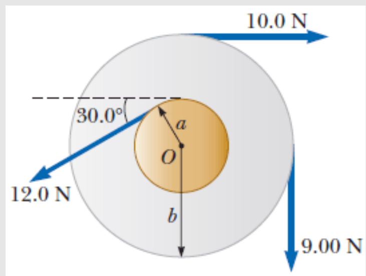
PHYSICS 111 HOMEWORK SOLUTION #10

April 8, 2013

0.1

Find the net torque on the wheel in the figure below about the axle through O, taking a = 16.0 cm and b = 30.0 cm.

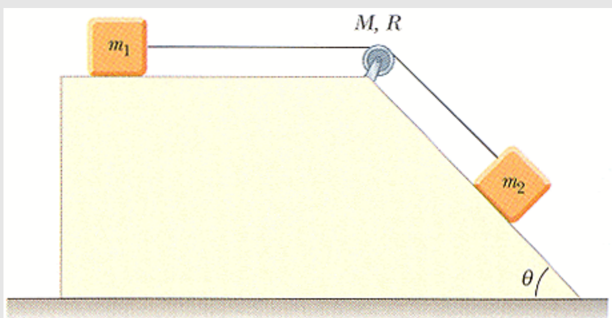


A torque that's produced by a force can be calculated from the expression: $\tau = F \cdot r \cdot \sin\theta$. All the forces acting on the wheel are perpendicular to the rotation axis with $\theta = 90°$ (the 30° is irrelevant in this picture). By taking the clockwise direction as positive, We should have:

$$\begin{aligned}\tau &= 10 \times 0.30 + 9 \times 0.30 - 12 \times 0.16 \\ &= 3.78 \text{ N.M}\end{aligned}$$

0.2

A block of mass $m_1 = 1.70$ kg and a block of mass $m_2 = 6.20$ kg are connected by a massless string over a pulley in the shape of a solid disk having radius R = 0.250 m and mass M = 10.0 kg. The fixed, wedge-shaped ramp makes an angle of $\theta = 30.0°$ as shown in the figure. The coefficient of kinetic friction is 0.360 for both blocks.

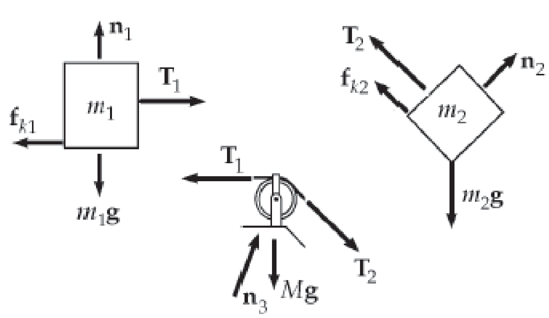


- a) Draw force diagrams of both blocks and of the pulley.
- b) Determine the acceleration of the two blocks. (Enter the magnitude of the acceleration.)
- c) Determine the tensions in the string on both sides of the pulley.

a)

Force diagrams:

(1)



b)

The two blocks will move with the same acceleration a

- Newtons 2^{nd} law for block m_1 gives :

$$\begin{aligned} T_1 - f_{k1} &= m_1 a \\ T_1 &= \mu m_1 g + m_1 a \end{aligned}$$

- Newtons 2^{nd} law for block m_2 gives :

$$\begin{aligned} -T_2 - f_{k2} + m_2 g \sin\theta &= m_2 a \\ T_2 &= m_2 g \sin\theta - \mu m_2 g \cos\theta - m_2 a \end{aligned}$$

- For the pulley (considered a disk with moment of inertia $\frac{MR^2}{2}$):

$$\begin{aligned} \tau_{net} &= I\alpha \\ T_2 R - T_1 R &= I\frac{a}{R} \\ a &= \frac{R^2}{\frac{MR^2}{2}}(T_2 - T_1) \end{aligned}$$

