PHYS 3324

Radioactive Decay

Required background reading
Attached are several pages from an appendix on the web for the Tipler-Llewellyn "Modern Physics" textbook. You do not have to read them all (unless you want to), but make sure you read the following:

- First paragraph after the words "Charged Particles". The basic idea is that charged particles of a given energy have a reasonably well-defined range when their kinetic energy gets absorbed by matter.
- The top paragraph on page 89; the main point there is that the energy loss of a charged particle per unit length in a material is just proportional to the density. So the energy loss in a given material of thickness x is essentially just $dE = K\,x\,\rho$, where K is a constant. So, for convenience, we often express the "thickness" of a material as $x\rho$, which has units of g/cm^2. This is a convenient way of comparing the energy of absorbing properties of pieces of material with different compositions.
- The section on photons at the end. The main point is that rather than having a well-defined range in materials, the intensity of photons decreases exponentially as the thickness of the shielding material is increased.

Read Thornton section 12-6 on radioactive decay, which contains the definition of half-life, etc.

Prelab Questions

1. Assume you have a monoenergetic source of gamma rays of intensity I_o and then you put some lead shielding between it and your detector. The transmitted intensity through the lead as a function of the thickness of the lead absorber is given by:

$$I = I_o\, e^{-\mu x} = I_o\, e^{-(\mu/\rho)\rho x}$$

where ρ = 11.35 g/cm^3 is the density of the lead and μ is the absorption coefficient of the lead for this energy photon. For the 0.662 MeV gamma rays that you will use in this experiment, the absorption coefficient is μ/ρ=0.1137 cm^2/g. What thickness of lead, x, is needed to cut down the transmitted intensity of 0.662 MeV photons by a factor of 2 (relative to the case with no shielding present)?

2. In this experiment, you will measure the half-life of a particular short-lived radioactive isotope, ^{137m}Ba. You will find that it has a half-life, $t_{1/2}$, of about 2.55 minutes. Assuming that the original intensity of the source is 1000 Bq = 1000 decays/second, how long do you have to wait until the amount of radioactivity is down to 1 Bq = 1 decay/second?

Introduction

Radioactivity is produced when the nucleus of an atom decays and emits radiation. The decay products can be electrons, positrons, gamma rays and alpha particles, all of which, due to the ionization they produce, are considered pollutants. They are clearly undesirable, as these rays or particles can penetrate the human body and produce mutation of cells, the results of which are not readily predictable. At very high radiation levels, the effects are well-known and can cause sickness and even death. Fortunately, the radioactive contaminants that surround us are very, very small. Common materials (stone, brick...) contain small concentrations of long-lived radioactive isotopes which produce low levels of radiation. An additional source of radioactivity is the ever-present cosmic radiation which bombards earth from outer space, producing a non-negligible part of the radioactivity in the biosphere. The radioactive sources you will use in this lab also consist of very small amounts of radiation, so they are not harmful to you.

The two types of radioactivity you will explore in this lab are those that produce gamma rays (energetic photons with energies ~ 1 MeV) and electrons (from so-called beta decay). We will describe each of these processes in a little more detail as you encounter them in this lab.

The equipment you will be using is shown in Figure 1. It consists of the following pieces; please check to be sure you have them all.

- Geiger tube mounted in cylindrical holder with shelves below them (please don't remove this Geiger tube without assistance from your TA; it is easily damaged)
- Source set with four sealed radioactive sources - ^{137}Cs, ^{90}Sr, ^{204}Tl, ^{60}Co
- Set of absorbers in wood box (plastic, polyethylene, aluminum, lead)
- Geiger tube control and counting unit (SPECTECH ST360 Counter)
- Minigenerator (at instructor's table; he or she will prepare this source for you)
- Stopwatch

Figure 1: Experimental equipment for this lab

The heart of this apparatus is the Geiger-Muller (GM) tube. It is attached to a control box that provides high voltage to the tube and processes the electrical signals from the tube. The GM tube is a gas-filled tube with a wire down the middle. Ionization from charged radioactive decay particles is amplified due to avalanche discharges in the gas in the presence of the large electric field from the applied high voltage. The control box counts the number of signals that the tube produces when penetrated with charged particles. With this apparatus, you can measure the counting rates of pulses produced by the particles emitted from a radioactive sample. (Note: even though gamma rays are uncharged, you can detect them with this apparatus too. The gammas can interact in the gas through one of three mechanisms – photoelectric effect, Compton scattering, and pair production. All three of these processes produce energetic charged particles that make a signal in the GM tube).

NOTE: Please do not remove the GM tube from its holder. It has a very thin window on the bottom end that is easily damaged. It needs to be thin so that low energy electrons from radioactive decay can penetrate it. The window is protected on the bottom side (top of the shelf assembly) with a small screen, but please still be careful when you are sticking sources and absorbers onto the shelves.

Procedure

Part 1: Measurement of the plateau curve for the GM tube and determination of the operating voltage

Every GM tube is a little different. Your first task is to determine the proper operating voltage for this tube. As the high voltage on the GM tube is increased the size of the collected pulses gets larger. Different energy particles give different size pulses. The processing electronics hooked to the GM tube only registers a "count" if a particle is above a certain threshold. We want to make sure that the high voltage is turned up high enough so that it will register counts for all radioactive particles that interact in it. On the other hand, if we turn up the high voltage too high the GM tube will operate in the "discharge" region meaning it will have a constant discharge, instead of individual pulses. The right operating point is somewhere in between, as you will see.

