Physical Pendulums and Small Oscillations

8.01
Week 12D2

Today's Reading Assignment:
MIT 8.01 Course Notes
Chapter 23 Simple Harmonic Motion
Sections 23.5
Chapter 24 Physical Pendulum
Sections 24.1-24.2

Announcements

Sunday Tutoring in 26-152 from 1-5 pm

Problem Set 10 consists of practice problems for Exam 3. You do not need to hand it in.

Exam 3 Information

Exam 3 will take place on Tuesday Nov 26 from 7:30-9:30 pm.

Exam 3 Room Assignments:

26-100 –Sections L01 and L03

26-152 –Section L02

50-340 – Sections L04, L05, L06, and L07

Conflict Exam 3 will take place on Wednesday Nov 27 from 8-10 am in room 26-204 or from 10-12 am in 4-315.

You need to email Dr. Peter Dourmashkin (padour@mit.edu) and get his ok if you plan to take the conflict exam. Please include your reason and which time you would like to take Conflict Exam 2.

Note: Exams from previous years have a different set of topics.

Exam 3 Topics

Collisions: One and Two Dimensions

Kinematics and Dynamics of Fixed Axis Rotation

Static Equilibrium

Angular Momentum of Point Objects and Rigid Bodies Undergoing Fixed Axis Rotation

Conservation of Angular Momentum

Experiment 3/4: Measuring Moment of Inertia; Conservation of Angular Momentum

Rotation and Translation of Rigid Bodies: Kinematics, Dynamics, Conservation Laws

Alert: Knowledge of Simple Harmonic Motion will not be tested on Exam 3

Summary: SHO

Equation of Motion:

$$-kx = m\frac{d^2x}{dt^2}$$

Solution: Oscillatory with Period

$$T = 2\pi / \omega_0 = 2\pi\sqrt{m / k}$$

Position:

$$x = C\cos(\omega_0 t) + D\sin(\omega_0 t)$$

Velocity:

$$v_x = \frac{dx}{dt} = -\omega_0 C\sin(\omega_0 t) + \omega_0 D\cos(\omega_0 t)$$

Initial Position at $t = 0$:

$$x_0 \equiv x(t = 0) = C$$

Initial Velocity at $t = 0$:

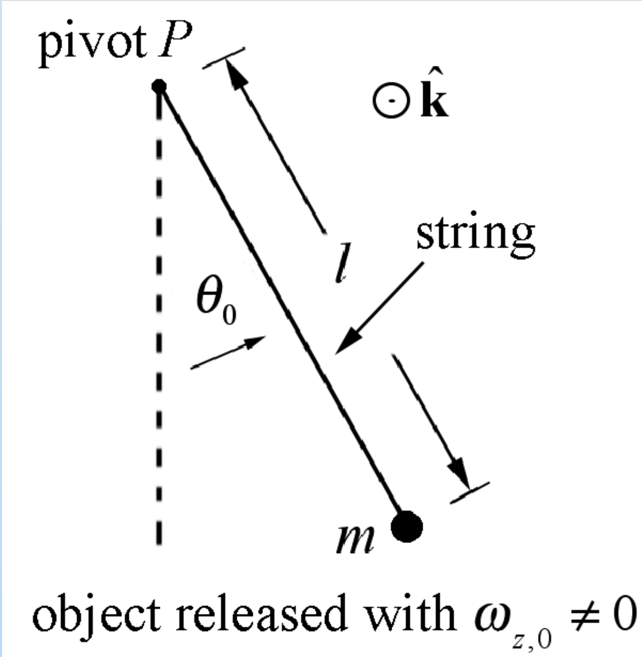
$$v_{x,0} \equiv v_x(t = 0) = \omega_0 D$$

General Solution:

$$x = x_0\cos(\omega_0 t) + \frac{v_{x,0}}{\omega_0}\sin(\omega_0 t)$$

Table Problem: Simple Pendulum by the Torque Method

A simple pendulum consists of a point-like object of mass m attached to a massless string of length l. The object is initially pulled out by an angle θ_0 and released with a non-zero z-component of angular velocity, $\omega_{z,0}$.



(a) Find a differential equation satisfied by $\theta(t)$ by calculating the torque about the pivot point.

