Before the process of rubbing, the two bodies were electrically neutral (having no charge). Therefore, the total charge is zero. After rubbing, one body becomes negatively charged while the other acquires a positive charge of equal magnitude. In effect, the algebraic sum of the equal and opposite charges on the two bodies is zero.

This shows that electric charge is a conserved quantity. In other words, conservation of charge implies that the total charge in an isolated system does not change. You should note that this does not mean that the total amount of positive or negative charge in a system is fixed. What we are saying is that for every additional positive charge created, there is always an equal amount of negative charge created.

The charge conservation law may be stated as follows:

The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, does not change

3.1.4 Quantization of charge

The smallest charge that is possible to obtain is that of an electron or proton. The magnitude of this charge is denoted by e. A charge smaller than e has not been found. If one determines the amount of charge on any charged body (e.g. a charge sphere) or any charged particle (e.g. α-particle) or any ion, its charge is always found to be an integral multiple of e, that is e, 2e, 3e, and so on. No charge will be a fractional multiple of e like 0.7e or 2.5e. This is true for both negative and positive charges and is expressed as

$$q = ne \qquad \ldots\ldots 3.2$$

where n is a positive or negative integer.

You have now learnt that charge exist in discrete packets rather than in continuous amount. Whenever a physical quantity possesses discrete values instead of continuous values, then the quantity is said to be quantized. Therefore, we say that charge is quantized.

Question

A conductor possesses a positive charge of $3.2\text{x } 10^{-19}$ C. How many electron does it have in excess or deficit ($e = 1.60 \text{ x } 10^{-19}$C)

3.2 Coulomb's law

A knowledge of the forces that exist between charge particles is necessary for a good understanding of the structure of the atom and of matter. The magnitude of the forces between charged spheres was first investigated quantitatively in 1785 by Charles Coulomb, a French scientist. He observed that the electrostatic force between the two sphere is proportional to the product of the charges and is inversely proportional to the square of their distance apart.

Coulomb's law may be stated in mathematical terms as

$$F \propto \frac{Q_1 Q_2}{r_2}$$

Where F is the electric force between the two charges Q_1 Q_2, distance r apart. We can turn the above expression of proportionality to as equation by writing

$$F = \frac{Q_1 \, Q_2}{r^2} \qquad \ldots\ldots 3.3$$

Where K is a constant.

$$K = \frac{1}{4\pi\varepsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

Where the constant ε depends on the material surrounding the charges, and is called permittivity.

Note

We shall see later that it is advantageous to have the additional constant 4π in any system having "spherical symmetry" i.e. any system in which effects are the same anywhere on the surface of a sphere.

The permittivity of a vacuum is denoted by εo (pronounced as epsilon nought) and is called the permittivity of free space

$$\varepsilon o = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$$

We can also write

$$\frac{1}{(4\pi \varepsilon o)} = 8.98 \times 10^9 Nm^2 C^{-2}$$

and also

$$F = 9 \times 10^9 Q_1 Q_2 / r^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

The permittivity of air at standard temperature and pressure (s.t.p) is $1.0005\varepsilon o$. Therefore, we can usually take Eo as the value for air.

We shall see in Module 2 that a more widely used unit for permittivity is the Farad per metre (Fm^{-1}).

You should note that Coulomb's law applies to point charges. Sub-atomic particles such as electrons and protons may be regarded as approximating to point charge. In practice two small spheres will only approximate to point charges when they are far apart and there must not be any charge nearby to disturb the uniform distribution of charge on each of them.

Example

A charge q_1, = 5.0μc is placed 30cm to the west of another charge q_2=12μc. What is the force exerted by the positive charge on the negative charge?

Solution

$q_1 = + 5.0\mu c$ $\qquad$ $q_2 = -12\mu c$

Coulomb's law gives force on negative charge due to the positive charge as follows:

$$F = \frac{1}{4\pi\varepsilon_o} \frac{q_1 \; q_2}{r^2}$$

$$F = \frac{(9 \times 10^9 \, N\, m^2 C^{-2}) \; (5 \times 10^{-6} C) \; (-12 \times 10^{-6} C)}{(0.30 m)^2}$$

$$= -6 \, N$$

The minus sign shows that the force is in the negative x-direction that is towards west. Therefore, it is a force of attraction.

3.3 Principle of superposition

In the last section, we considered the electrostatic forces between two charges. The question is how do we calculate the electrostatic force on a charge q_1 due to the presence of other two, three or more charges? This situation is shown in figure 3.1

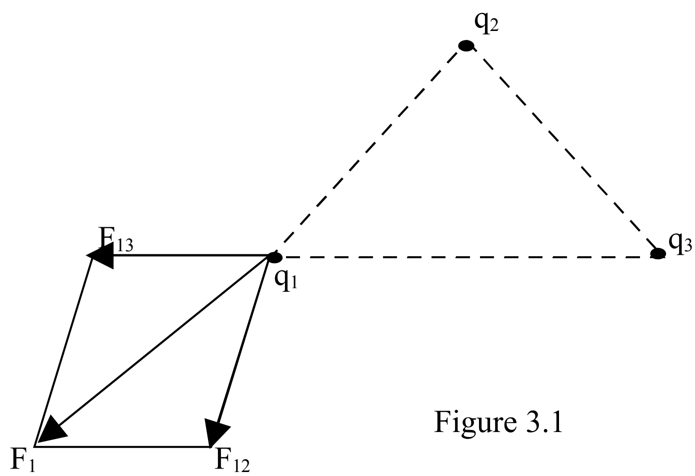

Figure 3.1

We can still calculate the force between different pair of charges using Coulomb's law. The total force on q_1 will be the vector sum of forces on q_1 due to q_2 and q_3 independently. This is the principle of Superposition. In the other words, the fact that electric forces add vectorially is known as the priciple of superposition.

To illustrate the principle, let us go through the following

Worked example

Example

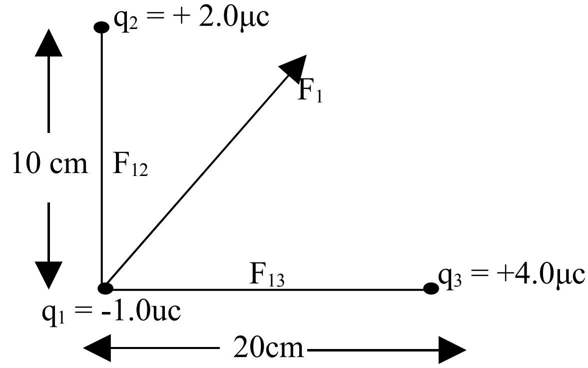

figure 3.2

In figure 3., q_1 = 1.0μc, q_2.0μc and q_3 = 4.0μc. Find the electrostatic force on q_1 to the two other charges. You should express your result as a magnitude and direction.

Solution

The three charges are located at the corners of a right angle triangle. The problem can be solved using the superposition triangle. The problem can be solved using the superposition principle.

The force on q_1 due to the charge q_2 is given by

$$F_{12} = \frac{1}{4\pi\varepsilon o} \frac{q_1 \quad q_2}{r^2}$$

$$= \frac{(9 \times 10^{9} \text{ Nm}^2 \text{ C}^{-2})(-1 \times 10^{-6} \text{ C})(2\text{x } 10^{-6}\text{C})}{(0.10\text{m})^2}$$

= 1.8N (attractive) in the +ve x-direction

Similarly, the force on q_1 due to q_2 is

$$F13 = \frac{(9 \text{ x}9 \text{ Nm}^2 \text{ C}^{-2})(-1 \times 10^{-6}\text{C})(4\text{x } 10^{-6}\text{C})}{(0.20\text{m})^3}$$

= 0.90N (attractive) in the +ve x-direction

According to the superposition principle, the resultant force F_1 acting on q, is the vector sum of the forces due to q_2 and q_3. The magnitude of F_1 is

$$\sqrt{(0.90)^2 + (1.8)^2 \quad 2.01N}$$

and it make an angle $\theta = \tan^{-1}(1.8/0.9) = \tan^{-1}2 = 63.5°$ with the positive x-axis.

3.4 The Electric Field

An electric field is a region where an electric charge experiences a force, just as a football field is an area where the game is played. If a very small, positive point charge q is place at any point in an electric field and it experiences a force F, then the field strength E 9also called the E-field) at that point is defined by the equation.

$$\varepsilon = f/q \quad \text{or} \quad F = qE \quad \ldots\ldots\ldots\ldots 3.5$$

The magnitude of E is the force per unit charge and its direction is that of F (i.e the direction of the force which acts on a positive charge).

Thus E is a vector.

3.4.1 Calculation of the Electric Field

In order to measure the electric field in a given region, we introduce a test charge and measure the force on it. However, we should realize that the test charge q exerts forces on the charge that produce the field, so it may change the configuration of the charges. In principle, the test charge should be so small as to have no significant effect on the charge configuration that produces the field.

Equation 3.5 shows that the electric field is measured in newtons Coulomb^{-1} (NC^{-1}). Since F is a vector quantity, E will also be a vector. If q is positive, the electric field E has the same direction as the force acting on the charge. If q is negative, the direction of E is opposite to that of the force F.

Let us consider the electric field of a point charge. We already know from coulomb's law that if we place a point charge q_1 at a distance r from another point charge q_1 the force on q_1 will be.

$$F = \frac{1}{4\pi\varepsilon o} \frac{qq_1}{r^2} \quad \ldots\ldots\ldots\ldots 3.6$$

Since the electric field is force per unit charge, we divide the force is equation 3.6 by the charge q1 to obtain the field due to q at the location of q1. That is

$$E = \frac{F}{q} = \frac{1}{4\pi\varepsilon o} = \frac{q}{r^2} \quad \text{………………..3.7}$$

Equation 3.7 gives the field arising due to the charge q at any location which is at a distance r from q.

What is the situation when the electric field is due to two or more charges? The answer is simple. Since the electric force obeys the superposition principle, so does the electric field (force per unit charge). Therefore, the field at a given point due to two or more charges is the vector sum of the fields of individual charges.

Example

An electric field is set up by two point charges q_1 and q_2 such that $q_1 = -q_2 = 12 \times 10^{-9}$ C and separated by a distance of 0.1m as shown in figure 3.3. Find the electric field at the points A and B.

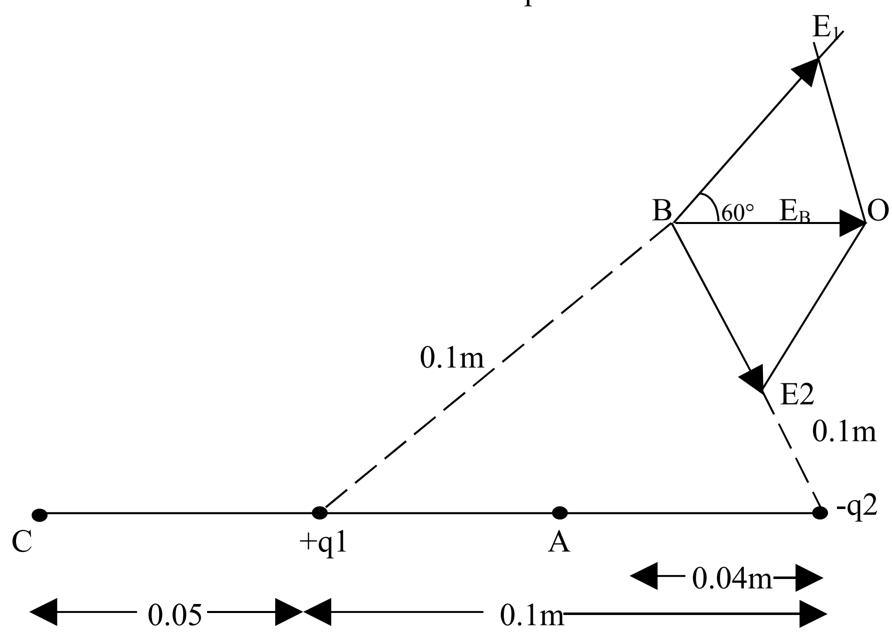

Fig 3.3 example 3.

Solution

(i) At point A_1 the electric field E_1 due to q_1 is

$$E_1 = \frac{(9 \times 10^9 \, Nm^2 C^{-2}) \; 12 \times 10^{-9} C}{(0.06)^2}$$

$$= \frac{(9 \times 10^9 \, Nm^2 C^{-2}) \times 12 \times 10^{-9} C}{36 \times 10^{-4} \, m^2}$$

$$= 3 \times 10^4 \, NC^{-1} \text{ in the +ve x-direction}$$

At A , the electric field E2, due to q_2 is

$$E_2 = \frac{(9 \times 10^9) \times (-12 \times 10^{-9} C)}{(0.04)^2}$$
$$= \frac{9 \times 10^9 \, Nm^2 C^{-2} \times 12 \times 10^{-9} C}{4 \times 10^{-4} m^2}$$
$$= 6.75 \times 10^4 \, NC^{-1} \text{ in the +ve x-direction}$$

Therefore, the net electric field, E_1 at point A is given by

$$E_A = E_1 + E_2$$
$$= (3 + 6.75) \times 10^4 \, NC^{-1}$$
$$= 9.75 \times 10^4 \, NC^{-1}$$

At point B, the electric field E, due to q, is

$$E_1 = (9 \times 10^9 \, Nm^2 C^{-2}) \quad \frac{12 \times 10^{-9} C}{(0.1m)^2}$$
$$= 1.08 \times 10^4 \, NC^{-1} \text{ in the direction shown in Fx, 3.}$$

Similarly the field E_2 due to q_2 is

$$E_2 = (9 \times 10^9 \, Nm^2 C^{-2})^2 \quad \frac{(-12^2 \times 10^{-9} C)}{(0.1m)^2}$$
$$= -1.08 \times 10^4 \, NC^{-1}$$

(The minus sign shows that the electric field points diagonally downward to the right)

START

Now we have to add the two forces vertically. If we resolve E_1 and E_2 into components along the x-axis and y-axis, it is clear from Fig. 3.3 that the y – components of vectors E_1 and E_2 cancel out and those along x-axis, i.e. BO add. You will observe that the angel between either vector and the x-direction is 60^0 because the triangle formed by B_1 q_1 and $-q_2$ is an equilateral triangle. The direction of the resultant field is, therefore, along BO and its magnitude is given by

$E_B = (1.08 \times 10^4 \cos 60^0 + 1.8 \times 10^4 \cos 60^0)$

$= \quad 1.08 \times 10^4 \text{ NC}^{-1}$

3.4.2 Field lines

An electric field can be represented by electric field lines or lines of force. The lines are drawn so that

(a) The field line at a point (or the tangent to it if it is curved) gives the direction of E at the point. This is the direction in which a positive charge would accelerate.

(b) The number of lines per unit cross-section area is proportional to E.

You should note that the field line is imaginary; the representation serves only the useful purpose of allowing us to know the general features of the electric field in the entire region at a glance. The tangent to the filed at point A in figure 3.4 shows the direction of electric field at that point. The field lines are continuous and extends throughout space depicting the electric field.

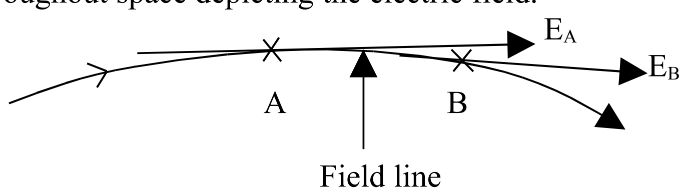

Figure 3.4 An electric field line

Since a field lines is also defined as a path along which a free, positive, point charge would travel in an electric field, it is always drawn with an arrowhead indicating the direction of travel of the position charge.

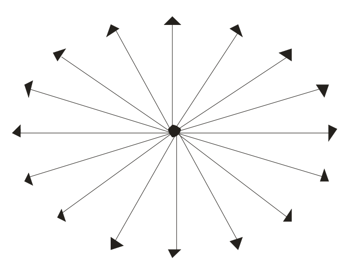

Figure 3.5 Field lines due to a positive point charge

Since the direction of an electric field is taken to be that of the direction of the forces on a positive charge, the field surrounding a point positive charge is radially outward, as shown in fig. 3.5

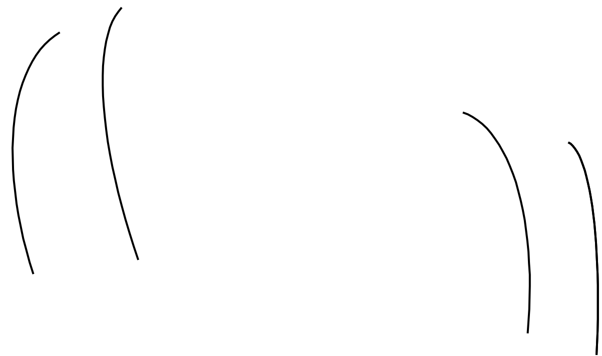

Figure 3.6 (a)

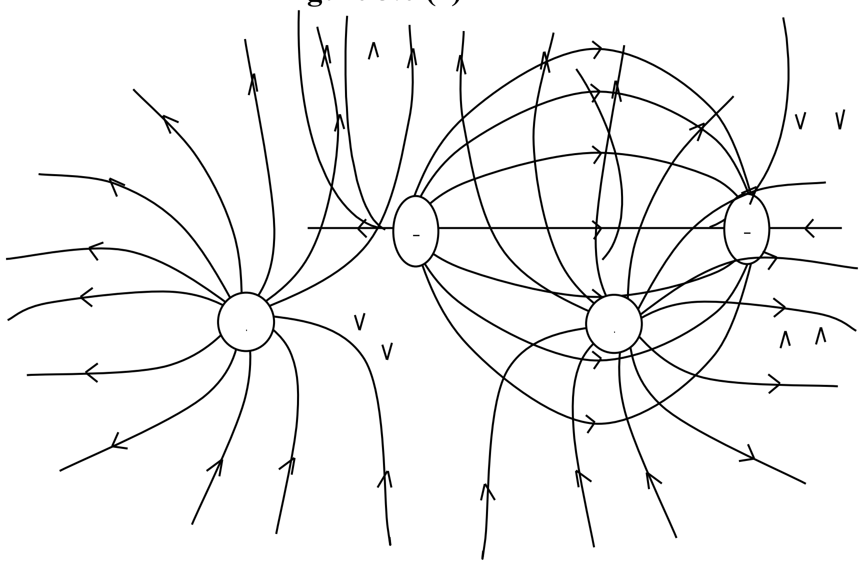

Figure 3.6(b)

Figure 3.6: The nature of field lines

(a) Two unlike charges

(b) Two like charges

4.0 Conclusion

You have now gone through the first unit of the course – electricity and magnetism. The most central point pertains to the electric charge. As you would see the concept of electric charge cuts across virtually all aspect of the course. You have also leant about the electric fields due to a single charge and those due to a system of charges. Do not forget that the electric force can be attractive or repulsive.

5.0 Summary

* There are only two types of electric charge and they are arbitrarily called positive and negative. Like charges repel and unlike charges attract each other.

* The unit of charge is the coulomb (C)

* Charge is always conserved. That is, the algebraic sum of the charges in a closed system does not change

* Electric charge is quantized, occurring only in discrete amounts.

* The force between two charges is proportional to the product of their charges and inversely proportional to the square of the distance between them. The force acts along the line joining the two charges.

$$\underline{F} = \frac{1}{4\pi\varepsilon o} \frac{(q_1 q_2)}{r_2}$$

The value of $1/4\pi\varepsilon o$ is 9×10^9 Nm^2C^{-2}

* The electric field at a point in space is defined as the electric force exerted on a test charge placed at that point.

$$\underline{E} = \frac{\underline{E}}{q}$$

* The electric field of a point charge q is given by

$$E = \frac{1}{4\pi\varepsilon o} \frac{q}{r^2} \hat{\underline{r}}$$

Where $\hat{\underline{r}}$ is a unit vector pointing from the point charge q to the location at which the electric field is being calculated.

* The electric field due to a distribution of charges, according to the superposition principle, is the vector sum of the fields of the individual charges making up the distribution

$$E = \sum_n \frac{1}{4\pi\varepsilon_o} \frac{q_n}{r_n^2} \underline{r}_n$$

* Electric lines of force are only a visual way of representing an electric field. The tangent to a line of force at any point shows the direction of the electric field at that point.

6.0 Tutor Marked Assignments

6.1 Two charges +4e and +e are fixed at a distance a. A third charge q is placed on the straight line joining the two charges so that q is in equilibrium. Find the position of q. Under what circumstances will the equilibrium be stable or unstable?

6.2 ABCD is a square of 0.04m side. Charges 16×10^{-9}, -16×10^{-9} and 32×10^{-9} coulomb are placed at the points A, C and D respectively. Find the electric field strength at point B.

6.3 A small object carrying a charge of -5×10^{-9} C experiences a force of 20×10^{-9} N in the negative x-direction when placed at a certain point in an electric field.

(a) What is the electric field at the point?

(b) What is the magnitude and direction of the force acting on a proton placed at the point ?

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UNIT 2

GAUSS'S LAW

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1.0 Introduction

Gauss's law expresses the relation between an electric charge and the electric field that it sets up. It is a consequence of Coulomb's law. Although it contains no additional information, its mathematical form enables us to solve many problems of electric field calculation far more conveniently than through the use of Coulomb's law.

In unit 1, you learnt that electric field at any point is given by the force experience by a unit positive charge placed at that point. In this unit, we will develop the concept of flux of an electric field and then arrive at the Gauss's law. We will also see how the gauss's law allows us to calculate the electric field far more easily than we could using Coulomb's law.

2.0 Objectives

After studying this unit, you will be able to:

* relate the electric flux through any surface to:
  - (i) the field strength
  - (ii) the surface area
  - (iii) the orientation of the surface relative to the Field
* write relation between the electric flax and the charge enclosed within the surface.
* compute the electric flux through any closed surface placed in the electric field.
* use Gauss's law to compute electric fields in the case of spherical, linear and planar symmetry.

3.1 Electric flux

The number of the lines of force crossing any surface depends on three factors – the field strength, E, the surface area, S and the orientation of the surface relative to the electric field.

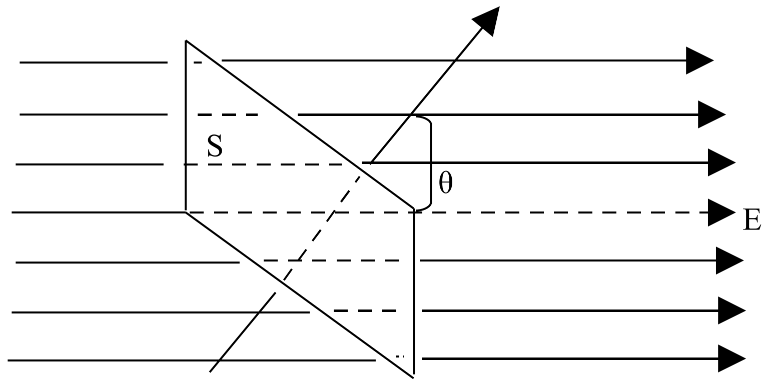

Figure 3.1

To specify the orientation of the surface, S we draw a perpendicular to the surface. If θ is the angle between the electric field and the perpendicular as shown in figure 3.1, then the number of lines of force passing through the surface ranges from maximum to minimum depending on θ. That is

When $\theta = 0$; the number of lines of force crossing the surface is maximum

When $\theta = 90^0$, the number of lines of force crossing the surface is zero.

We can now see that the number of lines of force crossing a surface is proportional to the projection of the field on to the perpendicular to the surface, i.e $\cos\theta$ -(Note $\cos 0^0 = 1$, $\cos 90^0 = 0$).

Putting together quantities on which the number of lines of force depends gives the relationship. Number of lines of force crossing a surface α $ES\cos\theta = \underline{E} \cdot \underline{S}$ – 3.1.

Where $\underline{E}$ is the electric field vector and $\underline{S}$ is a vector whose magnitude is equal to the area of the surface and whose direction is that of the perpendicular to the surface.

The quantity on the left side of Eq. is an indefinite number because we can draw as many lines of force as we like. But the quantity on the right hand side of the equation has a definite value. It is called electric flux. Let us denote it by ϕ. Hence

$$\Phi = \underline{E}.\ \underline{S} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

Eq. 3.2 shows that the electric flux is a Scalar, being the scalar product of two vectors. Since E is measured in NC^{-1}, the unit of flax is Nm^2C^{-1}

Example 1

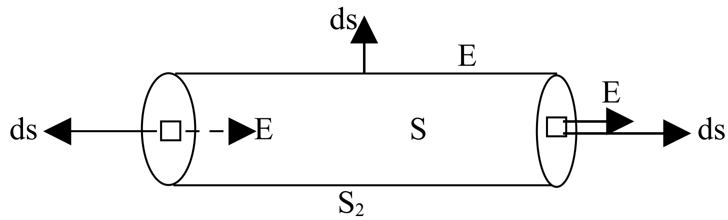

Figure 3.2

Figure 3.2 shows a closed surface S in the form of a cylinder of radius R situated in a uniform electric field $\underline{F}$, the axis of the cylinder being parallel to the field. What is the flux ϕ of the electric field through this closed surface?

Solution

We can write the total electric flux through the surface S as the sum of three terms, an integral over the surface, S_1 i.e the left cylinder cap. S_2, the cylinder surface, and S_3, the right cap. Thus from Eq. 3.2, we have

$$\Phi = \int_S \underline{E}.\ \underline{ds}$$

$$\Phi = \int_{S_1} \underline{E}.\ \underline{ds} + \int_{S_2} \underline{E}\ .\underline{ds} + \int_{S_3} \underline{E}.\ \underline{ds}$$

For the left cap, angle θ for all points is 180^0 , E is constant, and all the vectors ds are parallel.

Therefore, $\int_{S_1} \underline{E}\ .\ \underline{ds} = \int E\ (\cos 180^0)\ ds = -E\int ds = -\pi ER^2$

Since the area of the cap is πR^2.

Similarly, for the right hand cap

$$\int_{S_3} \underline{E}.\,\underline{ds} = +\ \pi ER^2$$

Since the angle θ = 0 for all points on the cap.
Finally, for the surface S_2.

$$\int_{S_2} \underline{E}.\underline{ds} = 0$$

Since the angle θ = 90^0 for all points on the cylindrical surface.

We can now sum up the total flux through the cylindrical surface S as

$$\Phi = -\pi ER^2 + 0 + \pi ER^2 = 0$$

Therefore, the total outward flux of the electric field through the closed surface of figure 3.2 is zero.

3.2 Gauss's Law

In section 3.1, we found that:

(i) The number of lines of force crossing any closed surface is proportional to the net charge enclosed by the surface

(ii) The electric flux through any closed surface is proportional to the total charge enclosed by the surface.

We can (ii) Mathematically as follows

$$\Phi \ \alpha \ q_{enclosed} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

Or $\varphi = \int E.\, ds\ \alpha\ q_{enclosed} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$

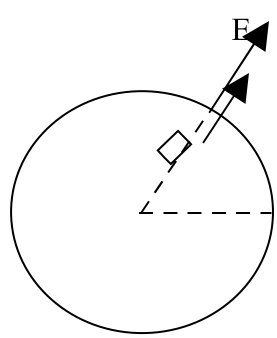

Figure 3.3

To evaluate the proportionality constant in equation 3.3 or 3.4, let us consider a positive point charge q in free space and a spherical surface of radius R centred on q as shown in fig. 3.3. The flax through any surface is given by equation, i.e

$$\Phi = \int \underline{E} . \underline{ds} = \int E .ds \cos \theta \text{ ………………….3.5}$$

Where θ is the angle between the direction of the electric field and the outward normal to the surface.

Now, the magnitude of the electric field at a distance R due to point charge q is given by

$$E = \frac{1}{4\pi\varepsilon o} \frac{q}{R^2}$$

The field point radially outward so that the electric field is everywhere parallel to the outward normal to the surface.

Then θ = 0 and cos θ = 1. Putting the values of E and cosθ in equation 3.5, the flux through the spherical surface of radius R becomes

$$\Phi = \int_{sphere} \frac{1}{4\pi\varepsilon o} \frac{q}{R^2} ds = \frac{1}{4\pi\varepsilon o} \frac{q}{R^2} \int ds$$

Note that the expression for the magnitude of the electric field has been taken outside the integral sign because it has the same value (it is constant) everywhere on the spherical surface. The remaining integral is just the areas of all infinitesimal elements, ds, on the sphere. In other word, the remaining integral is the surface area of the sphere, that is

$$\int ds = 4\pi R^2$$

Then the flax becomes

$$\Phi = \frac{1}{4\pi\varepsilon o} \frac{q}{R^2} 4\pi R^2 = q/Eo \text{ ……….3.6}$$

Comparing eqns 3.4 and 3.6, we observe that the proportionality constant is 1/εo. Hence we have

$$\Phi = \int \underline{E}. \underline{ds} = \frac{q_{enclosed}}{\varepsilon o} \text{ ……………………3.7}$$

Equation 3.7 is known as Gauss's law. It tells us that the electric flax through the sphere is proportional to the charge and is independent of the radius of the surface.

3.3 Application of Gauss's Law

Gauss's law applies to any hypothetical closed surface (called a Gaussian surface) and enclosing a charge distribution. However, the evaluation of the surface integral becomes simple only when the charge distribution has sufficient symmetry. In such situation, Gauss's law allows us to calculate the electric field far more easily than we could using Coulomb's law. Since gauss's law is valid for an arbitrary closed surface, we will use this freedom to choose a surface having the same symmetry as that of the charge distribution to evaluate the surface integral. We shall now illustrate the use of Gauss's law for some important symmetries.

3.3.1 Spherical Symmetry

A charge distribution is spherically symmetric if the charge density (that is, the charge per unit volume) at any point depends only on the distance of the point from a central point (called centre of symmetry) and not on the direction.

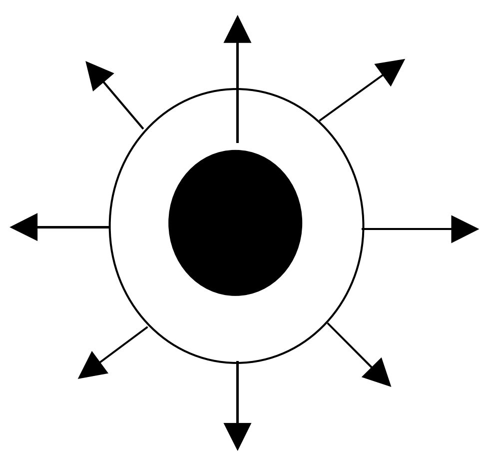

Figure 3.4

Figure 3.4 represents a spherically symmetric distribution of charge such that the charge density is high at the centre and zero beyond r. Spherical symmetry of charge distribution implies that the magnitude of electric field also depends on the distance r from the centre of symmetry. In such a situation, the only possible direction of the field consistent with the symmetry is the redial direction – outward for a positive charge (fig 3.4) and inward for a negative charge. Example of spherically symmetric charge distributions are:

(i) a point charge

(ii) a uniformly charged sphere, and

(iii) a uniformly charged thin spherical shell

Example 1. Use Gauss's law to derive the expression for the electric field of a point charge.

Solution

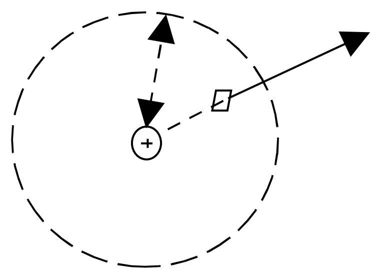

Figure 3.5

Figure 3.5 shows a positive point charge q. Using Gauss's law, let us find out the electric field at a distance r from the charge q.

Draw a concentric spherical Guass's surface of radius r. we know from symmetry that E points radially outward. If we divide the Gaussian surface into differential areas ds, then both E and ds will be at right angles to the surface, the angle θ between them being zero. Thus the quantity E.ds becomes simply Eds and Gauss's law becomes

$$\varepsilon o \oint \underline{E} . \underline{ds} = Eo \oint Eds = q$$

Since E has the same magnitude for all points on the Gaussian surface , we can write

$$\varepsilon o \oint E . ds = \varepsilon o \oint Eds = \varepsilon o \oint ds = q \ldots\ldots 3.8$$

However, the integral in equation 3.8 is simply the area of the spherical surface, i.e $4\pi r^2$ Hence

$\varepsilon o\ E\ (4\pi r^2) = q$

Or $E = q/4\pi\varepsilon o r^2$ ……………………3.9

Equation 3.9 is coulomb's law in the form we derived it in unit 1.

We can now see that Gauss's law and Coulomb's law are not two independent physical laws but the same law expressed in different ways.

3.3.2 The Electric Field of a Spherical Charge Distribution

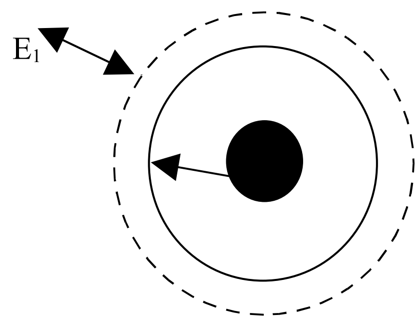

Figure 3.6

Let us consider a total charge Q which s spread uniformly throughout a sphere of radius R as shown is figure 3.6. We want to find the electric field at some point such as P_1 outside the distribution and at point P2 inside it.

(a) For points outside the charge distribution, let us draw a Gaussian surface $S_1$of radius r, through the point P_1.

◻ **How do we now take advantage of the spherical symmetry?**

Answer: Because of the spherical symmetry, the electric field is the same at all points on the Gaussian surface

◻ **What is the direction of the electric field?**

Answer: At any point on the Gaussian surface, the field is radically directed, i.e perpendicular to the surface, so that the angle between the normal to the surface and the electric field direction is zero. That is $\cos\theta = \cos 0 = 1$ (Here it is assumed that the sphere has a positive charge, if there is a net negative charge, the field will point radially (inward whereby $\theta = 180°$ and $\cos\theta = {}^{-1}$).

Then the flux through the Gaussian sphere S, becomes

$$\Phi = \int \underline{E}_1 \cdot \underline{ds}_1, = \int E_1 \cos\theta \, ds_1,$$

$$= E_1 \int ds_1 \qquad \text{(since } \cos\theta = 1)$$

$$= 4\pi r^2{}_1 E_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

Since $\int ds$, is just the surface area of the sphere S_1, i.e. $4\pi r^2$

Applying Gauss's law

$4\pi r_1{}^2 E_1 = Q/Eo$ Since the charge enclosed within the sphere S_1 is Q

$$E_1 = \frac{1}{4\pi Eo} \frac{Q}{r_1{}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.11$$

Equation 3.11 shows that the field at all points on surface S_1 is the same as if all the charges within the surface S_1 were concentrated at the centre.

(b) For points inside the charge distribution, the electric field depends on how the charge is distributed. This is because any Gaussian sphere with $r < R_1$ such as surface S_2 in figure 3.6 does not enclose the entire charge Q. The charge enclosed depends on the charge distribution.

Suppose a Gaussian sphere S_2 of radius r_2 is drawn passing through the point P_2 where we wish to find the electric field.

Let the field b denoted by E_2.

Inside the sphere S_2, eqn. 3.7 for the flux still holds. However, the charge enclosed is some fraction of Q. The volume of the charge sphere is $\frac{4\pi R^3}{3}$ and it

contains a total charge Q. Since the charge is spreading uniformly throughout the sphere, the volume charge density ρ is constant and is given by.

$$\rho = \frac{Q}{\frac{4\pi R^3}{3}}$$

Therefore, the charge enclosed by the sphere S_2 will be just the volume of that sphere multiplied by the volume charge density, that is

$$q_{enclosed} = \frac{4\pi}{3} r_2^3 \times \frac{Q}{4\pi R^3} = Q \frac{r_2^3}{R^3}$$

Applying the Gauss's law (eqn. 3.7), we have

$$4\pi r_2^2 E_2 = Q \; r_2^2/R^3$$

So that

$$E_2 = \frac{1}{4\pi E_o} \; \frac{Q r_2}{R^3} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

Illustrate with a sketch the variation of the electric fields both inside and outside a spherical charge distribution.

Answer:

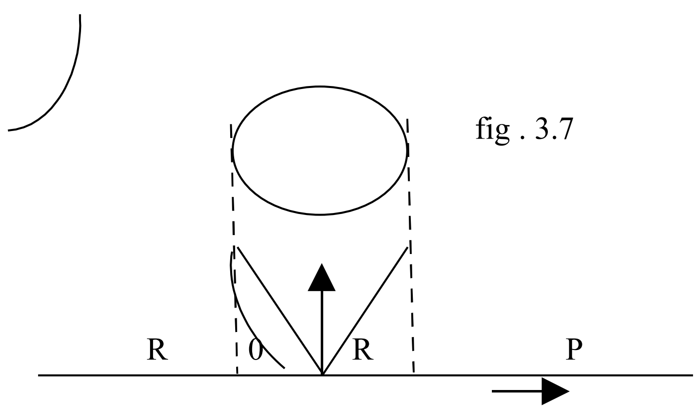

fig . 3.7

The electric field inside the distribution increases generally with distance from the centre (E α r). On the other hand, outside the charge distribution, the electric field falls off as 1/r2 figure 3.7 is a sketch of the fields inside and outside the sphere.

3.3.3 Line Symmetry

A charge distribution has cylindrical symmetry when it is infinity long and has a charge density that depends only on the perpendicular distance from a line called symmetry axis (see fig. 3.8

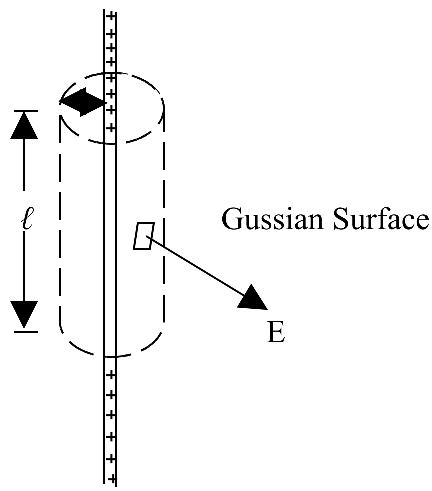

Figure 3.8

By symmetry, the electric field will point radially outward from the axis and its magnitude will depend only on the perpendicular distance from the axis. (We assume positive charge, for negative charge the field points inward). Let us find expression for the electric field, E at a distance r from the line charge (say a wire).

We draw a Gaussian surface which is a circular cylindrical of radius r and length l closed at each end by plane caps normal to the axis as shown in figure 3.8

To calculate the flux, you should note that the electric field also has cylindrical symmetry, which implies that its magnitude at a point depends only on the perpendicular distance of the point form the symmetry axis, and its direction has to be radially outwards.

The flax through the cylindrical surface is

$$\Phi = \int \underline{E}.\underline{ds} = \int E ds = E\int ds = 2\pi r l E \qquad \ldots\ldots\ldots..3.13$$

Where $2\pi r\ell$ is the area of the curved surface.

The flax through the end of the cylinder is zero because the field lines are parallel to the plane caps of the Gaussian surface. Mathematically, E and ds are perpendicular, so that cos θ = 0 in the scalar (dot) product E. ds. Therefore, the only flax is through the curved part of the cylinder. Gauss's law tells us that the flax is proportional to the charge enclosed within the cylindrical Gaussian surface, i.e.

$$2\pi r\iota E = \frac{q_{enclosed}}{\varepsilon o}$$

So that $$E = \frac{q_{enclosed}}{2\pi\varepsilon_o r\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.14$$

If the line charge density is λ, then the charge enclosed by the Gaussian cylinder of length $\lambda\ell$. Hence eqn 3.14 becomes

$$E = \frac{q_{enclosed}}{2\pi\varepsilon_o r\ell} = \frac{\lambda\ell}{2\pi\varepsilon_o r\ell} = \frac{\lambda}{2\pi\varepsilon_o r} \quad \ldots\ldots\ldots\ldots 3.15$$

With regard to the electric field inside the wire, we shall consider two cases:

(i) Suppose the charge is distributed uniformly within the wire with charge density ρ.

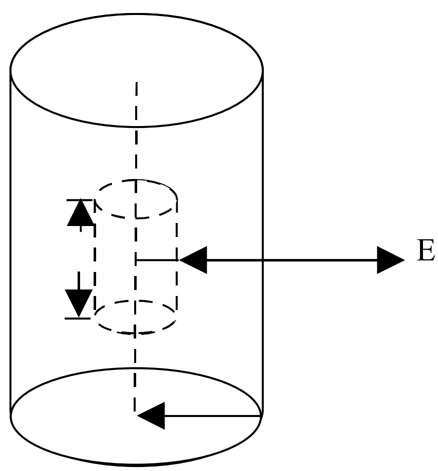

Figure 3.9 An enlarged view of the wire

Let the radius of the wire be r. to find E at the inner point P_1 a distance r apart from the axis of the wire, we draw a Gaussian surface (i.e a cylinder) of radius r and length ℓ passing through P as shown in figure 3.9. As we saw earlier, the flux is due to the curved surface only. Hence from Gauss's law.

$$\int \underline{E}.\ \underline{ds} = 2\pi\, r \ell\, E \;=\; q^1/Eo$$

The charge q^1 inside this Gaussian surface = π p r^2 ι

$$E\ 2\,\pi\, r\, \ell = \frac{\pi\, \rho\, r^2 \ell}{\varepsilon o}$$

$$\text{Or}\quad E = \frac{r\rho}{2\varepsilon o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.16$$

Thus the electric field at a point inside an infinite uniformly charged wire is radially directed and varies as the distance from its axis.

(ii) When the charge is on the surface of the wire or cylinder only, the electric field at any point inside it is zero because the net charge in the Gaussian surface through this point is zero.

⧈ **How does the electric field due to a charged wire or cylinder depend on its radius?**

Answer: Equations 3.15 and 3.16 show that electric field due to a charged wire or cylinder does not depend upon its radius. In effect, the field behaves as if the charge on the wire or cylinder were concentrated in a line along its axis.

3.3.4 Plane Symmetry

When the charge density depends only on the perpendicular distance from a plane, the charge distribution is said to have plane symmetry. The electric field is everywhere normal to the plane sheet as shown in figure 3.10, pointing outward, if positively charged and inward, if negatively charged.

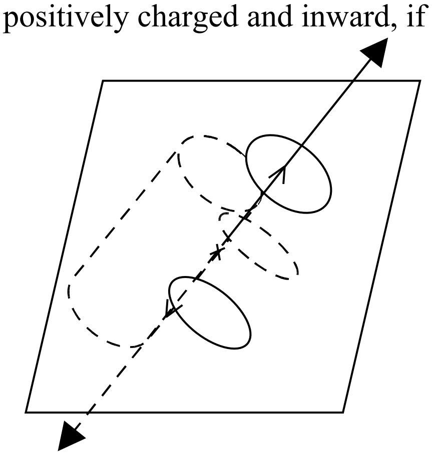

Fig 3.10 A charged distribution with plane Symmetry showing electric field

To find the electric field at a distance in front of the plane sheet, it is required to construct a Gaussian surface. A convenient Gaussian surface is a closed cylinder of cross-sectional area S and length 2r. The sides of the Gaussian surface are perpendicular to the symmetry plane and the ends of the surface are parallel to it. Since no lines of force cross the sides, the flux through the sides is zero. But the lines of force cross perpendicular to the ends, so that E and the area element vector ds on the ends are parallel. The cos θ in the product E.ds is 1 over both ends (-1 if charge is negative). Since the flux through the sides is zero, the total flux through our Gaussian surface then becomes

$$\Phi = \int_{\text{both ends}} Eds = 2ES$$

The factor, 2 arises because there are two ends. The Gauss's law gives

$$2\,ES = \frac{q_{enclosed}}{Eo}$$

If σ is the surface charge density, then the charge enclosed is σs.

Hence $$E = \frac{\sigma}{2\varepsilon o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.17$$

4.0 Conclusion

This unit is a follow-up of the previous unit. In particular, you have now learnt to apply the Gauss's law for the solution of problems involving electric charges with linear spherical and plane symmetry. You appreciate that the solution of problems with the Gauss's law is not as tedious as the use of coulomb's law. Always remember that Gauss's law applies to a closed surface, usually referred to as a Gaussian surface) and enclosing any charge distribution. For problems involving the application of Gauss's law, choose a surface having the same symmetry as that of the charge distribution to evaluate the surface integral. However we note that unlike Coulomb's law its not sufficient to determine the electric fields in all cases.

5.0 Summary

* the number of lines of force crossing a closed surface is proportional to the total charge enclosed by the surface.

* the concept of electric flux quantifies the notion "number of lines of force crossing a surface" the electric flux, ϕ is defined as the surface integral of the electric field $\underline{E}$ over a surface as follows:

$$\phi = \int \underline{E}.\underline{ds}$$

where $\underline{ds}$ is an infinitesimal vector whose direction at any point is towards the outward drawn normal to the surface at that point, its magnitude being the area of the surface.

* Gauss's law is $\int \underline{E}.\underline{ds} = q/\varepsilon o$

In which q is the net charge inside an imaginary closed surface (called a Gaussian surface) and εo is permittivity of free space. Gauss's law expresses an important property of the electric field.

* The electric field outside a spherically symmetrical shell with radius r and total charge q is directed radially and has magnitude

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \qquad (r > R)$$

The charge behaves as if it were all concentrated at the centre of the sphere.

* The electric field due to an infinite line of charge with uniform charge per unit length, λ, is a direction perpendicular to the line of charge and has magnitude

$$E = \frac{\lambda}{2\pi\varepsilon o r}$$

* The electric field due to an infinite sheet of charge is perpendicular to the plane of the sheet and has magnitude.

$$E = \sigma / 2\varepsilon o$$

Where σ is the surface charge density.

6.0 Tutor Marked Assignments

1. The electric field in a certain space is given by $\underline{E} = 200\ \underline{r}$. How much flux passes through an area A if it is a portion of

   (a) The xy – plane

   (b) The xz – plane

   (c) The yz – plane

2. A flat sheet of area $50cm^2$ carries a uniform surface charge density σ. An electron 1.5cm a point near the center of the sheet experience a force of $1.8x10^{-12}N$ directed away from the sheet. Find the total charge on the sheet.

3. Suppose that a positive charge is uniformly distributed throughout a spherical volume of radius R, the charge per unit volume being e.

   (a) Use Gauss's law to prove that the electric field inside the volume and at a distance r from the centre, is

   $$E = \frac{er}{3\varepsilon o}$$

   (b) What is the electric field at a point outside the spherical volume at a distance r from the centre ? Express your answer in terms of the total charge q within the spherical volume.

   (c) Compare your answer to (a) and (b) when r = R.

   (d) Sketch a graph of the magnitude of $\underline{E}$ as a function of r, from r = o to r = 3R.

7.0 Reference And Other Resources

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UNIT 3

ELECTRIC POTENTIAL

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1.0 Introduction

You will recall from your study of PHY 101. Elementary mechanics that work isI done when the point of application of a force undergoes a displacement in its own direction. If a body A exerts a force on another body B and work is done, a transfer of energy occurs which is measured by the work done. For example, if we lift a mass, m through a vertical height, h, the work done, W by the force we apply is W = mgh. The energy transfer is mgh and we consider that the system gains and store that amount of gravitational potential energy which is obtained from the conversion of chemical energy by our muscular activity. When the mass falls, the system loses gravitational potential energy, and, neglecting air resistance, there is a transfer of kinetic energy to the mass equal to the work done by gravity.

The meaning of electric potential can be illustrated by the above analogy from basic mechanics. A body when raised above the earth's surface is said to acquire potential energy because it can do work in falling. If it is free to move in a gravitational field it will fall to the position in which its potential energy is zero. Similarly, a charged body in an electric field has potential energy, and it will tend to move to those parts of the field where its potential energy is smaller. When a positive charge is repelled by another positively charged body and moves away, its potential energy decreases. It will be zero when it is completely away from the influence of the charged body, that is at infinity. We select as the zero of electric potential the potential at an infinite distance from any electric charges.

Definition of Electric Potential

The electric potential at a point in a field can be defined as the work done per unit charge moving from infinity to the point.

You should note that we are always assuming that the charge does not affect the field in any way.

The choice of the zero of potential is purely arbitrary and whilst infinity may be a few hundred metres in some cases, in atomic physics where distances of 10^{-10}m are involved, it need only be a very small distance away from the charge responsible for the field.

Electric potential is a property of a point in a field and is a scalar since it deals with a quantity of work done or potential energy per unit charge. The symbol for potential is V and the unit a joule per coulomb (JC^{-1}) or volts (V).

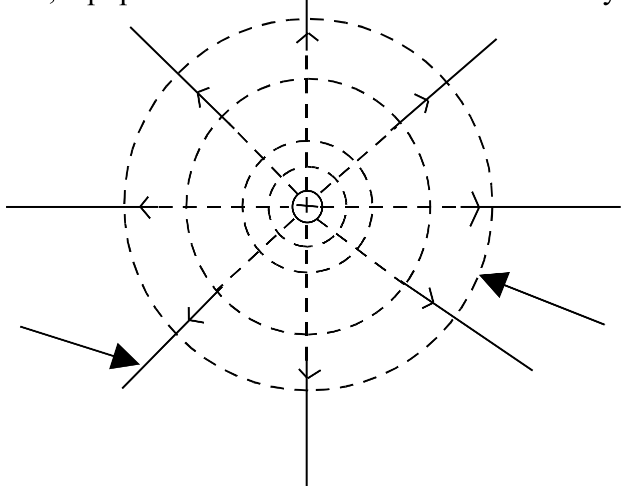
2.0 Objectives

After studying this unit, you will be able to:

* Compute the work done in taking a charge q from one point to another in an electric field.
* Compute the electric potential at a point due to a single charge
* Relate the electric potential and electric field, and thereby compute the electric field at a point knowing the electric potential.
* Compute the electric potential at a point due to a dipole and a quadrupole

3.1 Equipotential Surfaces

All point in a field which have the same potential can be imagined as lying on a surface, called an equipotential surface. When a charge moves on such a surface no energy change occurs and no work is done. The force due the field must therefore act at right angles to the equipotential surface at any point. Therefore, equipotential surfaces and field lines always interst at right angles.



Field line Equipeteateal Surface

Figure 3.1 Equipotential surface for a point charge

We can now see that as electric field can therefore be represented pictorially by field lines and by equipotential surface (or lines in two dimensional diagrams).

Figure 3.1 shows the equipotential surfaces for a point charge. These are concentric spheres (circles in two dimensions).

If equipotential are drawn so that the change of potential from one to the next is constant, then the spacing will be closer where the field is stronger. This follows from the fact that in order to perform a certain amount of work in such regions a shorter distance need be travelled.

3.2 Potential due to a point charge

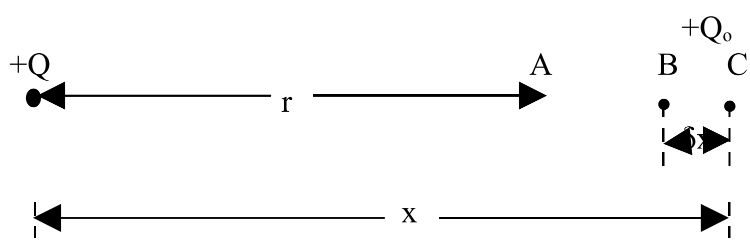


fig. 3.2

We wish to find the potential at point A in the field of an isolated point charge +Q situated at point 0, such that 0 A = r as shown in fig. 3.2.

Let us imagine a very small point charge $+Q_o$ is moved by an external agent from C distance x from A, through a very small distance δx to B without affecting the field due to +Q.

Assuming the force F on Qo due to the field remains constant over, δx, the work δW by the external agent over δx against the force of the field is

$$\delta W = F\ (-\delta x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

Do you know why we have to insert the negative sign for δx? This is to take account of the fact that the displacement δx is in the opposite direction to that in which F acts.

Applying coulomb's law.

$$F = \frac{Q\,Q_o}{4\pi\varepsilon o} \bullet \frac{1}{x^2}$$

$$\delta W = \frac{Q\,Qo}{4\pi\varepsilon o} \bullet \frac{(-\delta x)}{x^2}$$

The total work done, W in bringing Qo from infinite Qo from infinity to A is

$$W = \frac{-Q\,Qo}{4\pi\varepsilon o} \int_{\infty}^{r} \frac{dx}{x^2} = \frac{-\,Q\,Qo}{4\pi\varepsilon o} \left[{}^{1}/_{x}\right]_{\infty}$$

$$= \frac{Q\,Qo}{4\pi\varepsilon\,\Upsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

3.3 Potential due to a system of charges

Let us now consider a system of charges. Like in the previous case of electric field we shall find the superposition principle very useful. That is, the total potential, V_p at a point P due to a system of charges q_1, q_2, ……..,q_N is equal to the sum of the potentials due to the individual charges at that point.

If Υ_1, Υ_2, ….Υ_N are the distances of the charges q_1, q_2, ………. q_N respectively from the point P, the potential at that point is

$$\vee p = \frac{q_1}{4\pi\varepsilon o\Upsilon_1} + \frac{q_2}{4\pi\varepsilon o\Upsilon_2} + \ldots\ldots + \frac{q_N}{4\pi\varepsilon o\Upsilon_N} \ldots\ldots\ldots\ldots 3.4$$

You will note in eq. 3.4 that each charge is acting as if no other charge is present.

The potential at point P. may be written in a summation form as

$$\vee p = \frac{1}{4\pi\varepsilon o} \sum_{i=1}^{N} q_i/r_i \ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

Example 1

The following point charges are placed on the x - axis: 2 μc at x = 20cm, -3μc at x = 30cm, -4μc at x = 40cm. Find the potential on the x – axis at the origin.

Solution

We know that potential is a scalar quantity. Using the superposition principle, the potential at the origin (x = 0) is given by

$$V_o = \frac{1}{4\pi\varepsilon_o} \sum_{I=1}^{3} q_i/r_i$$

On substituting the numerical values of q_i and Υ_i, we obtain

$$V_o = 9 \times 10^9 \, Nm^2 C^{-2} \left(\frac{2 \times 10^{-6} C}{0.20m} - \frac{3 \times 10^{-6} C}{0.30m} - \frac{4 \times 10^{-6} C}{0.40m} \right)$$

$$= 9 \times 10^9 Nm^2 C^{-2} \quad 10^5 Cm^{-1} - 10^{-5} Cm^{-1} - 10^{-5} Cm^{-1}$$

$$= \quad -9 \times 10^4 \, Nm \, C^{-1}$$

$$= -9 \times 10^4 V$$

3.4 Potential Difference

Let us write down the amount of work done in bringing a unit positive charge from infinity, first to point A and then to point B, shown in figure 3.3. Remember that point A and B are within the field of charge q.

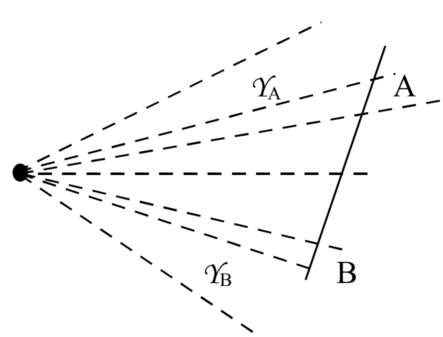


Fig. 3.3

Using equation 3.3, the potentials at point A and B are

$$V_A = \frac{q}{4\pi\varepsilon o \Upsilon_A}$$

And $$V_B = \frac{q}{4\pi\varepsilon o \Upsilon_B}$$

The difference of these two potentials (i.e $V_B - V_A$) is the work done in taking a unit charge from A to B, and it is called the potential difference the two points B and A. it is written as

$$V_{BA} = V_B - V_A = \frac{q}{4\pi\varepsilon}\left(\frac{1}{\Upsilon_B} - \frac{1}{\Upsilon_A}\right) \quad \ldots\ldots\ldots\ldots.3.6$$

We observe that the work done is carrying the charge in an electric field is independent of path. It is just this path independence that enables us to define the concept of potential.

By the way, you should note that if, instead of the unit positive charge, we transport a charge q^1 between A and B, then the work W done is given by:

$$W = q^1 V_{BA} = q^1 (V_B - V_A) \ldots\ldots\ldots.3.7$$

On a final note, I want you to bear in mind that the potential difference (p.d. for short) is a very importance concept in the field of electrostatics and current electricity. Its knowledge helps us to determine the exact value of the current which flows between any two points in an electric circuit, provided the resistance between the two points is known . We shall see this later in this course (module 3)

3.5 Relation Between Electric Field and Electric Potential

Fig 3.4

Let us consider a charge +Q at a point A in an electric field where the field strength is E. This configuration is illustrated in figure 3.4

The force, F on Q is given by

$$F = EQ$$

If Q moves a very short distance δx from A to B in the direction of E, then the work done δW by the electric force on q is

$$\delta W = \text{force x distance}$$
$$= F\delta x$$
$$= EQ\delta x \text{ (assuming E is constant over AB)}$$

If the p.d between B and A is δV, we have by the definition of p.d.

$$\delta V = \text{work done per unit charge}$$
$$= \frac{\delta W}{Q} = -\frac{EQ}{Q}\delta x$$

That is $\delta V = -E\delta x$

The negative sign is inserted to show that if displacements in the direction of E are taken to be positive, then when δx is positive, δV is negative, i.e the potential decreases.

In the limit, as δx ----0_1 E becomes the field strength at a point (A). In calculus notation

$$E = \lim_{\delta_x \to 0} \left(\frac{\delta V}{\delta_x} \right) = \frac{dv}{dx}$$

dv/dx is called the potential gradient in the direction and so the field strength at a point equals the negative of the potential gradient there.

Question: Is potential gradient a vector or a scalar?

Answer: You notice that potential gradient involves displacement . Therefore, it is a vector.

It is measured in volts per metre (Vm^{-1}). The Vm^{-1} and NC^{-1} are both units of E, but the Vm^{-1} is the one that is commonly used.

If the electric field E is uniform, that is it is constant in magnitude and direction at all points, it follows that

$$\frac{dv}{dx} \text{ is constant i.e } \frac{dv}{dx} = -E \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

$$\therefore V = \frac{Ex}{x} \quad \therefore \quad E = -v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

in other words, the potential changes steadily with distance (Recall the case of ds/dt = constant, for uniform velocity)

Example 2

Question: Two large horizontal, parallel metal plates are 2.0cm apart in air and the upper is maintained at a positive potential relative to the lower so that field strength between them is 2.5×10^5 $Vm^{-1.}$

(a) What is the p.d. between the plates?

(b) If an electron of charge 1.6×10^{-19}C and mass 9.1 x
10^{-31}kg is liberated from rest at the lower plates, what is its speed on reaching the upper plate?

Solution

(a) If E is the field strength (assumed uniform) and V is the p.d. between the plates which are at a distance d apart, we have (from equation 3.9)

$$\begin{aligned} V &= Ed \\ &= (2.5 \times 10^5 \, Vm^{-1}) \times (2.0 \times 10^{-2} m) \\ &= 5.0 \times 10^3 \text{ volts} \end{aligned}$$

(b) The energy change (i.e work done) W which occurs when a charge Q moves through a p.d of V volts in an electric field is given by

$$W = QV$$

There is a transfer of electrical potential energy from the field to k.e. of the electron, Hence we have

$$QV = \frac{1}{2} mv^2$$

where v is the required speed and m is the mass of the electron

Therefore, $\frac{1}{2} mv^2 = QV$

$$V = \sqrt{\frac{2qv}{m}}$$

i.e $$v = \sqrt{\frac{2x16x10^{-19}C) \ x \ (5.0 \ x \ 10^3 V)}{(9.1 \ x \ 10^{-31}kg)}}$$

$$= \sqrt{\frac{2 \ x \ 1.6 \ x \ 10^{-19} \ x \ 5.0 \ x \ 10^3}{9.1 \ x \ 10\text{-}31} \ \frac{C \ JC^{-1}}{kg}}$$

$(IV = 1 \ JC^{-1})$

$$= \sqrt{\frac{16}{9.1} x \ 10^{15} m^2 \ 5^{-2}}$$

$(I \ J = INm = 1 \ kgm \ S^{-2}m)$

$$= 4.2 \ x \ 10^1 ms^{-1}$$

3.6 Electric Field and Potential of an Electric Dipole

A pair of equal and opposite charge, ±q, separated by a vector distance a is called a dipole. (see figure 3.5). The vector a, which is also along the axis of the dipole, is drawn from the negative to the positive charge.

A molecule consisting of a positive and negative ion is an example of an electric dipole in nature. An atom consists of equal amount of positive and negative charges whose centre coincide, hence an atom is neutral for all points outside it. However, in the presence of an external electric field, the centres of positive and negative charges get separated. The atom then becomes a dipole.

As we shall see in module 2, the electric field and potential in the vicinity of a dipole forms the first step in understanding the behaviour of dielectrics under the influence of an external electric field.

3.6.1 Electric Field at a point P along the axis of the dipole

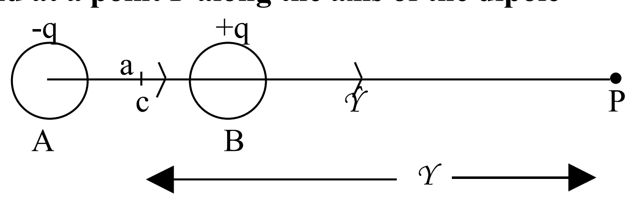


figure 3.5: Electric dipole AB with centre C and axis a .

The point P is along the axis.

Let the distance between the mid-point of the dipole and the point which is along axis be equal to r. We shall evaluate the electric field at P.

The electric field at P due to +q. is given by

$$E_+ = \frac{q\,\hat{r}}{4\pi\varepsilon o\,(r - a/2)^2}$$

$\underline{\Upsilon}$ is a unit vector in the direction Cp.

And that due to –q is

$$E = \frac{-q\,\hat{\underline{r}}}{4\pi\varepsilon o\,(r + {}^{a}/{}_{2})^2}$$

The resultant field at P is

$$E = E_+ + E_-$$

$$= \frac{q\,\hat{r}}{4\pi\varepsilon}\left(\frac{1}{\left(r - \frac{a}{2}\right)^2} - \frac{1}{\left(r + \frac{a}{2}\right)^2}\right)$$

$$= \frac{q\,(2a\,r)\,\hat{\underline{r}}}{4\pi\varepsilon o\left(r^2 - \frac{a^2}{4}\right)^2}$$

$$E \triangleq \frac{2\,P\,\underline{r}}{4\pi\varepsilon o r^3} \quad \text{for} \quad r >> a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

Where $p = qa\,\hat{\underline{r}}$ ……………….3.11

You will notice that, in the denominator, we have neglected $a^2/_4$ as compared to r^2, since a <<r is actual physical problems. In equation 3.10, qa$\underline{r}$ has been replaced by the vector quantity $\hat{P}$, which is known as the dipole moment. It will be of interest for you to know that in atomic and molecular dipole, a = 10^{-10}m and r >>a.

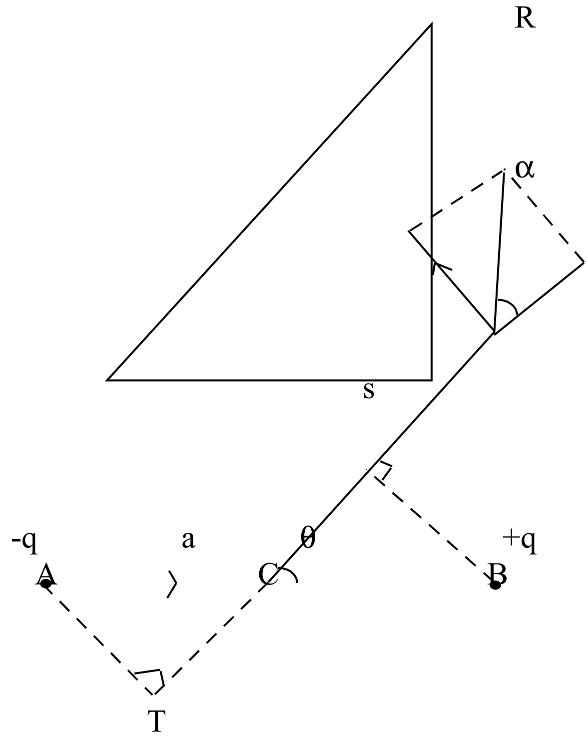
3.6.2 Potential due to a dipole

Let us evaluate the potential Vp at P, a distance r from the mid-point C of the dipole. (see figure 3.6). the line joining P to C makes an angle θ with the dipole axis, $\underline{a}$.

The distances of P from –q and +q are AP and BP respectively. From the geometry, you will notice that

$$BP = SP = PC - CS = r - a/2 \cos\theta$$

$$\text{And} \quad AP = TP = TC + CP = r + a/2 \cos\theta$$



Hence the potential at P is equal to

$$V_p = \frac{q}{4\pi E_o}\left(\frac{1}{(r - {}^a/_2 \cos\theta)} \quad \frac{1}{(r + a/2 \cos\theta)}\right)$$

$$= \frac{qa\cos\theta}{4\pi E_o \left(r^2 - \frac{a^2}{4}\cos^2\theta\right)}$$

When P is far away, r^2 is large compared to $a2/4 \cos^2\theta$ and neglecting $a^2/_4 \cos^2\theta$ in the denominator, we can write

$$V_p = \frac{p\,.\,\hat{r}}{4\pi\varepsilon_o r^2} = \frac{P\cos\theta}{4\pi\varepsilon_o r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

$= 0$ when $\theta = \pi/2$ (P lies on the perpendicular bisector dipole axis)

We can conclude from equation 3.10 and 3.12 that:

(i) The dipole potential varies as $^1/r^2$ and the field as $^1/r^3$ as compared to a point charge for which the potential varies as $^1/_r$ and the field as $^1/r^2$. Thus the potential and field decrease more rapidly with r a dipole than for a point charge.

(ii) The dipole potential vanishes on points which lie on the perpendicular bisector of the dipole axis. Hence no work is done in moving a test charge along the perpendicular bisector.

4.0 Conclusion

In this unit, you have learnt to compute the electrostatic potential of a charge distribution. You have seen that if you first derive the expression for the potential, being a scalar quantity, it facilitates the derivation of the expression for the electric field at a point. The concept of potential is also important because electric potential is loosely linked to the work done by the electric charge. For the computation of the total potential due to a system of charges, we have again found the superposition principle very handy. We have computed the electric field and potential of a dipole and you have been advised that this introduction to the electric field of a dipole will facilitate your understanding of the behaviour of dielectrics under the influence of an external electric field. You will see this in module 2.

5.0 Summary

* The work done in taking a unit positive charge from one point to another in electric field is independent of the path chosen between the two points

* The potential at a point is the work done in carrying a unit positive charge from infinity to the point against the electric field.

* The potential ϕ_r at a distance r from a point charge q is given by

$$\Phi_r = q/4\pi\varepsilon o r$$

* The potential difference Φ_{BA} between two point B and A is equal to the work done in taking a unit positive charge from A to B. if a charge q is taken from A to B, then the work done is .

$$W = q\Phi_{BA} = q\,(\phi_B - \phi_A)$$

* The unit of potential difference is the volt. The potential difference between point A and B is 1 volt when the work done in carrying unit positive charge between the two points is equal to 1 joule.

* The electric field of a dipole at a point along the axis of the dipole is given by

$$\underline{E} = 2P/4\pi Eor^3 \quad \text{for } r >> a$$

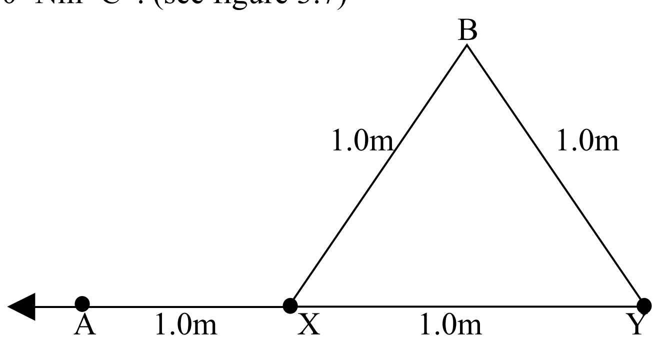
At a point on the perpendicular bisector of the dipole axis, the electric field is

$$\underline{E} = \frac{-p}{4\pi\varepsilon o r^3} \quad \text{for} \quad r >> a$$

Where r is the distance of the point from the centre of the dipole and P is the dipole moment

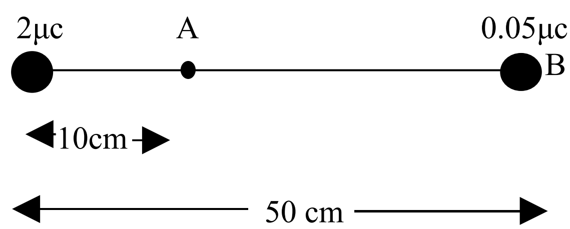
6.0 Tutor Marked Assignments

1. Find the electric potential at point A and B, due to two small sphere X and Y, 1.0m apart in air and carrying charges of $+2.0 \times 10^{-8}$C and -2.0×10^{-8} C respectively. Assume the permittivity of air = εo and $^1/4\pi\varepsilon o) = 9.0 \times 10^9$ $Nm^2 C^{-2}$. (see figure 3.7)

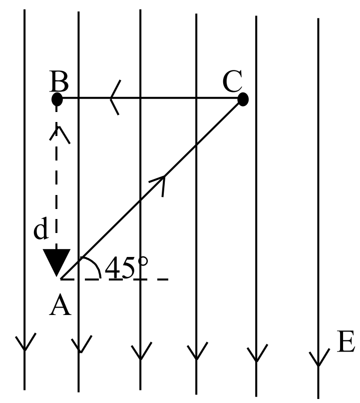


2. Find the potential at two points A and B at distances 10cm and 50cm from a charge of 2μc on shown in figure 3.8. Also find the work needed to be done in bringing a charge of 0.05μc from B to A.

Fig. 3.8



3. Compute the potential difference between points A and B assuming that a test charge q_o us moved without acceleration from A to B along the path shown in figure 3.9



7.0 References and other Resources

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UNIT 4

POTENTIAL FOR CONTINOUS CHARGE DISTRIBUTION AND ENERGY

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1.0 Introduction

In the previous units, we have discussed the electric field, $\underline{E}$ and the potential, ϕ due to discrete charge distributions. In the process, you had to evaluate a line integral. On the other hand, you can also calculate $\underline{E}$ from φ by a simple differentiation. In this unit, we shall extend these ideas to evaluate φ for some continuous charge distributions.

In this unit, you will also learn about the concept of electrostatic energy and the nature of the electrostatic force. These are basic concepts which will helping you in understanding not only the remaining part of this course but also many other course in Physics and Chemistry.

2.0 Objectives

After studying this unit, you will be able to:

* Obtain expressions for potential due to continuous and symmetric charge distributions.
* Calculate the electrostatic potential energy for a given charge distribution
* Show that the electrostatic force is conservative.

3.1 The Potential due to an Infinite Line Charge

In unit 2 of this module, we derived an expression for the electric field at a point near an infinitely long charged wire (or a line charge) as an application of gauss's law.

