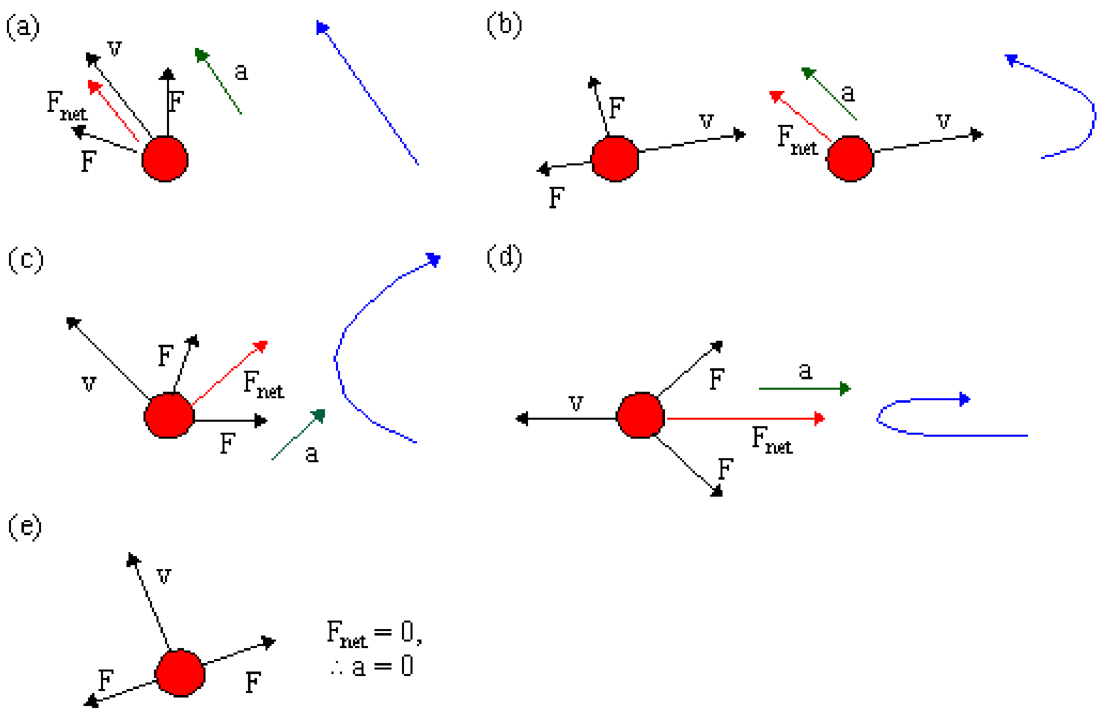
Physics 1100: 2D Kinematics Solutions

1. In the diagrams below, a ball is on a flat horizontal surface. The initial velocity and constant external forces acting on the ball are indicated. Describe qualitatively how motion the motion of the ball will change.

   First determine the direction of the net force on the ball. From Newton's Second Law, **F** = m**a**, so the direction of the net force and resulting acceleration are in the same direction. As time passes, the direction of the ball will tend to point in the direction of the acceleration.



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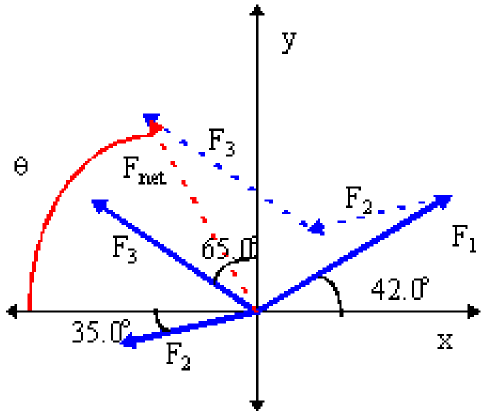
2. Three forces $\mathbf{F_1}$ = (25.0N, 42.5°), $\mathbf{F_2}$= (15.5N, 215°), and $\mathbf{F_3}$ = (20.5N, 155°) accelerate an 8.75 kg mass. What is the net force acting on the mass? What is the magnitude and direction of the mass's acceleration?

   This involves a 2D vector addition, so it is appropriate to sketch the addition of the forces and then add the components:



$\mathbf{F}_1 = \boldsymbol{i}[25cos(42.5°)] + \boldsymbol{j}[25sin(42.5°)] = \boldsymbol{i}[18.432] + \boldsymbol{j}[16.890]$

$\mathbf{F}_2 = \boldsymbol{i}[-15.5cos(35.0°)] + \boldsymbol{j}[-15.5sin(35.0°)] = \boldsymbol{i}[-12.697] + \boldsymbol{j}[-8.8904]$

$\mathbf{F}_3 = \boldsymbol{i}[-20.5sin(65.0°] + \boldsymbol{j}[-20.5cos(65.0°] = \boldsymbol{i}[-18.579] + \boldsymbol{j}[8.664]$

Adding the components of the vectors we get

$$\mathbf{F}_{net} = \boldsymbol{i}[18.432 + -12.697 + -18.579] + \boldsymbol{j}[16.890 + -8.8904 + 8.664]$$
$$= \boldsymbol{i}[-12.844] + \boldsymbol{j}[16.663]$$

The magnitude of the net force is given by $F_{net} = [(F_x)^2 + (F_y)^2]^{½} = 21.039$ N . The angle is determined by trigonometry to be $\theta = arctan(|F_y/F_x|) = 52.4°$. So putting the answer in the same form as that we were given, the net force is $\mathbf{F}_{net}$ = (21.0 N, 127.6°).

Since $\mathbf{a} = \mathbf{F}_{net}/m$, $\mathbf{a}$ = ([21.0 N / 8.75 kg], 127.6°) = (2.40 m/s^2, 127.6°).

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3. What would have to be the magnitude and direction of a fourth force $\mathbf{F_4}$ in question 2 so that the acceleration of the mass would be zero?

We are asked to find F_4 such that $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} + \mathbf{F_4} = \mathbf{F}_{net} + \mathbf{F_4} = 0$. In other words, we find to find $\mathbf{F_4} = -\mathbf{F}_{net}$. Equal and opposite vectors have the same magnitude but are 180° apart, so $\mathbf{F_4}$ = (21.0 N, 127.6° + 180°) = (21.0 N, 307.6°).

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4. A rifle bullet with a mass of 100 g leaves the rifle with a muzzle velocity of 500 m/s. If the barrel is 0.75 m long, calculate the average acceleration of the bullet. Calculate the average force that the exploding gunpowder exerts on the bullet. If the rifle has a mass of 7.50 kg, what is its average acceleration?

(a) To calculate the average acceleration, we make use of the kinematic data. To solve a kinematics problem, we list the given data and the unknown. We may reasonably assume that the bullet starts from rest.

$$v_0 = 0$$

$$v_f = 500 \text{ m/s}$$

$$\Delta x = 0.75 \text{ m}$$

$$a = ?$$

Examining the variables, we see that we can use the equation $2a\Delta x = (v)^2 - (v_0)^2$, to find the acceleration,

$$a = \frac{v^2 - v_0^2}{\Delta x} = \frac{(500 \text{ m/s})^2 - (0)^2}{0.75 m} = +1.667 \times 10^5 \text{ m/s}^2$$

The plus sign indicates that acceleration is in the same direction as the motion of the bullet.

(b) According to Newton's Second Law,

$$\boldsymbol{F}_{bullet} = m_{bullet}\boldsymbol{a}_{bullet} = (0.100 \text{ kg})(1.667 \times 10^5 \text{ m/s}^2) = +1.667 \times 10^4 \text{ N},$$

where the plus sign indicates that the force is in the same direction as the motion of the bullet.