To do this determination, you will put a radioactive source at a fixed position, and then you will register the number of counts in a fixed time period at a range of high voltage settings. An example of the type of data you will obtain is shown in Figure 2. This data is all taken for a radioactive sample at a fixed position. Below a certain high voltage we are clearly not counting all the counts. After a certain high voltage is achieved (about 800 V in Figure 2) the count rate is relatively flat as a function of high voltage (the "plateau" region). Eventually, when the high voltage is turned high enough, the count rate starts to rise dramatically. This is the onset of the "discharge" region. Once you determine your "plateau" curve, you want to choose an operating point that is somewhere left of the center of the plateau, as shown in Figure 2.

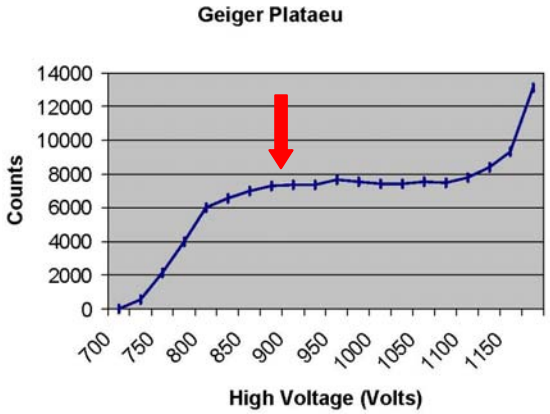


Figure 2: Typical Geiger plateau curve with recommended operating point (for this case) shown by the arrow

Follow this procedure to obtain your own plateau curve.

1. Turn on the GM tube control unit with the button on the back. Hit the "DISPLAY SELECT" button and toggle till you get to HIGH VOLTAGE. By clicking the UP button increase the GM tube high voltage to your starting point which will be 700 volts. Then set the count duration by clicking "SELECT DISPLAY" until you get to TIME and then toggle up until you get to 10 seconds. This means that the unit will count for a timed interval for 10 seconds after you hit the COUNT button.

2. Locate the ^{90}Sr source. This is a source that decays by "beta-decay". One of the products is electrons with kinetic energies up to 0.54 MeV. Place the source in the clear plastic holder with the yellow label down. This will give the highest count rate since the holder is less thick on the bottom (so more of the decay electrons penetrate through and get to the GM tube). Put the plastic holder on the top shelf, so it is as close as possible to the GM tube for the highest count rate. Click "SELECT DISPLAY" until you get to "COUNT" and then hit the COUNT button to start counting. The red LED on COUNT will go on and it will go off after your preset 10 seconds of counting.

3. Prepare a table to record high voltage and counts observed in 10 seconds. After each count you need to hit the RESET button to clear the counts back to zero. Take data for high voltages between 700 and 1200 in 20 volt steps. Be sure not to disturb the position of the radioactive source during data-taking. **DO NOT EXCEED a VOLTAGE of 1200 Volts applied to the GM tube!**

4. After you have taken your data make a quick plot of it using Excel on one of the computers available in the lab. Determine the operating voltage (somewhere to the left of the center of the plateau) such as shown in Figure 2. Your chosen operating voltage will probably end up somewhere in the range 850 – 940 volts. If it is not in that range, consult with your TA before proceeding. Don't disturb your set-up or counter settings; you will use exactly the same setup for part 2.

Part 2: Statistics of radioactive decay

When you take counts from a radioactive source for a fixed time interval, you get some number *N*. When you then take another count for the same time interval, you won't get exactly the same number of counts. If you do it many times you will see a distribution of values. For the case of radioactive decay, this is described well by a distribution called a *Poisson distribution*, which actually starts to become the same as a Gaussian distribution for a large number of counts. Statistical analysis shows that the standard deviation of this distribution is $\sigma = \sqrt{\overline{N}}$, where $\overline{N}$ is the average number of counts in that time interval. To test this you will take several fixed-time counts for a radioactive source; you will analyze it later and convince yourself that $\sqrt{\overline{N}}$ is a good estimate for the error on a given radioactive count.

1. Use the exact same setup as part 1:
   - 10 second counting interval
   - ^{90}Sr source on the top shelf (yellow label down)
   - GM tube set at your chosen operating voltage

2. Take 10 separate 10 second counts and note down your numbers. You will analyze these data later for your report.

Part 3: Absorption of Gamma Rays in Matter

This part of the experiment will examine the means by which nuclear radiation can be attenuated. First you will consider gamma rays (energetic photons in the MeV range that come from the decay of excited nuclear states). When a beam of gamma rays strikes a slab of material, reactions with the electrons and nuclei in the material (photoelectric effect, Compton scattering, and pair production) will remove some of the gamma rays from the beam. If *N* gamma rays are incident on a given slab of thickness *dx* in a given time interval, then that number is reduced by some amount *dN* through interactions in the material. The number *dN* is given by the relation

$$dN = -\mu N\, dx$$

where μ is a proportionality constant, called the absorption coefficient, which is characteristic of the material and also depends on the energy of the gamma rays. A negative sign appears because *dN* represents a removal of gammas from the beam. Solving this relation gives

$$N(x) = N_0\, e^{-\mu x} = N_0 e^{-(\mu/\rho)(\rho x)}$$

Here, *N(x)* is the number of gammas remaining in the beam after the beam has traversed a distance *x* into the material and *N* is the number or gammas entering the material at *x* =0.

