Useful Data

Symbol	*Description*	*Number*	
M_e	Mass of the earth	5.98×10^{24} kg	
R_e	Radius of the earth	6.37×10^{6} m	
g	Free-fall acceleration on earth	$9.80\,\mathrm{m/s^2}$	
G	Gravitational constant	6.67×10^{-11} N m^2/kg^2	
p_{atm}	Standard atmosphere	$101,300$ Pa	
T_0	Absolute zero	$-273°$ C	
N_A	Avogadro's number	6.02×10^{23} particles/mol	
R	Gas constant	8.31 J/mol K	
v_{sound}	Speed of sound in air at 20°C	343 m/s	
k_B	Boltzmann's constant	1.381×10^{-23} J/K	8.617×10^{-5} eV/K
σ	Stefan-Boltzmann constant	5.67×10^{-8} W/m^{2}K^4	
u	Unified mass of the proton and the neutron	1.67×10^{-27} kg	931.494 MeV/c^2
m_p	Mass of the proton	1.67×10^{-27} kg	938.2722 MeV/c^2
m_p	Mass of the neutron	1.67×10^{-27} kg	939.5653 MeV/c^2
m_e	Mass of the electron	9.11×10^{-31} kg	510.9989 keV/c^2
m_α	Mass of the α particle	6.64×10^{-27} kg	3727.38 MeV/c^2
K	Coulomb's law constant ($1/4\pi\epsilon_0$)	8.99×10^{9} N m^2/C^2	
ϵ_0	Permittivity constant	8.85×10^{-12} C^2/N m^2	
μ_0	Permeability constant ($4\pi \times 10^{-7}$ T m/A)	1.26×10^{-6} T m/A	
e	Fundamental unit of charge	1.60×10^{-19} C	
c	Speed of light in vacuum	3.00×10^{8} m/s	3.00×10^{17} nm/s
h	Planck's constant	6.63×10^{-34} J · s	4.136×10^{-15} eV · s
$\hbar$	Planck's constant	1.05×10^{-34} J · s	6.582×10^{-16} eV · s
a_R	Bohr radius $\left(\hbar^2/(m_e k e^2)\right)$	5.29×10^{-11} m	0.0529 nm
m_B	Bohr magneton $(e\hbar/(2m_e))$	9.27×10^{-24} J/T	5.788×10^{-5} eV/T
λ_c	Compton wavelength $(h/(m_e c))$	2.43×10^{-12} m	
α	Fine structure constant $\left(ke^2/(\hbar c)\right)$	$0.0072974 = 1/137$	
r_e	Classical Electron Radius $\left(\alpha^2 a_R\right)$	2.82×10^{-15} m	
hc		1.9864×10^{-25} J · m	1239.8 eV · nm
$\hbar c$		3.1615×10^{-26} eV · nm	197.33 eV · nm
ke^2			1.440 eV · nm
	Standard Temperature	0 °C	
	Standard Pressure	1 atm	

Unit Prefixes

Prefix	*Symbol*	*Factor*	*Prefix*	*Symbol*	*Factor*	*Prefix*	*Symbol*	*Factor*
atto-	a	10^{-18}	**micro-**	μ	10^{-6}	**mega-**	M	10^{6}
femto-	f	10^{-15}	**milli-**	m	10^{-3}	**giga-**	G	10^{9}
pico-	p	10^{-12}	**centi-**	c	10^{-2}	**tera-**	T	10^{12}
nano-	n	10^{-9}	**kilo-**	k	10^{3}	**peta-**	P	10^{15}

Conversion Factors

Length	*Volume*	*Velocity*	*Rotation*
1 in = 2.54 cm	1000 L = 1 m^3	1 mph = 0.447 m/s	1 rad = 180°/π = 57.3°
1 m = 39.37 in	1 L = 0.2642 G	1 m/s = 2.24 mph = 3.28 ft/s	1 rev = 360° = 2π rad
1 mi = 1.609 km			1 rev/s = 60 rpm
1 km = 0.621 mi			
1 Å = 1×10^{-10} m			
Mass and Energy	*Time*	*Pressure*	*Force*
1 u = 1.661×10^{-27} kg	1 day = 86,400 s	1 atm = 101.3 kPa = 760 mm of Hg	1 lb = 4.45 N
1 cal = 4.19 J	1 year = 3.16×10^7 s	1 atm = 14.7 lbs/in^2	
1 eV = 1.60×10^{-19} J			
1 hp = 745.7 W			
1 kW · h = 3.6 MJ			

Greek Letters

Alpha	A α	**Eta**	H η	**Nu**	N ν	**Tau**	T τ
Beta	B β	**Theta**	Θ θ	**Xi**	Ξ ξ	**Upsilon**	Υ υ
Gamma	Γ γ	**Iota**	I ι	**Omicron**	O o	**Phi**	Φ ϕ
Delta	Δ δ	**Kappa**	K κ	**Pi**	Π π	**Chi**	X χ
Epsilon	E ϵ	**Lambda**	Λ λ	**Rho**	P ρ	**Psi**	Ψ ψ
Zeta	Z ζ	**Mu**	M μ	**Sigma**	Σ σ	**Omega**	Ω ω

Astronomical Data

Object	*Mean distance from sun* (m)	*Period* (years)	*Mass* (kg)	*Mean radius* (m)	*Rotation Period* [a] (days)
Sun	—	—	1.99×10^{30}	6.96×10^8	25.38
Moon	3.84×10^8 [b]	27.3 days	7.36×10^{22}	1.74×10^6	27.3
Mercury	5.79×10^{10}	0.241	3.18×10^{23}	2.43×10^6	58.65
Venus	1.08×10^{11}	0.615	4.88×10^{24}	6.06×10^6	−243.02
Earth	1.50×10^{11}	1.00	5.98×10^{24}	6.37×10^6	1.00
Mars	2.28×10^{11}	1.88	6.42×10^{23}	3.37×10^6	1.03
Jupiter	7.78×10^{11}	11.9	1.90×10^{27}	6.99×10^7	0.41
Saturn	1.43×10^{12}	29.5	5.68×10^{26}	5.85×10^7	0.44
Uranus	2.87×10^{12}	84.0	8.68×10^{25}	2.33×10^7	−0.72
Neptune	4.50×10^{12}	165	1.03×10^{26}	2.21×10^7	0.67

[a] Sidereal
[b] Distance from earth

Coefficients of Friction

Material	*Static* μ_s	*Kinetic* μ_k	*Rolling* μ_r	*Material*	*Static* μ_s	*Kinetic* μ_k
Rubber on concrete	1.00	0.80	0.02	**Wood on wood**	0.50	0.20
Steel on steel (dry)	0.80	0.60	0.002	**Wood on snow**	0.12	0.06
Steel on steel (lubricated)	0.10	0.05		**Ice on ice**	0.10	0.03

Properties of Materials

Material[a]	ρ (kg/m^3)	c (J/kg K)	T_m (°C)	L_f (J/kg)	T_b (°C)	L_v (J/kg)	Resistivity[b] (Ω m)
N_2[c]	1.2	1039	−210	0.26×10^5	−196	1.99×10^5	2×10^{14}
Water	1000	4190	0	3.33×10^5	100	22.6×10^5	0.182
Ice	920	2090	—	—	—	—	—
Seawater	1030	3850	—	—	—	—	0.25
Ethyl alcohol	790	2400	−114	1.09×10^5	78	8.79×10^5	—
Gasoline	680	2220	—	—	—	—	10^9
Glycerin	1260	2430	18	2.00×10^5	290	2.3×10^8	2×10^7
Oil (typical)	900	2130	—	—	—	—	—
Carbon	2250	691	3727	9.74×10^6	4830	2.96×10^7	3.5×10^{-5}
Silicon	2330	703	1412	1.93×10^6	2680	1.37×10^7	4.00×10^3
Aluminum	2700	900	660	3.96×10^5	2450	1.05×10^7	2.8×10^{-8}
Copper	8920	385	1083	1.34×10^5	2595	5.07×10^6	1.7×10^{-8}
Gold	19300	129	1064	6.45×10^3	2970	1.58×10^6	2.4×10^{-8}
Iron	7870	449	1537	2.90×10^5	3000	6.37×10^6	9.7×10^{-8}
Lead	11300	128	328	0.25×10^5	1750	8.58×10^5	1.08×10^{-7}
Mercury	13600	140	−39	0.11×10^5	357	2.96×10^5	9.43×10^{-7}
Silver	10490	234	961	1.11×10^5	2210	2.32×10^6	1.6×10^{-8}
Tungsten	19600	134	3380	1.93×10^5	5930	4.48×10^6	5.6×10^{-8}
Nichrome	8400	450	1400	2.98×10^5	—	—	1.5×10^{-6}

[a] Some of the provided data points are summarized and averaged from various sources.
[b] Resistivity is the reciprocal of conductivity, $\rho = \frac{1}{\sigma}$.
[c] Standard temperature (0° C) and pressure (1 atm).