(c) According to Newton's Third Law, the force on the rifle must be equal but opposite to the force acting on the bullet, $\boldsymbol{F}_{rifle} = -\boldsymbol{F}_{bullet}$. Hence we know that the force on the rifle is -1.667×10^4 N, where the plus sign indicates that the force is in the opposite direction to the motion of the bullet.

(d) According to Newton's Second Law, $\boldsymbol{F}_{rifle} = m_{rifle}\boldsymbol{a}_{rifle}$. We thus find the accelartation of the bullet to be

$$\boldsymbol{a}_{rifle} = \boldsymbol{F}_{rifle} / m_{rifle} = (-1.667 \times 10^4 \text{ N}) / (7.50 \text{ kg}) = -2.22 \times 10^3 \text{ m/s}^2 .$$

where the minus sign indicates that the acceleration of the rifle is in the opposite direction to the motion of the bullet.

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5. You are thrown from a bicycle and skid in a straight line to a complete stop on a rough gravel path. A measurement of the bloody skidmark reveals that it is 3.50 m long. What average force did the gravel exert on your anatomy? Assume that your mass is 70.0 kg and that your initial speed was 30 km/h.

To find the force we would use Newton's Second Law, $\boldsymbol{F} = m\boldsymbol{a}$. We are given the mass but not the acceleration. However we are given kinematic data and we should be able to use this to find the acceleration.

$v_0 = 0$

$v_f = 30$ km/h (1000 m / 1 km) (1h / 3600 s)
$= 8.333$ m/s

$\Delta x = 3.50$ m

$a = ?$

Examining the variables, we see that we can use the equation $2a\Delta x = (v)^2 - (v_0)^2$, to find the acceleration,

$$a = \frac{v^2 - v_0^2}{\Delta x} = \frac{(0)^2 - (8.333\,\text{m/s})^2}{(2)(3.50\,m)} = -9.921\,\text{m/s}^2$$

The minus sign indicates that acceleration is in the opposite direction to your motion.

According to Newton's Second Law,

$$\boldsymbol{F} = m\boldsymbol{a} = (70.0\ kg)(-9.92\ m/s^2) = -694\ N\,,$$

where the minus sign indicates that the force is also in the opposite direction to your motion as you slide.

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6. The 80.0-kg male partner of a figure skating duo pushes his 60.0-kg female partner with a force of 70.0N. Find the acceleration of both partners.

According to Newton's Third Law, the woman must exert 70.0 N on the male skater but in the opposite direction, in other words, −70.0 N, where the minus sign indicates the direction opposite to the direction that the man pushes the woman.

To find the acceleration of the male skater we use Newton's Second Law,

$$\mathbf{a}_{man} = \mathbf{F}_{womanonman} / m_{man} = -70\text{ N} / 80.0\text{ kg} = -0.875\text{ m/s}^2.$$

The minus sign indicates that the man moves backwards which is what one would expect.

Similarly the acceleration of the female skater is

$$\mathbf{a}_{woman} = \mathbf{F}_{manonwoman} / m_{woman} = +70\text{ N} / 60.0\text{ kg} = +1.17\text{ m/s}^2.$$

The plus sign indicates that the woman moves backwards (forward from the man's perspective) which is what one would expect.

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7. An 800-kg artillery piece horizontally fires a 10.5-kg shell with a muzzle velocity of 450 m/s. The barrel is 3.75 m long. What is the acceleration of both?

To calculate the acceleration of the shell, we make use of the kinematic data. To solve a kinematics problem, we list the given data and the unknown. We may reasonably assume that the shell starts from rest.

$$v_0 = 0$$
$$v_f = 450 \text{ m/s}$$
$$\Delta x = 3.75 \text{ m}$$
$$a = ?$$

Examining the variables, we see that we can use the equation $2a\Delta x = (v)^2 - (v_0)^2$, to find the acceleration,

$$a = \frac{v^2 - v_0^2}{\Delta x} = \frac{(450\,\text{m/s})^2 - (0)^2}{3.75\,m} = +2.70 \times 10^4\,\text{m/s}^2$$

The plus sign indicates that acceleration is in the same direction as the motion of the shell.

We don't have any kinematic data for the artillery piece, but we do know from Newton's Third Law that the force on the cannon must be equal but opposite to the force acting on the shell, $\boldsymbol{F}_{cannon} = -\boldsymbol{F}_{shell}$, and we can calculate $\boldsymbol{F}_{shell}$.

According to Newton's Second Law,

$$\boldsymbol{F}_{shell} = m_{shell}\boldsymbol{a}_{shell} = (10.5\text{ kg})(2.70 \times 10^4\text{ m/s}^2) = +2.84\ 10^5\text{ N},$$

where the plus sign indicates that the force is in the same direction as the motion of the shell.

Since $\boldsymbol{F}_{cannon} = m_{cannon}\boldsymbol{a}_{cannon}$, we rearrange to find

$$\boldsymbol{a}_{cannon} = \boldsymbol{F}_{cannon}/m_{cannon} = -\boldsymbol{F}_{shell}/m_{cannon} = (-2.84 \times 10^5\text{ N})/(800\text{ kg}) = -354\text{ m/s}^2.$$

where the minus sign indicates that the acceleration of the cannon is in the opposite direction to the acceleration of the shell.

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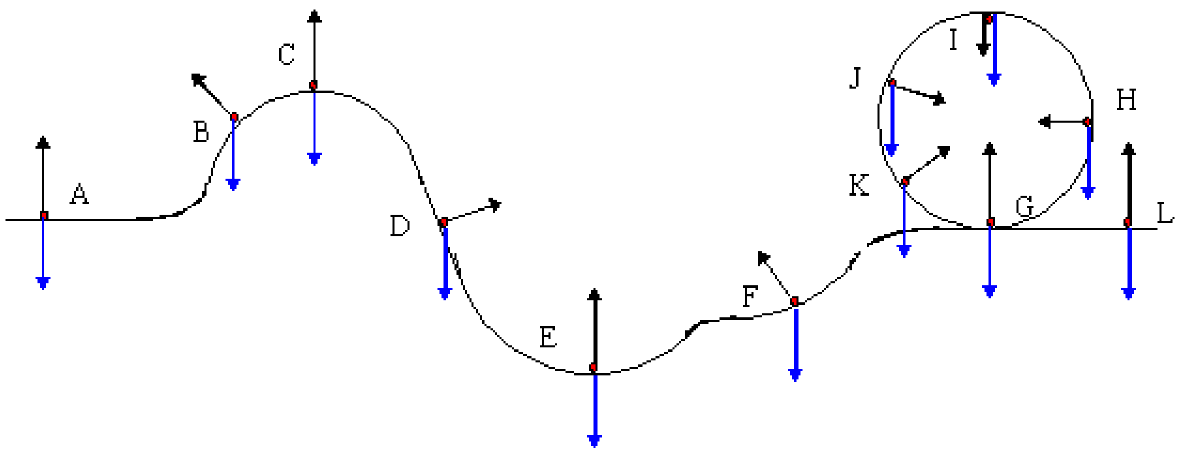
8. In the diagram below, an object travels over a hill, down a valley, and around a loop-the-loop. At each of the specified points draw a free body diagram indicating the directions of the normal force and of the weight.

Recall that weight always acts down which, by convention, is taken to be the bottom of the page. Normal forces act normal to the surface, from the surface through the object.

In the diagram, normal forces are represented by black (dark) arrow and weight by blue arrows



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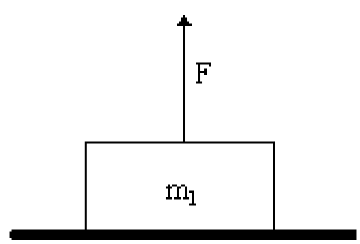
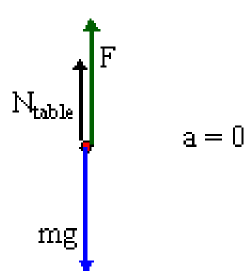
9. If m_1 = 25.0 kg, calculate the normal force on the bottom mass if the applied force, **F**, is (a) 100 N, (b) 245 N, and (c) 500 N.