In the second part of the expression we have expressed the "thickness" of the material as ρx where ρ is the density of the material. This has units of g/cm^2. In that scheme we represent the absorption coefficient as μ/ρ with units of cm^2/g.

Remove your ^{90}Sr source and get out the ^{137}Cs source. We are interested in the 0.662 MeV gamma rays that ^{137}Cs emits. You will study absorption of this gamma ray in two different types of absorbers – aluminum and lead. Take a look at the absorber set (in the wooden box). Note that for each different type of absorber, there are two numbers given for it – the thickness in inches (or in mils, which is 1/1000 of an inch) and the thickness given as ρx (in units of mg/cm^2). Also, there is a code for each one.

1. Start by setting up the counter for 30 second fixed counts and leave the high voltage of the GM tube at your desired operating voltage. Initially, take a "background" count by moving the other radioactive sources (including the ^{137}Cs) to the edge of the table. Take three 30 second counts and note them down. These are your counts of the background from radioactive material in the walls of the room and from cosmic rays.

2. Put the ^{137}Cs source onto the plastic tray with its yellow label UP (this is because ^{137}Cs emits an electron as part of a beta decay, but we are not interested in it here, so we orient the ^{137}Cs source with the most absorber in the way of the electron). Put the source on the **fifth** shelf from the top. This will leave room for you to put in absorbers. Prepare a data table that has columns for absorbers (and thicknesses) used and counts per 30 seconds. Take data for aluminum absorbers in the thickness range .040 inches up to a maximum of 0.395 inches (which can be obtained by putting the four thickest aluminum absorbers in the shelves above the ^{137}Cs source. Take data with at least seven different absorber combinations (for a total of 7 different thicknesses).

3. Now do the same as step 2, but for the lead absorbers. Here, you can put the ^{137}Cs source on the third shelf from the top. Take data for seven different combinations of lead absorbers for a thickness range of .032 inches up to .375 inches.

4. For your report, you will analyze the data you took above to determine the absorption coefficient for gamma rays of 0.662 MeV energy in aluminum and lead.

Part 4: Absorption of Energetic Electrons in Matter

Now you will consider how electrons are attenuated in matter. There is a fundamental difference between the absorption of gammas and electrons. Whereas a photon is completely removed from the beam in a single collision, the electron loses its energy gradually in many collisions and is characterized by a definite range. The absorption of electrons depends on the fact that they have charge as compared to the gammas which do not. As an electron passes through matter, the probability of a direct collision with an electron or nucleus is very small. But its electric field interacts with other electrons in the material exciting atomic electrons or removing (ionizing) them. Since the incident particle does work on these atomic electrons in the material, its energy decreases, and it slows down. The electron comes to rest when all of its kinetic energy has been

transferred to atoms of the material. For a given electron kinetic energy that means that electrons make it through until the thickness of material is enough to stop the electrons of that energy. Electrons of higher kinetic energy require more material to be stopped. As pointed out in the introduction, the amount of energy lost in a given thickness of material is just proportional to ρx, independent of the type of materials (for materials with low Z atoms), where ρ is the density and x is the thickness.

The actual source of electrons you will use in this part is a ^{204}Tl source which emits a spectrum of electrons with kinetic energies ranging from 0 to 0.765 MeV. So what you will actually see in this part is similar to Figure 3, which shows a typical "number of counts" versus "absorber thickness" plot. As the absorber thickness is increased, you will see the count rate decrease as the lower energy electrons are stopped. Eventually, there is enough absorber that even the electrons with the maximum kinetic energy are stopped. At that point you will essentially be seeing only the background count rate, independent of how much more absorber you add.

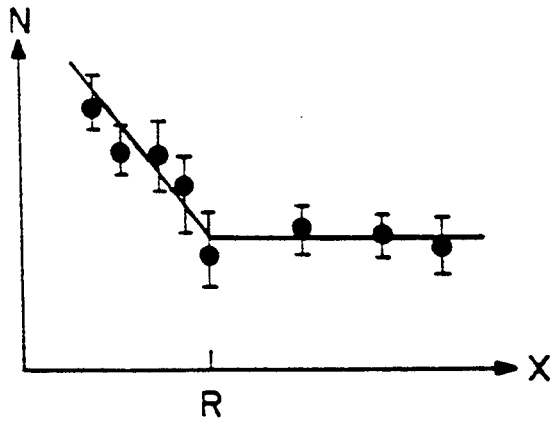


Figure 3: Number of electron counts versus absorber thickness (plotted on a semi-log graph, where the y-axis is the logarithmic axis).

1. Set the counting time interval to 20 seconds. Before inserting the source, take three 20 second measurements so you have a determination of the background count rate.