(b) For $\theta(t) \ll 1$, determine an expression for $\theta(t)$ and $\omega_z(t)$.

Concept Q.: SHO and the Pendulum

Suppose the point-like object of a simple pendulum is pulled out at by an angle $\theta_0 \ll 1$ rad. Is the angular speed of the point-like object

1. always greater than

2. always less than

3. always equal to

4. only equal at bottom of the swing to

the angular frequency of the pendulum?

Demonstration
Simple Pendulum:

Difference between Angular Velocity and Angular Frequency

Amplitude Effect on Period

When the angle is no longer small, then the period is no longer constant but can be expanded in a polynomial in terms of the initial angle θ_0 with the result

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{4}\sin^2\frac{\theta_0}{2}+\cdots\right)$$

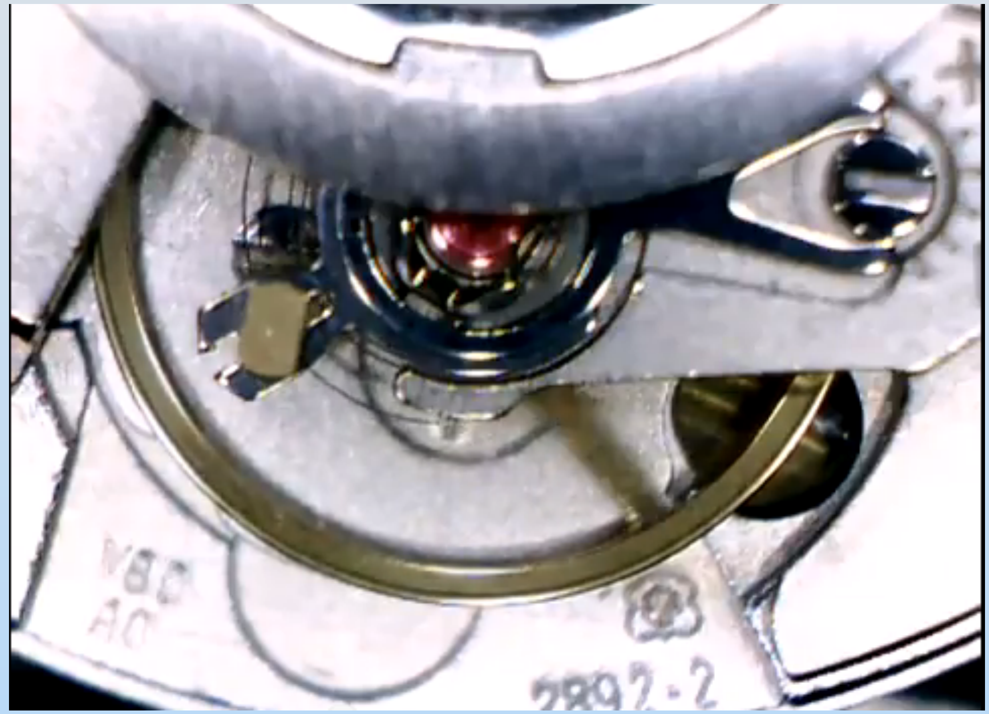
For small angles, $\theta_0 < 1$, then $\sin^2(\theta_0/2) \cong \theta_0^2/4$ and

$$T \cong 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{16}\theta_0^2\right) = T_0\left(1+\frac{1}{16}\theta_0^2\right)$$

Demonstration
Simple Pendulum:

Amplitude Effect on Period

Balance Spring Watch



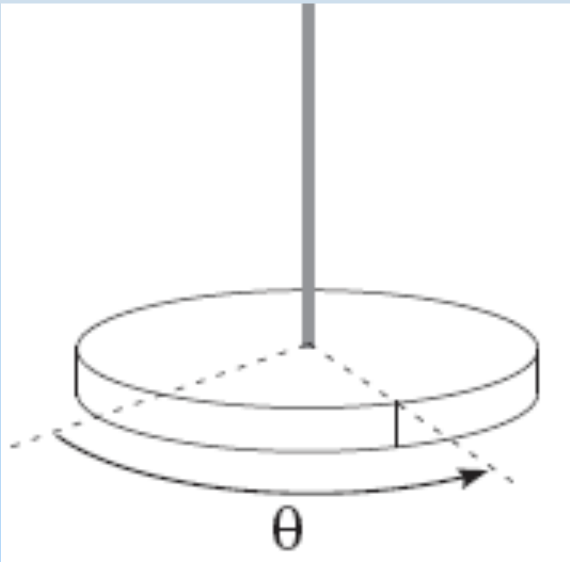
https://www.youtube.com/watch?v=jW_j4O1dyPo

