## Your Comments

I'm having a heck of a time wrapping my head around how this very fundamental stuff can have an "orientation". Why the heck does a magnetic field cause a force in a direction totally perpendicular to both the field and the charges movement, and whats to say the force is one way instead of the other? seems really arbitrary compared to gravity or coulomb's law

Magnetic fields are caused by charges in motion or current, right? How does current flow in something like a bar magnet? What force drives these charges to move in a bar magnet, why is it that they can't just settle out?
Can you play some Led Zeppelin before class again? When the levee breaks was awesome to listen to before class.
The word lost doesn't do justice to how I'm feeling about the material I just saw. Can I get some clarification up in this lecture?
I've got a right hand, and I kinda wanna use it....can we go over the right hand rule!!
Dear Professor Stelzor, I am very confused on the direction of a particle in a magnetic field. I would be so happy and overjoyed if you would be kind enough to explain that to me. If you did I would be the happiest man EVER! Sincerely, Confused Physics 212 Student

The typo for "north" in question 1 of the prelecture is really bothering me. Also, I meant to do this prelecture last night after my ECE190 exam, but I was caught up in some E-WEEK shenanigans. So it's currently 7:30am, and if I don't get a comment post for being clutch, I'll be very disappointed.

So what is a magnetic field? Please give us the real answer. Kind of like how you can "conceptually" understand what $\mathrm{E}=\mathrm{mc}^{\wedge} 2$ actually is.

There has been a serious lack of cool demos recently and it is making me sad. There better be some good demos for this stuff

## Physics 212 Lecture 12

## Today's Concept:

## Magnetic Force on Moving Charges

$$
\vec{F}=q \vec{v} \times \vec{B}
$$

## Key Concepts:

1) The force on moving charges due to a magnetic field.
2) The cross product.

## Today's Plan:

1) Review of magnetism
2) Review of cross product
3) Example problem

## Magnetic Observations

## Bar Magnets



Magnetic Charge?
$N \quad S \quad$ cut in half $\Rightarrow N S$

## Magnetic Observations

Compass needle deflected by electric current


Magnetic fields created by electric currents
Magnetic fields exert forces on electric currents (charges in motion)


## Magnetic Observations

$\dot{P}$
$I$ (out of the screen)
Case $I$
magnetic field at $P$ points
A. Case I: left, Case II: right B. Case I: left, Case II: left
C. Case I: right, Case II: left D. Case I: right, Case II: right

WHY? Direction of $\vec{B}$ : right thumb in direction of $I$, fingers curl in the direction of $\vec{B}$

## Magnetism \& Moving Charges

All observations are explained by two equations:

$$
\vec{F}=q \vec{v} \times \vec{B} \quad \text { Today }
$$

$$
d \vec{B}=\frac{\mu_{0} I}{4 \pi} \frac{d \vec{s} \times \hat{r}}{r^{2}}
$$

Next Week

## Cross Product Review

Cross Product different from Dot Product
$A \bullet B$ is a scalar; $A \times B$ is a vector
$A \bullet B$ proportional to the component of $B$ parallel to $A$
$A \times B$ proportional to the component of $B$ perpendicular to $A$

Definition of $A \times B$
Magnitude: $A B \sin \theta$
Direction: perpendicular to plane defined by $A$ and $B$ with sense given by right-hand-rule


Right-hand rule


## Remembering Directions: The Right Hand Rule

$$
\vec{F}=q \vec{v} \times \vec{B}
$$




Right-hand rule


## CheckPoint 1a

Three points are arranged in a uniform magnetic field. The B field points into the screen.

A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is
A. right
B. left
C. into the screen
D. out of the screen
E. zero

$$
\vec{F}=q \vec{v} \times \vec{B}
$$

The particle's velocity is zero.

There can be no magnetic force.


## CheckPoint 1b

Three points are arranged in a uniform magnetic field. The B field points into the screen.

The positive charge moves from $A$ toward $B$. The direction of the magnetic force on the particle is
A. right
B. left
C. into the screen
D. out of the screen
E. zero

$$
\begin{aligned}
& \vec{F}=q \vec{v} \times \vec{B}
\end{aligned}
$$



## Cross Product Practice

$\begin{array}{ll}\text { Protons (positive charge) coming out of screen } \\ \text { Magnetic field pointing down }\end{array} \quad \vec{F}=q \overrightarrow{\boldsymbol{v}} \times \vec{B}$
What is direction of force on POSITIVE charge?


## Motion of Charge $q$ in Uniform B Field

## Force is perpendicular to $v$

Speed does not change
Uniform Circular Motion
Solve for $R$ :

$$
\begin{array}{r}
\vec{F}=q \vec{v} \times \vec{B} \Rightarrow F=q v B \\
a=\frac{v^{2}}{R}
\end{array}
$$



## Uniform B into page

$$
q v B=m \frac{v^{2}}{R} \quad \longrightarrow \quad R=\frac{m v}{q B}
$$

now this is some cool stuff. i hope that there are going to be cool demos in lecture. for the motion in a uniform field, why doesn't the particle just start accelerating in theormsitive y direction, instead of going in a circle? shouldn't the magnetic force always be pointing in the positive direction since the entire plane is magnetized?