2. Mount the ^{204}Tl source (with yellow label DOWN so you the betas don't get absorbed in the source material) on the third shelf from the top. For this part of the experiment you should use the absorbers with the codes A – F. You don't have to worry about which type of material it is, just note down the total ρx thickness in mg/cm^2 for each combination of absorbers you use. Take counts for 20 seconds. Record the absorber thickness and count rate in a table. Gradually increase the absorber thickness, you should see the count rate decrease until it levels off at some thickness (when it should then be down near to the background count rate). (Note: make sure to handle the thin absorbers by their frame on the edges; don't grab them in the middle of the absorber.)

3. You will analyze your data for your report to determine the range in matter of the electrons with the highest kinetic energy (0.765 MeV) from ^{204}Tl.

Part 5: Measurement of a Radioactive Half-Life

So far, you have not been aware of the fact that the radioactive sources you have been using will eventually all decay away, and no more radioactive emissions will be observed. That is because all of the sources you have used so far have long half-lives; just look at the half-lives on their labels. In this part of the experiment, you will look at a radioactive isotope with a short half-life and use your GM tube to directly measure its half-life.

To prepare a short-lived isotope for you, your TA will obtain a radioactive sample from a miniature eluting system called a radionuclide "cow" (your TA will "milk" the cow for you). The cow contains a long-lived radionuclide which decays to a short lived, daughter radionuclide. It is designed so that the short-lived daughter can be chemically separated from the long-lived parent. The basic idea is shown in Figure 4. The parent nuclide, ^{137}Cs, has a half-life of 30.1 years, and it decays by beta decay to an excited state of the ^{137}Ba nucleus. The excited state has a half-life of 2.55 minutes, and it decays by emitting a 0.662 MeV gamma ray. This short-lived ^{137}Ba isotope is milked, chemically, from the cow in a few seconds. In this process, an acid is brought into contact with the ^{137}Cs. Whereas the barium is soluble in the acid, the cesium is not. Therefore, the radioactive barium isotope dissolves in the acid and can be separated from the cesium when the acid flows through the sample. Your TA will prepare a few drops of this with the barium dissolved in it and bring it to you. **BEFORE** asking your TA to do that, prepare for efficient data-taking by reading the procedure steps below.

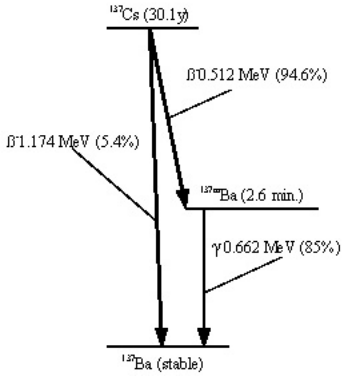


Figure 4: Decay scheme for the mini-generator. You will be measuring the half-life of the ^{137m}Ba isotope that is chemically separated from the cesium.

1. Read these instructions before obtaining your sample from your TA. The activity of the sample will decrease rapidly, so you need to be ready to start taking data immediately when your TA delivers your sample to you. Set the data-taking interval to 30 seconds.

Locate your stop watch on your tabletop. Before getting your sample, take a couple of 30 second "background counts" and record the numbers. Then prepare for getting your sample; basically you want to take a 30 second count every minute. The stop watch can be used to keep track of the minutes. You should do this for 10 minutes; so you will take a 30 second count, followed by 30 seconds of no counting over and over again for 10 minutes. When you have all this prepared, ask your TA to deliver your sample. He or she will put it in the shelf. Please don't remove it. Just take your data. The TA will remove the sample later and dispose of it after it has decayed away (after 30 minutes it will have decayed away to a level that is below the natural background radioactivity rate in the room; that was the point of one of your prelab questions).

2. You will analyze your count rate versus time data in your report to determine the half-life of this radioisotope. Based on what you can already see in your numbers, it probably won't surprise you that this radionuclide has a short half-life (about 2.55 minutes).

Before you leave, let your instructor survey your hands and work area with the radiation monitor. Unless you deliberately touched the liquid sample your TA brought you, you should have no contamination. But it is always safe to check and establish good habits in case you ever work in a lab with more significant quantities of radioactive materials. All of our sources are "license-exempt", so they really represent very small amounts of radiation.

Report

Make sure your report for this lab includes all of the following:

1. Introduction

2. Part1: Plateau curve: Include the plot of the plateau curve data you took and mark your chosen operating voltage on the plot.

3. Part 2: Statistics: As indicated in the text, $\sigma = \sqrt{\overline{N}}$ is a good measure of the statistical uncertainty (random error) for a radioactive sample with an average number of counts $\overline{N}$ in a given interval. Assume you measure the number of counts several different times, as you did in this part of the lab. If $\sqrt{\overline{N}}$ is a good estimate of the random error on each measurement, then 63% of the measurements should fall between $\overline{N} - \sqrt{\overline{N}}$ and $\overline{N} + \sqrt{\overline{N}}$. To check this, find the average, $\overline{N}$, of your ten 10-second counts. Do your results agree with the expectation that 63% of the measurements will fall within the range indicated?