Table Problem: Torsional Oscillator

A disk with moment of inertia I_{cm} about the center of mass rotates in a horizontal plane. It is suspended by a thin, massless rod. If the disk is rotated away from its equilibrium position by an angle θ, the rod exerts a restoring torque given by

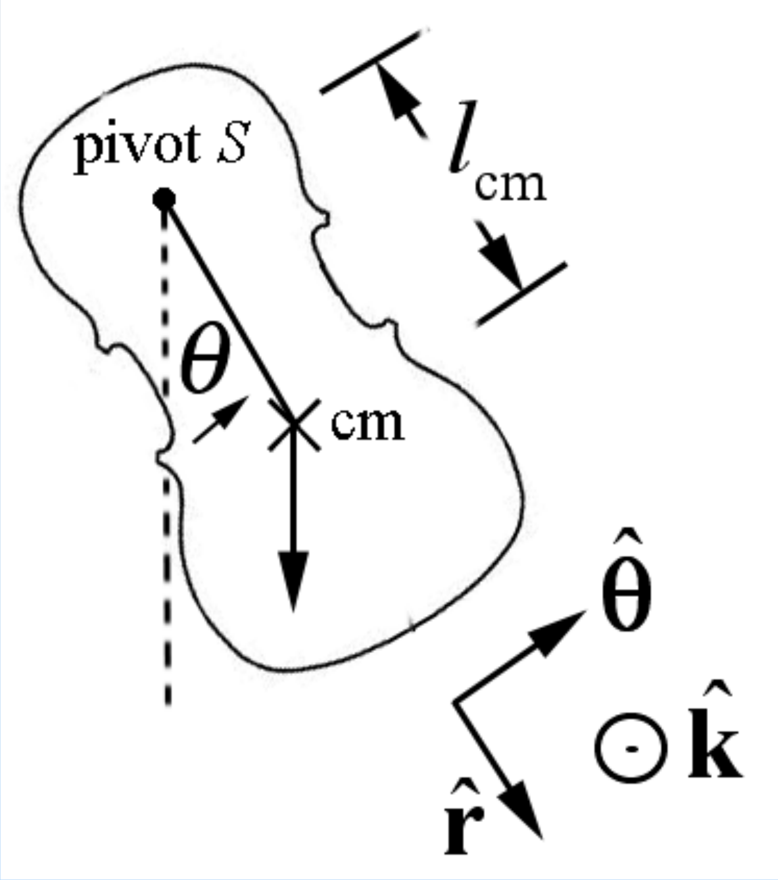
$$\tau_{cm} = -\gamma\theta$$

At $t = 0$ the disk is released at an angular displacement of θ_0 with a non-zero positive angular speed $\dot{\theta}_0$ Find the subsequent time dependence of the angular displacement $\theta(t)$.



Worked Example: Physical Pendulum

A physical pendulum consists of a body of mass m pivoted about a point S. The center of mass is a distance l_{cm} from the pivot point. What is the period of the pendulum for small angle oscillations, $\sin\theta \approx \theta$?