We saw that

$$E = \frac{\lambda}{2\pi\varepsilon_o r} \hat{r} \quad \ldots\ldots\ldots\ldots\ldots 3.1$$

Where λ is the charge per unit length on the wire, r is the perpendicular distance of the point from the wire, εo is the permittivity of free space, and $\hat{r}$ is a unit vector along the direction of increasing r. (see figure 3.1)

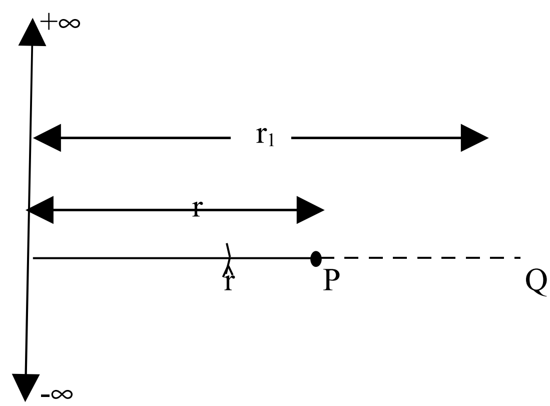


Fig. 3.1

We want to drive an expression for the potential due to the wire at a point P.

Define the Potential at point P.

We saw in unit 3 of this module that the negative of the line integral of the electric field between infinity and any point gives the value of the potential at that point, i.e,

$$\Phi_r = -\int_{\infty}^{r} \underline{E} \cdot \underline{dr} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

We shall evaluate the line integral by first taking a finite distance r, instead of infinity and then letting r_1 go to infinity. Here r_1 is the distant of the point Q from the wire (see fig 3.1) The integral then gives us the difference in potentials between p and Q, i.e

$$\Phi_r - \phi_{r1} = -\int_{r_1}^{r} \underline{E} \cdot \underline{dr}$$

Substituting the expression for E from eq. 3.1, we obtain

$$\Phi_r - \phi_{r1} = -\frac{\lambda}{2\pi\varepsilon o}\int_{r_1}^{r} \frac{\hat{\Upsilon}d\underline{r}}{r}$$

Since $\underline{\hat{\Upsilon}}$ and $\underline{dr}$ are in the same direction, we have

$$\Phi_r - \phi_{r_1} = \frac{-\lambda}{2\pi\varepsilon o}\int_{r_1}^{r} \frac{dr}{r}$$

$$= \frac{-\lambda}{2\pi\varepsilon o} \text{In}(r/r_1) = \frac{\lambda}{2\pi\varepsilon o}\text{In}(r_1/r) \quad \ldots\ldots\ldots\ldots 3.3$$

Now let us try to evaluate the potential with respect to infinity by letting r_1 go to infinity. We notice from equation 3.3 that ϕr_1 anywhere in the vicinity of the linear charge distribution (r finite), goes to infinity. This is because the assumption of a uniform and infinite charge per unit length over an infinitely long line invariably leads to an infinite amount of charge. Therefore, the sum of finite contributions form each part of an infinite amount of charge leads to an infinite potential.

Does this mean there is no Solution to the Problem?

The issue of an infinite potential does not pose any problem. In practical situation, we are interested only in the difference in potential. Do not forget that the choice of infinity for zero potential is only for convenience. Only potential differences have any real significance. The absolute value of potential does not have any physical significance.

What is then the Importance of Equation 3.3?

Equation 3.3 gives finite values of potential difference for finite distances of r and r_1.

3.1.1 The Equipotential surface of a Uniformly Line Charge

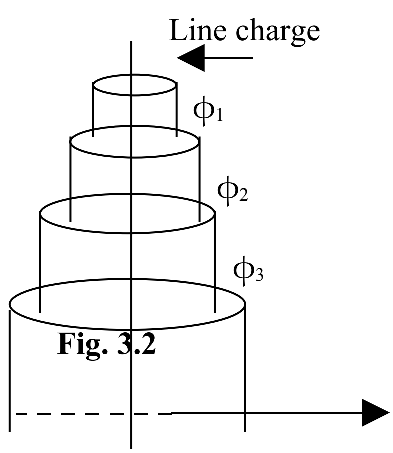


Fig. 3.2

☐ **What is an equipotetnial Surface ?**

By now you have understood that an equipontial surface is the locus all point having the same potential. For a uniform infinte line charge, the potential at a distnace r is given by equation 3.3 as

$$\Phi_r - \phi r_1 = \frac{\lambda}{2\pi\varepsilon o} \text{I n} (r_1/r)$$

From this you can see that the electric potetnial is the same for all points which are equidistant from the line of charge. Therefore, the equipotentials are cylindrical with the line of charges as the axis of the cylinder

3.2 The Potential of a Charged Circular disc

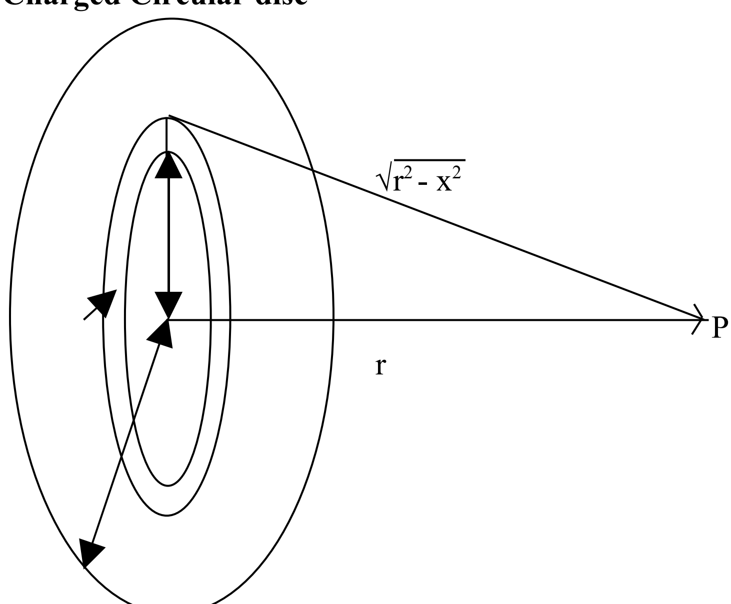


Fig 3.3

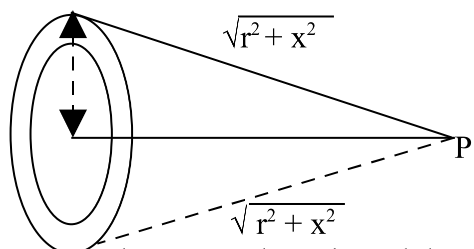
We wish to find the potential at some point P lying on the axis of a uniformly charged circular disc. The point P is at a distance r from the centre 0 of the disc and the line joining p to 0 is perpendicular to the plane of the disc.

For calculating the potential, first consider a narrow circular strip of the thickness dx at a distance x from its centre. Let the charge on the strip be dQ , where

$$dQ = (2\pi x dx)\sigma \quad \text{...3.4}$$

In equation 3.4, $2\pi x dx$ is the area of the strip. You can observe from fig. 3.2 that all parts of the strip are equidistant from the point P. The charge dQ on this strip can be written as a sum of a large number of

point charges, δqi such that $dQ = \sum_{i=1}^{n}\delta q_i$, n being very large.



The distance between all the points charges on the strip and the point P is $\sqrt{r^2 + x^2}$. The potential $d\phi$ at P due to the charge dQ (i.e. due to the whole strip) using the principle of superposition is

$$D0.\phi = \frac{\sum^{n}_{i-1} \delta q_i}{4\pi\varepsilon o\sqrt{r^2 + x^2}} = \frac{d\Phi}{4\pi\varepsilon o\sqrt{r^2 + x^2}}$$

$$= \frac{(2\pi x dx)\sigma}{4\pi\varepsilon o\sqrt{r^2 + x^2}} \quad \text{....................................3.5}$$

The total potetnial at point P due to the whole disc is obtained by dividing the disc into a very large number of similar but concentric stip and adding their contributions

Into how many concentric strips can we divide the disc?

A large number of terms (infinitesimals) are involved . Therefore, the summation has to be replaced by integration. In other words, we have to integrate equation 3.5 over all the concentric strips and obtained the total potential ϕ due to all the charges on the disc as follows:

$$\Phi = \int d\varphi = \frac{\sigma}{2\pi\varepsilon_o}\int_0^a \frac{x \, dx}{\sqrt{r^2 + x^2}} \quad \ldots\ldots\ldots\ldots..3.6$$

In eqn 3.6, σ and r are constants for a given charge density (and disc) and point P respectively. The limits of x are x = 0 and x = a, as we go from the centre of the disc to the edge.

Are you able to do the integration $\int_o^a \frac{x \, dx}{\sqrt{r^2 + x^2}}$ **?**

Answer: Let $x^2 + r^2 = y$
On differentiation, we have
$2xdx = dy$ since r is constant

$$\int_o^a \frac{x \, dx}{\sqrt{r^2+x^2}} = \int_{r^2}^{a^2+r^2} \tfrac{1}{2} y^{-1/2} dy$$

$$= \left[y^{1/2} \right]_r^{a^2+r^2}$$

$$= \sqrt{a^2+r^2} - r$$

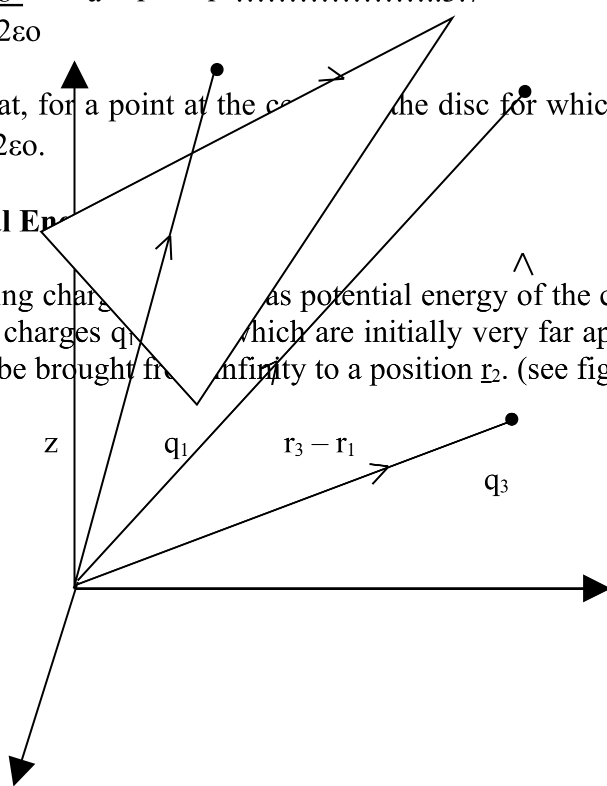
Using the result of the integration in equation 6, we have

$$\Phi = \frac{\sigma}{2\varepsilon o}\left(\sqrt{a^2 + r^2} - r\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots..3.7$$

Equation 3.7 shows that, for a point at the c... the disc for which r = 0, ϕ reduces to $\phi = a\sigma/2\varepsilon o$.

3.3 Electrostatic Potential En...

Work done is assembling char... as potential energy of the charges. Suppose there are two charges q_1 ... which are initially very far apart. Let q_1 be fixed at $\underline{r}_1$ and q_2 be brought fr... infinity to a position $\underline{r}_2$. (see fig 3.5



$\underline{r}_3 - \underline{r}_2$

$\underline{r}_1$

$\underline{r}_3$

q_2

$\underline{r}_2$

y

Fig 3.5

x

You recall that the amount of work required to bring q_2 from infinity is equal to the charge q_2 multiplied by the potential at r2 due to q_1, that is

$$dW = q_2 \frac{1}{4\pi\varepsilon_o} \frac{q_1}{|\underline{r}_2 - \underline{r}_1|} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

This is equal to the work done in assembling the two point charges q1 and q_2 at r_1 and r_2 by bringing them close together. The work done is stored in the system and is usually interpreted as the electrostatic potential energy of the system of two charges.

If q_1q_2 is positive, the potential energy is positive. A positive potential energy means that work has to be done to assemble the like charges together. If the product q_1 q_2 is negative (unlike charges) the potential energy is negative. A negative potential energy means that work has to be done to pull the charges away from each other. It follows that a positive potential energy corresponds to repulsive electric forces while a negative potential energy corresponds to attractive electric forces.

Let us now consider what happens when we have to assemble a system of many charges instead of only two. To start with, let us consider three charges q_1 ,q_2, and q_3 which have to be assembled at positions r_1, r_2, and r_3 as shown in figure 3.5. The assembling may be done step by step. First bring q_1 to r_1, and q_2 to r_2. For this work will be done as given in Eq. 3.8. Now bring q_3 to r_3 against the force that q_1 and q_2 exert on it. The work done for this stage is:

$$dW^1 = q_3 \frac{q_1}{4\pi\varepsilon_o |r_3 - r_1|} + q_3 \frac{q_2}{4\pi\varepsilon_o |r_3 - r_2|} \quad \ldots\ldots\ldots\ 3.9$$

This is because the total force on q_3 is equal to the sum of two individual forces. The total work done including the first stage is then

$$W = dW + dW$$

$$= q_2 \frac{q_1}{4\pi\varepsilon o /\underline{r}_2-\underline{r}_1/} + q_3 \frac{q_1}{4\pi\varepsilon o/\underline{r}_3-\underline{r}_1/} \quad q_3 \frac{q_2}{4\pi\varepsilon o/\underline{r}_3-\underline{r}_2/}$$

$$= \sum_{\text{all pairs}} \frac{q_j q_k}{4\pi\varepsilon o /\underline{r}_j-\underline{r}_k/} \qquad j = 1 \text{ to } 3 \text{ and } k = 1 \text{ to } 3 \text{ but } j = k$$

$$= \frac{1}{2} \sum_{j=1}^{3} \sum_{\substack{k=1 \\ j \neq k}}^{3} \frac{q_j \; q_k}{4\pi\varepsilon o / \underline{r}_1-\underline{r}_k/} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.10$$

Equation 3.10 requires some clarification. Please read the following explanation carefully.

There is a factor 2 before the summation sign to make sure that the contribution from each pair of charges is included only once. For example, for pair q_1 and q_2, we get contribution when j = 1 and k = 2, and similarly when k = 1 and j = 2. The factor ½ thus reduces this double contribution to a single contribution.

You should also note that we have written $j \neq k$ below the second summation sign. This is to avoid the force between a charge with its ownself, a situation which does not occur.

Generalising the situation for an assemblage of N point charges q_1 q_2, ………, q at r_1, r_2, ………, r_N, the expression for electrostatic potential energy may be written as:

Potential energy =

$$\frac{1}{2} \sum_{j=1}^{N} \sum_{\substack{k=1 \\ j \neq k}}^{N} \frac{q_j \; q_k}{4\pi\varepsilon o / \underline{r}_j-\underline{r}_k/} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.11$$

Note that in equation 3.11 for each value of j (as fixed for the first summation), the summation on k avoids that value of k which is equal to j. This amount to considering the potential at charge q_1. By all the other charges. In terms of potentials ϕj at the location of the charge qj, equation 3.11 may be written as

$$\text{Potential energy} = \tfrac{1}{2} \sum_{J=1}^{N} q_j \phi_j \quad \text{.............................3.12}$$

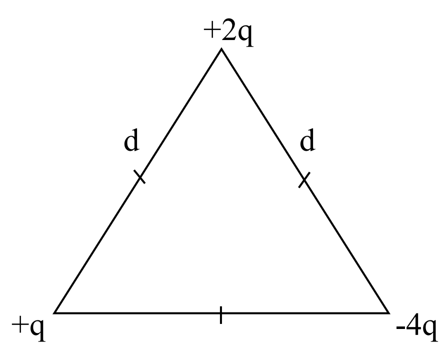
Equation 3.12 implies that for calculating the electrostatic potential energy for a group of point charges, we consider each charge in turn, and the corresponding potential at its position due to all other charges.

For the assemblage of three charges shown in figure 3.5, list the product $q_j\, q_k$ that come into the summation 3.11

Answer: q_1q_2, $q_1\, q_3$, $q_2\, q_1$, $q_2\, q_3$, $q_3\, q_1$ $q_3\, q_2$

Example 1

Three charges are arranged as shown in figure 3.6. What is their electrostatic potential energy? Assume $q = 1.0\text{x}10^{-5}$C, and d = 0.1m



Solution

The total potential energy of the system is the algebraic sum of the potential energies of all pairs of charges.

Potential Energy

$$= \frac{1}{4\pi\varepsilon o}\left[\frac{(+q)\,(-4q)}{d} + \frac{(+q)\,(+2q)}{d} + \frac{(-4q)\,(+2q)}{d}\right]$$

$$= \underline{\ 1\ } \left[\frac{-10q^2}{4\pi\varepsilon o}\right] d$$

$$= \frac{-\ (9.0 \times 10^9\ Nm^2\ C^{-2})\ (10) \times (1.0 \times 10^{-5}C)^2}{0.1m}$$

$$= -90J$$

3.4 Nature of Electric Force

You have seen in unit 3 that the work, W done is moving a charge q from point A to a point B in the region of the electric field $\underline{E}$ is

$$W = -\int_A^B \underline{F}.\underline{dr} = -q \int_A^B \underline{E}.\ \underline{dr} \qquad \ldots\ldots 3.13$$

Where $\underline{F}$ is the electrostatic force on q. You also saw in the same unit that the line integral of the electric filed, i.e.

$\int_A^B \underline{E}\ .\underline{dr}$ is independent of the path between A and B. This implies that the line integral of the electrostatic force, that is $\int_A^B \underline{E}.dr$, is also independent of the path between A and B.

In other words, the work done on a charged particle in moving it against the electrostatic force $\underline{E}$ is independent of the path between A and B, and depends only on the point A and B.

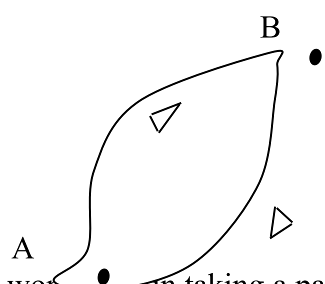


Figure 3.7

This also implies that work done in taking a path around a closed loop (figure 3.7) is zero. Of course, if this was not so, then one can find a loop, which when traversed yields a negative work. i.e. energy to us. The fact that this does not happen is related to the conservation of energy. Thus, the path independence of work done in an electrostatic field and the concept of potential are essentially related to the principle of energy conservation.

Name another type of conservative force in Physics?

Answer: The gravitational force

4.0 Conclusion

You have now learnt how to derive expressions for and evaluate the electric potential for some continuous charge distributions with simple geometry. In particular, we have derived the expressions for the potentials of an infinite line charge and a charged circular disc. We have discussed the concepts of electrostatic potential energy and the nature of the electrostatic force. The electrostatic force as well as the gravitational force which you came across in PHY 101 are conservative.

5.0 Summary

* The potential difference between two points at distances r and r_1 from an infinitely long charged wire is given by

$$\phi_r - \phi_{r1} = \frac{\lambda}{2\pi\varepsilon o} \ln (r_1/r)$$

Where λ is the charge per unit length of the wire.

* The potential ϕ at a point which is at a distance r on the axis of a charged circular disc of radius a is

$$\phi = \frac{\sigma}{2\pi\varepsilon o} \sqrt{(a^2+r^2} -r)$$

Where σ is the charge per unit area on the disc.

* The electrostatic potential energy is the energy stored in a system of charges. It is equal to the amount of work done in assembling the system together by bringing the charges from infinity.

* The electrostatic potential energy for a group of charges is written as

$$\text{Potential Energy} = \frac{1}{2} \sum_{j=1}^{N} q_j \phi_j$$

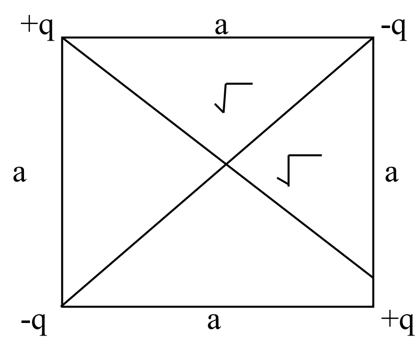
Where ϕ_j is the potential at the position of charge q_j due to all the other charges.

- * The electrostatic force is conservative. This is a consequence of the fact that the work done in taking a charge around a closed path is zero.

6.0 Tutor Marked Assignments (TMA)

1. If an electric field E equals zero at a given point, does it imply that the potential, ϕ equals zero at the same point? Give an example to illustrate your answer

2. An infinite charged sheet has a surface charge density σ of 1.0×10^{-7} Cm^{-2}. How far apart are the equipotential surfaces whose potentials differ by 5.0 volts?

3. 

Derive an expression for the work done in putting the four charges together as shown in figure 3.8.

7.0 References and Other Resources

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UNIT 5

DIELECTRICS AND CAPACITORS

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1.0 Introduction

In unit 1 of this course, we derived the Coulomb's law of electrostatic force for a situation in which the surrounding medium is vacuum or air. Of course, this is not always the case. In practice, is come across situations when the electric field is in a material medium. However, we must distinguish two different situation. The first is when the medium consists of insulating materials , also known as dielectric, that is those materials like glass, wood, mica, etc, which do not conduct electricity. The second is when the medium consist of conduction materials, i.e materials such as metals which are conductors of electricity. The conducting material contain electrons which are free to move within the material. These electrons move under the action of an electric field and constitute electric current. We shall study conducting materials and electric fields in conducting materials later in the course.

We shall now begin to study the electric field in the presence of an insulator. In these materials, there are practically no free electrons or, in some cases, the number of such electrons is so small that the conduction is not possible.

When a potential difference is applied to the insulators, no electric current flows. However, the study of their behaviours in the presence of an electric field gives us some useful information.

We shall also study a very important component used in electric circuits, the capacitor. Capacitor finds many applications in electric circuits. You will find radio receivers and for "smoothing" the rectified current delivered by a power supply. The ignition system of every automobile engine contain a capacitor to eliminate sparking of the 'points' when they open and close.

2.0 Objectives

After studying this unit, you will be able to

* Explain the properties of a dielectric medium
* Define capacitance and permittivity
* Compute the energy of a charged capacitor
* Distinguish between the various types of capacitor
* Solve problems involving series and parallel combinations of capacitors
* Explain dielectric strength and dielectric breakdown

3.1 The charge given to an isolated conductor can be regarded as being 'stored' on it. The amount of charge it will take depends on the electric field thereby created at the surface of the conductor. If this is too great there is a breakdown in the insulation of the surrounding medium, resulting in sparking and discharge of the conductor. The change in potential due to a given charge depends on the size of the conductor, the material surrounding it and the proximity of other conductors.

The idea that an insulated conductor in particular situation has a certain capacitance, or charge storing ability is very useful and is expressed as follows: if the potential of an insulated conductor change by V when given a charge Q, the capacitance C of the conductor is

$$C = Q/V \qquad \ldots\ldots\ldots\ldots 3.1$$

In words, capacitance is the charge required to cause unit change in the potential of a conductor.

The unit of C is coulomb per volt (CV^{-1}) also known as a Farad (F) in honour of Michael Faraday. The farad is a very large unit. Hence the microfarad (1 uF = 10^{-6}F), the nanofarad (1nF = 10^{-9}F) or the picofarad (1PF = 10^{-12} F) are generally used.

3.2 The Parallel Plate capacitor

The most common type of capacitor consists in principle of two conducting plates parallel to each other and separated by a distance which is small compared with the linear dimensions of the plates (see figure 3.1)

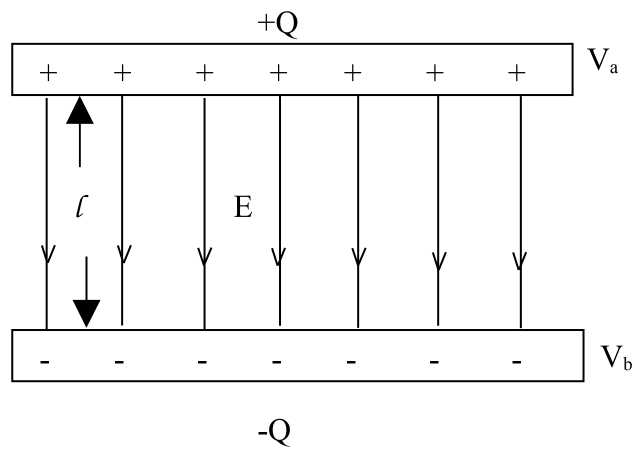


Figure 3.1 Parallel-plate capacitor

Let us assume that the plates are in vacuum, the surface area of each plate being A. If a charge +Q lines on the upper plate, the total flux from it to the lower plates is

$$EA = Q/\varepsilon o$$

But E is also given by, $E = Vab/\ell$

Where Vab is the potential difference between the plates and ι is their separation.

$$V_{ab} = E\ell = \frac{Q\ell}{\varepsilon_o A}$$

Hence the capacitance of a parallel –plate capacitor in vacuum is

$$C = \frac{Q}{Vab} = \varepsilon o A/\ell \text{ ……………………3.2}$$

Note:

In practice, equation 3.2 is not strictly true due to non-uniformity of the field at the edge of the plates. Since εo A and ℓ are constants for a given capacitor, the capacitance is a constant independent of the charge on the capacitor, and is directly proportional to the area of the plates and inversely proportional to their separation.

□ **Calculate the area of the plates of a 1F parallel plate capacitor in vacuum if the separation of the plates is 1mm. Comment on your answer regarding the farad as a unit of capacitance.**

Answer: $C = \varepsilon o A/\ell$

$$A = \frac{C\ell}{\varepsilon o} = \frac{1F \times 10^{-3}m}{8.85 \times 10^{-12}C^2N^{-1}m^{-2}}$$

$$= 1.13 \times 10^8 m^2$$

This corresponds to a square with side 10.6 x 10^3m= 10.6km. You can now appreciate that the farad is such a large unit of capacitance and it is necessary to have sub-multiples of it as discussed earlier.

3.3 Energy of a Charged Capacitor

The process of charging a capacitor consist of transferring charge from the plate at lower potential to the plate at higher potential. The charging process therefore requires the expenditure of energy. Let us imagine that the charging process is carried out by starting with both plates completely uncharged, and then repeatedly removing small positive charges from one plate and transferring them to the other plate. The final charge Q and the final potential difference V are related by

$$Q = CV$$

Since the potential difference increase in proportion with the charge, the average potential difference V_{av} during the charging process is just one-half the maximum value, or

$$V_{av} = Q/2C$$

This is the average work per unit charge so the total work required is just Vav multiplied by the total charge, or

$$W = V_{av} Q = Q^2/2C$$

Using the relation Q = CV, we have

$$\boxed{W = Q^2/2C = ½ CV = ½ QV \text{ joules (J)}} \quad ....3.3$$

3.3.1 Energy Density

We may consider the stored energy to be located in the electric field between the plates of the capacitor. The capacitance of a parallel-plate capacitor in vacuum is

$$C = \varepsilon o \ A/\ell$$

The electric field fills the space between the plates, of volume Aι, and is given by

$$E = V/\ell$$

The energy per unit volume, or the energy density, is given by

$$\text{Energy Density} = \frac{\frac{1}{2} CV^2}{A\ell}$$

$$= \frac{\frac{1}{2} \varepsilon_o A E^2 \ell^2}{A\ell^2}$$

$$\text{Energy density} = \frac{1}{2} \varepsilon_o E^2$$

3.4 Combination of Capacitors

3.4.1 Capacitors in Parallel

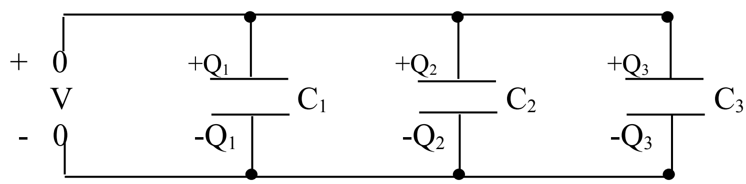


Fig. 3.2

Figure 3.2 shows three capacitors of capacitances C_1 C_2 and C_3 which are connected in parallel. The applied p.d, V is the same across each but the charges are different and are given by.

$$Q_1 = VC_1; \; Q_2 = VC_2; \; Q_3 = VC_3$$

The total charge, Q on the three capacitors is

$$Q = Q_1 + Q_2 + Q_3$$

$$Q = V\,(C_1 + C_2 + C_3)$$

If C is the capacitance of the single equivalent capacitor, it would have charge Q when the p.d across it is V.

Hence $Q = VC$

and $C = C_1 + C_2 + C_3$ ………………………3.4

You can also see that $Q_1 : Q_2 : Q3 = C_1 : C_2 : C_3$ ………3.5

In other words, the charges on capacitors in parallel are in the ratio of their capacitances.

3.4.2 Capacitors in Series (Cascade)

The capacitors in figure 3.3 are in series and have capacitances C_1, C_2 and C_3. Suppose a p.d of V volt applied across the combination causes the motion of charge from plate Y to plate A so that a charge +Q appears on A and an equal but opposite charge – Q appears on Y.

This charge –Q will induce a charge +Q on the plate Xif the plates are large and close together. The plates X ad M and the connection between them form an insulator conductor whose net charge must be zero and so +Q and X induces a charge _Q on M. In turn this charge induces +Q on L and so on.

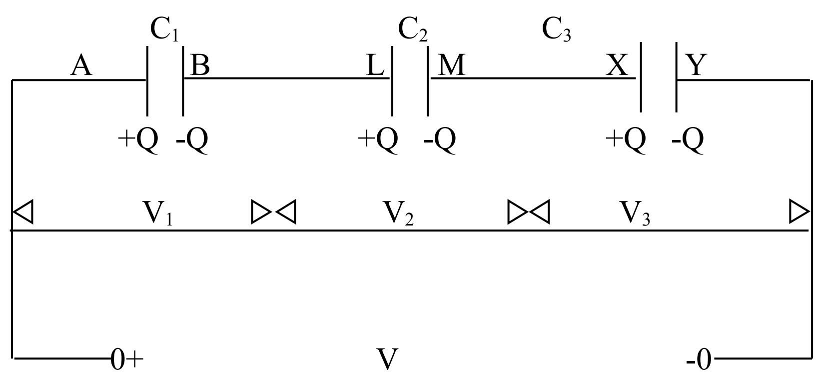


Fig 3.3

Capacitors in series thus all have the same charge and the p.d. across each is given by:

$$V_1 = Q/C_1 \qquad V_2 = Q/C_2 \qquad V_3 = Q/C_3$$

The total p.d V across the network is

$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

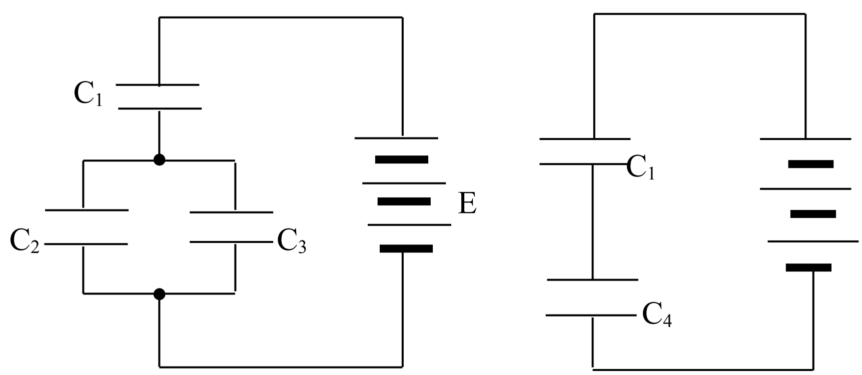
$$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

If C is he capacitance of the single equivalent capacitor, it would have a charge Q when the p.d. across it is V.

Therefore, $$\frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

i.e $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Note that: (1) For capacitors in parallel, the p.d across each is the same.**
  (2) For capacitors in series, each has the same charge.



(a) (b)

- **Example**

In the circuit of figure 3.4, $C_1 = 2$ μf, $C_2 = C_3 = 0.5$μf and E is a 6V battery. For each capacitor, calculate

(a) The charge on it, and

(b) The potential difference (p.d) across it.

Solution

The combined capacitance of the combination C_2 and C_3 is given by

$$C_4 = C_2 + C_3 = 0.5\mu F + 0.5\mu f = 1\mu f$$

Let us now redraw the circuit as in figure 3.4 (b) in which, C_1 and C_4 are in series. Their charges Q_1 and Q_4 will be equal, hence:

$$Q_1 = Q_4 = V_1 C_1 = V_4 C_4$$

Where V_1 and V_4 are the p.d.s across C_1 and C_4 respectively.

Therefore

$$\frac{V_1}{V_4} = \frac{C_4}{C_1} = \frac{1}{2}$$

But V_1 and $V_4 = 6$

$\therefore V_1 = 2V$, and $V_4 = 4V$

The p.d. across the combined capacitance C_4 equals that across each of C_2 and C_3. Hence

$$V_2 = V_3 = V_4 = 4V$$

Now $Q_2 = C_2 V_2$ and $Q_3 = C_3 V_3$

$$\therefore \quad \frac{Q_2}{Q_3} = \frac{C_2}{C_3} = \frac{0.5}{0.5} \quad (\text{since } V_2 = V_3)$$

$$= 1$$

But $Q_2 + Q_3 = Q_4 = 4 \times 10^{-6}$ C

$Q_2 = Q_3 = 2 \times 10^{-6}C = 2\mu c$

Therefore,

$Q_1 = 4\mu c \qquad Q_2 = Q_3 = 2\mu c$

$V_1 = 2V \qquad V_2 = V_3 = 4V$

3.5 Permittivity

In unit 1 we derived the coulomb's law of force between two point charges Q_1, and Q_2 at a distance r apart. Thus

$$F = \frac{Q\ Q^2}{4\pi\varepsilon_o r^2}$$

Where Eo is the permittivity of free space. This gave us an idea that the force depended on the intervening media. We shall now consider this idea especially in relation to capacitors.

3.5.1 Relative Permittivity, Er

Experiment shows that inserting an insulator or dielectric between the plates of a capacitor increases its capacitances. If Co is the capacitance of a capacitor when a vacuum is between its plates and C is the capacitance of the same capacitor with a dielectric filling the space between the plates, the relative permittivity Er of the dielectric is defined by.

$$Er = c/c_o$$

Taking a parallel-plate capacitor as an example, we have

$$Er = c/c_o = \frac{EA/\iota}{\varepsilon_o A/\iota} = {}^{E}/\varepsilon_o \quad \ldots\ldots\ldots\ldots\ldots..3.3$$

Where E is the permittivity of the dielectric and ε_o is that of a vacuum (i.e of free space). The expression for the capacitance of a parallel-plate capacitor with a dielectric of relative permittivity Er can therefore be written as

$$C = \frac{Er\ \varepsilon_o\ A}{\ell} \ldots\ldots\ldots\ldots3.4$$

Relative permittivity has no units, unlike E and Eo which have. It is a pure number without dimensions.

For air at atmospheric pressure Er = 1.0005 which is close enough to unity and so for most purposes E_{air} = Eo.

Dielectric	Relative Permittivity Er
Vacuum	1.0000
Air at s.t.p	1.0005
Polythene	2.3
Perspex	2.6
Paper (waxed)	2.7
Mica	7
Water (pure)	80
Barium titanate	1200

Table 3.1

The difficulty of removing all the impurities dissolved in water makes it unsuitable in practice as a dielectric.

☐ **A parallel capacitor consists of two square plates each of side 25cm, 3.0mm apart. If a p.d. of 200v is applied, calculate the charge on the plates with**

(i) Air; and

(ii) Paper of relative permittivity 2.5, filling the space between them ($Eo = 8.9 \times 10^{-12}\ Fm^{-1}$)

Solution

Fig. 3.5

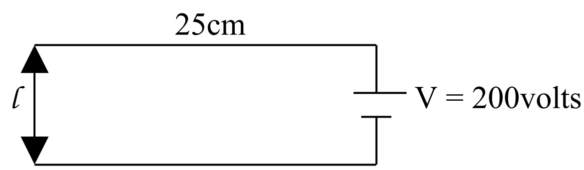


Figure 3.5 is a schematic representation of the parallel –plate capacitor.

Area of each plate = $(2.5 \times 10^{-1})^2 = 6.2^5 \times 10^{-2}\ m^2$
Plate separation = $3.0 \times 10^{-3} m$

(i) The capacitance of the capacitor with air filling the space between the plates is given by

$$C = \frac{\varepsilon_o A}{\ell}$$

where A is the area of the plate and ℓ is their separation

$$C = \frac{(8.9 \times 10^{-12}\ Fm^{-1})(6.25 \times 10^{-2}\ m^2)}{3.0 \times 10^{-3} m}$$

$$= 1.85 \times 10^{-10}\ F = 18.5\ nF$$

The charge on one plate is Q = CV

$$= (1.85 \times 10^{-10}\ F)(200\ V)$$

$$\therefore Q = 3.70 \times 10^{-8} C$$

(ii) The capacitance of the capacitor with paper of relative permittivity 2.5 filling the space between the plates is given by

$$C = \frac{E_r \varepsilon_o A}{\ell}$$

where Er is the relative permittivity of the paper Therefore, the charge on one plate is increased by the factor Er on the introduction of the paper between the plates.

$$Q = 2.5 \times 3.7 \times 10^{-8} C$$

$$Q = 9.25 \times 10^{-8} C$$

3.5.2 Dielectric Strength and Breakdown

When a dielectric material is subjected to a sufficiently strong electric field, it becomes a conductor. This phenomenon is known as dielectric breakdown. The onset of conductor, associated with cumulative ionization of molecules of the material, is often quite sudden, and may be characterized by spark or arc discharges.

When a capacitor is subjected to excessive voltage, an arc may be formed through a layer of dielectric, burning or melting a hole in it, permitting the two metal foils to come in contact, creating a short circuit, and rendering the device permanently useless as a capacitor.

The maximum electric field a material can withstand without the occurrence of breakdown is called dielectric strength

3.6 Types of Capacitor

Capacitors may be classified into two broad groups, that is fixed and variable capacitors. (see figure 3.5). They may be further classified according to their construction and use. As we shall see later, capacitors are used in electric circuits for various purposes. Different types have different dielectric. The choice of type depends on the value of capacitance and stability (i.e ability to retain the same value with age, temperature change, etc) needed and on the frequency of any alternating current (a.c) that will flow in the capacitor.

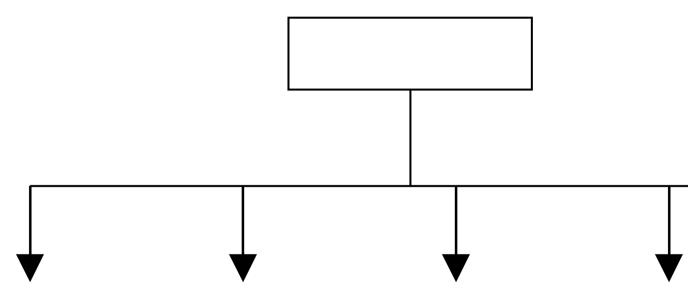
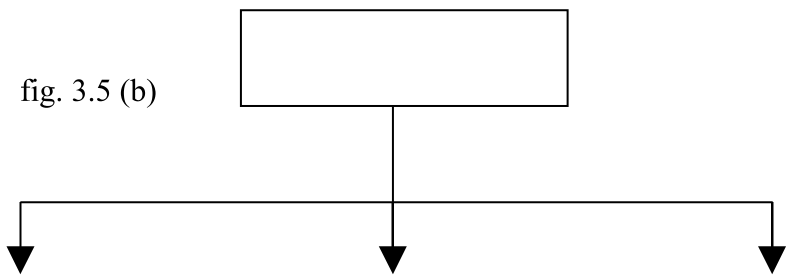


Fig. 3.5 (a)

Paper capacitor | Mica capacitor | Ceramic capacitor | Electrolytic capacitor

fig. 3.5 (b)



3.6.1 Paper, Plastic, Ceramic and Mica Capacitors

Waxed paper, plastics (e.g. polystyrene), ceramics e.g. talc with barium titanate added) and mica (which occurs naturally and splits into very thin sheets of uniform thickness) are all used as dielectrics. Typical constructions are shown in figure 3.6

The losses in paper capacitors limit their use to frequencies less than 1 MHz; also, their stability is poor – up to 10 percent changes occurring with age. Plastic, ceramic and mica types have better stability (1 percent) and can be used at much higher frequencies.

The values of the capacitances for these four types seldom exceed a few microfarads and in the case off mica the limit is about 0.01μF.

3.6.2 Electrolytic Capacitors

An electrolytic capacitor consists of two electrodes of aluminum, called the positive and the negative plates. The positive plate is electrolytically coated with a thin layer of aluminium oxide. This coating serves as the dielectric. The two electrodes are in contact through the electrolyte which is a solution of glycerine and sodium (or a paste of borates, e.g. ammonium borate). There are two types of electrolytic capacitors- the wet type and the dry type.

Electrolytic capacitors have capacitances up to $10^5 \mu F$ and are quite compact because the dielectric can have a thickness as small as 10^{-4}mm and net suffer breakdown even got applied p.d.s of a few hundred volts.

The are not used in alternating current circuits where the frequency exceeds about 10 KHz. Their stability is poor (10-20 percent) but in many cases this does not matter.

3.6.3 Variable Air Capacitor (Gang capacitor)

A very common capacitor whose capacitance can be varied continuously is used for tuning radio receivers.

The capacitor consists of two sets of semicircular aluminium plates (see figure 3.7) .One set of palters is fixed and the other set of plates can be rotated by the knob. As it is rotated, the moving set of plates gradually gets into (or comes out of) the interspace between the fixed set. The area of overlap between the two sets plates can thus be uniformly varied. In effect, this changes the capacitance of the capacitor. The air between the plates acts as the dielectric.

Losses in an air dielectric are very small at all frequencies. However, relatively large thickness are needed because breakdown occurs at a potential gradient which is compared with other of those dielectrics.

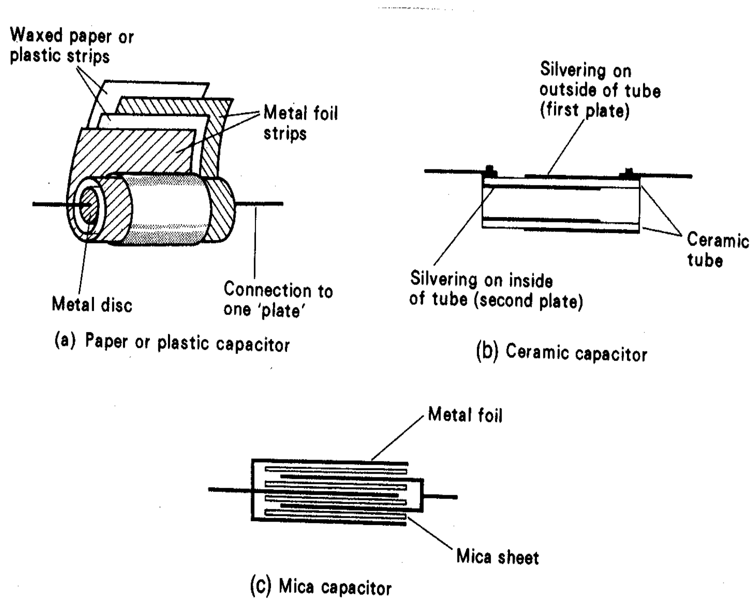

Waxed paper or plastic strips
Metal foil strips
Metal disc
Connection to one 'plate'
(a) Paper or plastic capacitor
Silvering on outside of tube (first plate)
Ceramic tube
Silvering on inside of tube (second plate)
(b) Ceramic capacitor
Metal foil
Mica sheet
(c) Mica capacitor

Fig 3.6 (a)

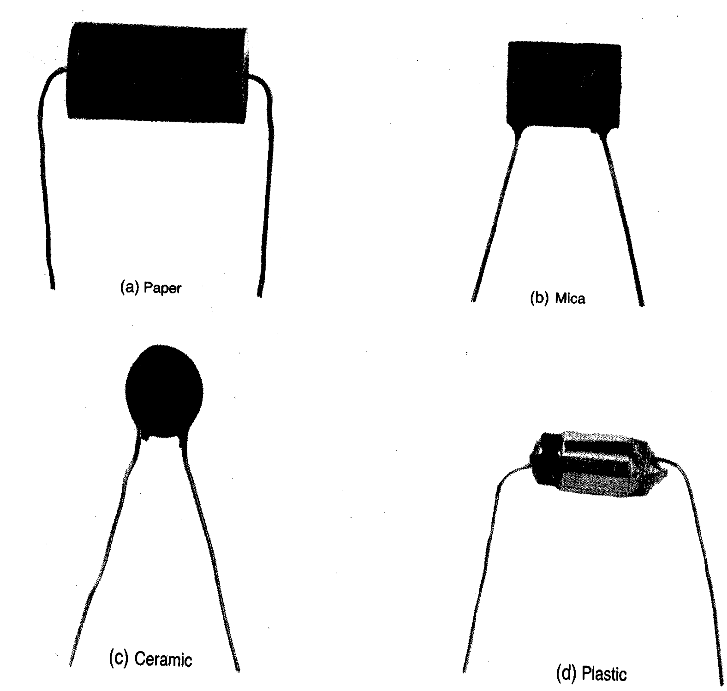
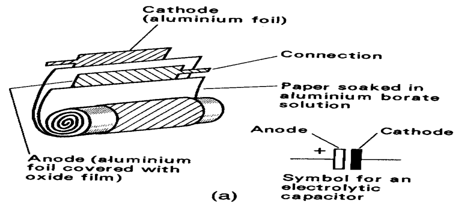

(a) Paper
(b) Mica
(c) Ceramic
(d) Plastic

Fig 3.6(b)



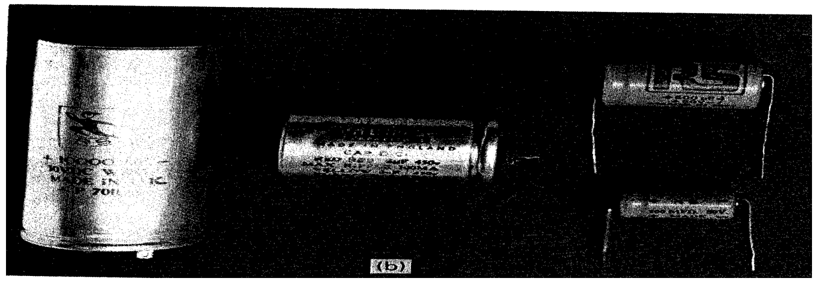


Fig. 3.6 (c)

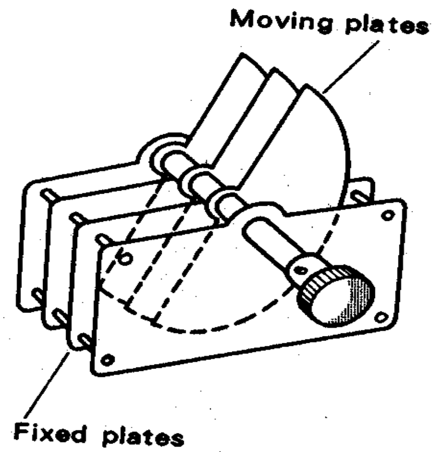


Fig. 3.7 – variable air capacitor

4.0 Conclusion

The following points should be borne in mind and, if you have not understood them very well, please go over the unit again for a better understanding.

The capacitance of a capacitor is a measure of its ability to store electric charge. The unit is coulomb per volt (CV^{-1}), commonly referred to as the farad.

You are now able to solve simple problems involving series and parallel combinations of capacitors. You have leant that when an insulator (dielectric) is inserted between the plates of a capacitor, the capacitance increases and you are now familiar with the following terms-permittivity, relative permittivity or dielectric constant, dielectric strength and breakdown.

We have studied various types of capacitor and their uses. In general, there are two groups – fixed and variable capacitors.

Capacitors are made in different ways, to suit the particular applications. Layers of conducting foil and paper rolled up give a cheap form of capacitor. Mica and meter foil can withstand high electric fields but are expensive. Electrolytic capacitors, in which the dielectric is a very thin oxide film deposited electrolytically, give very large capacitance. Ceramic capacitors are useful in transistor circuits where voltages are low but small size and compactness are desirable.

5.0 Summary

* A capacitor is a device which can store electric charge
* The energy stored in a capacitor is given by

$$W = \frac{1}{2} CV^2 = \frac{1}{2} Q^2/C \text{ joules}$$

where C, V and Q are the capacitance, potential difference and charge respectively.

* If two capacitors with capacitances C_1 and C_2 are connected in series, the effective capacitance is $C = C_1 C_2 / (C_1 + C_2)$

* The effective capacitance of the two capacitors c_1, and c_2, when they are connected in parallel is $C = C_1 + C_2$

* The permittivity, E of a dielectric medium is E = εr εo where Er is the relative permittivity (dielectric constant) and εo is the permittivity of vacuum

6.0 Tutor Marked Assignments (TMA)

1. A capacitor has n similar plates at equal spacing, with the alternate plates connected together. Show that its capacitance is equal to (n-1) Er Er A/ι where A is the plate area and ℓ is the plates separation.

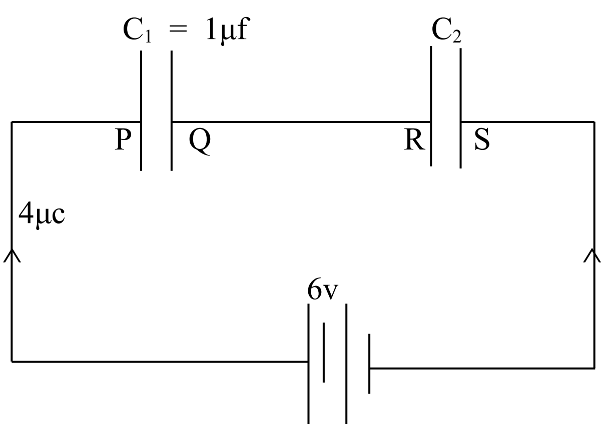
2. Calculate the potential difference between the plates of a parallel plate capacitor so that the gravitational force on a proton would be balanced

by the electric field (proton mass = 1.67 x 10^{-27}kg, plate separation 0.5cm).

3. In figure 3. if a charge of 4μc flows from the 6Vbattery to plate P of the 1μF capacitor, what charge flows from

(i) Q to R? ; and

(ii) S to the battery?



What is the p.d across (iii) C_1, and (iv) C_2?

What is the capacitance of (V) C_2 and (vi) the single capacitor which is equivalent to C_1 and C_2 in series, and what charge would it store?

4. The capacitance of a variable radio capacitor can be changed from 50PF to 950PF by turning the dial from 0^0 to 180^0. With the dial set at 180^0, the capacitor is connected to a 400-V battery. After charging, the capacitor is disconnected from the battery and the dial is turned to 0^0.

(a) What is the charge on the capacitor?

(b) What is the p.d across the capacitor when the dial reads 0^0?

(c) What is the energy of the capacitor in this position?

7.0 References And Other Resources

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UNIT 6

ELECTRIC CURRENT

Table of Contents

1.0 Introduction
2.0 Objectives
3.1 Electric Current and Current Density
3.1.1 Resistance, Resistivity and Conductivity
3.2 Electromotive force
3.3 Internal Resistance
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1.0 Introduction

We have seen that apart from gravity, the only force between two electric charges is the Coulomb force. In terms of applications, the importance of electrostatics is not well known to the ordinary person. Historically, things really got exciting when the charges started moving to form an electric current. It was by observing electrical currents that the connection between electricity and magnetism was irrevocably established.

All the electrical appliances we use, such as the radio, electric heater, electric fan refrigerator and so on depends on the flow of charge that is electric current.

The motion of charge usually occurs in conductors which contain free electrons; in the ionized gases of fluorescent lamps which contain charge carriers of both signs, and also in an evacuated region, for example, electrons in a TV picture tube.

2.0 Objectives

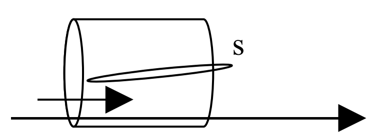
After studying this unit, you will be able to:

* Explain the concept of electric current and current density
* Distinguish between Ohmic and non-ohmic conductors.
* Define resistance, resistivity and conductivity
* Explain what are meant by the electromotive force and internval resistance of a battery or generator.
* Differentiate between an ‘open’ circuit and a ‘close’ circuit.
* Explain the terminal potential of a current source
* Do calculations involving resistances and e.m.f.s.

3.1 Electric Current and Current Density

When there is a net flow of charge across any area, we say there is a current across that area. For example, if the ends of a conductor, say copper wire are connected to a battery, an electric field E will be set up at every point within the conductor. As a result of the field, the electrons in the wire will move in the direction opposite to that of the field and give rise to an electric current in the wire.

The electric current is defined as the amount of charge passing through a given cross-section of the wire per unit time.



$$I = dq/dt$$

Fig 3.1

For the wire shown in figure 3.1, the current, 1 is defined as the rate at which charge passes through a plane perpendicular to the axis of the wire. For example, if a charge q crosses the cross-section, S in figure 3.1 in time t then the average current 1 is given by

$$I = \frac{\text{net charge transferred}}{\text{Time taken}} = q/t \text{ ....3.1}$$

When the current is not constant, i.e. the current varies with time; we define an instantaneous value I (t). If a charge of Δq crosses the shaded area, S in a time Δt, the instantaneous current is given by

$$I(t) = \lim_{\Delta t \to o} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \text{ ....3.2}$$

Equation 3.1 and 3.2 show that the unit of current is coulombs per second (C S). In the SI system of units, I coulomb per second is known as the ampere (A)

☐ **Current is a scalar quantity. Although you will soon see that a current in a wire is represented by an arrow. Such an arrow only shows direction of flow of charges along a conductor.**

By convention, the direction of current is defined as that direction in which a positive charge moves. If the moving charge is negative, as with electrons in a metal, then the current is opposite to the flow of the actual charges.

As explained earlier, current is the total charge passing through the wire per unit time across any cross-section. Therefore, the current is determined by the total charge that flows through the wire, whether or not the charge passing through every element of the cross-section of the wire is the same. To that extent, current is a macroscopic quantity that takes account of the flow of charge at every point of the conductor. This is called the current density, and it is denoted by the symbol $\underline{J}$

The current density, $\underline{J}$ is defined as the charge flowing per unit time per unit area normal to the surface, and has the same direction as a positive charge.

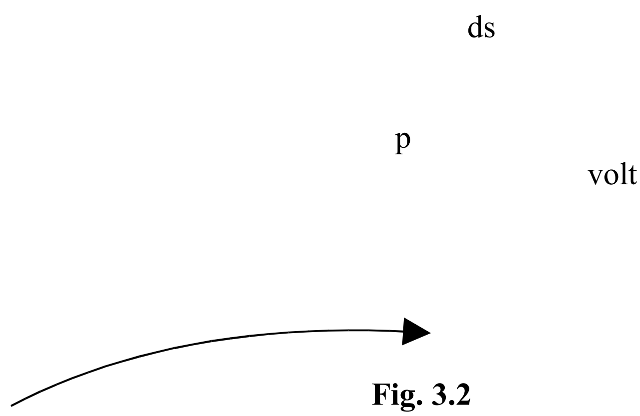


Fig. 3.2

Figure 3.2 illustrates a simple system in which particles, each of charge q, are moving to the right. Let us consider a small area ds around point P so that all the particles crossing this area may be assumed to have the same speed, V. Let us further imagine a cylinder of length vdt as shown in the figure. Then all the particles within this cylinder of volume ds V dt would cross the area ds in time dt. If n is the number of charged particles per unit volume, then the number of charged particles found in such a volume is ndsvdt. Therefore, the average rate at which the charge is passing through ds, that is the current through ds is given by

$$I = \frac{q(nds.v.dt)}{dt} = ndsvq \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

Since current density is defined as the current per unit area, perpendicular to the velocity of the charge carriers, we have

$$\underline{J} = \frac{I}{ds} = nqv \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

Since the direction of J is the direction of the actual flow of charges at the point, Equation 3.4 can be written in vector form as

$$\underline{J} = nq\underline{v} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

Thus J is a vector quantity. I SI units, J is expressed in amperes per square metre (Am^{-2}).

When the charge carriers are electrons, q = -e and Eq. 3.5 takes the form

$$\underline{J} = -ne\underline{v} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

The product nq is eq. 3.5 represents the volume charge density e of the charge carriers. Hence in terms of e the current density is expressed as

$$\underline{J} = e\underline{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

3.1.1 Resistance, Resistivity And Conductivity

In most materials and over a wide range of electric fields, experiments have shown that the current density, $\underline{J}$, at every point within a conductor in which there is a resultant electric field $\underline{E}$, is given by

$$\underline{E} = \rho\underline{J} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

It is often difficult to measure E and $\underline{J}$ directly, and it is useful to put the relation in a form involving readily measured quantities such as total current and potential difference.

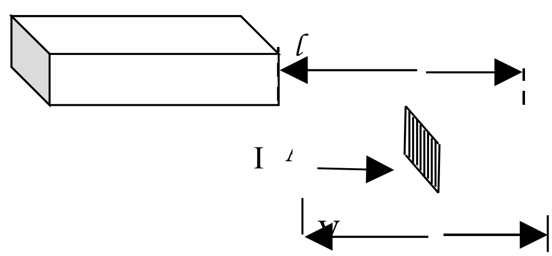


Figure 3.3

Let us consider a conductor with unifom cross-sectional area A and length ℓ as shown in fig. 3.3 .Assuming a constant current density over a cross-section, and a uniform eelctric field along the length of the conductor, the total current I is given by

$$I = JA$$

And the p.d., V between the ends is

$$V = E\ell \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

Solving these equations for J and E, respectively, and substituting the results in equation 3.8, we obtain

$$\frac{V}{\ell} = \frac{eI}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

Thus the total current, I is proportional to the potential difference. The quantity eℓ/A for a particular specimen of material is called its resistance R

$$R = \frac{e\ell}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.11$$

Equation 3.10 then becomes

$$V = IR \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

This is Ohm's law. Note that in this form, the law refers to a specific piece of material, not to a general property of the material.

Equation 3.11 show that the resistance of a wire or other conductor of uniform cross-section is directly proportional to its length and inversley proportional; to its cross-sectional area.

The unit of resistance is volt per ampere ($1VA^{-1}$). A resistance of 1 VA^{-1} is called 1 Ohm (IΩ)

Resistivity

In general, the dependence of J on E can be quite complex, but for some materials, especially metal, it can be represented quite well by eq. 3.8. For such materials the ratio of E to J is constant.

That is $E/J = \varrho \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.13$

ϱ is refrred to as the resistivity of the material and is defined as the ratio of electric field to current density

□ **Show that the unit of resistivity is ohm metre (Ωm)**

A 'perfect' conductor would have zero resistivity, and a perfecrt' insulator an infinite resistivity. Metals and alloys have the lowest resistivities and are the best conductors. The resistivities of insulators exceed those of metals by a factor of the order of 10^{22}.

You should note that resistivity depends on the nature of the conducting material whereas the resistance depends not only on the nature of the medium but on its physical dimensions.

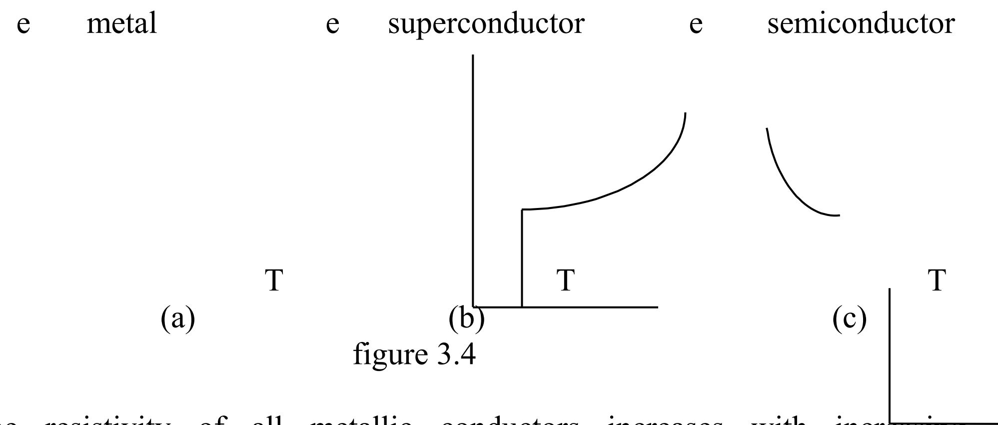


figure 3.4

The resistivity of all metallic conductors increases with increasing temperature as shown in fig. 3.4. (a). Over a temperature range that is not too great, the resistivity of a metal can be represented approximately by the equation

$$\varrho_T = \varrho_o [1 + \alpha (T - T_o)] \quad \ldots\ldots\ldots\ldots\ldots 3.14$$

Where ϱ_o is the resistivity at n reference temperature To and ϱ_r the resistivity at temperature T°C. The factor α is called the temperature coefficient of resistivity.

Laboratory experiments have shown that when rings of mercury, lead, tin and thallium are cooled to the temperature of liquid helium their resistance disappear, the resistivity at first decreases regularly, like that of any metal. At the helium temperature (critical temperature), usually in the range 0.1K to 20K, the resistivity suddenly drops to zero (see fig. 3.4 (b)). The materials are said to exhibit superconductivity. A current once established in a superconducting ring will continue of itself, apparently indefinitely, without the presence of any driving field.

The resistivity of a semiconductor decreases rapidly with increasing temperature as shown in fig. 3.4 (c).

□ **Example**

The resistance of 80.0cm of constantan wire, whose diameter of cross-section is 0.457mm, is 2.39Ω. Find the resistivity of constantan.

Solution

From equation 3.11, the resistivity of constantan is given by

$$\varrho = \underline{AR}$$

ℓ

with the symbols having their usual meanings

Now, $\ell = 0.80$m, $A = \pi \times \left(\frac{0.457}{2}\right)^2 \times 10^{-6}\ m^2$

$$\varrho = \frac{2.39 \times \pi \times 0.2285^2 \times 10^{-6}}{0.80}$$

$$= 4.90 \times 10^{-7}\Omega\ m$$

☐ **The resistance of a copper coil at 20°C is 300*l*. Calculate the resistance of the coil at 60°C if the temperature coefficient of resistance and for copper is 4.0 x 10^{-3}°C^{-1}.**

Solution

The variation of resistance with temperature is similar to that of resistivity. Therefore, we use equation like 3.14.

$$R_T = R_o [1 + \alpha (T - T_o)]$$

When using this equation where accuracy is importance the reference temperature To should be 0^0C and R_o, the resistance at 0^0C.

We have $R_{20} = R_o (1 + 20 \alpha)$

And $R_{6o} = R_o (1 + 60 \alpha)$

Dividing $\frac{R_{6o}}{R_{20}} = \frac{1 + 60 \alpha}{1 + 20 \alpha}$

$$R6o = \frac{30 (1 + 60 \times 4.0 \times 10^{-3})}{(1 + 20 \times 4.0 \times 10^{-3})}$$

$$= 34.5\Omega$$

Conductivity

We can write eq. 3.8 in the form

$\underline{J} = 1/\varrho\ \underline{E} = \sigma\ \underline{E}$ ……………………….3.15

Where σ is the reciprocal of resistivity and is known as the conductivity of the material. The value of σ is very large for metallic conductors and extremely small for good insulators.

The SI unit of conductivity is ($\Omega\ m^{-1}$)

3.2 Electromotive Force

There is no doubt that you are familiar with the two terms –batteries and generators which are used in our homes, officers, towns and villages. They are sources of electric current. Perhaps you are putting on a wrist watch that operates on a battery.

Batteries and generators are able to maintain one terminal positive (i.e. deficient of electrons) and the other negative (i.e with an excess of electrons). If we consider the motion of positive charges, then a battery, for example, moves positive charges from a place of low potential (the negative terminal) through the battery to a place of high potential (the positive terminal). Therefore, a battery or generator does work on charges and so energy must be changed within it. (You recall from PHY 101 that work is a measure of energy transfer). In a battery, chemical energy is transferred into electrical energy which we consider to be stored in the electric and magnetic fields produced. When there is a current in the external circuit, this stored electrical energy is changed, for example, to heat, but is replenished at the same rate at which it is transferred. The electric and magnetic fields thus act as temporary storage reservoir of electrical energy in the transfer of chemical energy to heat. A battery or dynamo is said to produce an electromotive force. (e.m.f). defined in terms of energy transfer.

Definition: The electromotive force of a source (a battery, generator, etc) is the energy (chemical, mechanical, etc) converted into electrical energy, when unit charge passes through it.

Unit of e.m.f like the unit of p.d is the volt.

⧈ **Note** 1. Although e.m.f and p.d. have the same unit, they deal with different aspects of an electric circuit. Whilst, e.m.f applies to a source supplying electrical energy, p.d refers to the conversion of electrical energy in a circuit.

2. The term e.m.f. might appear to be misleading to some extent, since it measures energy per unit charge and not force. The fact remains, however, that the source

of e.m.f is responsible for moving charges round the circuit. Hence it would appear that “force’ is not a misnomer here if we remember the principle of inertia – Newton’s first law.

3.3 Internal Resistance

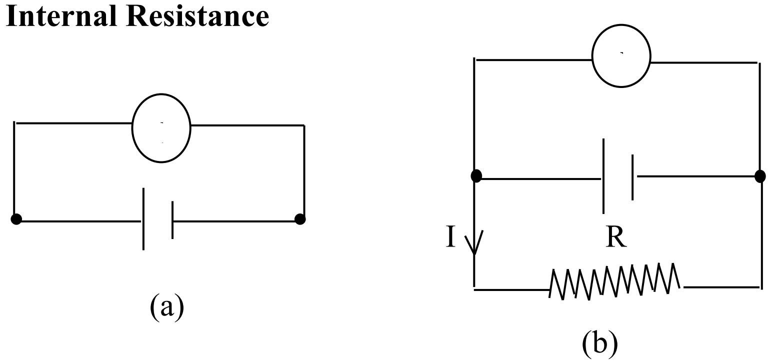


Fig. 3.5

A high-resistance **voltmeter** connected across a cell on open circuit records its electromotive force (very nearly) (see figure 3.5a). Let this be E. If the cell is now connected to an external circuit in the form of a resistor R and maintains a steady current I in the circuit., the voltmeter reading falls, let it be V, (see fig. 3.5b). V is the terminal p.d. of the cell (but not on open circuit) and it is also the p.d. across R (assuming the connecting leads have zero resistnace). Since V is less then E, not all the energy supplied per coulomb by the cell (i.e E) is chnged in the external ciruit to other forms of energy (often heat).

Now, let us find answers to these two questions?

(1) What do you undersand by a battery or generator being on open circuit ?

(2) What has happend to the “lost” energy per coulomb?

Answer 1 A battery or generator is on open circuit when it is not maintaing current.

2. The deficiency is due to the cell itself having some resistnace. A certain amount of electrical energy per coulomb is wasted in getting through the cell and so less is avilable for the extenal circuit.

The resistance of a cell is called its internal resistance (r). Taking stock of the energy changes in the complete circuit, including the cell, we have

Energy supplied per coulomb by call	=	Energy changed per coulomb by external circuit	+	Energy wasted per coulomb on internal resisnce of battery

e.m.f	=	p.d. across R	+	p.d. across r
E	=	V	+	v 3.15

Where V is the p.d. across the internal resistance of the cell, a quantity whcih cannot be measured directly but only by substracting V from E.

Eq. 3.15 shows that the sum of the p.d.s across all the resistance (external and internal) equals the e.m.f.

Since V = IR and v = Ir we can rewrite eq. 3.15 as

$$E = IR + Ir \quad \text{.......................................3.16a}$$

$$= I\ (R + r) \quad \text{....................3.16b}$$

□ **Can you now explain why i said at the begining of this section that a high resistance voltmeter connected across a cell on open circuit records its e.m.f only very nearly?**

Answer: This is becasue the volmeter must take some current, however small, to give a reding. A small part of the e.m.f is, therefore, lost in driving current through the internal resistance of the battery. A potentiometer is used to measure e.m.f to very high accuracy.

4.0 Conclusion

You have now been introduced to some basic concepts that will assist you to understand our subsequent study of Electricity and

Magnetism. Note that the current across an area is defined quantitatively as the net charge flowing across the area per unit time. We have also discussed some parameters which are characteristic of the current carrying conductors. These are resistance, resistivity and conductivity. These parameters vary with temperature.

We undertook a fairly detailed explanation of the elecromotive force of a source of current such as a battery or generator. You can now understand that e.m.f is the energy (chemical, mechnical, etc) which is converted into electrical energy when unit charge passes through the source.

5.0 Summary

- One ohm is the resistance of a conductor through which a current of 1A passes when a potential difference of 1 volt is maintianed across its ends.

- For temperature ranges that are not too great, the variation of resistance with temperature may be represented approximately as a linear relation

  $$R_T = Ro\ [\ 1 + \alpha\ (\ T - To)\ ]$$

  The variation of resistivity with temperature follows the same linear form.

- For a device obeying ohm's law, the current through the device depends on the p.d. between its terminals.

6.0 Tutor Marked Assignments (TMA)

1. The potential difference across the terminals of a battery is 8.5V when there is a current of 3A in the circuit frm the negative to the positive terminal. When the current is 2A in the reverse direction, the potential difference becomes 11V.

   (i) What is the internal resistance of the battery?

   (ii) What is the e.m.f of the battery?

2. A wire 100cm long and 2mm in diameter has a resistivity of 4.8x10

$^{-8}\Omega m$.

(i) Calculate the resistance of the wire?

(ii) A second wire of the same material has the same weight as the 100m length, but twice its diametric. Evaluate its resistance.

3. A wire has a resistance of 10.0 ohms at 20.0^0c and 13.1 ohms at 100^0c. Obtain a value for its temperature coefficient of resistance.

4. What is the potential difference between two points in a circuit if 200J of electrical energy are changed to other forms of energy when 25 coulombs of electric charge pass? If the charge flows in 10 seconds, what is the current?

5. A television set shoots out a beam of electrons. The beam current is 10uA. How many electrons strike the TV screen each second? How much charge strike the screen in a minute

6.

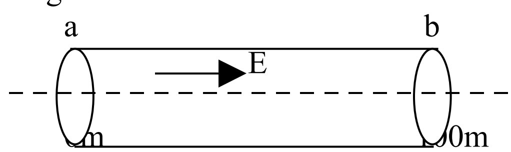


Fig. 3.6

Figure 3.6 shows a copper conductor of resistivity $\varrho = 1.72 \times 10^{-8}\ \Omega m$ having a current density $J = 2.54 \times 10^6\ Am^{-2}$. Calculate the electric field in the copper.

What is the potential difference between the two points a and b, 100m apart?

7.0 Reference and Other Resources

An Introduction to High School Physics. J.M. Das Sarma
Modern Book Agency Private Ltd.

College Physics. Sears F.W., Zemansky M.W. and Young H.D.
Addison-Wesley Publishing Company, London.

Electric and Magnetic Phenomena
Indira Gandhi National Open University PHE-07, 2001

Physics. A Textbook for Advanced Level Students. Tom Duncan

John Murray (Publishers) Ltd. London.

UNIT 7

DIRECT – CURRENT CIRCUITS AND INSTRUMENTS

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1.0 Introduction

Most electric circuits do not consist simply of a single source and a single external resistor as we saw in unit 6. Usually, an electric circuit may comprise a number of sources, resistors, or other elements such as capacitors, motors, etc. interconnected in a network.

We shall begin this unit with the study of some techniques of handling problems involving networks that cannot be reduced to simple series and parallel combinations.

The usual method of measuring resistance is by the principle of wheatstone's bridge, which embodies a method of comparison. Its practical form is the metre bridge. These will be studied in this unit. Another important measuring instrument discussed in this unit is the potentiometer. It is a simple physical apparatus which has a unique importance for its universal application in precision electrical measurements. It is used for the measurement of electromotive force, current or resistance and for the comparison of two e.m.f.s.