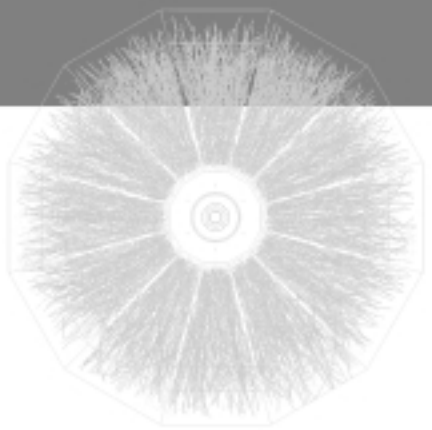
4. Part 3: Absorption of Gamma Rays in Matter: You should have two sets of data for this part – for aluminum and for lead. For each measurement, you need to subtract the background from the measurement and determine the error on each point using the fact

that the error is just $\sigma = \sqrt{N}$ for each measurement. As noted in the lab, the expected dependence of the count rate on absorber thickness is exponential - $N(x) = N_0 e^{-\mu x} = N_0 e^{-(\mu/\rho)(\rho x)}$. To make this appear as a straight line when you plot it, you need to plot the natural log (ln) of the number of counts *N* on the y-axis versus the material thickness on the x-axis. Do a least squares fit in each case to determine the absorption coefficient in units of cm^2/g (refer to the section VI on page A-18 of the error analysis appendix for how to do a least squares fit in a case like this where the errors on each point are different;also see the discussion in example 4 for an example of an exponential function like this one). Compare your results to the following accepted values: 0.0750 cm^2/g for aluminum and 0.1137 cm^2/g for lead for the 0.662 MeV gamma ray. The density of aluminum is 2.7 g/cm^3 and the density of lead is 11.35 g/cm^3.

5. Part 4: Absorption of Energetic Electrons in Matter: Correct each of your measurements for the background counts and assign an error using $\sigma = \sqrt{N}$. Plot your results on a plot like in step 4 (ln(N) versus absorber thickness). What range (in units of g/cm^2) corresponds to the point indicated in Figure 3 where the slope definitely changes? This is the range for the most energetic electrons (0.765 MeV) in your ^{204}Tl radioactive sample.

6. Part 5: Measurement of a radioactive half-life: Correct each of your measurements for the background counts and assign an error using $\sigma = \sqrt{N}$. Plot your results on a plot like in step 4 (ln(N) versus time) and do a least squares fit to determine the decay constant and the half-life. Refer to section VI on page A-18 of the error analysis appendix (and example 4 on page A-20) for details of how to do the least squares fit.

7. Conclusion



More
Interaction of Particles and Matter

In this section, we will discuss briefly the main interactions of charged particles, neutrons, and photons with matter. Understanding these interactions is important in the development of nuclear detectors, the design of radiation shielding, and the study of effects of radiation on living organisms. For the important latter case, we will examine the principal factors involved in stopping or attenuating a beam of particles.

Charged Particles

When a charged particle traverses matter, it loses energy mainly through collisions with electrons. This often leads to the ionization of the atoms in the matter, in which case the particle leaves a trail of ionized atoms in its path. If the energy of the particles is large compared with the ionization energies of the atoms, the energy loss in each encounter with an electron will be only a small fraction of the particle's energy. (A heavy particle cannot lose a large fraction of its energy to a free electron because of conservation of momentum, as we saw in Section 4-2. For example, when a billiard ball collides with a marble, only a very small fraction of the energy of the billiard ball can be lost.) Since the number of electrons in matter is so large, we can treat the loss of energy as continuous. After a fairly well-defined distance, called the *range,* the particle will have lost all its kinetic energy and will come to a stop. Near the end of the range, the view of energy loss as continuous is not valid because the kinetic energy is then small and individual encounters are important. For electrons, this can lead to a significant statistical variation in path length, but for protons and other heavy particles with energies of several MeV or more, the path lengths vary by only a few percent or less, for identical monoenergetic particles. The statistical variation of the path lengths is called *straggling.*

We can get an idea of the important factors in the stopping of a heavy charged particle by considering a simple model. Let ze be the charge and M the mass of a particle moving with speed v past an electron of mass m_e and charge e. Let b be the impact parameter. We can estimate the momentum imparted to the electron by assuming that the force has the constant value $F = kze^2/b^2$ for the time it takes the particles to pass the electron, which is of the order of $t \approx 2b/v$ (see Figure 12-18). The momentum given to the electron is equal to the impulse, which is of the order of magnitude

$$p \approx Ft = \frac{kze^2}{b^2}\frac{2b}{v} = \frac{2kze^2}{bv} \qquad \textbf{12-16}$$

(*Continued*)

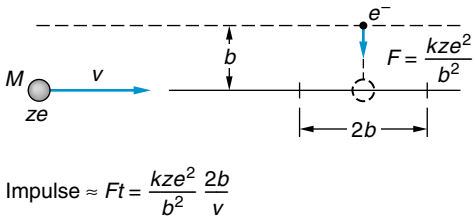


Fig. 12-18 Model for calculating the energy lost by a charged particle in a collision with an electron. The impulse given to the electron is of the order Ft, where $F = kze^2/b^2$ is the maximum force and $t = 2b/v$ is the time for the particle to pass the electron.

where k is the Coulomb constant. (The same result is obtained by integration of the variable impulse, assuming the particle moves in a straight line and the electron remains at rest.) The energy given to the electron is then

$$K_e = \frac{p^2}{2m_e} = \frac{2k^2z^2e^4}{m_ev^2b^2} \qquad \textbf{12-17}$$

This is the kinetic energy lost by the particle in one encounter.