Physical Pendulum

Rotational dynamical equation $\vec{\boldsymbol{\tau}}_S = I_S \vec{\boldsymbol{\alpha}}$

Small angle approximation $\sin\theta \cong \theta$

Equation of motion $\frac{d^2\theta}{dt^2} \cong -\frac{l_{cm} m g}{I_S}\theta$

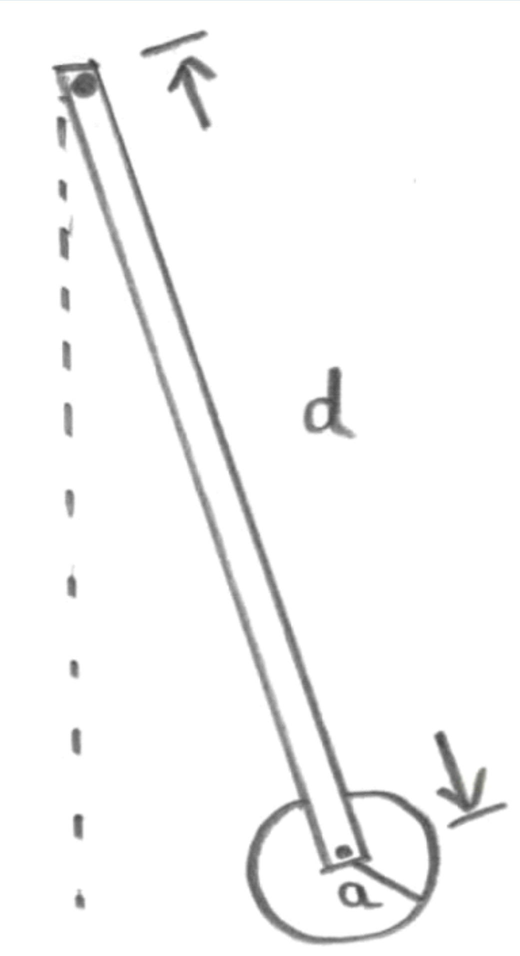
Angular frequency $\omega_0 \cong \sqrt{\frac{l_{cm} m g}{I_S}}$

Period

$$T = \frac{2\pi}{\omega_0} \cong 2\pi\sqrt{\frac{I_S}{l_{cm} m g}}$$

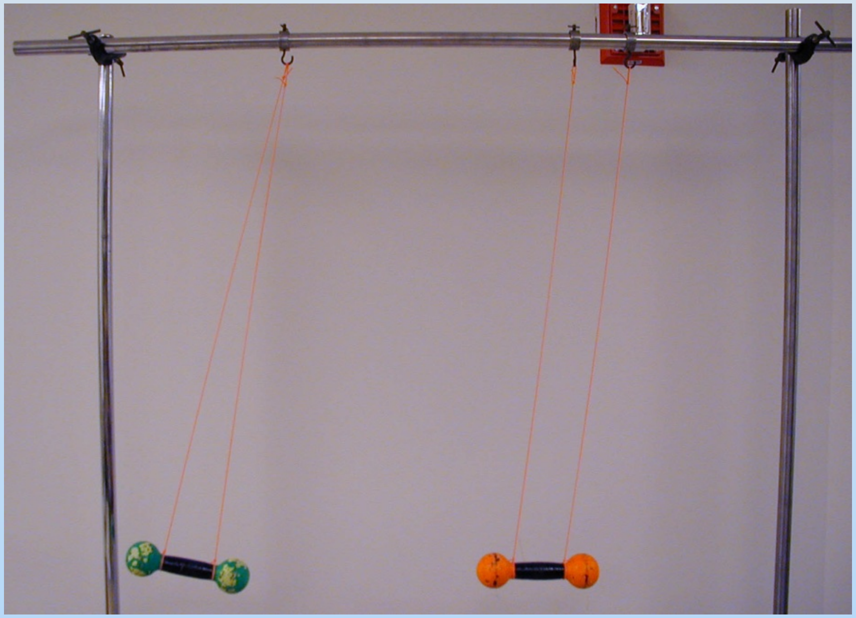
Concept Question: Physical Pendulum

A physical pendulum consists of a uniform rod of length d and mass m pivoted at one end. A disk of mass m_1 and radius a is fixed to the other end. Suppose the disk is now mounted to the rod by a frictionless bearing so that is perfectly free to spin. Does the period of the pendulum



1. increase?
2. stay the same?
3. decrease?