Properties of Gases

Gas	C_V Exact	C_V (J/mol K)	C_P Exact	C_P (J/mol K)	*Gas*	C_V Exact	C_V (J/mol K)	C_P Exact	C_P (J/mol K)
Monatomic	$3/2$R	12.5	$5/2$R	20.8	**Diatomic**	$5/2$R	20.8	$7/2$R	29.5

Elastic Properties

Material	*Young's Modulus* (N/m^2)	*Bulk Modulus* (N/m^2)	*Material*	*Young's Modulus* (N/m^2)	*Bulk Modulus* (N/m^2)
Aluminum	7×10^{10}	7×10^{10}	**Plastic (polystyrene)**	0.3×10^{10}	—
Concrete	3×10^{10}	—	**Steel**	20×10^{10}	16×10^{10}
Copper	11×10^{10}	14×10^{10}	**Water**	—	0.2×10^{10}
Mercury	—	3×10^{10}	**Wood (Douglas fir)**	1×10^{10}	—

Optics

Material	*Index of Refraction*	*Material*	*Index of Refraction*	*Material*	*Index of Refraction*
Vacuum	1.0000	**Water**	1.33	**Diamond**	2.42
Air	1.0003	**Glass**	1.50	**Cubic Zirconia**	2.16

The Periodic Table of the Elements

1 **H** Hydrogen 1.00794																	2 **He** Helium 4.003
3 **Li** Lithium 6.941	4 **Be** Beryllium 9.012182											5 **B** Boron 10.811	6 **C** Carbon 12.0107	7 **N** Nitrogen 14.00674	8 **O** Oxygen 15.9994	9 **F** Fluorine 18.9984032	10 **Ne** Neon 20.1797
11 **Na** Sodium 22.989770	12 **Mg** Magnesium 24.3050											13 **Al** Aluminum 26.981538	14 **Si** Silicon 28.0855	15 **P** Phosphorus 30.973761	16 **S** Sulfur 32.066	17 **Cl** Chlorine 35.4527	18 **Ar** Argon 39.948
19 **K** Potassium 39.0983	20 **Ca** Calcium 40.078	21 **Sc** Scandium 44.955910	22 **Ti** Titanium 47.867	23 **V** Vanadium 50.9415	24 **Cr** Chromium 51.9961	25 **Mn** Manganese 54.938049	26 **Fe** Iron 55.845	27 **Co** Cobalt 58.933200	28 **Ni** Nickel 58.6934	29 **Cu** Copper 63.546	30 **Zn** Zinc 65.39	31 **Ga** Gallium 69.723	32 **Ge** Germanium 72.61	33 **As** Arsenic 74.92160	34 **Se** Selenium 78.96	35 **Br** Bromine 79.904	36 **Kr** Krypton 83.80
37 **Rb** Rubidium 85.4678	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.90585	40 **Zr** Zirconium 91.224	41 **Nb** Niobium 92.90638	42 **Mo** Molybdenum 95.94	43 **Tc** Technetium (98)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.90550	46 **Pd** Palladium 106.42	47 **Ag** Silver 107.8682	48 **Cd** Cadmium 112.411	49 **In** Indium 114.818	50 **Sn** Tin 118.710	51 **Sb** Antimony 121.760	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90447	54 **Xe** Xenon 131.29
55 **Cs** Cesium 132.90545	56 **Ba** Barium 137.327	57 **La** Lanthanum 138.9055	72 **Hf** Hafnium 178.49	73 **Ta** Tantalum 180.9479	74 **W** Tungsten 183.84	75 **Re** Rhenium 186.207	76 **Os** Osmium 190.23	77 **Ir** Iridium 192.217	78 **Pt** Platinum 195.078	79 **Au** Gold 196.96655	80 **Hg** Mercury 200.59	81 **Tl** Thallium 204.3833	82 **Pb** Lead 207.2	83 **Bi** Bismuth 208.98038	84 **Po** Polonium (209)	85 **At** Astatine (210)	86 **Rn** Radon (222)
87 **Fr** Francium (223)	88 **Ra** Radium (226)	89 **Ac** Actinium (227)	104 **Rf** Rutherfordium (261)	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (263)	107 **Bh** Bohrium (262)	108 **Hs** Hassium (265)	109 **Mt** Meitnerium (266)	110 (269)	111 (272)	112 (277)	113	114				

58 **Ce** Cerium 140.116	59 **Pr** Praseodymium 140.90765	60 **Nd** Neodymium 144.24	61 **Pm** Promethium (145)	62 **Sm** Samarium 150.36	63 **Eu** Europium 151.964	64 **Gd** Gadolinium 157.25	65 **Tb** Terbium 158.92534	66 **Dy** Dysprosium 162.50	67 **Ho** Holmium 164.93032	68 **Er** Erbium 167.26	69 **Tm** Thulium 168.93421	70 **Yb** Ytterbium 173.04	71 **Lu** Lutetium 174.967
90 **Th** Thorium 232.0381	91 **Pa** Protactinium 231.03588	92 **U** Uranium 238.0289	93 **Np** Neptunium (237)	94 **Pu** Plutonium (244)	95 **Am** Americium (243)	96 **Cm** Curium (247)	97 **Bk** Berkelium (247)	98 **Cf** Californium (251)	99 **Es** Einsteinium (252)	100 **Fm** Fermium (257)	101 **Md** Mendelevium (258)	102 **No** Nobelium (259)	103 **Lr** Lawrencium (262)

Particle Properties

Particle	*Symbol*	*Mass* (kg)	*Mass* (MeV/c^2)	*Mass* (u)	*Spin* ($\hbar$)	*Lifetime* (s)
Electron	e	9.1094×10^{-31}	0.51100	5.4858×10^{-4}	½	Stable
Proton	p	1.6726×10^{-27}	938.27	1.00728	½	Stable
Neutron	n	1.6749×10^{-27}	939.57	1.00866	½	930 (free)
Muon	μ^-	1.8835×10^{-28}	105.66	0.11343	½	2.2×10^{-6}
Deuteron	^{2}H	3.3436×10^{-27}	1875.61	2.01355	0,1	Stable
α particle	α	6.6447×10^{-27}	3727.38	4.00151	0	Stable
Weak Boson	W	1.43×10^{-25}	80×10^3	85.9	1	3×10^{-25}
Z Boson	Z^0	1.63×10^{-25}	91.2×10^3	97.9	1	3×10^{-25}

Photoelectric Work Functions

Element	ϕ (eV)	*Element*	ϕ (eV)	*Element*	ϕ (eV)
Aluminum	4.28	**Gold**	5.10	**Platinum**	6.35
Cadmium	4.07	**Iron**	4.7	**Potassium**	2.28
Calcium	2.9	**Lead**	4.14	**Selenium**	5.11
Carbon	4.81	**Magnesium**	3.68	**Sodium**	2.75
Copper	4.65	**Nickel**	5.01	**Tungsten**	4.55