To find how many such encounters there are, consider a cylindrical shell of thickness db and length dx (see Figure 12-19). There are $Z\,(N_A/A)\rho 2\pi b\,db\,dx$ electrons in the shell, where Z is the atomic number, A the atomic weight, N_A Avogadro's number, and ρ the mass density. The energy lost to these electrons is then

$$-dK = \frac{2k^2z^2e^4}{m_ev^2b^2} Z \frac{N_A}{A} 2\pi b\rho\, db\, dx$$

If we integrate from some minimum b to some maximum b, we obtain

$$-\frac{dK}{dx} = \frac{4\pi k^2z^2e^4(Z/A)N_A\rho}{m_ev^2} L \qquad \textbf{12-18}$$

where

$$L = \ln\left(\frac{b_{\max}}{b_{\min}}\right) \qquad \textbf{12-19}$$

The range of the values of b can be estimated from general considerations. For example, this model is certainly not valid if the collision time is longer than the period the electron is in orbit. The requirement that $2b/v$ be less than this time sets an upper limit on b. The lower limit on b can be obtained from the requirement that the maximum velocity the electron can receive from a collision is $2v$ (obtained from the

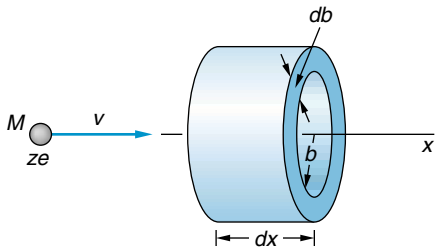


Fig. 12-19 In path length dx, the charged particle collides with $n2\pi b\,db\,dx$ electrons with impact parameters in db, where $n = Z(N_A/A)\rho$ is the number of electrons per unit volume in the material.

(*Continued*)

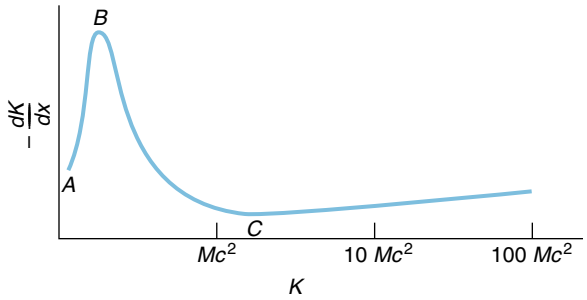


Fig. 12-20 Energy loss $-dK/dx$ vs. energy for a charged particle. The energy loss is approximately proportional to $1/v^2$, where v is the speed of the particle. Thus, in the nonrelativistic region B to C, $-dK/dx$ is proportional to K^{-1}, and in the relativistic region above C, $-dK/dx$ is roughly independent of K. At low energies, in the region A to B, the theory is complicated because the charge of the particle varies due to capture and loss of electrons.

classical mechanics of collisions of a heavy particle with a light particle). In any case, L is a slowly varying function of the energy, and the main dependence of the energy loss per unit length is given by factors other than L in Equation 12-18. We see that $-dK/dx$ varies inversely with the square of the velocity of the particle and is proportional to the square of the charge of the particle. Since $Z/A \approx 1/2$ for all matter, the energy loss is roughly proportional to the density of the material.

Figure 12-20 shows the experimentally measured rate of energy loss per unit path length $-dK/dx$ versus the energy of the ionizing particle. We can see from this figure that the rate of energy loss $-dK/dx$ is maximum at low energies and that at high energies it is approximately independent of the energy. Between points B and C on the curve, the energy loss is proportional to $1/v^2$, as suggested by Equation 12-18. For relativistic particles, those with energies beyond point C, the speed does not vary much with energy and the curve varies only because of the slow change of L. The low-energy portion of the curve, between A and B, is not given by our simple model. At very low energies the energy loss function is more complicated. Particles with kinetic energies greater than their rest energies mc^2 are called *minimum ionizing particles.* Their energy loss per unit path length is approximately constant, and their range is roughly proportional to their energy. Figure 12-21 shows the range versus energy curve for protons in air at very low energies, i.e., to the left of point A in Figure 12-20.

Since a charged particle loses energy through collisions with the electrons in a material, the greater the number of electrons, the greater the rate of energy loss. The

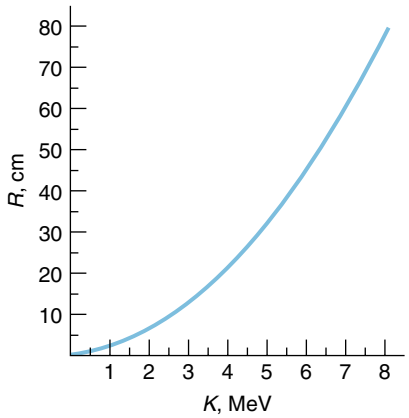


Fig. 12-21 Range vs. kinetic energy for protons in dry air. Except at low energies, the relationship between range and energy is approximately linear.