You will be introduced to two basic instruments for the measurement of current and potential difference. These are the ammeters and the voltmeter.

2.0 Objectives

By the time you have studied this unit, you will be able to

- * Do simple calculations involving series and parallel combinations of resistors in an electric circuit.
- * Solve problems pertaining to networks by the application of kirchoff's rules.
- * Understand the principle of the wheatstones bridge and the use of its practical form in the laboratory for the measurement or comparison of resistances.
- * The method of using the potentiometer for the comparison of e.m.f.s or the measurement of e.m.f, current or resistance.

3.1 Resistors in Series and in Parallel

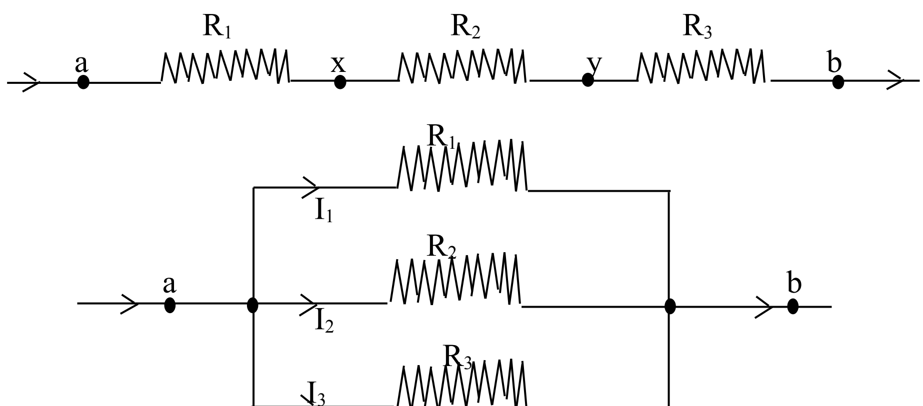


Figure 3.1

Figure 3.1 shows two different ways in which three resistors having resistances R_1, R_2 and R_3 might be connected between points a and b. In fig 3.1 (a) the resistors only provide a signle path between the points. A number of circuit elements such as resistors, cell, motors, etc are similarly said to be in series with one another between the two points of connect as in (a) so as to provide only a single path between the points. The current is the same in each element .

The resistors in fig 3.1 (b) are said to be in parallel between the point a and b. Each resistance provides an alternative path between the points and, any number of circuit elements similarly connected are in parallel with one another. The potential difference is the same across each element.

It is always possible to find a single resistor which could replace a combination of resistors in any given circuit and leave unchanged the potential difference a between the terminals of the combination and the current in the rest of the circuit. The resisitance of this single resistor is called the equivalent or effective resistance of the combination.

If any one of the networks in fig 3.1 were replaced by its equivalent resistance, we could write.

$$Vab = IR \text{ or } R = \frac{Vab}{J}$$

Where Vab is the p.d between the terminals of the network and I is the current at the point a or b. Therefore, the usual method of computing an equivalent resistance is to assume a p.d. Vab across the actual network, compute the corresponding current I (or vice versa), and take the ratio of one to the other.

For the series combination in fig. 3.1 (a), the current in each must be the same and equal to the current I.

Hence $V_{ax} = IR,\ V_{xy} = IR_2,\ V_{yb} = IR_3$

and $V_{ab} = V_{ax} + V_{xy} + V_{yb}$

$= I\,(R_1 + R_2 + R_3)$

$$\therefore \frac{V_{ab}}{I} = R_1 + R_2 + R_3$$

But V_{ab} / r is, by definitions, the equivalent resistance R.

Therefore, $R = R_1 + R_2 + R_3$ ………………………..3.1

Showing that the equivalent resistance of any number of resistors in series equals the sum of their individual resistances

For the parallel combination of resistances in fig. 3.1 (b), the p.d. between the terminals of each must be the same and equal to V_{ab}. If the currents in each are denoted by I_1, I_2 and I_3, respectively,

$$I_1 = \frac{V_{ab}}{R_1}\ ; \quad I_2 = \frac{V_{ab}}{R_2}\ ; \quad I_3 = \frac{V_{ab}}{R_3}$$

Charge is delivered to point a by the line current I_1 and removed from a by the currents I_1, I_2 and I_3. Since charge is not accumulating at a , it follows that

$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Or

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

But

$$\frac{I}{Vab} = \frac{1}{R}$$

Therefore

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad \ldots\ldots\ldots\ldots\ldots..3.2$$

Showing that for any number of resistors in parallel, the reciprocal of the equivalent resistance equals the sum of the reciprocals of their individual resistances.

Example

Compute the equivalent resistance of the network in figure 3.2, and find the current in each resistor.

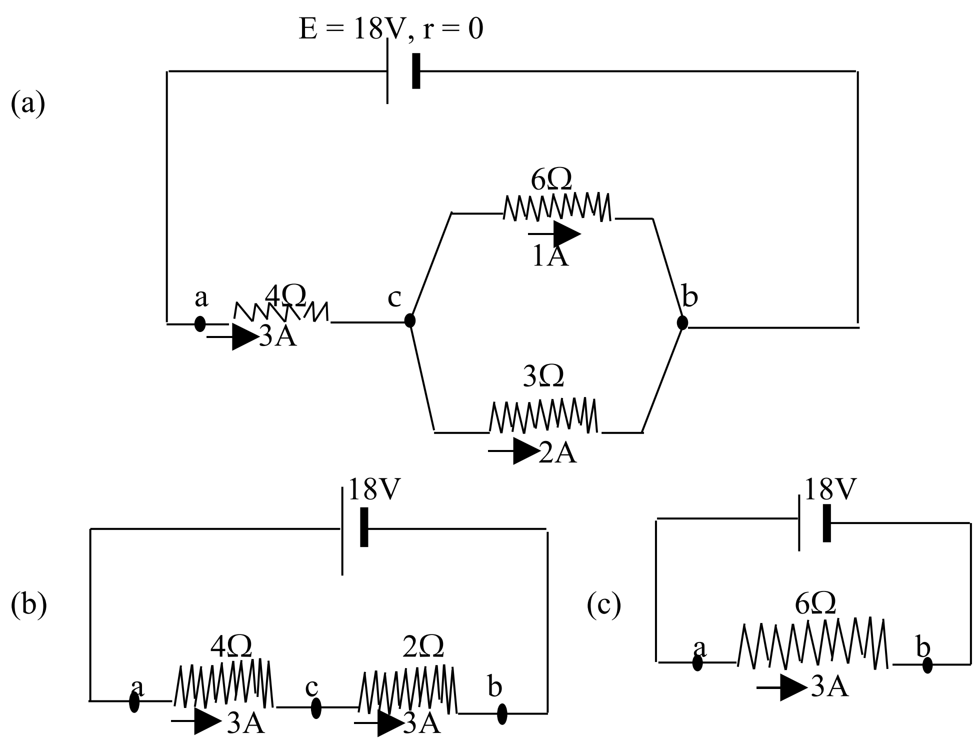


Figure 3.2

Figure 3.2 shows the successive steps on the reduction to a single equivalent resistance. The 6Ω and 3Ω resistors in part (a) are equivalent to the single 2Ω resistor in part (b) and the series combination of this with the 4Ω resistor results in the single equivalent 6Ω resistor in part (c).

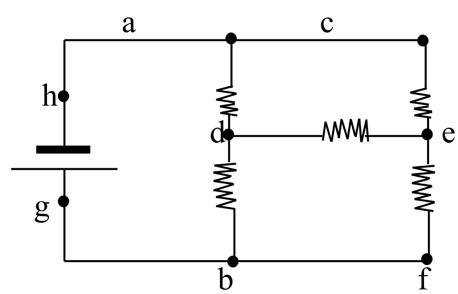
☐ **Work out and check the statements in the above paragraph by calculations**.

In the simple series circuit of part C, the current is 3A. Hence the currnt in the 4Ω and 2Ω resistors in part (b) is 3A The p.d Vab is therefore 6V. Since it msut be 6v in part (a) as well, the currents in the 6Ω and 3Ω resistor in part (a) are 1A and 2A, respectively.

Continue your calculations in order to confirm the results.

3.2 Kirchhoff's Rules

fig. 3.3



Unfuntunately, not all networks can be reduced to simple series-parallel combinations. An example is a resistance network with a cross connection (see fig. 3.3). No principles are required to compute the current in these networks, but there are a number of techniques that enable such problems to be handled systemtically. We shall discuss only one of them which was developed by G.R. Kirchohoff (1824-1887). Before we go to kirchhoff's law, let us define two relevant terms.

Branch Points: A branch point in a network is a point where three or more conductors are joined. For example, a, d, e, and b are branch points.

Loop: A loop is any closed conducting path.

Identify the branch points and loops in fig. 3.3

Answer: a, d, e, and b are branch points. Possible loops are the closed paths aceda, defbd, hadbgh, and hadefbgh.

Kirchhoff's rules consist of the following statements:

Point rule: The algebraic sum of the currents toward any branch point is zero.

$$\sum I = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

Loop rule: The algebraic sum of the e.m.f.s in any loop equals the algebraic sum of the IR products in the same loop.

$$\sum E = \sum IR \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

The point rule merely states formally that no charge accumulates at a branch point. The second rule is a generalization of the circuit equation E = IR, and reduces to this equation of the current I is the same in all resistances.

Example 1

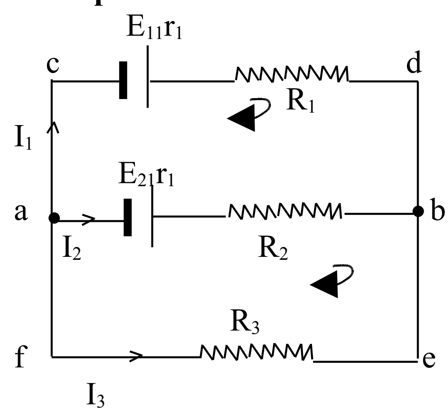


Fig 3.4

In the figure 3.4, let magnitudes and directions of the e.m.f.s and the magnitudes of the resistances be given. We wish to solve for the currents in each branch of the network.

Solution.

The network is prepared for solution by Kirchhoff's rules as follows. Assign a direction and a letter to each unknown current. The assumed directions are purely arbitrary. Note that the currents in sources 1 and resistor 1 are the same, and require only a single letter 1_1. The same is true for source 2 and resistor 2; the current in both is represented by I_2.

There are only two branch points, a and b.

At point b, $\sum I = I_1 + I_2 + I_3 = 0$ ……………………(i)

Since there are only two branch points, there is only one independent point equation. If the point rule is applied to the other branch point, point a, we have $\sum I = -I_1 - I_2 - I_3 = 0$.

Which is the same equation with signs reversed.

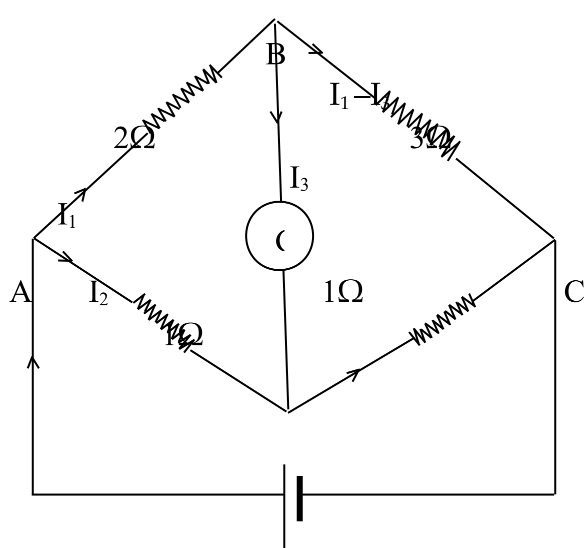
Let us consider the loops acdba, and abefa and apply the clockwise direction positive in each loop. The loop rule then furnishes the following equations.

$$E_1 - E_2 = I_1 r_1 + I_1 R_1 - I_2 R_2 - I_2 r_2 \text{ ……..(ii)}$$
$$E_2 = I_2 r_2 + I_2 R_2 - I_3 R_3 \text{ ………………………(iii)}$$

We then have three independent equations to solve for the three unknown currents.

Example 2

Calculate the currents in the network shown in fig. 3.5, assuming that the cell has a negligible internal resistance.



2Ω

$I_1 + I_2$ $1_2 + I_3$

D

F

2v

Fig. 3.5

Solution

Kirchhoff's point rule has been applied in the Figure by inserting the currents, I_1, I_2, I_3, etc.

Applying the loop rule to

$ABDA_1 \; 2I_1 + 1I_3 - 1I_3 = 0$

$BCDB_1 \; 3(I_1 - I_3) - 2(I_2 + I_3) - 1I_3 = 0$

$FABCF_1 \quad 2I_1 + 3(I_1 - I_3) = 2$

The rule can be applied to FADCF also, but only these equations are required. Moreover, as you would see, this fourth equation can be derived from the above so that it contributes nothing new.

Solving the equations,

$$I_1 = \frac{16}{43} \text{ A}, \quad I_2 = \frac{30}{43} \text{ A} \quad \text{and} \quad I_3 = -\frac{2}{43} \text{ A}$$

Since I_3 is negative it is clear that the arrow was marked in figures 3.5 in the wrong direction. Such an error is always automatically corrected by a minus sign in the solution.

3.3 Ammeters and Voltmeters

Most ammeters and voltmeters are basically galvanometers (i.e. current detectors capable of measuring currents of the order of milliamperes or micro amperes) of the moving – coil type which have been modified by connecting suitable resistors in parallel or in series with them. Moving coil instruments are accurate and sensitive.

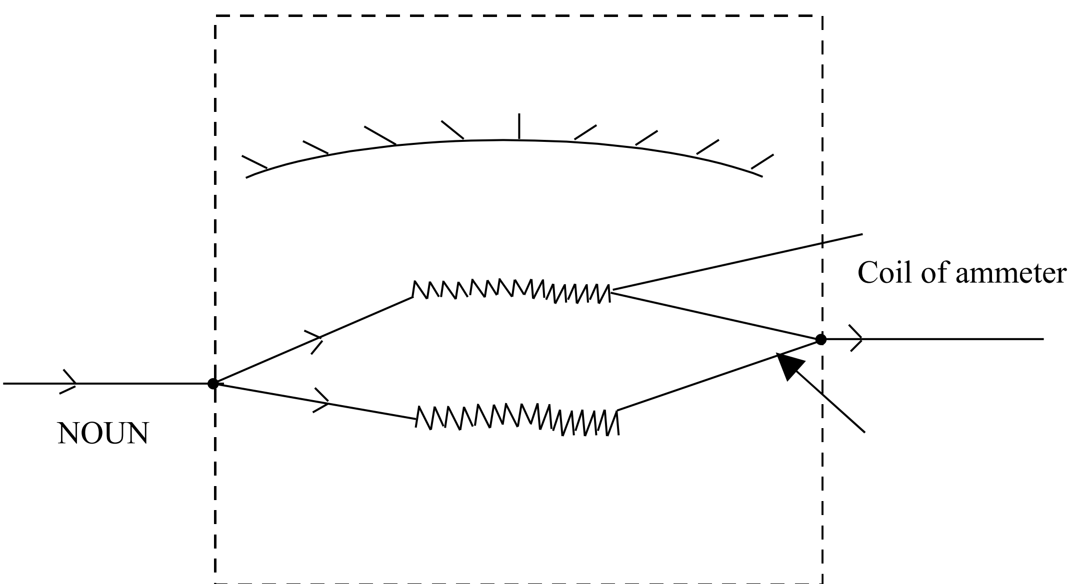
Connecting an ammeter or voltmeter should cause the minimum disturbance to the current or p.d it has to measure. An ammeter is normally connected in series so that the current passes through the meter. The resistance of an ammeter must therefore be small compared with the resistance of the rest of the circuit. Otherwise, inserting the ammeter changes the current to be measured. The perfect ammeter would have zero resistance, the p.d. across it would be zero and no energy would be absorbed by it.

The p.d between two point A and B in circuit is most readily found by connecting a voltmeter across the points, i.e., in parallel with AB. The resistance of the voltmeter must be large compared to the resistance of AB, otherwise the current drawn from the main circuit by the voltmeter (which is required to make it operate) becomes an appreciable fraction of the main current and the p.d across AB changes. A voltmeter can be treated as a resistor which automatically records the p.d. between its terminals. The perfect voltmeter would have infinity resistance, take no current and absorb no energy.

3.3.1 Shunts and Multipliers

(a) Conversion of a microammeter into an ammeter.

Let us consider a moving – coil meter which has a resistance (due largely to the coil of 1000Ω and which gives a full –scale deflection (f.s.d) when 100μ passes through it. If we wish to convert it to an ammeter reading 0-1A, this can be done by connecting a resistor of every low value parallel with it.



1000Ω
S
1A
0.9999A Ammeter terminal

Fig. 3.6

Such a resistor is called a shunt and must be chosen so that only 100μA (0.0001A) passes through the meter and the rest of the 1A, that is 0.9999A, passes through the shunt (see Fig. 3.6) A full-scale deflection of the meter will then indicate a current of 1A

To obtain the value S of the shunt, we use the fact that the meter and the shunt are in parallel. Therefore,

p.d. across meter = p.d. across shunt

Applying Ohm's law to both meter and shunt

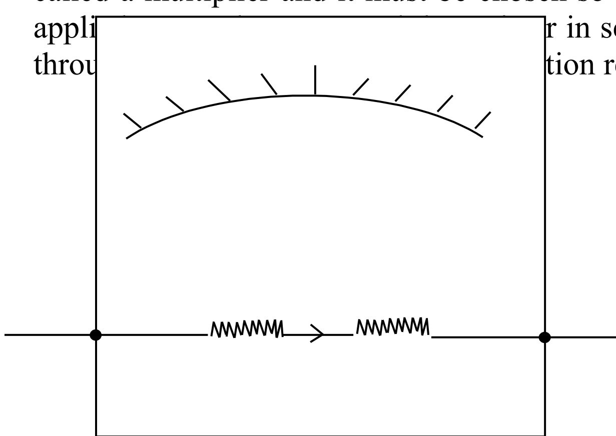
$$0.0001 \times 1000 = 0.9999 \times S \quad \text{(from } V = IR\text{)}$$

$$\therefore S = \frac{0.0001 \times 1000}{0.9999} = 0.1\Omega$$

As you would see, the combined resistance of the meter and the shunt in parallel will now be very small (less than 0.1Ω) and the current in a circuit will be virtually undisturbed when the ammeter is inserted.

(b) Conversion of a Microammeter into a Voltmeter

To convert the same moving-coil meter of resistance 1000Ω and full-scale deflection 100μA to a voltmeter reading 0-1V,a resistor of high value must be connected in series with the meter. The resistor is called a multiplier and it must be chosen so that when a p.d of 1V is appli r in series, only 0.0001A goes throu tion results (see fig. 3.7)



0 V 1

Coil of Meter

M 1000Ω

0.0001A Voltmeter terminal

To obtain the value M of the multiplier, we apply the Ohm's law when there is a f.s.d. of 0.0001A. Hence the p.d. across multiplier and meter in series .

$$= 0.0001\ (m + 1000)$$

But the meter is to give an f.s.d. when the p.d across it and the multiplier in series is 1V. Therefore,

$$0.0001\ (M + 1000) = 1$$

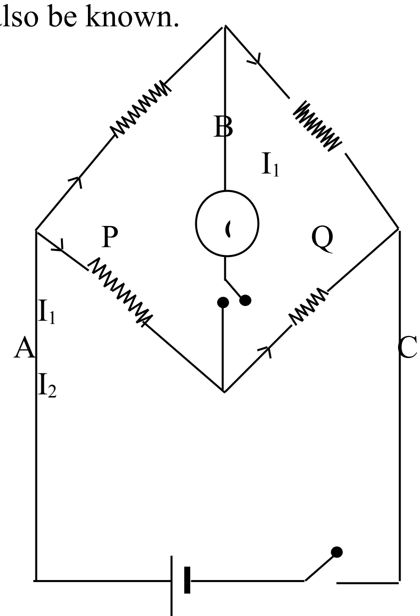
$$M + 1000 = \frac{1}{0.0001} = 10000$$

$$\text{and} \quad m = 9000\Omega$$

3.4 The Wheatstone Bridge

The most used method of measuring resistance is the Wheatstone bridge. This has the great advantage of being a null method; that is to say adjustments are made until a galvanometer is undeflected and hence the result does not depend on the accuracy of an instrument. However, reliable standard resistances are required.

Four resistances P_1, Q_1, R_1, S are arranged as in figure 3.8 one of these, say P, is the unknown standard resistance and the values of R and S or their ratio must also be known.



R S

I_2

D

Fig. 3.8

A sensitive galvanometer and a cell are connected as shown if the resistance R and S are adjusted so that no current flows in the galvanometer, the bridge is then said to be balanced.

Since no current flows through the galvanometer,

Current through P = current through Q = I_1

and current through R = current through S = I_2

Also potential at B = potential at D

i.e. $I_1 P = I_2 R$ ………………..(i)

Similarly p.d. between B and C = p.d between D and C

i.e. $I_1 Q = I_2 S$ …………………(ii)

Hence $\frac{(i)}{(ii)}$ $P/Q = R/S$

from where P can be calculated

3.4.1 The meter bridge

A practical form of the Wheatstone bridge is the meter bridge (fig. 3.9)

A wire AC of uniform cross-section and 1m long, made of some alloy such as constantan so that its resistance is of the order of 1.0hm, lies between two thick brass or copper strips bearing terminals, above a meter ruler. There is another brass strip bearing three terminals to facilitate connections and also a sliding contact D which can move along the meter wire.

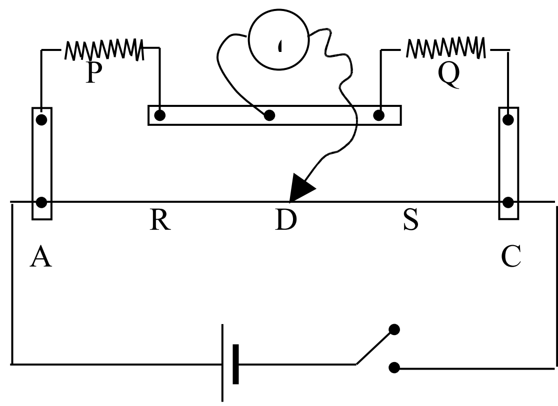


Fig. 3.9

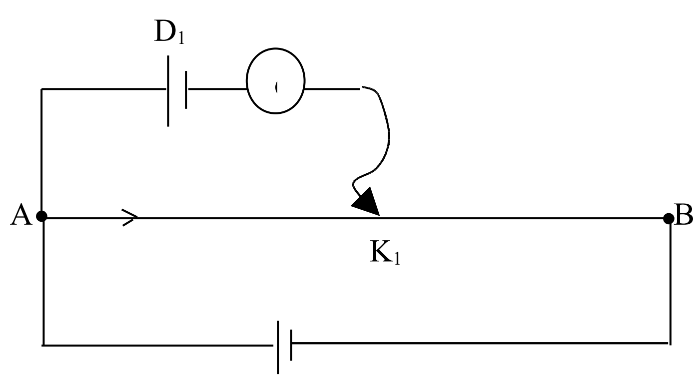
Figure 3.9 has been labeled identically with fig. 3.8 to show that the two circuits are similar.

The position of the sliding contact is adjusted until there is no current in the galvanometer.

$$\text{Then} \quad \frac{P}{Q} = \frac{R}{s} \quad \frac{\text{length AD}}{\text{length CD}}$$

3.5 The Potentiometer.

The potentiometer is a 'null' method of measuring p.d In its simplest form, the potentiometer consists of a resistance wire of uniform cross-section through which a steady current is passed.



C
Fig. 3.10

In fig. 3.10, A B represents the potentiometer wire and C the cell (usually an accumulator) supplying the steady current. There is a drop of potential down the wire from A to B; the p.d between two points on the wire is proportional to their distance apart, and can be used to counter-balance an unknown p.d.

Thus to compare the e.m.fs of two cells one of the cells D_1 is connected as in the figure and the sliding contact is moved along AB until there is no current in the galvanometer. The p.d between A and K_1, is then equal to the e.m.f of the cell D_1.

It is essential that the same poles of C and D_1, should be connected to A and that e.m.f of C should be greater than that of D_1.

The cell D_1, is now replaced by the second cell D_2 and the new position of the sliding contact, k_2 found.

$$\frac{E_1}{E_2} = \frac{AK_1}{AK_2}$$

Where E_1 is the e.m.f. of D_1
And E_2 is the e.m.f. of D_2

3.6 The R-C Circuit

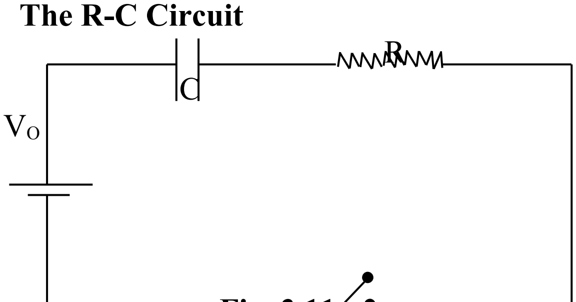


Fig. 3.11

Applying a voltage directly across a capacitor does not give the full charge instantaneously to the plates since as the charge builds up, it tends to repel the addition of any further charge.

When a resistor and a capacitor are connected in series to a source of voltage Vo, we have the e.m.f equation.

$$V_c + V_R = V_o \quad \ldots\ldots 3.5$$

Where V_c and V_R are the voltages across C and R.

Writing $V_c = q/c$

and $V_R = RI = R\frac{dq}{dt}$

Where q is the charge on the capacitor and I the current at a time t, we have

$$\frac{dq}{dt} + \frac{q}{RC} = \frac{V_o}{R}$$

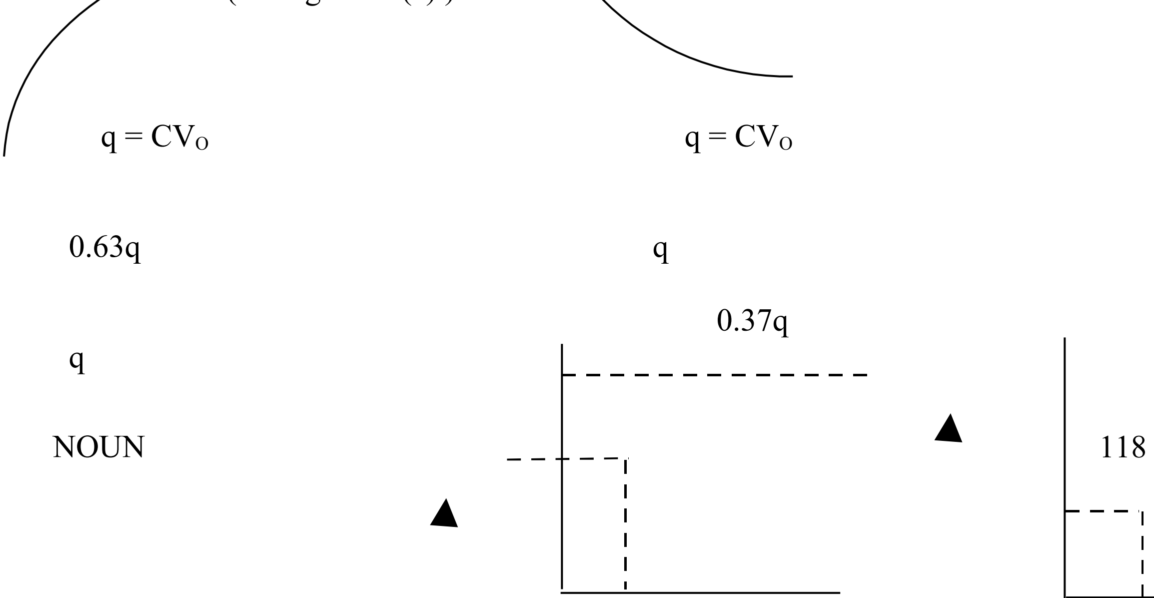
The solution of this equation, that is the charge on the capacitor at time t is given by

$$q = CV_o(1 - e^{-t/CR}) \quad \ldots\ldots 3.6$$

The term CR is called the time-constant of the circuit. After time t = CR, the charge on the capacitor will have risen to a value given by

$$q = CV_o(1 - e^{-1})$$

Since e = 2.718, $e^{-1} = 0.37$ approximately and $(1-e^{-1}) = 0.63$, so that after a time equal to the time-constant of the circuit, the charge on the capacitor (and hence the potential difference across it) will have reached 0.63 of its final value (see fig. 3.12 (a))



CR t→ CR t→

(a) (b)

Figure 3.12

The discharging process follows the inverse curve (figure 3.12 (b)), and the charge remaining on the capacitor at time t is given by

$$q = CV \; e^{-t/CR}$$

where V is the potential difference across the capacitor when it was fully charged.

Circuits with long time-constants are sued in many practical applications, e.g. to activate the flashing lights set up near roadwork's, and the regular sound pulses emitted by sonar.

4.0 Conclusions

We have studies some techniques of dealing with resistance in parallel and series combinations in electric circuits. Most importantly, we learnt to apply the two rules developed by kirchoff to networks that cannot be reduced to simple series and parallel combinations.

You are now familiar with the principle of the Wheatstone bridge and its practical form, the meter bridge that is, a reliable method of measuring resistances or comparing two unknown resistances. For the accurate measurement of e.m.f.s, you have been introduced to the potentiometer.

We have seen how a moving coil galvanometer can be converted to an ammeter or voltmeter. An ammeter is connected in series for the measurement of current at any point in an electric circuit, while a voltmeter is connected in parallel to measure the potential difference between two points.

5.0 Summary

* The equivalent resistance of n resistances in series is

$$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots + R_n = \sum_i^n R_i$$

* The equivalent resistance of n resistances in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots + \frac{1}{R_n} = \sum_i^n \frac{1}{R_i}$$

* Kirchhoff's rules consist of the following statements

1. The algebraic sum of the current toward any branch point in a network is zero. This is the point rule.

2. The algebraic sum of the e.m.f.s in a loop equals the algebraic sum of the IR products in the same loop – This is the loop rule.

* A moving coil galvanometer can be converted to an ammeter by connecting a resistor of very low value in parallel with it. Such a resistor is called a shunt.

* A moving coil galvanometer can be converted to a voltmeter by connecting a resistor of high value in series with it. Such a resistor is called a multiplier.

* In a series combination of a capacitor and a resistor to a d.c. source the building up of charge on the capacitor is given by equation

$$q = CV_o \left(1 - e^{-t/CR}\right)$$

where CR is the time-constant of the circuit. The time-constant is the time when the charge on the capacitor will have reached 0.63 of its final value.

* During the discharging process, the remaining charge on the capacitor at time t is given by

$$q = CVe^{-t/RC}$$

where V is the p.d. across the capacitor when it was fully charged.

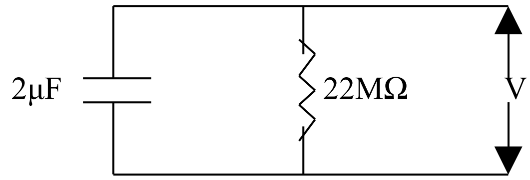
6.0 Tutor Marked Assignments (TMA)

1. Two resistance coils, P and Q are placed in the gaps of a metre bridge A balance point is found when the movable contact touches the bridge wire at a distance of 35.5cm from the end joined E_o P. When the coil P is shunted with a resistance of 10ohms, the balance point is moved through a distance of 15.5cm . Find the value of resistance P and Q.

2. A two-metre potentiometer wire is used in an experiment to determine the internal resistance of a voltaic cell. The e.m.f of the cell is balanced by the fall of potential along 90.6cm of wire. When a standard resistor of 10 0hms is connected across the cell the balance

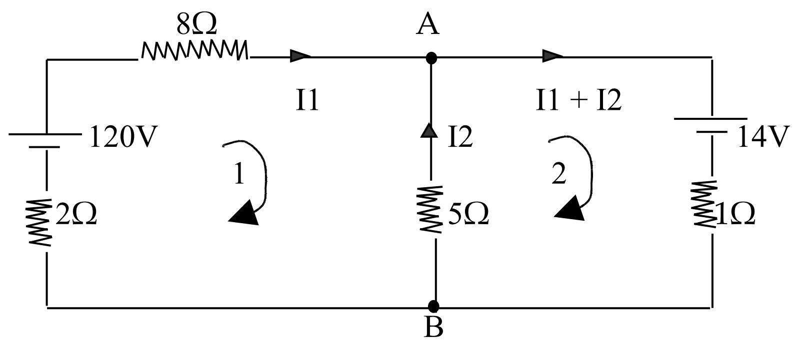
length is found to be 75.5cm . Draw a labelled circuit diagram and calculate from first principles, the internal resistance of the cell.

3.



A 2UF capcitor charged originally to a potential difference of 60v is discharged across a 22mΩ resistor as shown in the figure. Calcualte the time constant of the ccircuit and the potential differnce across the capacitor after this time constant time?

4. Find the currnt in every branch of the two loop circuit shown in the figure.



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UNIT 8

THE MAGNETIC FIELD

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1.0 Introduction

The first magnetic phenomena to be observed were those associated with natural magnets. These were rough fragments of iron ore found near the ancient town of Magnesia in Asia. In fact, the word magnet was derived from the name of that town. These natural magnets have the property of attracting to themselves unmagnetised iron, the effect being most pronounced at certain regions of the magnet known as its poles.

It was by observing electric currents that the connection between electricity and magnetism was firmly established. Thus in 1820 Haus Christian Oersted (1777 – 1851) at the University of Copenhagen, Demark found that a wire carrying an electric current deflected a nearby compass needle.

It is now well known that electric current produce magnetic fields and that a changing magnetic field produces an electric current. This connection between current and magnetism gave birth to electromagnetism, a subject to which modern civilization is heavily indebted.

In this unit, we shall consider the production of magnetic fields due to steady currents, and the forces they exert on circuits carrying steady currents.

2.0 Objectives

After studying this unit you will be able to:

* understand what is meant by he magnetic filed, the right-hand rule, Bio-Savart law and Fleming's left-hand (or motor) rule.

* define the magnetic field at a point in terms of the force on steady current element and also on a moving charged particle.

* use the formula for the force on a steady current element or on a charged particle due to a magnetic field to calculate the force on certain simple current-carrying circuits, and solve simple problems.

* use Bio-Savart law to describe and compute the magnetic field generated by a simple current-carrying conductor.

* compute the torque exerted by a steady magnetic field upon closed current loops.

* define the ampere on the basis of the force between two long, straight, parallel current –carrying conductors.

3.1 Fields Due to Magnets

The magnetic properties of a magnet appear to originate at certain regions in the magnet which are referred to as the poles. In a bar magnet the poles are the ends.

Here are some experimental findings about magnets

(i) Like poles are of two kinds

(ii) Like poles repel each other and unlike poles attract

(iii) Poles always seem to occur in equal and opposite pairs, and

(iv) When no other magnet is near, a freely suspended magnet sets so that the line joining its poles (i.e its magnet axis) is approximately parallel to the earth's north – south axis.

The fourth finding suggests that the earth itself behaves like a large permanent magnet and it makes it appropriate to call the pole of a magnet which points (more or less0 towards the earth's geographical North Pole, the north pole of the magnet and the other the south pole.

⧈ **Can you use your knowledge of the electric field to define a magnetic field?**

The space surrounding a magnet where a magnetic force is experienced is called a magnetic field.

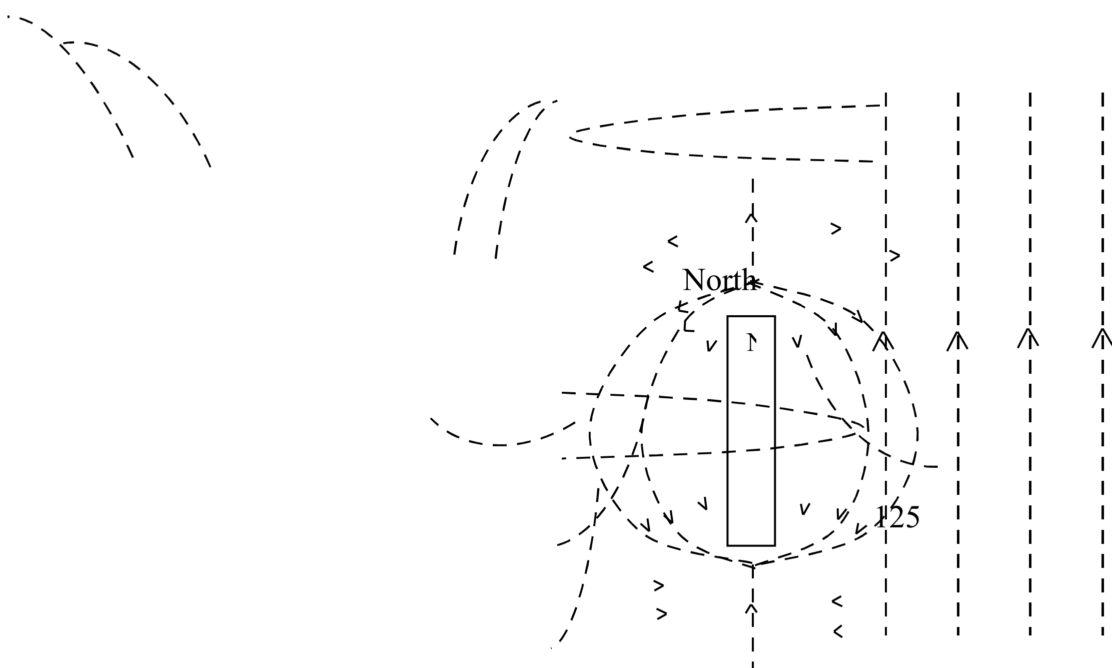
The direction of a magnetic field at a point is taken as the direction of the force that acts on a north magnetic pole there.

A magnetic field can be represented by magnetic field lines drawn so that:

(i) The line (or the tangent to it if is curved) gives the direction of the field at that point, and

(ii) The number of lines per unit cross-section area is an indication of the “strength” of the field.

Arrow on the lines show the direction of the field and since a north pole is repelled by the north pole of a magnet and attracted by the south, the arrows point away from the north poles and toward south poles.



Bar magnet Earth's local field

Fig. 3.1

Figure 3.1 shows some typical field patterns. The field round a bar magnet varies in strength and direction from point, that is it is not uniform. Locally, the earth's magnetic field is uniform; the lines are parallel, equally spaced and point north.

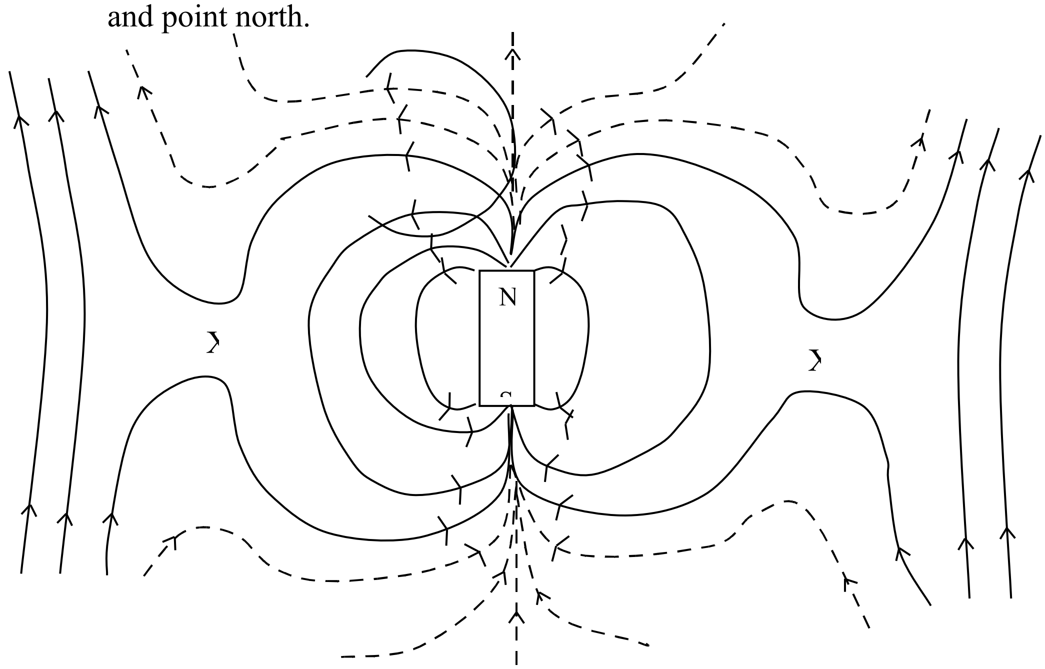


Figure 3.2

☐ **From your knowledge of vectors, what happens when two magnetic fields are equal and opposite?**

A neutral point is a place where two magnetic fields are equal and opposite and the resultant force is zero. The two points marked X in fig 3.2 in the combined field due to the earth and a bar magnet with its N pole pointing North.

Where will the neutral points be when the N pole of the magnet points south? Illustrate with a sketch similar to fig. 3.2

3.2 Field Due to Currents

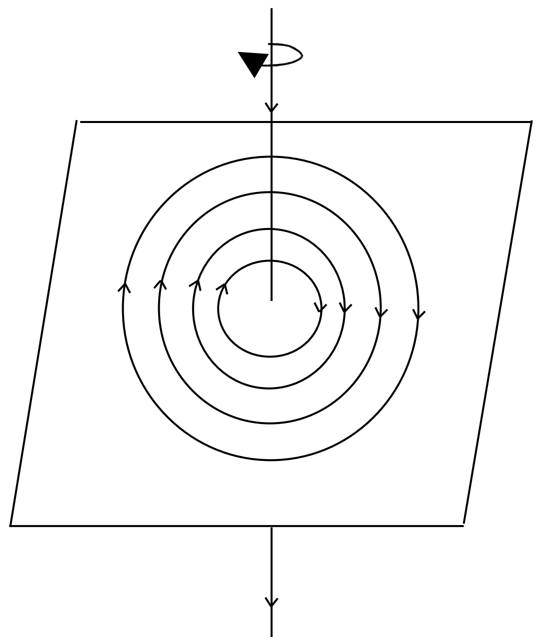
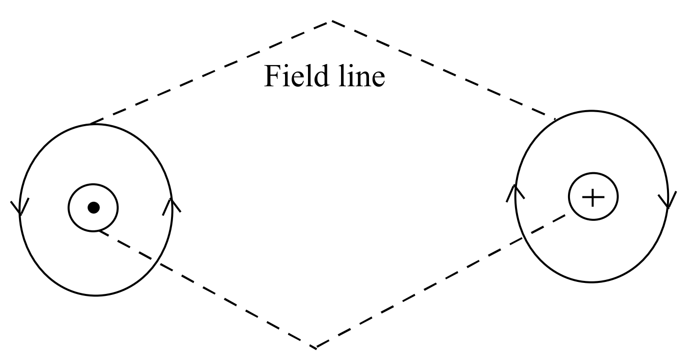


Fig. 3.3

A conductor carrying an electric current is surrounded by a magnetic filed. The lines due to a straight wire are circles, concentric with the wire as shown in fig. 3.3. The right-hand screw rule is a useful aid for predicting the direction of the field, knowing the direction of the current. It states that:

If a right-handed screw moves forward in the direction of the current (conventional), then the direction of rotation of the screw gives the direction of the lines.



wire

(a) (b)

Fig. 3.4

Figure 3.4 a and b illustrate the rule. In (a) the current is flowing out of the paper and the dot in the centre of the wire is the point of an approaching arrow; in (b) the current flowing into the paper and the cross is the tail of a receding arrow.

Sketch the field pattern due to a current in a plane circular.

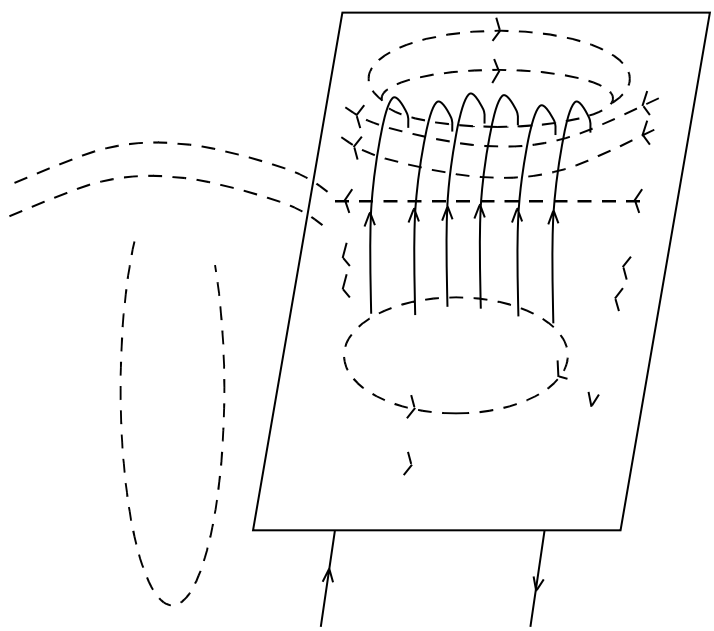


Fig. 3.5

Figure 3.5 shows the field pattern due to a current in a long cylindrical coil, called the solenoid. A solenoid produces field similar to that of a bar magnet. In the figure, the left-hand end behaves like the north pole of a bar magnet and the right-hand end like South Pole.

3.3 Force on a Current in a Magnetic Field.

When a current-carrying conductor lies in a magnetic field, magnetic force are exerted on the moving charges within the conductor. These forces are transmitted to the material of the conductor, and the conductor as a whole experienced a force distributed along its length. The electric motor and the moving coil galvanometer both depend on their operation on the magnetic force on conductor – carrying currents.

The force on a current-carrying conductor is:

(i) always perpendicular to the plane containing the conductor and the direction of the field in which it is placed and

(ii) greatest when the conductor is at right angles to the field.

.0

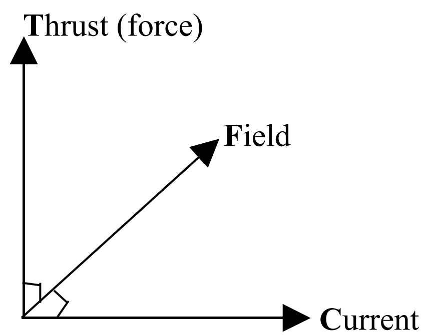


Fig. 3.6

Fleming's left-hand (or motor) rule – The facts about the relative directions of current, field and force are summarized by fleming's left-hand rule which states that:

If the thumb and first two fingers of the left-hand are held each at right angles to the other, with the first Finger pointing in the direction of the Field and the seCond finger in the direction of the Current, then the Thumb predicts the direction of the Thrust or force (see fig. 3.6).

Factors Affecting the Force

The force F on a wire lying at right angles to a magnetic field is directly proportion to the current I in the wire and to the length L of the wire in the field. It also depends on the magnetic field.

3.3.1 Magentic Flax Density

We shall use the fact that the force depends on the magnetic field to define the strenght of the field.

Can your recall the definitions of electric and the gravitational field strengths?

Electric field strength E is defined as the force per unit charge and the gravitaional field strenght, g (acceleration due to gravity) is the force per unit mass.

An analogous quantity for magnetic fields is the flux density or magnetic induction B (also called the B-field), defined as the force acting per unit current length, i.e. the force acing per unit on a conductor which carried unit current and is at right angles to the direction of the magnetic field.

In symbols, B is defined by the equation

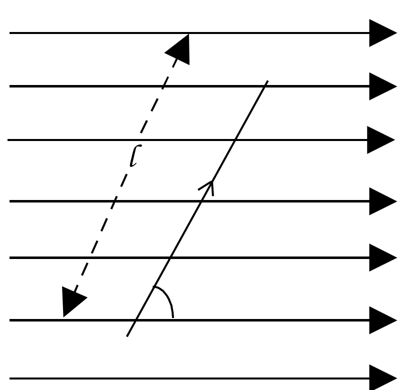
$$B = \frac{F}{I\ell} \quad \text{..................................3.1}$$

Thus if F = 1N when I = 1A and ι= 1m then B = $1NA^{-1}m^{-1}$. The unit 1 newton per ampere metre is given the special name of 1 tesla (T).

B is a vector whose direction at any point is that of the field line at that point. Its magnitude may be represented pictorially by the number of field lines passing through unit area; the greater this is, the greater the value of B.

Rearranging the equation 3.1 which defines B, the force F on conductor of length ℓ, carrying a current I and lying at right angle to a magnetic field of flux density B_1 is given by

$$F = BI\ell \quad \text{....................................3.2}$$



I B

θ

Fig 3.7

If, as shown in fig. 3.7, the field B is not perpendicular to the wire but makes an angel θ with it, the component of B (i.e B cos θ) parallel to the wire exerts no force; the componet perpendicular to the wire is given by B_i = Bsinθ, so, in general,

$$F = I \ell B_1 = I \iota B \sin\theta \ldots\ldots\ldots\ldots\ldots 3.3$$

Show that the unit B can also be written as $Vsm^{-2.}$ What is the force exerted on a straight wire of length 3.5cm, carrying a current of 5A, and situated at right angles to a magnetic field of flux density 0.2T ?

Solution: Form eq. 3.2

$$B = \frac{F}{I\ell}$$

Substituting F in newtons, I in amperes, and ℓ in metres gives us the units of B.

$$\frac{F}{I\ell\ell} = \frac{N}{Am} = \frac{Nm}{Am^2} = \frac{J}{Am^2} = \frac{AVS}{Am^2} = \frac{Vs}{m^2}$$

A ‘ Vs’ is called a “weber” (pronounced vayber’) and is abbreviated to Wb. So that unit of magnetic flux density B is Wb $m^{-2.}$ It is the same unit that has been named tesla and given the symbol T as you saw earlier.

Calculation: Force = $B I\ell$ (Eq. 3.2)

$= 0.2 \times 3.5 \times 10^{-2} \times 5$

$= 0.035$ N

Force on an Electron Moving in a Magnetic field

An electric current in a wire is convertionally regarded as a flow of positve charge, although it consists in fact of a flow of negative elecctrons in the opposite direction.

Suppose an electron of charge e is moving with velocity v at right angles to a magneitc field of flux density B.

The electron moves a distance ℓ in a time t, where t = l/v, and consitutes a current I.

Current = flow of charge per second

$$I = \frac{e}{t} = \frac{e}{\iota/v} = \frac{ev}{\iota}$$

$$I\ell = ev$$

But force on a current = BIι

Force on a moving electron = Bev …………………3.4

Torque on a rectangular coil

Figure 3.8 (a) and (b) represnets a vertical rectangular coil length and breadth a and b respectively, carrying a current I with its plane at an angle α to a horizontal magnetic field of magentic flux density B. Applying Flemings left-hand rule to figure 3-8 (a), it will be seen that the left-hand vertical side is urged out of the paper,

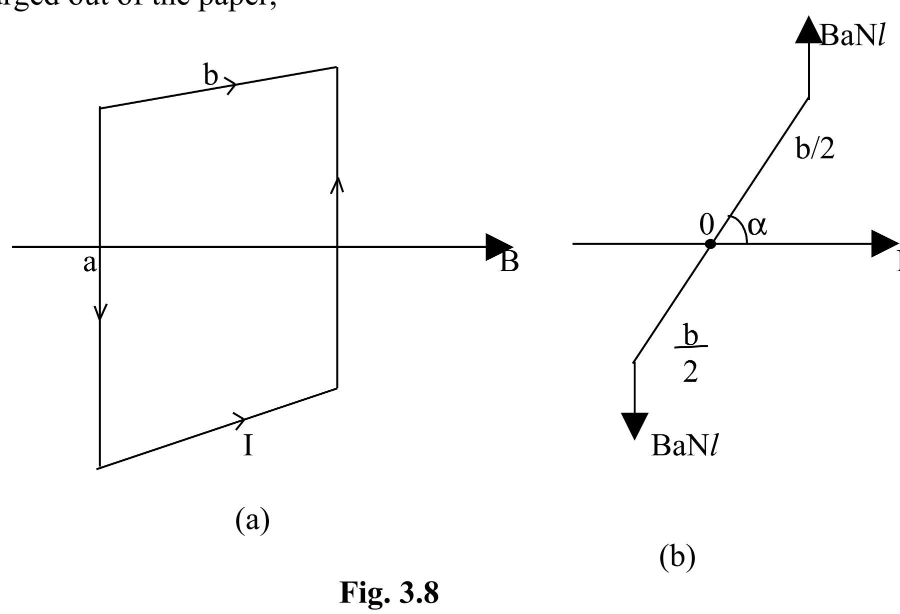


Fig. 3.8

The righ-hand vertical side into the paper, and the top and bottom are urged up and down respectively. If the coil is free to turn about a vertical axis, only the forces on its vertical sides will have a turning effect. The forces on these sides are each BaNI, where N is the number of turns of the coil (see fig. 3.8b)

Taking the moment of the forces about 0 we have

$$P = 2\,BaNI\,\frac{b}{2}\cos\alpha$$
$$= BANI\cos\alpha\ \text{Nm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

where A = area of coil = ab. Thus the torque on the coil is BANI cos α.

Use Eq. 3.5 to derive expressions for the maximum and minimium values of the torque on the coil.

The torque on the coil attains its maximum value when the plane of the coil is parallel to B and α = 0. The maximum value is BANI. Its minimum value, when the plane of the coil is perpendicular to B and $\alpha = 90^0$, is zero.

Note: The torque on a coil is always BANI cos α whatever the shape of the coil.

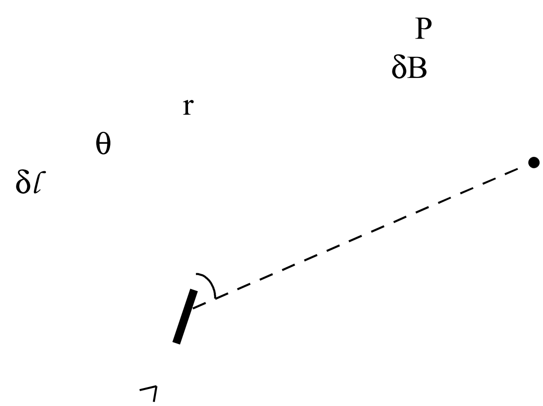
Find the torque on a galvanometer coil, 2cm square and containing 100 turns, when a current of 1mA passes through it. The radial field of the permanent magnet has a flux density of 0.2T.

From Eq. 3.5, the torrque is given by

$$\Gamma = BANI$$
$$= 0.2 \times 2^2 \times 10^{-4} \times 100 \times 10^{-3}$$
$$= 8 \times 10^{-6}\ \text{Nm}$$

3.3.2 The Biot-Savart Law

In section 3.3.1, we saw how the flux density B could be calculated for some simple shapes, we shall see in this section how it can be calculated using the Biot-Savart law.



I

Fig. 3.9

The calculation involves considering a conductor as consisting of a number of very short lenghts each of which contributes to the total field at any point.

Biot and Savart stated that for a very short length $\delta\ell$ of conductor, carrying a steady current I, the magnitude of the flux density δB at a point P distant r from $\delta\ell$ is

$$\delta B \quad \frac{\alpha \delta\ell \sin\theta}{r^2}$$

Where θ is the angle between $\delta\ell$ and the line joining it to the point P (see fig. 3.9). The product $I\delta\iota$ is called a " current element".

It is clear that before the Biot-Savart law can be used in calculation, it has to be expressed as an equation and a constant of proportionality introduced. The constant of proportionality is a property of the medium. It is called the **permeability** of the medium and is denoted by μ.

The permeability of a vacuum is denoted by μ_o, and its value is defined to be $4\pi \times 10^{-7}$ and its unit is the henry per metre (Hm^{-1}). Air and most other materials except Ferromagnetics have nearly the same permeability as a vacuum.

□ **Note that whilst the value of δo, the permittivity of free space is found by experiment, that for μ_o is by definiiton.**

Rationalization is achieved by introducing 4π in the denominator. The Biot-Savart equation for a current element becomes

$$\delta B = \frac{\mu_o I \delta\ell \sin\theta}{4\pi r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

Calculation of Flux Density

In most cases, the calcualtion of flux density requires the use of calculus. We shall consider some simple geometries.

(i) Circular Coil

Suppose the coil is in air, has radius r, carries a steady current I and is onsidered to consist of current elements of length $\delta\ell$. Each element is at disance r from the centre 0 of the coil and is at right angles to the line joining it to 0. i.e. $\theta = 90^0$ (see fig 3.10). At 0 the total flux density B is he sum of the flux densities δB due to all the elements.

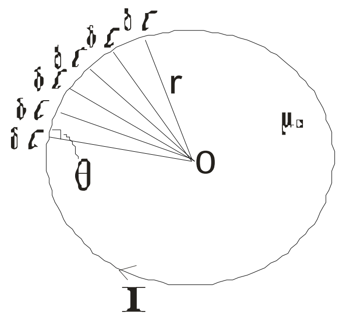


Fig 3.10

That is

$$B = \sum \frac{\mu_o I\delta\ell \sin\theta}{4\pi r^2} = \frac{\mu_o I \sin\theta}{4\pi r^2} \sum\delta\ell$$

But $\sum\delta\ell$ = total length of the coil = $2\pi r$ and $\sin\theta = \sin 90^0 = 1$

$$\text{Hence } B = \frac{\mu_o I\, 2\pi r}{4\pi r^2} = \frac{\mu_o I}{2r}$$

If the coil has N turns each of radius r

$$B = \frac{\mu_o N I}{2r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

(ii) B at a point on the axis of a circular coil

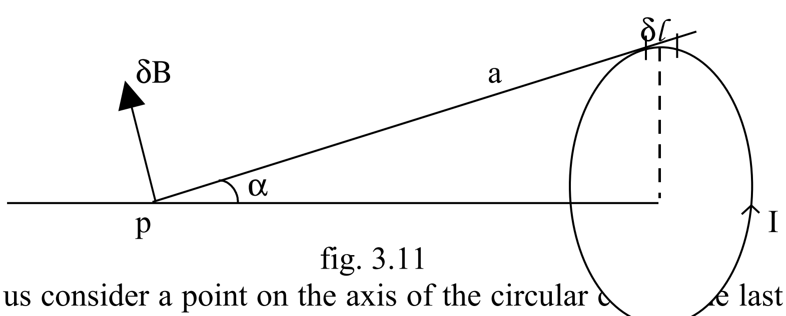


fig. 3.11

Let us consider a point on the axis of the circular coil of the last section. The magnetic flux density at p due to the current element δι is

$$\delta B = \frac{I\delta\ell \sin\theta}{4\pi a^2}$$

Where $\theta = 90^0$ and $\sin\theta = 1$. The direction of δB is at right angles to the line joining P to the current element and in the plane of the paper (the current element being perpendicular to the paper). δB can be resolved into a component along the axis, $(I\delta\ell/4\pi a^2)\sin\alpha$ and one at right angles. By considering pairs of current elements at opposite ends of a diameter, it is clear that the magnetic flux density at right angles to the axis vanishes.

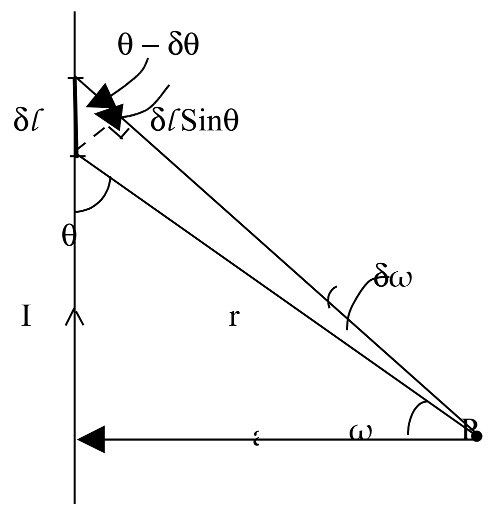
If the coil has N turns, $\sum I\delta\ell = I\,2\pi Nr$

Magnetic flux density along the axis at p due to the whole coil is given by

$$B = \frac{\mu I 2\pi Nr}{4\pi a2}$$

$$= \frac{\mu NI}{2r}\,\text{Sin}^3\alpha \quad \left(\frac{r}{a} = \sin\alpha\right) \quad \ldots\ldots..3.8$$

(iii) Very Long Straight Wire



We wish to find the flux density at P, perpendicular distance a in air from an infinitely long straight wire carrying current I.

The small element $\delta\iota$ in figure 3.12 contributes flux density δB at P. Applying the Biot-savart law

$$\delta B = \frac{\mu_o I\,\delta\iota\sin\theta}{4\pi r^2} = \frac{\mu_o I}{4\pi}\cdot\frac{\delta w}{r} \quad (\text{since } \delta\iota\sin\theta = r.\delta w)$$

The total flux density B at P is obtained by integrating this expression over the whole length of the wire between the limits $-\pi/2$ and $+\pi/2$, where these are the angles sutended at P by the ends of the wire.

We have cos w = a/r, hence

$$B = \int_{-\pi/2}^{+\pi/2} dB = \frac{\mu_o I}{4\pi a} \int_{-\pi/2}^{+\pi/2} \cos w\,dw \quad \ldots\ldots\ldots\ldots\ldots\ldots..3.9$$

$$= \frac{\mu_o I}{4\pi a} (\sin w)_{-\pi/2}^{+\pi/2}$$

$$B = \frac{\mu_o I}{2\pi a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.10$$

- **Note that equation 3.9 shows that the field is non-uniform and there is cylindrical symmetry (fig. 3.3).**

- **Example**

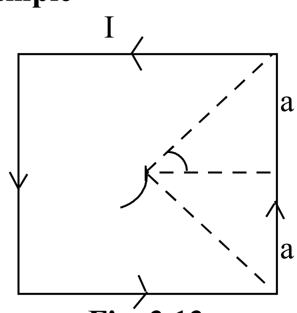


Fig. 3.13

Find the magnetic flux density, B, at the centre of a square coil, of side 2a, carrying a current I as shown in fig. 3.13

Solution:

The value of B due to one side of the coil

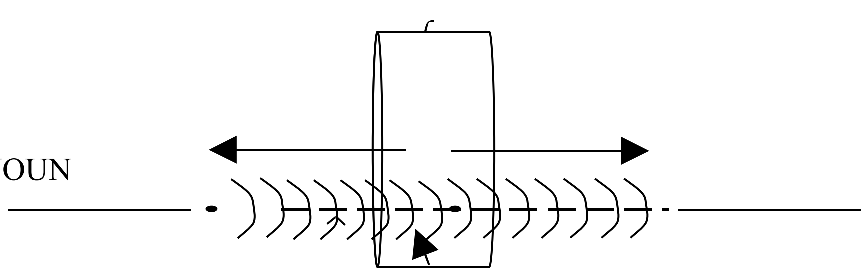
$$= \int_{-\pi/4}^{\pi/4} \frac{\mu I \cos w\,dw}{4\pi a} \quad \text{(see eq. 3.9)}$$

$$= \frac{\mu I}{4\pi a} [\sin \alpha]_{-\pi/4}^{\pi/4}$$

$$= \frac{\sqrt{2}\mu I}{4\pi a}$$

$\therefore$ Magnetic flax density, B, due to the foursides $= \dfrac{\sqrt{2}\,\mu I}{\pi a}$

(iv) Very Long Solenoid



P O

I

Solenoid of N turns

Fig 3.14

If the solenoid has N turns, length ℓ and carried a current I, the flux density B at a point 0 on the axis near the centre of the solenoid, figure 3.14, is found to be given by

$$B = \frac{\mu_o NI}{\ell}$$

$$= U_o n I \ldots\ldots\ldots\ldots 3.11$$

Where $n = N/\ell$ = number of turns per unit length. Thus B is equal to μ_o multiplied by the ampere-turns per metre.

At P, a point at the end of a long solenoid

$$B = \frac{\mu_o n I}{2} \ldots\ldots\ldots\ldots 3.12$$

That is, B at a point at the end of the solenoid's axis is half the value at the centre.

3.3.3 The Ampere

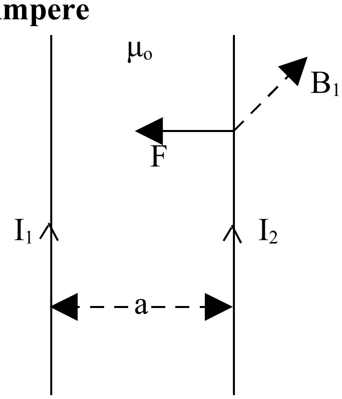


Fig 3.13

The ampere is the basic electrical unit of the SI systm. Therefore, it has to be defined, like any other unit, so that it is accurately reproducible. The definition is based on the force between two long, straight, parallel current-carrying conductors.

To derive an expression for the force, let us consider two long, straight, parallel conductors, distance a part in air, carrying current I_1 and I_2

respectively (see fig. 3.15). the magnetic field at the right-hand conductor due to the current I_1 in the left-hand one is directed into the paper and its flux density B_1 is given by

$$B_1 = \frac{\mu_o I_1}{2 \pi a}$$

The forces F acting on length ℓ of the right-hand conductor (carrying current I_2) is therefore

$$F = B_1 I_2 \ell = \frac{\mu_o I_1 I_2 \ell}{2\pi a} \quad \ldots\ldots\ldots\ldots\ldots.. 3.13$$

The left-hand conductor experiences an equal and opposite force due to being in the field of the right-hand conductor.

The definition of the ampere is based on Equation 3.13 as follows:

The ampere is the constant current which, flowing in two infintely long, straight, parallel conductors of negligible circular cross-section, placed in a vacuum 1 metre apart, produces between them a force of 2×10^{-7} newton per metre of their length.

Once the ampere has been defined, the value of μ_o follows. Thus we have from the definition,

$$I_1 = I_2 = IA$$
$$\ell = a = 1m$$
$$F = 2 \times 10^{-7} N$$

Substituting in $F = \mu_o I_1 I_2 \ell (2 \pi a)$, we have

$$2 \times 10^{-7} = \mu_o \times 1 \times 1 / (2 \pi \times 1)$$
$$\mu_o = 4\pi \times 10^{-7} 11m^{-1}$$

This is the value given earlier (see section 3.3.2)

4.0 Conclusion

This unit has introduced you to an important area of the course, the magnetic field. You should note that electric currents produce magnetic fields and a changing magnetic field produces an electric current.We have defined the magnetic field at a point in terms of the force on a steady current element. You are now able to use the derived formula for the force on a steady current element to calculate the force on some simple current-carrying circuits. The Biot-Savart law enables you to handle other shapes requiring the use of differential calculu. The ampere which is the basic electric unit of the SI system has been defined on the basis of the force between two long, straight, parallel current-carrying conductors. The value of the permeability of free space was deduced from the definition of ampere.

5.0 Summary

* The magnetic flux density, B, measure the strength of a magnetic field and is defined by the equation:

  $$F = B I\ell$$

  Its units are $W.b.m^{-2}$, or T.

* The force on an electron moving at right angles to a magnetic field is Bev

* The magnetic field due to a current element is

  $$\delta B = \frac{\mu I\, \delta\iota \sin\theta}{4\pi r^2}$$

  This is called the Biot-Sarvat law. The integrated, this law gives:

  B at the centre of a long solenoid $= \mu nI$

  B at the centre of a circular coil $= \frac{\mu NI}{2a}$

  B on the axis of a circular coil $= \frac{\mu NI}{2a} \sin^3 \alpha$

  B at a distnce, a, from a long straight wire $= \frac{\mu I}{2\pi a}$

* The force between two long straight parallel wires is then

  $$F = \frac{\mu I_1 I_2}{2\pi a}$$

* From the definition of the ampere in terms of a force between two long straight wires, the permeability of the vacuum, μ_o, is

  $$\mu_o = 4\pi \times 10^{-7}\ Hm^{-1}$$

6.0 Tutor Marked Assignments (TMA)

1. The magnetic flux density in the middle of a long solenoid carrying a current of 2.0A is 5.0 MT. Find the number of terms per metre of the solenoid ($\mu_o = 4\pi \times 10^{-7}\ H.m^{-1}$)

2. What is the magnetic flux density midway between two long parallel wires separated by 10cm, each carrying a current of 5A

(a) In the same direction

(b) In opposite directions

What is the force per cm length exerted by each wire on the other? ($\mu_o = 4\pi \times 10^{-7}$ Hm^{-1})

3. A current of 5A is caried by a straight wire in a uniform flux density of 2.0×10^{-3} T. Calculate the force per unit length on the wire if it is

(i) Perpendicular to the field

(ii) Inclined at 30^0 to it.

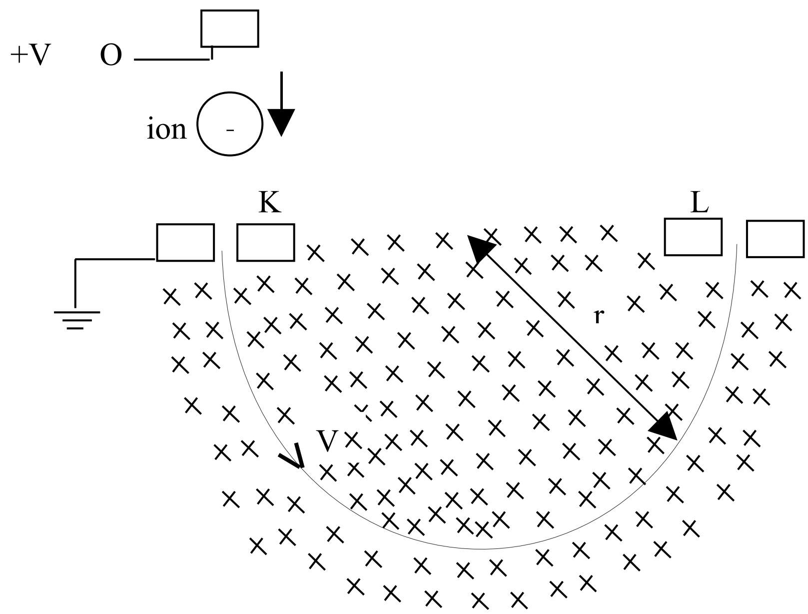


Fig. 3.16

4. What do you understand by a magnetic field?

Figure 3.16 shows an appratus that is used to compare the masses of ions of different isotopes of the same element. In one experiment magnesium ions of mass M and e from a hot source J were accelerate by a p.d. V and passed through a slit K with speed u.

(i) Show that

$$U = \sqrt{\underline{2eV}}$$

M

Once through the slit K, the ions move with constant speed u at right angles to a uniform magentic field of intesity B and along a cicular path of radius r.

(ii) Show that $r = \frac{mu}{Be}$

and that $M = \frac{eB^2 r^2}{2} \left(\frac{1}{V} \right)$

7.0 References and Other Resources

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A second Course of Electricity. J. Jenkins and W.H. Jarvis. University Press, Cambridge 1973.

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UNIT 9

MOTION OF CHARGED PARTICLES IN ELECTRIC AND MAGNETIC FIELDS

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1.0 Introduction

By now, you are familiar with three kinds of force – gravitational, electrical and magnetic forces. As you know, the forces are best described in terms of fields. All forces have a property by virtue of which they act on a particular kind of particle located in the region occupied by the field. Once we know exactly how the fields affect the particles on which they act, we are in position to understand the nature of the field and, hence, the nature of the force.