Since this problem deals with forces that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block. The block has mass, so it has weight. The table top exerts a normal on the block. A given force is also acting on the block. The table top normal is nonzero only if the block remains on the table, so we will assume that the acceleration is zero.



In component form the forces and acceleration vectors are;

$\mathbf{F} = \boldsymbol{i}0 + \boldsymbol{j}F$,

$\mathbf{N}_{table} = \boldsymbol{i}0 - \boldsymbol{j}N_{table}$,

$\mathbf{W} = \boldsymbol{i}0 - \boldsymbol{j}mg$, and

$\mathbf{a} = \boldsymbol{i}0 + \boldsymbol{j}0$.

Since $\Sigma F_y = ma_y$, we find $N_{table} + F - mg = 0$. We don’t find anything interesting from $\Sigma F_x = ma_x$. So the equation for the normal force is

$$N_{table} = mg - F\ .$$

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Examining the three cases yields,

F	N_{table}
(a) 100 N	145 N
(b) 245 N	0 N
(c) 500 N	−255 N

Since normal force can never be zero, case (c) is saying that the block has been lifted off the table, so that $N_{table} = 0$, and the block is actually accelerating upwards.

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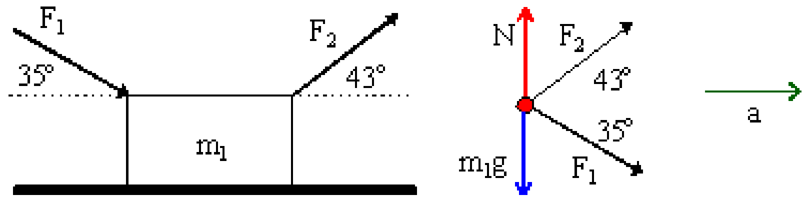
10. What is the normal force on the object of mass m_1 shown in the diagram? What is the acceleration of the object? In the diagram $\mathbf{F}_1 = 25$ N, $\mathbf{F}_2 = 15$ N, and $m_1 = 20$ kg.

This problem deals with forces and acceleration, which suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). In the diagram we show the givens forces F_1 and F_2. As well, since the object has mass, it has weight. Since the object touches the table top, there is a normal force from the table top through the object. We assume that the object will accelerate to the right.



Next we break the forces and acceleration into components;

$\mathbf{F}_1 = \boldsymbol{i}F_1 cos(35°) – \boldsymbol{j}F_1 sin(35°)$,

$\mathbf{F}_2 = \boldsymbol{i}F_2 cos(43°) – \boldsymbol{j}F_2 sin(43°)$,

$\mathbf{N} = \boldsymbol{i}0 – \boldsymbol{j}N$,

$\mathbf{W} = \boldsymbol{i}0 – \boldsymbol{j}mg$, and

$\mathbf{a} = \boldsymbol{i}a + \boldsymbol{j}0$.

We apply Newton's Second Law separately to the ***i*** and ***j*** components. Thus $\Sigma F_x = ma_x$ yields

$$F_1 cos(35°) + F_2 cos(43°) = m_1 a$$

and $\Sigma F_y = ma_y$ gives

$$N - m_1 g - F_1 sin(35°) + F_2 sin(43°) = 0$$

Thus the normal force is:

$$N = m_1 g + F_1 sin(35°) − F_2 sin(43°) = (20)(9.81) +25 sin(35°)− 15 sin(43°) = 200.3 \text{ N}$$

The acceleration is:

$$a = [F_1 cos(35°) + F_2 cos(43°)]/m_1 = [25 cos(35°) + 15 cos(43°)]/20 = 1.572 \text{ m/s}^2.$$

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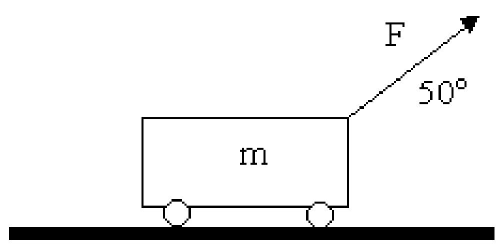
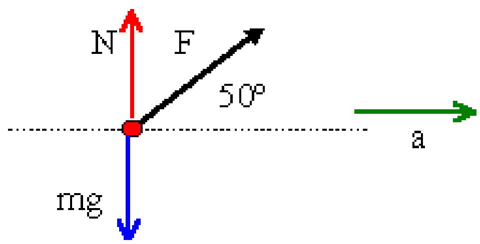
11. A physics instructor is pulling a string with a constant force of 1.00 Newtons. The string is attached to a toy on a frictionless surface. The string makes a 50° angle with the horizontal. The toy has a mass of 0.400 kg. Find the acceleration of the toy.



This problem deals with forces and acceleration, which suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). In the diagram we show the given force F. As well, since the object has mass, it has weight. Since the object touches the tabletop, there is a normal force from the tabletop through the object. We assume that the object will accelerate to the right.



We break the forces and acceleration into components;

$\mathbf{F} = \boldsymbol{i}\mathrm{F}\,cos(50°) - \boldsymbol{j}\mathrm{F}\,sin(50°)$,

$\mathbf{N} = \boldsymbol{i}0 - \boldsymbol{j}\mathrm{N}$,

$\mathbf{W} = \boldsymbol{i}0 - \boldsymbol{j}\mathrm{mg}$, and

$\mathbf{a} = \boldsymbol{i}\mathrm{a} + \boldsymbol{j}0$.

We apply Newton's Second Law separately to the ***i*** and ***j*** components. Thus $\Sigma F_x = ma_x$ yields

$$\mathrm{F}\,cos(50°) = \mathrm{ma}$$

and $\Sigma F_y = ma_y$ gives

$$\mathrm{N} - \mathrm{mg} + \mathrm{F}\,sin(50°) = 0$$

Thus the normal force is:

$$\mathrm{N} = \mathrm{mg} - \mathrm{F}\,sin(50°) = (0.400)(9.81) - 1.00\,sin(50°) = 3.16\ \mathrm{N}$$

The acceleration is:

$$\mathrm{a} = \mathrm{F}\,cos(50°)/\mathrm{m} = 1.00\,cos(50°)/0.400 = 1.61\ \mathrm{m/s^2}\ .$$

Since the acceleration is positive, our assumption about its direction was correct. The fact that the normal is positive, i.e. that the object has not lost contact with the tabletop, tells us that there is no acceleration in the y direction.

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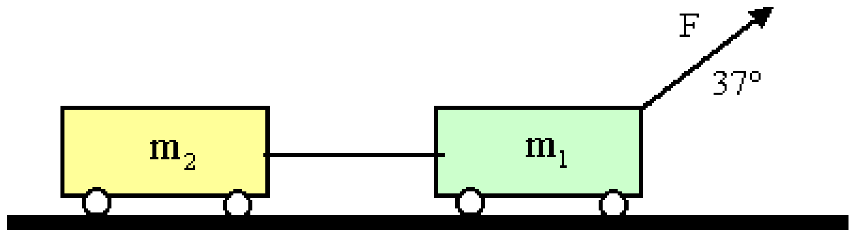
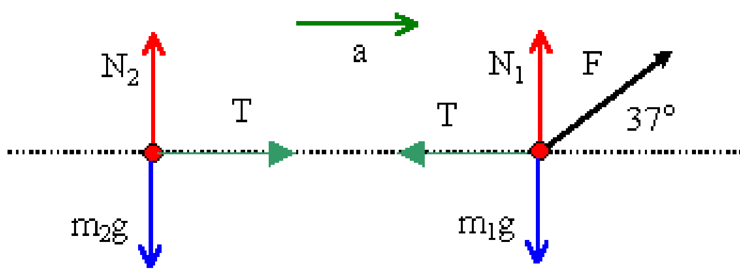
12. Two blocks are connected by a string. The blocks have masses of 1.50 kg and 1.20 kg. The heavier block is being pulled by a force of 8.00 N at angle of 37° to the floor. Assume the floor is frictionless. Find the tension in the string between the blocks.



