# PHY2049 - Fall 2016 - HW5 Solutions

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October 6, 2016

These are solutions to Halliday, Resnick, Walker Chapter 27, No: 4, 14, 30, 35, 38, 40, 50, 65

## 1 27.4

**Figure 27-27 shows a circuit of four resistors that are connected to a larger circuit.The graph below the circuit shows the electric potential V(x) as a function of position x along the lower branch of the circuit, through resistor 4; the potential $V_A$ is 12.0 V. The graph above the circuit shows the electric potential V(x) versus position x along the upper branch of the circuit, through resistors 1, 2, and 3; the potential differences are $V_B = 2.00$ V and $V_C = 5.00$ V. Resistor 3 has a resistance of 200$\Omega$ . What is the resistance of (a) resistor 1 and (b) resistor 2?**

Resistors $R_1$, $R_2$, $R_3$ all carry the same current, and 12V drops across the three in total. Let $V_i$ be the voltage drop across $R_i$. The graph is unnecessarily confusing, but looking closely we see the voltage across $V_1 = 2$V, $V_2 =$ 5V. Of 12V, that leaves 5V left to drop across $R_3$. The problem states $R_3 =$ 200 $\Omega$, so using ohm's law, the current

$$i = \frac{V_3}{R_3} = 0.25\text{A}$$

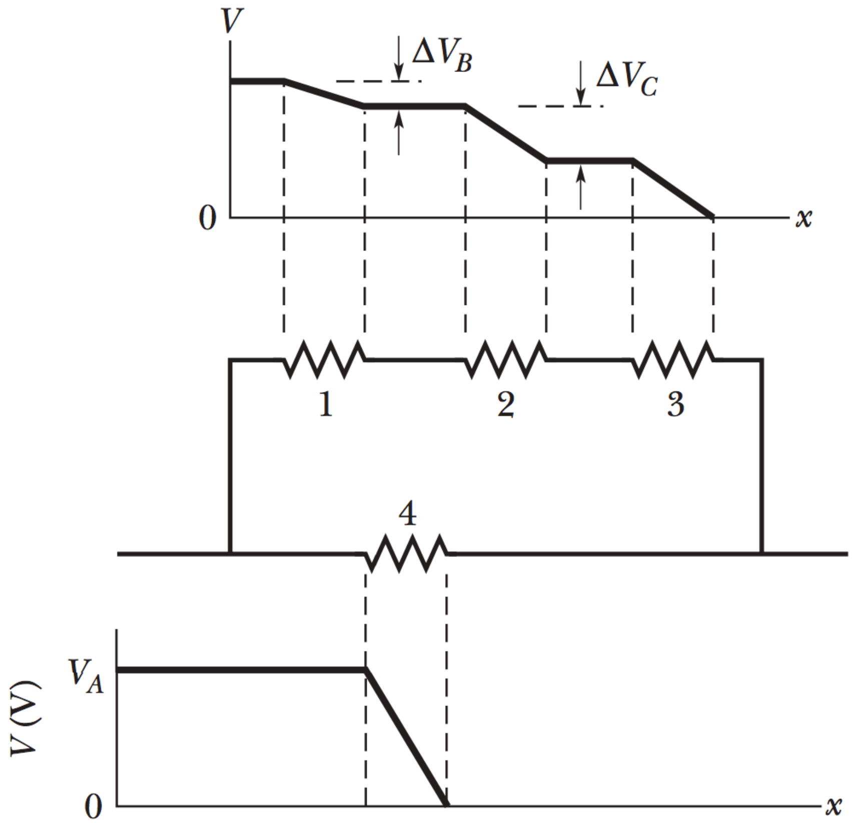


Figure 1: Fig. 27-27 Problem 4

Then,

$$R_2 = \frac{V_2}{i} = 200\Omega$$

$$R_1 = \frac{V_1}{i} = 80\Omega$$

# 2 27.14

**In Fig. 27-32a, both batteries have emf $\mathcal{E} = 1.20$ V and the external resistance $R$ is a variable resistor. Figure 27-32b gives the electric potentials $V$ between the terminals of each battery as functions**