Unit Summary

Concept	*Unit*	*Sub-units*	*Concept*	*Unit*	*Sub-units*
Time	second	s	Young's Modulus	—	N/m^2
Distance	meter	m	Bulk Modulus	—	N/m^2
Velocity	—	m/s	Specific Heat (Solid/Liquid)	—	J/(kg · K)
Acceleration	—	m/s^2	Heat of Transformation	—	J/kg
Mass	kilogram	kg	Specific Heat for Gas	—	J/(mol · K)
Temperature	Kelvin	K	Linear Density	—	kg/m
Force	Newton (N)	kg · m/s^2	Intensity	—	W/m^2
Momentum	—	kg · m/s	Electric Charge	Couloumbs	C
Angular Momentum	—	kg · m^2/s	Electric Field	—	N/C = V/m
Energy	Joule (J)	N · m	Linear Charge Density	—	C/m
Power	Watt (W)	J/s	Surface Charge Density	—	C/m^2
Angle	—	rad	Volume Charge Density	—	C/m^3
Angular Velocity	—	rad/s	Electric Flux	—	N · m^2/C
Angular Acceleration	—	rad/s^2	Electric Potential	Volt (V)	J/C
Moment of Inertia	—	kg · m^2	Capacitance	Farad (F)	C/V
Torque	—	N × m	Energy Density	—	J/m^3
Frequency	Hertz (Hz)	cycles / s	Current	Amps (A)	C/s
Angular Frequency	Hertz (Hz)	rad/s	Current Density	—	A/m^2
Phase	—	rad	Resistance	Ohms (Ω)	V/A
Mass Density	—	kg/m^3	Resistivity	—	Ω · m
Number Density	—	molecules/m^3	Conductivity	—	1/(Ω · m)
Pressure	Pascal (Pa)	N/m^2	Magnetic Field	Tesla (T)	N/(A · m)
Volume Flow Rate	—	m^3/s	Magnetic Flux	Weber (Wb)	T m^2
Time Constant		s	Inductance	Henry (H)	T · m^2 / A

Math Review

Unit Conversion

Use dimensional analysis to convert units. Be careful about converting area and volume.
To convert 60 cm^3 to cubic meters:

$$60\ \text{cm}^3 \times \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 60\ \cancel{\text{cm}}^3 \times \left(\frac{1\ \text{m}^3}{1000000\ \cancel{\text{cm}}^3}\right) = 0.00006\ \text{m}^3$$

Angular Concepts

Angle is measured in radians, and for motion with an arc whose radius is r:

$$\theta\ (\text{radians}) \equiv \frac{s}{r} \qquad 2\pi\ \text{rad} = 360^\circ$$

The Dot Product

For two vectors, $\vec{A}$ and $\vec{B}$, and the angle between $\vec{A}$ and $\vec{B}$ defined by α, the dot product $\vec{A}\cdot\vec{B}$ is:

$$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y = |A|\,|B|\,cos\,(\alpha)$$

This can be interpreted as the length of the projection of $\vec{A}$ on to $\vec{B}$ multiplied by the magnitude of $\vec{B}$, or *how much of $\vec{A}$ is in $\vec{B}$ multiplied by $|B|$*. The dot product is required in work calculations because the work cares about the amount force that causes movement in a particular direction.

Cross Product

The cross product of two vectors $\vec{A}$ and $\vec{B}$ can be illustrated by the geometric argument that $\vec{A}\times\vec{B}$ is the magnitude of $\vec{B}$ times the amount of $\vec{A}$ that is **not** in the same direction as $\vec{B}$. The cross products results in a vector with the direction given by the right-hand rule.

$$\vec{A}\times\vec{B} = AB\sin\phi \qquad \text{(direction given by the right hand rule)} \qquad \vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Important Integrals

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) \qquad \int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}} \qquad \int \frac{xdx}{(x^2 \pm a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 \pm a^2}}$$

Mathematical Approximations

Description	*Approximation*
Binomial Approximation ($x \ll 1$)	$(1+x)^n \approx 1 + nx$
Small-Angle Approximation ($\theta \ll 1$) radian	$\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$

References

Knight, Randall D., *Physics for Scientists and Engineers, Second Edition*, Pearson Addison Wesley, 2008.
Halliday, David and Resnick, Robert, *Fundamentals of Physics, Third Edition Extended*, John Wiley & Sons, 1988.
Tipler, Paul A. and Llewellyn, Ralph A., *Modern Physics, Fifth Edition*, W. H. Freeman and Company.
Zwillinger, Daniel, *Standard Mathematical Tables and Formulae, 30th Edition*, CRC Press, 1996.

`http://www.engineeringtoolbox.com` `http://www.environmentalchemistry.com`
`http://wikipedia.org` `http://www.wisegorilla.com/images/chemstry/chemstry.html`

Chapter 20: Traveling Waves

Sinusoidal Waves

$$D(\vec{r},t) = \underbrace{A(\vec{r})}_{\text{Amplitude}} \sin\underbrace{(k\vec{r} - \omega t + \phi_0)}_{\text{Phase}} \qquad \text{(Wave in Three Dimensions)}$$

$$D(x,t) = A\sin(kx - \omega t + \phi_0) \qquad \text{(Wave in One Dimension)}$$

Space Domain	BRIDGE	*Time Domain*
λ Wavelength (m)		T Period (s)
		$f = \frac{1}{T}$ Frequency (Hz)
$k = \frac{2\pi}{\lambda}$ Wave Number (Angular)		$\omega = \frac{2\pi}{T} = 2\pi f$ Angular Frequency
	$v = \lambda f = \frac{\omega}{k}$	
	$\Delta\phi = k\Delta x = 2\pi\frac{\Delta x}{\lambda}$	
Snapshot – *Fixed Time*		History – *Fixed Place*
$D(x,t_0) = A\sin(kx - \omega t_0 + \phi_0)$		$D(x_0,t) = A\sin(kx_0 - \omega t + \phi_0)$
	A Amplitude (m) *Max Displacement*	
	ϕ_0 Phase Constant (rad) *Initial Conditions*	

Note that $kx - \omega t$ is a wave moving right $(+x)$ and $kx + \omega t$ is a wave moving left $(-x)$.

Phase Difference

The phase difference between two points on a wave, $\Delta\phi$, is:

$$\Delta\phi = \phi_2 - \phi_1 = (kx_2 - \omega t + \phi_0) - (kx_1 - \omega t + \phi_0) = k(x_2 - x_1) = k\Delta x = 2\pi\frac{\Delta x}{\lambda}$$

Waves on a String (*Transverse Waves*)

$$\mu = \frac{m}{L}\ \text{kg/m} \quad \text{(Linear Density)} \qquad v = \sqrt{\frac{T_s}{\mu}}\ \text{m/s} \quad \text{(Wavespeed)}$$

Sound Waves (*Longitudinal Waves*)

Doppler Shift – Transmitting frequency f_0. Note that the top sign is for approaching, the bottom sign is for receding. v is the speed of sound in air.

$$\underbrace{f_\pm = \left(\frac{1}{1 \mp v_s/v}\right) f_0}_{\text{Moving Source at } v_s} \qquad \underbrace{f_\pm = \left(1 \pm \frac{v_o}{v}\right) f_0}_{\text{Moving Observer at } v_o \text{ and Stationary Source}} \qquad \underbrace{f_\pm = \left(\frac{v \pm v_o}{v \mp v_s}\right) f_0}_{\text{Moving Observer at } v_o \text{ and Moving Source at } v_s}$$

$$v_{\text{sound}} = 343\ \text{m/s dry air, sea level, } 20^\circ\text{C} \qquad \text{(Wavespeed)}$$

Electromagnetic Waves (*Transverse Waves*)

$$v_{\text{light}} = c = 3.00 \times 10^8 \text{ m/s vacuum} \quad \text{(Wavespeed)}$$

Index of refraction n:

$$n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in the material}} = \frac{c}{v_{mat}} = \frac{\lambda_{vac}}{\lambda_{mat}} \qquad f_{vac} = f_{mat}$$

Doppler – Red shift is for a receding source and Blue shift is for an approaching source, emitted wavelength λ_0, emitted frequency f_0.