Demo: Identical Pendulums, Different Periods

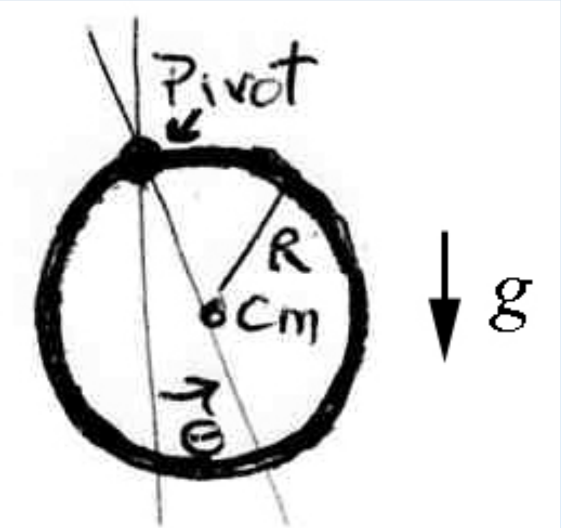


Single pivot: body rotates about center of mass.

Double pivot: no rotation about center of mass.

Table Problem: Physical Pendulum

A physical pendulum consists of a ring of radius R and mass m. The ring is pivoted (assume no energy is lost in the pivot). The ring is pulled out such that its center of mass makes an angle θ_0 from the vertical and released from rest. The gravitational constant is g.



a) First assume that $\theta_0 << 1$. What is the angular frequency of oscillation?
b) What is the angular speed of the ring at the bottom of its swing?

Simple Harmonic Motion

$$U(x) \simeq U(x_0) + \frac{1}{2}k_{eff}(x - x_0)^2$$

Energy Diagram: Example Spring Simple Harmonic Motion

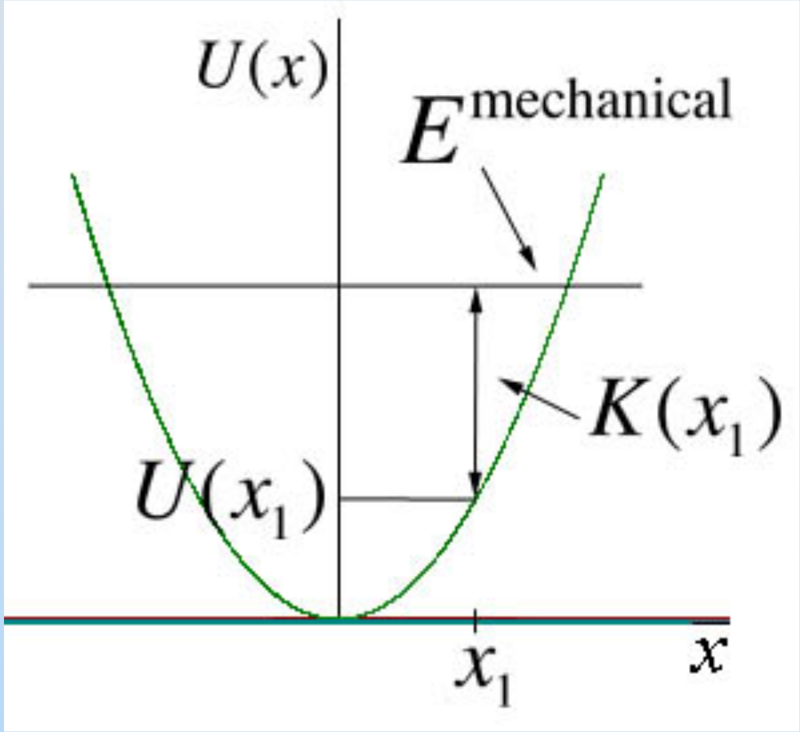
Potential energy function:

$$U(x) = \frac{1}{2}kx^2, \quad U(x=0) = 0$$

Mechanical energy is represented by a horizontal line

$$E = K(x) + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

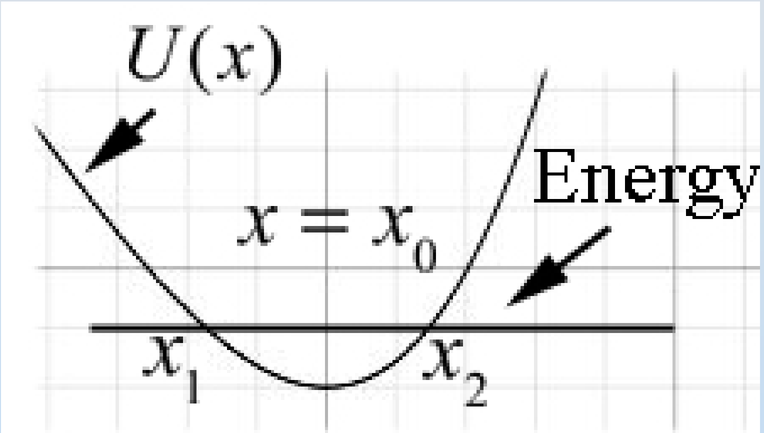
$$K(x) = E - U(x)$$



Small Oscillations

Small Oscillations

Potential energy function $U(x)$ for object of mass *m*



Motion is limited to the region

$$x_1 < x < x_2$$

Potential energy has an extremum when

$$\frac{dU}{dx} = 0$$

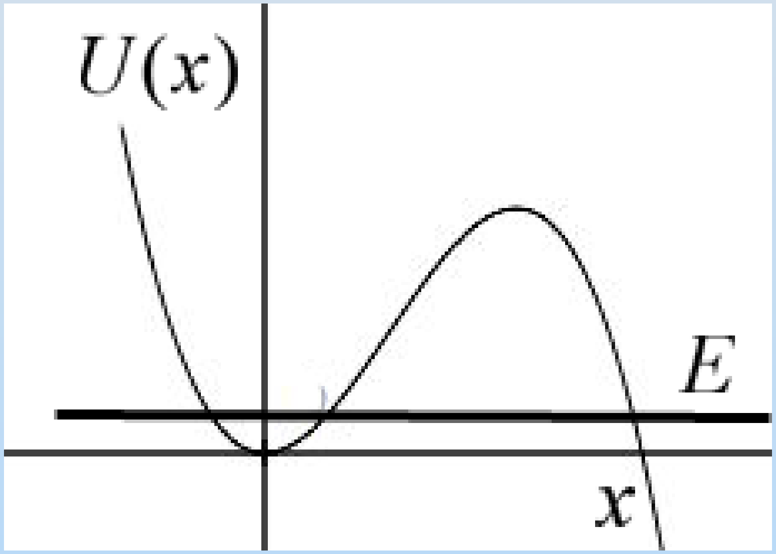
Small Oscillations

Expansion of potential function using Taylor Formula

$$U(x) = U(x_0) + \frac{dU}{dx}(x_0)(x - x_0) + \frac{1}{2!}\frac{d^2U}{dx^2}(x_0)(x - x_0)^2) + \cdots$$

When x_0 is minimum then

$$\left.\frac{dU}{dx}\right|_{x=x_0} = 0$$



When displacements are small

$$|x - x_0| << 1$$

Approximate potential function as quadratic

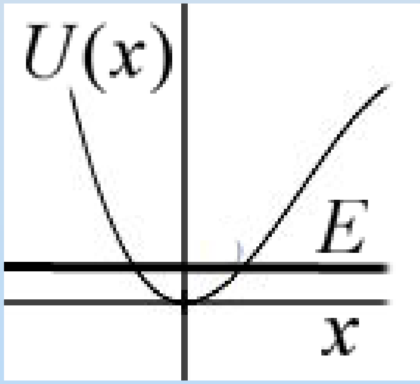
$$U(x) \simeq U(x_0) + \frac{1}{2}\frac{d^2U}{dx^2}(x_0)(x - x_0)^2 = U(x_0) + \frac{1}{2}k_{eff}(x - x_0)^2$$

Small Oscillations: Period

When displacements are small $(x - x_0) << 1$
Approximate potential function as quadratic function

$$U(x) \simeq U(x_0) + \frac{1}{2}\frac{d^2U}{dx^2}(x_0)(x - x_0)^2 = U(x_0) + \frac{1}{2}k_{eff}(x - x_0)^2$$

Angular frequency of small oscillation



$$\omega_0 = \sqrt{k_{eff} / m} = \sqrt{\frac{d^2U}{dx^2}(x_0) / m}$$

Period:

$$T = 2\pi / \omega_0 = 2\pi\sqrt{m / k_{eff}} = 2\pi\sqrt{m / \frac{d^2U}{dx^2}(x_0)}$$