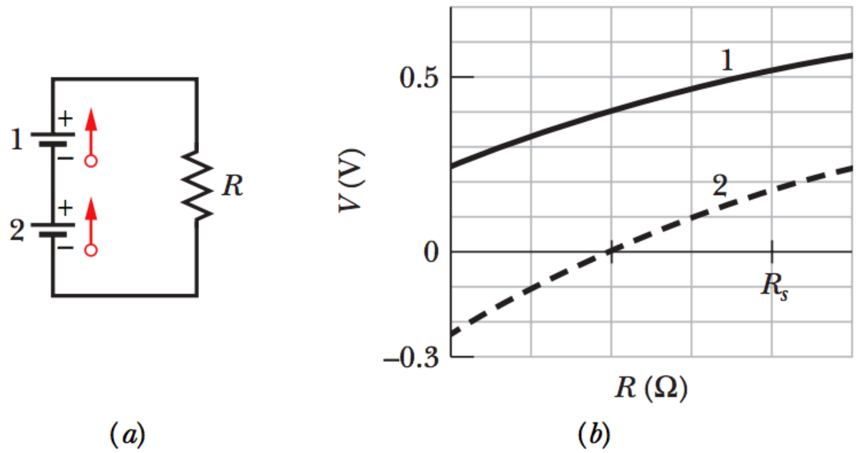


**Fig. 27-32** Problem 14.

Figure 2: Fig. 27-32 Problem 14

**of $R$: Curve 1 corresponds to battery 1, and curve 2 corresponds to battery 2.The horizontal scale is set by $R_s$= 0.20Ω.What is the internal resistance of (a) battery 1 and (b) battery 2?**

The voltage across the terminals of a battery is its EMF minus the potential due to the parasitic internal resistance.

$$V_{\text{battery}} = \mathcal{E} - iR_{\text{internal}}$$

Thus we model a real battery as an ideal voltage source in series with its parasitic internal resistance. Lets call the internal resistance of EMF $\mathcal{E}_i = r_i$. Then the kirchoff's law in this loop yields

$$-iR + (\mathcal{E}_2 - ir_2) + (\mathcal{E}_1 - ir_1) = 0$$

Can you see that the terms in parenthesis are the voltages of the batteries? I hope so. That is what is plotted here, as a function of $R$.

$$-iR + V_1 + V_2 = 0$$

Lets use the point where $R = \frac{1}{2}R_s = 0.1\Omega$. I read from the graph that $V_1 = 0.4\text{V}$ and $V_2 = 0\text{V}$. Now I can write down the loop equation using these values:

$$-iR + V_1 + V_2 = 0$$

$$-i\,(0.1\Omega) + 0.4\text{V} + 0\text{V} = 0$$

Therefore I know the current at that moment:

$$|i| = \frac{.4\text{V}}{.1\Omega} = 4\text{A}$$

Now I can get the internal resistances:

$$\mathcal{E}_1 - ir_1 = 0.4$$

$$r_1 = \frac{0.8\text{V}}{i} = 0.2\Omega$$

$$\mathcal{E}_2 - ir_2 = 0$$

$$r_2 = \frac{1.2V}{4A} = 0.3\Omega$$

# 3 27.30

**In Fig. 27-41, the ideal batteries have emfs $\mathcal{E}_1 = 10.0$ V and $\mathcal{E}_2 = 0.500\ \mathcal{E}_1$, and the resistances are each 4.00 $\Omega$.What is the current in (a) resistance 2 and (b) resistance 3?**

Imagine the current flowing out of each emf: out of $\mathcal{E}_1$ clockwise through $R_1$, and out of$\mathcal{E}_2$ counterclockwise through $R_2$. Then the current through $R_3$ must be just the sum of $i_1$ and $i_2$:

$$i_3 = i_1 + i_2$$

We have 3 loops to apply kirchoff's law here, but we only need two because we have two unknowns. I'll choose the two loops which flow through $R_3$, but you can choose any 2 loops:

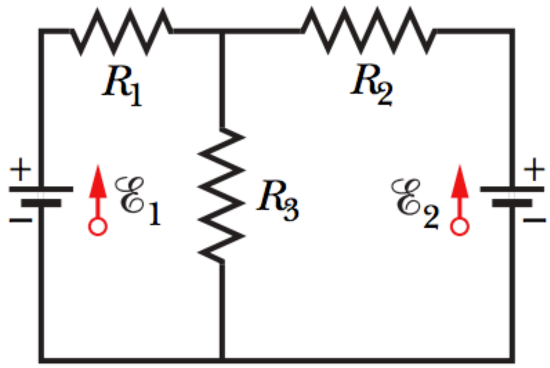


**Fig. 27-41** Problems 30, 41, and 88.

Figure 3: Fig. 27-41 Problem 30

$$\mathcal{E}_1 - i_1 R_1 - i_3 R_3 = 0$$

But wait ... all the R's are the same, 4V. And also, $i_3 = i_1 + i_2$ so lets rewrite that one, and express the other loop the same way. So my system of equations is

$$\text{loop1: } 10\text{V} - i_1 4\Omega - (i_1 + i_2)4\Omega = 0$$

$$\text{loop2: } 5\text{V} - i_2 4\Omega - (i_1 + i_2)4\Omega = 0$$

You'll find $i_1 = 1.25A$ and $i_2 = 0A$. If you are too lazy to solve the simultaneous equations, Wolfram Alpha will do it for you. See Figure 4.

# 4 27.35

**In Fig. 27-46, $\mathcal{E}$ =12.0 V, $R_1 = 2000\Omega$ , $R_2 = 3000\Omega$ , and $R_3 = 4000\Omega$ . What are the potential differences (a) $V_A - V_B$, (b) $V_B - V_C$, (c) $V_C - V_D$, and (d) $V_A - V_C$?**

Because of the symmetry of the problem, we can presume that the current $i_1$ is the same current flowing through both 2k resistors; $i_2$ flows through both 3k resistors and $i_3$ is the current throgh $R_3$. If you apply the junction rule,

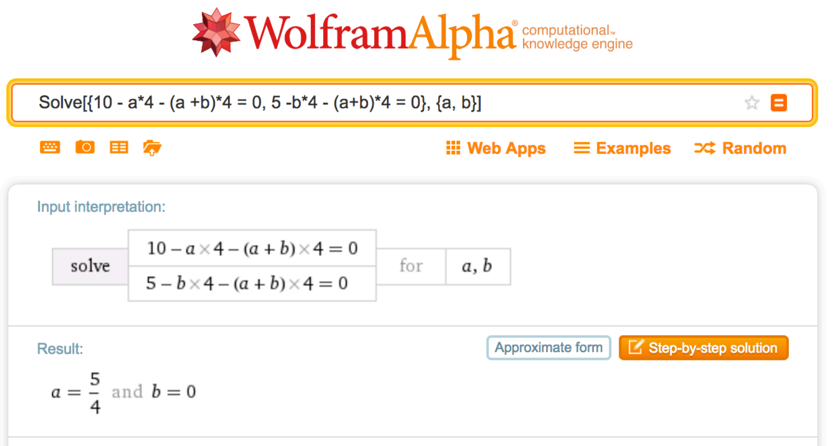


Figure 4: Wolfram alpha can do systems of equations with the syntax Solve[ {eq1,eq1},{var1,var2} ]

you'll see that the current flowing through $R_3$ must be the difference $i_1 - i_2$ so we only have two independent loop equations

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0$$
$$\mathcal{E} - i_1 R_1 - (i_1 - i_2) R_3 - i_1 R_1 = 0$$

$$12 \text{ V} - i_2 (2\text{k}\Omega) - i_1 (2\text{k}\Omega) = 0$$
$$12\text{V} - 2 i_1 (2\text{k}\Omega) - (i_1 - i_2)(4\text{k}\Omega) = 0$$

We find $i_1$ =2.625 mA, $i_2$ = 2.25 mA, and therefore $i_3 = i_1 - i_2 = 375$ $\mu$A.