$$\lambda_{\substack{\text{red}\\\text{blue}}} = \sqrt{\frac{1 \pm v_s/c}{1 \mp v_s/c}}\,\lambda_0 \qquad f_{\substack{\text{red}\\\text{blue}}} = \sqrt{\frac{1 \mp v_s/c}{1 \pm v_s/c}}\,f_0$$

Power, Intensity, and Decibels

$$I = \frac{\text{Power (W)}}{\text{Area (m}^2\text{)}} \qquad I_{\text{spherical source}} = \frac{P_{\text{source}}}{4\pi r^2} \qquad \beta = 10\text{ dB}\log_{10}\left(\frac{I}{I_0}\right) \qquad I_0 = 1.0 \times 10^{-12}\text{ W/m}^2$$

Chapter 21: Superposition

Superposition

$$D_{net}(x,t) = D_1(x,t) + D_2(x,t) + \cdots = \sum D_i(x,t)$$

Standing Waves

Superposition of waves such that they appear to be fixed in place. Nodes, spaced $\lambda/2$ apart, are points that do not move. Antinodes oscillate back and forth and vary by $2A$. For a space of length, L:

Half Wavelengths	*Nodes*	*Antinodes*	λ
m	$m+1$	m	$\frac{2L}{m}$

$$\lambda_m = \frac{2L}{m} \qquad f_m = \frac{v}{\lambda_m} = m\frac{v}{2L} \qquad m = 1, 2, 3, 4, \ldots$$

The **Fundamental Frequency**, f_1, is the frequency where $m = 1$. All other frequencies are multiples of the fundamental frequency, $f_m = m f_1$.

Standing Waves on a String

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Standing Sound Waves

For a pipe of length, L:

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\frac{v}{2L} = m f_1 \qquad m = 1, 2, 3, 4, \ldots \qquad \text{(open-open or closed-closed pipe)}$$

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\frac{v}{4L} = m f_1 \qquad m = 1, 3, 5, 7, \ldots \qquad \text{(open-closed pipe)}$$

Interference

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0$$

The **Path-length Difference** is the difference between the distance of a point to the wave sources, $\Delta x = x_2 - x_1$. The **Inherent Phase Difference**, $\Delta\phi_0$, is the actual phase difference of the sources themselves.

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2m\pi \text{ rad} \qquad m = 1, 2, 3, 4, \ldots \qquad \text{(Maximum Constructive Interference)}$$

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\left(m + \frac{1}{2}\right)\pi \text{ rad} \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Perfect Destructive Interference)}$$

If there is no inherent phase difference, or in other words, $\Delta\phi_0 = 0$:

$$\Delta x = m\lambda \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Constructive Interference, Strong Reflection)}$$

$$\Delta x = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Destructive Interference, Weak Reflection)}$$

Optical Coatings

With no inherent phase difference in the process, starting with light with wavelength, λ in vacuum:

$$\Delta\phi = 2\pi\frac{2d}{\lambda/n} = 2\pi\frac{2nd}{\lambda}$$

Note that a reflection from a surface with increasing n introduces a π **rad** phase shift in the returning wave (the peak just before the boundary will become a trough on reflection). A reflection from a surface with decreasing n has no phase shift at all.

$n_{\text{surface}} > n_{\text{film}}$	$n_{\text{surface}} < n_{\text{film}}$		
$\lambda_C = \dfrac{2nd}{m}$	$\lambda_C = \dfrac{2nd}{m - \frac{1}{2}}$	$m = 1, 2, 3, 4, \ldots$	(Constructive Interference)
$\lambda_D = \dfrac{2nd}{m - \frac{1}{2}}$	$\lambda_D = \dfrac{2nd}{m}$	$m = 1, 2, 3, 4, \ldots$	(Destructive Interference)

Notice that the conditions for constructive and destructive interference are reversed when the film index of refraction, n_{film}, is greater than the surface index of refraction, n_{surface}. These are because a light wave that reflects from a boundary at which the index of refraction increases has a phase shift of π rad. This assumes that initial light is hitting the film from air ($n_{\text{air}} \approx 1$).

Beat Frequency

For two frequencies close together, the frequency of the resulting *wah-wah*, f_{beat} is:

$$f_{\text{beat}} = |f_1 - f_2|$$

Chapter 22: Wave Optics

Double-Slit Interference

For a double-slit spaced apart by d, L meters from a screen, the angle, θ_m, of the *bright* fringes and θ'_m of the *dark* fringes, and the position, y_m of the *bright* fringes and y'_m of the *dark* fringes are:

$$\theta_m = m\frac{\lambda}{d} \qquad y_m = m\frac{\lambda}{d}L = \theta_m L \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Bright Fringes)}$$

$$\theta'_m = \left(m + \frac{1}{2}\right)\frac{\lambda}{d} \qquad y'_m = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}L = \theta'_m L \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Dark Fringes)}$$

The allowed numbers for m are $m = 0, 1, 2, 3, \ldots$, $m = 0$ is the **central maximum**. These apply for small angles, θ_m only. **Note that the m of the Dark Fringe is labeled so that it matches the m of the bright fringe nearest $y = 0$.** There are two positions for y'_0. The intensity, I_{double}, of the double-slit interference pattern as a function of position, y, is:

$$I_{double} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

These formula stem from the fact that, for constructive interference, the path length difference between a point on the screen and each slit must satisfy $\Delta r = d\sin\theta_m = m\lambda$ for $m = 0, 1, 2, 3, \ldots$. The destructive interference arises from a half-wavelength offset of $\Delta r = d\sin\theta_m = \left(m + \frac{1}{2}\right)\lambda$ for $m = 0, 1, 2, 3, \ldots$. The small angle approximation simplifies the formulas because for small angle, $\sin\theta \approx \theta$.

Diffraction Grating

For a diffraction with line-spacing of d, L meters from a screen, the angle, θ_m, of the *bright* fringes and θ'_m of the *dark* fringes, and the position, y_m of the *bright* fringes and y'_m of the *dark* fringes are:

$$\sin\theta_m = m\frac{\lambda}{d} \qquad y_m = L\tan\theta_m \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Bright Fringes)}$$

$$\sin\theta'_m = \left(m + \frac{1}{2}\right)\frac{\lambda}{d} \qquad y'_m = L\tan\theta_m \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(Dark Fringes)}$$

The allowed numbers for m are $m = 0, 1, 2, 3, \ldots$. These apply for all angles, θ_m. The maximum intensity, I_{max}, of the diffraction grating with N slits is:

$$I_{max} = N^2 I_1$$

Single-Slit Diffraction

For a single-slit of width a, $L \gg a$ meters from a screen, the angle, θ_p, of the *dark* fringes, and the position, y_p of the *dark* fringes:

$$\theta_p = p\frac{\lambda}{a} \qquad y_p = p\frac{\lambda}{a}L = \theta_p L \qquad p = 1, 2, 3, \ldots \qquad \text{(Dark Fringes)}$$

The allowed numbers for p are $p = 1, 2, 3, \ldots$. These apply for small angles, θ_p only. The width, w, of the central maximum is:

$$w = \frac{2\lambda L}{a}$$

Circular-Aperature Diffraction

For a circular-aperature of diameter D, L meters from a screen, the width, w, of the central maximum is:

$$w = 2L\tan\theta_1 \approx \frac{2.44\lambda L}{D}$$

Chapter 23: Ray Optics

The Law of Reflection

The **Angle of Incidence**, θ_i, is equal to the **Angle of Reflection**, θ_r. For a flat mirror, the object distance, s, is equal to the image distance, s_i

$$\theta_i = \theta_r \quad (\theta \text{ is always measured from normal}) \qquad\qquad s' = -s \quad \text{(flat mirror)}$$

The Law of Refraction (Snell's Law)

Refraction occurs when light enters a medium in which its index of refraction, n, changes.