This problem deals with forces and acceleration, which suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for <u>each</u> block. In the diagram for m_1 we show the given force F. As well, since the object has mass, it has weight. Since the object touches the tabletop, there is a normal force from the tabletop through the object. There is a string attached to m_1 so there is a tension from the heavier block to the lighter. In the diagram for m_2 we show the tension of the string acting on it which is the same as on m_1 but opposite in direction. It also has weight and is on a surface. Both objects should have the same acceleration since they are connected by a string. We will assume that they will accelerate to the right.



For each block, we break the forces and acceleration into components

Left Block	**Right Block**
	$\mathbf{F} = \boldsymbol{i}F\,cos(37°) + \boldsymbol{j}F\,sin(37°)$
$\mathbf{T} = \boldsymbol{i}T + \boldsymbol{j}0$	$\mathbf{T} = -\boldsymbol{i}T + \boldsymbol{j}0$
$\mathbf{N}_2 = \boldsymbol{i}0 - \boldsymbol{j}N_2$	$\mathbf{N}_1 = \boldsymbol{i}0 - \boldsymbol{j}N_1$
$\mathbf{W}_2 = \boldsymbol{i}0 - \boldsymbol{j}m_2g$	$\mathbf{W}_1 = \boldsymbol{i}0 - \boldsymbol{j}m_1g$
$\mathbf{a}_2 = \boldsymbol{i}a + \boldsymbol{j}0$	$\mathbf{a}_1 = \boldsymbol{i}a + \boldsymbol{j}0$

We apply Newton's Second Law separately to the ***i*** and ***j*** components for each block.

Object	***i***	***j***
	$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
Right Block	$F\,cos(37°) - T = m_1a$	$N_1 - m_1g + F\,sin(37°) = 0$
Left Block	$T = m_2a$	$N_2 - m_2g = 0$

Thus the normal forces are:

$$N_1 = m_1g - F\,sin(50°) = (1.50)(9.81) - 8.00\,sin(37°) = 9.90\text{ N},$$

and

$$N_2 = m_2g = (1.20)(9.81) = 11.77\text{ N}.$$

The fact that both normals are positive indicates that the blocks do not lose contact with the tabletop and are therefore not accelerating in the y direction. This is consistent with our assumption about the acceleration being only in the x direction.

To find the tension T we need to eliminate the acceleration a from the pair of x equations. The second gives

$$a = T / m_2.$$

We substitute this into the first equation

$$F\cos(37°) - T = m_1 \, [T / m_2] \,.$$

We put terms with T on one side and everything else on the other side

$$F\cos(37°) = m_1 \, [T / m_2] + T \,.$$

We collect T and find

$$F\cos(37°) = T \, [m_1/m_2 + 1] \,.$$

So we have

$$T = F\cos(37°)/[m_1/m_2 + 1] = 8.00\cos(37°)/[1.50/1.20 + 1] = 2.84 \text{ N} \,.$$

So the tension in the string between the blocks is 2.84 N.

The acceleration of the blocks is

$$a = T/m_2 = 2.84 / 1.20 = 2.37 \text{ m/s}^2 \,.$$

This is positive so our assumption about the direction of the acceleration was correct.

The acceleration is:

$$a = F\cos(50°)/m = 1.00\cos(50°)/0.400 = 1.61 \text{ m/s}^2 \,.$$

Since the acceleration is positive, our assumption about its direction was correct. The fact that the normal is positive, i.e. that the object has not lost contact with the tabletop, tells us that there is no acceleration in the y direction.

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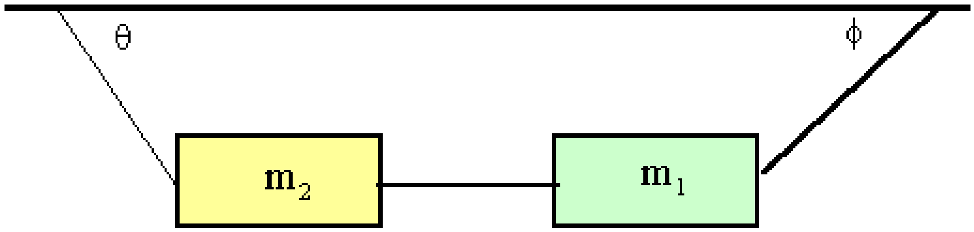
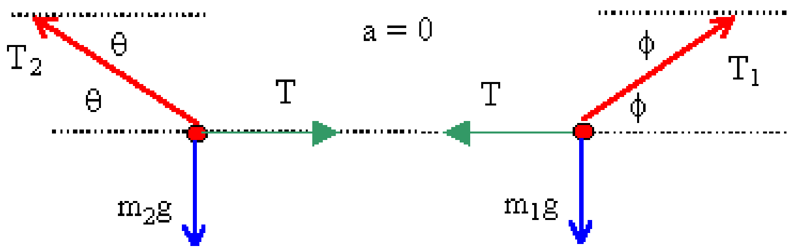
13. Two blocks are connected by strings to the ceiling as shown in the diagram below. The left block has mass m_2 = 3.35 kg. The mass of the right block m_1 is unknown. The strings make angles θ = 60º and φ = 45º with the ceiling. Find the tension in the level string connecting the blocks. Find m_1.



This problem deals with a force, tension, which suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each block. In the diagram for m_1 we show two tension forces, one for each string. As well, since the object has mass, it has weight. In the diagram for m_2 we again show two tension forces and its weight. We would not expect the blocks to move so we will assume that there is no acceleration.



In the free-body diagrams, we also use a little geometry to find more useful angles to work with.

For each block, we break the forces and acceleration into components

Left Block	Right Block
$\mathbf{T}_2 = -\boldsymbol{i}T_2 cos(\theta) + \boldsymbol{j}T_2 sin(\theta)$	$\mathbf{T}_1 = \boldsymbol{i}T_1 cos(\varphi) + \boldsymbol{j}T_1 sin(\varphi)$
$\mathbf{T} = \boldsymbol{i}T + \boldsymbol{j}0$	$\mathbf{T} = -\boldsymbol{i}T + \boldsymbol{j}0$
$\mathbf{W}_2 = \boldsymbol{i}0 - \boldsymbol{j}m_2 g$	$\mathbf{W}_1 = \boldsymbol{i}0 - \boldsymbol{j}m_1 g$
$\mathbf{a}_2 = \boldsymbol{i}0 + \boldsymbol{j}0$	$\mathbf{a}_1 = \boldsymbol{i}0 + \boldsymbol{j}0$

Next we apply Newton's Second Law separately to the ***i*** and ***j*** components for each block:

Object	***i***	***j***
	$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
Left Block	$T_2\, cos(\theta) - T = 0$	$T_2 sin(\theta) - m_2 g = 0$
Right Block	$T_1\, cos(\varphi) - T = 0$	$T_1 sin(\varphi) - m_1 g = 0$

Considering the equations for the left block, we see that

$$T = T_2\, cos(\theta)\ .$$

We don't know T_2 however. But we do have another equation with T_2 in it and it indicates that

$$T_2 = m_2 g \,/\, sin(\theta)\ .$$

Hence the tension in the string between the blocks is

$$T = T_2\,cos(\theta) = m_2 g\,cos(\theta) / sin(\theta) = 3.35(9.81)cos(60^o)/sin(60^o) = 18.97\text{ N} .$$

From the equations for the right block we see that

$$m_1 = T_1 sin(\varphi)/g .$$

However, we don't know T_1. We do have a second equation that tells us that

$$T_1 = T / cos(\varphi) .$$

Combining these two results yields

$$m_1 = [T / cos(\varphi)]\, sin(\varphi)/g = (18.97)sin(45^o) / 9.81cos(45^o) = 1.93\text{ kg.}$$

The first block has a mass of 1.93 kg.