Once you have the currents, you are almost done because the various voltage drops are given simply by Ohm's law: for example, point A and point B are connected by $R_1$; therefore, $V_A - V_B$ is exactly just the current times the resistance in $R_1$:

$$V_A - V_B = i_1 R_1 = 5.25\text{V}$$
$$V_B - V_C = i_3 R_3 = 1.5\text{V}$$
$$V_C - V_D = i_1 R_1 = 5.25\text{V}$$
$$V_A - V_C = i_2 R_2 = 6.75\text{V}$$

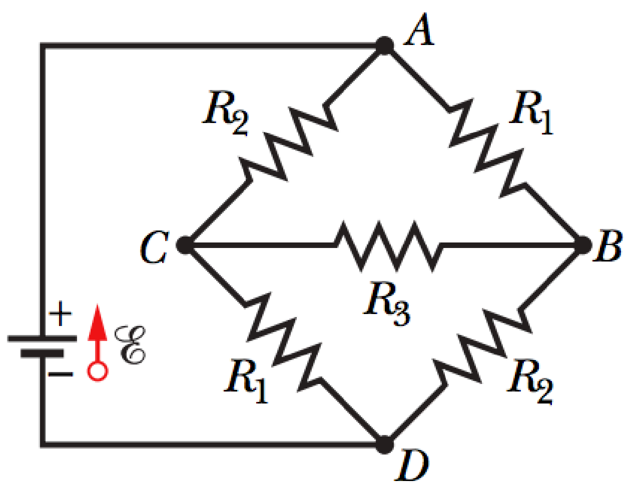


Figure 5: Fig. 27-46 Problem 35

# 5 27.38

**Figure 27-49 shows a section of a circuit. The resistances are $R_1$ = 2.0 $\Omega$, $R_2$ = 4.0 $\Omega$, and $R_3$ = 6.0$\Omega$ , and the indicated current is $i$ = 6.0 A. The electric potential difference between points A and B that connect the section to the rest of the circuit is $V_A - V_B$= 78 V. (a) Is the device represented by Box absorbing or providing energy to the circuit, and (b) at what rate?**

We can find the voltage across $R_3$ right away

$$V_3 = iR_3 = (6\text{A})(6\Omega) = 36\text{V}$$

The total potential from A to B is given as 78V. If 36V is across the parallel part, that leaves 78V-36V = 42V left over. This is the voltage across $R_1$, so the current there must be

$$i_1 = \frac{V_1}{R_1} = 42\text{V}/2\Omega = 21\text{A}$$

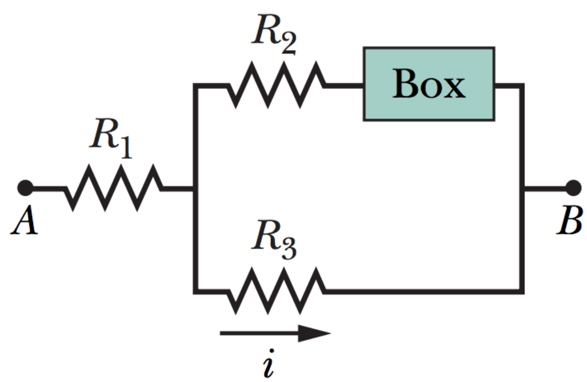


**Fig. 27-49** Problem 38.

Figure 6: Problem 38

Then because 21A must be the sum of currents through both branches of the parallel section ("the junction rule"), the current through $R_2$ must be 21A - 6A = 15A.

Now we can calculate the power dissipated by all the resistors

$$P = i_1^2 R_1 + i_2^2 R_2 + i^2 R_3 = 1998 \text{ W}$$

Let's compare that to the power supplied to the circuit externally

$$P = iV = (21\text{A})(78\text{V}) = 1638 \text{ W}$$

Because the resistors are using energy faster than it is being supplied externally via the 21A input current, we must conclude that the box is supplying the additional energy. The rate is at least the difference between the resistor power consumption of 1998 W and the external power supply of 1638 W leaving **360 W.**

# 6 27.40

**Two identical batteries of emf $\mathcal{E}$ =12.0 V and internal resistance $r$ = 0.200 $\Omega$ are to be connected to an external resistance $R$, either**