You have studies in PHY101 the motion of objects in the earth's gravitational field. For example, the path of the projectile in air is a parabola. In this unit we shall study the motion of a charged particle in an electric field or a magnetic field. We shall also examine what happens when a charged particle moves through a space, in which both magnetic and electric fields exist simultaneously.

2.0 Objectives

After studying this unit you should be able to :

* carrying out simple calcualtions involving the motion of charged particles in a uniform electric field.
* describe the main features of the motion of charge in a magnetic field and define the term cyclotron frequency
* explain the helical trajectory of a charged particle moving in a uniform magnetic field.
* Explain the working principle of the Cathode Ray Oscilloscope

* understand the applications of the combined electric and magnetic fields acting perpendicular to each other.

3.1 Motion in an Electric Field

figure 3.1 shows a uniform electric field $\underline{E}$, which is set up between two = charged plates. Let us consider a positive charge q moving in the direction of the field with velocity $\underline{V}$. You will remenber that the force acting on the particle is given by

$$\underline{F} = q\,\underline{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

Showing that the force is independent of both the velocity and position of the particle. This constant force gives the particle a constant acceleration. From Newton's second law ($\underline{F} = m\underline{\alpha}$), this constant acceleration is given by

$$\underline{a} = F/m = q\underline{E}/m \ldots\ldots\ldots 3.2$$

where m is the mass of the particle.

It follows from Eq. 3.2 that the acceleration is in the same direction as the electric field. The equation also shows that it is the ratio of charge to mass that determines a particle acceleration in a given electric field. You can now understand why electrons which are much less in mass (about 2000 times) than protons but carrying the same charge, are readily accelerated in electric fields. This is why many practical devices, like television tubes, electron microscope, etc makes use of the high accelerations which are possible with electrons.

⊡ **Example 1**

Suppose the electric field shown in fig. 3.1 is of strength 2.0 NC^{-1}. An electron is released from rest in this field. How far and in what direction does it move in 1.0µs?

⊡ **Solution**

You know how to solve problems involving constant acceleration from your PHY 101 where you used equations.

$$V = u + at$$
$$S = ut + \frac{1}{2}at^2$$
$$V^2 = u^2 + 2as$$

In this case, let us assume that distance is measured along the vertical direction, then

$$Y = 0 + \frac{1}{2}at^2$$

With the acceleration given by Eq. 3.2, have

$$Y = \frac{1}{2}\frac{qE}{M}$$

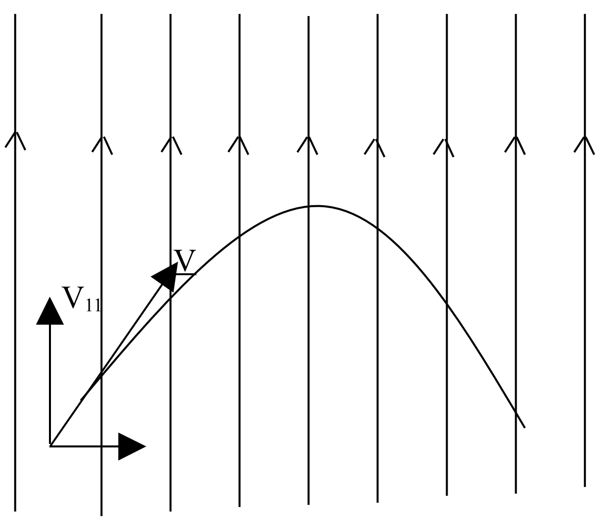
$$= \frac{(-1.6 \times 10^{-19}\,C)(2.0\,NC^{-1})(1.0 \times 10^{-6}S)^2}{2 \times (9.1 \times 10^{-31}\,kg)}$$

$$= -0.18m$$

The minus sign indicates that motion is downward, opposite to the field direction. This is expected because an electron carries a negative charge.

We may also have cases of a charged particle moving in an electric field with an initial velocity in any direction that is not along the electric field. For example, let us consider the case in which the charged particle is moving in the electric field with a velocity in the direction shown in figure 3.2.

You already know that when a particle (or a projectile) moves with constant acceleration under the earth's gravitational field, it follows a parabolic path.



V_6

Fig 3.2

A similar sitaution arises when the direction of the initial velociy of the particle is not in the direction of the field. In such cases, the velocity $\underline{v}$ is regarded as the sum of two other velocities; one parallel to the field denoted by V_{11}, and the other perpendicular to it denoted by V_1. Therefore, the total velocity $\underline{V}$ can be written as follows:

$$\underline{V} = \underline{V}_{11} + \underline{V}_1$$

The horizontal position of the charged particle at any time t is given by .

$$x = V_1 t \text{ ....}3.3$$

The vertical position of the charged particle at any time t is

$$y = V_{11}t + \frac{1}{2}\frac{qE}{m}t^2 \text{ ....}3.4$$

Substituting the value of t from Eq. 3.3 into 3.4, we obtain

$$y = \frac{V_{11}}{v_1}x + \frac{1}{2}\frac{qE\,x^2}{mv_1^2} \text{ ....}3.5$$

This is the equation of the particle in the electric field. Since $\underline{V}$, $\underline{q}$, E and m are constant, Eq. 3.5 is of the form $y = ax + bx^2$, a and b are constants.

This is the equation of a parabola.

3.2 Motion in a Magnetic Field

The force exerted by a magnetic field $\underline{B}$ on a moving charged particle is

$$\underline{F} = q\ \underline{v} \wedge \underline{B} \text{ ....}3.6$$

The magnitude of this magnetic force is

$$F = qvB \sin\theta$$

where θ is the angle between V and B . The direction of the magnetic force is perpendicular to both V and B.

Form Eq. 3.6, it follows that magnetic force always act perpendicular to the direction of motion. This means that the magnetic field can do no work on a charged particle. Because no work is done, the kinetic energy of the particle cannot change, both the speed and kinetic energy remain constant. Therefore, the magnetic force changes only in the direction of particle's motion but not its speed.

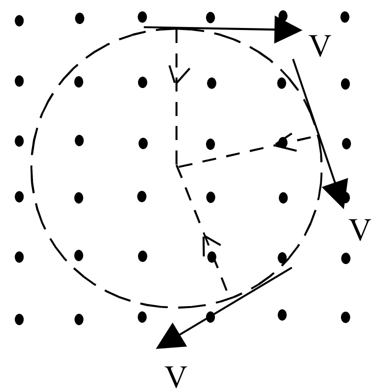


Fig 3.3

To understand how the direction of the particle's motion is changed, let us consider the case of a particle of charge q moving at right angles to a uniform magnetic field as shown in fig. 3.3. Suppose at some instant at the point A, the velocity V points to the right, so with the field being out of the page, the cross product V ∧B points downward according to the right hand rule. If the particle is positve it will experience a downward force. This force changes the direction of the particle's motion, but not its speed . A little while later, the particle is moving downward and to the right. Now the force point downwards and to the left. Since the speed of the particle is still v and the velocity is still at right angles to the field, so the magnitude of the force remains the same. Thus the particle describe a path in which the force always has the same magnitude and is always at right angles to its motion. Each time, under the influence of the force the particle is deflected from the rectilinear path resulting in the simplets possible curved path – a circle. Now, in any circular path, the particle expereinces a centripetal force F directed towards the centre of the cirlce. It is given by.

$$F_c = \frac{mv^2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.7$$

Where r is the radius of the circular orbit and v is the tangential speed of the particle. Therefore, in the presnet case, the centripetal force being the magnetic force we can write.

$$F_c = qvB = \frac{mv^2}{r}$$

So that $r = mv/qB$ ……………………………….3.8

The larger the particle's momentum mv, the larger the radius of the orbit. On the other hand, if the field or charge is made larger, the orbit becomes smaller. Therefore, the observation of a charged particle's trajectory in a magnetic field is the standard technique for measuring the movement of the particle. A charged particle can traverse a circular path either in clockwise direction or anticlockwise direction.

Since the circumference of the orbit is $2\pi r$, the time taken by the particle to complete one full orbit is

$$T = \frac{2\pi r}{V} \quad \text{……………………………………….3.9}$$

Using Eq. 3.8 for the radius r, we have

$$T = \frac{2\pi r}{V} \quad \frac{2\pi}{v} \quad \frac{mv}{qB} = \frac{2\pi m}{qB} \quad \text{………………..3.10}$$

The frequency of rotaioin of a moving charge is given by

$$f = \frac{1}{T} = \frac{qB}{2\pi m} \quad \text{………………………………….3.11}$$

The frequency, f is called the cyclotron frequency. It is so called because it is the frequency at which the charged particles circulate in a cyclotron particle accelerator.

We shall now consider what path, a charged particle will have, if initially its velocity is neither perpendicular nor parallel to the field. In this case , the velocity can be resolved into two vectors: $v_{\perp}$ perpendicular to the field and v_{11} along the field. Then Eq. 3.6 becomes:

$$F = q\,(v_{\perp} + v_{11}) \wedge \underline{B} = q\,v_{\perp} \wedge \underline{B} + q\,v_{11} \wedge \underline{B}$$

Since the second term on the right hand side of this equation is the cross product of two parallel vectors, it is zero. Therefore,

$$F = m\frac{dv}{dt} = q\, v_1 \wedge \underline{B}$$

Or $$m\frac{dv_{\perp}}{dt} + m\frac{dv_{11}}{dt} = qv_{\perp} \wedge \underline{B}$$

The force F is clearly perpendicular to $\underline{B}$ i.e there is no acceleration in the diretion parallel to $\underline{B}$. This means.

$$m \frac{dv_{11}}{dt} = 0$$

$$m \frac{dv_{\perp}}{Dt} = qv_{\perp} \wedge \underline{B} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

Equation 3.12 shows that the force is perpendicular to the field. i.e. it influences the particles motion in a plane perpendicular to the field. But we know that the particle's motion perpendicular to the magnetic field is circular. Equation 3.12 further shows that no force acts along the magnetic field. Therefore, the component of the velocity which is along the field remains unaffected by the field. Thus the particle moves with a uniform velocity V_{11} along the magnetic field even as it executes a circular motion with velocity $V_{\perp}$ perpendicular to the field. The resulting path is a helix (see fig 3.4)

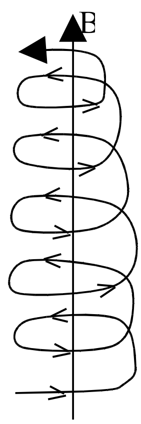


Fig 3.4

The radius of the helix is given by equation 3.8 if we replace v by $v_{\perp}$.

3.3 Cathode Ray Oscilloscope (CRO)

The force on a moving charge due to a magnetic field is used to create pictures on a television screen. The main component of a television is the cathode-ray tube, which is essentially a vacuum tube in which electric fields are used to form a beam of eletrons. This beam causes phosphor on the television screen to glow when struck by the electrons in the beam. Without magnetism, however, only the centre of the screen would be illuminated by the beam.

The CRO is based on the following two principles

(i) When fast moving electrons strike the glass screen coated with zinc sulphide, they cause fluoresence.

(2) Since the mass of electrons is very small, they are easily deflected by the electric and magnetic fields and follows their variation with practically no time lag.

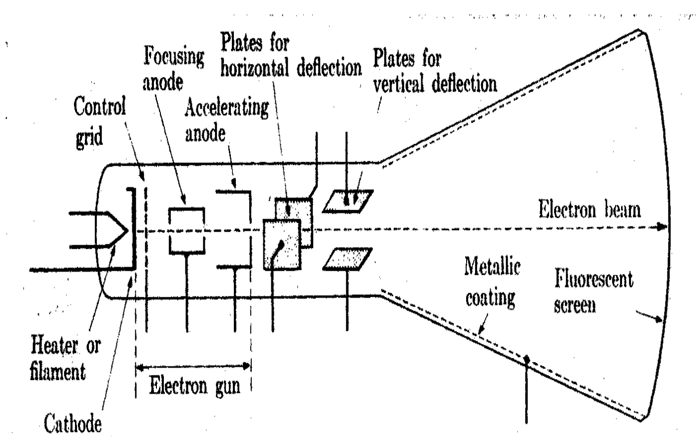
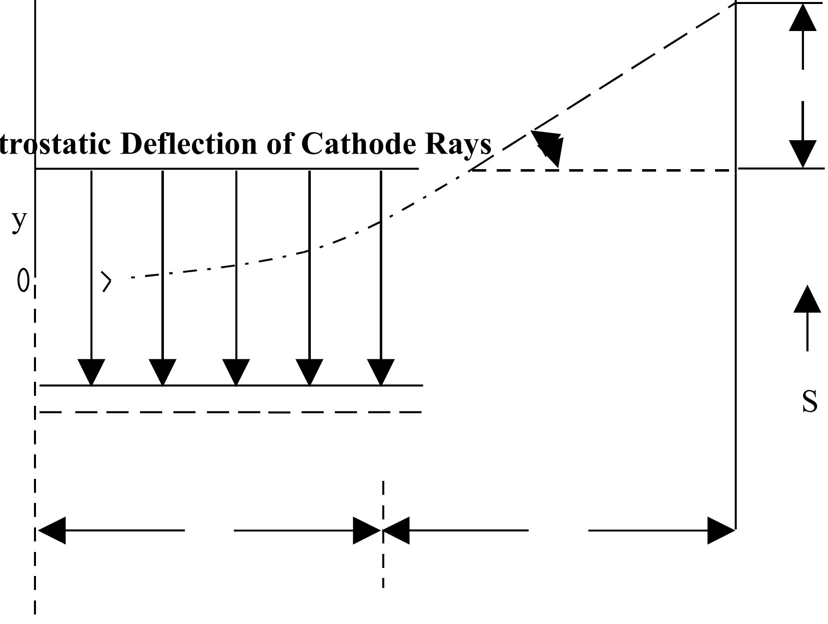


Fig 3.5

Figure 3.5 shows the basic elements of a cathode-ray tube.
Electrostatic deflection of an electron beam is used in the cathode-ray tubes of modern oscilloscopes. There are two sets of deflecting plates, so that the electron beam can be deflected right and left as well as up and down. These tubes utilize the fact that he deflection is proportional to the electric field between the plates. Television tubes, on the other hand, commonly utilize magnetic deflection to cause the beam to sweep over the face of the picture area.

Electrostatic Deflection of Cathode Rays



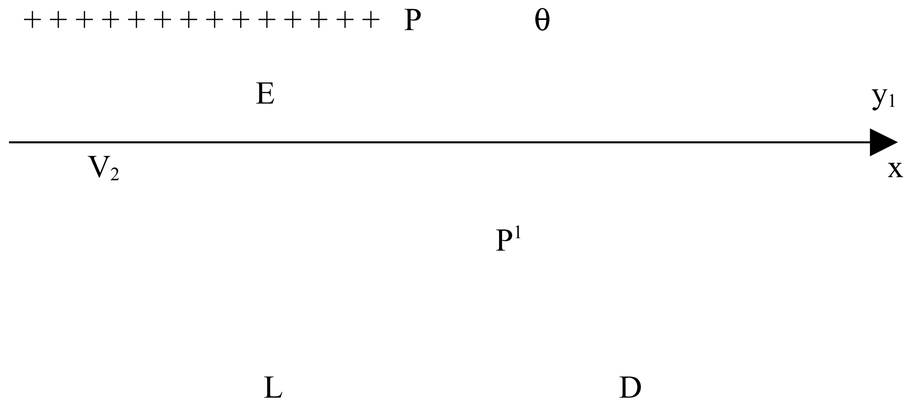


Fig. 3.5

The cathode rays enter the region between the plates at the origin 0 with a velocity vx, this velocity will continue to be the horizontal component of the rays (electrons). Thus the horizontal displacement after a time t is

$$\chi = V_x t \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

Between the plates, however, the rays experience an upward acceleration

$$a_y = qE/m \text{ (see eq. 3.2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.14$$

Neglecting the fringing effect of the electric field, we can assume that E is constant, and it is equal to the potential difference between the deflection plates divided by their separation. Hence the general displacement equation becomes

$$y = \frac{qEt^2}{2m} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.15$$

Elimination of t between Eqs. 3.13 and 3.15 yields the equation for the parabolic trajectory,

$$\frac{y}{2mv^2_x} = qE\,x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.16$$

The quantity y_1, defined in fig. 3.5, is the value of y when x = L. Beyond the plates, the trajectory is a straight line because the charge is then moving in a field-free space. The value of y_2 is Dtan θ, where D and θ are defined as in fig. 3.5. The slope of the line is

$$\left(\tan\theta\right) = \underline{dy} = \left(\quad\right) \underline{qEx} = \underline{qEL} \quad \ldots\ldots 3.17$$

dx mv$_x^2$ mV^2_x
x=L x=L

The total deflection of the beam, Y_E, is Y_1, + Y_2, so that

$$Y_E = Y_1 + Y_2 = \frac{qEL^2}{2mv^2_x} + \frac{qELD}{mv_x^2} = \frac{qEL}{mv^2_x} = \left(\frac{L}{2} + D\right) \ldots\ldots. \quad 3.18$$

3.4 Lorentz Force and its Applications

Suppose a particle having a charge q is moving with a velocity $\underline{v}$ through a space , in which both magnetic and electric fields exist simultaneously, then the force exerted on such a particle is given by

$$\underline{F} = q\underline{E} + q\,\underline{V} \wedge \underline{B} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.19$$

Equation 3.19 is the vector sum of the electric force qE and the magnetic force q $\underline{v} \wedge \underline{B}$. It is called the Lorentz force equation and $\underline{F}$ is the lorentz force.

We shall now consider an important application of the combined electric and magnetic fields, acting perpendicularly to each other.

3.4.1 The Cyclotron

The cyclotrons is the most familiar of all machines for accelerating charged particles and ions to a high velocity. A sketch of the cyclotron is shown in fig. 3.6

The machine consists of two circular boxes, D_1 and D_2, called 'dees' because of their shape, enclosed in a chamber C containing gas at low pressure. The chamber is arranged between the poles of an electromagnet so that a nearly uniform magnetic field acts at right angles to the plane of the dees. A hot filament Femits electrons so which ionize the gas present, producing protons from hydrogen, deuterons from deuterium, etc. An alternating electric field is created in the gap between the dees by using them as electrodes to apply a high frequency alternating p.d. Inside the dees, there is no electric field, only the magnetic field.

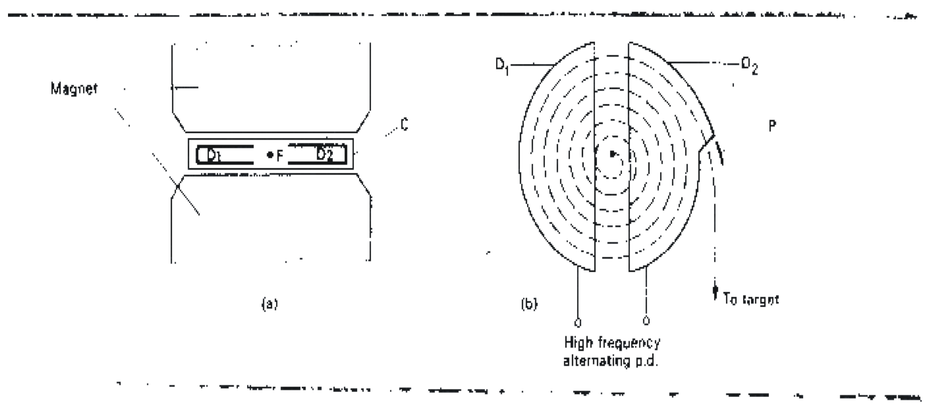


Fig. 3.6

Suppose at a certain instant D1 is positive and D_2 is negative. A positively charged particle starting from F will be accelerated toward D_2 and when inside this dee it describes a semi-circular path at constant speed since it is under the influence of the magnetic field alone. The radius r of the path is given by eq. 3.8

$$r = \frac{mv}{qB}$$

where B is the magnetic flux density, q is the charge on the particle, v its speed inside D_2 and m its mass.

If the frequency of the alternating p.d is such that the particle reaches the gap again when D_1 is negative and D_2 positive , it accelerates across the gap and describes another semi-circle inside D_1 but of greater radius since its speed has increased. The particle thus gains kinetic energy and moves in a spiral of increasing radius, provided that the time of one complete half-oscillation of the p.d equals the time for the particle to make one half-revolution (see fig. 3.6a). We shall now show that this condition generally holds.

Let T be the time for the particle to describe a semi-circle of radius r with speed r then.

$$T = \frac{\pi r}{V}$$

Substituting for r, we have

$$T = \frac{\pi m}{qB}$$

T is therefore independent of V and r and constant if B_1 m and q do not change. Hence for paths of larger radius the increased distance to be covered is exactly compensated by the increased speed of the particle. After about 100 revolutions, a plate P_1 at a high negative potential, draws the particles out of the dew before it bombards the target under study.

4.0 Conclusion

We have seen that the force exerted on a moving charged particle is determined entirely by the electric field $\underline{E}$ and the magnetic field $\underline{B}$ at the location of the particle. We have studies the effect of the combined electric and magnetic fields on the motion of charged particles. The general equation for the force containing both electric and magnetic fields is called Lorentz equation. We discussed the cyclotron which is a very important practical application of the lorentz equation.

You have also been introduced to the basic elements and principles of the cathode ray oscilloscope (CRO), a very versatile laboratory instrument used for display, measurement and analysis of waveforms and other phenomena in electrical and electronic circuits.

5.0 Summary

* A charge q moving with a velocity v experiences a magnetic force

$$\underline{F} = q\,\underline{v} \wedge \underline{B}$$

at a point in space where the magnetic field is $\underline{B}$.

* A charged particle moving in a uniform electric field follows a parabolic path because it is subjected to a constant acceleration.

* A charged particle, velocity of which is $\underline{v}$ in a plane perpendicular to a magnetic field B, describes a circular trajectory with radius r given by

$$r = \frac{mv}{qB}$$

where m is the mass of the charged particle.

* When the direction of motion of the charged particle is neither parallel nor perpendicular to the direction of the magnetic field, it describes a helical trajectory.

* In the cathode – ray oscilloscope (CRO), the electron beam can be deflected either by an electric field or by a magnetic field. In both cases, the deflection of the electron beam is proportional to the applied electric (or magnetic) field.

* The motion of a charged particle, moving through a combination of the electric and magnetic field, is described by the Lorentz force.

$$\underline{F} = q\underline{E} + q\,\underline{v} \wedge \underline{B}.$$

6.0 Tutor Marked Assignments (TMA)

1. An electron is moving horizontally to the right at a speed or $4.0 \times 10^6 ms^{-1}$. It enters a region of length 20cm in which there is an electric field of $1.0 \times 10^3 NC^{-1}$ pointing downward as shown in fig. 3.7. Answer the following questions.

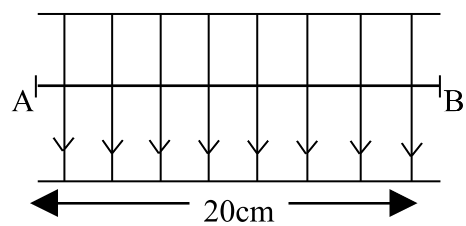


Fig. 3.7

(i) How long does it spend travelling from A to B?

(ii) By how much and in what direction is the electron deflected, when it leaves the electric field? Describe the motion of the particle in the electric field and draw a rough sketch of the path.

(iii) What is the magnitude of the vertical component of the velocity of the electron, when it leaves the electric field?

(iv) What is the speed of the electron when it leaves the electric field?

(v) Through what angle has the electron been deflected when it leaves the electric field?

(vi) Describe and draw the rough sketch of its subsequent motion.

2. The pole faces of a cyclotron magnet are 120cm in diameter; the field between the pole faces is 0.80T. The cyclotron is used to accelerate protons. Calculate the kinetic energy in eV, and the speed of a proton as it emerges from the cyclotron. Determine the frequency of the alternating voltage that must be applied to the dees of this accelerator ($1eV = 1.6 \times 10^{-19}$ J); (mass of the proton = 1.67×10^{-27}kg).

UNIT 10

ELECTROLYSIS AND CELLS

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1.0 Introduction

Most liquids which conduct electricity are spilt up chemically by electric current. This process is called electrolysis and the liquids are called electrolytes. Most solutions of acids, salts and basis are electrolytes; the commonest liquid conductor which is not an electrolyte is mercury. Electrolytes are molten ionic compounds or solutions containing ions, i.e. solutions of ionic salts or of compounds that ionize in solution. Liquid metals, in which the conduction is by free electrons, are not usually regarded as electrolytes. Solid conductors of ions, as in the sodium-sulphur cell, are also known as electrolytes.

The electrode by which the flow of electrons enters the electrolyte is called the cathode, and that by which the flow of electrons leaves, the anode . In effect, the circuit is complete, owing to the action of the charge carriers of the ionized electrolyte.

Electrolysis has several industrial applications. It is used for the refining of impure copper. The anode is made from impure copper and the cathode from a thin sheet of pure copper. They are immersed in copper sulphate solution. Electrolysis transfers copper but not the impurities from the anode to the cathode. The cathode becomes coated with a thick layer of pure copper. Electroplating makes use of the same principle. A layer of one metal may be deposited on another metal. For example, cutlery made from brass may be plated with silver or gold. Steel or copper can be plated with chromium or cadmium to give protection against corrosion. Chromium-plated steel is frequently used for metal parts of bicycles, automobiles and household goods.

2.0 Objective

After studying this unit you should be able to:

* explain the phenomenon of electrolysis
* define Faraday’s laws of electrolysis
* do simple calculations involving the mass of an element liberated from an electrolytic solution by the transfer of electric charge.
* describe the principles of the operations of primary and secondary cells.
* explain the Arrbenius or ionic theory of electrolytic dissociation

3.1 Electrolysis

The conduction of electricity by an electric solution results in a net migration of positive ions or cations which constitute the hydrogen or metallic radical of the electrolyte) toward the cathode, and of negative ions or anions (which constitute the acidic radicals) towards the anode. This process is known as electrolysis, and is sometimes referred to as the chemical effect of electric current.

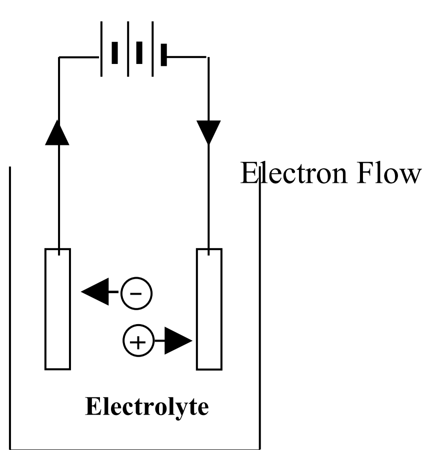


Fig. 3.1

Ions, on arrived at he appropriate electrode, receive or give up electrons to form neutral radicals, the electron being given to or absorbed from the current in the external circuit. It should be noted that there are no free electrons in the electrolyte and that the ions cannot enter the electrodes as current.

We shall describe electrolysis with the process of copper plating. When an electric current is passed between copper electrodes through an aqueous solution of copper sulphate, it is found that copper is removed from the anode and deposited on the cathode. It is assumed that the copper sulphate molecule in water becomes copper ions with a positive charge (shortage of electrons), and sulphate ions with a negative charge:

$$CuSO_4 \; CU^{2+} \rightarrow + SO_4^{2-1}$$

The external electricity supply is taking electrons from the anode, so the $SO^{2-}4$ ion is attracted towards it. At the anode it gives up its two electrons, at the same time combining with part of the anode copper to produce more copper sulphate

$$CU + SO_4 \rightarrow CUSO_4$$

This is called a 'secondary reaction' Meanwhile the CU^{2+} ions are attracted to the cathode, which is being supplied with electrons by the external electricity supply. Two such electrons will neutralize a CU^{2+} ion, leaving metallic copper which adhere to the cathode.

If platinum electrodes are used, it is found that the $SO^{2-}{}_4$ ions do not combine with the anode; instead the SO_4 radical combines with some of the water:

$$2S0_4 + 2\, H_20 \rightarrow 2\, H_2\, S0_4 + 0_2$$

Oxygen is seen bubbling from the anode, and an indicator will confirm the generation of an acid in the vicinity of the anode.

3.1.1. Faraday's Laws of Electrolysis

The quantitative investigation of electrolysis was performed by Faraday who summarized his results in two laws:

(i) The mass of a substance liberated in electrolysis is proportional to the quantity of electricity passed, i.e. to the product of current and time.

(ii) When the same quantity of electricity is passed thought difference electrolytes the masses of substances liberated are in the ratio of their equivalent weights (relative atomic masses charge on each ion)

The quantity of electricity which liberates 1 mole of all singly-charged ions, ½ mole of doubly-charged ions, and so on, is approximately 96,500C is called the faraday. (The faraday is NOT an SI unit).

3.1.2 The Electrochemical Equivalent

The electrochemical equivalent of a substance is the mass liberated in electrolysis by 1 coulomb.

This is a quantity which can be determined accurately by experiment and from which the faraday may be calculated:

$$\text{Faraday} = \frac{\text{equivalent weight of substance}}{\text{Electrochemical equivalent of substance}}$$

The mass m of a substance deposited or liberated by electrolysis when a current 1 flows for time t is thus given by:

$$M = ZIt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

where Z is equal to a universal constant multiplied by the chemical equivalent of the substance and is called its electrochemical equivalent.

Before we go further, let us explain some terms

(a) The molecular (atomic) weight of a substance is the number of times the average mass (allowing for isotopes) of one of the molecules (atoms) is greater than the atomic mass unit.

(b) The number of grammes of a substance equal to its molecular (atomic) weight is called one grammes molecule (atom) or one mole.

(c) The chemical equivalent of a substance is its molecular weight divided by its valency.

(d) A gramme equivalent of a substance is the number of grammes of it equal to its chemical equivalent.

It follows that a certain quantity of charge F, called the faraday, is required to liberate one gramme equivalent of any radical during electrolysis. We may therefore express faraday's laws of electrolysis as :

$$m = \left\{\frac{m}{z}\right\} \qquad \left\{\frac{It}{F}\right\} \quad \ldots\ldots\ldots\ldots.3.2$$

where m is the mass in grammes of a radical having molecular weight m and valence z liberated when a current of I ampere flows for t second.

$$Z = M/2F \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.3$$

In the case of an element, the atomic weight, A, replaces M in equations 3.2 and 3.3.

☐ **Example 1**

In an experiment analyzing an aqueous solution of $CUS0_4$ between CU electrodes, 0.477g of CU are deposited on the cathode when a current of 0.80A flows for 30 minutes

Calculate:

(a) The electrochemical equivalent of CU

(b) Its atomic weight (CU is divalent under these condition

Solution

(a) When 0.80A flows for 30 minutes, the amount of coulombs passing is 0.80 x 30 x 60 = 1440

Now, m = ZIt = Z x 1440 = 0.477

$$\therefore Z = \frac{0.477}{1440}$$

$$= 3.31 \times 10^{-4} g\, C^{-1}$$

(b) $Z = \frac{A}{2} \times \frac{1}{F} = \frac{A}{2 \times 96500}$ (see eq. 3. 3)

and A = 63.8

Note:

(i) A cell designed for the study of electrolysis is called a voltameter.

(ii) Electroplating is the use of electrolysis to coat one metal with another

□ **Example 2**

A current of 0.8 A is passed for 30 minutes through a water voltmeter, consisting of platinum electrodes dipping into dilute sulphuric acid, and it is found that 178.0cm^3 of hydrogen at a pressure of 75.0cm of mercury at 288k are liberated. Calculate the electrochemical equivalent of hydrogen, given that its density at s.t.p. is
8.99 x 10^{-2}kgm$^{-3.}$

If a copper voltmeter, consisting of copper plates dipping into a solution of copper sulphate, had been connected in series with the water voltmeter, calculate the mass of copper deposited on the cathode, given that the equivalent weights of hydrogen and copper are 1 and 31.8 respectively.

Solution

Volume of hydrogen at s.t.p. $= 178 \times \frac{75}{76} \times \frac{273}{273 + 15}$

$= 166.5 cm^3$

mass of hydrogen $= 166.5 \times 8.99 \times 10^{-2}\ 10^{-6}$
$= 1.50 \times 10^{-5} kg$

Charge of electricity passed $= 0.800 \times 30 \times 60$
$= 1440C$

Electrochemical equivalent of hydrogen $= \dfrac{1.50 \times 10^{-5}}{1440}$

$= 1.04 \times 10^{-8}$ kg C^{-1}

$\dfrac{\text{mass of copper deposited}}{\text{mass of hydrogen liberated}} = \dfrac{31.8}{1}$

$\therefore$ mass of copper deposited $= 31.8 \times 50 \times 10^{-5} = 4.77 \times 10^{-4}$kg.

3.1.3 The Faraday and the Electronic Charge

Let us consider the direct deposition of an element. The number of atoms of the element in one mole = N_A (Avogadro's number) and, as each ion associated with the liberation of each atom carries a charge ze, it follows that the total charge carried by the ions associated with the liberation of one gramme equivalent = (N_A/z) (ze) = N_Ae. This total charge is clearly equal to the faraday, giving us the important relationship.

$$F = N_A e \qquad \ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

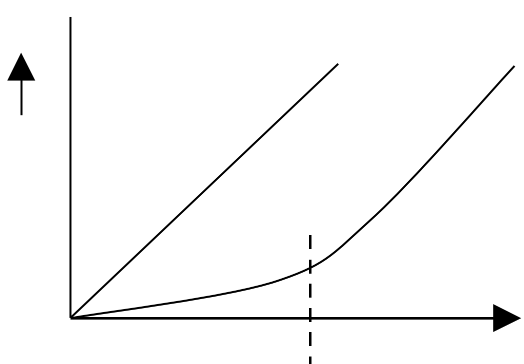
Accurate determinations of the faraday give its value as 96519 coulombs, but for most problems F = 96500 coulombs may be used

Note:

The relation $F = N_Ae$ has been used in the determination of the electric charge. F can be found by experiments with electrolysis and N_A by measurements on Brownian motion or X-ray diffraction, the latter method being much more accurate than the former. The ratio of these quantities thus gives the electronic charge e.

3.1.4 Polarisation

Electrolytic cells or voltameters in which there is the evolution of gas at the electrodes generally exhibit the phenomenon of polarization. It is found that if the voltage applied to the cell is less than a critical value V_1 the induced current will soon die away to a very small value. This is owing to the fact that the freshly formed gases which surround the electrodes effectively form an electric cell providing a back e.m.f. whose polarity is opposed to that applied externally. Unless, therefore, the e.m.f. applied to the cell is greater than that produced by the effective "internal battery" no appreciable current will flow.



Current B A

V_1 applied voltage

Fig 3.1

A graph of current against voltage for a voltameter containing very dilute suphuric acid and platinum electrodes is shown in fig. 3.1 (curve A). In this case $V^1 = 1.7$ volt ; it arises from the liberation of hydrogen at the cathode and oxygen at the anode if the applied voltage V is greater than V^1 then the current 1 α $(V - V^1)$.

Cells in which no gas is evolved are free of polarization effects and in them the current 1 is proportional to the applied voltage (curve B).

3.1.5 Ionic Theory of Electrolysis

This theory was put forward by Arrhenius in 1887. Known as the theory of electrolytic dissociated into ions. Thus when sodium chloride, NaCl, is dissolved in water some of the molecules dissociate into positively charged sodium ions, Na+, and negatively charged chlorine ions, Cl^- Ions are continually recombining and molecules dissociating, there being a dynamic equilibrium represented by the equation.

$$NaCl \rightleftharpoons Na^+ Cl^-$$

When an e.m.f is applied between the electrodes dipping into an electrolyte the positive ions, or cations, are attracted to the cathode while the negative ions, or anions, are attracted to the anode. The two streams of oppositely charged ions, travelling in opposite directions, carry the current through the electrolyte. Anions give up their surplus electrons to the anode, and cations receive electrons from the cathode, thus maintaining the flow of electrons in the external circuit. Having given up their charges the ions are liberated as uncharged atoms and molecules.

3.2 Cells

A cell is a system in which two electrodes are in contact with an electrolyte. The electrodes are metal or carbon plates or rods, in some cases, liquid metals (e.g. mercury). In an electrolytic cell a current from an outside source is passed through the electrolyte to produce chemical change. In a voltaic cell, spontaneous reactions between the electrodes and electrolyte (s) produce potential difference between the two electrodes.

Cells in use nowadays are divided into two groups: unrechargeable (primary) and rechargeable (secondary).

3.2.1 Primary Cells

The commonest 'dry' cell is the dry Leclarche (see fig. 3.2). It consists of a carbon rod (positive or electron – receiving terminal), surrounded by a paste of magnesium dioxide. Outside this is a concentric paste of ammonium chloride (the electrolyte), contained in zinc case which is also the negative terminal. The chemical reaction which take place can be represented by the equation.

$$Zn + 2NH_4Cl + 2\ Mn0_2 \rightarrow Zn\ (NH_3)_2Cl_2 + H_20 + Mn_20_3$$

The manganese dioxide is referred to as the 'depolarizer' because without it, bubbles of hydrogen would form on the carbon rod, forming an insulating layer around it with the $Mn0_2$ paste, this hydrogen is at once converted into water, which does not interfere with the cell's action.

Hydrogen ions give up their + charge (i.e. acquire a neutralizing electron) at the carbon rod. When the ammonium chloride reacts with the zinc case, the latter is let with an extra electron. Thus the electron flow in the external circuit is from zinc (- terminal) to carbon (+ terminal).

The e.m.f of the cell is about 1.5 volts; in use, its internal resistance rises until it is of no further use.

One of the main disadvantage of the dry Lechanche' cell lies in the fact that the zinc case takes part in the reaction , and is therefore slowly eaten away. Eventually, it will puncture, and the corrosive electrolyte will spill out, endangering the equipment it is supposed to be energizing 'leak-proof" cells have an extra outer covering, but are only relatively safe in this respect.

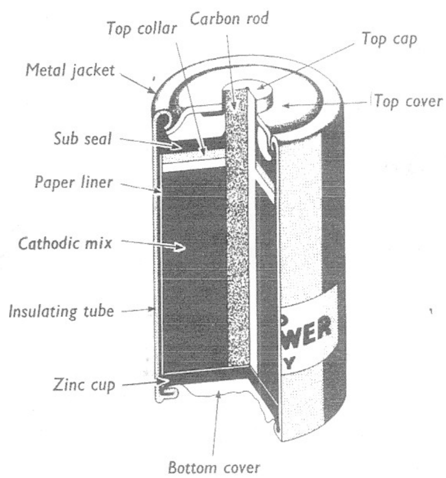


Fig 3.2

3.2.2 Secondary Cells

The two important 'rechargeable' cells in common use are the lead-acid type, and the nickel-cadmium alkaline type. Secondary cells are also referred to as accumulators.

Lead Acid Accumulator

In its basic form, the positive terminal is a perforated lead plate filled with lead peroxide, and the negative plat is lead. Both hang in fairly concentrated sulphuric acid. The reversible reaction is

$$Pb + pbO_2 + 2H_2SO_4 \rightleftharpoons 2pbSO_4 + 2H_2O$$

During discharge the electrolyte loses sulphuric acid, and its density falls. Thus a hydrometer can be used to check the charge of a lead-acid cell. The electrolyte density of a fully charged cell is 1250kgm^{-3}, and a fully discharged one, 1100kg m^{-3}.

During the repeated charge and discharge, solid reaction products collect below the plates. When the level of these products reaches the suspended plats, the cell fails, so it is quite easy to predetermine roughly the range at which a lead – acid cell will fail.

The fully-charged e.m.f. of the lead cell accumulator is 2.05v, and it falls only slightly during use.

Nickel-Cadmium Accumulator

The nickel- cadmium accumulator is lighter and more robust than the lead-acid its e.m.f, which falls during discharge, is 1.2v. Both plates are made of perforated steel; the negative plate is filled with nickel hydrate and graphite. The charge and discharge reaction is

$$2Ni\,(0H)_3 + Cd \rightarrow 2\,Ni\,(0H)_2 + Cd\,(0H)_2$$

In a fully-charged cell, the nickel hydrate is highly oxidized and the negative material is reduced to pure cadmium. On discharge the nickel hydrate is reduced to a lower degree of oxidation, and the cadmium in the negative plate is oxidized. Thus the reaction may be regarded as the transfer of $(0H)^-$ ions from one plate to the other, and the density of the electrolyte (21 per cent potassium hydroxide solution) does not change being 1200kg m^{-3} at normal temperature.

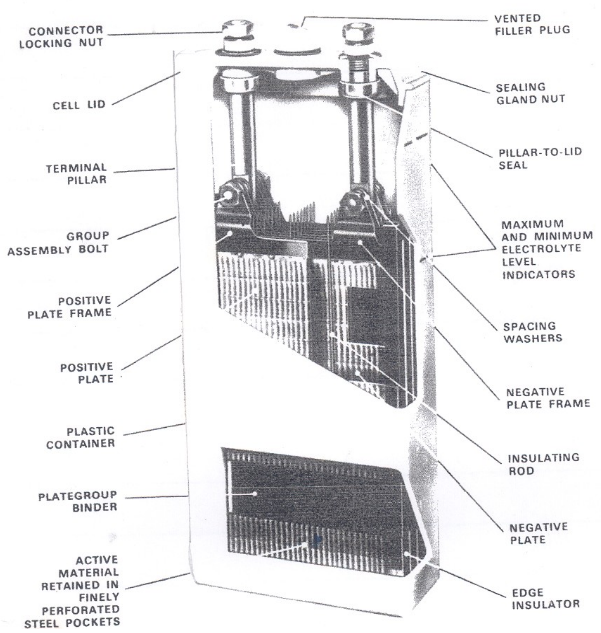


Fig 3.3

Figure 3.3 shows a nickel-cadmium accumulator with plastic containers. (The figure shows a cutaway section of the plastic-cased cell).

4.0 Conclusions

Solutions that conduct electricity are called electrolytes. In an electrolyte the metallic and hydrogen ions are positive. The other ions are negative. Currents in an electrolyte consist of positive ions moving to the cathode (negative terminal) and negative ions moving to the anode (positive terminal).

Univalent atoms gain or lose one electron each in ionization, and bivalent atoms gain or lose two electrons.

Electric cells convert chemical energy into electric energy and consist of two different metals (or metal and carbon) separated from each other by an electrolyte.

Many different cells have been invented since the first was made by Volta at the end of the eighteenth century. Volta's simple cell consisted of plates of copper and zinc in dilute sulphuric acid and had an e.m.f of about 1.0v.

In general, primary cells have to be discarded after use and are popularly called 'dry' batteries, though this description is not strictly correct. Some types used today are carbon-zinc, alkaline – manganese, mercury, silver oxide and Weston standard.

Secondary cells can be charged and discharges repeatedly (but not indefinitely) and are generally called accumulator. They supply 'high' continuous currents depending on their capacity, which is expressed in ampere-hours (Ah) for a particular discharge rate. For example, a cell with a capacity of 30 Ah at the 10 hour rate will sustain a current of 3A for 10 hours, but whilst 1A would be supplied for more than 30 hours, 6A would not flow for 5 hours.

5.0 Summary

* The mass of a substance liberated in electrolysis is proportional to the quantity of electricity passed, i.e. to the product of current and time.

* When the same quantity of electricity is passed through different electrolytes the masses of substances liberated are in the ratio of their equivalent weights (relative atomic mass ÷ charge on each ion)

* The electrochemical equivalent of a substance is the mass liberated in electrolysis's by 1 coulomb.

* The quantity of electricity which liberates 1 mole of all singly-charged ion, ½ mole of doubly-charged ions, and so on, is approximately 96500 coulombs and is called the faraday. The faraday is not an SI unit.

* Arrhenius theory of dissociation assumes that when an electrolyte enters solution part of it is dissociated into ions, the extent of dissociation increasing with dilution.

* There are several industrial applications of electrolysis. These include electroplating, electrotyping and the production of pure metals.

* The two terminals of an accumulator should never be short-circuited , i.e. connected directly by a wire of low resistance. Distilled water must be added from time to time to make up for the water lost by evaporation as time passes on during the action of the cell.

6.0 Tutor Marked Assignments (TMA)

1. State Faraday's Laws of Electrolysis
   A brass plate of total area $100cm^2$ is to be given a copper plating of thickness 0.01mm. How long will the process take if the current is 1A? (Density of copper = $8930kgm^{-3}$; electrochemical equivalent of hydrogen = $1.04x10^{-8}kgC^{-1}$; chemical equivalent of copper = 31.6)

2. Calculate the electrochemical equivalent of hydrogen, given that 1A deposits 0.65gm of copper from a solution of copper sulphate in 33 mins. (atomic weight of copper = 63, valency = 2).

3. Describe briefly the lead-acid accumulator and explain the chemical reactions which take place in it during charging and discharging.

4. Give an account of the elementary theory of electrolysis and show that it is consistent with Faraday's laws of electrolysis.

   What current will liberate $100cm^3$ of hydrogen at 293k and 78cm of mercury pressure in 5 minutes? (electrochemical equivalent of hydrogen = $1.044 \times 10^{-8}kg\ C^{-1}$, density of hydrogen = $0.09kgm^{-3}$ at N.T.P).

7.0 References and Other Resources

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UNIT 11

THERMAL EFFECTS OF ELECTRIC CURRENTS AND

ELECTRIC POWER

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1.0 Introduction

In PHY 101, you learned that power is the rate of doing work and may be expressed in such units as joules per second and kilowatts. The power of a waterfall depends upon the height of the fall and upon the number of kilogram-weights of water transferred per unit time. Similarly, in electric circuits the power expended in heating a resistor, charging a storage battery, or turning a motor depends upon the difference of potential between the terminals of the device and the electric current through it.

Electric power is the rate of expending energy or doing work in an electrical system. For a direct-current, it is given by the product of the current passing through a system and the potential difference across it. Electric currents can do work in many ways, as charging storage batteries, running electric motors, and generating heat.

2.0 Objective

After studying this unit, you should be able to:

* understand the conversion of electrical energy to thermal energy
* state joule's law of generation of heat

* define the kilowatt-hour, the commercial unit of electrical energy
* distinguish between electric power and electromotive force.
* explain the principle of the design and construction of incandescent lamps.
* explain the rule of a fuse in an electric circuit
* solve problems involving the conversion of electrical energy into other forms of energy.
* describe a simple experiment to illustrate the electrical equivalent of heat.

3.1 Current Power

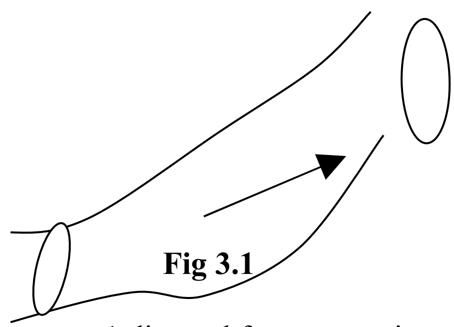

Fig 3.1

Let us consider a current 1 directed from an equipotential at potential V_a to an equipotential at potential V_b (fig. 3.1) (In a time interval dt, a charge dQ pases the V_a equipotential while the same amount of charge passes the V_b equipotential. The charge passing hrough he region between these equipotentials therefore experiences a change in potential energy given by.

$$dW = V_a dQ - V_b\, dQ = V_{ab}\, 1dt \quad \ldots\ldots\ldots\ldots\ldots\ldots.3.1$$

This is the work done by the eelctric force on the charge between the V_a the and V_b equipotentials. The power supplied by the elctric field to the charge moving between the V_a and V_a equipotentails is

$$P = \frac{dW}{dt} = V_{ab}I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.2$$

This equation will be applied to many different physical situations. We consider first an example in which the charge move through a vacuum.

□ **Example**

In an electrostatic accelerator, protons emerge with neglegible kinetic energy from an ion source maintianed at a potential of $2.0x10^6$V. These protons are accelerated through a vacuum toward an electrode at zero potential. The proton beam emerging from the accelerator contains 10^{13} protons per second. What is the power supplied to the protons by the electric field?.

Solution

Since the charge on each proton is $1.60x10^{-19}$C,. the beam current is

$$I = 1.60 \times 10^{-19} \times 10^{13} \text{ A} = 1.60 \times 10^{-6}\text{A}$$

The power supplied by the electric field to the proton beam is

$$p = V_{ab} \; I = (2.00 \times 10^{6} \text{ V})(1.60 \times 10^{-6} \text{ A}) = 3.2 \text{ W}.$$

3.2 Power in Electric Circuits

The interpretation of the power equation (eq. 3.2)

$$P = V_{ab}I$$

is of particular interest for the case when Fig 3.1 represents a portion of an electric circuit. The electric potential energy of the circulating charges changes at the rate $V_{ab}I$ as they drift through this portion of the circuit with a kinetic energy that is negligibly small. In this circumstance, $P = V_{ab}I$ is a general expression for the power input or output of this portion of the circuit.

(i) If $V_a > V_b$, the circulating charges give up energy and there is a power input. $Pin = V_{ab}I$

(ii) If $V_a < V_b$, the circulating charges gain energy and there is a power output. $P_{out} = V_{ab}I$

3.3 Power Dissipation in a Resistor, Joule's Law

If the portion of the circuit is a pure resistance R, the potential drop $V_{ab} = V_a - V_b$ is always positve (an IR drop in the direction of the current), so there is a power input to the resistor, $P = V_{ab}I$.

Mobile charged particles are accelerated by the electric field within the conductor, but the kinetic energy gained by the charge carriers is transferred by collisions to the atoms of the conductor. The net result is that electric potential energy of the mobile charged particles is converted into internal energy (or thermal enrgy) of the condcutor. As the internal energy of the conductor increases, its temperature rises unitl there is an outward flow of heat at the same rate as the enrgy input. In this process, called Joule heating, the input power is dissipated within the conductor.

Various expressions for the power dissipated are:

$$P = VI = I^2 R = V^2/R \quad \text{.........................} 3.3$$

For an ohmic resistor, Eq. 3.3 is called Joule's law.

Example

A wire – wound resistor with a resistance of 250 ohm will overheat if the power dissipated exceeds 10W. What is the maximum constant voltage V that can be applied across the terminals of this resistor?

Solution

The maximum power is 10W, therefore the maximum voltage is given by

$$V^2 = PR \quad \text{(see Eq. 3.3)}$$

$$V = \sqrt{10 \times 250}$$

$$= 50 \text{ volts}$$

Note:

Equation 3.3 gives three alternative expressions for power but the last two are only true when all the electrical energy is changed to heat. The first, P = VI, gives the rate of production of all forms of energy. For example, if the current in an electric motor is 5A when the applied p.d. is 10V then 50W of electric power is supplied to it. However, it may only produce 40 W of mechanical power, the other 10W being the rate of production of heat by he motor windings due to their resistance.

3.4 Electric Power and Energy

Electrical energy U is the electric power, P times the time, t:

$$U = Pt \quad \text{.....................................} 3.4$$

The kilowatt-hour is a unit of energy. You can buy electrical energy at a certain price per kilowatt-hour. In Nigeria, the National Electric Power

Authority (NEPA) sells electrical energy at N6.00 per unit, that is a kilowatt-hour. The electric bill of a consumer depends upon the energy supplied, which, in turn, depends on the current, the time, and the voltage.

Suppose that the current through the heater coils of a stove is 10A at 220 V, so that the power is 2,200 JS^{-1}, that is 2,200 watts or 2.2 kw. Then in 1 hour (3600s), the total input enegy is 2.2 kilowatt-hour or 7,920,000 joules.

- **Note**

The energy converted by a device in kilowatt-hours is thus calculated by multiplying the power of thedevice in kilowatts by the time in hours for which it is used. Hence a 3kw electric radiator working for 4 hours uses 12kwh of electrical energy – often called 12 units.

- **Example**

An electric refrigerator requires 200W and operates 8 hours per day. What is the cost of the energy to operate it for 30 days at 6 naira per unit?

Solution

$$200 \text{ W} \times 8.0 \text{ hr/day} \times 30 \text{ days} = 48000 \text{ watt-hr} \times \frac{1\text{kw} - \text{hr}}{1000 \text{ watt-hr}}$$

$$= 48 \text{ kw-hr}$$

$$\therefore \quad 48 \text{ kw-hr} \times \text{N}6.00/\text{kw-hr} = \text{N}288.00$$

3.5 Electric Power and Electromotive Force

Let us calcualte the electrical power required to charge a storage battery. You would recall that the terminal voltage of a storage battery during the charging process is greater than its electromotive force E by the amount of the internal voltage drop within the battery. Thus if the battery charger sets up a terminal voltage V in sending a charuing current I through the internal resitance r we have

$$V = E + Ir \quad \text{.................................3.5}$$

The power delivered to the battery is V times I:

$$P = EI + I^2 r \quad \text{...............................3.6}$$

Question:

What do the terms on the right – hand side of eq. 3.6 mean?

Answer:

The first term, the product of the e.m.f and the current, is the rate at which energy is transformed from electrical energy into stored chemical energy – useful energy. The second term is the rate at which electrical energy is converted into unusable thermal energy in the battery electrolyte and plates.

Example

By means of a battery charger, a potential difference of 7.0 V is applied to the termianls of a storage battery whose e.m.f is 6.0 V and whose internal resistace is 0.5 ohm.

(a) Wat is the charging current?

(b) At what rate is energy being converted into chemical energy, and at what rate is heat produced?

Solution

(a) On charging, the terminal voltage is

$$V = E + Ir$$

$$7.0 = 6.0 + 0.50I$$

and $I = 2.0$ A, the charging current

(b)

$$P = EI + I^2 r$$

$$P = (6.0)(2.0) + (2.0)^2(0.50)$$

$$= 12 \text{ watts} + 2 \text{ watts}$$

The rate of converting electrical energy into stored chemical energy is 12W and the rate of heating is 2W.

We can also calculate the power deliver by a discharging battery to an external circuit. Upon discharging, the batery has a terminal voltage V which is also the voltage drop across the external circuit.

$$V = E - Ir \quad \ldots\ldots\ldots 3.7$$

Or the power delivered by the battery is

$$P = EI - I^2 r \quad \ldots\ldots\ldots 3.8$$

As you now know, the first term on the right-hand side of Eq. 3.8 is the rate at which chemical energy is converted into electrical enegy and the second term is the rate of heating. The difference between the two is the rate at which useful energy is delivered to the external circuit.

Although we have considered only batteries so far, the foregoing discussion applied to any source of e.m.f, including electrical generators, thermocouples, and other devices. All such devices have an internal resistance that must be considered in calculating the useful power that they will deliver.

It is worth noting that the power delivered by such a device is a maximum when the external, or load resistance equals the internal resistnace. The proof of this statement requires the mathematical techniques of calculu. And will not be given here. However, we can illustrate its application. From Eq. 3.7, we have

$$IR = E - Ir$$

Where R is the load resistance. Hence,

$$I = \frac{E}{R + r}$$

and Eq. 3.8 becomes

$$P = \frac{E^2}{R + r} - \frac{E^2 r}{(R + r)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

where P is the power delivered to the load. You can easily show that when R = 0 (no load), the power delivered is zero. When R is very large, the denominators of both terms in Eq. 3.9 become very large, and the terms themselves become very small, again P = 0. But when R = r, the power delivered is $E^2/4r$; this is the largest deliverable power from that battery. Fig. 3.2 is a graph of power delivered plotted against load resistance.

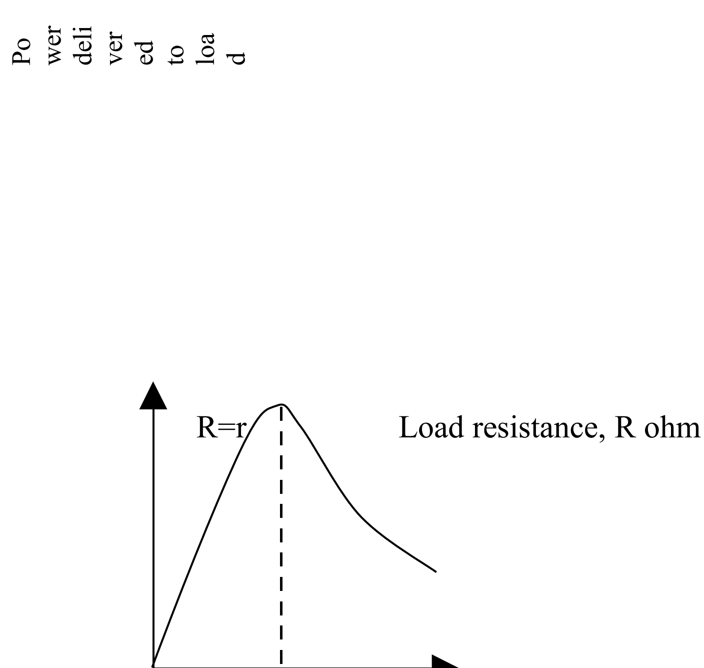

Fig. 3.2

3.6 The Incandescent Lamp and Heating Elements

The first incandescent lamps, devised more than a century ago, were platinum wires heated red hot by currents from voltaic cells. The lamps had little practical use, both because of their small luminous efficiencies and because the batteries were expensive and inconvenient. The development of the generator, based on the scientific discovery of electromagnetic induction by Michael Faraday in 1831, provided an economical source of electrical energy and led to a search for filament materials that could be operated at a higher temperature than platinum.

The expression $P = V^2/R$ shows that for a fixed supply p.d of V_1 the rate of heat production by a resistor increases as R decrease. Now, $R = \sigma l/A$, therefore $P = V^2A/\varrho l$ and so where a high rate of heat production at constant p.d. is required, as in an electric fire on the mains, the heating element should have a large cross-section area A, a small resistivity P and a short length ι. It must also be able to withstand high temperatures without oxidizing in air (and becoming brittle). Nichrome is the material which best satisfied all these requirements.

Electric lamp filaments have to operate at even higher temperatures if they are to emit light. In this case, tungsten, which has a very high melting point (3400^0C), is used either in a vacuum or more often in an inert gas (nitrogen or argon). The gas reduces evaporation of the tungsten and prevents the vapour condensing on the inside of the bulb and blackening it. In modern projector lamps, there is a little iodine which forms tungsten iodide with the tungsten vapour and remains as vapour when the lamp is working, thereby preventing blackening.

3.7 Fuses

In buildings, electrical devices are connected in parallel across the supply lines. The resistance of high-power devices is smaller than that of low power ones. The resistance of a 30W, 220V lamp is twice that of a 60W 220 V lamp.

When electrical supply wires are accidentally short-circuited by being brought into contact with each other, the resistance of the circuit so formed may be only a few hundredths of an ohm. The current becomes very large and heat the wire to dangerously high temperatures. To avoid this danger, fuses are connected in series with the supply lines. A fuse is a short length of wire, often tinned copper, selected to melt when the current through it exceeds a certain value. It thereby protects a circuit from exceessive currents. When the current attains a prescribed value, for instance 15A, the metal melts and the circuit is opened.

The fuse must be replaced after the short circuit has been repaired. The fuse wire is mounted in a receptacle, which prevents the melted metal from setting fire to the surroundings.

Circuit breakers- electromagnetic devices that open the circuit when the current exceeds a preset value and that can be reset when the overload is removed – are being increasingly used in place of fuses in buildings.

Note:

(i) The temperature reached by a given wire depends only on the current through it and is independent of its length (provided it is not so short for heat loss from the ends where it is supported, to matter).

(ii) The current required to reach the melting point of the wire increases as the radius of the wire increases.

It follows that fuses which melt at progressively higher temperatures can thus be made from the same material by using wires of increasing radius.

3.8 The Electrical Equivalent of Heat

There is one immediate and importance application of the results which we have obtained in this unit, an application that serves as a check on the correctness of what we have done and as another confirmation of the law of conservation of energy.

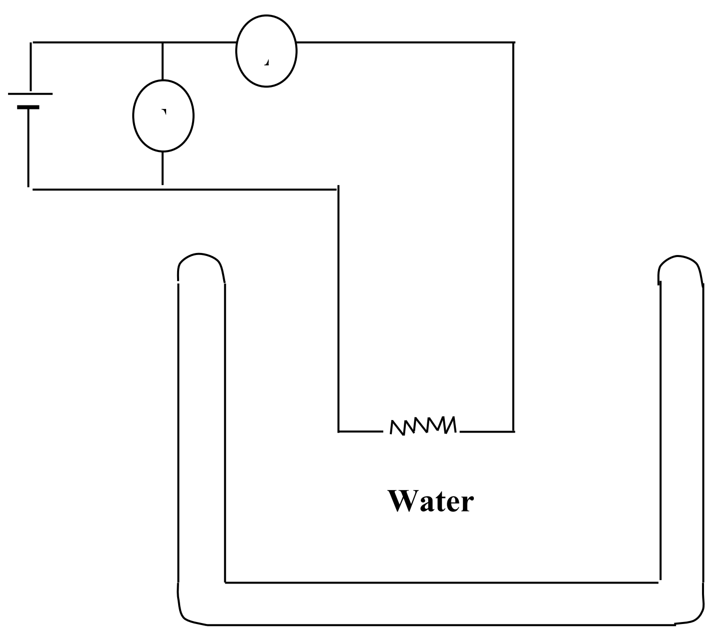

Fig 3.3

If electrical energy is being converted into the thermal energy in a wire and if the wire is sorround by a container of water, we can directely measure the amount of thermal energy by observing the temperature rise of the water (fig. 3.3.). Since we can also calculate the amount of electrical energy lost, the two figures can be compared to find the number of joules equivalent to one calorie. The electrical power is P = IV; if the experiment is continued for a time, t, the total amount of electrical energy lost is equal to IVt.

Such an experiment is similar to the experiments of Joule J. in which gravitational potential energy or other forms of machanical energy were used up and the resulting rise in temperature of the water observed.

Those experiments were described as a dertermination of the "mechanical equavalent of heat"; the experiment which we described here is a determination of the "electrical equivalent of heat: Incidentally Joule himself performed both kinds of experiments.

These experiments do turn out as expected and led to the same value for the conversion factor between joules and calories:

$$1 \text{ cal} = 4.184\text{J}$$

Note:

The calorie is not an SI unit of heat. However, it is found especially in order works of physics and chemistry.

Example:

A coffee percolator is rated at 800 W.

(a) How many calories of heat does it generate in 100s ?

(b) What time would be required for this percolator to heat 1 litre of water from 20^0C to 100^0C ? Neglect the heat loss to the percolator and its surroundings.

(c) If, in fact, 450s were required, what was the efficiency of device?

Solution:

(a) $P = 800\ W$
$= 800\ Js^{-1} \times cal / 4.18\ J$
$= 191\ cal\ s^{-1}$

Therefore the heat generated is given by

$H = 191\ cal\ s^{-1} \times 100s$
$= 19,100\ cal$

(b) The quantity of heat require to heat the water is

$Q = 1.00\ cal / gm\text{-}C° \times 1000gm \times 80°C$

$= 80,000\ cal$

$\therefore\ 191\ cal / sec \times t = 80000\ cal$

$t = 419s = 6.97min$

(c) $$\text{Efficiency} = \frac{191\ cal/sec \times 419sec}{191\ cal/sec \times 450sec} \times 100\%$$
$= 91\%$

4.0 Conclusions

When a battery is used to maintain an electric current in a conductor, chemical energy stored in the battery is continously converted to the electrical energy of the charge carriers.As the charge carriers move through the conductor, then electrical energy is convereted into internal energy (heat) due to collision between the charge carriers and other particles in the conductor. Electric power is the rate of conversion of electrical energy, that is the rate at which the charge carriers do work. Since the currnet, I, is the rate of charge movement, we can express electric power as current multiplied by potential difference.

The SI unit of power is the watt. When considering the dissipation of electrical energy, IW is equivalent to 1J of electrical energy being converted to other forms of energy per second.

Many of the electrical devices used in homes are simple resistances, in which electrical energy is converted directely into thermal energy and the thermal energy is what we desire, stoves, tonsters, and electric ions are obvious examples.

Perhaps the simplest and surely the most common household 'appliance' is the standard incandescent light bulb. This invention has done as much as anything else to change our style of life since its introduction in the late 19th century.

Every house-hold circuit includes a fuse or a circuit breaker, connected in series with everything else in order to limit the total current to a safe value.

5.0 Summary

* A portion of a circuit with a potential drop $V_{ab} = V_a – V_b$ carrying a current I directed from a to b, has:

  (i) A power input, $P_{in} = V_{ab}I$ if V_{ab} is positive

  (ii) A power output, $P_{out} = V_{ab}I$ if V_{ab} is negative

* In a conductor with resistance R = V/I, the power dissipated is

  $$P = VI = I^2 R = V^2/R$$

* The commercial unit of electrical energy is the kilowatt-hour (KWh) which is the energy expended in an electric circuit at the rate of one kilowatt for ine hour. It is the legal unit by which the consumption of electrical energy is measured and charged by the Power Holding Company of Nigeria (PHCN).

* A fuse is a device for protecting an apparatus or wiring from damage by overload. It acts as a cut-out by fusion. It consists of a short piece of wire, of some metal or alloy, connected in series with the apparatus or wiring to be protected.

* When a source of electrical energy is connected to a load, maximum power is transferred from source to load when the load resistance is equal to the internal resistance of the source. The source and load are then said to be matched.

6.0 Tutor Marked Assignments (TMA)

1. By considering a wire of radius r, length *l* and resistivity e, through which a current I flows, show that

   (a) The rate of production of heat by it is $I^2Pl/\pi r^2$

   (b) The rate of loss of heat from its surface is 2πrιh, where h is the heat lost per unit area of surface per second.

2. (a) A flashlamp bulb is marked '2.5V, 0.30A' and has to be operated from a dry battery of e.m.f. 3.0V for the current p.d of 2.5V to be produced across it. Why?

   (b) How much heat and light energy is produced by a 100W electric lamp in 5 minutes?

   (c) What is the resistance of a 240V, 60 W bulb?

3. An incandescent lamp is marked "120V, 75W".

   (a) What is the current in the lamp under normal conditions of operation?

   (b) How much does it cost per hour to operate the lamp if the price of electrical energy is N6.00 per unit?

4. A 200W lamp is totally immersed in 1500 gm of water. How much will the temperature of the water rise in 3 mins? Neglect heat losses from the water.

7.0 References and Other Resources

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UNIT 12

MAGNETIC PROPERTIES OF MATTER

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1.0 Introduction

You have leant in unit 8 that magnetic fields are produced by electric currents. When matter is present the magnetic field at any point is generally the superposition of the magnetic fields produced by two types of currents:

(a) Bound currents arising from the the circulating charges in atoms and molecules as well as from electron spin.

(b) Free electrons, such as the familiar conduction currents in wires, that are due to the drift of mobile charges.

The general theory of unit 8 relates the magnetic field B to all currents that are present. In this unit, we shall apply the theory to develop an efficient method of accounting for the average magnetic effects of bound currents.

2.0 Objectives

After studyiung this unit, you should be able to:

* define the magnetization vector, M

* understand the relationship between the magnetization vector and bound current linkages

* define magnetic susceptibility and permeability

* distinguish between diamagnetic, paramagnetic and ferromagnetic materials

* express the magnetization and demagnetization of ferromagnetic materials.

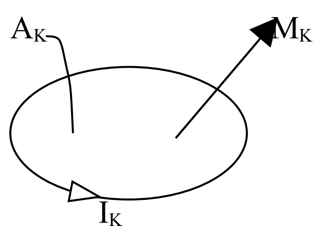
* state the curie's law

* explain the magnetization curve and hysteresis loop

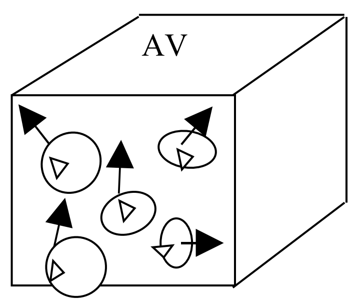
* understand the choice of magnetic materials for different purposes.

3.1 Ampere's Line Integral Law and the Magnetic Intensity H

We shall consider matter as containing a distribution of bound currents consisting of tiny current loops.



(a)



(b)

For example, let us consider a piece of matter containing current loops with the k th loop having a current I_k that encloses as area A_k (fig. 3.1)

The magnetic moment of this loop has a magnitude

$M_k = I_k A_k$ ……………………………….3.1

The magnetic moment of a system containing magnetic moments m_1. m_2, …..m_n, is defined as the vector sum $\sum_{k-1}^{n} m_k$ of all the magnetic moments of the system.

A basic physical quantity for the description of the magnetic state of a piece of matter is the magnetic moment per unit volume M, called the magnetization. To define M at a given point in a material, we consider a volume element dV that includes the point and is macroscopically small but microscopically large. Then the magnetization is defined as

$$M = \frac{\sum_{K=1}^{n} mk}{dV} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

where $m_1, m_2, \ldots\ldots, m_n$ are the magnetic moments within dV.

Fig. 3.1b shows a macroscopically small volume element dV, which is large enough to contain millions of molecules.

3.1.2 Relationship Between M and Bound Current Linkages

To investigate the relationship between M and the bound current linked by the path of integration, we consider for simplicity a path of integration which is a flux line. We also assume that all the magnetic moments within the matter are aligned with B. Then the bound current linked by the length ds of the path of integration shown in fig. 3.2 is

$$I_2 + I_4 + \ldots\ldots\ldots\ldots\ldots$$

where a given current loop within the volume element dV – Ads contributes to this sum if and only if the path passes through the loop.

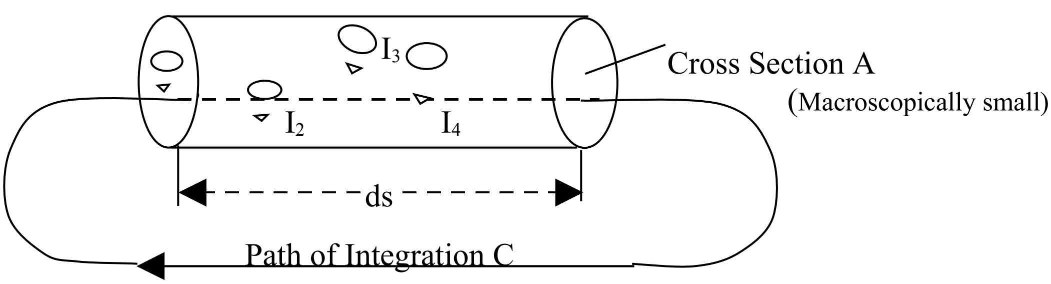


Fig 3.2

The value of this sum obviously depends on the precise location of the path of integration. Let us therefore consider all paths through the small macroscopic area which are parallel to ds. We compute the average of the net bound current linked by these paths. A fraction A_k/A of these paths will pass through a current loop of area. A_k. The contribution of this current loop to the average net current linked by a path is I_k (A_k/A). Therefore, the average value of the net current linked by paths through A and parallel to ds is

$$I_1 \ \underline{A_1} + I_2 \ \underline{A_2} + \ldots\ldots\ldots\ldots I_n \ \underline{A_n} = \sum^{n} m_k = \underline{Mdv} = mds \quad \ldots 3.3$$

A A A $\frac{k=1}{A}$ A

It follows that the net bound current linked by the closed path C, average over a family of parallel paths in the vicinity of C, has the value $\oint_c$ mds. For an arbitrary closed path and an arbitrary orientation of the magnetic moments, the same arguments lead to the general relationship.

Average net bound current linked = $\oint_c M_s d_s$ …………………..3.4

where Ms is the component of M in the direction of the path of integration.