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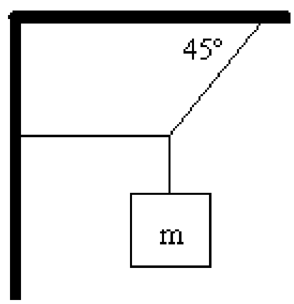
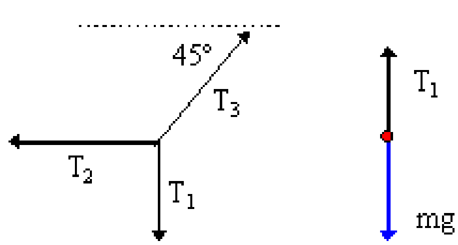
14. In the diagram below, the mass of the object is 50.0 kg. What are the tensions in the ropes?



Since this problem deals with forces, the tensions, and we known that the acceleration is zero since nothing is moving, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block and the knot. We need a FBD for the knot since this is where the tensions all meet. A knot is massless since we are dealing with massless ropes. The block has mass, so it has weight. Each rope represents a different tension.



For each object, we break the forces and acceleration into components

knot	***block***
$\mathbf{T_3} = \boldsymbol{i}T_3\,sin\,(45^\circ) + \boldsymbol{j}T_3\,cos\,(45^\circ)$	
$\mathbf{T_2} = -\boldsymbol{i}T + \boldsymbol{j}0$	$\mathbf{T_1} = \boldsymbol{i}0 + \boldsymbol{j}T_1$
	$\mathbf{W} = \boldsymbol{i}0 - \boldsymbol{j}mg$

$\mathbf{T}_1 = \boldsymbol{i}0 - \boldsymbol{j}\mathrm{T}_1$

$\mathbf{a}_{knot} = \boldsymbol{i}0 + \boldsymbol{j}0$ $\mathbf{a}_{block} = \boldsymbol{i}0 + \boldsymbol{j}0$

Applying Newton's Second law:

	knot	***block***
$\Sigma F_y = ma_y$	$\Sigma F_y = ma_y$	$\Sigma F_y = ma_y$
$T_2 - T_3 \cos(45°) = 0$	$T_3 \sin(45°) - T_1 = 0$	$T_1 - mg = 0$

From the above we see that $T_1 = mg = 490.5$ N, $T_3 = T_1/\sin(45°) = 693.7$, and $T_2 = T_3 \cos(45°) =$ 490.5 N.

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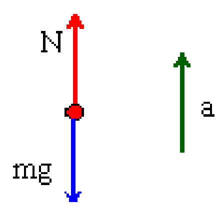
15. The apparent weight of a person in an elevator is 7/8th of his actual weight. What is the acceleration (including the direction) of the elevator?

This problem deals with a force, weight, and acceleration. That suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). Since the person has mass, he has weight. Since the person touches the floor of the elevator, there is a normal force from the floor. We will assume that the person will accelerate upwards.



Applying Newton's Second Law yields,

$$\Sigma F_y = ma_y$$
$$N - mg = ma.$$

This isn't enough to solve the problem, since we have one equation but two unknowns, N and a.

However, the apparent weight is just N, so the first sentence of the problem is N = (7/8)mg. Now we can find a,

$$a = (N - mg) / m = [(7/8)mg - mg]/m = -g/8 = -1.23 \text{ m/s}^2.$$

The minus sign indicates that, contrary to our initial assumption, the elevator accelerates downwards at 1.23 m/s^2.

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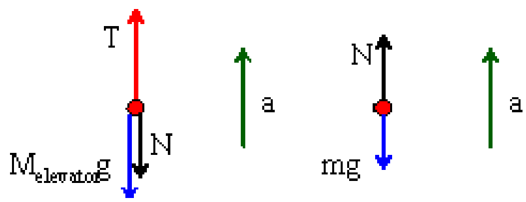
16. A 1200-kg elevator is carrying an 80.0-kg passenger. Calculate the acceleration of the elevator (and hence of the partner) if the tension in the cable pulling the elevator is (a) 15,000 N, (b) 12,557 N, and (c) 10,000 N. What is the apparent weight of the passenger in each case. Assume g = 9.81 m/s^2.

This problem deals with a force, tension, and acceleration. That suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each object, the person and the elevator. Each has mass, so each has weight. Since the person stands on the elevator, there is a an equal but opposite normal force on each. The rope, and thus the tension, acts directly only on the elevator. Both object have the same acceleration if they remain in contact. We will assume that the acceleration is upwards.



Applying Newton's Second law:

elevator	***passenger***
$\Sigma F_y = ma_y$	$\Sigma F_y = ma_y$
$T - Mg - N = +Ma$	$N - mg = ma$

So we have two equation in two unknowns, N and a. First we add the two equations together to get $T - (M+m)g = (M+m)a$. This our equation for the acceleration is $a = T / (M+m) - g$. The apparent weight, that is the normal force acting on the passenger, is given by $N = mg + ma$. When we plug in the numbers we find:

	acceleration	***apparent weight***
(a)	1.91 m/s^2	938 N
(b)	0	785 N
(c)	−2.00 m/s^2	625 N

The negative acceleration in part (c) indicates that the elevator is actually accelerating downwards.

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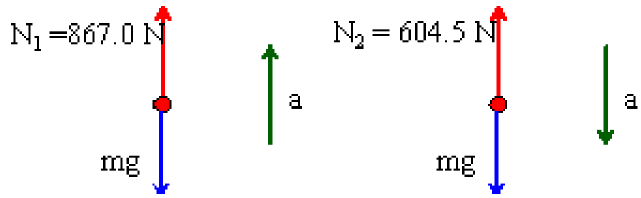
17. A person is standing on a weigh scale in an elevator. When the elevator is accelerating upwards with constant acceleration a the scale reads 867.0 N. When the elevator is accelerating downwards with acceleration a, the scale reads 604.5 N. Determine the acceleration a, the true weight of the person, and the mass of the person.

This problem deals with a force, the scale reading, and acceleration. That suggests we should apply

Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). Since the person has mass, he has weight. Since the person touches the scale, there is a normal force from the scale. The scale reading is a measure of this normal force. Since there are two cases, we draw an FBD for each case.



up	***down***
$\Sigma F_y = ma_y$	$\Sigma F_y = ma_y$
$N_2 - mg = +ma$	$N_2 - mg = -ma$

So we have two equation in two unknowns, m and a. First we add the two equations together to get $N_1 - mg + N_2 - mg = 0$. This becomes $m = (N_1 + N_2) / 2g = 75.0$ kg.

Using the first equation, we get $a = (N_2 - mg) / m = 1.75$ m/s^2.

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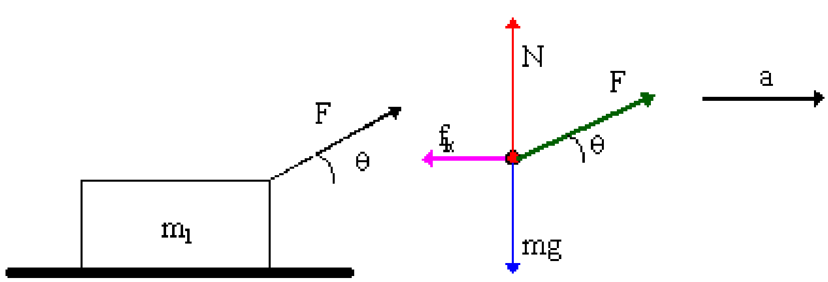
18. In the diagram below, m = 10.0 kg, θ = 25.0°, $\mu_s = 0.70$, and $\mu_k = 0.50$. What is the frictional force if **F** is (a) 25 N, (b) 65 N, and (c) 100 N? What is the acceleration in each case?