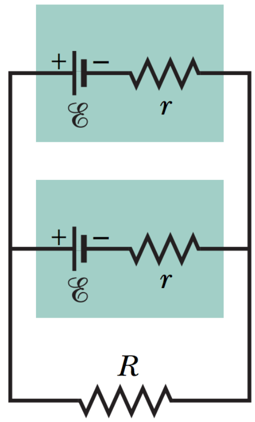


**Fig. 27-50** Problems 39 and 40.

Figure 7: Problem 40

**in parallel (Fig. 27-50) or in series (Fig. 27-51). If $R$ =2.00$r$, what is the current $i$in the external resistance in the (a) parallel and (b) series arrangements? (c) For which arrangement is $i$ greater? If $R = r/$2.00, what is $i$ in the external resistance in the (d) parallel and (e) series arrangements? (f) For which arrangement is $i$ greater now?**

Let $i$ be the current through one of the batteries. Looking at the parallel case, convince yourself that the two batteries are indistinguishable, and therefore have the same current through each. Therefore the current through the external resistance $R$ is twice the current through one of the batteries, or $2i$. Recalling $R = 2r$, a loop equation gives

$$\mathcal{E} - (i + i)R - ir = 0$$
$$\mathcal{E} - (2i)(2r) - ir = 0$$
$$i = \frac{\mathcal{E}}{5r} = 12\text{A}$$

Thus the current through the resistor $R$ is $2i$ or **24A**.

For the series case, we only have one loop to write down, so let's get on with it

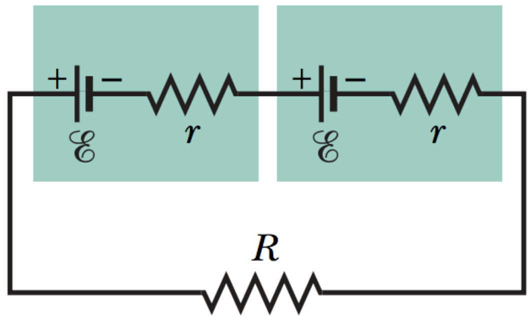


Fig. 27-51 Problem 40.

Figure 8: Problem 40

$$(\mathcal{E} - ir) + (\mathcal{E} - ir) - iR = 0$$
$$(\mathcal{E} - ir) + (\mathcal{E} - ir) - 2ir = 0$$
$$2\mathcal{E} - 4ir = 0$$
$$i = \frac{12\text{V}}{0.4\Omega} = \mathbf{30A}$$

Repeating the calculatons for the case $R = \frac{r}{2}$, we find the current through $R$ is **60A** in the parallel case, and **48A** in the series case.

# 7 27.50

**In Fig. 27-57,$R_1$ =2.00$R$, the ammeter resistance is zero, and the battery is ideal.What multiple of $\mathcal{E}/R$ gives the current in the ammeter?**

The current through the bottom two resistors must be equal. Let us call this current $i$. The total circuit current is thus $2i$. Draw the equivalent circuit which is $(2R \parallel R)$ in series with $(R \parallel R)$. You should find

$$R_{eq} = \frac{7R}{6}$$

Then Ohm's law with Thevenin's theorem says

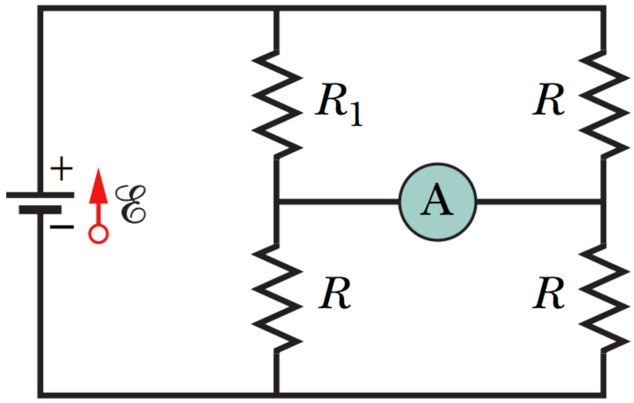


**Fig. 27-57** Problem 50.

Figure 9: Problem 50

$$(\text{voltage}) = (\text{total current})(\text{total equivalent resistance})$$