With this result Ampere's line integral law, expressed in terms of the macroscopic field. B and the macroscopic quantity M ,is

$\oint$Bsds = μ_o (net free current linked + $\oint_c$ Msds) ……3.5

3.1.3 The Magnetic Intensity H

Rewriting Eq. 3.5, we obtain

$$\oint \left(\frac{Bs}{\mu_o} - Ms\right) ds = I_f$$

where I_f is the net free current linked by the path of integration .

We shall now introduce a new macroscopic quantity, the vector called the magnetic intensity H, defined by

$\underline{H} = \underline{B}/\mu_o - \underline{M}$ ……………………………..3.6

Ampere's line integral law, in term of H, is

$\oint_c$ Hsds = I_f ……………………………..3.7

The line integral of H is determined entirely by the net free current I_f passing through the path of integration C.

The unit of H is ampere per metre (Am^{-1})

Example 1

A toroid with an iron core is formed by winding N turns of wire uniformly on a thin uniform iron ring with a circumference of $2\pi R$ (figure 3.3). The iron is initially unmagnetized (M=0). A current I is established in the wire and the magnetic field B within the iron is measured. Find the magnetic intensity H and the magnetization M within the iron core of this toroid if N= 500 turns, $2\pi R = 0.400$m, I = 800mA, and B = 1.58T

Solution

The symmetry of the current distribution in the wire and the symmetry of the distribution of the iron imply that the magnetic flux lines within the core will be circles concentric with the core and that B, H and M will be parallel and have a constant magnitude along the flux line.

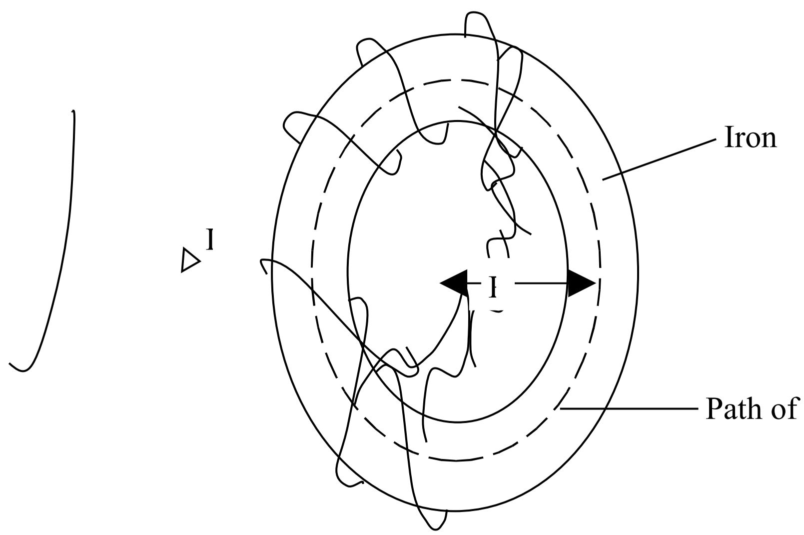


Fig 3.3

Selecting a circular flux line within the core as the path of integration, gives

$$\oint_c H_s\, ds = H \oint_c ds = H\,(2\pi R)$$

Since each turn of the coil carries a free current I through the path of integration, the net free current I_f passing the path C is given by

$$I_f = NI$$

Ampere's line integral law for H gives

$$H\,(2\pi R) = NI$$

Therefore, $H = \frac{NI}{2\pi R} = \frac{500 \times 800 \times 10^{-3} A}{0.400 m} = 10^3\ Am^{-1}$

Since $M = \frac{1}{\mu_o} B - H$, and the direction of B and H are the same, the magnitude of the magnetization is

$$\underline{M} = \frac{\underline{B}}{\mu_o} - \underline{H} = \frac{1.58\ T}{4\pi \times 10^{-7}\ T.m/A} - 1000\ Am^{-1}$$

$$= 1.26 \times 10^{6}\ Am^{-1}\ Am^{-1}$$

$\underline{M}$ is in the direction of $\underline{B}$.

3.2 Magnetic Susceptibility And Permeability

The magnetic properties of an isotropic material are characterized by a dimensionless parameter called the susceptibility x of the material defined by

$$M = x H \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

Or by the magnetic permeability U, or the relative permeability Ur, defined by

$$\mu_r = \frac{\mu}{\mu_o} = 1 + x \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

These definitions, together with $\underline{H} = \underline{B}/\mu_o - \underline{M}$, imply that

$$\underline{B} = \mu\ \underline{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

In a vacuum, $\underline{M} = 0$ and $\underline{B} = \mu o \underline{H}$. Consequently, a vacuum has a permeability μ_o, a relative permeability of exactly 1, and a susceptibility of zero.

3.3 Classification of Materials according to their Magnetic Properties

Contrary to the general notion that only iron, its compounds and steel can be magnetized, Faraday had shown that all materials can be magnetized, weakly or strongly, when subjected to a strong magnetizing field. They may, however, be divided into three categories.

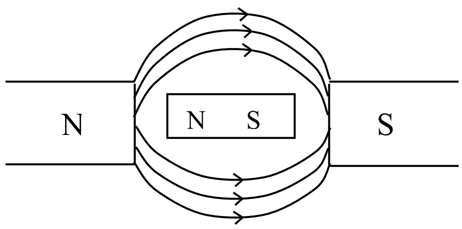
(i) Diamagnetic

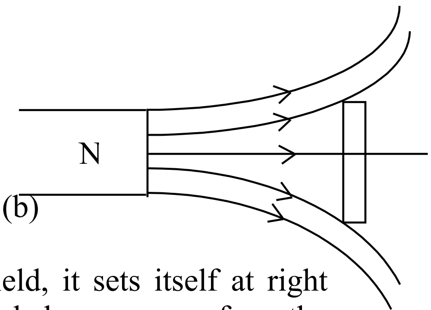
(ii) Paramagnetic and

(iii) Ferromagnetic materials.

3.3.1 Diamagnetic Materials

A diamagnetic material exhibit the following characteristics (i) When placed in between the N and S poles of a powerful magnet, it is magnetized with similar poles towards N and S respectively, as shown in fig. 3.3 (a). Its magnetization (M) is thus negative and hence its susceptibility (x= M/H) is also negative. That is why it is repelled when brought close to a magnetic pole. For the same reason, the flux density (B) inside it is less than in air.





(a) **Fig 3.3** (b)

(ii) When placed in a non-uniform magnetic field, it sets itself at right angles to the field as shown in Fig.3.3 (b), and always moves from the stronger to the weaker part of the field.

(iii) Its behaviour remains un-affected by any changes of temperature. Examples of diamagnetic material are Bismuth, Cadmium, Copper, Germanium, Gold, Lead and Zinc.

Diamagnetism arises because an external magnetic field B applied atom induces in the atom a magnetic moment m in the direction opposite to B. In essence, the mangetisation is a direct consequence of the induced e.m.f set up in the electron-orbits due to the applied field, its direction being opposite to that of the latter, in accordance with Lenz's law. Thus diamagnetism is really the reaction of matter to an applied magnetic field and is, as such, a fundamental magnetic phenomenon.

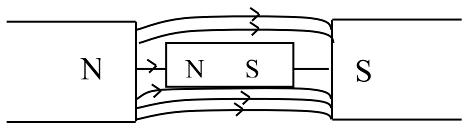
Since electrons revolve around the nucleus in the atoms of all materials, all of them should (and actually do) exhibit diamagnetism. However, the induced magnetic moments are so small that their effects are masked if the atoms of the material have a permanent magnetic moment.

3.3.2 Paramagnetic materials

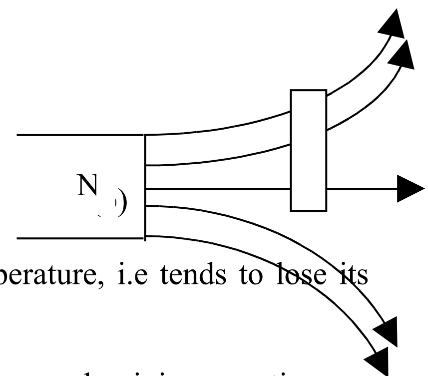
A paramagnetic material is weakly attracted by a magnet and shows the following general behaviour.

(i) When placed in between the N and S poles of the powerful magnet it gets feebly magnetized, with its opposite poles towards N and S respectively, that is in the direction of the magnetic field, as shown in Eq. 3.4a. Its Magnetization, M and susceptibility (x = M/H) are positive, though quite small. That is why it is attracted by a magnet when brought close to it.

(ii) When placed in a non-uniform magnetic field, it sets itself along or parallel to the field, as shown in fig. 3.4b, and moves from the weaker to the stronger part of the field.



(a) **Fig. 3.4**

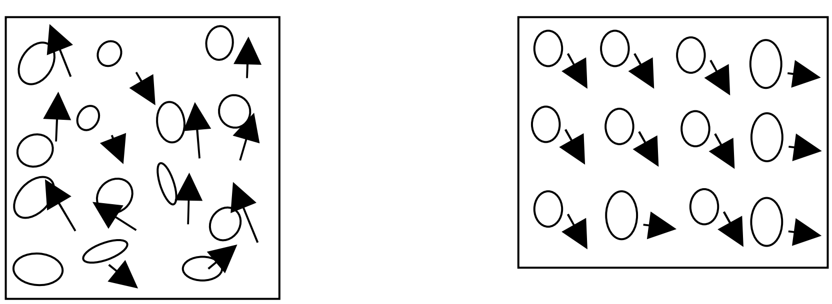


(iii) It is adversely affected by a rise in temperature, i.e tends to lose its magnetization.

Examples of paramagnetic materials are aluminium, antimony, chromium manganese, platinum, tantalum, alum, and gadolinium. In general, the transition elements and rare earths a paramagnetic.

Explanation of Paramagnetism

In the atoms of a paramagnetic material, the inner shells are not all completely filled. There is thus either an unpaired electron left in a shell having an odd number of electrons, or the spin magnetic moments of the electron are not completely cancelled, mostly the latter. There is thus always a small, residual, permanent magnetic moment in the atoms. The atomic current loops thus all functions as tiny bar magnets, interacting only very weakly with other. Hence in the unmagnetised material, tiny current loops, with magnetic moments perpendicular to their planes, lie haphazardly as shown in fig. 3.5a, cancelling each other out, with the result that the material shows no signs of any magnetism.



(a) **Fig. 3.5** (b)

When a magnetic field is applied to the material, it tends to align the magnetic moments of the tiny loops along its own direction (the potential energy being the least in this position), as shown in fig. 3.5b. The alignment, however, is only partial because of the thermal motion of the atoms. The higher the temperature, the greater the thermal motion of the atoms and hence the greater the lack of proper alignment.

A sufficiently strong magnetic field can, of course, align all the atomic loops along its own direction at a given temperature. When the loops have been so aligned, the magenetisation of the material reaches its saturation point and it has the maximum magnetic moment, M_N. Before the saturation stage is reached, the total magnetic moment is proportional to the magnetizing field B and inversely proportional to the temperature, T. Therefore, if we plot M against B/T, we obtain a curve of the form shown in fig. 3.6, indicating that at a certain value of B/T, the material attains its maximum magnetic moment, M_N, or that the saturation point of magnetization of the material is reached.

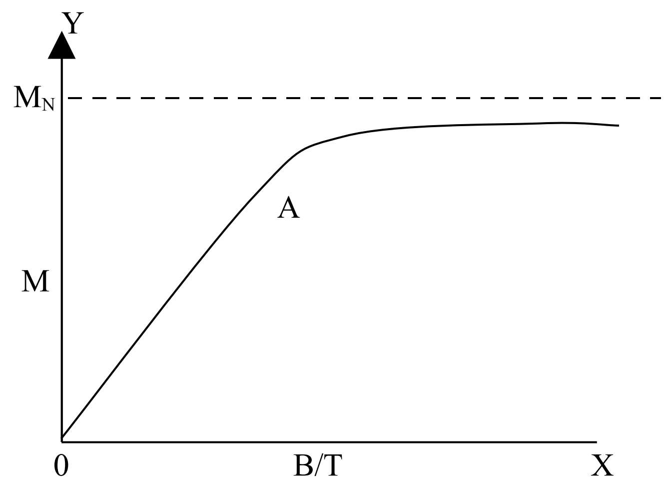


Fig. 3.6

As you can see, for lower values of B/T, say up to the point A, the curve is almost a straight line. Therefore, considering unit volume of the material, we obtain the relation.

$$M = C \frac{B}{T} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.11$$

where M is the magnetic moment of the material per unit volume, i.e, its mangetisation, and C, a constant.

The relationship of Eq. 3.11 is known as Curie's law, since it was first discovered by Pierre Curie . The constant C is called the Curie constant. The law breaks down at low temperatures, though in some cases, it continues to be valid up to as low a temperature as 13K.

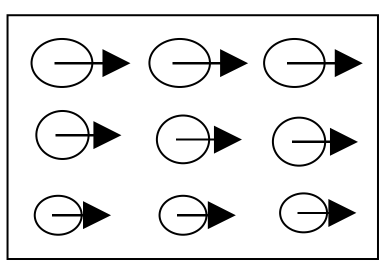
Although paramagnetic effects are generally stronger than diamagnetic effects, both are extremely weak. The alteration in a magentic field produced by the presence of such materials is less than 0.01%

3.3.3 Ferromagnetic Materials

Ferromagnetic materials behave just like paramagnetic materials but the effect is much more intense. Thus they are attracted by a magnet much more strongly. They always settle down in the direction of the magnetic field and their magnetization is positive and very much greater.

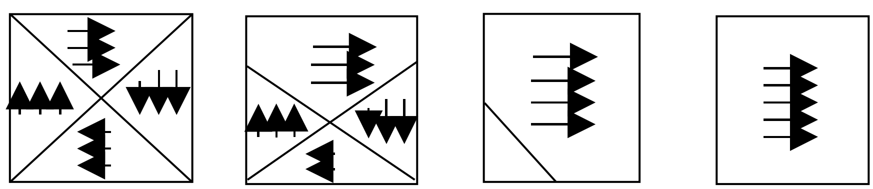
They comprise iron, nickel, cobalt , gadolinium and certain alloys. The value of μ_r though high, is not constant but varies with $\underline{B}$. For cast iron, which is not a very good magnetic material, μ_r has a maximum value of about 350. A silicon steel, stalloy, which is widely used in a.c. generator and transformers, has a maximum μ_r of about 6000. Some nickel-iron alloys have values of μ_r, up to 100,000, but they require careful heat treatment and are susceptible to mechanical strians.

In the atoms of ferromagentic materials, there are vacancies in the inner electron shells. The electrons in these shells are, therefore, not paired off with equal and opposite orbital magnetic moments and anti-parallel spins. In the case of iron, for example, as many as 5 out of 6 electrons in the n = 3, l = 2 sub-shell have parallel spins. The atoms of these elements, therefore, possess appreciable magnetic moments.



Another important property of these materials, is that an unpaired electron in atom internets strongly with the unpaired electron in the atom adjustment to it.Hence the magnetic moment get all aligned in the same direction as shown in fig. 3.7. This is known as exchange interaction.

The atoms in all these elements (which are crystalline in nature) seem to group themselves togather in small and separate assemblies, called domains, each about 5 x 10^{-5}m across. The magnetic moments in one domain are parallel to each other but not necessarily in the same direcion as those in a neighbouring domain.



(a) (b) (c) (d)

Fig. 3.8

In the unmagnetised state, the domains are oriented randomly, as it is in fig. 3.8 (a). Therefore, the material shows no sign of magnetization. When a magnetic field is applied to the material, alignment may occur in one or two ways: either (i) the magnetic moment in all the domains line up in the same direction as the field or (ii) if the material be pure and homogenous, the domain in which the magnetic moments are in the direction of the field continually expands at the expense of the others, as shown in fig. 3.8 (b), (c) and (d). The first procedure requires a much stronger magnetic field than the second.

Ferromagnetic materials retain their magnetization even after the magnetising field is removed. This is why permanent magnets are made of such materials. With temperature rise, the atomic alignment within the domains gets disturbed and at temperature near about 750^0C, called the curie temperature, the material is reduced to a paramagnetic one.

You should note that ferromagnetism is not an atomic property but just s special arrangement of groups of atoms into magnetic domain.

An unmagnetised ferromagnetic material, placed in a magnetic field, becomes magnetised and thereby makes substantial alteration in the magnetic field that would otherwise be present, typically increasing the field by a factor of a thousand at points within or near the material. Permanent magnets retain the alignment of their different domains. Other 'softer' ferromagnetic material tends to revert to random domain alignment when a magnetising field is removed.

Figure 3.9 show the rather complicated relationship between B and H in ferromagnetic material. If the sample is initially unmagnetised and H is steadily increased from zero, a B-H graph called the magnetization curve is obtained. This is the graph from state 0 to sate 1 in Figure 3.9. The permeability, μ = B/H, is not constant. A typical ferromagnetic material, annealed ion, has a relative permeability with an initial value of $3x10^2$, a maximum value of 5×10^3, and a limiting value of 1 (the value for a vacuum) as H approaches infinity.

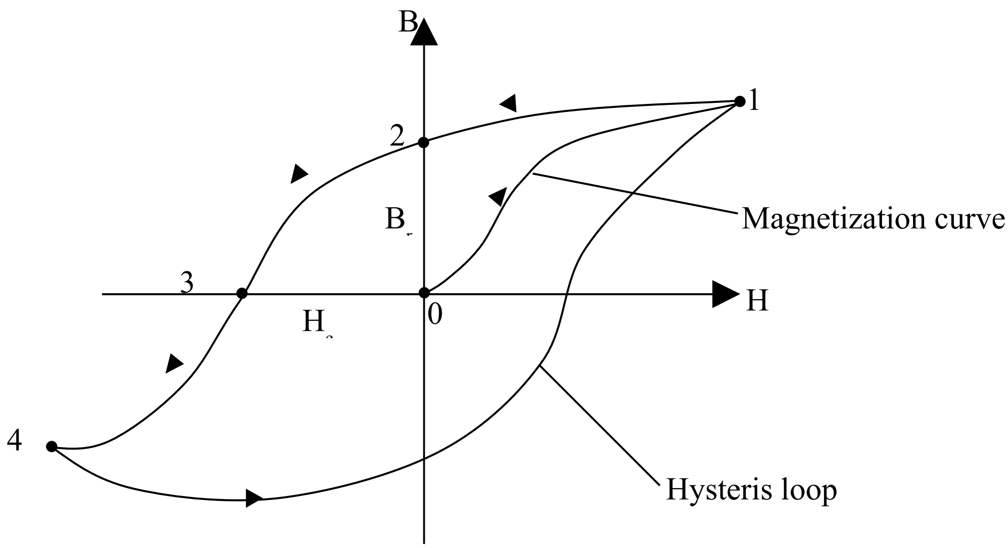


Fig 3.9

With the sample magnetised state 1, reduction in H does not result in B values lying on the magnetization curve. As H decreases B decreases, but along a different curve. When H has been decreased to zero (state 2), a magnetic field still remains. This magnetic field Br is called the remanence. Continuing with changes in the same direction, the field H is now established in the reverse direction, and b continues to decrease reaching the value zero at state 3. The corresponding magnetic intensity Hc is called the coercive force. With further change in the same direction we reach state 4, where B and H both have directions opposite to their directions in state 1. If we now reverse the direction of change of H we trace out the lower curve back to the original state 1.

This closed B-H curved is called a Hysteris loop and the phenomenon that the magnetization is not retraced is referred to as hysteresis (or lagging of the magnetic effect behind the magnetic field).

The cause of hysteresis has been traced to the fact that domain boundaries, instead of shifting freely when H and M change tend to become stuck at crystal imperfections.

Hysteresis makes possible the existence of permanent magnetism. A good material for a permanent magnet should have both a large remanence Br so that the magnet will be strong and a large coercive force Hc so that the field will not be greatly reduced by modest values of reverse magnetic intensity.

Because of hysteresis, the B-H relationship in a ferromagnetic material is always dependent on the history of the material. Such material has a memory, a fact that is exploited in magnetic types.

3.4 Demagnetizing a Material

Perhaps you already have the impression, based on this aspect of our study of magnetic materials, that a ferromagnetic material, once magenetized, can never be demagnetised, so that B may be zero when H is zero. For even if the magnetising field is reduced to zero, there still is a residual magnetism in the specimen. Demagnetization of the specimen can, however, be brought about in the following manner.

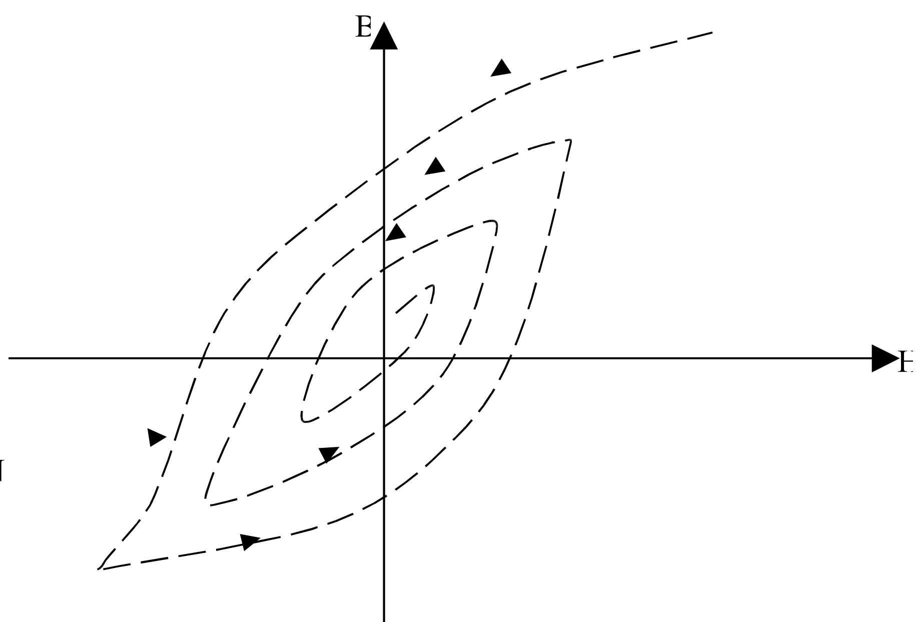


Fig. 3.10

The specimen is subjetced to cycles of magnetization with successively decreasing values of H. For this purpose, it is placed inside a coil and an alternating current passed through the coil from the mains, whose frequncy, as we know, is 50Hz. The specimen thus goes through hysteresis curves at the rate of 50 cycles per second. As the current is gradually reduced to zero, the hysteresis loops go on shrinking in area, as shown in figure 3.10, unitl it is reduced to zero. The specimen then gets completely demagnetised.

3.5 Choice of Magnetic Materials for Different Purposes

The B-H loop enabble us to judge the suitability of a material for use in several electrical devices.

(i) Transformer and generator cores.

The cores are taken through 50 cycles of magnetisation per second. They must, therefore, have narrow hysteresis loops, high permeability for low values of H and low coercivity . The materials suitable for this purpose are silicon irons and mumetal (76% nickel, 17% iron and small percentages of copper and aluminium).

(ii) Permanent Magnets

Since the materials used for permanent magnets have never to be taken through cycles of magnetization, hysteresis loss is not of any importance. Their remanence and coercivity, however, must be large.

Materials suitable for the purpose are cobalt steel (alloy of cobalt, tungsten and carbon) and tincal (containing titaniun, cobalt, nickel and aluminium).

(iii) Electromagnets

The main requirements are large vlues of magnetization, M for a given field H and low coercivity. The commonest material used is silicon steel (containing 4% of silicon).

(iv) Ferrites

These are alloys of iron oxide with other materials. One form of ferrites has high permeability, low hysteresis and high resistivity. It is used for receiver aerial in transistor receivers.

4.0 Conclusions

In diamagnetism the magnetization is in the opposite direction to that of the applied field, i.e the susceptibility is negative. Although all sustances are diamagnetic, it is a weak form of magnetism and may be masked by other, stronger, forms. It results from changes induced in the orbits of electrons in the atoms of a substance by the applied field, the direction of change (in accordance with Lenz's law) opposing the applied flux.

In paramagnetism the atoms or molecules of the substance have net orbital or spin magnetic moments that are capable of being aligned in the direction of the applied field. They therefore have a positive (but small) susceptibility and a relative permeability slightly in excess of one.

In ferromagnetic sustances, within a certain temperature range, there are net atomic magnetic moments, which line up in such a way that magnetization persists after the removal of the applied field. Below the curic temperature an increasing magnetic field appled to a ferromagnetic sustanes will cause increasing magnetization to a high value, called the saturation magnetization. The existence of domains is crucial for ferromagnetic behaviour. In a very strong field all the domain are lined up in the direction of the field and provide the high observed magnetization.

The shape and size of the hysteresis loop depends on the nature of the material of the specimen; the loop is narrow for soft iron and wider for steel. The area of the loop represents the energy expended in the material in going the cycle; this energy appears in the form of heat in the specimen.

A simple but effective way of demagnetizing a magentic matrial is to insert it in a multi-turn coil carrying alternating current and then either to reduce the current to zero or to withdraw the specimen from the coil. In both cases, the material is taken through a series of ever-diminishing hysteresis loops.

5.0 Summary

* The magnetization $\underline{M}$ at a point within a material is the magentic moment per unit volume defined by

$$\underline{M} = \frac{\sum_{K=1}^{n} \underline{m}k}{dv}$$

Where $\underline{m}_1$, $\underline{m}_2$, ……………, m_n are the magnetic moments of the bound current loops within a macroscopic volume element dV that includes the point P.

* If b is the average value of the magnetic field throughout dv, the magentic intensity $\underline{H}$ at point P is the quatity

$$\underline{H} = \frac{\underline{B}}{\mu_o} - \underline{M}$$

* Ampere's line integral law for $\underline{H}$ is

$$\oint_c Hsds = I_f$$

where I_f is the free current passing through the closed curve C.

* The magnetic susceptibility x of a material is defined by

$$\underline{M} = x\,\underline{H}$$

and its permeability μ by

$$\underline{B} = \mu\underline{H}$$

The relative permeability is $\mu r = \mu/\mu o = 1 + x$

* Magntic materials are classified as follows:

Diamagentic: μr slightly less than 1
Paramagnetic: μr slightly greater than 1
Ferromagentic: μr much greater than 1 and not constant.

The B-H relationship is given by a magentization curve which is not linear and by a hysteresis loop.

6.0 Tutor-Marked Assignments (TMA)

1. Explain the phenomenon of paramagnetism and give the important properties of paramagnetic materials.

2. It is said that diamagnetism is a fundamental atomic property but not ferromagnetism. Explain.

3a. What are magnetic domains in a ferromagntic material

(b) What is the importance of the hysteresis loop of a magnetic material?

(c) Explain how a magnetised material may be demagnetised , making use of hysteresis loop?

4. A toroid with 1500 turns is wound on an iron ring 360mm^2 in cross-sectional area, of 0.75m mean circumference and of 1500 relative permeability. If the windings carry 0.24A, find

   (a) The magnetizing field H

   (b) The magnetic induction B

   (c) The magnetic flux

   (d) The magnetomotive force

$$(\mu_o = 4\,\pi \text{ x } 10^{-7}\text{ Hm}^{-1})$$

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UNIT 13

TERRESTRIAL MAGNETISM

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1.0 Introduction

A magnetic needle, when suspended freely so that it can turn in both a horizontal and a vertical plane, turns until it comes to rest in one definite direction. This suggests that the Earth must have some of the properties of a magnet. The Earth's magnet has a magnetic field which act on suspended magnets. Comared with the field of a bar magnet the Earth's field is very weak. The magnetic needle turns only if it is suspended on a thread; the Earth's field is not strong enough to make the magnet rotate when it is lying on the table, for the magnetic force is not enough to overcome friction.

The magnitude and direction of the Earth's field varies with position over the Earth's surface and it also seems to be changing gradually with time. The pattern of field lines is similar to that which would be given if there was a strong bar magnet at the centre of the Earth.

At present there is no generally accepted theory of the Earth's magnetism but is may be casued by electric currents circulating in its core due to convection currents arising from radioactive heating inside the earth.

2.0 Objectives

After studying this unit, you should be able to:

* describe the earth's magnetic field

* define the magnetic elements
* describe the determination of declination at a place using an earth inductor
* explain the measurement of the horizontal component of the Earth's magnetic flux density using deflection magnetometer.
* describe the variation of the magnetic elements over the earth's surface .
* explain the drawing of magnetic maps.
* discuss the cencept of geomagnetic latitudes
* do simple calculation based on the geometry of the magnetic elements and the use of deflection magnetometer.

3.1 The Magnetic Field of the Earth

3.1.1 Description of the Earth's Field

The pattern of the earth's mangetic field lines is similar to that which would be given if there was a strong bar magnet at the centre of the earth (see Fig. 3.1).

The points of the Earth at which the magnetic field intensity is directed vertically are called magnetic poles. The Earth has two such poles; the norh magentic pole (in the sourthern hemisphere) and the south magnetic pole (in the northern hemisphere).

The striaght line passing through the magnetic poles is called the Earth's magnetic axis. The circumference of the great circle in the plane perpendicular to the magentic axis is called the magnetic equator. The magnetic field intensity at point on the magnetic eqautor is directly horizontally (see fig. 3.2).

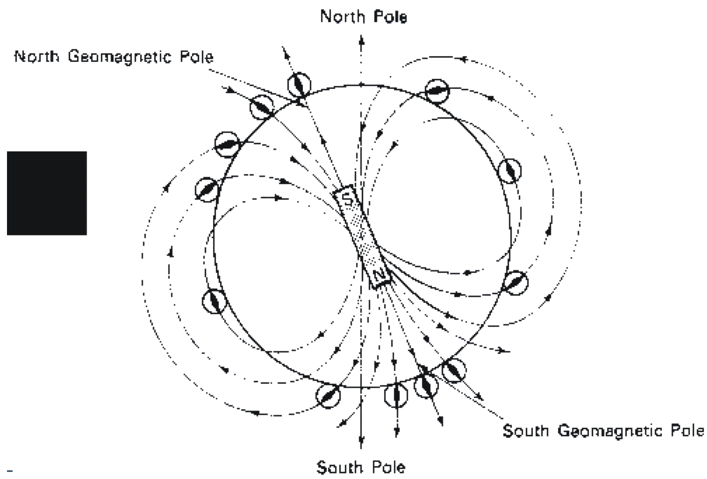


Fig .3.1

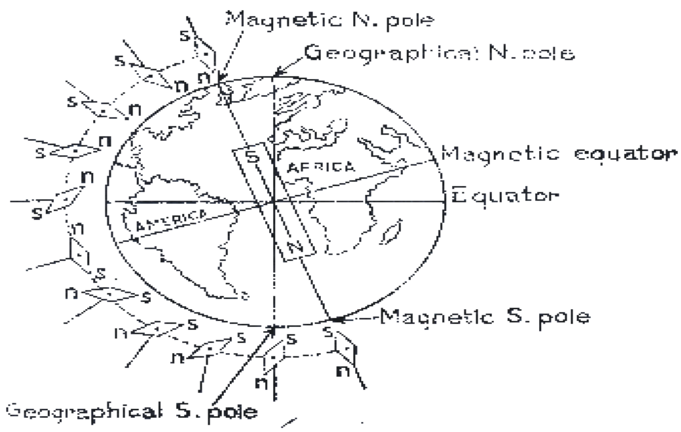


Fig.3.2

3.1.2 Definitions of some Relevant Terms

The following terms are in common use (see Fig. 3.3)

* The geographic meridian is the vertical plane in a direction, geographic N and S, i.e. which passes through the earth's geographic poles.

- The magnetic meridian is the vertical plane in which a magnet sets itself at a particular place.

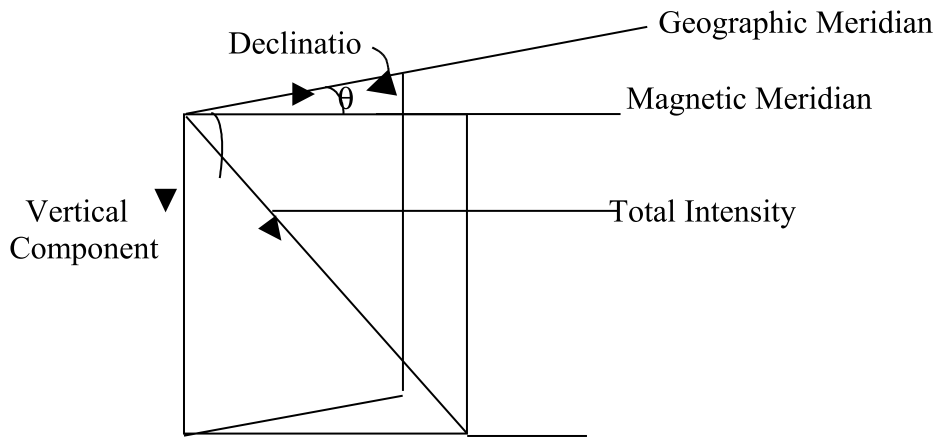


Fig. 3.3

The angle of declination (or variation of the compass) is the angle between the magnetic and geographic meridians.

The angle of dip (or inclination) is the angle between the horizontal and the magnetic axis of a magnet free to swing in the magnetic meridian about a horizontal axis. It is the angle between the direction of the earth's magnetic field and the horizontal.

The magnetic field of the earth often called the total intensity, is resolved for convenience into a horiontal component and a vertical component.

The quantities, declination, dip, total intensity, horizontal component and vertical component, are known as the magnetic elements. The earth's magnetic field at a particular place may be specified by the declination and any two of the magnetic elements.

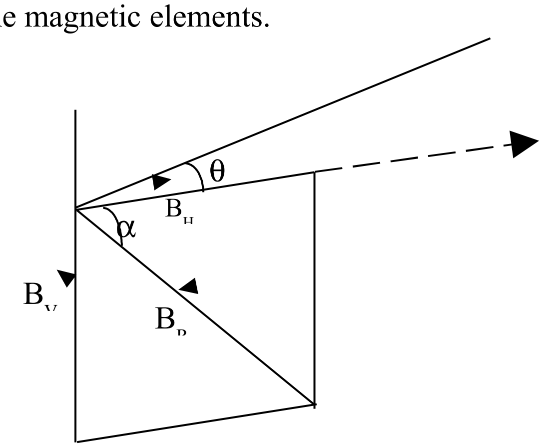


Fig. 3.4

It is convenient to resolve the earth's field strenght ' B_R into horizontal and vertical components, B_H and Bv respectively. We then have from fig. 3.4 that

$B_H = B_R \cos \alpha$ ………………..3.1
and $B_v = B_R \sin \alpha$ …………..3.2
Also $\tan \alpha = B_v/B_H$ ……………….3.3

Where α is the angle of dip and θ is the variation or declination.

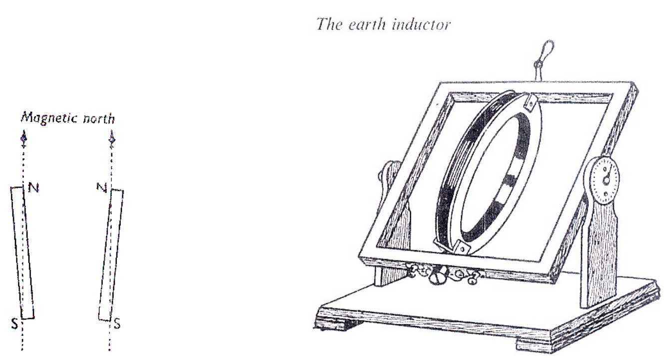
Note: Instruments such as compass needles whose motion is confined to a horizontal plane are affected by B_H only.

3.1.3 Determination of Declination

The determination of the declination at a place involve finding two direction, geographic N and magnetic North.

The former can be found accurately only by an astronomical method – observation of the sun and star. It can be found with fair accuracy from the fact that the shadow of a vertical slide cast by the sun at mid-day is due to N.

Magnetic North is found by suspending a bar magnet freely on a vertical axis. Since the magnetic axis of the magnet may not coincide with its geometric axis, the magnet must be turned over and the mean of the two directions found (see fig. 3.5a).



(a) (b)

Fig 3.5

The horizontals and vertical omponents of the earth's magnetic flux density can be measured by the earth inductor (fig 3.5b). The instrument consists of a coil of wire which can be rotated about an axis capable of being set in any direction by turning a movable frame. The principle of the instrument is that the coil is turned through a right angle between positions when it is threaded by maximum magnetic flux and zero magnetic flux, and the quantity of electricity, induced is measured by a ballistic galvanometer connected in series with the coil.

The quality of electricity induced is given by

$$Q = - N\phi/R$$

where N is the number of turns in the coil, ϕ is the change in the magnetic flux threading it and R is the total resistance of the coil and ballistic galvanometr circuit.

Suppose that the coil is perpendicular to a field of magnetic flux density B and is then turned through a right angle so that no magnetic flux thread it.

Then, $\phi = BA$

where A is the area of the coil.

$$\therefore B = -\frac{QR}{NA} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

To determine the horizontal component of the earth's magnetic flux density, the frame is made vertical and the whole instrument set magnetic E and W. In this position, the coil is perpendicular to the earth's magnetic field and the maximum horinzontal mangetic flux threads it The coil is turned through 90^0 (about its vertical axis) so that its plane then lies in the mangnetic meridian and none of the earth's magneic flux thread it.

To determine the vertical component of the earth's magnetic the field frame is turned so that the coil can move about a horizontal axis in the magnetic meridian. When the coil is rotate through 90^0, say from a vertical to a horizontal position, the quatity of electricity induced is proportional to the vertical component.

3.1.4 Determination of Dip

The angle of dip, D, can be calculated from the values of the horizontal and vertical components using Eq. 3.3

$$\text{Tan } D = \frac{\text{vertical component}}{\text{horizontal component}} = \frac{\theta_v}{\theta_h} \quad \ldots\ldots 3.5$$

Where θ_v and θ_h are the throws of the ballistic galvanometer in the earth inductor experiment.

3.2 The Deflection Magnetometer

Two magntic fields can be compared by means of a deflection magnetometer, which consists of a small magnet, pivoted on a vertical axis and carrying a light pointer which can move over a circular scale.

Normally one of the fields is the earth's horizontal component and the other field is arranged to be at right angles to this. The pivoted magent sets itself along the resultant of the two fields at an angle θ to its direction when it is in the earth's field alone. If B_H is the magnetic flux density of the earth's horizontal component and B is the magnetic flux density of the other field.

$$B = B_H \tan\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots..3.6$$

As shown in fig. 3.6

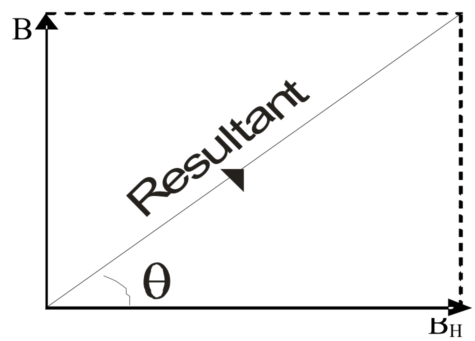


Fig 3.6

The horizontal component of the earth'e magnetic flux density can be measured using a deflection magnetometer.

The magnetic flux density at the centre of a circular coil of known radius and known number of turns, when a measured current is passing through the coil can be calculated. This can be compared by means of a deflection magnetometer, with the horizontal component of the earth's magnetic flux density, enabling the latter to be determined.

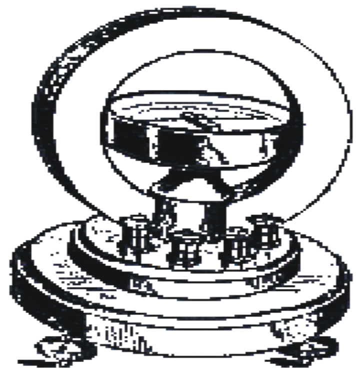


Fig. 3.7

A convinient instrument for the purpose is a tangent galvometer, which consits of a circular coil, at the centre of which there is a deflection magnetometer (fig. 3.7). If N is the number of turns in the coil, a its radius, I the current, and B the magntic flux density at the centre of the coil,

$$B = \frac{\mu_o NI}{2a}$$

Using the same nomenclature as above

$$B = B_H \tan\theta$$

$$B_H = \frac{\mu_o NI}{2a \tan \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

Example

A tangent galvanometer has a coil of 2 turn of mean radius 7.5cm, which is set with its plane in the magnetic meridian. A current is passed through the coil and produces a deflection of a magnet, pivoted at the centre of the coil, of 45^0. Calculate the current if the horizontal component of the earth's flux density is 1.8×10^{-5} Wb m^{-2}.

Solution

Let I = the current in the coil
The magnetic flax density at the centre of coil is given by

$$B = \frac{\mu_o NI}{2a}$$

$$= \frac{2 I \mu_o}{2 \times 7.5 \times 10^{-2}}$$

Magnetic flux density due the earth = 1.8×10^{-5}

Using eqn. 3.6

$$B = B_H \tan\theta$$

$$\therefore \frac{2 I \mu_o}{2 \times 7.5 \times 10^{-2}} = 1.8 \times 10^{-5} \tan 45^0$$

$$I = 1.07 \text{ A}$$

3.3 Variation of Dip Over the Earth's Surface

The angle of dip is 0^0 approximately at the geomagnetic equator. It increases steadily Northward or Southward, until it becomes 90° at the magnetic pole. In the northern hemisphere, the N pole dips, and in the southern hemisphere, the S pole dips. Fig. 3.8 is a map showing isoclinic lines, which are lines joining place at which the angle of dip is the same.

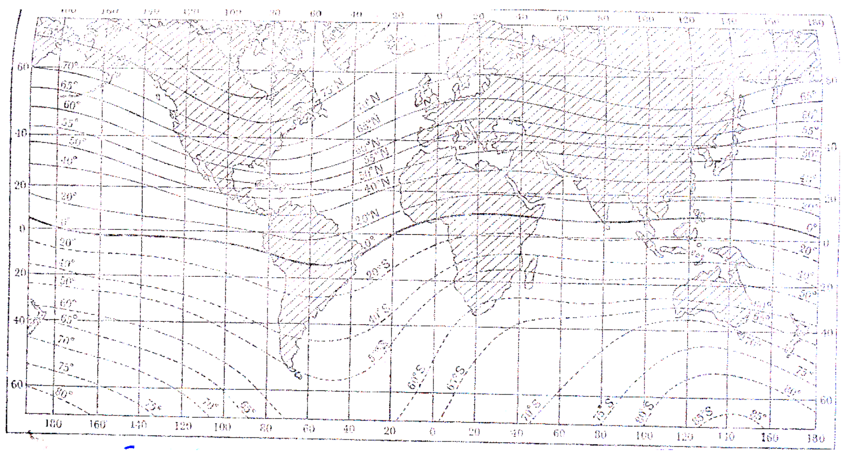


Fig. 3.8

3.4 Changes in the values of the Magnetic Elements

The magnetic field of the earth at any place is not constant but is subject to changes which may be classified as follows:

(i) Secular Change

The magnetic elements undergo a gradual cycle of changes which extent over a long interval after which they return to their original values. These changes are relatively large and take place steadily.

(ii) Annual Change

Such changes are periodic and the value of an element varies gradually between a maximum value if the declination at a place attains a maximum value in February and a minimum value in the course of a year. As an example and the minimum in August every year, then it is an annual change.

(iii) Daily Change

A periodic change extending over 24 hours in the value of an element is also noticed. An element reaches the maximum value at some hour of the day and the minimum value at some other hour, characteristic of the element.

(iv) Magnetic Storm

It has been found that during volcanic eruptions, display of Aurora Borealis, appearance of sunspots, etc, sudden and violent changes occur in the indications of recording instruments measuring the magnetic elements. These changes are said to be due to magnetic storms. They are not periodic.

3.5 Magnetic Maps

The values of the magnetic elements at different places are not usually same, and magnetic maps have been drawn by joining those places on the geographical maps in which a magnetic element has equal values. In magnetic maps, we have the following lines.

(i) Isogonic and Agonic Line –

Isogonic lines are lines joining places on the map of the earth where the declination is the same. Agonic lines are those which pass through places having zero declination

(ii) Isoclinic and Aclinic Lines –

Isoclinic lines (fig. 3.8) are lines joining places on the map of the earth where the magnetic dip is the same. A line passing through places having no dip is called aclinic line.

(iii) Isodynamic Lines

These lines join up places on the map of the earth where the value of horizontal intensity is the same.

The belt round the earth's surface passing through places of no dip is the magnetic equator. The portion of the earth's surface included between the magnetic pole and the magnetic equator has been divided into 90 equal parts. Through each such point of division a circle has been drawn round the earth's surface parallel to the great circle of the magnetic equator. These circle are known as the geomagnetic latitudes.

4.0 Conclusions

If a bar magnets is suspended at any point on the earth's surface so that it can move freely in all planes, the north –seeking end of the magnet (N-pole) will point in the northern direction. The angle between the horizontal direction in which it points and the geographic meridian at that point is called the magnetic declination. This is taken to be positive to the east of the geographic north and negative to the west. The needle will not, however, be horizontal except on the magnetic equator. In other positions it will make an angle (I) with the horizontal, called the inclination (or magnetic dip). At the magnetic poles, $I = 90^0$ ($+90°$ at the N-pole, -90^0 at the S-pole) and the needle will be vertical.

The source of the earth's magnetic field and the cause of the variations are not known with any certainty but the source is believed to be associated with dynamo action in the earth's liquid core.

5.0 Summary

* The earth is a great magnet. Each magnetic pole is over a thousand kilometer from the nearer geographic pole.

* At the magnetic equator a dipping compass needle is horizontal and at the magnetic poles it is vertical.

* The variation of the compass from a north-south direction is called the declination.

* The declination at any location changes slowly from year to year smaller changes occur from hour to hour.

* The quantities, declination, dip, total intensity, horizontal and vertical components, are known as the magnetic elements.

* The earth's magnetic field at a particular place may be specified by the declination and any two of the other magnetic elements.

* There is a slow unpredictable change in the local values of the magnetic elements called the secular magnetic variations.

* The earth's magnetic field varies irregularly, hour by hour, during the day, the changes in declination being a few hundredths of a degree. These fluctuations are usually large during magnetic storms.

6.0 Tutor Marked Assignments (TMA)

1. A circular coil having 50 turns of mean radius 8.0 cm is set with its coil in the magnetic meridian and a smaller magnet is pivoted at its centre. When a current is passed through the coil it is found that the coil must be rotated through 40^0 before the magnet is once again in the plane of the coil. Calculate the current in the coil if the horizontal component of the earth's magnetic flux density is $1\text{-}8x10^{-5}$ Wb m^{-2}.

2. A compass needle of electro-magnetic moment 2.0×10^{-6} A m^2 is held at right angles to the earth's horizontal magnetic field of flux density 1.8×10^{-5} Wb m^{-2}. Calculate the torque exerted by the field on the needle.

   If the needle is 1.5cm long, calculate the force in newtons on each pole.

3. A circular coil of 50 turns of mean radius 10cm stands in the magnetic meridian, and a small magnetic needle at its centre is deflected through 60^0. Assume that the horizontal component of the earth's flux density to be 2.0×10^{-5} Wb m^{-2}, calculate the current in the coil.

4 (a) What are the magnetic elements of a place?

  (b) Show how from a knowledge of the horizontal component of the earth's field and the dip, the total intensity of the earth's magnetic field at any place is determined.

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UNIT 14

ELECTROMAGNETIC INDUCTION I

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2.0 Objectives
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1.0 Introduction

We now know that an electric current creates a magnetic field. The reverse effect of productinng electricity by magentism was discovered independently in 1831 by Faraday in England and Henry in America and is called electromagnetic induction. Indeed, the development of electrical enginering as we know it today began with Faraday and Henry who independently discovered the principles of induced e.m.f.s and the methods by which mechanical energy can be converted directly to electrical energy. You would realise that out present day large scale production and distribution of electrical energy would not be economically fensible if the only sources of e.m.f. available were those of a chemical nature, such as dry cells.

2.0 Objectives

After stduying this unit you will be ble to

* understand that in a region where the magentic field is changing, there is an electric field which is related to the time rate of change of the magnetic field.

* state the faraday's laws of electromagnetic induction

* know that induced e.m.f.s may be generated in a coil or a circuit by changing the magnetic field (the transformaer effect) or by relative movement (the dynamo effect).

* explain the principle of the design and operation of a.c. and d.c. generators.

3.1 The Electric Field Accompanying a time –Varying Magnetic Field.

In a region where a magnetic field is changing, there is an electric field which is related to the time rate of change of the magnetic field.

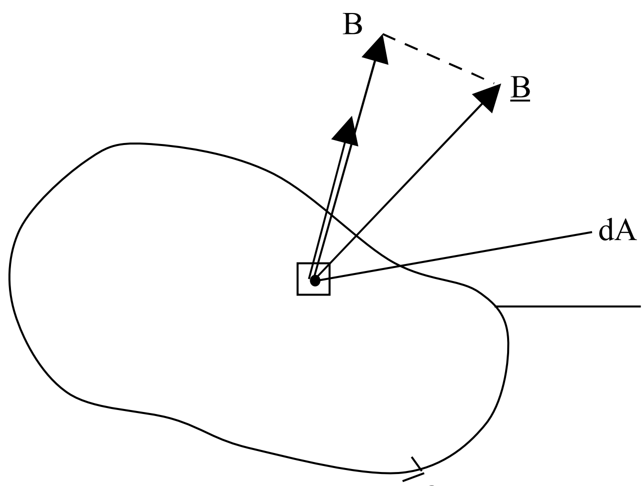


Fig 3.1

The fundamental relationship between the electric field E and the time rate of change ∂B/∂t of the magntic field which is deduced from the results of a number of experiments in Faaday's law

$$\oint_c E_s \, ds = - \int_s \frac{\partial B_N}{\partial +} dA \text{ ………………3.1}$$

where S is any (open) surface boounded by an arbitrary closed curve C. B_N is the component of B along the normal n to S, where the direction of n is related to the positive direction for the path of integration C as shown in fig. 3.1 .

Note:
The magnetic field B is a function of position (x_1 y_1 z) and time t: that is, B = B (x_1 y_1 z, t). The time rate of change of B at a given point (x_1 y_1 z) in space is the partial derivative ∂B /∂t which is the derivative with respect to t when the values of the position coodinates are held constant

We observe that Faraday's law implies that the fundamental relationship of electrostatics,

$$\oint_c Es\ ds = 0 \text{ ……………………3.2}$$

is true if and only if B is not time – dependent.

The surface integral in Faraday's law can be interpreted in terms of the magentic flux enclosed by C:

$$\phi = \int_s B_N\ dA \text{ …………………………3.3}$$

The rate of charge of this flux associated with time variations of B is

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial t} \int_s B_N\ dA = \int_s \frac{\partial B_N}{\partial t}\ dA \text{ ……………..3.4}$$

Therefore, Faraday's law can be written as

$$\phi_c\ Es\ ds = -\frac{\partial \phi}{\partial t} \text{ ………………..3.5}$$

We shall now discuss the direct experimental evidence for Faraday's law and its practical applications.

Example

A single turn circular coil is placed at the location of path C in figure 3.1. The coil encloses an area of $0.40m^2$. The average value of the magnetic field within the coil increses at a uniform rate from 0.10 T to 0.30T in 5.0×10^{-2}s. Find the line integral of E about the path C.

Solution

Taking conunterclockwise direction as the positive direction for the path C, and using the right hand rule, we find that the direction of n in figure 3.1 is out of the page. B_N and the magnetic flux ϕ enclosed by C are consequently negative. Initially

$$\phi = 0.10T \times 0.40m^2 = -4.0 \times 10^{-2} \text{ Wb}$$

and finally, $\phi = -0.30T \times 0.40m^2 = -12.0 \times 10^{-2}$ Wb

therefore, $\frac{\partial \phi}{\partial t} = \frac{-8.0 \times 10^{-2} \text{ Wb}}{5.0 \times 10^{-2} \text{ s}} = -1.6$ wb/s

Faraday's law gives

$$\oint_c E_s \, ds = \frac{-\partial \phi}{\partial t} = 1.6 \text{ V}$$

3.2 Induced e.m.f.s and Currents.

Induced e.m.f.s can be generated in two ways.

(a) By relative movement (the dynamo effect)

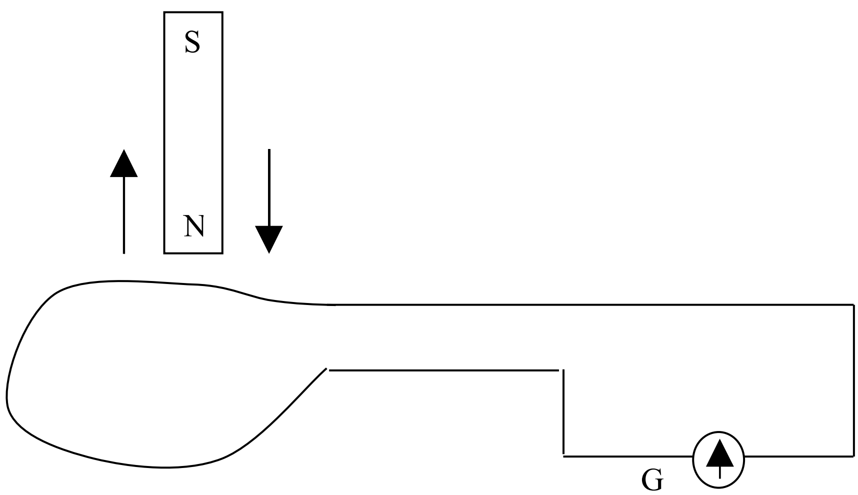


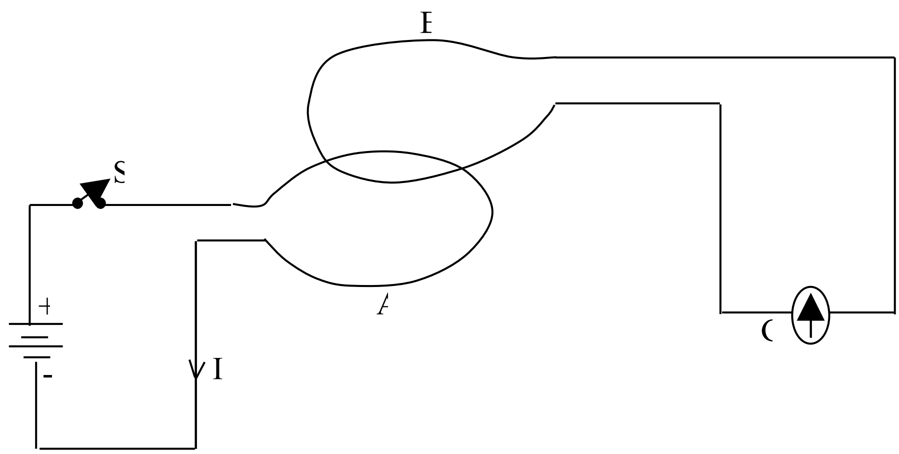
Fig 3.2

If a bar magnet is moved into and out of a stationary coil of wire connectd to a centre – zero galvanometer, fig. 3.1, a small current is recorded during the motion but not at other times. Movement of the coil towards or away from the stationary magnet has the same results. Relative motion between the coil and magnet is necessary.

Observation shows that the direction of the induced current depends on the direction of relative motion. Also the magnitude of the current inceases with the speed of motion, the number of turns on the coil and the strenght of the magnet.

You should note that although it is current we detect in this demonstration, an e.m.f must be induced in the coil to cause the current. The induced e.m.f is the more basic quantity and is always present even when the coil is not in a complete circuit. The value of the induced current depends on the resistance of the circuit as well as on the induced e.m.f.

(b) By Changing a Magnetic Field (the transformer effect)



In this case two coils are arranged near each other as shown in fig. 3.3. It is observed that a transient current is induced if:

1. The steady current in the adjacent circuit is turned on or off.

2. The adjacent circuit with a steady current is moved relative to the first circuit.

   It is worth noting that the induced current is in one direction when the current in coil A increase and in the opposite direction direction when it decreases.

   Cases of electromagentic induction in which current changes in one circuit cause induced e.m.f in a neighbouring circuit, not connected to the first, are examples of mutual induction – the transformer principle.

3.2.1 Statement of Faraday's Law

The induced e.m.f is directly proportional to the rate of change of flux linkage.

In calculus notation it can be written as

$$E \propto \frac{d}{dt}(N\phi)$$

$$\text{or } E = \text{constant} \times \frac{d}{dt}(N\phi) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

where E is the induced e.m.f and $d(N\phi)/dt$ is the rate of change of flux linkage . The law is found to be true for the dynamo and the transformer types of induction.

There are further experimental investigations that are concerned with properties of the coil itself which affect the e.m.f. induced in it. They will help you further with the understanding of Faraday's law.

(i) Number of turns, N

(ii) Area, A

(iii) Orientation θ

☐ **Example:** suppose a single –turn coil of cross-sectional area 5.0 cm^2 is at right angles to a flux density of 2.0×10^{-2}T, which is then reduced steadily to zero in 10s.

The flux-linkage change d (N ϕ) = (number of turns) x (change in B) x (area of coil)

$$= (1) \times (2.0 \times 10^{-2}) \times (5.0 \times 10^{-4}\ m^2)$$
$$= 1.0 \times 10^{-5}\ Wb$$

The change occurs in time dt = 10s, hence the e.m.f , E induced in the coil is given by

$$E = \frac{d}{dt}(N\phi) = \frac{1.0 \times 10^{-5}}{10}\ \frac{Wb}{s}$$
$$= 1.0 \times 10^{-6}\ V$$

If the coil had 5000 turns, the flux linkage would be 5000 times as great (i.e $Nd\phi = 5.0 \times 10^{-2}$ Wb), hence

$$E = 5000 \times 10^{-6} = 5.0 \times 10^{-3}\ V$$

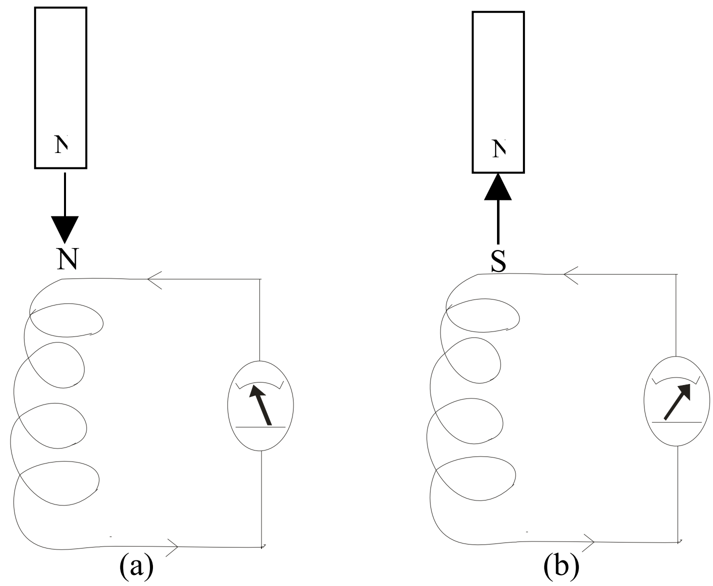
If the normal to the plane of this coil made an angle of 60^0 (instead of 0^0) with the field then $N\,d\phi = 5.0 \times 10^{-2} \cos 60°$

$$= 2.5 \times 10^{-2}\ Wb$$
$$\text{and}\quad E = 2.5 \times 10^{-3}\ V$$

3.2.2 Lenz's Law

While the magnitude of the induced e.m.f. is given by Faraday's law, its direction can be detrmined by a law due to the Russian Scientist Lenz. It may be stated as follows.

The direction of the induced e.m.f. is such that it tends to oppose the flux-change causing it, ans does oppose it if induced current flows.



Thus in fig. 3.4a a bar magnet is shown approaching the end of a coil, north pole first. If Lenz's law applies, the induced current should flow in a direction which makes the coil behave like a magnet with a north pole at the top. The downward motion of the magnet and the acompaying flux linkage will then be opposed. When the magnet is withdrawn, the top of the coil should behave like a south pole, fig. 3.4b, and attract the north pole of the magnet, so hindering its removal and again opposing the flux-change. The induced current is therefore in the opposite direction to that when the magnet approaches.

For straight conductors moving at right angles to a magnetic field a more useful version of Lenz's law is Fleming's right-hand rule (also called the dynamo rule; his left-hand rule is often refrred to as the motor rule).

Fleming's right – hand rule

It state that if the thumb and the first two fingers of the right hand are held so that each is at right angles to the other with the First finger pointing in the direction of the Field and the thuMb in the direction of Motion of the conductor, then the seCond finger indicates the direction (conventional) of the induced current.

Lenz's law is incorporated in the mathematical expression of Faraday law by including a negative sign to show that Current due to the induced e.m.f produces an opposing flux-change, thus we write:

$$E = \frac{d}{dt}(N\phi)$$

Example

In figure 3.6, parallel conducting tracks ab and cd are joined at one end by a conductor ac and are located in a constant uniform magnetic field perpendicular to the plane of the tracks. A conducting bar ef moves at a constant speed v. Find the e.m.f. induced in the circuit eface.

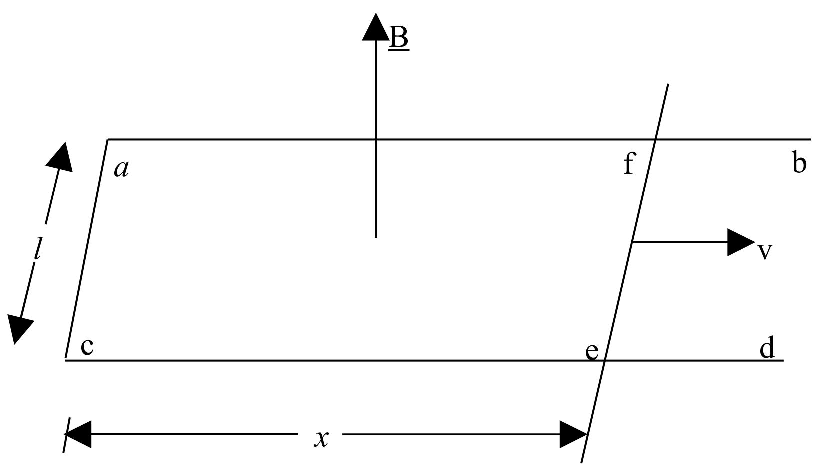


Fig. 3.6

Solution

The flux enclosed by the circuit is

$$\phi = B\,l\,x$$

The speed of the moving conductor is

$$V = \frac{dx}{dt}$$

As a result of this motion the enclosed flux changes at the rate

$$\frac{d\phi}{dt} = BlV$$

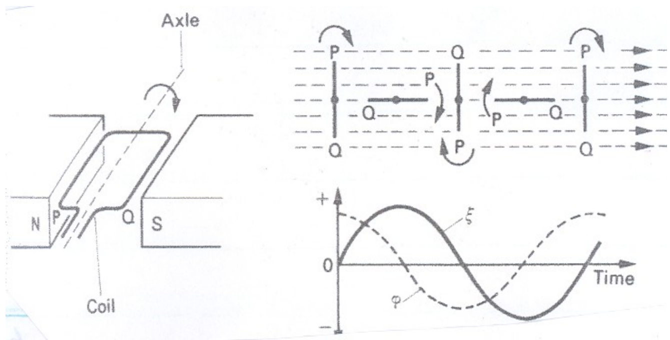
From Farady's flux rule, the magnitude of the e.m.f is

$$E = BlV$$

Since the upward flux through the circuit is inceasing, Lenz's law implies that the induced e.m.f is clockwise looking down on the circuit.

3.3 Generators (a.c and d.c)

A generator or dynamo produces electrical energy by electromagnetic induction. In principle, it consists of a coil which is rotated between the poles of a magnet so that the flux-linkage changes (see fig. 3.7).



(a) (b)

Fig 3.7

The flux linking each turn of a coil of area A having N turns, rotating with angular velocity ω is a uniform flux density B, is given at a time t (measured from the vertical position) by

$$\phi = BA \cos wt$$

By Faraday's law, the induced e.m.f E is

$$E = \frac{d}{dt}(N\phi) = BAN\,w \sin wt$$

Both ϕ and E alternate sinnusiodally and their variation with the position of the coil is shown in fig. 3.7b.

We see that although ϕ is a maximum when the coil is vertical (i.e. perpendicular to the field), E is zero because the rate of change of ϕ is zero at that instant, i.e the tangent to the ϕ-graph is parallel to the time axis and so has zero gradient.

The expression for E shows that its instantaneous values increse with B, A, N and the angular velocity of the coil. If the coil makes one complete revolution, one cycle of alternating e.m.f is generated, i.e for a simple, single-coil generator the frequency of the supply equals the number of revolutions per second of the coil.

In an a.c. generator (or alternator) the alternating e.m.f is taken off and applied to the external circuit by two spring-loaded graphite blocks (called 'brushes') which press against two copper slip-rings. These rotate with the axle, are insulated from one another and each is connected to one end of the coil (fig. 3.8)

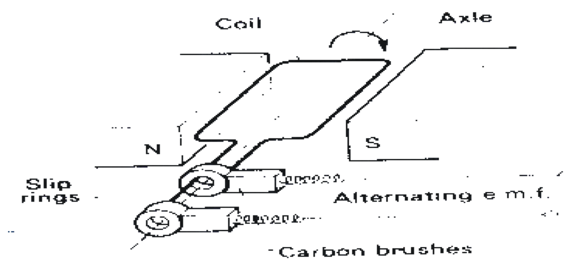
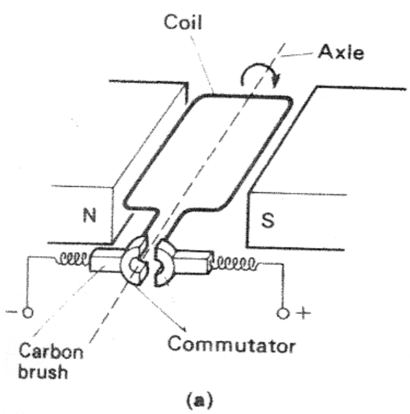


Fig 3.8

In a d.c generator a commutator is used instead of slip rings. This consists of a split-ring of copper, the two halves of which are insulated from each other and joined to the ends of the coil (see fig, 3.9a). The brushes are arranged so that the change –over of contact from one split ring to the other occurs when the coil is vertical. In this position the e.m.f induced in the coil reverses and so one brush is always positive and the other negative.



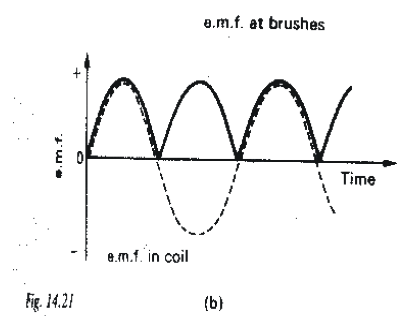


Fig 3.9 a **Fig 3.9 b**

The graphs of fig. 3.9b show that e.m.f.s in the coil and at the brushes; the latter, although varying, is undirectional and produces d.c. in an external circuit. However, the use of many coils and a correspondingly greater number of commutator segments gives a much steadier e.m.f

4.0 Conclusion

As a follow-up to our previous studies of the laws of electricity and magnetism for time-independent fields, we now understand some new basic physical laws which express the interconnection between time-varying electric and magnetic field.

To induce an e.m.f. in a wire loop, part of the loop must move through a magnetic field or the entire loop must pass into or out of the magnetic field. No e m.f. is induced if the loop is static or the magnetic field is constant. The magnitude of the induced e.m.f and current depends on how the loop is oriented to the magnetic field.

The a.c. generator (alternator) and the d.c. generator are devices which convert mechanical energy into electrical energy. The principle of their operation is electromagnetic induction.

5.0 Summary

* The e.m.f E induced in a circuit that encloses a flux ϕ given by the Faraday flux rule

$$E = -\frac{d\phi}{dt}$$

where the changes of the enclosed flux may be due to motion of the circuit as well as a time variation of the magnetic field.

* According to Faraday's law of electromagnetic induction, whenever there is a change in the magnetic flux linking a coil or a circuit, and e.m.f (and hence a current) is induced in the circuit, it lasts only so long as the change is taking place. The magnitude of the induce e.m.f. is to the rate of change of magnetic flux linked with the circuit.

* Faraday's law tells us nothing about the direction of the induced e.m.f. and the current set up in the coil or the circuit in which the change in magnetic flux is brought about. This is given by Lenz's law which states that the direction of the induced e.m.f and current set up in a coil or a circuit due to a change in the magentic flux linked with it is such that it opposes the very cause to which it is due.

6.0 Tutor Marked Assignment

1. A coil of 500 turns and area $10cm^2$ is placed with its plane perpendicular to a magnetic field of 2×10^{-3} T. If the field be uniformly reduced to zero in 10^{-2}s, what will be the e.m.f induced in

the coil? If the resistance of the coil is 50 ohm, calculate the values of the current and the charge induced in the coil.

2. Find the magnitude of the induced e.m.f. in a 200-turn coil with cross-sectional area of $0.16m^2$ if the magnetic field through the coil changes from 0.10T to 0.50T at a auniform rate over a period of 0.02s.

3.(a) Under what circumstances is an e.m.f induced in a conductor? What factors govern the magnitude and direction of the induced e.m.f.?

(b) A straight wire of length 50cm and resistance 10ohm moves sideways with velocity of $15ms^{-1}$ at right angles to a uniform magnetic field of flux density 2.0×10^{-3}T. What current would flow if it ends were connected by leads of negligible resistance?

7.0 Reference and Other Resources

University Physics for Science and Engineering. Donald E. Tilley. Cummings Publishing Company . Inc. London.

Physics for Class XI. D.S Mathur. Sultan Chand & Sons Publishers. New Delhi. India.

Modern University Physics. J. A. Richards, F. W. Sear, M.R. Wehr and M.W. Zemansky. Addison-Wesley Publishing co. Inc. London.

UNIT 15

ELECTROMAGNETIC INDUCTION 11

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1.0 Introduction

This Unit is a continuation of our study of electromagnetic induction. The matual inductance of two circuit, and the self-inductance of a single circuit, are quantities that are introduced so that an induced e.m.f can be related directly to a changing current, rather than to a changing flux. We investigate the relationship between the energy stored in a current – carrying circuit and the circuit self-inductance, as well as the influence of self-inductance on the transient behaviour of resistive circuits.

We shall also discuss in this unit a particularly useful application of electromagnetic induction, the transformer. It is a device used to convert an alternating current at a low voltage into one at a high voltage or vice versa.

2.0 Objectives

After studying this unit you will be able to

* explain the principles of the operation of a transformer

* define the mutual inductance of two coupled circuits such as the primary and secondary circuits of a transformer

* describe the energy losses in a transformer

* define the self-inductance of a coil

* calculate the time constant of an R-L circuit

* describe the transients in R-L circuits.