Since this problem deals with forces, the applied force and friction, and we are asked about the acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block. We will set up the axes along the incline. The block has mass, so it has weight. There is a normal for the block from the incline. There is friction but it is not immediately clear if it is static or kinetic.

Let's assume that the block is moving so that there is kinetic friction.



Next we break all the vectors into components:

$\mathbf{F} = \boldsymbol{i}F\,cos(\theta) + \boldsymbol{j}F\,sin(\theta)$,

$\mathbf{N} = \boldsymbol{i}0 + \boldsymbol{j}N$,

$\mathbf{f_k} = -\boldsymbol{i}f_k + \boldsymbol{j}0$

$\mathbf{W} = \boldsymbol{i}0 - \boldsymbol{j}mg$, and

$\mathbf{a} = \boldsymbol{i}a + \boldsymbol{j}0$.

We apply Newton's Second Law separately to the ***i*** and ***j*** components. Thus $\Sigma F_x = ma_x$ yields

$$F\cos(\theta) - f_k = ma$$

And $\Sigma F_y = ma_y$ produces

$$N + F\sin(\theta) - mg = 0$$

By definition, $f_k = \mu_k N$.

From the equation in the second column, we have $N = mg - F\sin(\theta)$. Thus $f_k = \mu_k[mg - F\sin(\theta)]$. Putting this into the first equation yields

$$F\cos(\theta) - \mu_k[mg - F\sin(\theta)] = ma\ .$$

Rearranging and solving for a yields,

$$a = (F/m)[\cos(\theta) + \mu_k \sin(\theta)] - \mu_k g\ .$$

Solving for the three cases:

	F	***a***
	(N)	(m/s^2)
(a)	25	−2.11
(b)	65	+2.36
(c)	100	+6.27

Clearly, our answer to part (a) violates our assumption that the friction is kinetic. Hence the friction must have been static and a = 0 for case (a).

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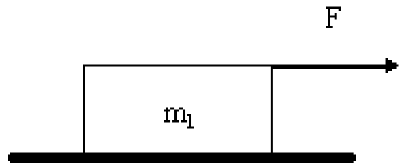
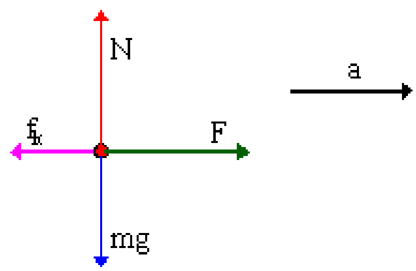
19. (a) If m = 10.0 kg, $\mu_k = 0.3$, and **F** = 50 N **x**, find **a**.

(b) If m = 10.0kg, **a** = 2.5m/s^2 **x** and **F** = 75 N **x**, find μ_k.



Since this problem deals with forces, friction and **F**, and acceleration is involved, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block. We will set up the axes along the direction of acceleration. The block has mass, so it has weight. There is a normal for the block from the table top. Presumably the block is moving so there is kinetic friction opposed to the motion.



i	***j***
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$F - f_k = ma$	$N - mg = 0$

By definition, $f_k = \mu_k N$. From the equation in the second column, we have $N = mg$. Thus $f_k = \mu_k mg$. Putting this into the first equation yields

$$F - \mu_k mg = ma\ .$$

(a) Solving for the acceleration

$$a = F/m - \mu_k g = (50.0\ \mathrm{N})/(10.0\ \mathrm{kg}) - 0.3(9.81\ \mathrm{m/s^2}) = 2.06\ \mathrm{m/s^2}\ .$$

(b) Solving for μ_k

$$\mu_k = F/mg - a/g = (75\ \mathrm{n})/(10.0\ \mathrm{kg} \times\ 9.81\ \mathrm{m/s^2}) - 2.5/9.81 = 0.51\ .$$

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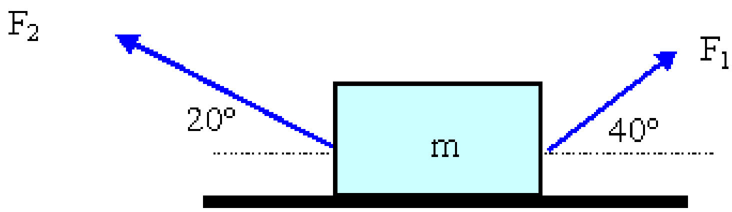
20. A 15.0-kg block sitting on a floor is pulled by two forces as shown in the diagram below. The block does not move. Find the type, magnitude, and direction of the frictional force acting on the block. Note F_1 = 48.0 N and F_2 = 42.0 N.



Since the block does not move, we know that we are dealing with static friction. It can either be to the right or to the left in the diagram. To determine which, we must examine the magnitude of each force's x component:

$$F_{1x} = F_1 cos(40^o) = 36.77 \text{ N},$$

and

$$F_{2x} = F_2 cos(20^o) = 39.47 \text{ N}.$$

Since F_2x is bigger, the frictional force must be directed to the right. Note that nothing in the problem says that the object is about to slip, so we are not dealing with f_s^{max}. Considering the forces in the x direction

$$-F_{2x} + F_{1x} + f_s = 0 .$$

So

$$f_s = F_{2x} - F_{1x} = 39.47 - 36.77 \text{ N} = 2.70 \text{ N} .$$

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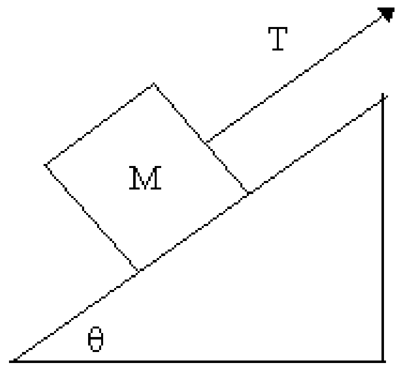
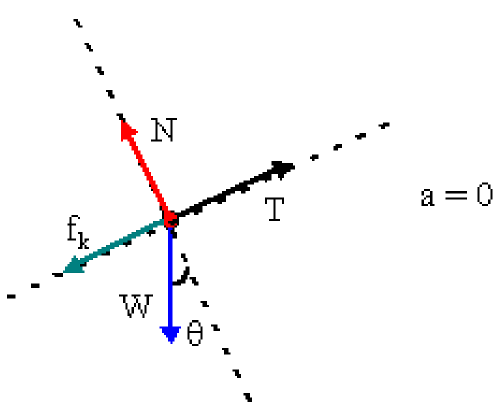
21. A block weighing 100 N is placed on a plane with slope angle 30.0° and is pulled by a rope which is parallel to the incline and under tension T as shown below. The coefficient of static friction is 0.52, and the coefficient of kinetic friction is 0.20.
(a) Find the tension T for which the block moves up the plane at constant speed, once it has been set in motion.
(b) Find the tension T for which the block moves down the plane at constant speed, once it has been set in motion.
(c) What is the maximum value of T for which the block will remain at rest?
(d) What is the minimum value of T for which the block will remain at rest?



Since this problem deals with forces, weight and friction, and we known that the acceleration is zero since the velocity is constant or zero, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for the block. We will set up the axes along the incline. The tension in the rope points along the rope away from the block. The block has mass, so it has weight. There is a normal for the block from the incline. There is friction but it is not immediately clear if it is static or kinetic.

(a) Here the block is moving up the incline, so we have kinetic friction down the incline.