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad \begin{cases} n_{\text{small}} \to n_{\text{big}} & \text{bend towards normal} \\ n_{\text{big}} \to n_{\text{small}} & \text{bend away from normal} \end{cases} \quad n = \frac{c}{v_{\text{medium}}} = \frac{\lambda_{vacuum}}{\lambda_{medium}} \quad f_{vacuum} = f_{medium}$$

Total Internal Reflection

The **Critical Angle**, θ_c, is the angle at which light can no longer be transmitted through the boundary:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Images from Refraction

The image distance when refraction occurs between the boundary of two flat surfaced media is:

$$s' = -\frac{n_2}{n_1}s \qquad \text{for a flat surface, the object is immersed in } n_2$$

Magnification

$$M = -\frac{h'}{h} = -\frac{s'}{s} \quad \text{(+ is an upright image, − is an inverted image)}$$

Thin Lens/Mirror Equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{(thin lens/mirror)} \qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(lens maker's equation)} \qquad f = \frac{R}{2} \quad \text{(mirror)}$$

Summary of Signs

Quantity	*Positive* (+)	*Negative* (−)
Lens Radius R	Convex toward object	Concave toward object
Mirror Radius R	Concave toward object	Convex toward object
Lens Focal Length f	Converging Lens	Diverging Lens
Mirror Focal Length $f = R/2$	Concave toward object	Convex toward object
Image Distance (lens) s'	Real Image, Opposite side	Virtual Image, Same side
Image Distance (mirror) s'	Real Image, Same side	Virtual Image, Opposite side
Magnification M	Upright Image	Inverted Image

Chapter 24: Optical Instruments

Cameras

The **aperture** is the effective diameter, D, of a lens. The light intensity I is related to the **f-number** by the following:

$$f\text{-number} = \frac{f}{D} \qquad I \propto \frac{D^2}{f^2} = \frac{1}{(f\text{-number})^2} \qquad \text{Power} = P = \frac{1}{f}$$

Optical Resolution

Because of diffraction of light, the minimum spot size to which light can be focused through a lens of Diameter, D, and object distance, f, or the mimimum angular distance two objects can be apart from each other and the images of the objects be successfully resolved is (Rayleigh's Criterion):

$$\theta_{\text{min}} \approx 1.22\frac{\lambda}{D} \qquad d_{\text{min separation}} \approx \frac{1.22 f\lambda}{D} \qquad d_{\text{min microscope}} \approx \frac{0.61\lambda}{\text{NA}}$$

Chapter 25: Modern Optics and Matter Waves

Colors and Light Wavelengths

Color	*Approximate Wavelength*	*Color*	*Approximate Wavelength*
Radio	meters	Visible: Red	650 nm
Microwave	millimeters and centimeters	Visible: Orange	590 nm
Infrared	micrometers	Visible: Yellow	570 nm
Visible			
Ultraviolet	nanometers	Visible: Green	510 nm
X-ray	fraction of a nanometer	Visible: Blue	475 nm
Gamma ray	picometer	Visible: Violet	400 nm

Hydrogen Spectrum

$$\lambda_{\text{m,n}} = \frac{91.18\text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad (m = 1, 2, 3, \ldots,\ n = m+1, m+2, m+3\ldots)$$

X-Ray Diffraction (Bragg Diffraction)

The atom spacing in crystaline structures is like a transmission grating for x-rays. The conditions for constructive interference between layers requires that angle of incidence, θ_m, and therefore the path-length through the layers (space distance d apart) be:

$$\Delta r = 2d\cos\theta_m = m\lambda \qquad m = 1, 2, 3, \ldots$$

Note that in this circumstance the sample is literally rotated within the beam of x-rays and the θ_m of constructive interference are noted on the detector allowing for the calculation of d.

Photons

The energy of one photon, E_{photon} is quantized and is $E_{\text{photon}} = hf$ where $h = 6.63 \times 10^{-34}$ J s.

Particle Wavelength

The wavelength of one particle with momentum, called the de Broglie wavelength, p is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$.

Quantized Momentum and Quantized Energy

Since matter has wave properties, and when confined in a box of length L, it cannot share space with the ends of the box (i.e. the ends of the box are the nodes of the particle's standing waves). The possible wavelengths, momentum, and energy are limited to (note that $n = 0$ is **not** allowed):

$$\lambda_n = \frac{2L}{n} = \frac{h}{p_n} \qquad p_n = n\frac{h}{2L} \qquad E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 = n^2 E_1 \qquad n = 1, 2, 3, \ldots$$

The minimum possible energy allowed by the particle is always greater than zero indicating that a confined particle cannot be at rest. E_n represents a possible **energy level**, and n is considered a **quantum number**.

Chapter 26: Electric Charges and Forces

Electric Charge and Coulomb's Law

Particles with opposite charges attract each other and particles with the same charge repel each other. For two particles seperated by distance r, with charges q_1 and q_2, the force between them is:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \qquad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

The unit vector $\hat{r}$ is one unit outward from a source charge. Electrons are the source of negative charge and protons are the source of positive charge. The electron and proton charges are equal in magnitude and is called the **fundamental unit of charge**.

$$1\ e = 1.6 \times 10^{-19} \text{ C}$$

The Coulomb (C) is a unit of electrical charge. Common illustrations of charging objects are:

Glass rod rubbed with silk: The glass rod loses electrons and as a result is positively charged.

Plastic rod rubbed with fur: The plastic rod gains electrons and as a result is negatively charged.

Electric Dipole

When two equal, yet opposite charges are located a distance, $\vec{s}$, from each other, the situation is called an **Electric Dipole**. The **dipole moment** is important in the study of electricity and magnetism and has the value:

$$\vec{p} = q\vec{s} \quad \text{(from the negative to the positive charge)}$$

Chapter 27: The Electric Field

Comparison of Gravity and Electricity

	Gravity	Electricity
Force	$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}$	$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$
Field	$\vec{g} = \frac{\vec{F}}{m_2} = G\frac{m_1}{r^2}\hat{r}$	$\vec{E} = \frac{\vec{F}}{q_2} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}$

Charge Distribution

For an object of evenly distributed charge, Q, and with dimensions of either length L, area A or volume V, the following distribution relationships hold.

Distribution	Symbol	Relationship	Differential	Units	Integral
Linear Charge Distribution	λ	$\lambda = Q/L$	$dQ = \lambda dL$	C/m	$Q = \int dq = \int \lambda\, dl$
Surface Charge Distribution	η	$\eta = Q/A$	$dQ = \eta dA$	C/m^2	$Q = \int dq = \int \eta\, dA$
Volume Charge Distribution	ρ	$\rho = Q/V$	$dQ = \rho dV$	C/m^3	$Q = \int dq = \int \rho\, dV$

Electric Field Due to Point Charges

$$\vec{E}_{\text{point charge}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad E_{\text{dipole on axis}} = \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3} \qquad E_{\text{dipole}\perp\text{axis}} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3} \qquad p = qs$$

Electric Field Due to a Continuous Distribution of Charge

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \cdots = \sum \vec{E}_n \qquad \vec{E} = \int \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

Electric Field Perpendicular to a Charged Rod

$$E_{\text{rod}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2 + (L/2)^2}} \quad \text{(finite rod)} \qquad E_{\text{line}} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r} \quad \text{(infinite rod)}$$

Electric Field on Rings and Disks

$$E_{z\text{-axis of a charged ring}} = \frac{1}{4\pi\epsilon_0}\frac{zQ}{(z^2 + R^2)^{3/2}} \qquad E_{z\text{-axis of a charged disk}} = \frac{\eta}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Electric Field of a Charged Objects

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant} \qquad \vec{E}_{\text{outside sphere}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \quad (\text{r} \geq \text{R})$$

Parallel-Plate Capacitor

A **Parallel-Plate Capacitor** is two metal plates with equal area, A, spaced a distance, d, apart from each other. When the net charge on each surface is equal and opposite, then the electric field between the plates is **uniform** and has the value:

$$E_{\text{capacitor}} = \frac{\eta}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \qquad \text{(from positive to negative)}$$

Constant Acceleration – The BIG-3 – Review

$$v_f = v_i + a\Delta t \quad (1) \qquad s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2 \quad (2) \qquad v_f^2 = v_i^2 + 2a\Delta s \quad (3)$$