Rearranging should give:

$$\begin{aligned} a &= \frac{2}{M}(m_2 g \sin\theta - \mu m_2 g \cos\theta - \mu m_1 g) - \frac{2}{M}(m_1 + m_2)a \\ &= \frac{2g(m_2 \sin\theta - \mu m_2 \cos\theta - \mu m_1)}{M + 2m_1 + 2m_2} \\ &= \frac{2 \times 9.81(6.20 \sin 30 - 0.360 \times 1.70 \times \cos 30 - 0.360 \times 1.70)}{10 + 2 \times 1.70 + 2 \times 6.20} \\ &= 0.422 \text{ m/s}^2 \end{aligned}$$

b)

To the LEFT of the pulley, the string tension is T_1:

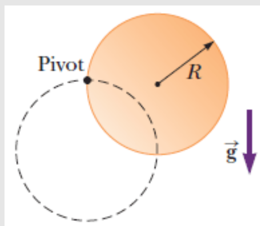
$$\begin{aligned} T_1 &= \mu m_1 g + m_1 a \\ &= 1.70(0.360 \times 9.81 + 0.422) \\ &= 6.72 \text{ N} \end{aligned}$$

To the RIGHT of the pulley, the string tension is T_2:

$$\begin{aligned} T_2 &= m_2 g \sin\theta - \mu m_2 g \cos\theta - m_2 a \\ &= 6.20(9.81 \sin 30 - 0.360 \times 9.81 \times \cos 30 - 0.422) \\ &= 8.83 \text{ N} \end{aligned}$$

0.3

A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (see figure below). The disk is released from rest in the position shown by the copper-colored circle.



- a)What is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (Use any variable or symbol stated above along with the following as necessary: g.)
- b) What is the speed of the lowest point on the disk in the dashed position? (Use any variable or symbol stated above along with the following as necessary: g.)
- c) Repeat part (a) using a uniform hoop of mass M. (Use any variable or symbol stated above along with the following as necessary: g.)

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a)

Let's determine the moment of inertia of the disk about the axis of rotation which is going through a point on the rim.

- About the center the Moment of inertia is $\frac{1}{2}MR^2$
- Parallel axis theorem will give the moment of inertia about the axis in consideration:

$$\begin{aligned} I &= \frac{1}{2}MR^2 + MR^2 \\ &= \frac{3}{2}MR^2 \end{aligned}$$

A frictionless rotation implies energy conservation which can be written as:

$$\begin{aligned}
\Delta E &= 0 \\
\Delta Ki &= -\Delta P \\
\frac{1}{2}I\omega^2 - 0 &= -(-MgR) \\
\frac{1}{2} \times \frac{3}{2}MR^2 \times \frac{V^2}{R^2} &= MgR \\
V^2 &= \frac{4}{3}gR \\
V &= 2\sqrt{\frac{Rg}{3}}
\end{aligned}$$

b)

The lowest point on the disk and the center have the same angular speed but different speeds (located at different distances from the rotation axis)

$$\begin{aligned}
\omega' &= \omega \\
\frac{V'}{2R} &= \frac{V}{R} \\
V' &= 2V \\
&= 4\sqrt{\frac{Rg}{3}}
\end{aligned}$$

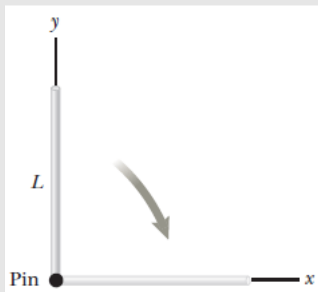
c)

If it was a hoop instead of a disk, the only change will be on the moment of inertia which is MR^2 about the center and $MR^2 + MR^2 = 2MR^2$ about the axis through the point on the rim:

$$\begin{aligned}
\frac{1}{2}I\omega^2 - 0 &= -(-MgR) \\
\frac{1}{2} \times 2MR^2 \times \frac{V^2}{R^2} &= MgR \\
V^2 &= gR \\
V &= \sqrt{Rg}
\end{aligned}$$

0.4

A uniform solid disk of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (see figure below). The disk is released from rest in the position shown by the copper-colored circle.