3.1 Transformers

E.m.f.s associated with a changing magnetic flux are used to transform a varying voltage in one circuit into a large or smaller voltage in another circuit. This is accomplished using a transformer consisting of two coils electrically insulated from each other and wound on the same ferromagnetic core (see fig. 3.1)

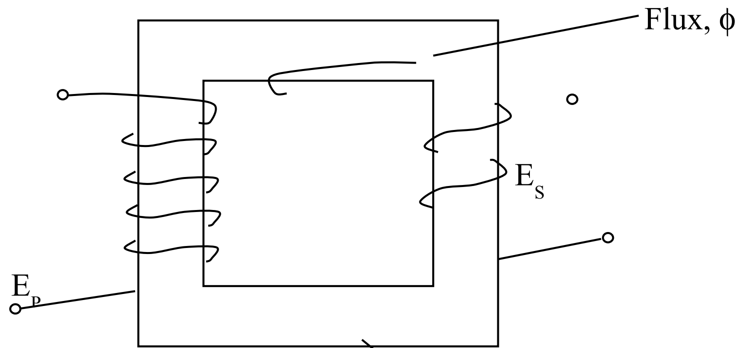


Fig. 3.1

Power is supplied to one coil named the primary coil. A varying curent in this coil sets up a varying magnetic flux, largely confined to the ferromagnetic core. The other coil, called the secondary coil, thus enloses a varying flux. While this flux is changing at the rate $d\phi/dt$, each of the Ns turns in the secondary coil experiences, according to Faraday's flux rule, an induced e.m.f. equal to $d\phi/dt$, and the e.m.f for the entire secondary coil is

$$E_s = - N_s \frac{d\phi}{dt} \qquad \text{.................................3.1}$$

We neglect the small amount of leakage flux and assume that the magnetic flux at any instant is the same through the primary and secondary coils. Then there is an e.m.f. $d\phi/dt$ induced in each of the N_P turns of the primary coil and the total e.m.f. for this coil is

$$E_P = -N_P \frac{d\phi}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

Dividing Eq. 3.1 by Eq. 3.2 we obtain

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

According to this result, when there are varying currents, the ratio of the e.m.f. induced in the secondary and primary coils at any instant is equal to the "turn ratio", Ns/Np.

When Ns is greater than Np, the secondary e.m.f exceeds the primary e.m.f and the transfermer is called a step-up transformer. In a step-down transformer, Ns is less than Np.

It is desrable to transmit electrical power at high voltages and small currents in order to minimize the I^2R poer dissipation in the transmission line. But this power must be generated and ultimately delivered at relatively low voltages to avoid problems of insulation and safety. A very useful feature of alternating current is the fact that the voltage and the current can be changed with ease and efficiency by the use of transformers.

□ **What happens if we apply a d.c. across the primary terminals of a transformer?**

No e.m.f is induced in the secondary. It should be emphasized that only varying voltage (a.c or pulses) should be applied across the primary terminals of a transformer. A steady d.c. voltage established across the primary coil does not produc a changing magnetic flux. Moreover, since no opposing self-induced e.mf. (section 3.3) is induced in the primary, the primary current will be limited only by the low primary coil resistance. The result is that the primary coil overheats and burns out.

3.1.1 Energy Losses in a Transformer

Although transformers are very efficient devices, small energy losses do occur in them due to four main causes.

(i) Resistance of windings. The copper wire used for the windings has resistance and so ordinary (I^2R) heat losses occur. In high-current, low p.d. windings these are minimized by using thick wire.

(ii) Eddy currents. The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

(iii) Hysteresis. The magnetization of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material (such as mametal) which has a low hysteresis loss.

(iv) Flux leakage. The flux due to the primary may not all link the secondary if the core is badly designed or has air gaps in it.

3.2 Mutual Inductance

To faulitate the analysis of couple circuits such as the primary and secondary circuits of a transformer, we should know the relationship between the changing current in one circuit and the e.m.f that this induces in the other circuit.

Suppose a current Ip in the primary circuit produces a flux ϕ_s through each turn of the secondary coil. The mutual inductance M of the primary and secondary circuits is defined by

$$Ns\, \phi_s = M\, Ip \quad \text{.........................3.4}$$

If there are no ferromagnetic materials present, ϕ_s is proportional to Ip and m is then a constant, with a value that depends only on the geometry of the circuits. For a transformer with a ferromagnetic core, M depends also on the permeability of the core which is a function of the magetic field B established within the core.

The SI unit of mutual inductance is the weber per ampere (Wb. A^{-1}), which is called the henry (H).

$$IH = I \text{ Wb } A^{-1}$$

A current I_s in the secondary will create a flux ϕ_p enclosed by each turn of the primary. Using an eqaution analogous to Eq. 3.4, we define a mutual inductance M' by

$$N_p\, \phi_p = M\, I_s \quad \text{..........................3.5}$$

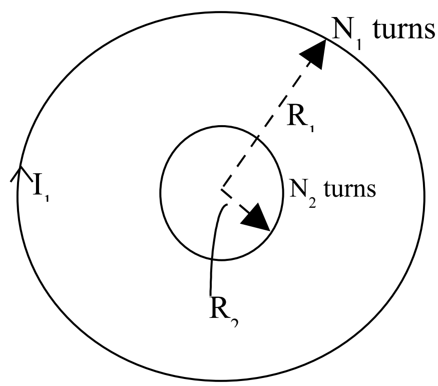
In general M = M, so that there is a single mutual inductance of any two circuits.

When M is constant, $N_p\, d\phi_p/dt = M\, dI_s/dt$, and Faraday's flux rule gives

$$E_p = -M \frac{dI_s}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

Similarly,
$$E_s = -m \frac{dI_p}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

These two equations accurately express the coupling between any two distinct circuits in the absence of ferromagnetic materials



Example

Find the mutual inductance M of a large circular coil with N_1 turns of radius R_1 and a very small concentric coil with N_2 turns of radius R_2 (see fig. 3.2).

Solution:

The flux ϕ_2 through the small coil produced by a current I_1 in the large coil can be calculated easily, since

$$\phi_2 = B_1 \pi R_2^2$$

where B_1 is the field produced by I_1 at the centre of the first coil.

Now $B_1 = \mu_o N I_1 / 2R_1$

Therefore,
$$M = \frac{N_2 \phi_2}{I_1} = \frac{\pi \mu_o N_1 N_2 R_2^2}{2R_1}$$

We can see that M is a constant, independent of the currents, and is determined by geometrical parameters- the coil radii and number of turns.

3.3 Self-Inductance

When the current through a coil is changing, the flux produced by this current and enclosed by the turns of the coil is also changing,; consequently there is an e.m.f induced in the coil. Such an e.m.f. is called a self-induced e.m.f.

Suppose a coil current I produces a flux ϕ through each of N turns of a coil. The self-inductance L of the coil is defined by the equation

$$N \phi = L I \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

If there are no ferromagnetic materials present, ϕ is proportional to I. Then L is a constant with a value detrmined by the coil geometry. The self-inductance of a given coil can be increased greatly by providing a ferromagentic core, but, because the core's magnetic permeability μ depends on B_1 the self-inductance is then a complicated function of the coil current.

When L is constant, $Nd\phi/dt$, = L dI/dt , and Faraday's flux rule gives

$$E = L \frac{dI}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

Eq. 3.9 gives a useful expression for the self-induced e.m.f in terms of the changing current.

Lenz's law implies that the direction of the self-induced e.m.f is such as to oppose the change in current that gives rise to the change in flux. Therefore, the induced e.m.f is in the direction of the current if the current is decreasing, but in the opposite direction of the current if increasing. After a direction in an electric circuit is selected as the positive direction for both currents and e.m.fs, the minus sign in Eq. 3.9 ensures that E and I have the proper relative directions at all times.

Circuit elements that are specifically designed to have appreciable self-inductance are called inductors.

The unit of inductance is the henry (H), defined as the inductance of a coil (or circuit) in which an em.f. of 1 volt is induced when the current changes at the rate of 1 ampere per second. That is, IH = 1 Vs A^{-1}

3.3.1 Inductance of Solenoid

Let us consider a long, air-cored solenoid of length l, cross-sectional area A having N turns and carrying current I. the flux density B is almost constnat over A and , neglecting the ends, is given by

$$B = \mu_o \frac{N}{l} I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

The flux ϕ through each turn of the solenoid is BA and for the flux-linkage we have

$$N\phi = BAN$$
$$= \left(\frac{\mu_o N I}{l}\right) AN$$
$$= \frac{\mu_o AN^2}{l} \cdot I$$

If the current changes by dI in time dt causing a flux-linkage change d(Nϕ) then by Faraday's law the induced e.m.f is

$$E = \frac{-d}{dt}(N\phi)$$
$$= -\frac{\mu_o AN^2}{l} \cdot \frac{dI}{dt}$$

If L is the inductance of the solenoid, then from the defining equation we have

$$E = -L\frac{dI}{dt}$$

Comparing these two expressions at follows that

$$L = \frac{\mu_o AN^2}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.11$$

showing that L depends only on the geometry of the solenoid.
If N = 400 turns, l = 25cm = 25 x 10^{-2}m, A 50cm^2 = 50 x 10^{-4} m^2 and $\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}, then L = 4.0 x 10^{-3} H = 4.0mH.

A solenoid having a core of magnetic material would have a much greater inductance but the value would vary depending on the current in the solenoid.

3.3.2 Energy Stored by an Inductor

When an e.m.f. E = - L dI/dt, is induced in an inductor by a changing current I, the rate at which energy is supplied by the inductor is EI.. When E is in the direction of I, the inductor supplies energy to the external circuit. And when E and I have opposite directions , energy is supplied to the inductor.

When the current I in an inductor is in the positve direction and is increasing, energy is being transferred to the inductor at a rate

$$\frac{dv}{dt} = \left(L\frac{dI}{dt}\right) I$$

The total energy supplied while the current increase from 0 to I is therefore

$$U = \int_o^I L\ I^1\ dI^1\ \ \tfrac{1}{2} LI^2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.12$$

We conclude that the enrgy stored in the magnetic field of an inductor carrying a current I is

$$U = \tfrac{1}{2} L\, I^2 \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.13$$

Since every current produces a field, every circuit must have some self-inductance. On switching on any circuit some time is necessary to provide the energy in the magnetic field and so no current can be brought instantaneously to a non –zero value. Similary, on switching off any circuit, the energy of the magnetic field must be dissiputed somehow, hence the spark.

3.4. Transients in R – L Circuits

An inductor in which there is an inceasing current is a seat of e.m.f whose direction is opposite to that of the current. As a consequence of this 'back' e.m.f, the current in the inductive circuit will not rise to its final value at the instant when the circuit is closed, but will grow at a rate which depends on the inductance and resistance of the circuit.

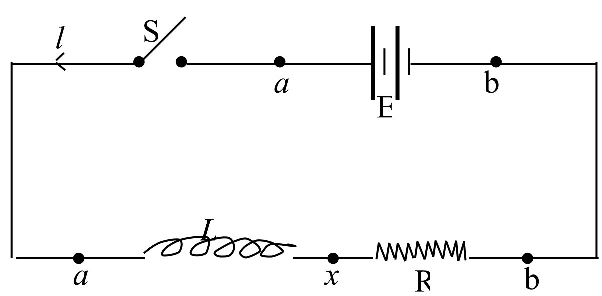


Fig 3.3

Fig. 3.3 shows a series circuit consisting of a 'pure' inductor (an inductor without resistance), a non inductive resistor, a battery of e.m.f and negligible internal resistnace and a switch S. At some instant after the switch is closed, let i represent the current in the circuit and dt/dt its rate of increase. The potential difference across the inductor is

$$E_{ax} = -L \frac{di}{dt}$$

When the switch is closed both *i* and di/dt are counterclockwise in fig. 3.1 and we have

$$\left(E - L \quad \right)$$

$$V_{ax} = \sum Ri - \sum E = 0 = Ri - \frac{di}{dt}$$

$$\text{Or} \quad \frac{E}{R} = \frac{L}{R}\frac{di}{dt} + i$$

$$\text{and} \quad \frac{di}{(E/R - i} = \frac{R}{L}\, dt$$

Since i = 0 when t = 0, we integrate between limits, as follows:

$$\int_o^i \frac{di}{(E/R)-i} = \frac{R}{L}\int_o^t dt$$

$$\text{and get } - \text{In}\, \frac{(E/R) - i}{E/R} = \frac{R}{L}t$$

$$\text{Hence} \quad \frac{E}{R} - i = \frac{E}{R} e^{-Rt/L}$$

$$\text{That is} \quad i = \frac{E}{R} - \frac{E}{R} e^{-Rt/L} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.14$$

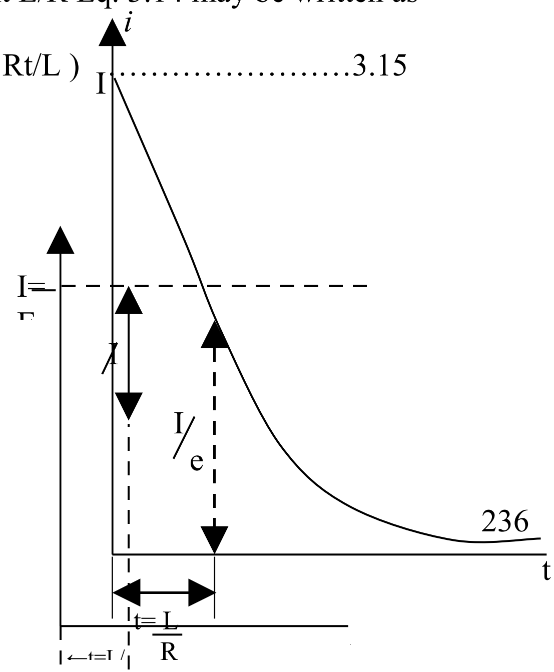
The first term on the right, E/R, is constant, while the second term depends on the time. At time t = 0, the second term equals E/R and the current i is zero. As time goes on, the second term deceases and approaches zero. The current therefore increases and approaches the constant value E/R given by the first term.

The second term is called a transient current and the first term the steady-state current.

You should note that the steady-state current does not depend on the self-inductance and is the same as it would be in a pure resistance R connected to a cell of e.m.f. E.

If I represents the steady state current E/R Eq. 3.14 may be written as

$$i = I \;(1 - e^{-Rt/L}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.15$$



(a) (b)

Fig 3.4

Figure 3.2 is a graph of Eq. 3.15. The instantaneous current i first rises rapidly, then increases more slowly and approached a symptotically the final value I = E/R. the time constant of the circuit is defined as the time at which Rt/L =1, or when

$$t = L/R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.16$$

When $t = L/R$, $i = I\,(1-e^{-1}) = I\,[1 - ($½$.718)] = I\,[1-0.369]$
$= 0.631\, I$
or about 63 percent of I.

For a circuit with a given resistance, the time is longer the larger the inductance, nd vice versa. Thus although the graph of I vs. t has the same general shape whatever the inductance, the current rises rapidly to its final value if L is small, and slowly if L is large.

Example If R = 100 ohms and L = 10 henrys

$$\frac{L}{R} = \frac{10}{100} = 0.1\text{s}$$

and the current increases to about 63% of its value in 0.1s. On the other hand, if L = 0.01 henry

$$\frac{L}{R} = \frac{0.01}{100} = 10^{-4}\text{s}$$

and only 10^{-4}s is required for the current to increase to 63% of its final value.

If there is a steady current I in the circuit of fig. 3.3 and the battery is short-circuited, the decay of the current shown in Fig. 3.4b follows a curve which is the exact inverse of Fig. 3.4a. The equation of the decaying current is

$$i = I e^{-Rt/L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.17$$

and the time constant, L/R, is the time for the current to decrease to 1/e of its original value.

4.0 Conclusion

A transformer is a device which changes an alternating p.d from one value to another of greater or smaller value using the principle of mutual induction.

In mutual induction, current changing in one coil or circuit (the primary) induces an e.m.f. in a neighbouring coil or circuit (the secondary).

The flux due to the current in a coil links that coil and if the current changes the resulting flux change induces an e.m.f. in the coil itself. This changing magnetic field type of electromagnetic induction is called self-induction, the coil is said to have self-inductance or simply inductance (symbol L) and is called an inductor.

5.0 Summary

* The ratio of the e.m.f.s induced in the secondary and primary coils is equal to the transformer turns ratio Ns/Np,

$$\frac{Es}{Ep} = \frac{Ns}{Np}$$

  Provided the leakage flux is negligible.

* The coupling of any two circuits is detrmined by their mutual inductance M defined by

$$Ns\ \phi_s = MI_p$$

  where the curent Ip in coil produces a flux ϕs enclosed by each of the Ns turns of the other coil. When M is constant

$$Ep = -M\frac{dIs}{dt} \text{ and } Es = -M\frac{dIp}{dt}$$

* The self – inductance l of an N-turn coil is defined by

$$N\phi = LI$$

where a current I in the coil produces a flux ϕ through each turn. The self-induced e.m.f. in an inductor with self-inductance L is

$$E = -L \frac{dI}{dt}$$

* The energy stored in the magnetic field of an inductor carrying a current I is $U = \frac{1}{2} L I^2$

* The time constant of an R-L circuit is

$$\tau = \frac{L}{\text{circuit resistance}}$$

* When a constant e.m.f E is applied across an inductor the growth of the current toward its final value I_f is descibed by

$$I = I_f \ (1 - e^{-t}/\tau)$$

Replacement of this constant e.m.f by resistor results in an exponental decay of the current

$$I = Io \ e^{-t/t}$$

6.0 Tutor Marked Assignments (TMA)

1. An inductor of inductance 3H and resistance 6Ω is connected to the terminals of a battery of e.m.f 12V of negligible internal resistance.

   (a) Find the initial rate of increase of current in the circuit.

   (b) Find the rate of increase of current at the instant when the current is 1A.

   (c) What is the instantaneous current 0.2s after the circuit is closed?

   (d) What is the final steady-state current?

2. An N-turn coil is part of a circuit with a total resistance R. While the flax through the coil is changing, a current I is induced. Show that the total charge that passes any point in the circuit, while the flax enclosed by the coil changes from ϕ_1 to ϕ_2 is

$$Q = \int_{t1}^{t2} I \, dt = N \frac{(\phi_2 - \phi_1)}{R}$$

3. A 10-turn coil of radius 0.50cm is concentric with a 20-turn coil of radius 50cm.

(a) What is the mutual inductnace of these coils?

(b) At the instant t = 1/240 s, what is the e.m.f. in the larger coil by a current in the smaller coil given in SI unit by

$$I = 5.0 \cos 377t ?$$

7.0 References and Other Resources

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UNIT 16

ALTERNATING CURRENT THEORY I

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1.0 Introduction

You learned in one of the earlier units that a generator or dynamo produces energy by electromagnetic induction and that the current alternate sinusoidally. In most commercial power plants., mechanical enrgy is provided in the form of rotational motion. For example, at kainji, Niger state where Nigeria has its biggest hydroelectric plant, falling water directed against the blades of a turbine causes the turbine to turn. Basically, a generator uses the turbine's rotary motion to turn a wire loop in a magentic field. Thus a generator produces a continuously changing e.m.f or (current).

The current supplied to most of the buildings in this country comes from the National Grid and it is alternating, with a frequency of 50Hz. Alternating current is used in preference to direct current because it is easier to generate, and much easier to transform from one voltage to another.

Alternating current theory is important in two areas of engineering, the transmission of electrical energy and in signal communication (telephone, radio, etc.) Radio waves, for example, are generated by high freqeuncy alternating current in a transmission antenna.

To facilitate a proper understanding of the alternating current theory which is the subject of this unit, it is necessary for you to have a thorough grounding in circular motion, oscillation and electric circuit theory. The subject links all of these topics. Therefore, you are advised to start by revising your work on circular motion and simple harmonic motion.

2.0 Objectives

After studying this unit you should be able to:

* define an alternating current or e.m.f.
* calculate the peak and root-mean square values of an alternating current or e.m.f.
* understand the principles governing the operation of a.c. meters
* describe the procedure for the transmission of electric power from power stations to towns and villages
* understand the rectification of alternating current.

3.1 Alternating currents and Voltage

The effects of a.c are essentially the same as those of d.c. Both are satisfactory for heating and lighting purposes. As we have seen, a.c. is more easily generated and distributed than d.c and for this reasons the main supply is a.c.

3.1.1 Definition of Alternating Current or Voltage

An alternating current or e.m.f varies periodically with time in magnetude and direction. One complete alternation is called a cycle (see Fig. 3.1) and the number of cycles occurring in one second is termed the frequency (f) of the alternating quantity (current or voltage).

The unit of frequency is the hertz (Hz) and was previously the cycle per second (c.p.s). The frequency of electricity supply in Nigeria is 50Hz which means that duration of one cycle, known as the period (T) , is1/50 = 0.02s. In general, $f = 1/T$.

The simplest and most important alternating e.m.f. can be represented by a sine curve and is said to have a sinusoidal wave form (see Fig 3.1). It can be represented by the equation

$$E = Eo \sin wt \ldots\ldots 3.1$$

where E is the e.m.f . at time t, Eo is the peak or maximum e.m.f and W is a constant which equals 2πf where f is the frequency of the e.m.f. similarly, for a sinsoidal alternating current, we have

$$I = Io \sin wt \ldots\ldots 3.2$$

In Eq. 3.2, I is the instantaneous value of the current at a given instant t and Io its maximum or peak value or the current amplitude.

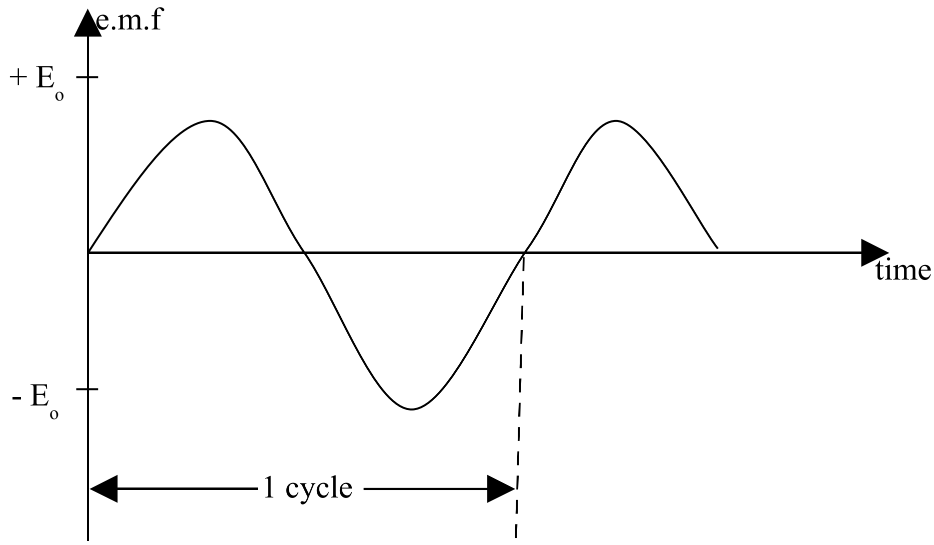


Fig 3.1

3.1.2 Effective Value of an Alternating Current or Volage.

Let us begin this section by finding the answer to the following question. Can we measure alternating current by means of an ordinary d.c. ammeter?

The answer is No. This is because during one half cycle of the current the pointer will move in one direction and during the other half cycle, to the same extent in the opposite direction. All this movement on either side will be so rapid that the pointer will appear to be stationary at zero which is the average value of the current over a complete cycle.

$$\text{Average current} = \frac{\int_0^T I dt}{\int_0^T dt} \qquad \frac{\int^T I_o \sin wt}{T} = 0 \ldots\ldots 3.3$$

The same will be the case if we wish to measure an alternating potential difference by means of an ordinary d.c. voltmeter. This too will show zero defletion of the pointer, indicating zero average value of the alternating p.d. over a whole cycle.

Therefore, in the case of alternating current and p.d, we take the average over half a cycle or half a time period.

$$I_{av} = \frac{\int_o^{T/2} I dt}{\int_o^{T/2} dt} = \frac{\int_o^{T/2} I_o \sin wt\, dt}{T/2} = \frac{2}{\pi} I_o \quad \ldots\ldots\ldots\ldots 3.4$$

(since $T = \frac{2\pi}{W}$)

Similarly, average p.d., $V_{av} = \frac{2}{\pi} V_o$ …………………..3.5

The average value of the alternating current or. P.d. is obviously the same for either half of a cycle but opposite in direction. For a whole cycle, therefore, if follows that the average is zero as stated in Fig. 3.3.

You may then ask at this junction, how do we measure an alternating current? The answer is that we should base our measuring device on some such effect of the current which is indpendent of its direction (or which depends on the square of the current, i.e, I^2, so that there will be no negative value of I^2). One of such effects is the heating effect of a current. For, although the current flows in one direction in a resistance during one half cycle and in the opposite direction during the other half, the latter does not reverse the heating effect produced by the former but itself produces an equal heating effect. Thus a heating effect is produced by the cycle or the altenating current as a whole. We therefore, define an alternating current in terms of a direct or steady current which produces the same heating effect as the given alternating current.

The effective value of an alternating current is defined as that value of a diret current which produces the same heating effect in a given resistance as the alternating current.

Now, heat produced by a direct current I in a resistnce R in time r is equal to I^2Rt. Let the same heating effect be produced in the same resistance R in time t by an alternating current I_{eff}

Then $I_{eff}^2 Rt = I^2 Rt$

I_{eff}^2

Or $I^2_{eff} Rt = I^2 Rt$

Since alternating current varies sinusoidally with time, I^2_{eff} is really the average of the square of the alternating current over a whole cycle or a whole time – period and is therefore given by the relation.

$$I^2_{eff} = \frac{\int_o^T I_o^2 \sin^2 wt}{_o\int^T dt} = \frac{I_o^2}{2}$$

$$I_{eff} = \frac{I_o}{\sqrt{2}} = 0.707 I_o \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

Equation 3.6 shows that the effective value of an alternating current is ½ times its maximum or peak value. The effective value is also called the root mean square (r.m.s.) value of the current.

Another way of looking at the problem of effective value is to plot the square of the alternating current against time as shown in Fig. 3.2

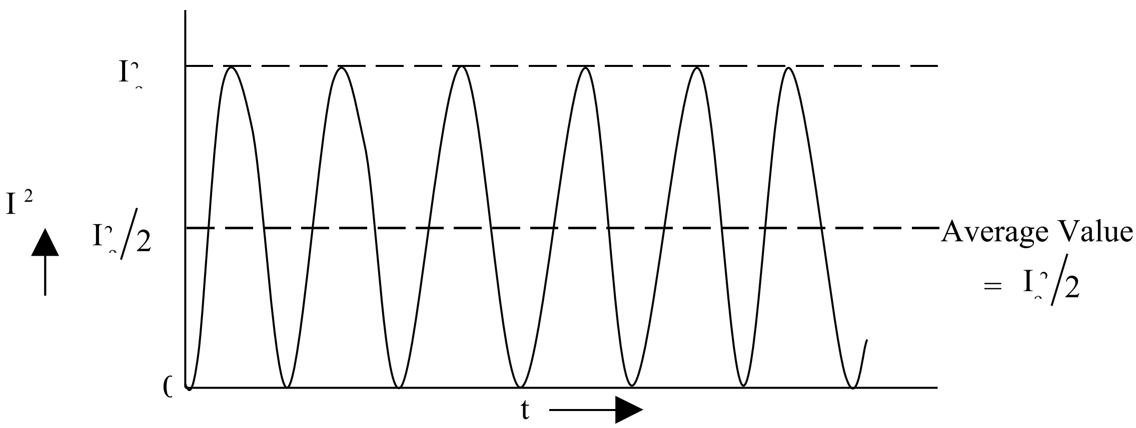


Fig. 3.2

The mean or avaerage value of the square of the current is $I_o^2/2$.

Similarly, the mean or average value of the square of the potential difference $= V_o^2/2.$

Effective value of p.d.. $V_{eff} = V_o/\sqrt{2} = 0.707V_o$
= 0.707 x peak value of p.d..

All instrument for measuring alternating current and p.d depend for their working on the square of the current and voltage repectively. They are calibrated to read the effective values of current and p.d. in ampere and volts respectively. In other words, the peak value is obtained by multiplying the value read on the instrument by $\sqrt{2}$.

Ordinarily, when we say that the voltage of the domestic a.c. supply is 220, it means that the r.m.s or the effective value of the voltage is 220. The voltage amplitude or the peak value of voltage is 220 x $\sqrt{2}$ = 311 volts.

Example

A 100W lamp is used on a domestic a.c. supply at 220 volt.
What is

(i) The effective value of the current consumed by the lamp?

(ii) The r.m.s value of the voltage

(iii) The peak value of the voltage

(iv) The average value of the voltage.

Solution

(i) $I_{eff} = \frac{\text{watt}}{\text{volt}} = \frac{100}{220} = 0.45A$

(ii) r.m.s or effective voltage = 220 volt

(iii) The peak value of voltage = $\sqrt{2}$ x effective value

$= \sqrt{2} \times 220$

$V_o = 311$ volt

(iv) Average value of voltage $= \frac{2V_o}{\pi} = \frac{2}{\pi} \times 311 = 198$ volt

NB: Remember that the average value of an alternating current or potential difference is taken over half a cycle.

3.2 Alternating Current Meters

We saw in section 3.1.2 that the deflection of an a.c. meter must not depend on the direction of the current. Most voltmeters and ammeters for a.c. use are calibrated to read r.m.s values and give correct readings only if the waveform is sinusoidal.

3.2.1 Moving – Iron Meter.

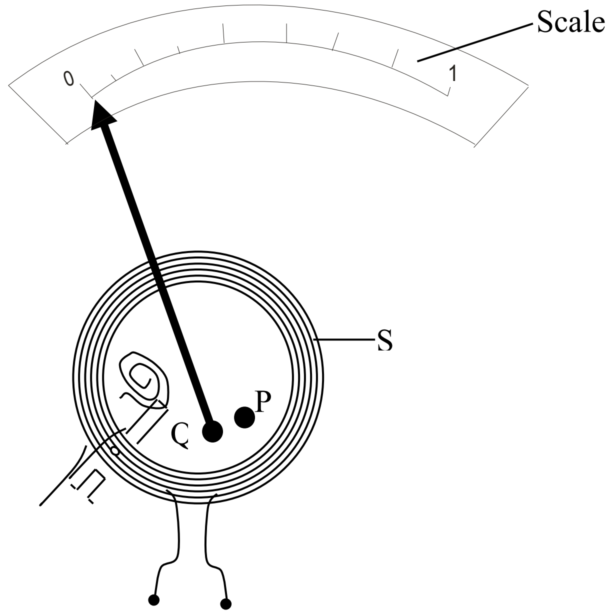


Fig 3.3

The repulsion type of moving-iron meter consists of two soft iron rods P and Q mounted inside a solenoid S and parallel to its axis (see fig. 3.3). P is fixed and Q is carried by the pointer. Current passing either way through S magnetizes P and Q in the same direction and they repel each other. Q moves away from P until stopped by the restoring couple due, for example, to hair-springs. In many cases, air damping is provided by attaching to the movement an aluminium pointer or vane which moves inside a curved cylinder.

The deflecting force is a function of the average value of the square of the current. Hence a moving-iron meter can be used to measure either d.c. or a.c. and in the latter case r.m.s values are recorded. The scale is not divided uniformly, being closed up for smaller currents.

A moving-iron voltmeter is a moving-iron milli-ammeter with a suitable (non-induction) multiplier connected in series.

3.2.2 Thermocouple Meter

One junction of a thermocouple (i.e. two wires of dissimilar metals) is joined to the centre of the wire XY carrying the current to be measured and is heated by it, the other junction is at room temperature. (see fig. 3.4).

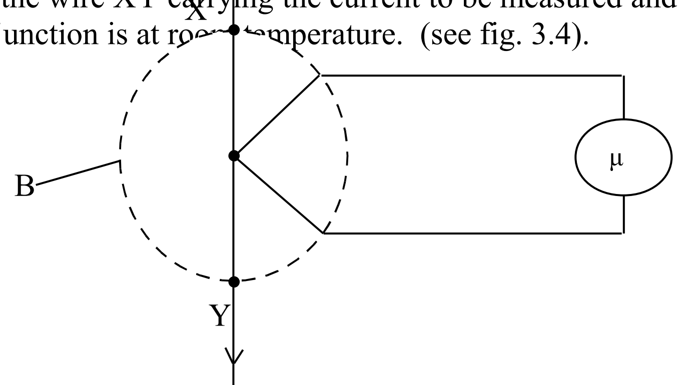


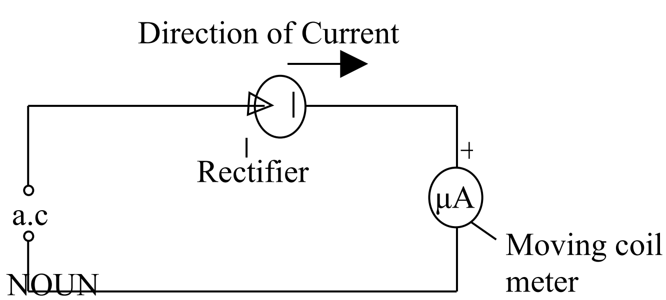
Fig. 3.4

When a.c. flows in XY, a thermoelectric e.m.f is generated and produces a direct current that can be measured by a moving coil micro-ammeter (previously calibrated by passing known values of d.c. through XY). The hot junction is enclosed in an evacuated bulb B to shield it from draughts.

This type of meter relies on the heating effect of current. It therefore measures r.m.s values and can be used for alternating currents of high frequency (up to several MHz) because of its low inductance and capacitance compared with other meters.

3.2.3 Rectifier Meter

A rectifier is a device with a low resistance to current flow in one direction and a high resistance for the reverse direction. When connected to an a.c. supply it allows pulses of varying but direct current to pass. In a rectifier-type meter the average values of these is measured by a moving-coil meter (see figs. 3.5 amd 3.6). Rectification, i.e the conversion of a.c. to d.c, thus occurs . There are various kinds of rectifier, those that are commonly used are semi-conducting (germaniun) diodes.



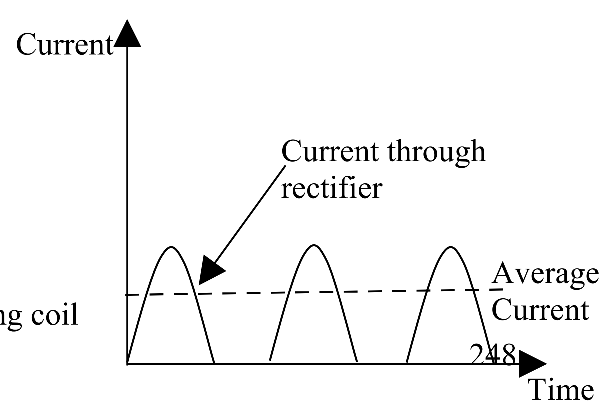


Fig 3.5

Rectifier instruments, being used on the moving-coil meter, are much more sensitive than other a.c. meters and are used in multimeters that have a.c as well as d.c. ranges. The scale of a rectifier meter is calibrated to read r.m.s values of currents and p.d.s with sinusoidal waveforms.

3.3 Long Distance Power Transmission

We have seen that the principle of electromagnetic induction makes altrnating current prefereable to direct current for large-scale production of electricity.

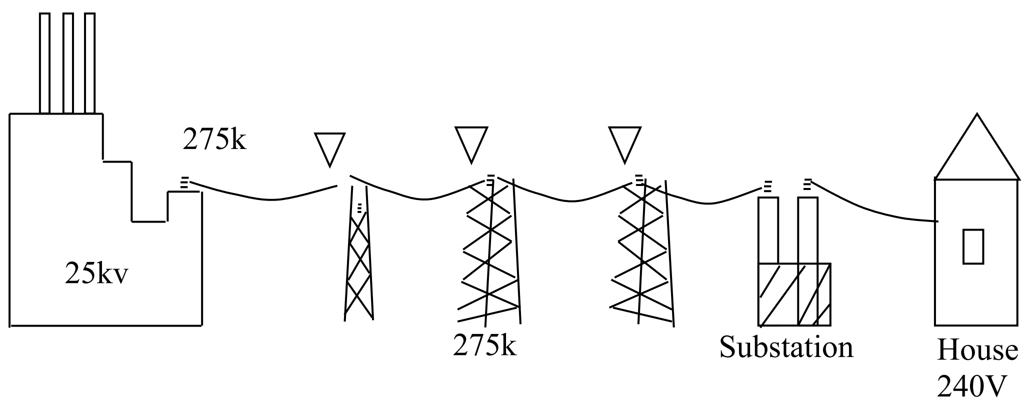


Fig 3.6

Electric power has to be transmited to towns and villages from power stations, usually located at fairly distant places where better and cheaper facilities for power generation are available. Long cables, called transmission lines are used. The arrangement is referred to as a Grid system. (see Fig. 3.6)

You msut have observed as you travel from one place to another in your Local Government Area or State, that the Grid system in Nigeria is a netweork of cables (wires), most of it supported on pylons, which connects very many power (PHCN) stations throughout the country and carrying electrical energy from then to consumers. Consider, for a moment, the length of cables from Kainji Dam in Niger State to Port Harcourt in Rivers State.

If the current were to be transmitted at comparatively low voltage at which it is generated at the power station, the current strenght would be high and would result in considerable loss of energy in the form of heat in the transmission lines.

For example, suppose a power station generates 500KW of power at 250V. If it were to be supplied diretley to the towns and cities, as it is produced, the current that the cables will have to carry will be $I = 500 \times 10^3/250 = 2000$ amperes. This is too much high a current to be carried by the transmission lines, not only safety for but also because of the following reasons.

(i) It will result into very high I^2R losses i.e. losses due to heating up of the wires).

(ii) If thick wires are used in order to decrease the resistance, the cost of the material as well as the supporting pylons would be extremely high.

(iii) There will be a large fall of potential per unit length of the wire.

The problem is, therefore, solved by first stepping up the voltage at the power station itself, by means of a transformer, so that the current to be carried by the transmission lines may be small.

In a typical power station, electricity may be generated at about 25KV ($50H_2$) and stepped up in a transformer to 275KV or 400KV for transmission over long distances. The p.d. is subsequentely reduced in sub-station by other transformers for distribution to local users at suitable p.d.s – 33 KV for heavy industry, 11 KV for light industry and 240V for homes, schools, shops, etc. (see Fig. 3.6).

3.4 Rectification of Alternating Current

Rectification is the conversion of a.c to d.c by a rectifier. The major source of d.c. voltages are d.c. power supplies. They differ from batteries in that they obtain energy from an a.c power source. The conversion of a.c. into d.c. can be accomplished in several different ways depending on the requirements of d.c. power. Half-wave rectifiers, full-wave rectifiers, filtered poewr supplies, and regulated power supplies are used. The present study will be limited to half-wave and full-wave rectification.

3.4.1 Rectifiers

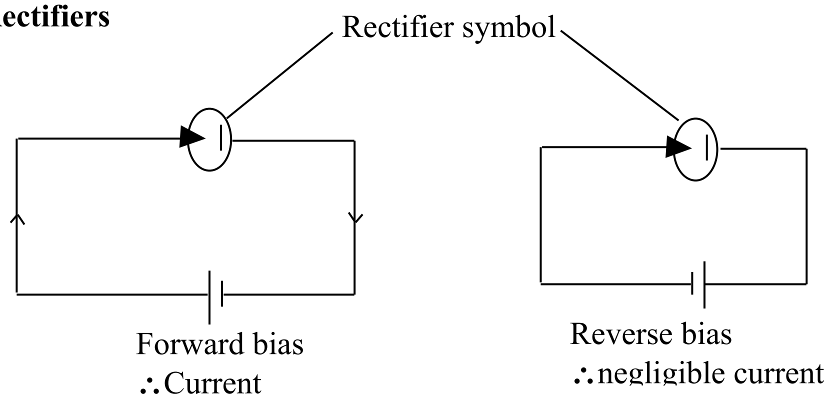


Fig 3.7

Rectifiers have a low resistance to current flow in one direction, known as the forward direction , and a high resistance in the opposite or reverse direction. They are conductors which are largely
unidirectional.

When connection is made to a supply so that a rectifier conducts, it is said to be forward biased; in the non-conducting state it is reverse based (see fig. 3.7).

The arrowhead on the symbol for a rectifier indicated the forward direction of conventional current.

3.4.2 Half-wave Rectifier

There are applications where all that is needed is a voltage that is always one polarity with respect to a reference terminal. Variations in voltage may not be important. In such applications the half-wave rectifier can be used to provide a fluctuating, unidirectional voltage. The circuit for such a supply is shown in Fig. 3.8.

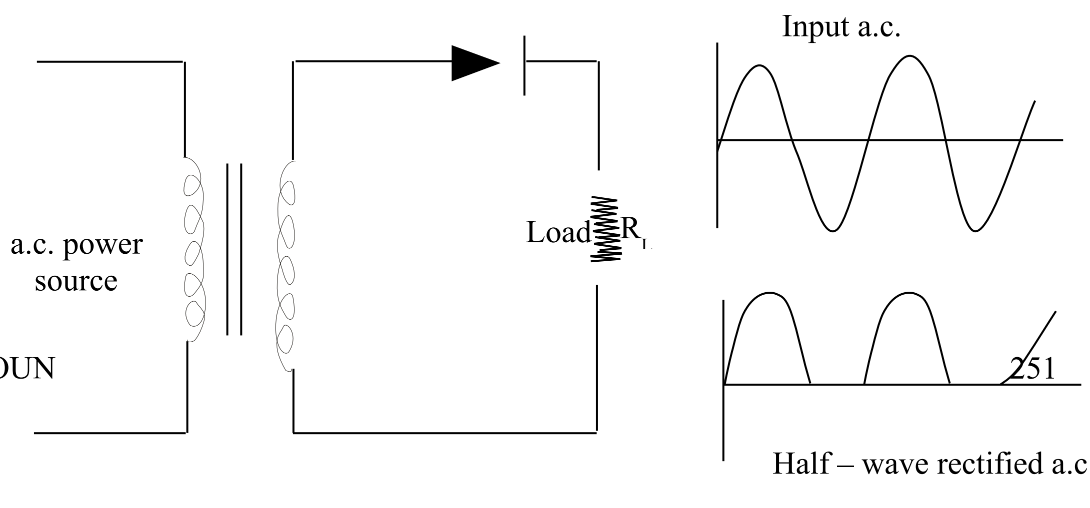


Fig 3.8

The half-wave rectifier acts like a switch that opens and closes the path from the transformer to the load resistor. In fig 3.8b the alternating input p.d. applied to the rectifier is shown. If the first half-cycle acts in the forward direction of the rectifier, a pulse of current flows round the circuit, creating a p.d across the load R_L. The second half-cycle reverse biases the rectifier, little or no current flows and the p.d. across R_L is zero. This is repeated for each cycle of a.c. input. The current pulses are unidirectional and so the p.d. across the load is direct, for although it fluctuates it never changes direction.

3.4.3 Full-Wave Rectifier

A full-wave rectifier can be used if half-wave rectifiers prove to be unsatisfactory because there are times when there is no current. Two diodes act as switches which are turned on and off alternately. This operation can be explained by referring to Fig. 3.9. Two rectifiers and a transformer with a centre-tapped secondary are used. The centre

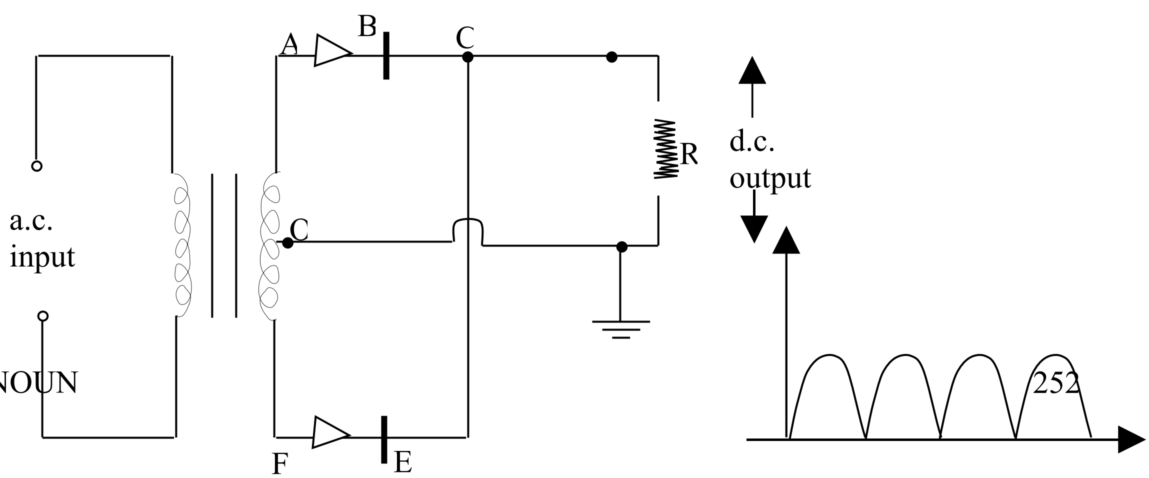


Fig 3.9

tap 0 has a potential half-way between that of A and F and it is convenient to take it as a reference point having zero potential.

If the first half-cycle of input makes A positive, rectifier B conducts, giving a current pulse in the circuit ABC, R, OA. During this half-cycle, the other rectifier E is non-conducting since the p.d across FO reverse biases it. On the other half of the same cycle F becomes positve with respect to 0 and A negative. Rectifier E conducts to give current in the circuit FEC, R, OF; rectifier B is now reverse biased.

In effect. The circuit consists of two half-wave rectifiers working into the same load on alternate half-cycles of the aplied p.d. The current through R is in the same direction during both half-cycles and a fluctuating direct p.d. is created across R as shown in Fig. 3.9.

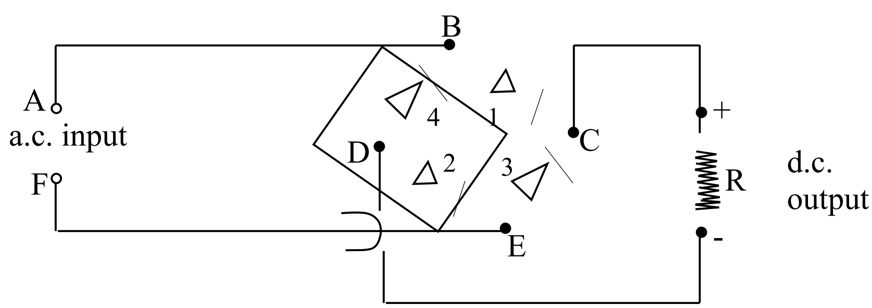


Fig 3.10

Fig 3.10 shows the bridge full-wave rectifier, which is another type of full-wave rectification. Four rectifiers arranged in a bridge network. If A is positive during the first half-cycle, rectifiers 1 and 2 conduct and current takes the path ABC, R, DEF. On the next half-cycle when F is positve, retifiers 3 and 4 are forward biased and current follows the path FEC, R, DBA. Once again there is a unidirectional current through R during both half-cycles of *input* p.d. and a.d.c. output is obtained.

4.0 Conclusion

The output e.m.f of a typical generator has a sinusoidal wave form, which alternates from positive to negative. Hence the output current (or e.m.f) from the generator changes its direction at regular intervals. This type of current is called alternating current, or more commonly, a.c.

Alternating current is much preferred to direct current for the transmission of electrical power. This is because it can be transformed from one voltage to another. The loss due to transmission line heating is I^2R where I is the current and R the resistance of the transmission cables. So it is better to transmit electricity, where possible, at very high voltages and small currents. Power is generated at relatively low voltages which are ‘ stepped down’ for transmission purposes and finally ‘ stepped up’ again for domestic consumption using transfomers.

To measure an a.c. an instrument must be selected whose indication is independent of the direction of the current, for example a moving-iron or a moving-coil instrument fitted with a rectifier and a thermocouple.

5.0 Summary

* The effective value of an alternating current (also called the r.m.s value) is the steady direct current which converts electrical energy to other forms of energy in a given resistance at the same rate as the a.c. An ammeter shows the effective or r.m.s value of the current.

* An alternating e.m.f is given by the relation

  $$E = E_o \sin wt$$

  Where E is the instantaneous value of the e.m.f at a given instant t, Eo its maximum or peak value or the e.m.f . amplitude and ω is the angular velocity in radians per second.

* Calculation of the r.m.s of sinusoidal voltage is a simple application of calculus, and gives the result.

  $$V_{r.m.s} = \frac{1}{\sqrt{2}} \times \text{(peak value of voltage)}.$$

* The frequency of the mains supply in Nigeria is 50Hz.

* The voltage of the mains is stated as 240V; this is the r.m.s value, though in practice this varies according to the load.

* Rectification is the process of obtaining a direct current from an alternating electrical supply. A rectifier is an electrical device that allows more current to flow in one direction than the other, thus

enabling alternating e.m.fs to drive only direct current. The device most commonly used for rectification is the semiconductor diode.

* In half-wave rectification, achieved with one diode, a pulsating current is produced. In full-wave rectification two diodes are used, one pair conducting during the first half cycle and the other conducting during the second half. The bridge recttifier also gives full-wave rectification.

6.0 Tutor Marked Assignments

1a. A sinusoidally varying voltage supply has an r.m.s. value of 12 volts. Exlain what the term root mean square means and calculate the amplitude of the supply.

(b) With the aid of a circuit diagram, show how the supply in (a) could be rectified to give a full-wave direct current.

2. When a certain a.c. supply is connected to a lamp it lights with the same brightness as it does with a 12V battery.

(a) What is the r.m.s value of the a.c. supply?

(b) What is the peak p.d. of the a.c. supply?

(3) The instantaneous current I across a 200-ohm resistor is given in SI units by $I = 10\cos 377t$

(a) Find the frequency and the period.

(b) Find the voltage across the resistor when t = 1/240 second.

7.0 References and the Resources

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UNIT 17

ALTERNATING CURRENT THEORY I1

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1.0 Introdution

This unit builds on the concepts introduced in the earlier unit on alternating current theory 1. Emphasis is given to the fundamental idea about a.c. circuit.s

The capacitor and the inductor are introduced in a way that will enable you to understand why these elements have useful characteristics. Basic definitions and waveforms are used as aids in the development of the impedance relations for the inductor and the capacitor. The influence of element size and source frequency on imedance is given.

The mathematical models of the a.c. circuits are given to show the usefulensss of these models. Problems with solutions are given to help you learn how to use the basic models for the circuits element used in a.c circuits.

2.0 Objectives

After studying this unit you should be able to :

* distinguish between passive and active circuit elements.
* represent sinusoidal functions such as I and V by phasors or a phasor diagram.

- * determine the current at instant t in terms of the appropriate parameters of the circuit element and of the voltage across the circuit element.
- * analyse a.c. circuits by applying kirchhoff's rules to instantaneous voltages and currents.
- * explain the phenomenon of resistance in electric currents.
- * define the power factor of a circuit element
- * calculate power in a.c. circuits.

3.1 Phasor and Phasor Diagram

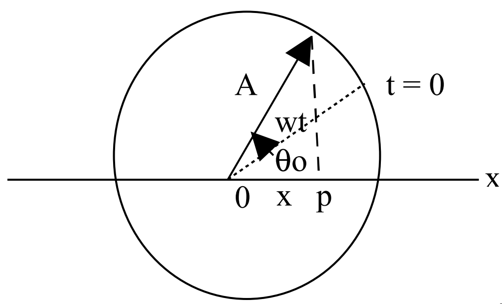


fig. 3.1

You would recall that simple harmonic motion (s.h.m) is the particular type of oscillatory motion executed, for example, by the point P in fig. 3.1 as the arrow of constant length A rotates about the origin with a constant angular speed ω. The point P is always at the foot of the perpendicular from the tip of the arrow to the x-axis. At the instant t, the arrow makes an anlge

$$\theta = \omega t + \theta_o \text{ ....3.1}$$

with the x-axis.

The arrow in fig. 3.1 does not have a direction in space. It is an arrow drawn on the diagram to repreent the length A and the angle θ. Such a quantity is called a phasor to avoid confusion with vectors that do have direction in space. Formally, we define a phasor to be a mathematical quantity characterised by a magnitude (A) and an angle (θ) with algebraic properties that will be presented as required. (you will see in your mathematics course that phasors are called complex numbers).

As time elapses, θ changes and the phasor rotates about a perpendicular axis through its 'tail', which is fixed at the point 0. The point P moves along the X axis, and its position coordinates x has the time dependence that is characteristic of s.h.m.

$$x = \text{projection of A on X-axis}$$

$$= A \cos \theta$$

$$= A \cos (\omega t + \theta_o) \text{..................3.2}$$

where A, θ_o and w are constants.

3.2 Passive and Active circuit Elements

An electronic component, such as a capacitor or resistor, that is incapable of amplification is referred to as a passive element. On the other hand, an electronic component, such as a transistor, that is capable of amplification is said to be an active element.

3.3 Phasor Representation of Alternating current

Suppose that the voltage V across a given circuit element at the instant t is a sinusoidal function of time with angular frequency $\omega = 2\pi f$, given by

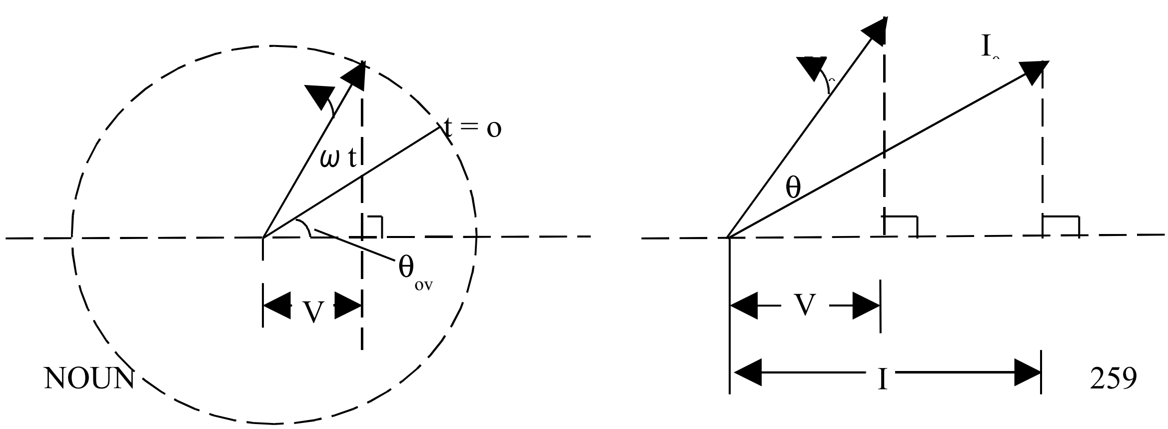
$$V = V_o \cos(\omega t + \theta_{ov}) \quad \text{.....................} \; 3.3$$

You recall that the positive constant V_o is the amplitude of this function. This is the largest value attained by the instantaneous voltage. The constant θ_{ov} is the initial phase angle of the voltage function.

The current I in the circuit element is a sinusoidal function of the same angular frequency ω as that of the voltage V across the circuit element. The current at the instant t is therefore a function of the form.

$$I = Io \cos(\omega t + \theta_{oI}) \quad \text{.....................} 3.4$$

This function has an amplitude Io and an initial phase angle $\theta_{oI.}$



(a) (b)

Fig. 3.2

Following the discussion in section 3.1 of this unit, we can represent sinusoidal functions such as I and V by phasors on a phasor diagrm. The voltage phasor is represented by an arrow of length Vo that makes an angle wt + θov with the horizontal axis (Fig. 3.2a). The instantaneous voltage V is equal to the projection of this rotating phasor on the horizontal axis. Similarly the current phasor is represented by an arrow of length Io at an angle $\omega t + \theta_{oI}$ with the horizontal axis.

The angle θ between the voltage phasor and the current phasor is called the phasor difference between the voltage and the current (Fig. 3.2b). This phasor difference θ is the difference of the phase angles of the functions V and I:

$$\theta = (\omega t + \theta_{ov}) - (\omega t + \theta_{oI}) = \theta_{ov} - \theta_{oI} \quad \ldots\ldots\ldots\ldots 3.5$$

If θ = 0, the voltage and the current are said to be in phase, otherwise they are out of phase. For circuit elements that include capacitors and indicators, we shall find that the current and the voltage are generally out of phase. The phase difference θ is the most important quantity in a.c. circuit analysis.

The difference $\theta_{ov} - \theta_{oI}$ of the initail phase angles is significant (being the phase difference θ), but actual values of these angles are determined by just which instance is selected as the initial instant, t = 0 for simplicity we can choose the initial instant to be a time when the current I reaches its peak value I_o. Then $\theta_{oI} = 0$, $\theta_{ov} = \theta$, and the functions V and I are

$$V = V_o \cos(\omega t + \theta) \quad \ldots\ldots\ldots\ldots 3.6$$
$$I = I\cos \omega t \quad \ldots\ldots\ldots\ldots 3.7$$

Example

The instantaneous voltage V and current I are given in SI units by

$$V = -155\cos(377t - \pi/2)$$
$$I = 2.0\sin 377t$$

Find the phase difference between the voltage and the current.

Solution

Perhaps the simplest method is to locate the phasor representing V and I at some particular instant. For example, at the time t such that $377t = \pi/2$, $V = -155\cos(\pi/2 - \pi/2) = 155$ volts and I = 2.0A.

The phasor diagram at this instant is shown in Fig. 3.3.

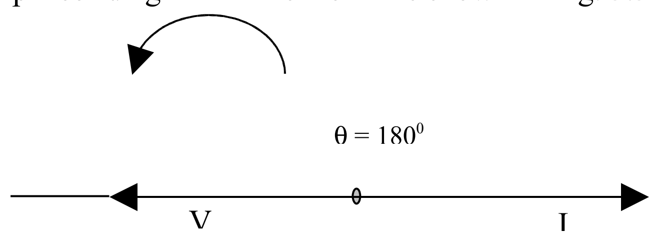


Fig. 3.3

The angle between the voltage phasor and the current phasor, which is the phase difference θ, is seen to be 180^0 or π rad.

Another method (Analytical)

To find the phase difference analytically, we first write V and I in the same standard form with a positve constant multiplying a cosine function of time. Using cos (α + π) = -cos α, we can write the voltage function as

$$V = 155 \cos (377t - \pi/2 + \pi).$$

Using cos (α - π/2) = sin α, we can rewrite the current function as

$$I = 2.0\cos (377t . \pi/2).$$

Then the phase difference is

$$\theta = (377t - \pi/2 + \pi) - (377t - \pi/2).$$

$$= \pi.$$

3.4 Impedance of Resistor Capacitors and Inductors.

In an a.c. circuit, any positive circuit element is characterised by two numbers, Z and θ, where θ is the phase difference between the voltage across the element and the current through it, and Z is the ratio of the voltage amplitude V_o to the current amplitude I_o.

$$Z = \frac{V_o}{I_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

These two numbers, Z and θ, determine what is called the impedance of the circuit elements, Z is called the magnitude of the impedances and θ the angle of the impedance .

Fig. 3.4

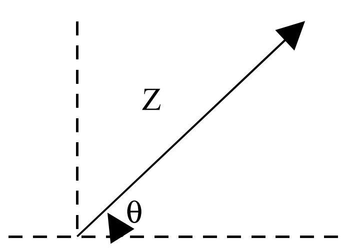


Fig. 3.4 shows that the impedance of a circuit element is a phasor with a magnitude Z and an angle θ. (Note that the angle θ is fixed; impedance phasors do not rotate).

The relationship between the current and the voltage is determined by the impedance of a circuit element. If Z and θ are known, the current function $I = I_o \cos \omega t$ can be determined if the voltage function. $V = V_o \cos(\omega t + \theta)$ is known, and vice versa.

3.4.1 Impendance of a Resistor

We shall consider only resistors which obey 0hm's law. If the current through the resistor of resistance R is I = Iocoswt, the voltage across the resistor is

$$V = RI = R I_o \cos \omega t.$$

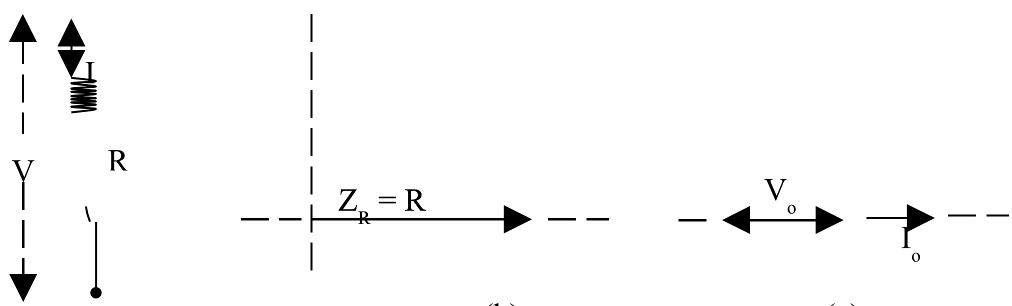
This voltage function has an amplitude given by

$$V_o = RI_o$$

Therefore the magnitude of the impedence of a resistor is

$$Z = \frac{V_o}{I_o} = R \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

The phase difference θ between the voltage function and the current function is zero, that is voltage and the current are in phase.



(a) (b) (c)

These results show that the impedance of a resistor is a phasor of magnitude R and angle zero, which is represented on an impedance diagram as shown in fig. 3.5 (b).

The phasors representing the current and the voltage are shown in figure 3.5 (c) at the instant t = 0. These phasors each rotate counter-clockwise with an angular speed ω.

3.4.2 Impedance of an Inductor

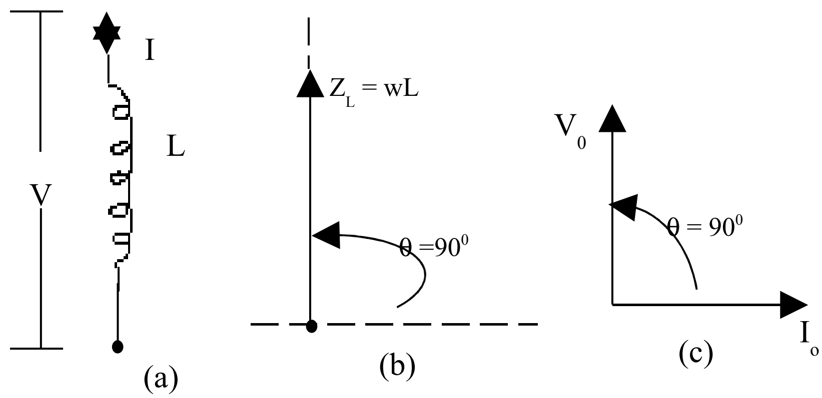


Fig. 3.6a shows a pure inductor which has a self-inductance L but no resistance. If the current through the induction is I = Io coswt, the voltage across the inductor is

$$V = L \frac{dI}{dt} = -\omega L I_o \sin \omega t = \omega L I_o \cos(\omega t + \pi/2)$$

The voltage function has the amplitude

$$V_o = wL\, I_o$$

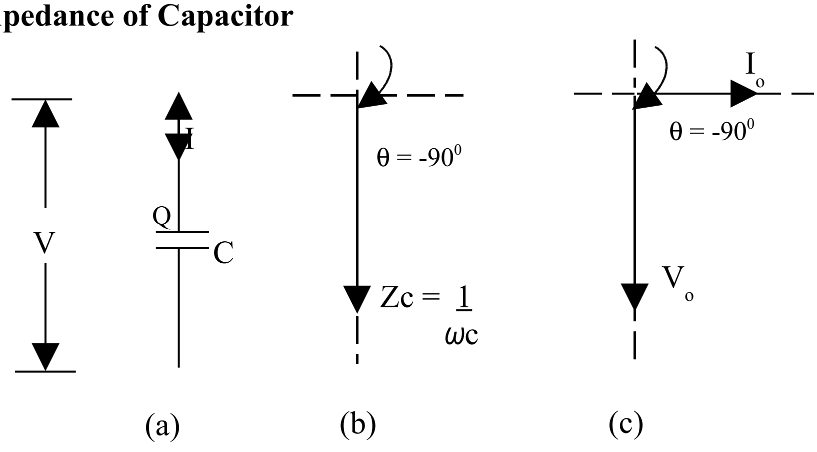
Therefore, the magnitude of the impedance of a pure inductor is

$$Z_L = \frac{V_o}{I_o} = \omega L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

The phase difference between the current function and the voltage function is $\pi/2$. The impedance of a pure inductor is therefore a phasor of magnitude ωL and angle $\pi/2$, as shown if Fig. 3.6b. Notice that Z_L is proportional to the frequency ($\omega = 2\pi f$).

Rotating phasors representing the voltage and current functions are shown in Fig. 3.6 at the instant t = 0. The current and the voltage are 90^0 out of phase, with the voltage leading the current.

3.4.3 Impedance of Capacitor



If charge flows to the upper plate of the capacitor of Fig. 3.7 (a) at the rate

$$\frac{dQ}{dt} = I = I_o \cos \omega t$$

The charge on this plate at the instant t is given by the function

$$Q = \int_0^t I\, dt' = I_o \int_0^t \cos \omega t'\, dt' = \frac{I_o}{\omega} \sin \omega t$$

The voltage across the capacitor is

$$V = \frac{Q}{C} = \frac{I_o}{\omega c} \sin \omega t = \frac{I_o}{\omega c} \cos (\omega t - \pi/2)$$

The voltage function has the amplitude

$$V_o = I_o/\omega C$$

Therefore, the magnitude of the impedance of a capacitor is

$$Zc = \frac{Vo}{Io} = \frac{1}{\omega c} \quad ....3.11$$

The phase difference θ between the voltage function and the current function is - π/2. The impedance of a capcitor is therefore a phasor of magnitude $1/\omega C$ and angle -π/2, as shown in Fig. 3.7 (c) . Notice that the magnitude of the impedance of a capacitor is large at low frequencies an small at high frequencies.

Rotating phasors represening the voltage and the current functions are shown in fig. 3.7 (c) at the instant t = 0. The voltage lags the current by 90^0.

Example

(i) Find the magnitude Zc of the impedance of a 1.00-mf capacitor at f = 60Hz.

(ii) A 60 – Hz alternating voltage with an amplitude of 4.00 V is applied across a 1.00 – mH inductor. Find the current amplitude.

Solution

(i) Using equation 3.11, at f = 60Hz

$$Z_c = \frac{1}{2\pi\,(6oH_z)\,(1.00 \times 10^{-6}\ F)} = 2.65 \times 10^3 \text{ ohm.}$$

(ii) From Eq. 3.10, at f = 60 Hz

$$Z_L = wL = 2\pi\,(60Hz)\,(1.00 \times 10^{-3}H) = 0.377 \text{ ohm}$$

Therefore, the current amplitude is given by

$$Io = \frac{Vo}{Z_L} = \frac{4.00V}{0.377\Omega} = 10.6A$$

3.5 Impedance of Series Circuits

3.5.1 Impedence of an R-L-C Series combination

At any instant, the current I is the same in each element of the series combination of Fig. 3.8.

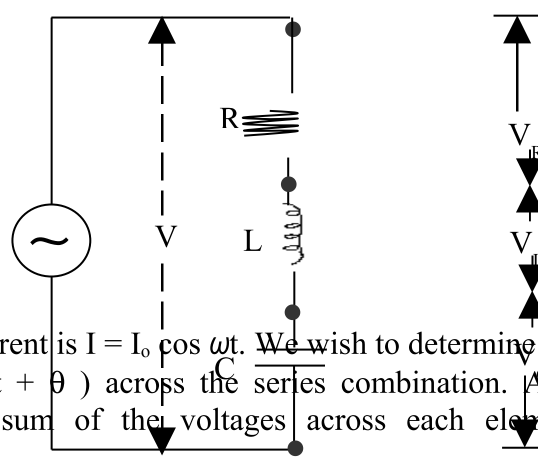


Fig. 3.8

Suppose this current is $I = I_o \cos \omega t$. We wish to determine the voltage $V = V_o \cos(\omega t + \theta)$ across the series combination. At any instant, this voltage is the sum of the voltages across each element of the series combination:

$$V = V_R + V_L + V_c \text{ ....3.12}$$

Using previous results, we can express the voltage functions for each circuit element as

$$V_R = V_{oR} \cos \omega t$$
$$V_L = V_{oL} \cos(\omega t + \pi/2)$$
$$V_c = V_{oc} \cos(\omega t - \pi/2)$$

Where the subscript 0 denotes the amplitude of the corresponding function.

Equation 3.12 requires that

$$V_o \cos(\omega t + \theta) = V_{oR} \cos wt + V_{oL} \cos(\omega t + \pi/2) + Voc \cos(wt - \pi/2) \quad \ldots .3.13$$

The problem is to detrmine V_o and θ in terms of V_{oR}, V_{oL} and V_{oc}. For this purpose, we shall use the method of phasor additon.

Since the current is the same for each element in a series circuit, the current phasor provides a convenient reference phasor which is conventionally shown pointing to the right in the phasor diagram (corresponding to the instant that we have selected as t = 0). The direction of each voltage phasor is determined from the phase difference between the voltage and the current as we saw in section 3.4 (see Fig. 3.9.)

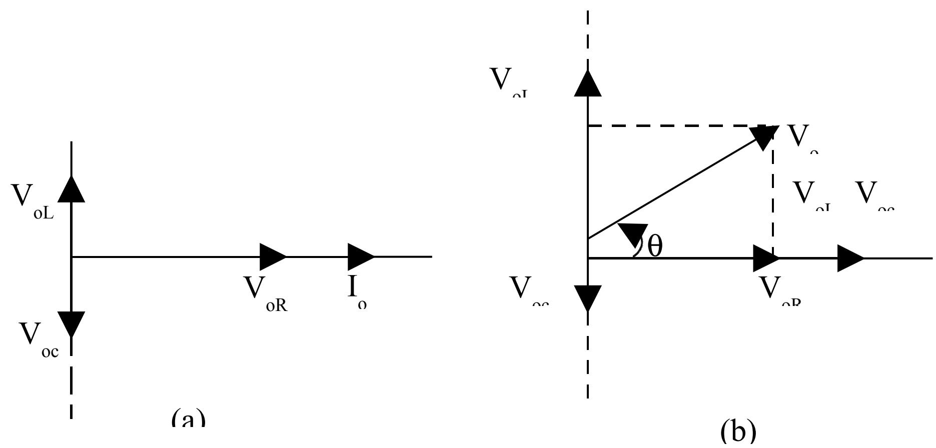


Fig. 3.9

We can interpret Eq. 3.12 as an equation relating the components of phasors along the direction of the current phasor, we conclude that the phasor representing V is the vector sum (or phasor sum) of the phasor representing V_R V_L and V_c.

In Fig. 3.9 (b), the phasor representing V has a magnitude V_o and makes an angle θ with the current phasor. Applying Pythagoras's theorem,

$$V_o = \sqrt{V_{oR}^2 + (V_{oL} - V_{oc})^2}$$

$$= I_o \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$$

Which shows that the magnitude Z = Vo/Io of the impedance of the series combination is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2} \quad \ldots\ldots\ldots 3.14$$

From the phasor diagram, fig 3.9, we see that the angle θ of this impedence is such that

$$\tan\theta = \frac{V_L - V_c}{V_R} = \frac{\omega L - (1/\omega c)}{R} \quad \ldots\ldots\ldots\ldots 3.15$$

This impedance is represented on the impedance diagram of Fig. 3.10

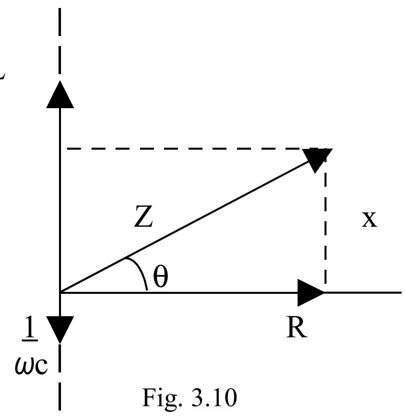


Fig. 3.10

3.5.2 Resistive and Reactive Componets of an Impedance

Instead of specifying an impedance by giving it magnitude Z and angle θ, we can give its horizontal and vertical components on an impedance diagrm. These components are called resistive component (or the resistance R) and the reactive component (or the reactance X). Thus a pure inductor of inductance L has an impedance specified by R = O, $X_L = \omega L$. The impedance of a capacitor of capacitance C has the components R = 0, Xc = -1/wc.