Charged Particle Motion in an Electric Field

The acceleration, a_c, or torque, τ, of a charged particle of charge q and mass m_c placed in an electric field E is:

$$\vec{a}_c = \frac{q\vec{E}}{m_c} \qquad \vec{\tau} = \vec{p} \times \vec{E} \qquad \vec{p} = q\vec{s}$$

Chapter 28: Gauss's Law

Flux and Gauss's Law

$$\Phi_e = E_\perp A = EA\cos\theta = \oint \vec{E}\cdot d\vec{A} \qquad \Phi_e = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad \text{(Gauss's Law)}$$

When the gaussian surface is chosen correctly in situations where the field exhibits useful symmetry, the closed surface integral can generally become:

$$\Phi_e = EA = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Chapter 29: The Electric Potential

Potential Energy of Charge Configurations

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \quad \text{(two point charges)} \qquad U = -\vec{p}\cdot\vec{E} \quad \text{(dipole)}$$

Electric Potential

Electric Potential, measured in Volts (V), is a measure of the Electric Potential Energy per unit charge at a particular place. To compare with gravity (remember that g has earth's mass or m_{cause} already in it just like E has q_{cause} inside it):

Concept	*Gravity*	*Electricity*
Force	$F_G = m_{\text{feel}}g$ (N)	$F_E = q_{\text{feel}}E$ (N)
Field	$g = \frac{F_G}{m_{\text{feel}}} = \frac{m_{\text{feel}}g}{m_{\text{feel}}}$ (N/kg)	$E = \frac{F_E}{q_{\text{feel}}} = \frac{q_{\text{feel}}E}{q_{\text{feel}}}$ (N/C)
Potential Energy	$U_G = m_{\text{feel}}gy$ (J)	$U_E = k\frac{q_{\text{cause}}q_{\text{feel}}}{r}$ (J)
Potential	$V_G = \frac{U_G}{m_{\text{feel}}} = \frac{m_{\text{feel}}gy}{m_{\text{feel}}} = gy$ (J/kg)	$V_E = \frac{U_E}{q_{\text{feel}}} = \frac{k\frac{q_{\text{cause}}q_{\text{feel}}}{r}}{q_{\text{feel}}} = \frac{kq_{\text{cause}}}{r}$ (J/C = V)

The definition for Electric Potential is:

$$V \equiv \frac{U_E}{q_{\text{feel}}} \quad \text{(Definition)} \qquad U_E = q_{\text{feel}}V \qquad dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} \qquad \Delta K = -\Delta U = -q\Delta V \quad \text{(Conservation of Energy)}$$

The Electric Potential of a Point Charge and a Uniformly Charged Sphere

For a point charge or a uniformly charged sphere of radius R with total charge Q charged to potential V_0:

$$V = \frac{U}{q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(point charge)} \qquad V = \frac{U}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} = \frac{R}{r}V_0 \qquad (r \geq R) \quad \text{(uniformly charged sphere)}$$

The Electric Potential inside a Parallel-Plate Capacitor

$$V = Es$$

The Electric Potential of Several Charges

$$V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i} \qquad V = \int dV$$

Chapter 30: Potential and Field

Potential and Field

Electric Potential Energy	Electric Potential
$\Delta U = -\int_{s_i}^{s_f} \vec{F}\cdot d\vec{s}$	$\Delta V = -\int_{s_i}^{s_f} \vec{E}\cdot d\vec{s}$
$\vec{F}_s = -\frac{dU}{ds}\hat{s}$	$\vec{E}_s = -\frac{dV}{ds}\hat{s}$
$\vec{F}_s = -\vec{\nabla}U = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right)$	$\vec{E}_s = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$

$$\Delta V_{\text{loop}} = \oint dV = 0 \quad \text{(voltage is conserved)}$$

Constant Potential and Field

$$\Delta U = -F\Delta s \quad \text{(electric potential energy)} \qquad \Delta V = -E\Delta s \quad \text{(electric potential)}$$

Capacitance

Remember the Electric Field and Potential inside a capacitor is:

$$E_{\text{cap}} = \frac{\eta}{\epsilon_0} = \frac{Q}{A\epsilon_0} \qquad V_{\text{cap}} = E_{\text{cap}}d \qquad Q = C\Delta V \qquad C_{\text{parallel-plate capacitor}} = \frac{A\epsilon_0}{d}$$

C is Capacitance (1 farad = 1 C/V) and completely depends on the geometry of the capacitor.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \quad (\textbf{Series: } \textit{constant} \text{ charge}) \qquad C_{\text{eq}} = C_1 + C_2 + \cdots \quad (\textbf{Parallel: } \textit{constant} \text{ voltage})$$

The energy U_{cap} and energy density u_{E} stored in a capacitor is:

$$U_{\text{cap}} = \frac{1}{2}C(\Delta V)^2 \qquad u_{\text{E}} = \frac{1}{2}\epsilon_0 E^2$$

Dielectrics

The **dielectric constant** is the factor by which the electric field is weakened by a substance.

$$\kappa \equiv \frac{E_0}{E} \qquad C = \kappa C_0$$

The dielectric strength of a substance is the amount of electric field a substance can sustain.

Chapter 31: Current and Resistance

Current

Loads that draw on current use energy, but do not use up current. Electrons travel through a wire at v_d with the net electron speed in a metal wire much slower than the speed at which the energy travels. The electron current i is the number of electrons per second that pass through a wire and is related to the electron density, n_{e}, which is the number of available electrons per unit volume. The number of electrons, N_e to pass by any point in a wire of cross-sectional area A during a time interval, Δt is:

$$N_e = i\Delta t \qquad i = n_{\text{e}}Av_d$$

The cause of electron current in all points of the wire is an internal electric field created by a battery or capacitor. The *nonuniform* distribution of surface charges along a wire creates a net electric field $\vec{E}$ *inside* the wire that points from the more positive end of the wire toward the more negative end of the wire. The electron current is directly proportional to the electric field strength, i.e. $i = \frac{n_{\text{e}}e\tau A}{m}E$.

Conduction

Collisions of electrons in a metal wire are the source of "friction" that heat up a wire. The mean-time between electron collisions can be estimated and is called τ. The drift speed in terms of τ is $v_d = \frac{e\tau}{m}E$.

Current Density

The current I and the current density J (current per square meter) is the amount of charge that flows through a point in a wire per second.

$$I \equiv \frac{dQ}{dt} = ei \qquad J \equiv \frac{I}{A} = n_e e v_d$$

Conductivity and Resistivity

The conductivity of a material, σ, and the resistivity (inverse of conductivity), ρ, depends only on the material and is:

$$\sigma = \frac{ne^2\tau}{m} \qquad \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad J = \sigma E$$

Current in a Wire and Ohm's Law

For a circuit with low resistance and high voltage, the current must be high. For a curcuit with high resistance and low voltage, the current must be low. The current I in a wire is modeled by Ohm's Law. The electric field inside a wire containing resistance R (measured in Ohms Ω), resistivity ρ, length L, and cross sectional area A, connected to a battery with potential difference ΔV is:

$$I = \frac{\Delta V}{R} \quad \text{(Ohm's Law)} \qquad E_{\text{wire}} = \frac{\Delta V}{L} \qquad R = \frac{\rho L}{A}$$

Chapter 32: Fundamentals of Circuits

Two Rules for Circuits

$$\sum I_{\text{in}} = \sum I_{\text{out}} \quad \text{(Kirchhoff's Junction Law)} \qquad \Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0 \quad \text{(Kirchhoff's Loop Law)}$$

Circuit Types

Series: Two elements in series follow each other in a circuit. In a series circuit, the two elements must have the *same* charge or current, but can have *different* voltage (adding up to the total voltage).

Parallel: Two elements in parallel are adjacent to each other in a circuit. In a parallel circuit, the two elements must have the *same* voltage, but can have *different* charge or current (adding up to the total charge or current).