Next we break all the vectors into components:

$\mathbf{T} = \boldsymbol{i}\mathrm{T} + \boldsymbol{j}0$,
$\mathbf{N} = \boldsymbol{i}0 + \boldsymbol{j}\mathrm{N}$,
$\mathbf{f_k} = -\boldsymbol{i}f_k + \boldsymbol{j}0$
$\mathbf{W} = -\boldsymbol{i}\mathrm{W}\sin(\theta) - \boldsymbol{j}\mathrm{W}\cos(\theta)$,
and

$\mathbf{a} = i0 + \boldsymbol{j}0$.

We apply Newton's Second Law separately to the $\boldsymbol{i}$ and $\boldsymbol{j}$ components. Thus $\Sigma F_x = ma_x$ yields

$$T - f_k - W\sin(\theta) = 0$$

And $\Sigma F_y = ma_y$ produces

$$N - W\cos(\theta) = 0$$

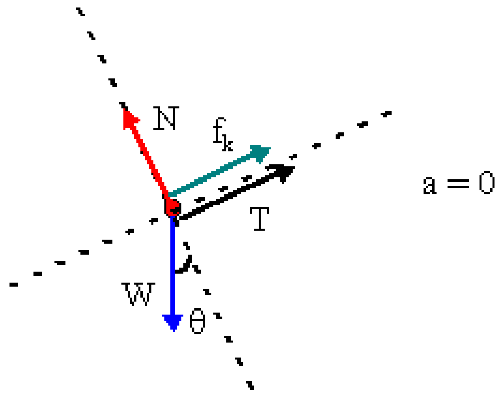
By definition, $f_k = \mu_k N$. From the equation in the second column, we have $N = W\cos(\theta)$. Thus $f_k = \mu_k W\cos(\theta)$. Putting this into the first equation yields

$$T - \mu_k W\cos(\theta) - W\sin(\theta) = 0 .$$

Rearranging gives an expression for T,

$$T = W[\sin(\theta) + \mu_k\cos(\theta)] = (100\text{ N})[\sin(30°)+0.20\cos(30°)] = 67.3\text{ N} .$$

(b) Here the block is moving down the incline, so the kinetic friction is up the incline.



i	*j*
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$T + f_k - W\sin(\theta) = 0$	$N - W\cos(\theta) = 0$

By definition, $f_k = \mu_k N$. From the equation in the second column, we have $N = W\cos(\theta)$. Thus $f_k = \mu_k W\cos(\theta)$. Putting this into the first equation yields,

$$T + \mu_k W\cos(\theta) - W\sin(\theta) = 0 .$$

Rearranging gives an expression for T,

$$T = W[\sin(\theta\ ) - \mu_k\cos(\theta)] = (100\text{ N})[\sin(30°)-0.20\cos(30°)] = 32.7\text{ N} .$$

(c) The block doesn't move, so we must be dealing with static friction. If we pull too hard the block will move up the incline, so the static friction must point down the incline. So we only need to redo (a) with static friction. Nothing will change except the coefficient of friction, so are results will be the same but with μ_k replaced by μ_s. Thus our results will be

$$T_{max} = W[\sin(\theta) + \mu_s\cos(\theta)] = (100\text{ N})[\sin(30°)+0.52\cos(30°)] = 95.0\text{ N} ,$$

The block doesn't move, so we must be dealing with static friction. If we don't pull hard enough the block will slide down the incline, so the static friction must point up the incline. So we only need to redo (b) with static friction. Nothing will change except the coefficient of friction, so are results will be the same but with μ_k replaced by μ_s. Thus our results will be

$$T_{min} = W[\sin(\theta)- \mu_s\cos(\theta)] = (100\text{ N})[\sin(30°)-0.52\cos(30°)] = 5.0\text{ N} .$$

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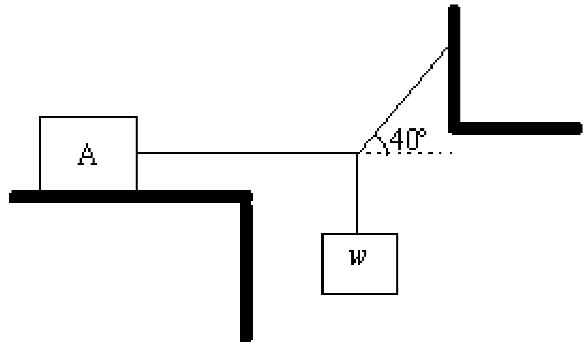
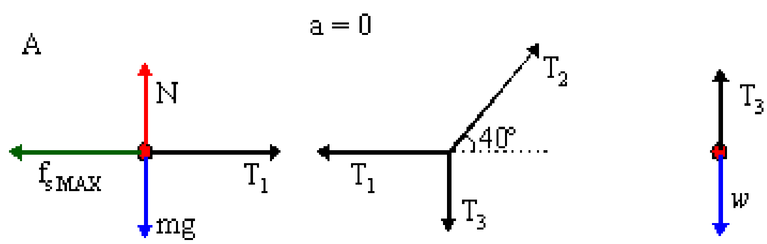
22. In the diagram below, block A weighs 100 N. The coefficient of static friction between the block and the surface on which it rests is 0.40. Find the maximum weight *w* for which the system will remain in equilibrium.



A system in equilibrium does not accelerate, so a = 0. Since this problem deals with forces, friction and weight, and we known that the acceleration is zero, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD) for each block and the knot. We need a FBD for the knot since this is where the tensions all meet. A knot is massless since we are dealing with massless ropes. Each block has mass, so each has weight. Each rope represents a different tension. Block A is on a table so there is a normal force from the table through A. If there were no friction, Block A would move forward. We deduce that the maximum static friction points left.



For each object, we break the forces and acceleration into components

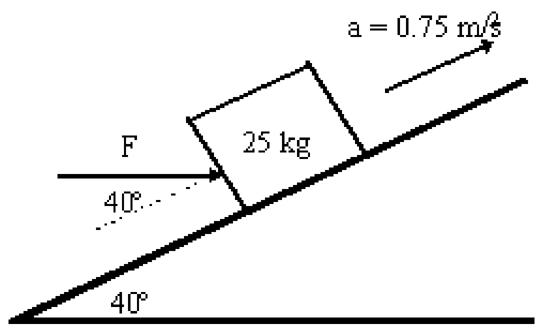
block A	***knot***	***weight***
$\mathbf{T}_1 = \boldsymbol{i}0 + \boldsymbol{j}T_1$	$\mathbf{T}_1 = \boldsymbol{i}0 - \boldsymbol{j}T_1$	
$\mathbf{f} = -\boldsymbol{i}f_{sMAX} + \boldsymbol{j}0$	$\mathbf{T}_2 = \boldsymbol{i}T_2 cos(40°) + \boldsymbol{j}T_2 sin(40°)$	
$\mathbf{N} = \boldsymbol{i}0 + \boldsymbol{j}N$	$\mathbf{T}_3 = \boldsymbol{i}0 - \boldsymbol{j}T_3$	$\mathbf{T}_3 = \boldsymbol{i}0 + \boldsymbol{j}T_3$
$\mathbf{W}_A = \boldsymbol{i}0 - \boldsymbol{j}mg$		$\mathbf{W} = \boldsymbol{i}0 - \boldsymbol{j}w$
	$\mathbf{a} = \boldsymbol{i}0 + \boldsymbol{j}0$	

Applying Newton's Second law:

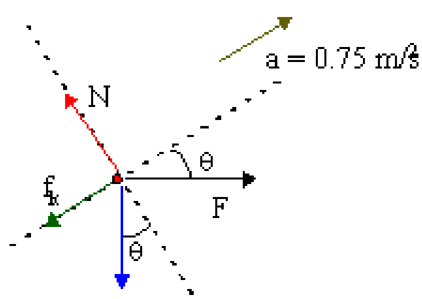
Block A		***knot***		***w***
i	***j***	***i***	***j***	***j***
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$	$\Sigma F_y = ma_y$
$T_1 - f_{s\,MAX} = 0$	$N - mg = 0$	$T_2 cos(40°) - T_1 = 0$	$T_2 sin(40°) - T_3 = 0$	$T_3 - w = 0$

Besides our equations in the last row above, we also have the definition $f_{s\,MAX} = \mu N$. Our equation in the second column tells us that N = mg = 100 N, so we know $f_{s\,MAX} = \mu N = 0.40100$ N = 40 N. The first equation tells us that $T_1 = f_{s\,MAX} = 40$ N. The third equation yields, $T_2 = T_1 / cos(40°) = 52.2$ N. The fourth equation yields $T_3 = T_2 sin(40°) = 33.6$ N. The equation in the last column indicates that $w = T_3 =$ 33.6 N. So the maximum weight of the hanging block is 33.6 N.