From Fig. 3.10 it can be seen that the relationship between ($Z_1\theta$) and and (R_1 X) are

$$Z = \sqrt{R^2 + X^2} \text{ and } \tan\theta = \frac{X}{R} \quad \ldots\ldots\ldots 3.16$$

$$\text{Or } R = Z\cos\theta \text{ and } X = Z\sin\theta \quad \ldots\ldots\ldots\ldots 3.17$$

3.5.3 Phasor Addition of Impedance in Series

If an impendance having components (R_1 X_1) is connected in series with an impedance having components (R_2, X_2), this combination is equivalent to a single impedance with components (R_s X_s) given by:

$$R_s = R_1 + R_2 \text{ and } X_s = X_1 + X_2 \quad \text{...........3.18}$$

□ **Example**

Consider the circuit in Fig. 3.11

(a) From the amplitudes of the voltages across individual circuit elements, determine the amplitude V_o of the voltage across the entire part of the circuit.

(b) Calculate the impendace Z of this external circuit and use this to dertermine V_o.

(c) Determine the phase difference between the a.c. voltage and the current in the circuit

(d) Draw the impedance diagram for the R – L – C circuit element.

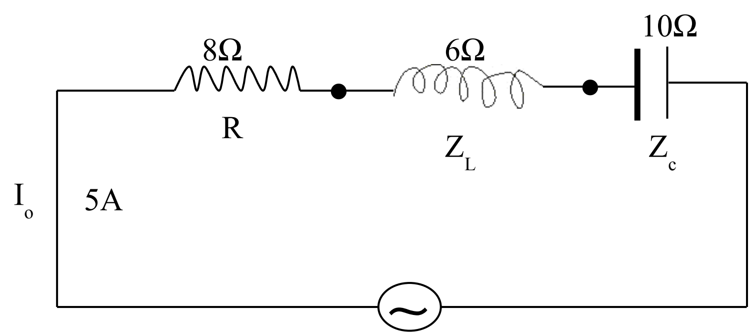


Fig. 3.11

Solution

$V_{oR} = IoZ_R = 5.0 \text{ x } 8 = 40.0V$, and the phase angle $\theta_R = 0$

$V_{oL} = Io\ Z_L = 5.0 \text{ x } 6.0 = 30.0V$, and the phase angle $\theta_L = +\ 90^0$

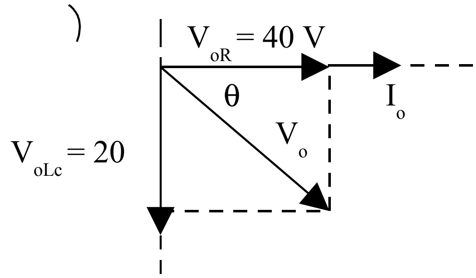
$V_{oc} = Io\ Z_c = 5.0 \text{ x } 10.0 = 50.0$ V and the phase angle $\theta_c = -90^0$

Because of the 180^0 phase difference between the voltage across the inductor and the voltage across the capacitor, the amplitude of the voltage across these two component is given by

$$V_{oLc} = V_{oc} - V_{oL} = (50.0 - 30.0\ )V = 20.0V$$

Since V_{oc} is greater than V_{oL}, the phasor of magnitude V_{oLc} points downward on a phasor diagram. Now, evaluating the phasor sum of the phasors of magnitude V_{oLc} and V_{oR}, we have

$$V_o = \sqrt{40.0^2 + 20.0^2} = 44.7V$$



(b) $$Z = \sqrt{R^2 + (X_L + Xc)^2}$$
$$= \sqrt{8^2 + (6 - 10)^2}$$
$$= 8.94\ \Omega$$

Using $Vo = I_o Z$, we obtain

$$Vo = (5.0A)(8.94\ \Omega) = 44.7V$$

In agreement with the result of part (a)

(c) From the phasor diagram, we see that

$$\tan\theta = \frac{-20}{40} = -0.500 \text{ and } \theta = -26.6^0$$

(d) The impedance diagram is shown in Fig. 3.12

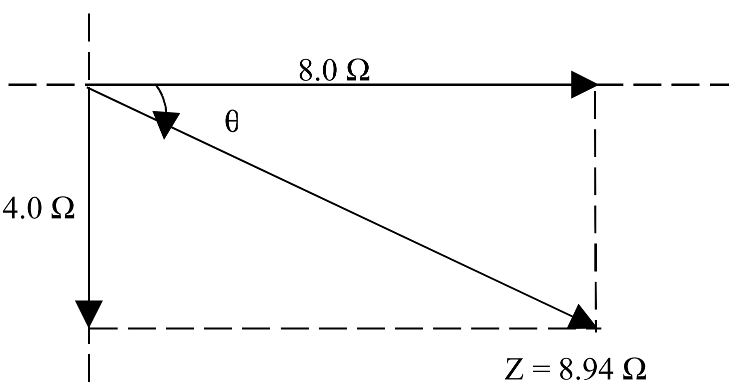


Fig. 3.12

3.6 Series Resonance

Eq. 3.4 shows that in the L-C-R series circuit, the magnitude of the impendance is a function of the frequency:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..3.18$$

If an a.c. generator of constant voltage amplitude Vo but variable frequency is connected across the R-L-C circuit, the current amplitude, $I_o = V_o/Z$, depends on the frequency.

The frequency f at which the current amplitude I_o is a maximum is called the resonant frequency f_o of the circuit. Maximum I_o will be obtained at the frequency which minimizes Z.

It follows from Eq. 3.18 that Z is a minimum at an angular frequency ω_o such that

$$\omega_o L - \frac{1}{\omega o\ C} = O$$

The resonant frequency therefore is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}} \quad \ldots\ldots\ldots\ldots\ldots\ldots.3.19$$

and at resonance, Z = R

Now, the r.m.s. p.d. across the capacitor at resonance is

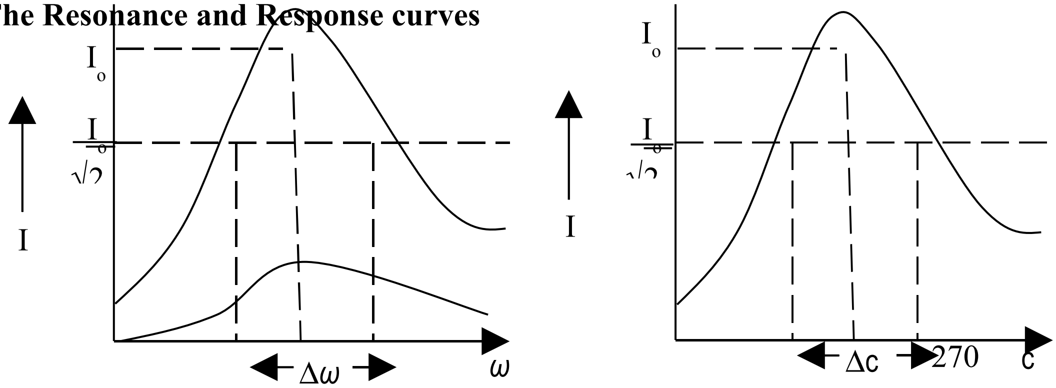
$$V_c = I_o \times \frac{1}{\omega C} = \frac{V_o}{R} \times \frac{1}{\omega_o C}$$

$$\therefore V_c/V_o = \frac{1}{W_o CR} = \frac{W_o L}{R} \quad \ldots\ldots\ldots..3.20$$

The quantity V_c/V_o is known as the Q of the circuit, it can be much greater than 1, therefore V_c can be much greater than V_o. This is important, as if V_c is too large, it may cause the capacitor to break down.

Similarly, the r.m.s p.d. across the inductor at resonance is $\omega_o L/R$. This p.d. is in opposite phase to V_c, which is why it is only V_o that drives the resonant current through R.

3.6.1 The Resonance and Response curves



(a) (a)

Fig. 3.13

Resonance in an L – C – R circuit may be achieved in two ways:

(i) by keeping L, C and R fixed and vary ω (that is the frequency, f)

(ii) by keeping ω fixed and varying L or C (usually C).

In case (i), the graph of the r.m.s, I against ω is called the resonance curve (Fig. 3.13a). In case (ii) the graph of I against C is called the response curve (Fig. 3.13b).

The sharpness of both curves depends on the amount of resistance and therefore on the Q of the circuit. It can be shown that if the width of the resonance curve at the half-power points $(I = I_o/\sqrt{2})$

is ΔW then $\frac{\omega}{\Delta\omega} = Q$ ………………….3.21

where I_o is the value of the current at resonance.

Similarly, it can be shown that if the width of the response curve at the half power points is ΔC, then

$$\frac{2C}{\Delta C} = Q \quad \text{……………3.22}$$

3.7 Power in a.c. Circuits

At an instant when there is a voltage $V = V_o \cos(\omega t + \theta)$ across a circuit element carrying a current I = Io coswt, the instantaneous power supplied to the circuit element is

$$P = VI = V_o I_o \cos \omega t \cos(\omega t + \theta)$$

Using the identity, $\cos(\omega t + \theta) = \cos \omega t \cos \theta - \sin \omega t \sin\theta$, we have

$$P = V_o I_o \cos\theta \cos^2 \omega t - V_o I_o \sin\theta \, \omega t \sin \omega t$$

For a complete cycle, the average value of coswtsinwt is zero and the average value of $\cos^2 \omega t$ is ½ . Therefore the average powe input to the device is

$$P_{av} = \tfrac{1}{2} V_o I_o \cos\theta$$
$$= \left(\frac{V_o}{\sqrt{2}}\right) \left(\frac{I_o}{\sqrt{2}}\right) \cos\theta$$

$$= V_e I_e \cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.23$$

where V_e and I_e are effective value of voltage and current respectively.

The term cos θ is called the power factor of the circuit element. For a resistnace, $\theta = 0$, $\cos\theta = 1$, and $P_{av} = V_e I_e = I^2_e R$.
For a pure inductance or capacitance, $\theta = \pm 90^0$, $\cos\theta = 0$, and $P_{av} = 0$. In other words, although power is supplied to an inductor or a capacitor during one part of a cycle, this power is returned during the other part of the cycle and the average power input is zero.

All the average power supplied to a passive circuit element is dissiputed in the reistance component R of the element's impedance. You can verifying this by noting from Fig. 3.10, for example, that for a passive circuit element. $\text{Cos}\ \theta = R/Z_1\ Ve = I_e Z$, and therefore.

$$P_{av} = V_e I_e \cos\theta = RI_e^{2}.$$

4.0 Conclusion

We have introduced a new set of passive elements into the a.c. circuit and their characteristic responses to transient currents have been described by phasors and phasor diagrams. A series circuit containing all the three elements, R, C and L has been analysed and used to explain the phenomenon of resonance in an electrical circuit. It is shown that series resonant circuits have low impedance over a narrow band of frequencies and relatively high impedances at other frequencies.

5.0 Summary

* A passive element has an impedance characterised by a magnitude $Z = V_o/I_o$ and an angle θ, or by a resistive component $R = Z \cos\theta$ and a reactive component $X = Z \sin\theta$

$$Z^2 = \sqrt{X^2 + R^2} \qquad \text{and } \tan\ \theta \quad X/R$$

The impedance of an R – L – C combination has a resistive component R and an inductive component $X = \omega L - 1/\omega C$

* For circuit elements connected in sereis, instantaneous voltages add algebraically but voltage amplitudes add vectorially. Each voltage $V = V_o \cos(\omega t + \theta)$ is represented by a phasor of magnitude V_o at an

angle θ with the current phasor. The phasor representing the voltage across a series combination is the vector sum of the phasors respresenting the voltages across the individual circuit elements.

* Impedance in series add like vectors.

* When the frequency of the voltage across an R –L-C series circuit is varied, maximum current (and minimum impedance $Z_{min} = R$) occur at the resonant frequency $f_o = 1/2\pi \sqrt{LC}$.

* When the voltage across a circuit and the current through it have a phase difference θ and have effective vluaes V_e and I_e, the average power input to the circuit element is

$$P_{av} = V_e I_2 \cos \theta.$$

6.0 Tutor Marked Assignments (TMA)

(1a) Find the phase difference between the voltage V and the current I given in SI units by

$$V = 10 \sin 20t$$
$$I = 0.30 \cos 20t$$

(b) The instantaneous current 1 across a 200-ohm resistor is given in SI unit by $I = 10 \cos 377t$

(i) Find the frequency and the period

(ii) Find the voltage across the resistor when $t = \frac{1}{120}$ second

(2a) In an R-L series circuit, the voltage across the resistor has an amplitude of 50V and the voltage across the inductor has an amplitude of 120V. Find the amplitude of the voltage across the R-L series combination. What is the phase difference between this voltage and the current?

(2b) What is the capacitance of the capacitor required in series with a 40-mH inductance coil to provide a circuit which resonates at a frequency of 60Hz?

(3) A 60 – Hz alternating voltage of 110V effective value is applied to a series circuit having an inducatance L of 1.00H, a capacitance C of 2.00mf, and a resistance R of 400Ω. Calculate the:

(a) Inductive reatance

(b) Capacitive reactance

(c) Total impedance

(d) Effective current in the circuit

(e) Power factor of the circuit.

7.0 References and Other Resources

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UNIT 17

ALTERNATING CURRENT THEORY I1

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1.0 Introdution

This unit builds on the concepts introduced in the earlier unit on alternating current theory 1. Emphasis is given to the fundamental idea about a.c. circuit.s

The capacitor and the inductor are introduced in a way that will enable you to understand why these elements have useful characteristics. Basic definitions and waveforms are used as aids in the development of the impedance relations for the inductor and the capacitor. The influence of element size and source frequency on imedance is given.

The mathematical models of the a.c. circuits are given to show the usefulensss of these models. Problems with solutions are given to help you learn how to use the basic models for the circuits element used in a.c circuits.

2.0 Objectives

After studying this unit you should be able to :

* distinguish between passive and active circuit elements.
* represent sinusoidal functions such as I and V by phasors or a phasor diagram.

* determine the current at instant t in terms of the appropriate parameters of the circuit element and of the voltage across the circuit element.
* analyse a.c. circuits by applying kirchhoff's rules to instantaneous voltages and currents.
* explain the phenomenon of resistance in electric currents.
* define the power factor of a circuit element
* calculate power in a.c. circuits.

3.1 Phasor and Phasor Diagram

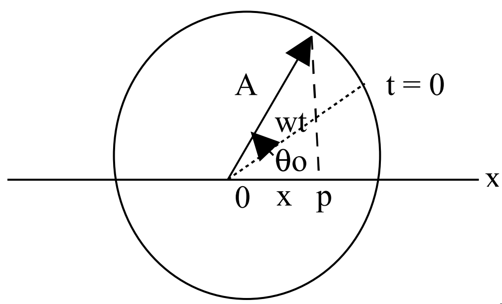


fig. 3.1

You would recall that simple harmonic motion (s.h.m) is the particular type of oscillatory motion executed, for example, by the point P in fig. 3.1 as the arrow of constant length A rotates about the origin with a constant angular speed ω. The point P is always at the foot of the perpendicular from the tip of the arrow to the x-axis. At the instant t, the arrow makes an anlge

$$\theta = \omega t + \theta_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

with the x-axis.

The arrow in fig. 3.1 does not have a direction in space. It is an arrow drawn on the diagram to repreent the length A and the angle θ. Such a quantity is called a phasor to avoid confusion with vectors that do have direction in space. Formally, we define a phasor to be a mathematical quantity characterised by a magnitude (A) and an angle (θ) with algebraic properties that will be presented as required. (you will see in your mathematics course that phasors are called complex numbers).

As time elapses, θ changes and the phasor rotates about a perpendicular axis through its 'tail', which is fixed at the point 0. The point P moves along the X axis, and its position coordinates x has the time dependence that is characteristic of s.h.m.

$$x = \text{projection of A on X-axis}$$
$$= A\cos\theta$$
$$= A\cos(\omega t + \theta_o) \ldots\ldots\ldots\ldots\ldots 3.2$$

where A, θ_o and w are constants.

3.2 Passive and Active circuit Elements

An electronic component, such as a capacitor or resistor, that is incapable of amplification is referred to as a passive element. On the other hand, an electronic component, such as a transistor, that is capable of amplification is said to be an active element.

3.3 Phasor Representation of Alternating current

Suppose that the voltage V across a given circuit element at the instant t is a sinusoidal function of time with angular frequency $\omega = 2\pi f$, given by

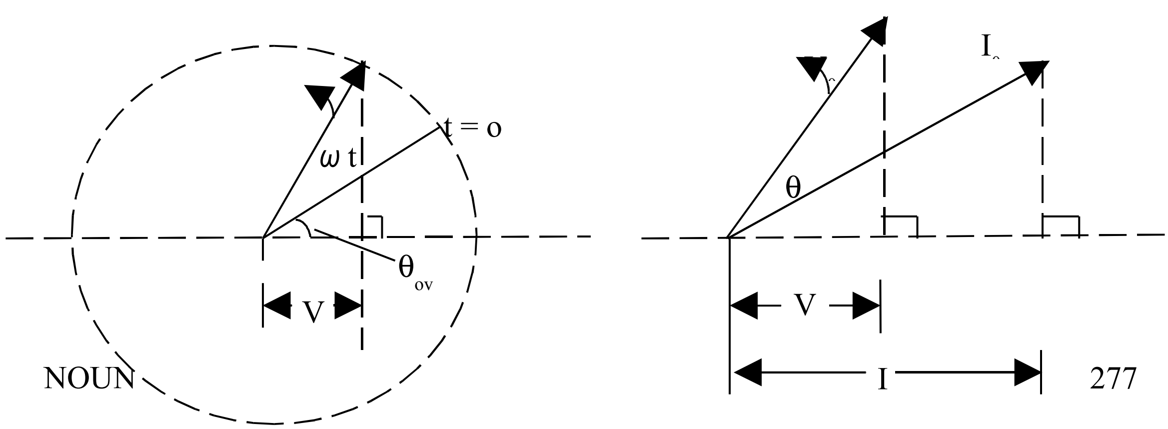
$$V = V_o \cos(\omega t + \theta_{ov}) \qquad \text{..................... 3.3}$$

You recall that the positive constant V_o is the amplitude of this function. This is the largest value attained by the instantaneous voltage. The constant θ_{ov} is the initial phase angle of the voltage function.

The current I in the circuit element is a sinusoidal function of the same angular frequency ω as that of the voltage V across the circuit element. The current at the instant t is therefore a function of the form.

$$I = Io \cos(\omega t + \theta_{oI}) \qquad \text{.....................3.4}$$

This function has an amplitude Io and an initial phase angle $\theta_{oI.}$



(a) (b)

Fig. 3.2

Following the discussion in section 3.1 of this unit, we can represent sinusoidal functions such as I and V by phasors on a phasor diagrm. The voltage phasor is represented by an arrow of length Vo that makes an angle wt + θov with the horizontal axis (Fig. 3.2a). The instantaneous voltage V is equal to the projection of this rotating phasor on the horizontal axis. Similarly the current phasor is represented by an arrow of length Io at an angle $\omega t + \theta_{oI}$ with the horizontal axis.

The angle θ between the voltage phasor and the current phasor is called the phasor difference between the voltage and the current (Fig. 3.2b). This phasor difference θ is the difference of the phase angles of the functions V and I:

$$\theta = (\omega t + \theta_{ov}) - (\omega t + \theta_{oI}) = \theta_{ov} - \theta_{oI} \quad \ldots\ldots\ldots\ldots 3.5$$

If θ = 0, the voltage and the current are said to be in phase, otherwise they are out of phase. For circuit elements that include capacitors and indicators, we shall find that the current and the voltage are generally out of phase. The phase difference θ is the most important quantity in a.c. circuit analysis.

The difference $\theta_{ov} - \theta_{oI}$ of the initail phase angles is significant (being the phase difference θ), but actual values of these angles are determined by just which instance is selected as the initial instant, t = 0 for simplicity we can choose the initial instant to be a time when the current I reaches its peak value I_o. Then $\theta_{oI} = 0$, $\theta_{ov} = \theta$, and the functions V and I are

$$V = V_o \cos(\omega t + \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.6$$
$$I = I\cos \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.7$$

Example

The instantaneous voltage V and current I are given in SI units by

$$V = -155\cos(377t - \pi/2)$$
$$I = 2.0\sin 377t$$

Find the phase difference between the voltage and the current.

Solution

Perhaps the simplest method is to locate the phasor representing V and I at some particular instant. For example, at the time t such that 377t = π/2, V = - 155cos (π/2 - π/2) = 155 volts and I = 2.0A.

The phasor diagram at this instant is shown in Fig. 3.3.

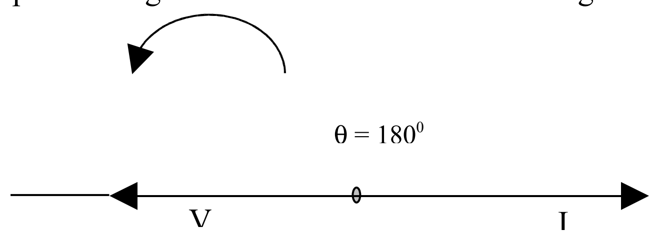


Fig. 3.3

The angle between the voltage phasor and the current phasor, which is the phase difference θ, is seen to be 180^0 or π rad.

Another method (Analytical)

To find the phase difference analytically, we first write V and I in the same standard form with a positve constant multiplying a cosine function of time. Using cos (α + π) = -cos α, we can write the voltage function as

$$V = 155 \cos (377t - \pi/2 + \pi).$$

Using cos (α - π/2) = sin α, we can rewrite the current function as

$$I = 2.0\cos (377t . \pi/2).$$

Then the phase difference is

$$\theta = (377t - \pi/2 + \pi) - (377t - \pi/2).$$

$$= \pi.$$

3.4 Impedance of Resistor Capacitors and Inductors.

In an a.c. circuit, any positive circuit element is characterised by two numbers, Z and θ, where θ is the phase difference between the voltage across the element and the current through it, and Z is the ratio of the voltage amplitude V_o to the current amplitude I_o.

$$Z = \frac{V_o}{I_o} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

These two numbers, Z and θ, determine what is called the impedance of the circuit elements, Z is called the magnitude of the impedances and θ the angle of the impedance .

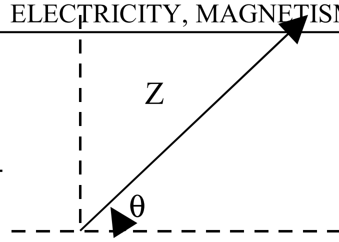


Fig. 3.4

Fig. 3.4 shows that the impedance of a circuit element is a phasor with a magnitude Z and an angle θ. (Note that the angle θ is fixed; impedance phasors do not rotate).

The relationship between the current and the voltage is determined by the impedance of a circuit element. If Z and θ are known, the current function $I = I_o \cos \omega t$ can be determined if the voltage function. $V = V_o \cos(\omega t + \theta)$ is known, and vice versa.

3.4.1 Impendance of a Resistor

We shall consider only resistors which obey 0hm's law. If the current through the resistor of resistance R is I = Iocoswt, the voltage across the resistor is

$$V = RI = R I_o \cos \omega t.$$

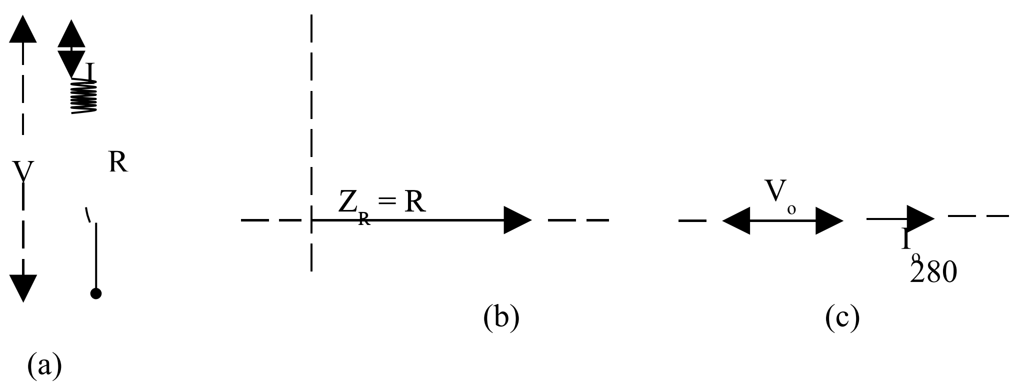
This voltage function has an amplitude given by

$$V_o = RI_o$$

Therefore the magnitude of the impedence of a resistor is

$$Z = \frac{V_o}{I_o} = R \quad \ldots\ldots\ldots\ldots\ldots\ldots..3.9$$

The phase difference θ between the voltage function and the current function is zero, that is voltage and the current are in phase.



These results show that the impedance of a resistor is a phasor of magnitude R and angle zero, which is represented on an impedance diagram as shown in fig. 3.5 (b).

The phasors representing the current and the voltage are shown in figure 3.5 (c) at the instant t = 0. These phasors each rotate counter-clockwise with an angular speed ω.

3.4.2 Impedance of an Inductor

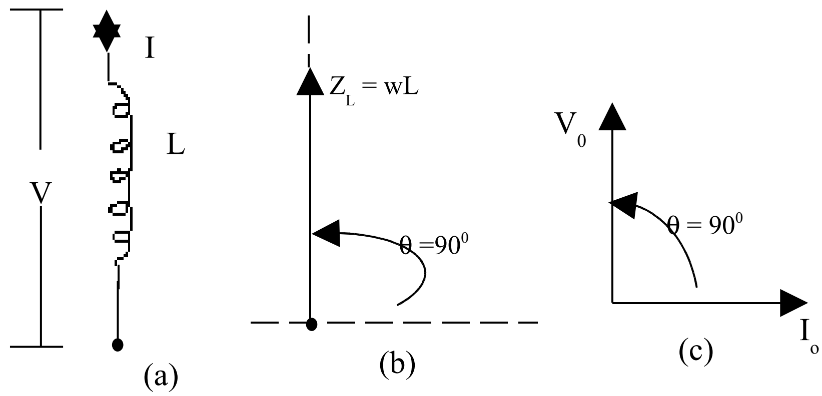


Fig. 3.6a shows a pure inductor which has a self-inductance L but no resistance. If the current through the induction is I = Io coswt, the voltage across the inductor is

$$V = L\frac{dI}{dt} = -\omega L I_o \sin \omega t = \omega L I_o \cos(\omega t + \pi/2)$$

The voltage function has the amplitude

$$V_o = wL\, I_o$$

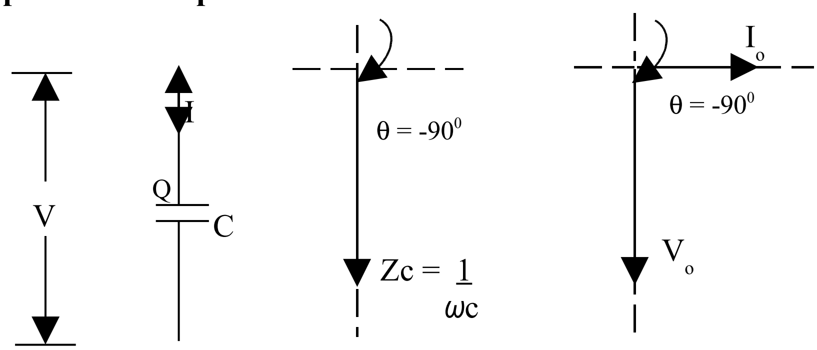
Therefore, the magnitude of the impedance of a pure inductor is

$$Z_L = \frac{V_o}{I_o} = \omega L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

The phase difference between the current function and the voltage function is $\pi/2$. The impedance of a pure inductor is therefore a phasor of magnitude ωL and angle $\pi/2$, as shown if Fig. 3.6b. Notice that Z_L is proportional to the frequency ($\omega = 2\pi f$).

Rotating phasors representing the voltage and current functions are shown in Fig. 3.6 at the instant t = 0. The current and the voltage are 90^0 out of phase, with the voltage leading the current.

3.4.3 Impedance of Capacitor



(a) (b) (c)

If charge flows to the upper plate of the capacitor of Fig. 3.7 (a) at the rate

$$\frac{dQ}{dt} = I = I_o \cos \omega t$$

The charge on this plate at the instant t is given by the function

$$Q = \int_0^t I\, dt' = I_o \int_0^t \cos \omega t'\, dt' = \frac{I_o}{\omega} \sin \omega t$$

The voltage across the capacitor is

$$V = \frac{Q}{C} = \frac{I_o}{\omega c} \sin \omega t = \frac{I_o}{\omega c} \cos (\omega t - \pi/2)$$

The voltage function has the amplitude

$$V_o = I_o/\omega C$$

Therefore, the magnitude of the impedance of a capacitor is

$$Zc = \frac{Vo}{Io} = \frac{1}{\omega c} \qquad ....3.11$$

The phase difference θ between the voltage function and the current function is - π/2. The impedance of a capcitor is therefore a phasor of magnitude $1/\omega C$ and angle -π/2, as shown in Fig. 3.7 (c) . Notice that the magnitude of the impedance of a capacitor is large at low frequencies an small at high frequencies.

Rotating phasors represening the voltage and the current functions are shown in fig. 3.7 (c) at the instant t = 0. The voltage lags the current by 90^0.

Example

(i) Find the magnitude Zc of the impedance of a 1.00-mf capacitor at f = 60Hz.

(ii) A 60 – Hz alternating voltage with an amplitude of 4.00 V is applied across a 1.00 – mH inductor. Find the current amplitude.

Solution

(i) Using equation 3.11, at f = 60Hz

$$Z_c = \frac{1}{2\pi\,(6oH_z)\,(1.00 \times 10^{-6}\ F)} = 2.65 \times 10^3 \text{ ohm.}$$

(ii) From Eq. 3.10, at f = 60 Hz

$$Z_L = wL = 2\pi\,(60Hz)\,(1.00 \times 10^{-3}H) = 0.377 \text{ ohm}$$

Therefore, the current amplitude is given by

$$Io = \frac{Vo}{Z_L} = \frac{4.00V}{0.377\Omega} = 10.6A$$

3.5 Impedance of Series Circuits

3.5.1 Impedence of an R-L-C Series combination

At any instant, the current I is the same in each element of the series combination of Fig. 3.8.

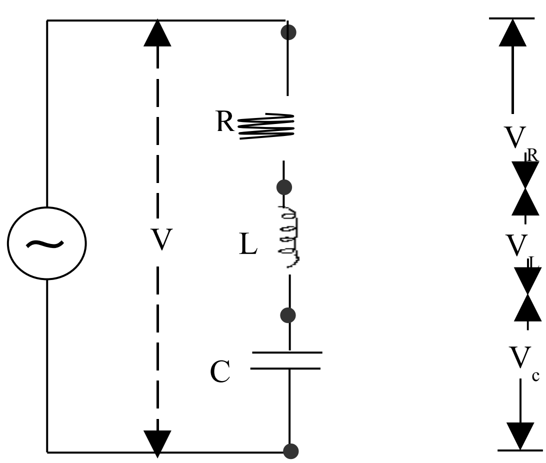


Fig. 3.8

Suppose this current is $I = I_o \cos \omega t$. We wish to determine the voltage $V = V_o \cos (\omega t + \theta)$ across the series combination. At any instant, this voltage is the sum of the voltages across each element of the series combination:

$$V = V_R + V_L + V_c \ldots\ldots\ldots\ldots\ldots 3.12$$

Using previous results, we can express the voltage functions for each circuit element as

$$V_R = V_{oR} \cos \omega t$$
$$V_L = V_{oL} \cos (\omega t + \pi/2)$$
$$V_c = V_{oc} \cos (\omega t - \pi/2)$$

Where the subscript 0 denotes the amplitude of the corresponding function.

Equation 3.12 requires that

$$V_o \cos (\omega t + \theta) = V_{oR} \cos wt + V_{oL} \cos (\omega t + \pi/2) + Voc \cos (wt - \pi/2) \ldots. 3.13$$

The problem is to detrmine V_o and θ in terms of V_{oR}, V_{oL} and V_{oc}. For this purpose, we shall use the method of phasor additon.

Since the current is the same for each element in a series circuit, the current phasor provides a convenient reference phasor which is conventionally shown pointing to the right in the phasor diagram (corresponding to the instant that we have selected as t = 0). The direction of each voltage phasor is determined from the phase difference between the voltage and the current as we saw in section 3.4 (see Fig. 3.9.)

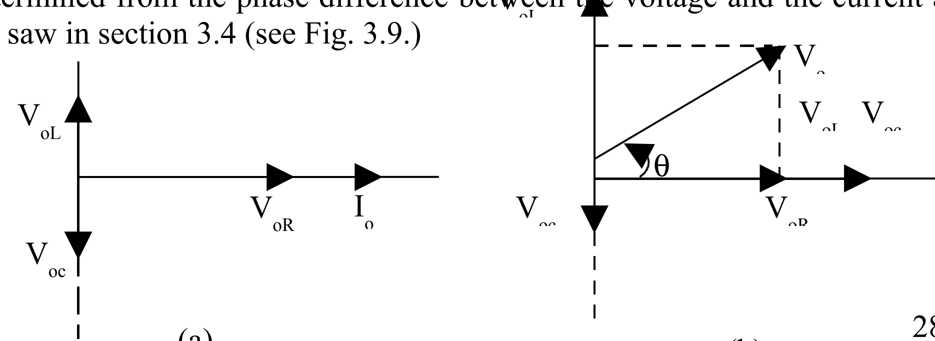


Fig 3.9

We can interpret Eq. 3.12 as an equation relating the components of phasors along the direction of the current phasor, we conclude that the phasor representing V is the vector sum (or phasor sum) of the phasor representing V_R V_L and V_c.

In Fig. 3.9 (b), the phasor representing V has a magnitude V_o and makes an angle θ with the current phasor. Applying Pythagoras's theorem,

$$V_o = \sqrt{V_{oR}^2 + (V_{oL} - V_{oc})^2}$$

$$= I_o \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$$

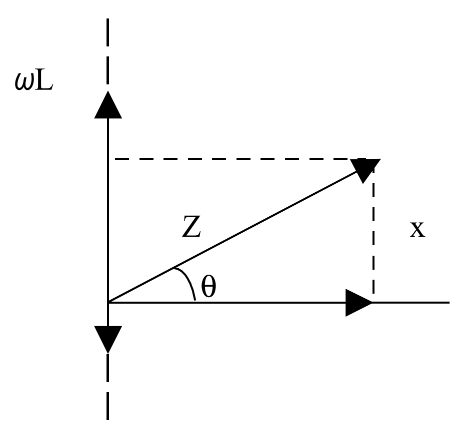
Which shows that the magnitude Z = Vo/Io of the impedance of the series combination is given by

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2} \quad \text{.........3.14}$$

From the phasor diagram, fig 3.9, we see that the angle θ of this impedence is such that

$$\tan\theta = \frac{V_L - V_c}{V_R} = \frac{\omega L - (1/\omega c)}{R} \quad \text{............3.15}$$

This impedance is represented on the impedance diagram of Fig. 3.10



$\frac{1}{\omega c}$ R

Fig. 3.10

3.5.2 Resistive and Reactive Componets of an Impedance

Instead of specifying an impedance by giving it magnitude Z and angle θ, we can give its horizontal and vertical components on an impedance diagrm. These components are called resistive component (or the resistance R) and the reactive component (or the reactance X). Thus a pure inductor of inductance L has an impedance specified by R = O, $X_L = \omega L$. The impedance of a capacitor of capacitance C has the components R = 0, Xc = -1/wc.

From Fig. 3.10 it can be seen that the relationship between $(Z_1\theta)$ and and $(R_1$ X) are

$$Z = \sqrt{R^2 + X^2 \text{ and } \tan\theta} = \frac{X}{R} \quad \ldots\ldots\ldots 3.16$$

$$\text{Or } R = Z\cos\theta \text{ and } X = Z\sin\theta \quad \ldots\ldots\ldots 3.17$$

3.5.3 Phasor Addition of Impedance in Series

If an impendance having components $(R_1\ X_1)$ is connected in series with an impedance having components (R_2, X_2), this combination is equivalent to a single impedance with components $(R_s\ X_s)$ given by:

$$R_s = R_1 + R_2 \text{ and } X_s = X_1 + X_2 \quad \ldots\ldots\ldots\ldots 3.18$$

⧈ **Example**

Consider the circuit in Fig. 3.11

(a) From the amplitudes of the voltages across individual circuit elements, determine the amplitude V_o of the voltage across the entire part of the circuit.
(b) Calculate the impendace Z of this external circuit and use this to dertermine V_o.

(c) Determine the phase difference between the a.c. voltage and the current in the circuit

(d) Draw the impedance diagram for the R – L – C circuit element.

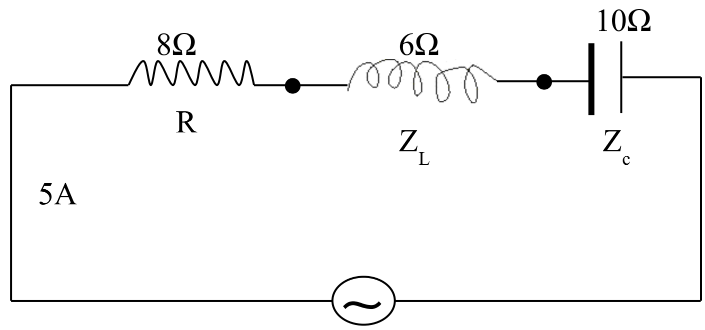


Fig. 3.11

Solution

$V_{oR} = IoZ_R = 5.0 \times 8 = 40.0V$, and the phase angle $\theta_R = 0$

$V_{oL} = Io\, Z_L = 5.0 \times 6.0 = 30.0V$, and the phase angle $\theta_L = +90^0$

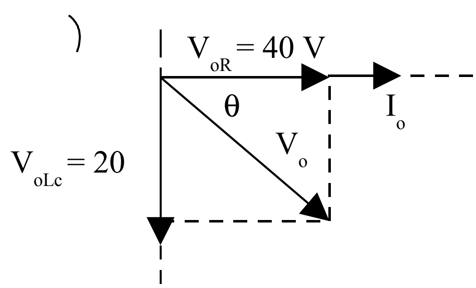
$V_{oc} = Io\, Z_c = 5.0 \times 10.0 = 50.0$ V and the phase angle $\theta_c = -90^0$

Because of the 180^0 phase difference between the voltage across the inductor and the voltage across the capacitor, the amplitude of the voltage across these two component is given by

$$V_{oLc} = V_{oc} - V_{oL} = (50.0 - 30.0)V = 20.0V$$

Since V_{oc} is greater than V_{oL}, the phasor of magnitude V_{oLc} points downward on a phasor diagram. Now, evaluating the phasor sum of the phasors of magnitude V_{oLc} and V_{oR}, we have

$$V_o = \sqrt{40.0^2 + 20.0^2} = 44.7V$$



(b) $Z = \sqrt{R^2 + (X_L + Xc)^2}$

$= \sqrt{8^2 + (6 - 10)^2}$

$= 8.94\ \Omega$

Using $V_o = I_o Z$, we obtain

$$V_o = (5.0A)(8.94\ \Omega) = 44.7V$$

In agreement with the result of part (a)

(c) From the phasor diagram, we see that

$$\tan\theta = \frac{-20}{40} = -0.500 \text{ and } \theta = -26.6^0$$

(d) The impedance diagram is shown in Fig. 3.12

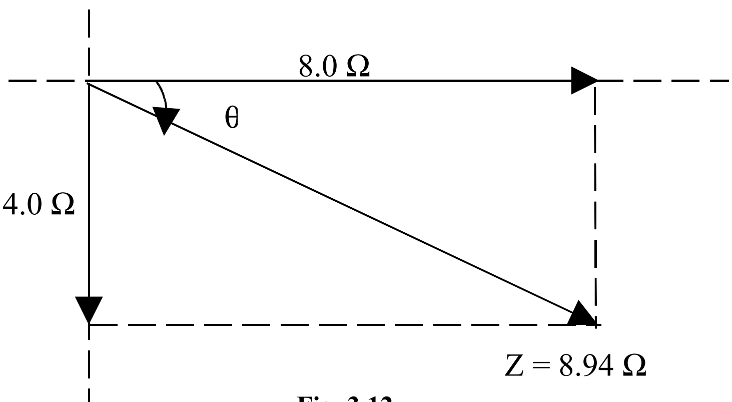


Fig. 3.12

3.6 Series Resonance

Eq. 3.4 shows that in the L-C-R series circuit, the magnitude of the impendance is a function of the frequency:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.18$$

If an a.c. generator of constant voltage amplitude Vo but variable frequency is connected across the R-L-C circuit, the current amplitude, $I_o = V_o/Z$, depends on the frequency.

The frequency f at which the current amplitude I_o is a maximum is called the resonant frequency f_o of the circuit. Maximum I_o will be obtained at the frequency which minimizes Z.
It follows from Eq. 3.18 that Z is a minimum at an angular frequency ω_o such that

$$\omega_o L - \frac{1}{\omega_o C} = O$$

The resonant frequency therefore is

$$f_o = \frac{\omega_o}{} = \frac{1}{} \quad \ldots\ldots\ldots\ldots\ldots\ldots.3.19$$

$$2\pi \qquad 2\pi\sqrt{LC}$$

and at resonance, $Z = R$

Now, the r.m.s. p.d. across the capacitor at resonance is

$$V_c = I_o \times \frac{1}{\omega C} = \frac{V_o}{R} \times \frac{1}{\omega_o C}$$

$$\therefore V_c/V_o = \frac{1}{W_o CR} = \frac{W_o L}{R} \quad \ldots\ldots\ldots.3.20$$

The quantity V_c/V_o is known as the Q of the circuit, it can be much greater than 1, therefore V_c can be much greater than V_o. This is important, as if V_c is too large, it may cause the capacitor to break down.

Similarly, the r.m.s p.d. across the inductor at resonance is $\omega_o L/R$. This p.d. is in opposite phase to V_c, which is why it is only V_o that drives the resonant current through R.

3.6.1 The Resonance and Response curves

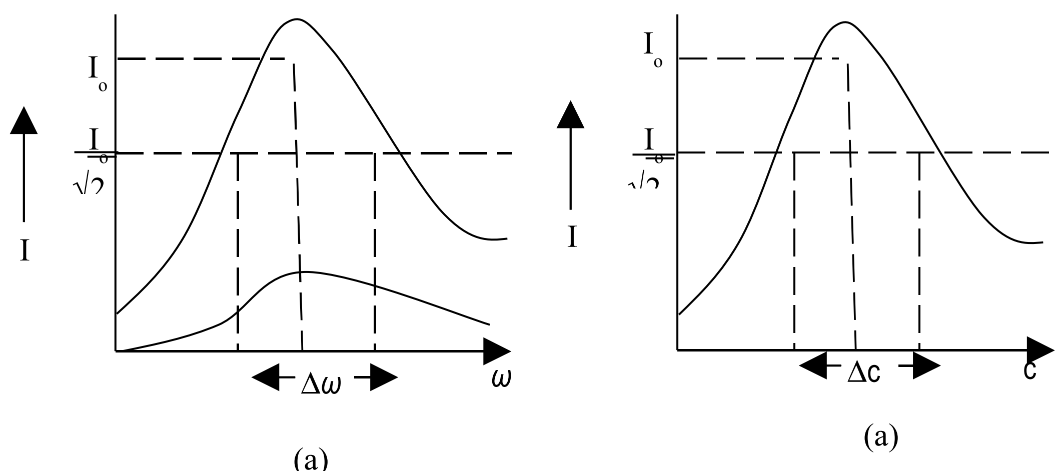


Fig. 3.13

Resonance in an L – C – R circuit may be achieved in two ways:

(i) by keeping L, C and R fixed and vary ω (that is the frequency, f)

(ii) by keeping ω fixed and varying L or C (usually C).

In case (i), the graph of the r.m.s, I against ω is called the resonance curve (Fig. 3.13a). In case (ii) the graph of I against C is called the response curve (Fig. 3.13b).

The sharpness of both curves depends on the amount of resistance and therefore on the Q of the circuit. It can be shown that if the width of the resonance curve at the half-power points $(I = I_o/\sqrt{2})$

is ΔW then $\frac{\omega}{\Delta\omega} = Q$ …………………3.21

where I_o is the value of the current at resonance.

Similarly, it can be shown that if the width of the response curve at the half power points is ΔC, then

$$\frac{2C}{\Delta C} = Q \quad \ldots\ldots\ldots\ldots\ldots 3.22$$

3.7 Power in a.c. Circuits

At an instant when there is a voltage $V = V_o \cos(\omega t + \theta)$ across a circuit element carrying a current I = Io coswt, the instantaneous power supplied to the circuit element is

$$P = VI = V_o I_o \cos\omega t \cos(\omega t + \theta)$$

Using the identity, $\cos(\omega t + \theta) = \cos\omega t \cos\theta - \sin\omega t\sin\theta$, we have

$$P = V_o I_o \cos\theta\cos^2\omega t - V_o I_o \sin\theta\, \omega t \sin\omega t$$

For a complete cycle, the average value of coswtsinwt is zero and the average value of $\cos^2\omega t$ is ½ . Therefore the average powe input to the device is

$$P_{av} = \tfrac{1}{2} V_o I_o \cos\theta$$
$$= \left(\frac{V_o}{\sqrt{2}}\right)\left(\frac{I_o}{\sqrt{2}}\right)\cos\theta$$

$$= V_e I_e \cos\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.23$$

where V_e and I_e are effective value of voltage and current respectively.

The term cos θ is called the power factor of the circuit element. For a resistnace,$\theta = 0$, $\cos\theta = 1$, and $P_{av} = V_e I_e = I^2_e R$.

For a pure inductance or capacitance, $\theta = \pm 90^0$, $\cos\theta = 0$, and $P_{av} = 0$. In other words, although power is supplied to an inductor or a capacitor during one part of a cycle, this power is returned during the other part of the cycle and the average power input is zero.

All the average power supplied to a passive circuit element is dissiputed in the reistance component R of the element's impedance. You can verifying this by noting from Fig. 3.10, for example, that for a passive circuit element. Cos $\theta = R/Z_1$ $Ve = I_e Z$, and therefore.

$$P_{av} = V_e I_e \cos \theta = RI_e^{2.}$$

4.0 Conclusion

We have introduced a new set of passive elements into the a.c. circuit and their characteristic responses to transient currents have been described by phasors and phasor diagrams. A series circuit containing all the three elements, R, C and L has been analysed and used to explain the phenomenon of resonance in an electrical circuit. It is shown that series resonant circuits have low impedance over a narrow band of frequencies and relatively high impedances at other frequencies.

5.0 Summary

* A passive element has an impedance characterised by a magnitude $Z = V_o/I_o$ and an angle θ, or by a resistive component $R = Z \cos \theta$ and a reactive component $X = Z \sin\theta$

  $$Z^2 = \sqrt{X^2 + R^2} \qquad \text{and } \tan \theta \quad X/R$$

  The impedance of an R – L – C combination has a resistive component R and an inductive component $X = \omega L - 1/\omega C$
* For circuit elements connected in sereis, instantaneous voltages add algebraically but voltage amplitudes add vectorially. Each voltage $V = V_o \cos (\omega t + \theta)$ is represented by a phasor of magnitude V_o at an angle θ with the current phasor. The phasor representing the voltage across a series combination is the vector sum of the phasors respresenting the voltages across the individual circuit elements.

* Impedance in series add like vectors.

* When the frequency of the voltage across an R –L-C series circuit is varied, maximum current (and minimum impedance $Z_{min} = R$) occur at the resonant frequency $f_o = 1/2\pi \sqrt{LC}$.

* When the voltage across a circuit and the current through it have a phase difference θ and have effective vluaes V_e and I_e, the average power input to the circuit element is

  $$P_{av} = V_e I_2 \cos \theta.$$

6.0 Tutor Marked Assignments (TMA)

(1a) Find the phase difference between the voltage V and the current I given in SI units by

$$V = 10 \sin 20t$$
$$I = 0.30 \cos 20t$$

(b) The instantaneous current 1 across a 200-ohm resistor is given in SI unit by $I = 10 \cos 377t$

(i) Find the frequency and the period

(ii) Find the voltage across the resistor when $t = \frac{1}{120}$ second

(2a) In an R-L series circuit, the voltage across the resistor has an amplitude of 50V and the voltage across the inductor has an amplitude of 120V. Find the amplitude of the voltage across the R-L series combination. What is the phase difference between this voltage and the current?

(2b) What is the capacitance of the capacitor required in series with a 40-mH inductance coil to provide a circuit which resonates at a frequency of 60Hz?

(3) A 60 – Hz alternating voltage of 110V effective value is applied to a series circuit having an inducatance L of 1.00H, a capacitance C of 2.00mf, and a resistance R of 400Ω. Calculate the:

(a) Inductive reatance

(b) Capacitive reactance

(c) Total impedance

(d) Effective current in the circuit

(e) Power factor of the circuit.

7.0 References and Other Resources

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UNIT 18

ELECTRIC, PHOTOELECTRIC AND THERMIONIC EFFECTS

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1.0 Introduction

We have seen that electric energy is readily changed into heat, but the inverse process was discovered in 1821 by T.J. Seebeck (1770 – 1831), who actually found that a magnetic field surrounded by a circuit consisting of two metal conductors only if the junctions between the metals were maintained at different temperatures. The generation of an e.m.f in a circuit contaning two different metals or semiconductors, when the junctions between the two are maintained at different temperatures is known as Seebeck or thermoelectric effect. The magnitude of the e.m.f depends on the nature of the metals and the difference in temperature.

Another process of producing electricity from other sources of energy is the photoelectric effect. This is the liberation of electrons from a substance exposed to electromagnetic radiation. An electron emitted from a sustance by irradiation as a result of the photoelectric effect is called a photoelectron. In the photoelectric effect, the energy of photons is converted into electrical energy .

We shall conclude this unit with the study of thermionic emission and we shall see that at high temperature a number of the conduction electrons have enough energy to escape from the metal.

2.0 Objectives

After studying this unit, you should be able to:

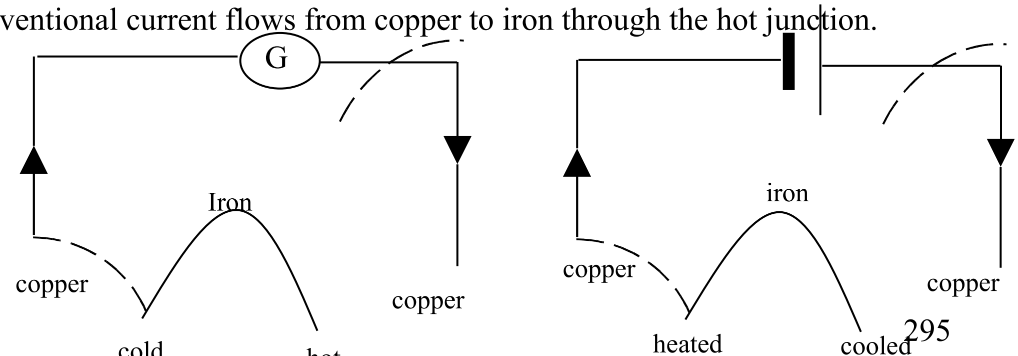
* distinguish between Seebeck and Peltier effects
* define thermoelectric e.m.f
* explain the thermoelectric phenomenon
* state the laws of intermediate metals and intermediate temperatures.
* sescribe the thermoelectric series
* sxplain the photoelectric effect
* define the work function, stapping potential, threshold frequency and electron -volt
* explain simple problems involving the use of the photoelectric equation.

3.1 Thermoelectric Effect

3.1.1 Seebeck and Peltier Effects

If two different metals such as copper and iron are joined in a circuit and their juctions are kept at different temperatures, a small e.m.f is produced and current flows (see Fig. 3.1). The effect is known as the thermoelectric or Seebeck effect and the pair of junctions I called a thermocouple.

If the junctions of the iron-copper thermocouple in Fig. 3.1a are maintained at 0^0C and 100^0C respectively, the thermo-e.m.f is of the order of 1 mV; the conventional current flows from copper to iron through the hot junction.



(a) Fig. 3.1 (b)

The inverse effect was discovered by peltier in 1834. If a current passes through a circuit containing two different metals, heat is generated at one junction and absorbed at the other (Fig. 3.1b). The effect occurs whether the current is driven by an external cell or whether it is generated by the thermocouple itself. Thus when the Seebeck effect occurs, heat is continually absorbed at the hot junction and developed at the cold junction; likewise when the Peltier effect occurs it tends to set up a Seebeck e.m.f. which opposes the current.

The Seebeck efect is due to the migration of electrons from one of the metals to the other at the junctions of a thermocouple. In the copper-iron thermocouple (fig.3.1a), for example, electrons move from iron to copper at both the hot and cold junctions, establishing a potential difference at each junction. However, at the hot junction, more electrons migrate from iron to copper than at the cold junction. The potential differences at the junctions oppose each other, like batteries connected positive to positive and negative to negative, but the potential difference at the hot junction is the greater. The total e.m.f in the circuit is such as to send an electric current from the hot junction to the cold in the iron.

Several thermocouples connected in series constitute a thermopile, which is more sensitive to temperature changes than a single thermocouple and can be used as a thermometer after it has been calibrated by being subjected to known temperature differences. Temperature differences as well as a millionth of a celsius degre can be measured by these devices. Thermopiles have been used in biological research to study the heat evolved by living tissue.

3.1.2 Laws of Intermediate Metals and Intermediate Temperatures

The following two laws have been established experimentally

(a) If A, B and C are three different metals, the thermoelectric e.m.f of the couple AC is equal to the sum of the e.m.f's of the couples AB and BC over the same temperature range.

It follows that the junctions of a thermocouple may be soldered without affecting the e.m.f.

(b) The e.m.f. of a thermocouple, with junctions at tmperatures θ_1 and θ_3, is the sum of the e.m.f's of two couples of the same metals with junctions at θ_1 and θ_2, and at θ_2 and θ_3, respectively.

It follows that when a galvanometer is connected to a thermocouple, as in fig. 3.1a, the e.m.f is independent of the temperature of the galvanometer.

3.1.3 Measurement of Thermoelectric e.m.f.

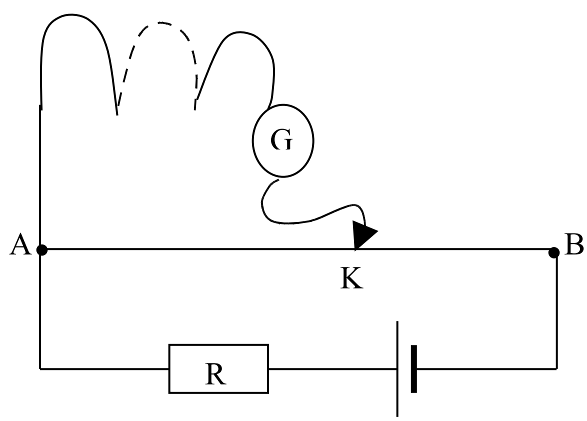


Fig. 3.2

The e.m.f of a thermocouple may be measured by means of a potentiometer. Since the thermo-e.m.f. is of the order of a millivolt, it is necessary to arrange that the potential drop across the potentiometer wire is only slightly more than this by putting a resistance R in series (see Fig. 3.2). Then if the e.m.f of the cell is E, and r is the resistance of the potentiometer wire, the potential drop across the potentiometer wire is

$$\left(\frac{r}{\quad}\right) E$$

The internal resistance of the cell has been ignored, but R must be several hundred ohms and hence no appreciable error is introduced.

The junctions of the thermocouple are maintaned at the desired temperatures, for example, those of melting ice and boiling water, while the balance point K is found.

$$\text{Then e.m.f of thermocouple} = \frac{AK}{AB} \cdot \frac{rE}{R+r}$$

The value of E may be found with moderate accuracy by means of a voltmeter.

3.1.4 Factors Affecting the Thermo e.m.f.

We can use the arracngement of Fig. 3.2 to study the Seebeck effect in more details by measuring the e.m.f of the thermocouple at several temperature differences of the junctions, maintaining the cold junction at 0^0C.

By measuring the e.m.f generated by various thermocouples under different conditions, it is observed that:

(i) The e.m.f depends upon the metals of which the thermocouple is made, being relatively large for iron-constantan thermocouples, for example, and small for copper-iron at a given temperature difference

(ii) The e.m.f is not large for most thermocouples, amounting to not more than 40-50 mV for a temperature difference of 1000^0C.

(iii) The e.m.f is not directly proportional to temperature difference but often rises to a maximum and then decreases, assuming approximately the shape of a parabola (see fig. 3.3).

The temperature of the hot junction at which the e.m.f is a maximum is called the neutral temperature. On the other hand, the temperature of the hot junction at which the e.m.f is zero after attaining a maximum value is called the temperature of inversion.

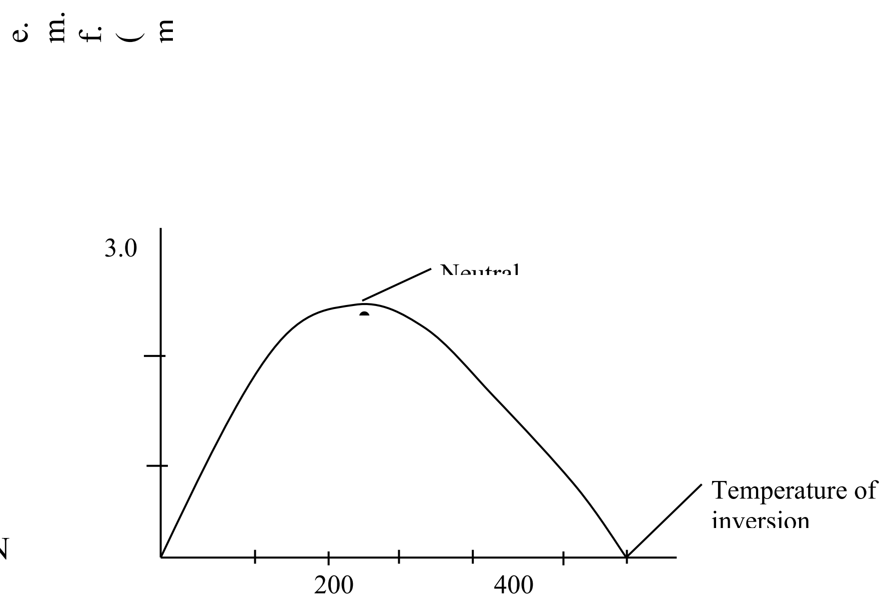


Fig. 3.3

3.1.5 Thermoelectric Series

For small values of temperature difference between the junctions, the metals can be arranged in a thermoelectric series as follows:

antimony, iron, zinc, copper, silver, lead, aluminium. Mercury, platinum-rhodium, platinum, nickel, constantan (60% copper, 40% nickel), bismuth

When two of these metals are combined to form a thermocouple, the conventional current flows from the one earlier in the list to the other across the cold, junction; thus the current flows from antimony to bismuth through the cold junction.

The thermo-e.m.f. is greater the farther apart the metals are in the list.

3.2 Photoelectric Effect

Photoelectric effect is the emission of electrons from metal surfaces when electromagnetic radiation of high enough frequency falls on them. The effect is given by zinc when exposed to X-rays or ultraviolet. Sodium gives emission with X-rays, ultraviolet and all colours of light except orange and red, while preparations containing caesium respond to infrared as well as to high frequency radiation.

The effect was first noticed by the German Physicist Heinrich Hertz in 1887 during the course of his experiments with the first simple radio transmitter. He observed that a spark jumped more readily between the terminals of his high-voltage source when they were irradiated by ultraviolet rays. It was soon shown that negatively charged particles were emitted by some metals under irradiation. The charge-to-mass ratio of the particles was that of electrons, and physicists soon agreed that the particles were indeed electrons.

3.2.1. Some Experimental Results Concerning Photoelectricity

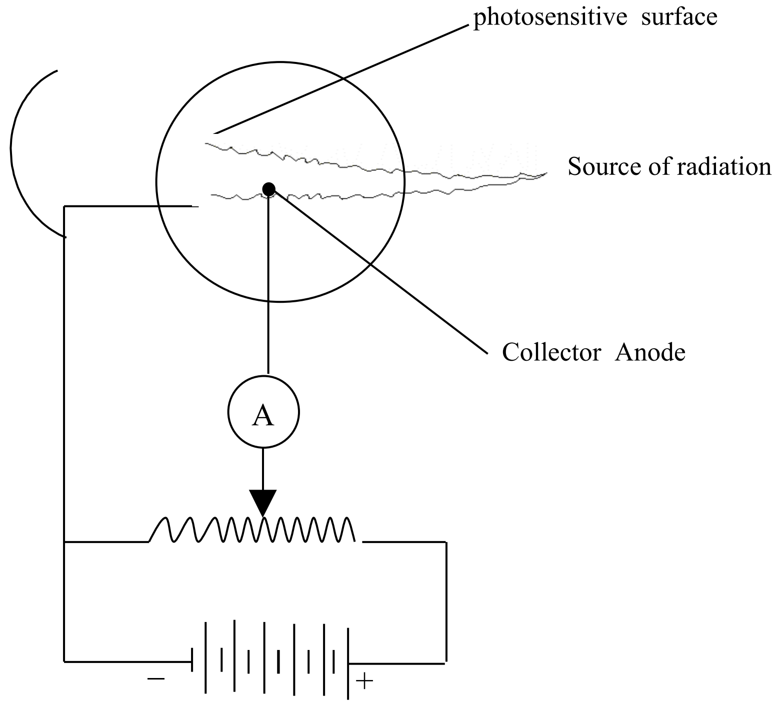


Fig. 3.4

A number of facts about the photoelectric effect were established by experiments by Millikan and some other physicists. In these investigations, different metallic surfaces were irradiated with electromagnetic radiations of different wavelenghts, and the emissoin of photoelectrons was studied. Photoelectrons can be collected by a positively charged collector or anode (see Fig. 3.4). The photoelectric current is measured by an ammeter A-generally an exceedinly sensitve ammeter, called an electrometer, that can measure currents as small as 10^{-12} A. The main results of such experiments are as follows:

(1). At a given frequency, the photoelectric current is accurately proportional to the illumination (intensity of the incident light) os the photosensitive surface over a very wide range of illuminations. Moreover, the maximum kinetic energy with which the photoelectrons leave the surface is entirely indepedendent of the illumination.

(2) Photoelectrons are emitted by most metals provided that the frequency of the radiation exceeds a certain critical threshold frequency. The threshold frequency for the metals sodium and potassium lies in the visible region of the spectrum.

(3) The kinetic enrgy of the photoelectrons emited ranges from zero to a maximum. If the maximum kinetic energy is plotted against the frequency of the radiations, a straight-line graph is obtained.

Observation 3 was obtained by measuring the stopping potential of photoelectrons for light of known wavelenghts. The stopping potential is the negative collector –to –cathode voltage that just barely stops all photoelecrons emitted by these radiations.

As one reduces the normally positve voltage between the anode and the photosensitive surface, approaching zero voltage, the photoelectric current deceases. If the voltage is then made increasinly negative, a voltage is reached at which the current is just barely detectable by the electrometer. This voltage is the stopping potential.

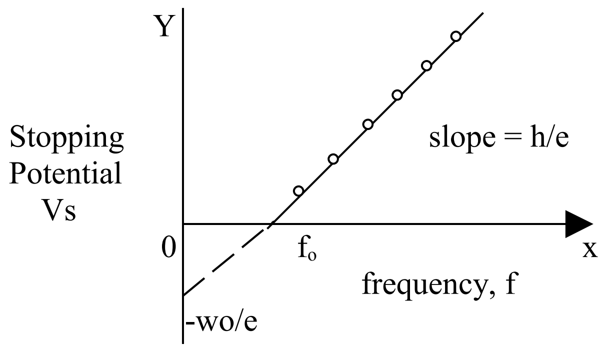


Fig. 3.5

Figure 3.5 shows the graph of the stopping potential versus the frequency for a given photosurface.

It is clear from the graph that for a given surface, there is a certain minimum frequency, f_o of the incident radiation for which the stopping potential is zero. It is called the threshold freqency for the given surface. This means that there can be no photo-electric emission from a surface if the frequency of the incident radition be less than the threshold frequcny for that surface, whatever the intensity of radiation.

For a frequency even slightly higher than the threshold frequency, there is almost instantaneous photoelectric emission from the surface of the material even if the intensity of the radiation be low.

The intercept on the potential axis is interpreted as meaning that small amount of energy is needed to draw electrons out of the surface, away from the altraction of the ions in the lattice. This energy is called the work function ϕ of the paricular surface. It is calculated by multiplying the value of the intercept by the charge on an electrons, and is stated in electron-volts.

3.2.2 Einstein's Interpretation of the Photoelectric Effect

The wave theory of light can offer no explanation for the existence of a threshold frequency. Emission of an electron would be expected as soon as sufficient wave energy had been absorbed by the surface.

Einstein, 1905, suggested that the explanation could be found if electromagnetic radiation were to be considered as made up of particles (photons) whose energy was related to the frequency of the radiation. This was a development of an earlier proposal by Planck (1902) who was attempting to find a mathematical basis for the curves of the continous spectrum.

Einstein's photoelectric theory gives

$$E = hf \quad \ldots\ldots\ldots\ldots 3.1$$

For the enrgy of a quantum of radiation, i.e a single photon of frequency f . h = 6.63×10^{-34} Js, and is known as Planck's constant.

If is represented the work function by ϕ, the maximum kinetic energy with which the photoelectron emerges is

$$\frac{1}{2} mV^2 = hf - \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.2.$$

Why then are photoelectrons with smaller velocities observed?

Some photoelectrons are liberated beneath the surface of the metal at a depth of a few atomic layers, for example. They lose greater amounts of energy than ϕ in leaving the metal, emerging finally with less than the maximum kinetic energy given by Eq. 3.2

□ **Example**

Quanta of wavelenght 6000 A^0 strike the surface of a metal whose work function is 1.0 eV. What is the maximum kinetic energy that a photoelectron can have?

Solution

The energy of quanta at 6000A^o is given by

$$E = hf = hc/\lambda = (6.6 \times 10^{-34}\ J.s)\frac{(3.0 \times 10^{8}\ ms^{-1})}{6.0 \times 10^{-7} m}$$

$$= 3.3 \times 10^{-19}\ J$$

From Eq. 3.2, the maximum kinetic energy of the photoelectrons is

$$½\, mV^2_{max} = E - \phi$$

$$= 3.3 \times 10^{-19}\ J - (1.0\ eV0\ (1.6 \times 10^{-19}\ joule /eV)$$

$$= 3.3 \times 10^{-19}\ J – 1.6 \times 10^{-19}\ J$$

$$= 1.7 \times 10^{-19}\ J$$

Referring back to idea of a 'stopping potential' (section 3.2.1), since the most energetic photoelectrons are just stopped by this negative voltage, the kinetic energy with which they leave the photosurface just equals the work they do in moving against the opposing electric field. In equaton form:

$$½\, mV^2_{max} = V_s e \quad \ldots\ldots\ldots\ldots \quad 3.3$$

where m and e are the mass and charge of the electron , V_{max} is the maximum velocity with which photoelectrons are emitted, and V_s is the stopping potential for that frequency.

One can readily see that Eintein's $½\, mV^2_{max} = hf - \phi$ fits the straight line experimental, plot of stopping potential, V_s versus frequency (fig. 3.5). Since

$$V_s e = ½\, mV^2_{max}$$

Einstein equation becomes

$$V_s e = hf - \phi$$

$$\text{Or} \quad V_s = \frac{h}{e} f - \frac{\phi}{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 3.4$$

Here h/e, the ratio of Planck's constant to th charge on the electron, is the slope of the line in Fig. 3.5, and ϕ/e, the ratio of the work function to the electronic charge, is the intercept of the line on the vertical axis. Eqs. 3.2 and 3.4 are referred to as Einstein's photo-electric equations.

⊡ **Example**

If a photoemissive surface has a threshold wavelenght of 0.65mm, calculate

(i) Its work function in electronvolts, and

(ii) The maximum speed of the electrons emitted by violet light of wavelenght 0.40mm.

(Speed of light, $C = 3.0 \times 10^8 ms^{-1}$, $h = 6.6 \times 10^{-34}$ Js,
$e = 1.6 \times 10^{-19}$ C and mass of electron $m = 9.1 \times 10^{-31}$ kg).

Solution

(i) $\lambda_o = 0.65 \text{ mm} = 6.5 \times 10^{-7}$ m

$$f = c/\lambda_o$$
$$= \frac{3.0 \times 10^8 \text{ ms}^{-1}}{6.5 \times 10^{-7} \text{ m}}$$
$$= 4.6 \times 10^{14} \text{ Hz}$$

(ii) We have $\phi = hf_o$

$$= (6.6 \times 10^{-34} \text{ j.S})\ (4.6 \times 10^{14} \text{ s}^{-1})$$

But $1 \text{ eV} = 1.6 \times 10^{-19}$ J

$$\phi = \frac{6.6 \times 4.6 \times 10^{-20} \text{ eV}}{1.6 \times 10^{-19}}$$

$$= 1.9 \text{ eV}$$

(iii) For violet light $\lambda = 0.40 \text{ mm} = 4.0 \times 10^{-7}$ m

$$f = c/\lambda$$
$$= \frac{3.0 \times 10^{-8} \text{ ms}^{-1}}{4.0 \times 10^{-7} \text{ m}}$$
$$= 7.5 \times 10^{14} \text{ Hz}$$

From the photoelectric equation $\frac{1}{2} MV^2_{max} = hf - \phi$

$$\frac{1}{2} MV^2_{max} = (6.6 \times 10^{-34} \times 7.5 \times 10^{14} - 1.9 \times 1.6 \times 10^{-19}) \text{ J}$$

$$= 1.9 \times 10^{-19} \text{ J}$$

$$\therefore V_{max} = \sqrt{\frac{2 \times 1.9 \times 10^{-19}}{9.1 \times 10^{-31}}} \text{ ms}^{-1}$$

$$= 6.5 \times 10^5 \text{ Ms}^{-1}$$

3.2.3 Applications of Photoelectric Effect

There are many technological applications of photoelectric effect. Photocells are ava cuated tubes in which the negative electrode is coated with potassium, cesium, or some combinations of these elements in a light-sensitive layer. Electrons ejected from this layer by photons are collected by the collector anode; the photoelectric current is proportional to the illumination. Photocells are used as control devices. Whenever the illumination of the photocell is changed, the photoelectric current changes, the voltage across a resistor in series with the cell changes, and an electric signal is sent to an amplifier. Thus the photo-cell can be used as a burglar alarm, as a safety device on industrial machinery, and as a counter of objects moving on a converyor belts.

Thus for example, in the case of burglar alarm, as the burglar enters a room, he intercepts the invisible radiation falling on a photocell, without his knowing it, by a relay device, the circuit containing the burglar alarm is thus completed and the alarm sounded.

Similarly, in the case of a fire alarm, a number of photocells are located at important strategic points in various parts of the building. In case of fire in any part of the building, some light falls on one or the other of the photocells. Thus completes an electrical circuit containing the fire alarm which then starts sounding.