Capacitors and Resistors in a Circuit

Capacitors and resistors in a circuit should be simplified into an equivalent capacitance C_{eq} or resistance R_{eq} using the series/parallel equations along with the basic equation $Q = C\Delta V$ or Ohm's Law $I = V/R$. The capacitor equations were given in Chapter 30 and the resistor equations follow here.

$$R_{\text{eq}} = R_1 + R_2 + \cdots \quad (\textbf{Series: } \textit{constant} \text{ current}) \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \quad (\textbf{Parallel: } \textit{constant} \text{ voltage})$$

Power in a Circuit

$$P_{\text{bat}} = I\mathcal{E} \qquad P_{\text{R}} = I\Delta V_{\text{R}} = I^2 R = \frac{(\Delta V_{\text{R}})^2}{R}$$

Interesting Notes about Circuits

Energy: Energy as measured in the electrical industry is given in **kilowatt hours**.

Ammeter: An ammeter is a device that measures the current in a circuit. To work the voltmeter must placed in *series* with the circuit.

Voltmeter: A voltmeter is a device that measures the voltage difference between two points in a circuit. To work the voltmeter must be placed in *parallel* with the circuit.

Ground: In electricity, the **ground** can either be an infinite source of negative charge (the ground will give up electrons) or an infinite sink of positive charge (the ground will take electrons). A grounded point is forced to a potential of 0 Volts.

RC Circuits

An RC circuit is a circuit with a resistor and a charged capacitor in it. The loop rule for an RC circuit exposes the following equations:

$$Q(t) = Q_0 e^{-t/RC} \quad \text{(Discharging)} \qquad Q(t) = Q_{\max}\left(1 - e^{-t/RC}\right) \quad \text{(Charging)}$$

Chapter 33: The Magnetic Field

Magnetic Force

Sources of magnetic force are poles where opposite magnetic poles attract and like magnetic poles repel. A current carrying wire has magnetic field around it with the direction of the field lines given by the right-hand rule, i.e. the thumb pointing in the direction of the current and the fingers curling in the direction of the magnetic field. On paper, a magnetic field pointing into a page is given by the symbol ⊗ and for a magnetic field pointing out of a page it is given by the symbol •. Permeability is like permittivity for magnetism with the permeability constant, μ_0 given by $\mu_0 = 4\pi \times 10^{-7}$ T·m/A = 1.257×10^{-6} T·m/A.

Biot-Savart Law

The Biot-Savart Law is the magnetic equivalent of Coulumb's Law. The magnetic field symbol is $\vec{B}$. The units for magnetic field are given in **tesla** (N/A m). Note that $dq\,\vec{v} = dq\,\frac{d\vec{s}}{dt} = \frac{dq}{dt}\,d\vec{s} = I\,d\vec{s}$. The $\hat{r}$ vector points from the source of the magnetic field to the point of observation.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{dq\,\vec{v}\times\hat{r}}{r^2} \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2} \qquad \mu_0 = 1.257\times 10^{-6}\ \text{T·m/A}$$

Magnetic Fields for Common Current Distributions

$$\vec{B}_{\text{wire}} = \frac{\mu_0}{2\pi}\frac{I}{d} \qquad \vec{B}_{\text{loop}} = \frac{\mu_0}{2}\frac{IR^2}{(z^2+R^2)^{3/2}} \qquad \vec{B}_{\text{coil}} = \frac{\mu_0}{2}\frac{NI}{R} \qquad \vec{B}_{\text{curved segment}} = \frac{\mu_0}{4\pi}\frac{I}{R}\theta$$

$$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3} \qquad \vec{\mu} = I\vec{A}$$

Ampere's Law

Ampere's Law is the equivalent of Gauss's Law for magnetism. The integral is a conveniently chosen line integral where contributions to the integral only come as the line is parallel with the magnetic field.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

Solenoid

A solenoid is a coil of wires with number of turns N and length L. The magnetic field inside a solenoid is given by:

$$\vec{B}_{\text{solenoid}} = \frac{\mu_0 N I}{L} = \mu_0 n I \qquad n = \frac{N}{L}$$

Magnetic Force on a Moving Charge and on a Current Carrying Loop

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B} \qquad \vec{\tau}_{\text{loop}} = \vec{\mu} \times \vec{B}$$

Cyclotron Motion

$$r_{\text{cyc}} = \frac{mv}{qB} \qquad f_{\text{cyc}} = \frac{qB}{2\pi m}$$

Hall Effect

For a conductor of thickness t, and electron density n, the Hall Effect differentiates the particle responsible for current and is represented by a voltage difference due to forces on current moving through a conductor in the presence of a magnetic field. The force on the charge carrier deflects it in the direction F_{mag} and causes a voltage difference between the sides of the conductor. The Hall Voltage is given by:

$$\Delta V_{\text{H}} = \frac{IB}{tne} \qquad \Delta V_{\text{H}} = Blv \quad \text{(motional EMF)}$$

Force on Current Carrying Wires

The force on current carrying wires of length L in the following situations are:

$$F_{\text{wire in magnetic field}} = L\left(\vec{I} \times \vec{B}\right) \qquad F_{\text{parallel wires}} = \frac{\mu_0 L}{2\pi} \frac{I_1 I_2}{d}$$

Between parallel wires if the current is in the same direction, the wires will attract each other. If the current is in opposite directions, the wires will repel each other.

Chapter 34: Electromagnetic Induction

Magnetic Flux

$$\Phi_{\text{m}} = \oint \vec{B} \cdot d\vec{A} \qquad \left(\text{weber } \left[\text{T m}^2\right]\right)$$

Lenz's Law

The direction of any induced current is such that the induced magnetic field opposes the change in the flux.

Faraday's Law

The induced current in a closed loop with N turns is given by the following:

$$I_{\text{induced}} = \frac{\mathcal{E}}{R} \qquad \mathcal{E} = N\left|\frac{d\Phi_{\text{m}}}{dt}\right| \qquad \mathcal{E} = \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{\text{m}}}{dt} \quad \text{(Faraday's Law)}$$

Transformers

Transformers convert electricty into a magnetic field via a closed loop primary coil, then recreate electricity in a secondary coil with a different number of turns. The relationship between the voltage and turns in the transformers is $V_1/N_1 = V_2/N_2$ while the relationship between the voltage and current in the transformers is $P = V_1 I_1 = V_2 I_2$. Transformers that increase voltage are called *step-up* and transformers that reduce voltage are called *step-down* transformers.

Inductors

Closed loop coils called solenoids store energy by creating magnetic fields as the current through them changes. The inductance L, energy U_{L}, and energy density u_{B} of a circuit with flux Φ_{m}, current I, and N turns is:

$$L = N\frac{\Phi_{\text{m}}}{I} \qquad \text{(henry (H) } \left[\text{T m}^2/\text{A}\right]) \qquad U_{\text{L}} = \frac{1}{2}LI^2 \qquad u_{\text{B}} = \frac{1}{2\mu_0}B^2$$

The inductance and induced emf in a solenoid with N turns is:

$$L_{\text{solenoid}} = N\frac{\Phi_{\text{m}}}{I} = \frac{\mu_0 N^2 A}{l} \qquad \mathcal{E}_{\text{solenoid}} = -L\frac{dI}{dt} \qquad \Phi_{\text{solenoid}} = N\Phi_{\text{per coil}}$$

LC Circuits

An LC Circuit is a circuit with an inductor and a capacitor.