Can you take us to the LHC (while it is shutdown) to see some of the big magnets in the ring and the detectors there?


Electricity \& Magnetism Lecture 12, Slide 14

## CheckPoint 2

The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.


What is the direction of the magnetic field in chamber 1 ?
A. up

$$
\begin{gathered}
\begin{array}{c}
\text { B. down } \\
\vec{F}=q \vec{v} \times \vec{B} \\
{ }_{B} \longrightarrow_{F}^{v} \uparrow
\end{array}
\end{gathered}
$$

C. into the page
D. out of the page


## CheckPoint 8

The drawing below shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1 , and observed to follow the dashed path shown in the figure.


Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2
A. $\left|\mathrm{B}_{1}\right|>\left|\mathrm{B}_{2}\right|$
B. $\left|B_{1}\right|=\left|B_{2}\right|$
C. $\left|B_{1}\right|<\left|B_{2}\right|$

Observation: $R_{2}>R_{1}$

$$
R=\frac{m v}{q B} \longrightarrow\left|B_{1}\right|>\left|B_{2}\right|
$$



## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

What is $B$ ?


Conceptual Analysis
What do we need to know to solve this problem?
A) Lorentz Force Law

$$
(\vec{F}=q \vec{v} \times \vec{B}+q \vec{E})
$$

D) Conservation of Energy/Newton's Laws
E) All of the above

Absolutely! We need to use the definitions of $V$ and $E$ and either conservation of energy or Newton's Laws to understand the motion of the particle before it enters the $B$ field.
We need to use the Lorentz Force Law (and Newton's Laws) to determine what happens in the magnetic field.

## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

What is $B$ ?


B

## Strategic Analysis

Calculate $v$, the velocity of the particle as it enters the magnetic field Use Lorentz Force equation to determine the path in the field as a function of $B$ Apply the entrance-exit information to determine $B$

## Let's Do It !

## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

What is $B$ ?

-What is the change in the particle's potential energy after travelling distance d?

$$
\Delta U=-q E d
$$

(A)
$\Delta U=-E d$
(B)
$\Delta U=0$
(C)

Why??

- How do you calculate change in the electric potential given an electric field?

$$
\Delta V=-\int \vec{E} \cdot d \vec{\ell}=-E d
$$

- What is the relation between the electric potential amd the potential energy?

$$
\Delta U=q \Delta V
$$

## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

What is $B$ ?


What is $v_{0}$, the speed of the particle as it enters the magnetic field ?

$$
v_{o}=\sqrt{\frac{2 E}{m}}
$$

$$
E
$$

Why?

$$
\begin{gathered}
v_{o}=\sqrt{\frac{2 q E d}{m}} \\
\mathrm{~B}
\end{gathered}
$$

$$
\begin{gathered}
v_{o}=\sqrt{2 a d} \\
\mathrm{C}
\end{gathered}
$$

$$
v_{o}=\sqrt{\frac{2 q E}{m d}}
$$

$$
v_{o}=\sqrt{\frac{q E d}{m}}
$$

$$
D
$$

Conservation of Energy

$$
\begin{aligned}
& \text { Initial: Energy }=U=q V=q E d \\
& \text { Final: } \text { Energy }=K E=1 / 2 m v_{0}{ }^{2} \\
& \text { ton's Laws }
\end{aligned} \quad \longrightarrow \frac{1}{2} m v_{o}^{2}=q E d \longrightarrow v_{o}=\sqrt{\frac{2 q E d}{m}}
$$ Newton's Laws

$$
\begin{aligned}
& a=F / m=q E / m \\
& v_{0}^{2}=2 a d
\end{aligned}
$$

$$
\longrightarrow v_{o}^{2}=2 \frac{q E}{m} d \longrightarrow v_{o}=\sqrt{\frac{2 q E d}{m}}
$$

## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.
What is $B$ ? $\quad v_{o}=\sqrt{\frac{2 q E d}{m}}$


What is the path of the particle as it moves through the magnetic field?

# $\mathrm{xx} \mathcal{X}^{\boldsymbol{X}} \mathrm{XXXXXX}$ x $x$ XXXXXXX XXXXXXXX <br> A XXXXXXXXX 

|  | $\mathrm{x} \times \hat{\chi} \times \mathrm{XXXXX}$ |
| :---: | :---: |
|  | XXXXXXXXX |
|  | XXXXXXXXX |
| B | X X X X X X X |

$$
\begin{array}{r}
X X X X X X X X X \\
X X X X X X X X X \\
X X X X X X X X X \\
C X X X X X X X X
\end{array}
$$

## Why?

Path is circle!
Force is perpendicular to the velocity
Force produces centripetal acceleration
Particle moves with uniform circular motion

## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.
What is $B$ ? $v_{o}=\sqrt{\frac{2 q E d}{m}}$


B

What can we use to calculate the radius of the path of the particle?