In television transmission, a photocell is an absolute necessity. The light reflected from the persons speaking or performing in the studio, as also from the other objects, falls on a photocell, which converts the light energy into electrical energy. The latter is converted back into electromagnetic radiation and transmitted all over the television network.

Photocells are also used for the automatic control of traffic signals, automatic switching on and off the street lighting system, automatic opening and closing of doors, controlling the temperature of furnaces and so on.

3.2.4 The Electron Volt

In the Einstein equation (Eq. 3.2 or 3.4), we measured the maximum kinetic energy of the emitted electrons by noting the potential energy difference ($V_s e$), which was equivalent to the electron kinetic energy. This method of determining and expressing electron energies is a particularly convenient one, and it suggests a new unit of energy. This new unit of energy is called the electron volt, eV, which is defined as the amount of energy equal to the change in energy of one electronic charge when it moves through a potential difference of one volt.

Energies in joule's can be converted into electron volts by dividing by e = 1.60 x 10 $^{-19}$. In this case e is not a charge but a conversion factor having the units of joules per electron volt.

Example

Light having a wavelenght of 5000A^{o} falls on a material having a photoelectric work function of 1.90eV.

Find (a) The energy of the photon in eV.

(b) The kinetic energy of the most photoelectron in eV and in joule, and

(c) The stopping potential

Solution

(a) From Eq. 3.1, the energy of the photon is given by

$$E = hf \text{ joules}$$

$$= \frac{hf}{e_c} \text{ eV}$$

Since C = fλ, where λ is the wavelenght of the photon

$$E \quad \frac{hc}{e_c\lambda}$$

$$= \frac{(6.63 \times 10^{-34} \text{ J.s})(3 \times 10^{8} \text{ ms}^{-1})}{(1.60 \times 10^{-19} \text{ J})(5 \times 10^{3} \times 10^{-8}\text{m})}$$

$$= 2.47\text{eV}$$

(b) The law of conservation of energy gives

Maximum kinetic energy = photon energy – work function

$$\text{Or } E_k = 2.47\text{eV} - 1.90\text{eV} = 0.57\text{eV}$$

$$\text{Also } E_k = 0.57\text{eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{1\text{eV}} = 9.11 \times 10^{-20} \text{ J}$$

(c) The stopping potential is given by

$$V_s = \frac{0.57eV}{1 \text{ electronic charge}} = 0.57 \text{ volt}$$

3.3 Thermionic Emission

As you have learned from your study of elementary chemistry, in a metal each atom has a few loosely – attached outer electrons which move randomly through the material as a whole. The atoms that exist as positive ions in a 'sea' of free electrons. If one of these electrons near the surface of the metal tries to escape, it experiences an attractive inward force from the resultant positve charge left behind. The surface cannot be penetrated by an electron unless an external source does work against the attractive force and thereby increases the kinetic energy of the electron. If this is done by heating the metal, the process is called thermionic emission.

The work function ϕ of a metal is the energy which must be supplied to enable an electron to escape from its surface. It is conveniently expressed in electron to escape from its surface. It is conveniently expressed in electron – volts.

The smaller the work function of a metal the lower the temperature of thermionic emission, in most cases the temperature has to be very near the melting point. Two materials used are (i) thoriated tungsten having ϕ = 2.6 eV and giving good emission at about 200K and, in most cases, (ii) a metal coated with barium oxide for which ϕ = 1eV, copious emission occuring at 1200K.

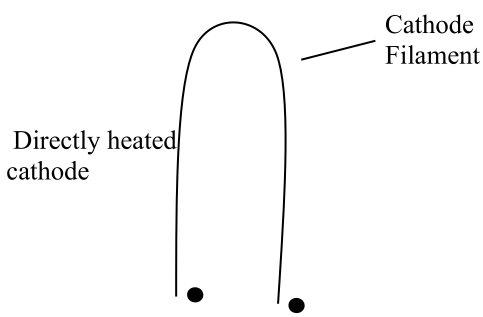


Fig. 3.6a

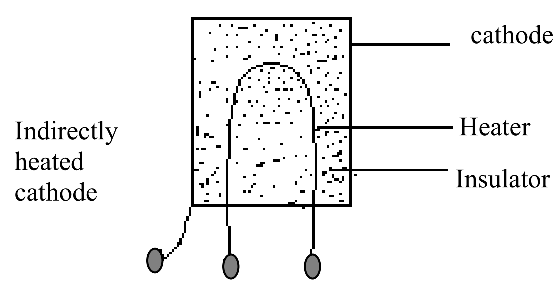


Fig. 3.6b

In many thermionic devices a plate, called the anode, is a positive potential with respect to the heated metal and attracts electrons from it. The latter is then called a hot cathode. Hot cathodes are heated electrically either directly or indirectly. In direct heating current passes through the cathode (or filament) itself which is in the form of a wire (Fig. 3.6). An indirectly heated cathode consists of thin, hollow metal tube with a fine wire, called the heater, inside and separated from it by an electrical insulator (see fig. 3.6b). indirect heating is most common since it allows alternating current to be used without the potential of the cathode continually varying. A typical heater supply for many thermionic devices is 6.3V a.c. 0.3A.

The electron current per unit area is given by

$$I = AT^2 e^{-\phi/KT} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

where A is a constant for the metal, ϕ is the thermionic work function (the least energy required to extract an electron when supplied thermally) and K the boltzmann constant. The thermionic and photoelectric values for ϕ tally, fairly closely.

Thermionic emission is the basis of the thermionic value and the electron gun in cahode –ray tubes.

4.0 Conclusion

The thermoelectric effect (Seebeck effect) is the generation of an e.m.f in a circuit containing two different metals or semiconductors, when the junctions beteen the two are maintained at different temperatures. The magnitude of the e.m.f depends on the nature of the metals and the difference in temperature. The Seebeck effect is the basis of the thermocouple.

Photoelectric effect is the liberation of electrons from a substance exposed to electromagnetic radiation. The number of electrons emitted depends on the intensity of the radiation. The kinetic energy of the electrons emitted depends on the frequency of the radaition.

Thermionic effect refers to the emission of electrons from a heated metal. Thermionic emission is used to produce the electron supply in cathode-ray tubes and x-ray tubes.

5.0 Summary

* When wires of two different metals are joined at both ends and the two junction are maintained at different temperatures, an e.m.f is set up in the circuit. This e.m.f depends on the temperature difference between the junctions. Thus the e.m.f can be used to measure this temprature difference.

* The law of intermediate metals states that if A,B,C are three different metals, the thermoelectric e.m.f of the couple AC is equal to the sum of the e.m.f of the couple AB and BC over the same temperature range.

* The law of intermediate temperature states that the e.m.f of a thermocouple with junctions at temprature θ_1 and θ_3, is the sum of the e.m.f's of two couple of the same metals with junctions at θ_1 and θ_2, and at θ_2 and θ_3, respectively.

* Einstein's photoelectric theory gives

  $$E = hf$$

  For the energy of a quantun of radiation , i.e. a single photo of frequency f . h = 6.63×10^{-34} Js, and is known as Planck's constant

* Einstein's photoelectric equation is

  $$\frac{1}{2} mV^2_{max} = hf - \phi$$

  where $\frac{1}{2} MV^2_{max}$ is the maximum kinetic energy of the electrons ejected from a metal surface which has a work function ϕ, and hf is the energy of the photons striking the surface.

* The electron-volt is the amount of energy equal to the change in energy of one electronic change when it moves through a potential difference of one volt.

* In thermionic emission, the electron current per unit area is given by

  $$I = AT^2 {}_{exp} (- \phi/KT)$$

  where A is a constant for the metal, ϕ is the thermionic work function and K = 1.380×10^{-23} JK^{-1} is the Boltzmann's constant.

6.0 Tutor Marked Assignment (TMA)

1. A thermojunction of iron and copper connected to a galvanomete cannot be used to measure temperatures above about 450K, whereas one of iron-constantan may be used up to at least 875K. What is the explanation?

2. Give an account of the Seebeck and Peltier effects, and explain how they are related.
   Explain, with diagrams, how you would set up a potentiometer circuit suitable for measuring the e.m.f of a thermocouple. Indicate the approximate voltages and resistances of the parts of the appraratus if

the e.m.f to be measured is about 0.1V and the potentiometer wire has a resistance of 5 ohms.

3. Write down Einstein's equation for photoelectric emission. Explain the meanings of the terms in the equation and discuss their significance.

   When the incident light is monochromatic and of wavelenght 600 nm, the kinetic energy of the faster electrons is 2.56×10^{-20} J. For light of wavelenght 400nm, it is 1.92×10^{-19} J. Use this information to estimate the value of Planck's constant, h and the work function, ϕ of the metal.

   (Take the velocity of light to be 3×10^8 ms $^{-1}$)

4. In what ways are the experimental facts regarding the photoelectric effect in disagreement with the predictions of classical electromagnetic theory?

   The work function of cesium surface is 2.0eV:

   (a) find the maximum kinetic energy of the ejected electrons when the surface is illuminated by violet light with $\lambda = 4.13 \times 10^3 A^o$ ($h = 6.63 \times 10^{-34}$ Js).

   (b) what is the threshold wavelenght (the largest λ) for the photoelectric effect to occur with this metal?

7.0 References and Other Resources

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UNIT 19

MODERN PHYSICS 1

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1.0 Introducton

To begin our study of modern physics we have to address an old question: "Is light composed of waves or particles? Indeed, experiments on interference and diffracton early in the nineteenth century led physicists to decide in favour of the wave theory. But surprises were in store for them, beginning with a revolutionary new interpretation of the process of radiaton by a black body. (An ideal system that absorbs all incoming radiation is called a blackbody).

In this unit we will deal with some of the changes that occurred during the transition to Modern Physics. We shall introduce you to new experimental knowledge and to new theories about the atom and its constituent particles, about radiations, and about physical systems containing these things.

The ideas of physics you have already encountered generally apply in modern physics, but many of them must be reinterpreted . We shall still use the principles of conservation of energy and momentum, for example, and the concepts of velocity, mass, position and time. However, our literal everyday interpretation of these concepts often fails as in the atomic world.

2.0 Objectives

After studying this unit you should be able to

* explain the description of a black body and how the spectrum of black body radiation was used by planck to formulate quantum theory.

* describe a photon and derive expressions for its energy, momentum and wavelenght.

* understand the structure of the nuclear atom and explain the Bohr model of the atom.

* explain the emission of the visible spectral lines of hydrogen and other atoms .

* describe the production of X-rays

* list some properties and applications of X-rays

* do simple calculations involving photon energies, wavelenght and momentum.

* solve problems on X-ray production.

3.1 The Photon and the Atom

3.1.1. Blackbody Radiation and The Quantum Theory

A major surprise to physicists at the end of the nineteenth century concerned the distribution of wavelength's emitted by a blackbody. As you know, most objects absorb some incoming radiation and reflect the rest. An ideal system that absorbs all the incoming radiation is called a blackbody

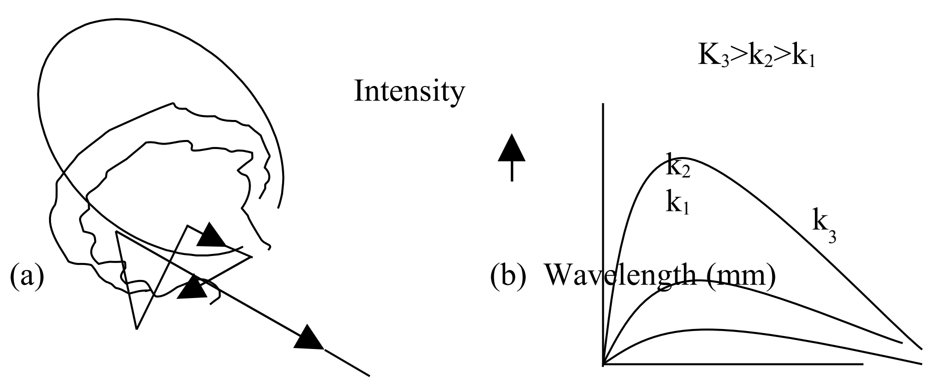


Fig. 3.1

Physicists study blackbody radiation by observing a hollow object with a small opening as shown in fig. 3.1a. The system is a good approximation to a blackbody because it traps radiation. The light emitted by the small opening is in equilibrium with the walls of the object because it has been absorbed and re-emitted many times.

Figure 3.1b shows the intensity of blackbody radiation at three dfferent temperatures. You can see that as the temperature increases, the total energy emitted by the body (the area under the curve) also increases. In addition, as the temperature increases, the peak of the distribution shifts to shorter wavelengths.

Scientist could not account for this experimental results with classical physics. Figure 3.2 compares an experimental plot of the blackbody radiation spectrum with the theoretical picture of what this curve should look like based on classical theories.

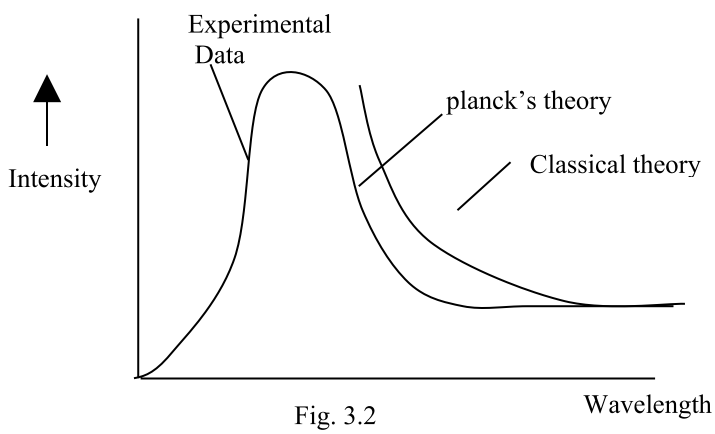


Fig. 3.2

Classical theory predicts that as the wavelength approaches zero, the amount of energy being radiated should become infinte. This is contrary to the experimental data, which show that as the wavelength approaches zero, the amount of energy being radiated also approaches zero. This contradiction is often referred to as the ultraviolet catastrophe because the disagreement occurs at the ultraviolet end of the spectrum.

In 1900, Max Planck (1858 – 1947) developed a formula for blackbody radiation that was in complete agreement with experiment data at all wavelenghts.

Planck proposed that blackbody radiation was produced by submicroscopic electric oscillators, which he called resonators. He assumed that the walls of a glowing cavity were composed of billions of these resonators, all vibrating at different frequencies.While most scienctist naturally assumed that the energy of these resonators was continuous, Planck made the radical assumption that these resonators could only absorb and then re-emit certain discrete amounts of energy. With this method, planck found that the total energy of a resonator with frequency f is an integral multiple of hf , as follows

$$E = nhf \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

where n is a positive integer called a quantum number, and the factor h is planck's constant.

Because the energy of each resonator comes in discrete units, it is said to be quantized, and the allowed energy states are called quantum states or energy levels. With the assumption that energy is quantized, planck was able to derive the curve shown in Fig. 3.2.

According to planck's theory, the resonators absorb or emit energy in discrete units of light energy called quanta (now called photons) by "jumping" from one quantum state to another adjacent state. It follows from Eq. 3.1 that if the quantum number, n changes by one unit, the amount of energy radiated changes by hf. Hence, the energy of a light quantum, which corresponds to the energy difference between two adjacent levels, is given by.

$$E = hf \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

A resonator will radiate or absorb energy only when it changes quantum states. If a resonator remains in one quantum state, no energy is absorbed or emitted.

3.1.2 Photon Energy, Momentum and Wavelength

A beam of electromagnetic radiation, considered as an electromagnetic wave, is characterised by its frequency, f or its wavelength λ wich are related by

$$f = \frac{c}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.3$$

The same beam, considered as a stream of photons, is characterised by the energy E or the momentum p of the individual photons. A photon has no mass and travels at a speed, $c = 3.00 \times 10^8$ ms $^{-1}$. Its energy and momentum are related by

$$E = cp \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

The vital connecting link between these two descriptions of the same beam of radiation, proposed by Einstein, is that the photon energy E is proportional to the freqeuncy f of the electromagnetic wave.

$$E = hf \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.5$$

The constant of proportionality h is the planck's constant.

Electromagnetic radiation can be classified according to the energy of its photons, or the wavelength, or the frequency, whichever is most convenient. For example, from f = cλ, Eq. 3.5 can be written to show the relationsgip between photon energy E and wavelength.

$$E = \frac{hc}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.6$$

Photon energies are usually specified in electrovolts, and wavelengths in angstrom unit. Inserting the numerical values of h and c and the required conversion factors, we write Eq. 3.6 in a form which is convenient for calculations:

$$E = \frac{12.4 \times 10^3 A^0 .eV}{\lambda} \quad \ldots\ldots\ldots 3.7$$

which gives the photon energy E in electrovolts when the wavelength λ is in angstrom units.

□ **Example**

Find the energy of the photons in a beam whose wavelength is:

(a) $6.2 \times 10^3\ A^0$ (orange light)

(b) $4.13 \times 10^3\ A^0$ (violet light)

Solution

(a) A photon beam of arange light has an energy which is, from Eq. 3.7

$$E = \frac{12.4 \times 10^3\ A^0.\ eV}{6.2 \times 10^3 A^0} = 2.0\ eV$$

(b) A photon of the beam of violet light has an energy

$$E = \underline{12.4 \times 10^3 A^0\ eV} = 3.0\ eV$$

$4.13 \times 10^{3} A^0$

3.1.3 The Nuclear Atom

The model of the atom in the days of Newton was that of a tiny, hard indestructible sphere. This model was a good basis for the kinetic theory of gases. However, new models had to be devised when experiments revealed the electrical nature of atoms. The discovery of the electron in 1897 prompted J.J. Thomson (1856 – 1940) to suggest a new model of the atom. In Thomson's model, electrons are embedded in a sperical volume of positve charge like seeds in a watermelon, as shown in Fig. 3.3.

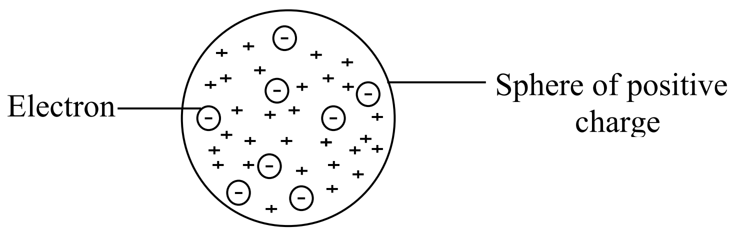


Fig. 3.3

In 1911 a " planetary model," in which the electrons revolve like planets round a small, massive, positively charged nucleus resulted from Rutherford's experiments on the scattering of α -particles by gold or platinum foil.

The nucleus is now believed to consist of protons of mass 1 and charge + 1e, and neutrons of mass 1 and charge 0. Surrounding the nucleus are planetary electrons of mass 1/1840 and charge - 1e. The constitutions of the three lightest elements are as follows:

	Nucleus	Planetary Electrons
Hydrogen	Charge 1 mass 1 (1 proton)	1
Helium	charge 2 mass 4 (2 protons and 2 neutrons)	2
Lithium	charge 3 mass 7 (3 protons and 4 neutrons)	3

The atoms of the remainder of the elements are built up in a similar way. The atomic number gives the charge on the nucleus and also the number of planetary electrons; the atomic mass minus the atomic number gives the number of neutrons in the nucleus.

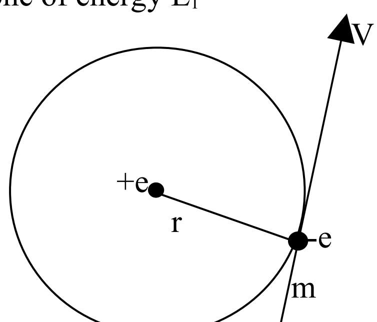
The Bohr Atom

The difficulty with Ruthford's model atom is that, according to the laws of classical physics, it cannot exist. The electrons rotating round the nucleus must have an acceleration towards the nucleus and consequently should radiate energy continuously, spiralling towards the nucleus to provide the energy.

In 1913 Niels Bohr applied the quantum theory to the nuclear atom. Bohr assumed that the planetary electrons in an atom can exist only in a limited number of stable orbits or stationsary states having definite amount of energy but not emitting radiation, and that radiation occurs only when an electron jumps from one stable orbit to another . He assumed

$$hf = E_2 - E_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.8$$

Where is the frequency of the energy radaited when the electron jumpls from orbit of energy E_2 to one of energy E_1



The hydrogen atom consists of a single electron of charge – e revolving round a nucleus of charge +e. Suppose the electron has a mass m, that it revolves in a circle, and that when the radius of its orbit is r its velocity is v (Fig. 3.4). The electrostatic altraction between the nucleus and the electron must equal the centrifugal force; thus

$$\frac{e^2}{4\pi \varepsilon_o r^2} = \frac{mv^2}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.9$$

At this point Bohr made a further assumption which seems purely arbitrary; on the new wave mechanics, however, its meaning becomes more understandable. He assumed that the angular momentum of the electron, mvr, is always an exact multiple of $h/2\pi$; thus

$$mvr = \frac{nh}{2\pi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.10$$

when n is an integer called the quantum number (see section 3.1.1). The angular momentum is then said to be quantized.

The next step is to find the energy of the electron on its orbit. The potential energy of the electron is the work done in bringing it from infinity to its orbit and this is – ($e^2/4\pi r\varepsilon_o$); the negative sign indicates that work is done by the electron as it approaches the oppositely charged nucleus. The kinetic energy of the electron is ½ mv^2 and from Eq. 3.9 this is equal to ½ ($e^2/4\pi r\varepsilon o$).

$$\therefore \text{Total energy of electron} = k.e + p.e$$

$$= \frac{1}{2} \frac{e^2}{4\pi\,\varepsilon or} - \frac{e^2}{4\pi\,\varepsilon or}$$

$$= \frac{1}{2} \frac{e^2}{4\pi\varepsilon or}$$

Form Eq. 3.9

$$mv^2r = \frac{e^2}{4\pi\varepsilon o} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.11$$

Squaring Eq. 3.10

$$m^2v^2 r^2 = \frac{n^2h^2}{4\pi^2} \ldots\ldots\ldots\ldots\ldots 3.12$$

Dividing 3.12 by 3.11, $r = \frac{n^2h^2\varepsilon o}{\pi e^2 m}$

$$\text{Energy of electron} = -\frac{1}{2} \frac{e^2}{4\pi r\varepsilon o} = \frac{e^4 m}{8n^2h^2\varepsilon o^2} \ldots\ldots \quad 3.13$$

$$hf = E_2 - E_1 = \frac{e^4 m}{8h^2\varepsilon o^2} \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\}$$

$$f = \frac{e^4 m}{8h^3\,\varepsilon o^2} \left\{ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.14$$

In 1884 Balmer, a swiss schoolmaster, had discovered a formula representin the series of visible spectral lines of hydrogen. The formula can be written in the form

$$f = R \left(\frac{1}{2^2} - \frac{1}{m^2} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.15$$

where m is an integer greater than 2. The value R, called the Ry dberg constant, was known from the measured frequencies of the spectral lines. Bohr was able to calculate the value of the constant corresponding to R in his formular , i.e $e^4m/8h^3\varepsilon o^2$, from the known values of e, m and h. The agreement was perfect within the limits of experimental error.

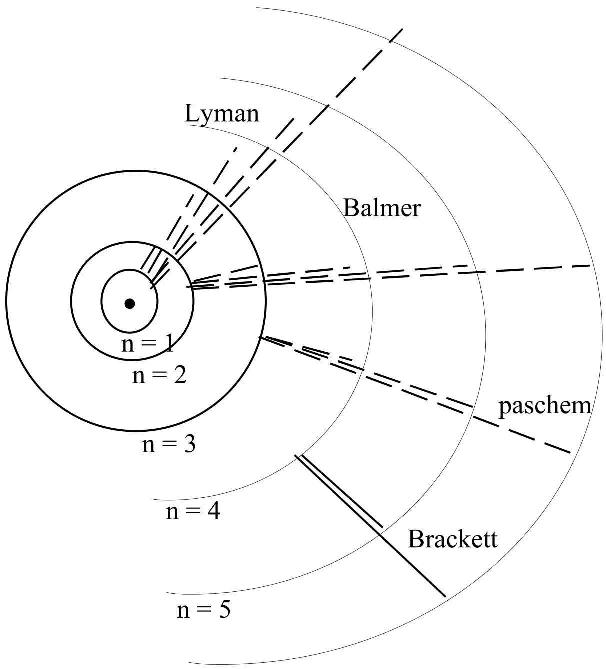


Fig. 3.5

Bohr explained the emission of the visible spectral lines of hydrogen as due to electrons jumping from outer orbits to the second orbit (fig. 3.5). Other spectral series for hydrogen were also known, the Lyman series in the ultraviolet, the Paschen and the brackett series in the infra-red. These are due to electrons jumping into the first, third and fourth orbits respectively.

In the normal condition of the hydrogen atom the electron is in its innermost orbit, for which n = 1. As a result of collision, say in a discharge tube the electron may be knocked into orbits for which n = 2 or 3, etc; the atom is then said to be excited. The electron will jump back to the innermost orbit, possibly in one jump or in stages, given out the appropriate radiation. If the electron is completely removed the atom is said to be ionized.

When an atom emits a photon, the law of conservation of energy implies that the enrgy of the atom must change from an initial value E_u (the subscript u denotes the upper energy level, as in Fig. 3.6) to a lower value E_l such that

$$E_{photon} = E_u - E_l \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.16$$

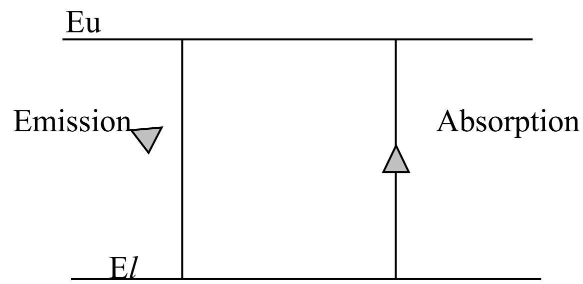


Fig. 3.6

This "Bohr frequency condition" determines the photon frequency f, since $E_{photon} = hf$. The empirically determined sequence of terms, whose differences determine the frequencies in the hydrogen atom spectrum, must then be proportional to the possible values of the energy of the hydrogen atom. These energies called the energy levels of the hydrogen atom, are given by (see Eq. 3.13)

$$E_n = \frac{-21.8 \times 10^{-19}}{n^2} \text{ J}$$
$$= \frac{-13.6 \text{ eV}}{n^2}$$

where n is the number of the level, or the principle quantum number. That is $E_1 = -13.6$eV, $E_2 = -3.40$eV, $E_3 = -1.51$eV, and so on, as shown in Fig. 3.7.

For example, a hydrogen atom can exist for a shortwhile ($\simeq 10^{-8}$s) in a state with energy $E_3 = -1.51$ eV. If after the emissin of a photon, the atom is left in the state with the lower energy, $E_2 = -3.40$ eV (Fig. 3.7), the photon emitted must have an energy, according to Eq. 3.16, given by

$$E_{photon} = E_3 - E_2 = (-1.51 \text{ eV}) - (-3.40 \text{cV}) = 1.89\text{eV}$$

The wavelength of this photon is

$$\lambda = \frac{hc}{E} = \frac{12.4 \times 10^3 \text{ A}^o. \text{ eV}}{1.89\text{eV}}$$

$$= 6.56 \times 10^3 \text{A}^o$$

($h = 6.63 \times 10^{-34}$ J.s, $C = 3 \times 10^8 ms^{-1}$, $1eV = 1.6 \times 10^{-19}$ J, $1A^0 = 10^{-10}$ m)

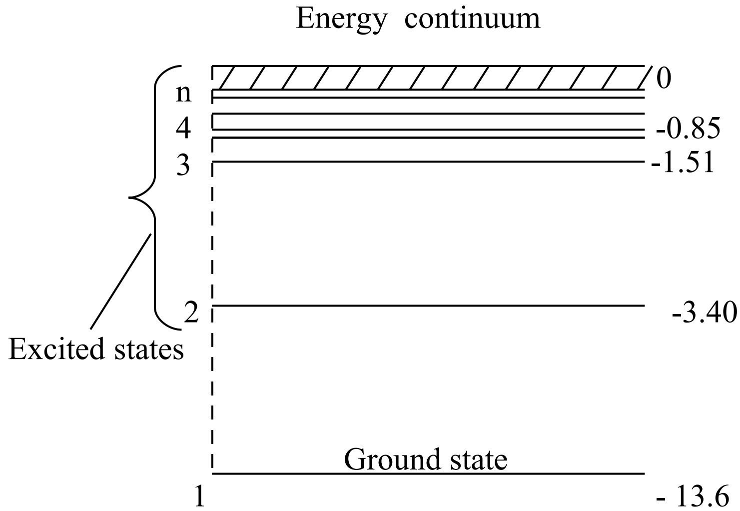


Fig. 3.7

The novel idea thus advanced by Bohr is that the energy of the hydrogen atom (and infact, all atoms molecules, and any bound system) can have only certain discrete values (instead of a continous range of values) in its bound states. That is, there exist discrete energy levels.

The lowest of these energy levels is called the ground state, and all the higher levels are called excited states. The value E = 0 is the energy when the electron and the proton are completely separated and at rest. Since this energy level is 13.6 eV above the ground state, we see that 13.6eV must be supplied to a hydrogen atom in its ground state in order to remove the electron, that is, to ionize the atom. In other words, the binding energy of a hydrogen atom against separation into a proton and an electron is 13.6 eV.

When the electron and proton are separated, they can have any amount of kinetic energy. Corresponding to these states, which are not bound states, the energy level diagram (Fig. 3.7) shows a continous range of possible values of the energy of the electron proton system.

When an atom gives off energy it passes from an upper to a lower energy level. If an atom absorbs energy it passes from a lower to a higher level (Fig. 3.6). For the absorption of a photon, the Bohr frequency condition still applies, but now the lower energy Eι is the initial energy of the atom.

3.2 X-rays

3.2.1 X-ray Spectra

X-rays are a form of electromagnetic radiation having short wavelengths and high frequencies, about $10^{18} – 10^{19}$ Hz.

Their production can be explained using the energy levels theory, since X-rays are produced by streams of high-energy electrons in collision with atoms of high atomic number, such as tangsten or molybdenum.

X-rays are produced, by two distinct processes when the electron hit a metal target.

(i) The electrons suddenly lose energy when they collide with the target nuclei. A large percentage of the energy is converted into heat, but some is converted into X-ray photons. Each electron gains the same energy from being accelerated by the tube voltage, but varying fractions of this energy are converted into photons. As usual, the energy of the photon created = hf. The maximum frequency (minimum wavelength) will be produced when all the energy gained by an electron is converted into a photon. A continous range of smaller frequencies (greater wavelengths) is created by smaller fractions of the electrons energies being converted into photons. This continous X-ray spectrum is typical of the tube voltage but independent of the target material.

(ii) The electron gives some of its energy to an electron in a target atom. The electron in the atom ‘jumps’up to a higher level and X-rays are emitted as the electron falls back to the lower level. The energy hf of the photons produced is equal to the energy level difference. The energy of an X-ray photon is very large, and so the energy levels involved must have a large separation. The frequencies (and wavelength’s) emitted have discrete values, which are typical of the target material. This gives rise to the peaks in Fig. 3.8.

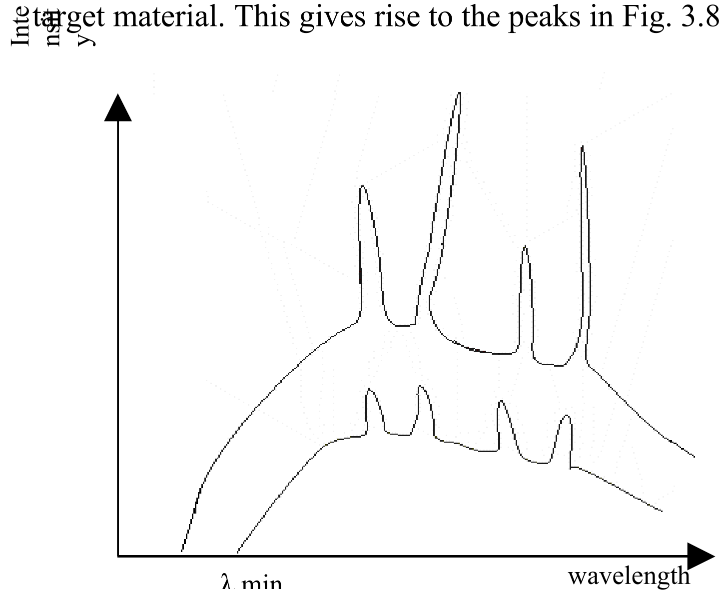


Fig. 3.8

3.2.2 The X-ray Tube

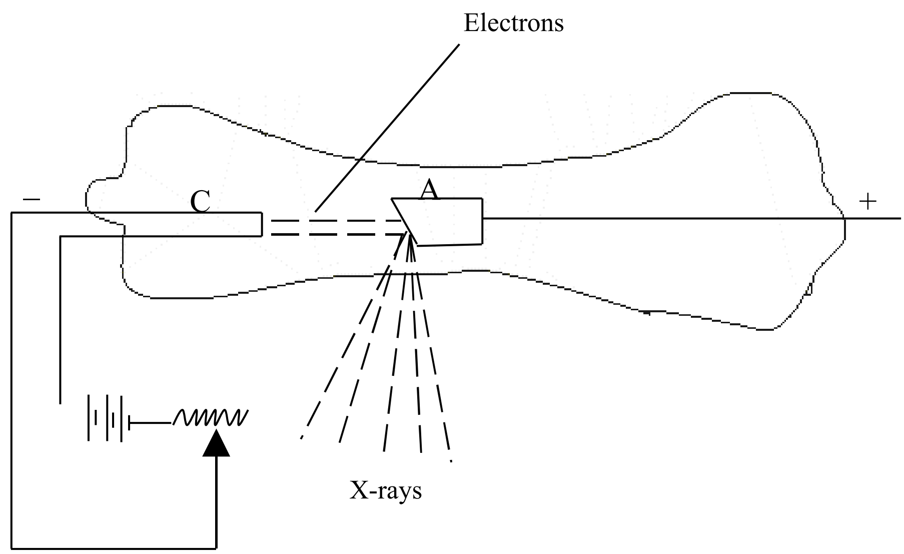


Figure 3.9 shows the essentials of the x-rays tube invented by Coolidge in 1913. the tube is exhausted as highly as possible so that no discharge would pass through it if it were used as a gas tube. The electrons are emitted by a tungsten filament C and the rate of their emission, which determines the intensity of the X-rays, can be controlled by the heating current through the filament.

The hardness of the X-rays is controlled separately by the p.d. between A and C. Their hardness depends on their frequency and if this is greater the faster the speed of the electrons hitting the target. The phenomenon is a kind of reverse photoelectric effect and a similar relation holds:

$$Ve = hf \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.17$$

where V is the p.d. through which the electrons fall, e = charge of the electron, h = Planck's constant, f = maximum frequency of the X-rays emitted.

As mentioned earlier, only about 1% of the energy of the electrons is converted into X-rays. The rest of the energy appears as heat, and hence the target is made of a metal of high melting – point, such as tungsten or molybdenun, embedded in a solid block of copper which is a good thermal conductor.

3.2.3 Nature and Properties of X-rays

X-rays are electromagnetic waves, like light, having the same speed of 3 x $10^8 ms^{-1}$, but having a wavelength about a thousand times shorter than that of light waves. Their wavelengths range from 0.06 to 100 Angstrom units againsts 3,900 to 7,600 Angstroom units of visible light.

(1) X-rays travel in free space with a speed of 3 x $10^8 ms^{-1,}$ the same that of light.

(2) They affect a photographic plate much more intensely than light because of their very much shorter wavelength.

(3) They are unaffected by magnetic and electric fields, clearly showing that they are not a stream of charged particles.

(4) Like light, they liberate photoelectrons when allowed to fall on certain metals.

(5) Scattered X-rays show a marked degree of polarization like the scattered sky light.

(6) They undergo reflection, refractions, dispersion and diffraction.

(7). They can ionise a gas (or air) through which they are allowed to pass.

(8) They cause fluorescence in substances like barium plastinocyanide, zinc sulphide and cadmium tun state.

(9) As a direct consequence of their extemely small wavelenght, they can easily pass through matter, opaque to ordinary light, as for example, paper, card board, wood, thin sheets of metal and also through human flesh. They are, however, absorbed by denser substances, like iron, lead and bones (due to their high calcium content) and therefore cast their shadows on fluorescent screens or photographic plates.

Thus if I_o and I are the respective intensities of X-rays before and after passing through a material of thickness x (where intensity is the amount of energy carried in unit time across unit area, perpendicular to the direction of flow of enrgy), we have

$$I = I_o e^{-mx} \quad \text{.....................3. 18}$$

Where u is called the linear absorption coefficient of the material and its dimensions are, therfore, those of reciprocal length.

Example

Monochromatic X-rays ($\lambda = 1A^0$) are reduced to 1/3 of their original intensity in passing through a gold foil (Z = 79) of 3mm thickness. Calculate the absorption coefficient for the X-rays. What are the dimensions of the absorption coefficient?

Solution

We use the relation

$$I = I_o e^{-mx} \quad (\text{Eq. } 3.18)$$

That is $^I/I_o = e^{-mx}$

Here $^I/I_o = 1/3$ and x = 3mm = 0.3 cm

$$\therefore \quad ^1/_3 = e^{-mx\,0.3}$$

$$3 = e^{0.3m}$$

$$\log 3 = 0.3m, \text{ whence } m = \frac{\log_e 3}{0.3}$$

$$\text{That is } m = \frac{\log10^3}{0.3\log_{10}e} = \frac{2.3026 \times 0.4771}{3.0}$$

$$= 2 \quad 3.7\ cm^{-1}$$

The demensions of the absorption coefficient are those of the reciprocal length, that is $M^o L^{-1} T^0$.

3.2.4 Uses of X-rays

The usefulness of X-rays is largely due to their penetrating power.

(i) Medicine . Radiographs or X-ray photographs are used for a variety of purposes. As mentioned earlier, X-rays can pass through flesh but not through bones. Therefore, sharp dark shadows of the bony parts of the body are obtained against a lighter background on a flourescent screen or a photographic plate, if these be interposed in the path of the Xx-rays. Such X-ray photographs are called radiographs. Dislocation, fractures and the presence of foreign bodies like pins, bullets, etc. inside the human body can thus be easily deteted. In radio therapy, periodic X-ray exposures, in properly controlled doses, are given for treatment of obstinate skin diseases and malignant or cancerous growths or tumours.

(ii) Industry. Casting and welded joints can be inspected for internal imperfections using X-rays. A complete machine may also be examined from a radiograph without having to be dismantled.

(iii) X-ray crystallography. The study of crystal structure by X-rays is now a powerful method of scientific research. The first crystals to be analysed were of simple compounds such as sodium chloride but in current years the structure of very complex organic molecules has been unravelled.

4.0 Conclusions

A blackbody is a hypothetical body that absorbs all the radiations falling on it. A small hole in the wall of an enclosure at uniform temperature is the nearest approach that can be made to it in practice.

Black-body radiation is the electromagnetic radiation emitted by a black body. It extends over the whole range of wavelengths and the distribution of energy over this range has a characteristic form with a maximum at a certain wavelength. The position of the maximum depends on temperature, moving to shorter wavelengths with increasing temperature.

The quantum theory devised by Max Planck in 1900 to account for the emission of the black-body radiation from hot bodies has it that energy is emitted in quanta, each (quantum) of which has an energy equal to hf, where h is Planck's constant and f is the frequency of the radiation.

X-ray are electromagnetic radiation of shorter wavelenght than ultraviolet radiation produced by bombardment of atoms by high quantum – energy particles. The range of wavelengths is 10^{-11} m to 10^{-9} m. Atoms of all the elements emit a characteristic X-ray spectrum when they are bombarded by electrons. The X-ray photons are emittd when the incident electrons knock an inner orbital electron out of an atom. When this happens an outer electron falls into the inner shell to replace it, losing potential energy (ΔE) in doing so. The a avelength λ of the emitted photon will then be given by $\lambda = ch/\Delta E$, where c is the speed of light and h is the Planck's constant.

X-rays can pass through many foprms of matter and they are therefore used medically and inductrially to exermine internal structures. X-rays are produced for these purposes by an X-ray tube.

5.0 Summary

* A beam of electromagnetic radiation of frequency f and wavelength λ consists of identical photons each with an energy

$$E = hf$$

and a momentum

$$P = h/\lambda$$

* The energy and hence the frequency of a photon emitted or aborbed when the energy of an atom changes, is given by the Bohr frequency condition

$$E_{photon} = E_u - E_\iota$$

where E_u and E_ι are the values of the atom's upper and lower energy levels involved in the transition. In a bound state, the possible values of the energy (the discrete energy levels) of a hydrogen atom are

$$E_n = \frac{13.6eV}{n^2} \qquad (n = 1, 2, \ldots\ldots\ldots)$$

* The model of the atom we have considered consists of electrons surrounding the nucleus. The laws of quantum mechanics allow those electrons to have only discrete amounts of energy, leading to the idea of energy levels.

* X-rays are form of electromagnetic radiation having short wavelengths and high frequency, about $10^{18} - 10^{19}$ Hz. Their production can be explained using the energy levels theory since X-rays are produced by stream of high-energy electrons in collision with atoms of high atomic number. X-rays find useful applications in medicine, engineering and industry because of their penetrating power.

6.0 Tutor Marked Assignments

1. A 10 W Sodium lamp radiates entirely with wavelength 590nm.

   (a) Assuming its efficiency is 10%, calculate the number of photons produced each second by the lamp.

   (b) If all the light produced is incident on metallic caesium and the photoelectric current is 48 nA, what fraction of the photons produces electrons?
   ($h = 6.6 \times 10^{-34}$ J s, $C = 3 \times 10^{8} ms^{-1}$, $e = 1.6 \times 10^{-19}C$)

(2.) A hydrogen atom emits light of wavelength 121.5nm and 102.5nm when it returns to its ground state from its first and second excited states respectively. Calculate:

   (a) The corresponding photon energies, and

   (b) The wavelength of light emitted when the atom passes from the second excited state to the first.
   (speed of light , c = 3.00 $10^{8}ms^{-1}$, the Plauck's constant, $h= 6.63 \times 10^{-34}$ J.s)

(3) X-rays are electromagnetic maves in the frequency range 10^{17} Hz – 10^{21} Hz.

   (i) When a fast-moving electron is brought to a halt, X-rays may be produced. If all the kinetic energy of the electron travelling at $2.3 \times 10^{7} ms^{-1}$ is transformed into 1 photon of X-radiation, calculate the frequency of the X-radiation (The Plank's constant = 6.6×10^{-34} Js, mass of electron = 9.1×10^{-31}kg).

   (ii) Why is it possible that X-rays may be produced at the screen of a cathode ray tube?

   (iii) X-rays can also be produced when an atom reverts from an excited state to its ground state. What value of the transition energy between these states would produce X-radiation of frequency 2.0×10^{17} Hz?

(4) The Einstein photo-electric equation may be written in the form

$$hf = \frac{1}{2} mV^2 + W.$$

Explain the physical meanings of the three terms in this expression. A metal surface is illuminated with light of varying frequencies and electrons are emitted. The maximum energy of the emitted electrons is measured and the following results obtained:

Frequency/Hz x 10^{14}	10.00	9.0	8.0	7.0	6.0
Maximum energy /eV	2.04	1.60	1.21	0.78	0.26

By ploting a suitable graph, use this data to determine a value for the Planck's constant, h. (1eV = 1.6 x 10^{-19} J)

7.0 References and Other Resources

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UNIT 2O

MODERN PHYSICS 11

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1.0 Introduction

In unit 19 we saw that the nucleus of the hydrogen atom consists of a single particle, called proton, with just a single electron going round it in a circular orbit. The mass of a proton is about 1836 times that of an electron and it carries a positive charge (+e) equal in magnitude to the negative charge (-e) on an electron. The atom is thus electrically neutral, with practically the whole of it mass concentrated in the nucleus. In this unit we shall consider the structure of the atoms of other elements. In particular, we will see that the occurreance of discrete energy levels in a hydrogen atom is only a particular instant of a very general phenomenon. The energy of any bound system is restricted to certain discrete values which are called the energy levels of the system. For instance, the nucleus 60/28N_i has a stable ground state and many discrete energy levels corresponding to excited states. We will also see that the excited states of an atom are unstable with a characteristic mean life τ.

The spontaneous of a nucleus is called radioactivity and is classified according to the particle emitted in α decay an α-particle $(^4_2H_e)$ is emitted, are often referred to as γ rays.

2.0 Objectives

At the end of your study of this unit you should be able to:

* identify alpha, beta and gamma particles/radiations and how their emission from the nucleus is explained.

* recognise the random aspect of the decay

* understand the terms activity, decay constant, and half-life and their interrelation.

* know the structure of the nucleus and the ideas of the strong force, binding energy and mass defect.

* distinguished between the have a knowledge of the basic principles of fision and fusion.

3.1 Radioactivity and The Nuclear Atom

3.1.1 The Nuclear Model of the Atom.

The protons and neutron both being constituents of the nucleus are called nucleons and their total number gives what is called the mass number A of the atom. The number of protons in the nucleus
(because this is equal to the number of electrons) gives the atomic number, Z of the atom. Therefore, the number of neutrons in the nucleus, N = A – Z, i.e equal to the difference between the mass number and the atomic number of the atom. To summarize,

Atomic number of an atom, Z = number of protons or number of electrons in the atom

Mass number of an atom, A = number of nucleous in the atom
= number of protons + number of neutrons in the nuleus

Number of neutrons in an atom, N = A –Z = mass number minus atomic number.

Nuclei with identical number of protons (i.e. with the same value of Z) or identical number of neutrons (i.e. the same value of N) belong to the same species and a nuclear species is called a nuclide. The notation $_ZX^A$, where X stands for the chemical symbol of the atom or the element, the subcript A for the mass number and the subscript Z for the atomic number of the atom.

Nuclides with the same atomic number, Z (i.e with the same number of protons) are called isotopes; those with the same value of mass number A (i.e. with the same number of nucleous) are called isobars and those with the same value of N = A-Z (i.e with the same number of neutrons) are called isotoes. Thus, for example, $17Cl^{37}$ is an isotope of Cl^{35}, because Z = 17. It is an isobar of $_{16}S^{37}$, because for both a = 37 and it is an isotope of $_{19}K^{39}$ because for both N = A-Z = 20 (i.e. 37 – 17 = 39 – 19 = 20).

3.1.2 Atomic Mass Unit

Hitherto, we have given the masses of protons and neutrons in kilogram. It will interest you to learn that the International Union of Pure and Applied Physics (IUPAP) decided in 1960 to adopt a new masss scale for the measurement of mases in nuclear Physics. It is called the atomic mass scale and the atomic unit on this scale (written as amu) is $^1/_{12}$ of the mass of $_6C^{12}$, the most abundant and the most stable isotope of carbon. It is always preferable to express the masses of atoms on the atomic mass scale rather than in kilogram, because it is more suitable for the magnitude of atomic masses and is far more accurate, since atomic masses can be determined very accurately relative to the carbon atom $_6C^{12}$.

Now, since the mass of an atom is equal to its atomic weight divided by Avogadro number (6.02×10^{23}), we have

$$1 \text{ amu} = \frac{(^1/_{12} \times 12)}{6.02 \times 10^{23}} \text{ g} = 1.66 \times 10^{-27} \text{ kg}$$

which is nearly the mass of a hydrogen atom.

In accordance with Einstein's mass energy equation $Eo = MoC^2$, where Eo is the energy of a resit mass Mo (c being the velocity of light in free space = $3.0 \times 10^8 ms^{-1}$).

Hence $1 \text{ amu} = 1.66 \times 10^{-27} \times (3.0 \times 10^8)^2 = 1.49 \times 10^{-10}$ J

And since 1.602×10^{-19} J = 1 eV, we have

$$1 \text{ amu} = \frac{1.49 \times 10^{-10}}{1.602 \times 10^{-19}} = 9.31 \times 10^8 \text{ eV}$$

$$= \frac{9.31 \times 10^8}{10^6} = 931 \text{ MeV}$$

$$\text{Thus } 1 \text{ amu} = 1/12\ {}_6C^{12} = 1.66 \times 10^{-27} \text{ kg} = 1.49 \times 10^{-10} \text{ J}$$

$$= 9.31 \times 10^8 \text{eV} = 931 \text{ MeV}$$

The atomic mass scale is also referred to as the isotropic mass scale and hence the mass of an atom in amu is its isotropic mas.

3.1.3 Nuclear Binding Energy

The mass, M of an atom can be determined directly by the mass spectrograph. (Here M stands for the mass in kg of an individual atom).

The mass that an atom ought to have as an assembly of neutrons and protons and electrons can be calculated, for there are Z protons (mass Mp), Z electrons (mass Me) and N = (A – Z) neutrons (mass Mn), giving a total mass of $Zm_p + Zm_c + NM_n$.

But the measured mass M is less than this by a differene $\Delta M = (Zm_p + Zm_c + Nm_n - M)$, which is called the mass defect.

The mass defect ΔM represents the energy ΔMc^2 that would be liberated if the nucleons and the electrons were assembled, and it is therefore the energy which would have to be supplied in order to dismember the atom again. So the greater the value of ΔM, the greater is the stability of the atom against this kind of breaking – up. A better stability criterion is the mass deflect per nucleon, ΔM/A, which represents the binding energy per nucleon.

Note

The mass of a nucleon, about 1.6×10^{-27}kg, is roughly 1.5×10^{-10} J, say about 1 eV; one millionth of this is a difference of 1 KeV per nucleon. Therefore, we need to used very accurate mass values if the calculation is to Mean much.

As we saw in the last section, masses are expressed in terms of the unified mass constant , M_u, which is the mass of the ^{12}C atom (amu), and some relevant values are:

Unified atomic mass constant, Mu	=	1.660566×10^{-27}kg
	=	931.478 MeV
Mass of proton, M_p	=	1.00727647Mu
Mass of electron, M_e	=	5.485803×10^{-4} Mu
Mass of neutron, M_n	=	1.008665 Mu

The graph of Fig. 3.1 shows how the binding energy per nucleon varies with A. The only three points that do not lie on the smooth curve are the particularly stable (even-evn and multiple of 4) nuclides ^4He, ^{12}C and ^{16}C. The curve has a maximum at about A = 50, but over the range A = 80 to A = 250 the gradient is almost uniform and is not very considerable. The higher a nuclide's place on the curve, the more stable it is, and any nuclear change which ascends the curve librates energy. The two ways up are fusion from the left-hand-side, and fission (or on a smaller scale the disintegratioin that gives α-emission) form the right –hand side.

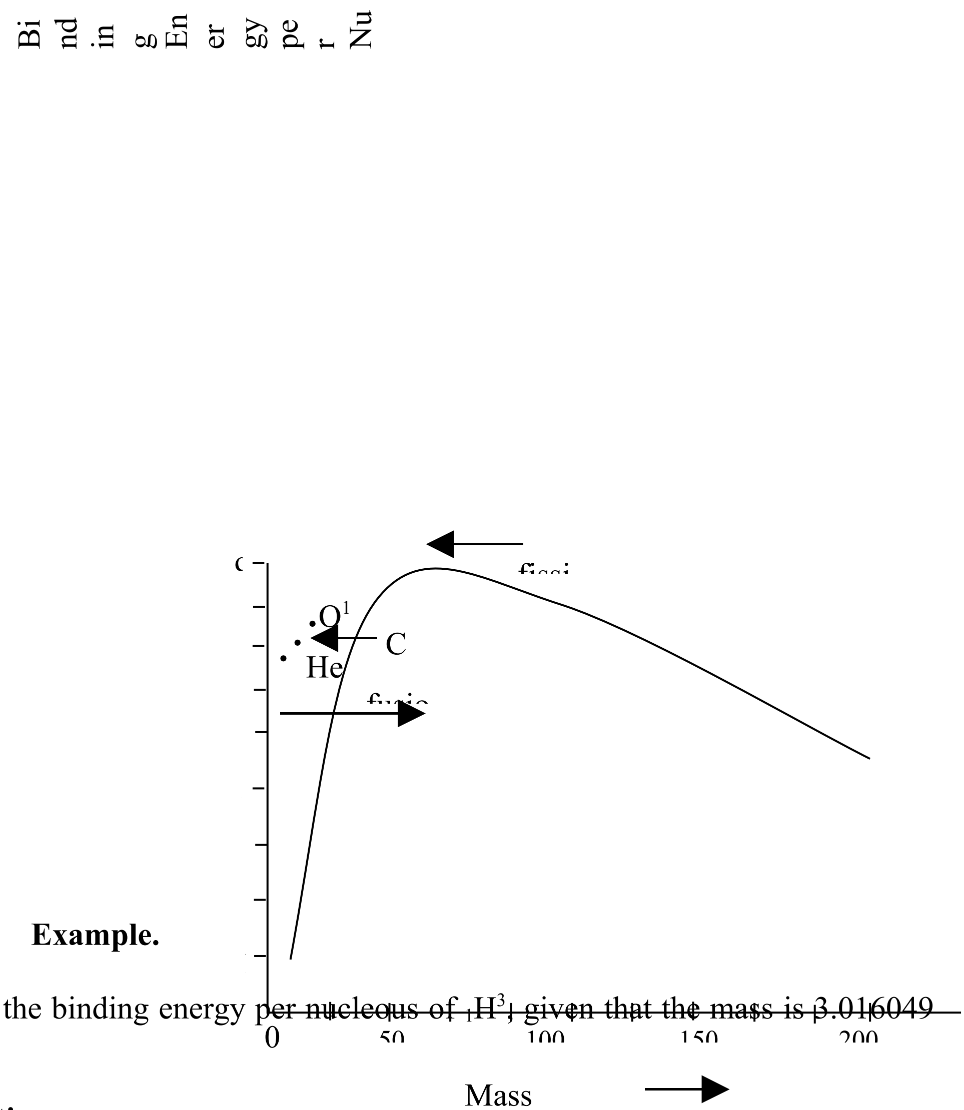


Fig. 3.1

Example.

Find the binding energy per nucleous of $_1H^3$, given that the mass is 3.016049 amu.

Solution

$_1H^3$ consists of 1 proton and (3-1) = 2 neutrons.
Mass of 1 proton = 1.007825amu
and mass of 2 neutrons = 2x 1.008665 = 2.017330 amu
Total mass of 1 proton and 2 neutrons = 1.007825 +2.017330
= 3.025155amu

Mass difference, $\Delta M = 3.025155 - 3.016049 = 0.007106$ amu
Hence binding energy of the nucleus ($_1H^3$), $E_6 = 0.007106 \times 931MeV$
$= 6.615656MeV$

Binding Energy of 1H3 per nucleon, $E_o/A = \frac{6.615656}{3}$
$= 2.20522MeV$

3.1.4 Nuclear Forces

Now that we know that a nucleus consists of protons carrying +e charges) and neutron (carrying no charge), the question arises as to what keeps the nucleus from falling apart in view of the fairly large force of repulsion between the protons. Definitely, the gravitational force of attraction between the nucleous is much too weak to hold them together. There must, therefore be some other very strong force of attraction, binding the protons and neutrons so compactly together, quite different from the forces with which we are familiar in classical physics.

Various experiments on scaltering of nuclei by one another, on collision, clearly show that there are indeed very strong attractive forces which are effective only within a very small range of the order of 10^{-15} m. It is, therefore, not a mere coincidence that the radius of a nucleus too is of the same order (10^{15}m). These short range forces are called nuclear forces and are effective only when two nuclei just touch each other and fall to zero as soon as they are separated.

Another significant point about these attractive forces is that they are the same between protons and protons (p-p force), between protons and neutrons (p-n forces) and between neutrons and neutrons (n-n forces), in spite of the fact that there is also a repulsive force between protons and protons. This latter force must obviously be negligible compared to the attractive nuclear force between them. Hence, so far as nuclear forces are concerned, protons and neutrons are one and the same thing, the positive charge on the protons being of no consequence at all. This fact is referred to as the charge-independence character of the nuclear forces.

3.1.5 Three Main Tyoes of Rediation

Alpha Radiation:

This is a particle, comprising two protons and two neutrrons. Hence it has a mass about 8000 times that of the electron and a charge of $+3.2 \times 10^{-19}$C.

Beta Radiation:

There are, infact, two B particles, the B_- and the B_+. The B_- is the B_- particle normally referred to in Nuclear Physics and it is an electron. Electrons do not infact exist in the nucleus, but the beta particle is created and ejected from the nucleus when a neutron changes into a proton. The B_+ particle (a position same mass as electron, same charge as proton) is created and ejected when a proton changes into a neutron.

Gamma Radaition:

This is a photon of electromagnetic radiation sometimes ejected by nuclei following beta or a alpha emission, when the nucleus adjusts its energy levels. It has no mass and no charge.

3.1.6 Radioactive Decay

When the nucleus of a radioactive atom disintegrates, it may emit an alpha particle or a beta particle. Gamma rays may precide or follow either kind of particle. When an alpha particle is emitted, the mass number A decreases by 4 and the atomic number Z by 2, because the positively charged alpha particle carries off two electronic units of charge, leaving the positve nuclear charge less by two electron units (conservation of energy). Emitting a P-particle does not alter the mass number, it increases the atomic number by one, because the negatively charged B-particle carries off one electronic unit of charge, leaving the positive nuclear charge greater by one electronic unit.

The disintegration of an individual nucleus is a random event. The word decay (or rate of decay) is used for the rate at which the number N of serviving nuclei in a given sample of a pure radiative nuclide diminishes with time. Since the decay is andom, this rate depends only on itself. The rate $-dN/dt$ at any given time is proportional to the number of surviving nuclei at that time.So $-dN/dt = \lambda N$ where λ is a constant which depends on the nuclide called the decay constant.

Integrating gives $\log_e (N/N_o) = -\lambda t$, where $N = N_o$ at $t = 0$, so that at time t

$$N = N_o e^{-\lambda t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.3.1$$

The number of nuclei that have disintegrated at time t is given by

$$N_o - N = N_o (1- e^{-\lambda t}) \quad \ldots\ldots\ldots\ldots\ldots..3.2$$

The half-life $T_{1/2}$, of a radioactive nuclide is defined as the time, from the original observation, for the number of surviving nucleir to be reduced to one-half. Thus, for N/No = ½ = - $\log_e 2$ = $-\lambda T_{½}$, and

$$T_{½} = (\log_e 2) / \lambda = \frac{0.693}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots 3.3$$

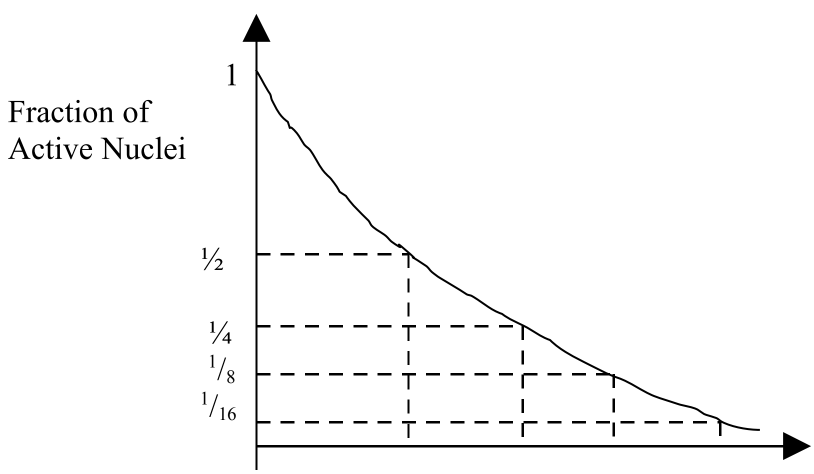
The quantity that is actually observed as the ‘ activity’ is a count-rate, or the equivalent of an ionisation current, which gives the rate of decay – dN/dt at that instant. But dN/dt is proportional to N, whence

$$-\frac{dN}{dt} = -\left(\frac{dN}{dt}\right)_o e^{-\lambda t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4$$

and the half-life is therefore also the time, from the initial observation, for the activity to be reduced to one-half. However, unless the product is a stable nuclide, or one with a very long half-life, there is more than one contribution to the activity. So it is only in suitable cases that the measured activity enables the half-life to be found directly.

It follows from Eq. 3.3 that a large λ means a short half-life because at any particular time there is a large rate of decay for a given number of atoms. After one half-life both the number of atoms and the activity have halved. After two half-lives they both have quartered, and so on. Always remember that the half-life of a nuclide is the average time it takes for half its atoms to decay.

An activity of 1 disintegration per second is 1 Bq (becquerel) Half-lives vary from millionths of a second to thousands of millions of years. Radium 226 has a half-life of 1622 years, therefore starting with 1 g of pure radium, ½ g remains as radium after 1622 years, ¼ g after 3244 years and so on. An exponental decay curve, like that for the discharge of a capacitor through a high resistor, is shown in fig. 3.2 to illustrate the idea of half-lives.



0 1 2 3 4

Fig. 3.2

Number of half-lives

The first member of a series decays into a daughter product, but this in turn will probably itself decay until a stable non-decaying isotope is produced. The list of all the members of the family is called a radioactive series. Some time after the production of the original source all the members of the series will be in equilibrium, i.e they will be produced from their parent at the same rate at which they are decaying, i.e $N_1 \lambda_1 = N_2 \lambda_2 = N_3 \lambda_3$ ………., where N_1, N_2, …….are the equilibrium numbers of atoms of each memebr of the series.

An example of a series is:

$$^{23}_{90}\text{Th} \xrightarrow{\alpha} {}^{228}_{88}\text{Ra} \xrightarrow{\beta} {}^{228}_{89}\text{Ac} \xrightarrow{\beta} {}^{228}_{90}\text{Th} \xrightarrow{\alpha} {}^{224}_{88}\text{Ra} \xrightarrow{\alpha} {}^{220}_{80}\text{Ra} \xrightarrow{\alpha} {}^{216}_{84}\text{Po}$$

Note the effects on Z and A of the emission of alphas and betas. Note also that 232_{Th} and 228_{Th} are isotopes.

Example

If 8 x 10^{10} atoms of radon are separated from radium, how many disintegrations will occur in 11.46 days ? (Half-life radon = 3.82 days).

Solution

First, notce that 11.46 days is three times the half-life of radon. We could solve the problem as follows: during the first 3.82 days, half of the radon atoms will decay, leaving 4 x 10^{10}; during the second 3.82 days, half of these will decay, leaving 2 x 10^{10} and during the final 3.82 days, half of these will decay, leaving 1 x 10^{10}. Thus 7 x 10^{10} disintegrations would occur in 11.46 days.

Alternatively,

Using the formular fo radioactive decay (Eq. 3.1), we have

$$N = 8 \times 10^{10}\, e \ - \ (\ 0.693/3.82 \text{ days})\ (11.46 \text{ days})$$

$$= 8 \times 10^{10} e^{-2.079}$$

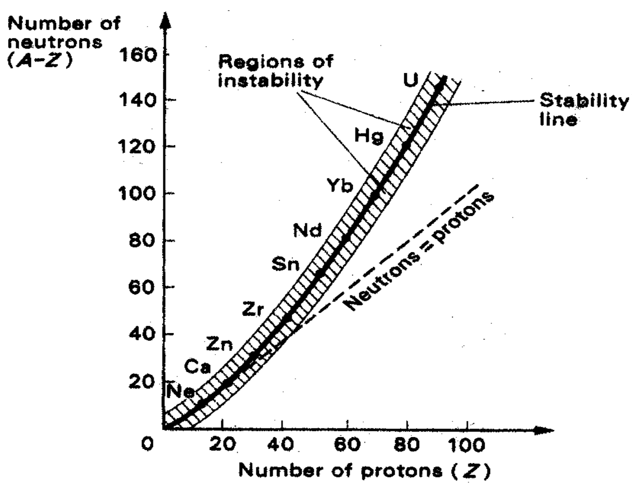
$$= 8 \times 10^{10} \times 0.125$$

$$= 1 \times 10^{10}$$

Hence the number of disintegrations is 7×10^{10}.

3.1.7 Nuclear Stability

Whilst the chemical properties of an atom are governed entirely by the number of protons in the nucleus (i.e the atomic number Z), the stability of an atom appears to depend on both the number of protons and the number of neutrons.



In Fig. 3.3. the number of neutrons (A – Z) has been plotted against the number of protons for all known nuclides, stable and unstable, natural and man-made. A continous line has been drawn approximately through the stable nuclides (only a few are labelled) and the shading on either side of this line shows the region of unstable nuclides.

For stable nuclides the following points emerge:

(i) The lightest nuclides have almost equal numbers of protons and neutrons.

(ii) The heavier nuclides require more neutrons than protons, the heaviest having about 50 per cent more.

(iii) Most nuclides have both an even number of protons and an even number of neutrons. The implication is that two protons and two neutrons. i.e an alpha particle, form a particularly stable combination and in this connection, it is worth noting that oxygen ($^{16}_{8}O$), silicon ($_{14}^{28}Si$) and iron ($^{56}_{28}Fe$) together account for over three quarters of the earth's crust.

For unstable nuclides the following points can be made:

(i) Disintegrations tend to produce new nuclides nearer the 'stability' line and continue until a stable nuclide is formed.

(ii) A nuclide above the line decays so as to give an increase in atomic number, i.e by beta emission (in which a neutron changes to a proton and an electron). Its neutro-to-proton ratio is thereby increased.

(iii) A nuclide below the line disintegates in such a way that its atomic number decreases and its neutron-to-proton ratio increases. In heavy nuclides this can occur by alpha emission.

3.1.8 Nuclear Fission and Fusion

If a nucleus of large mass splits (fissions) into two nuclei of smaller mass, then bearing in mind that the total number of nucleous remains constant, the total energy in the nuclei is less, and the energy difference is released as kinetic energy of the fragment. In 235U, spontaneous fission does not occur, but fission can be caused by bombarding it with thermal (low energy) neutrons.

$$^{235}U + {}^{1}n \rightarrow X + Y + K\ {}^{1}n$$

X and Y represent the fission fragments whose proton and nucleon numbers are not same values for each fission; K is the number of neutrons released in the process, K is not always the same, but the total number of protons and nucleons must be the same on both sides of the equation . K is usually 2 or 3 with an average value of 2.47. The phenomenon may be represented as in fig 3.4

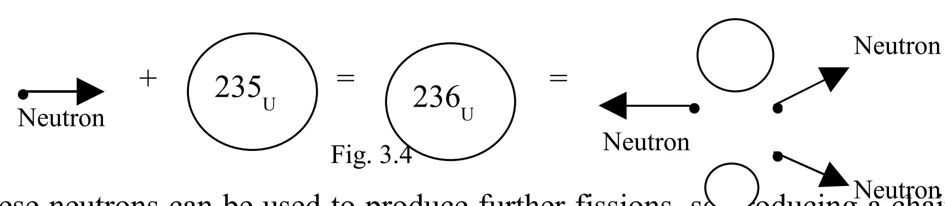


Fig. 3.4

These neutrons can be used to produce further fissions, so producing a chain reaction which will run out of control unless the number of neutrons produced is kept under control. Infact, the neutrons produced in a fission reaction have considerable energies and are known as ‘fast neutrons which do not fision 234U. The neutrons have to be slowed down to thermal energies.

Several different nuclei have been identified as the result of fission, and all that can be said is that the nucleus splits into parts with masses in the approximate ratio 5:7.

Energy can also be produced by the fusion of two nuclei of small mass to produce a more massive nucleus e.g.

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + ^{1}_{0}n$$

This reaction takes place in the sun. Fig. 3.1 shows how this is possible. The difficulty arises in providing the very high temperatures needed to give the two positve nuclei sufficient kinetic energy to overcome their electrostatic repulsion.

4.0 Conclusion

Radioactivity is the spontaneous distegration of certain atomic nuclei accompanied by the emission of alpha-particle (helium nuclei), beta – particles (electrons and positrons), or gamma radiation, which are short-wavelength electromagnetic waves.

Nucleat fussion is a type of nuclear reaction in which atomic nuclei of low atomic number fuse to form a heavier nucleus with the release of large amount of energy. In nuclear fission reactions, a neutron is used to break up a large nucleus , but in number fussion the two reacting nuclei themselves have to be brought into collision. In a nuclear reaction, a chain reaction takes place, the splitting of one uranium nucleus yielding neutrons that cause the splitting of other nuclei.

5.0 Summary

* The nucleus contains protons and neutrons (collectively called nucleons). The total number of neutrons and protons is known as the mass number A.

* The number of protons (the atomic number Z) determines which element the atom is.

* In a neutral atom there are as many orbiting electrons as there are protons in the nuleus and so the atom is uncharged.

- Isotopes are atoms or ions of the same element but with different numbers of nuleous, i.e the same number of protons buyt different number of neutrons.

- A radioactive source is a collection of such atoms whose nuclei have the property of randomly but spontaneouly emitting radiation.

- The radioactive decay law states that a radioactive substance decays exponentially with time. If there are N_a undecayed nuclei at some time t = 0 and a smaller N at a later time t, then $N = N_o e^{-\lambda t}$ where λ is the radioactive decay constant.

- The half-life is the time for the number of active nuclei present in a source at a given time to fall to half its value.
  It is given by $T_{½} = \frac{0.693}{\lambda}$

- An alpha particle is a helium nucleus consisting of 2 protons and 2 neutrons and when an atom decays by alpha emission, its mass number decreases by 4 its atomic number by 2.

- When beta decay occurs, a neutron changes into a proton and an electron. The proton remains in the nucleus and the electron is emitted as a beta particle. The new nucleus has the same mass number, but its atomic number increases by one since it has one more proton.

- The emission of gamma rays is explained by considering that nuclei (as well as atoms) have energy levels and that if an alpha or beta particle is emitted, the nucleus is left in an excited state. A gamma ray photon is emitted when the nucleus returns to the ground state.

6.0 Tutor Marked Assignment

1. Explain the terms “ radioactive decay constant” and “half-life”, and show how they are related to one another.

   The half-lives of two radioactive isotopes X and Y are 25 minutes and 40 minutes respectively. Initially a sample of the isotopes X has the same activity as a sample of the isotope Y. Compare the number of radioactive atoms present in the two samples.

   Compare the number of radioactive atoms present in the two samples and also the activities of the two samples at a time 2 hours later.

(2a) Find the binding energy per nucleon of $^{31}_{15}P$ of its mass is 30.973763 amu.

b) A nucleus of $^{9}_{4}Be$ is struck by an alpha particle. A nuclear reaction takes place with the emission of a neutron. Give the atomic number, mass number and chemical name of the resulting nucleus.

(3a) The decay law for a radioactive disintegration is $-dN/dt = \lambda N$, where λ is a constant and N is the number of unchanged atoms at time t. Given that $N = N_o$ when $t = 0$, obtain the expression for N as a function of t. Deduce the formula giving the number of atoms that have distegrated during the interval of time t.

(b) Define the half-life of a radioactive substance, and show that it equals $(\log_e 2) / \lambda$.

(c) The half-life of an α -emitter is 10 days, and a certain sample has an initial activity of 10^9 disintegrations per second. Find the total number of α - particles emitted during the next 60 days and the activity of the sample at the end of this period.

7.0 References and Other Resources

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