(*Continued*)

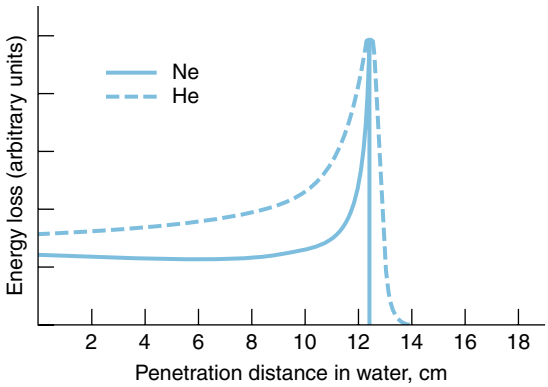


Fig. 12-22 Energy loss per unit length of helium ions and neon ions in water vs. depth of penetration. Most of the energy loss occurs near the end of the path in the Bragg peak. In general, the heavier the ion, the narrower the peak.

energy loss rate $-dK/dx$ is approximately proportional to the density of the material. For example, the range of a 6-MeV proton is about 40 cm in air; but in water, which is about 800 times more dense than air, its range is only 0.5 mm.

It is often convenient to divide out the density dependence in Equation 12-18 by defining thickness parameter:

$$l = \rho x \qquad \text{g/cm}^2 \qquad \textbf{12-20}$$

If we then express the energy loss as dK/dl, it does not vary much from one material to another.

If the energy of the charged particle is large compared with its rest energy, the energy loss due to radiation as the particle slows down is important. This radiation is called *bremsstrahlung*. The ratio of the energy loss by radiation and that lost through ionization is proportional to the energy of the particle and to the atomic number Z of the stopping material. This ratio equals 1 for electrons of about 10 MeV in lead.

The fact that the rate of energy loss for heavy charged particles is very great at low energies (as seen from the low-energy peak in Figure 12-20) has important applications in nuclear radiation therapy. Figure 12-22 shows the energy loss per unit length versus penetration distance of charged particles in water. Most of the energy is deposited near the end of the range. The peak in this curve is called the *Bragg peak.* A beam of heavy charged particles can be used to destroy cancer cells at a given depth in the body without destroying other, healthy cells if the energy is carefully chosen so that most of the energy loss occurs at the proper depth.

Neutrons

Since neutrons are uncharged, their interaction with electrons in matter is via the magnetic moments of the two particles, rather than the Coulomb force. This interaction is used extensively to investigate the magnetism of bulk matter; however, it does not result in the transfer of kinetic energy to the electrons. Neutrons are removed from a beam by scattering from the nuclear potential or by capture. For kinetic energies that are large compared with thermal energies (kT), the most important processes are elastic and inelastic scattering. If we have a collimated neutron beam, any scattering or absorption will remove neutrons from the beam. This is very different from the case of a heavy charged particle, which undergoes many collisions that decrease the energy of the particle but do not remove it from the beam until its energy is essentially zero. A neutron is removed from the beam at its first collision.

(*Continued*)

The chance of a neutron's being removed from a beam within a given path distance is proportional to the number of neutrons in the beam and to the path distance. Let σ be the total cross section for the scattering plus the absorption of a neutron. If I is the incident intensity of the neutron beam (the number of particles per unit time per unit area), the number of neutrons removed from the beam per unit time will be $R = \sigma I$ per nucleus (Equation 12-5). If n is the number density of the nuclei (the nuclei per unit volume) and A is the area of the incident beam, the number of nuclei encountered in a distance dx is $nA\ dx$. The number of neutrons removed from the beam in a distance dx is thus

$$-dN = \sigma I(nA\ dx) = \sigma nN\ dx \qquad \textbf{12-21}$$

where $N = IA$ is the total number of neutrons per unit time in the beam. Solving Equation 12-21 for N, we obtain

$$N = N_0 e^{-\sigma n x} \qquad \textbf{12-22}$$

If we divide by each side of Equation 12-22 by the area of the beam, we obtain a similar equation for the intensity of the beam:

$$I = I_0 e^{-\sigma n x} \qquad \textbf{12-23}$$

We thus have an exponential decrease in the neutron intensity with penetration. After a certain characteristic distance $x_{1/2}$, half the neutrons in a beam are removed. After a second equal distance, half of the remaining neutrons are removed, and so on. Thus, there is no well-defined range.

At the half-penetration distance $x_{1/2}$, the number of neutrons will be $(1/2)N_0$. From Equation 12-22,

$$\begin{aligned} \frac{1}{2} N_0 &= N_0 e^{-\sigma n x_{1/2}} \\ e^{\sigma n x_{1/2}} &= 2 \\ x_{1/2} &= \frac{\ln 2}{\sigma n} \end{aligned} \qquad \textbf{12-24}$$

The main source of energy loss for a neutron is usually elastic scattering. (In materials of intermediate weight, such as iron and silicon, inelastic scattering is also important. We shall neglect inelastic scattering here.) The maximum energy loss possible in one elastic collision occurs when the collision is head-on. This can be calculated by considering a neutron of mass m with speed v_L making a head-on collision with a nucleus of mass M at rest in the laboratory frame (see Problem 12-24). The result is that the fractional energy lost by a neutron in one such collision is

$$\frac{-\Delta E}{E} = \frac{4mM}{(M + m)^2} = \frac{4(m/M)}{[1 + (m/M)]^2} \qquad \textbf{12-25}$$

This fraction has a maximum value of 1 when $M = m$ and approaches $4(m/M)$ for $M >> m$.