23. When a force of 500 N pushes on a 25-kg box as shown in the figure below, the acceleration of the box up the incline is $0.75\ m/s^2$. Find the coefficient of sliding friction between box and incline.



Since this problem deals with forces, F, weight, and friction, and we are given the acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). We are told that there is an applied force F acting horizontally. Since the block has mass, it has weight. It is on the incline, so there is a normal. The object is accelerating, so we must be dealing with kinetic friction which will be opposite to the direction of motion. Note that we will choose a set of axes such that one axis points along the incline.



Next we break all the vectors into components:

$\mathbf{F} = \boldsymbol{i}F\,cos(\theta) - \boldsymbol{j}F\,sin(\theta)$,
$\mathbf{N} = \boldsymbol{i}0 + \boldsymbol{j}N$,
$\mathbf{f_k} = -\boldsymbol{i}f_k + \boldsymbol{j}0$
$\mathbf{W} = -\boldsymbol{i}mg\,sin(\theta) - \boldsymbol{j}mg\,cos(\theta)$, and

$\mathbf{a} = \boldsymbol{i}a + \boldsymbol{j}0$

We apply Newton's Second Law separately to the $\boldsymbol{i}$ and $\boldsymbol{j}$ components. Thus $\Sigma F_x = ma_x$ yields

$$F\,cos(\theta) - f_k - mg\,sin(\theta) = ma$$

And $\Sigma F_y = ma_y$ produces

$$N - mg\,cos(\theta) - F\,sin(\theta) = 0$$

Since we know $f_k = \mu_k N$, and the second equation says that $N = mg\,cos(\theta) + F\,sin(\theta)$, we have $f_k = \mu_k[mg\,cos(\theta) + F\,sin(\theta)]$. Substituting this into the first equation yields,

$$F\,cos(\theta) - \mu_k[mg\,cos(\theta) + F\,sin(\theta)] - mg\,sin(\theta) = ma\ .$$

Isolating the term involving μ_k yields

$$-\mu_k[mg\,sin(\theta) + F\,sin(\theta)] = -F\,cos(\theta) + mg\,sin(\theta) + ma\ .$$

So we find that

$$\begin{aligned}\mu_k &= \frac{F\cos(\theta) - mg\sin(\theta) - ma}{mg\cos(\theta) + F\sin(\theta)} \\ &= \frac{500\cos(40°) - (25)(9.81)\sin(40°) - 25(0.75)}{(25)(9.81)\cos(40°) + 500\sin(40°)} \\ &= \frac{383.02 - 157.647 - 18.75}{187.87 + 321.39} \\ &= \frac{206.63}{509.27} \\ &= 0.406\end{aligned}$$

The kinetic coefficient of friction is 0.41 .

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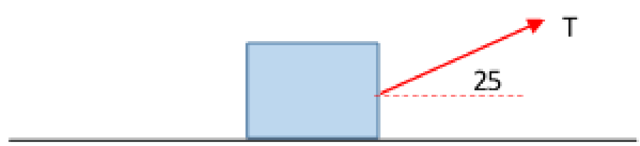
24. A 20.0-kg block sits on a horizontal surface where the coefficients of friction are $\mu_s = 0.35$ and $\mu_k = 0.25$. A string with a tension of 50.0 N pulls the block at 25.0° above horizontal. Will the block move? If it does slip, what will be its acceleration?



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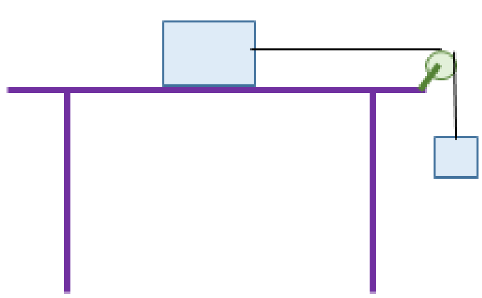
25. Two blocks are connected by a string. One of the blocks has a mass of 10.0 kg and is on a table where the coefficients of friction are $\mu_s = 0.15$ and $\mu_k = 0.10$. The string between the blocks runs over a massless pulley. The second 4.0-kg block is held and then gently released. Do the blocks move? If so, what is the acceleration of the blocks?



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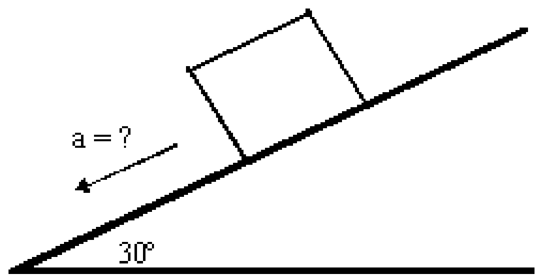
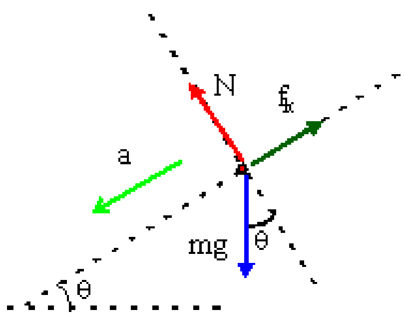
26. A 20-kg box sits on an incline as shown below. The coefficient of kinetic friction between the block and incline is 0.30. Find the acceleration of the box down the incline.



Since this problem deals with a force, friction, and we are asked about the acceleration, that suggests we should apply Newton's Second Law. To apply Newton's Second Law we draw a free-body diagram (FBD). Since the box has mass, it has weight. It is on the incline, so there is a normal. For a moving object, we must be dealing with kinetic friction. Kinetic friction opposes the motion, so it will point up the incline. Note that we should choose a set of axes such that one axis points along the acceleration as this simplifies our equations.



Next we break all the vectors into components:

$\mathbf{N} = \boldsymbol{i}0 + \boldsymbol{j}N$,
$\mathbf{f_k} = \boldsymbol{i}f_k + \boldsymbol{j}0$
$\mathbf{W} = -\boldsymbol{i}mg\,sin(\theta) - \boldsymbol{j}mg\,cos(\theta)$, and

$\mathbf{a} = -\boldsymbol{i}a + \boldsymbol{j}0$.

We apply Newton's Second Law separately to the $\boldsymbol{i}$ and $\boldsymbol{j}$ components. Thus $\Sigma F_x = ma_x$ yields

$$f_k - mg\,sin(\theta) = -ma$$

And $\Sigma F_y = ma_y$ produces

$$N - mg\,cos(\theta) = 0$$

Since we know $f_k = \mu_k N$, and the second equation says that $N = mg\,cos(\theta)$, we have $f_k = \mu_k mg\,cos(\theta)$. Substituting this into the first equation yields,

$$\mu_k mg\,cos(\theta) - mg\,sin(\theta) = -ma\ .$$

Solving for a yields

$$a = g\,sin(\theta) - \mu_k mg\,cos(\theta) = (9.81\ m/s^2)[sin(30°)-0.30cos(30°)] = 2.36\ m/s^2.$$

The box accelerates down the incline at 2.36 m/s^2.

Questions? mike.coombes@kwantlen.ca

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