Concept Question: Energy Diagram 3

A particle with total mechanical energy E has position $x > 0$ at $t = 0$

1) escapes to infinity

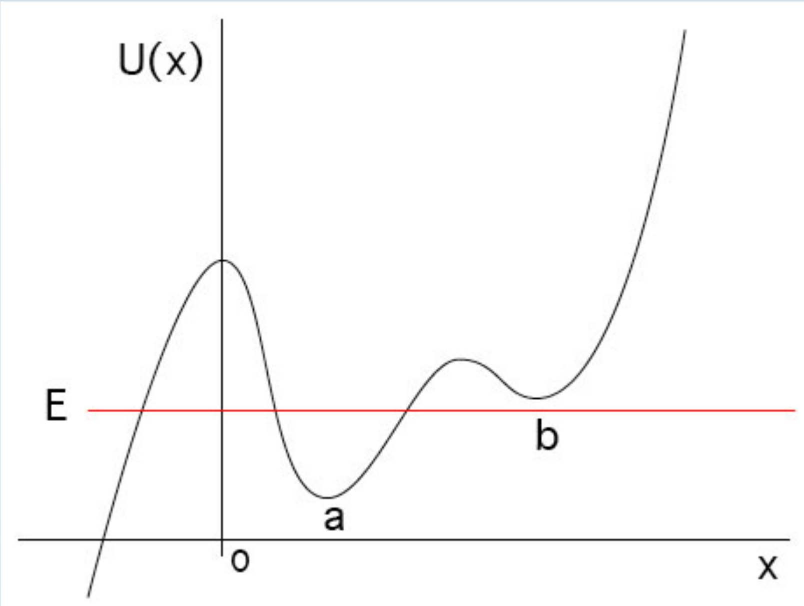
2) approximates simple harmonic motion

3) oscillates around a

4) oscillates around b

5) periodically revisits a and b

6) two of the above



Concept Question: Energy Diagram 4

A particle with total mechanical energy E has position $x > 0$ at $t = 0$

1) escapes to infinity

2) approximates simple harmonic motion

3) oscillates around a

4) oscillates around b

5) periodically revisits a and b

6) two of the above

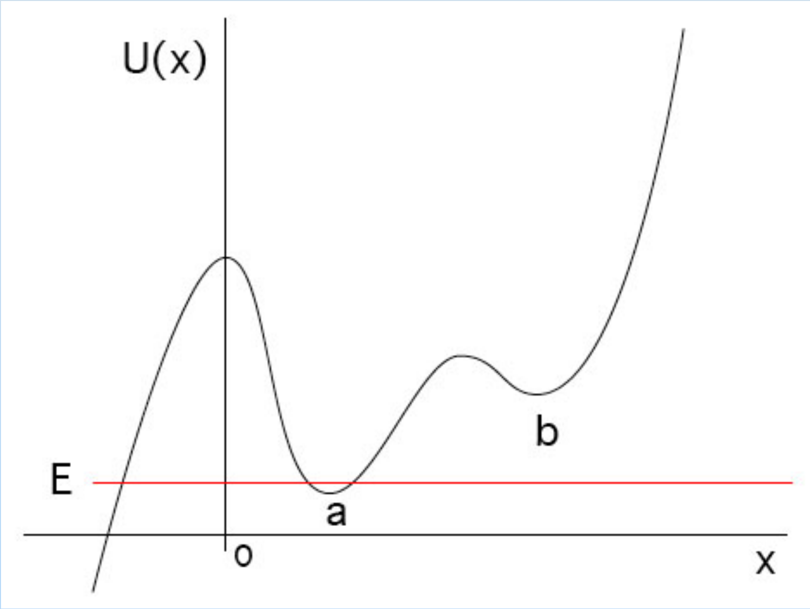


Table Problem: Small Oscillations

A particle of effective mass *m* is acted on by a potential energy given by

$$U(x) = U_0\left(-ax^2 + bx^4\right)$$

where U_0, a, and b are positive constants

a) Find the points where the force on the particle is zero. Classify them as stable or unstable.

b) If the particle is given a small displacement from an equilibrium point, find the angular frequency of small oscillation.