## Calculation

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.
What is $B$ ? $\quad v_{o}=\sqrt{\frac{2 q E d}{m}} \quad R=\frac{1}{2} x_{0}$


B

$$
\begin{array}{c|cccc}
B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}} & B=\frac{E}{v} & B=E \sqrt{\frac{m}{2 q E d}} & B=\frac{1}{x_{o}} \sqrt{\frac{2 m E d}{q}} & B=\frac{m v_{o}}{q x_{o}} \\
\mathrm{~A} & \mathrm{~B} & \mathrm{C} & \mathrm{D} & \mathrm{E}
\end{array}
$$

## Why?

$$
\begin{array}{r}
\vec{F}=m \vec{a} \longrightarrow q v_{o} B=m \frac{v_{o}^{2}}{R} \longrightarrow B=\frac{m}{q} \frac{v_{o}}{R} \longrightarrow B=\frac{m}{q} \frac{2}{x_{o}} \sqrt{\frac{2 q E d}{m}} \\
B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}}
\end{array}
$$

## Follow-Up

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.
What is $B$ ? $\quad B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}}$


B
Suppose the charge of the particle is doubled ( $Q=2 q$ ), while keeping the mass constant. How does the path of the particle change?


A

No slam dunk.. As Expected!
Several things going on here


1. $q$ changes $->v$ changes
2. $q$ \& $v$ change -> $F$ changes
3. $v \& F$ change $->R$ changes

## Follow-Up

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.
What is $B$ ? $\quad B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}}$


B

Suppose the charge of the particle is doubled ( $Q=2 q$ ), while keeping the mass constant. How does the path of the particle change?

## How does $v$, the new velocity at the entrance, compare to the original velocity $v_{0}$ ?

A $v=\frac{v_{o}}{2}$
B $v=\frac{v_{o}}{\sqrt{2}}$
C $v=v_{o}$
D $v=\sqrt{2} v_{o}$
E $\quad v=2 v_{o}$

## Why?

$$
\frac{1}{2} m v^{2}=Q E d=2 q E d=2 \frac{1}{2} m v_{o}^{2} \longrightarrow v^{2}=2 v_{o}^{2} \longrightarrow v=\sqrt{2} v_{o}
$$

## Follow-Up

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

What is $B$ ?

$$
B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}}
$$



B
Suppose the charge of the particle is doubled ( $Q=2 q$ ), while keeping the mass constant. How does the path of the particle change?

$$
v=\sqrt{2} v_{o}
$$

How does $F$, the magnitude of the new force at the entrance, compare to $F_{0}$, the magnitude of the original force?
A $F=\frac{F_{o}}{\sqrt{2}}$
B $F=F_{o}$
C $F=\sqrt{2} F_{o}$
$\mathrm{D} F=2 F_{o}$
E $F=2 \sqrt{2} F_{o}$

Why?

$$
F=Q v B=2 q \cdot \sqrt{2} v_{o} \cdot B \quad F F=2 \sqrt{2} F_{o}
$$

## Follow-Up

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

What is $B$ ?

$$
B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}}
$$



Suppose the charge of the particle is doubled ( $Q=2 q$ ), while keeping the mass constant. How does the path of the particle change?

$$
v=\sqrt{2} v_{o} \quad F=2 \sqrt{2} F_{o}
$$

How does $R$, the radius of curvature of the path, compare to $R_{0}$, the radius of curvature of the original path?
A $R=\frac{R_{o}}{2}$
$\mathrm{B} R=\frac{R_{o}}{\sqrt{2}}$
C $R=R_{o}$
D $R=\sqrt{2} R_{o}$
E $R=2 R_{o}$

Why?

$$
F=m \frac{v^{2}}{R} \quad \longrightarrow R=m \frac{v^{2}}{F} \quad \longrightarrow \quad R=m \frac{2 v_{o}^{2}}{2 \sqrt{2} F_{o}}=m \frac{v_{o}^{2}}{\sqrt{2} F_{o}}=\frac{R_{o}}{\sqrt{2}}
$$

## Follow-Up

A particle of charge $q$ and mass $m$ is accelerated from rest by an electric field $E$ through a distance $d$ and enters and exits a region containing a constant magnetic field $B$ at the points shown. Assume $q, m, E, d$, and $x_{0}$ are known.

$$
\text { What is } B \text { ? } \quad B=\frac{2}{x_{o}} \sqrt{\frac{2 m E d}{q}}
$$



Suppose the charge of the particle is doubled ( $Q=2 q$ ), while keeping the mass constant. How does the path of the particle change?


$$
R=\frac{R_{o}}{\sqrt{2}}
$$



A Check: (Exercise for Student)
Given our result for B (above), can you show:

$$
R=\frac{1}{B} \sqrt{\frac{2 m E d}{Q}} \longrightarrow R=\frac{R_{o}}{\sqrt{2}}
$$