For us, this means

$$\mathcal{E} = (2i)\frac{7R}{6}$$

Therefore

$$\frac{\mathcal{E}}{R} = \frac{7i}{3}$$

Let us now look at the top part of the circuit, and call the current through the $2R$ branch $i_1$ and the current through the $R$ branch $i_2$. Clearly, these currents sum to the total current so

$$2i = i_1 + i_2$$

The voltage across the $2R$ and $R$ resistors are the same, so

$$i_1(2R) = i_2R$$

Therefore we can see that $\frac{i_2}{i_1} = 2$, which means $i_2$ makes up $\frac{2}{3}$ of the total current, and $i_1$ makes up $\frac{1}{3}$ of the total current. Recall we defined the total current to be $2i$ so

$$i_2 = \left(\frac{2}{3}\right)(2i) = \frac{4i}{3}$$
$$i_1 = \left(\frac{1}{3}\right)(2i) = \frac{2i}{3}$$

Finally, convince yourself using the junction rules that the current flowing through the ammeter $i_A$, which is what we want all along, must make up half the difference between $i_2$ and $i_1$

$$i_A = \frac{1}{2}(i_2 - i_1)$$
$$i_A = \frac{1}{2}\left(\frac{4i}{3} - \frac{2i}{3}\right) = \frac{1}{2}\frac{2i}{3} = \frac{i}{3}$$

Since we earlier concluded $\frac{\mathcal{E}}{R} = \frac{7i}{3}$ we can assert

$$\frac{1}{7}\frac{\mathcal{E}}{R} = \frac{i}{3} = i_A$$

# 8 27.65

**In Fig. 27-66, $R_1$= 10.0 k$\Omega$, $R_2$ = 15.0 k$\Omega$, $C$ = 0.400 $\mu$F, and the ideal battery has emf $\mathcal{E}$ = 20.0 V. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time $t = 0$. What is the current in resistor 2 at $t$ = 4.00 ms?**

In the steady state, there is no current through the capacitor and the resistors divide the voltage as usual. The inital capacitor voltage $V_0$ is just the voltage across $R_2$

$$V_2 = \mathcal{E}\frac{R_2}{R_1 + R_2} = V_0$$
$$V_0 = (20\text{V})\frac{15\text{ k}\Omega}{25\text{k}\Omega} = \mathbf{12V}$$

If the capacitor's voltage were forced to change, so too would the voltage across $R_2$ because they are in parallel.

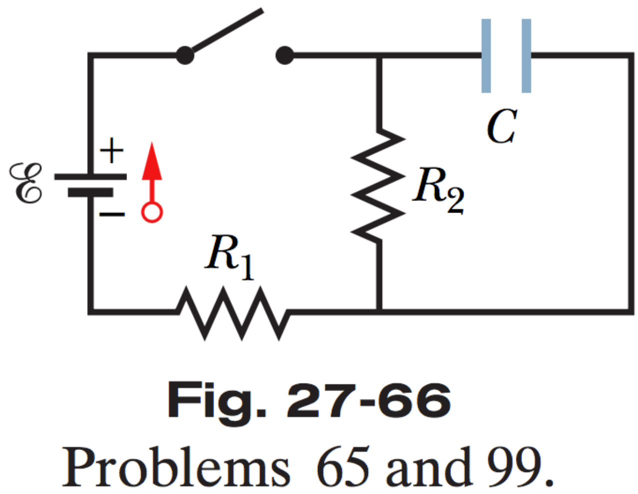


Figure 10: Problem 65

When the switch is closed, the capacitor will begin to discharge. The time dependent voltage across this discharging capacitor (and thus $R_2$ is given by)

$$V(t) = V_0 e^{-\frac{t}{RC}}$$

The $RC$ time constant is $(15\text{k}\Omega)(0.4\mu\text{F}) = 0.006$ seconds. The time of interest is $t' = 4.00$ ms $= 0.004$ seconds

$$V(t = 0.004) = (12V)\, e^{-\frac{0.004}{0.006}} = \mathbf{6.16V}$$

The current through $R_2$ is then just given by ohms law

$$i_2 = \frac{V_2}{R_2} = \frac{6.16\text{V}}{15\text{k}\Omega} = \mathbf{0.41\ mA}$$