EXAMPLE 12-10 **Penetration of Neutrons in Copper** The total cross section for the scattering and absorption of neutrons of a certain energy is 0.3 barns for copper. (*a*) Find the fraction of neutrons of that energy that penetrates 10 cm in

(*Continued*)

copper. (*b*) At what distance will the neutron intensity drop to one-half its initial value?

Solution

(*a*) Using $n = 8.47 \times 10^{28}$ nuclei/m^3 for copper, we have

$$\sigma nx = (3.0 \times 10^{-28}\ \text{m}^2)(8.47 \times 10^{28}/\text{m}^3)(0.10\ \text{m}) = 0.254$$

According to Equation 12-22, if we have N_0 neutrons at $x = 0$, the number at $x = 0.10$ m is

$$N = N_0 e^{-\sigma nx} = N_0 e^{-0.254} = 0.776 N_0 \qquad \textbf{12-26}$$

The fraction that penetrates 10 cm is thus 0.776, or 77.6 percent.
(*b*) For $n = 8.47 \times 10^{28}$ nuclei/m^3 and $\sigma = 0.3 \times 10^{-28}$ m^2, we have from Equation 12-24

$$x_{1/2} = \frac{\ln 2}{(0.3 \times 10^{-28}\ \text{m}^2)(8.47 \times 10^{28}/\text{m}^3)} = \frac{0.693}{2.54}\ \text{m} = 0.273\ \text{m} = 27.3\ \text{cm}$$

Photons

The intensity of a photon beam, like that of a neutron beam, decreases exponentially with distance in an absorbing material. The intensity versus penetration is given by Equation 12-23, where σ is the total cross section for absorption and scattering. The important processes that remove photons from a beam are the photoelectric effect,

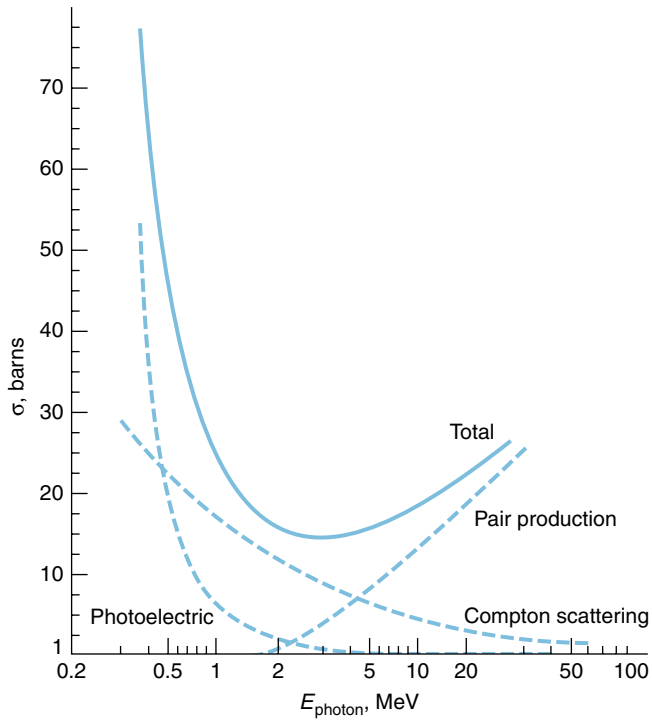


Fig. 12-23 Photon interaction cross sections vs. energy for lead. The total cross section is the sum of the cross sections for the photoelectric effect, Compton scattering, and pair production.

(*Continued*)

Compton scattering, and pair production. The total cross section for absorption and scattering is the sum of the partial cross sections for these three processes: σ_{pe}, σ_{cs}, σ_{pp}. These partial cross sections and the total cross section are shown as functions of energy in Figure 12-23. The cross section for the photoelectric effect dominates at very low energies, but it decreases rapidly with increasing energy. It is proportional to Z^4 or Z^5, depending on the energy region. If the photon energy is large compared with the binding energy of the electrons (a few keV), the electrons can be considered to be free, and Compton scattering is the principal mechanism for the removal of photons from the beam. The cross section for Compton scattering is proportional to Z. If the photon energy is greater than $2m_ec^2 = 1.02$ MeV, the photon can disappear, with the creation of an electron-positron pair. This process, called *pair production,* was described in Section 2-4. The cross section for pair production increases rapidly with the photon energy and is the dominant component of the total cross section at high energies. As was discussed in Section 2-4, pair production cannot occur in free space. If we consider the reaction $\gamma \rightarrow e^+ + e^-$, there is some reference frame in which the total momentum of the electron-positron pair is zero; however, there is no reference frame in which the photon's momentum is zero. Thus, momentum conservation requires that a nucleus be nearby to absorb momentum by recoil. The cross section for pair production is proportional to Z^2 of the absorbing material.