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \qquad Q(t) = Q_0\cos\omega t \qquad \omega = \sqrt{\frac{1}{LC}} \qquad I(t) = -\frac{dQ}{dt} = \omega Q_0\sin\omega t = I_{\max}\sin\omega t$$

LR Circuits

An LR Circuit is a circuit with an inductor and a resistor.

$$I(t) = I_0\left(1 - e^{\frac{-t}{\tau}}\right) \quad (\textit{Charging}) \qquad I(t) = I_0 e^{-t/\tau} \quad (\textit{Discharging}) \qquad \tau = \frac{L}{R}$$

Chapter 35: Electromagnetic Fields and Waves

Relativity and Galilean Field Transformation Equations

The electric field $\vec{E}$ measurement is dependent on the frame it is measured in. In another reference frame that is moving, some of the electric field will be measured as magnetic field. For the Galilean Field Transformation Equations, the primed quantities are in a frame moving at speed V relative to the unprimed frame.

$$\vec{E}\,' = \vec{E} + \vec{V}\times\vec{B} \qquad \vec{B}\,' = \vec{B} - \frac{1}{c^2}\vec{V}\times\vec{E} \qquad \vec{E} = \vec{E}\,' - \vec{V}\times\vec{B}\,' \qquad \vec{B} = \vec{B}\,' + \frac{1}{c^2}\vec{V}\times\vec{E}\,'$$

Intensity and Radiation Pressure

Light intensity is a function of radiated Power P and area A that the light falls upon. Because light carries momentum, light falling onto a surface produces radiation pressure.

$$I = \frac{P}{A} = S_{\text{avg}} = \frac{1}{2c\mu_0}E_0^2 = \frac{c\epsilon_0}{2}E_0^2 \qquad I = \frac{P_{\text{source}}}{4\pi r^2} \quad \text{(radially emitted)} \qquad p_{\text{rad}} = \frac{F}{A} = \frac{P/A}{c} = \frac{I}{c} \quad \text{(radiation pressure)}$$

Maxwell's Equations

Maxwell's equations condense all of the laws of electricity and magnetism into four plus one concise equations.

Maxwell's Equation	*Description*
Gauss's Law: $\oint \vec{E}\cdot d\vec{A} = \frac{Q\text{enc}}{\epsilon_0}$	Charged particles create an electric field.
Gauss's Law for Magnetism: $\oint \vec{B}\cdot d\vec{A} = 0$	There are no magnetic monopoles.
Faraday's Law: $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_\text{m}}{dt}$	Changing magnetic fields create an electric field.
Ampère-Maxwell Law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_\text{through} + \epsilon_0\mu_0\frac{d\Phi_\text{e}}{dt}$	Currents and changing electric fields create magnetic fields. The second term is defined as the displacement current $I_\text{disp} = \epsilon_0\frac{d\Phi_\text{e}}{dt}$.
Lorentz Force Law: $\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$	The force a charged particle feels moving in the presence of fields.

Consequences of Maxwell's Equations

$$\frac{\partial B_z}{\partial x} = -\epsilon_0\mu_0\frac{\partial E_y}{\partial t} \quad \text{(Wave Equation for Light)} \qquad c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3.00\times 10^8 \text{ m/s}$$

$$E(x,t) = E_0\sin(kx-\omega t) \qquad B(x,t) = B_0\sin(kx-\omega t) \qquad \frac{E_0}{B_0} = \frac{E_{rms}}{B_{rms}} = c \qquad E_{rms} = \frac{E_0}{\sqrt{2}} \qquad B_{rms} = \frac{B_0}{\sqrt{2}}$$

Poynting Vector

The poynting vector describes energy flow in light and points in the direction the light is traveling.

$$\vec{S} \equiv \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

Polarization

Unpolarized light hitting a polarizing filter allows half of the light through, i.e. $I_\text{transmitted} = \frac{1}{2}I_0$. The transmitted intensity of polarized light going through a polarizing filter aligned at an angle θ is $I_\text{transmitted} = I_0\cos^2\theta$.

Chapter 36: AC Circuits

Alternating Current and Phasors

Alternating Current (AC) is current where the voltage changes throughout time. Standard AC current operates in the U.S.A. at 60 Hz (120 V_rms) while many other countries use 50 Hz AC current (both 120 V_rms and 220 V_rms). The voltage as a function of time for sinusoidal AC is given by $\mathscr{E} = \mathscr{E}_0\cos\omega t$. A phasor is a vector that literally rotates counterclockwise around the origin at frequency ω. Its length corresponds to the maximum value of the quantity. The instantaneous value is the projection of the phasor onto the x-axis.

AC Circuits: The General Idea

To describe AC Circuits, we delineate between instantaneous values (*lower case*) and peak values (*upper case*). The analysis seeks to make AC equations look like the standard equations of electricity. It is important to note that when comparing peak values, the times of the peak values may be different.

$$I = \frac{\Delta V}{R} \quad \text{(Ohm's Law)} \qquad P = I\Delta V \quad \text{(Power)} \qquad I_\text{rms} = \frac{I_\text{peak}}{\sqrt{2}} \qquad V_\text{rms} = \frac{V_\text{peak}}{\sqrt{2}}$$

Resistor Circuits

$$v_R = i_R R = V_R \cos\omega t \qquad i_R = \frac{v_R}{R} = \frac{V_R \cos\omega t}{R} = I_R \cos\omega t \qquad I_R = \frac{V_R}{R} \qquad \text{(Ohm's Law)}$$

In the case of a Resistor Circuit the max voltage occurs at the same time as the max current so the voltage and the current are *in phase*.

Inductor Circuits

$$v_L = L\frac{di_L}{dt} \qquad di_L = \frac{v_L}{L}dt = \frac{V_L \cos\omega t}{L}dt \qquad i_L = \frac{V_L}{L}\int cos\omega t\, dt = \frac{V_L}{\omega L}\sin\omega t$$

In the case of an Inductor Circuit the peak current *lags* the peak voltage by $\pi/2$ or $90°$. In these equations X_L is called the **inductive reactance**.

$$I_L = \frac{V_L}{X_L} \qquad X_L \equiv \omega L \qquad \text{(Ohm's Law)}$$

Capacitor Circuits

$$v_C = V_C \cos\omega t \qquad q = Cv_C = CV_C \cos\omega t \qquad i_C = \frac{dq}{dt} = \frac{d}{dt}(CV_C \cos\omega t) = -\omega C V_C \sin\omega t$$

In the case of a Capacitor Circuit the peak current *leads* the peak voltage by $\pi/2$ or $90°$. In these equations X_C is called the **capactive reactance**.

$$I_C = \frac{V_C}{X_C} \qquad X_C \equiv \frac{1}{\omega C} \qquad \text{(Ohm's Law)}$$

Series RLC Circuits

The Series RLC Curcuit has a resister, an inductor, and a capacitor in series. In this circuit there is still an X_C and an X_L defined the same as above.

$$I_{RLC} = \frac{\mathscr{E}_0}{Z} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \tan\phi = \frac{X_L - X_C}{R} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{(Resonance)}$$

Z is called the **impedance** of the circuit and ϕ is the **phase angle**, and ω_0 is the **angular resonance frequency** for any RLC Circuit, which signifies the frequency allowing the maximum current in the circuit.

Power in an RLC Circuit

The average power in the RLC circuit and resistor with phase angle ϕ and **power factor** $\cos\phi$ is:

$$P = I_{rms}^2 Z\cos\phi = \frac{\mathscr{E}_{rms}^2}{Z}\cos\phi = I_{rms}\mathscr{E}_{rms}\cos\phi = I_{rms}^2 R = \frac{V_{R_{rms}}^2}{R}$$

RC Filter Circuit

The **crossover frequency** $\omega_c = \dfrac{1}{RC}$ in an RC filter circuit is the angular frequency at which the circuit is in a resonant condition, and no filtering is expected. Connecting an alternating current source across the capacitor leads in an RC Circuit constitutes a *Low-Pass Filter* allowing transmission of frequencies well below the cross-over frequency ($\omega \ll \omega_c$). Connecting an alternating current source across the resistor leads in an RC Circuit constitutes a *High-Pass Filter* allowing transmission of frequencies well above the cross-over frequency ($\omega \gg \omega_c$).

$$V_C = \frac{\mathscr{E}_0 X_C}{\sqrt{R^2 + X_C^2}} \quad \text{gain}_{\text{low-pass}} = \frac{V_C}{\mathscr{E}_0} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \qquad V_R = \frac{\mathscr{E}_0 R}{\sqrt{R^2 + X_C^2}} \quad \text{gain}_{\text{high-pass}} = \frac{V_R}{\mathscr{E}_0} = \frac{R}{\sqrt{R^2 + X_C^2}}$$