- a)At the instant the rod is horizontal, find its angular speed. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)
- b)At the instant the rod is horizontal, find the magnitude of its angular acceleration. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)
- c)At the instant the rod is horizontal, find the x and y components of the acceleration of its center of mass. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)
- d) At the instant the rod is horizontal, find the components of the reaction force at the pivot. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)

a)

The center of mass of the rod will change its height by $\frac{L}{2}$ as the rod moves from vertical to horizontal. Using energy conservation and taking into account that the moment of inertia of the rod about its end is $\frac{mL^2}{3}$, we should have :

$$\begin{aligned}
\frac{1}{2}I\omega^2 - 0 &= mg\frac{L}{2} \\
\frac{1}{2}\frac{mL^2}{3}\omega^2 &= mg\frac{L}{2} \\
\omega^2 &= \frac{3g}{2} \\
\omega &= \sqrt{\frac{3g}{L}}
\end{aligned}$$

b)

In Newton 2^{nd} law for rotation, only gravity will be involved since the force on the axis contributes zero:

$$\begin{aligned}
\sum \tau &= I\alpha \\
Mg\frac{L}{2} &= \frac{mL^2}{3}\alpha \\
\alpha &= \sqrt{\frac{3g}{2L}}
\end{aligned}$$

c)

When the rod is horizontal, its overall acceleration $\vec{a} = a_x\vec{i} + a_y\vec{j}$. It's easy to see that $a_x = -a_r$ and $a_y = -a_t$, where a_r and a_t are the radial acceleration and tangential acceleration respectively.

$$\begin{aligned}
a_x &= -a_r \\
&= -\frac{V^2}{r} \\
&= -r\omega^2 \\
&= -\frac{L}{2}\frac{3g}{L} \\
&= -\frac{3g}{2}
\end{aligned}$$

Similarly,

$$\begin{aligned}
a_y &= -a_t \\
&= -\frac{dV}{dt} \\
&= -r\frac{d\omega}{dt} \\
&= -r\alpha \\
&= -\frac{L}{2}\frac{3g}{2L}
\end{aligned}$$

$$= -\frac{3g}{4}$$

d)

The only two forces acting on the rod are gravity $m\vec{g}$ and the reaction force on the pivot $\vec{R}$.

$$M\vec{g} + \vec{R} = M\vec{a}$$

By projection on x-axis:

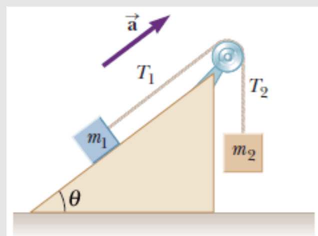
$$\begin{aligned} R_x &= Ma_x && (1) \\ &= M \times \frac{-3g}{2} && (2) \\ &= \frac{-3Mg}{2} && (3) \end{aligned}$$

On the y-axis:

$$\begin{aligned} R_y - Mg &= Ma_y \\ R_y &= M(g + a_y) \\ &= M(g - \frac{3Mg}{4}) \\ &= \frac{Mg}{4} \end{aligned}$$

0.5

As shown in the figure below, two blocks are connected by a string of negligible mass passing over a pulley of radius 0.220 m and moment of inertia I. The block on the frictionless incline is moving with a constant acceleration of magnitude a = 1.40 m/s2. (Let $m_1 = 13.5$ kg, $m_2 = 18.0$ kg, and $\theta = 37.0°$.) From this information, we wish to find the moment of inertia of the pulley.



- a)What analysis model is appropriate for the blocks?
- b)What analysis model is appropriate for the pulley?
- c)From the analysis model in part (a), find the tension T_1.
- d) From the analysis model in part (a), find the tension T_2.
- e) From the analysis model in part (b), find a symbolic expression for the moment of inertia of the pulley in terms of the tensions T_1 and T_2, the pulley radius r, and the acceleration a.
- f) Find the numerical value of the moment of inertia of the pulley.

a)

The two blocks are moving with the same constant acceleration. Therefore, the model that's appropriate is that of a particle under constant acceleration.

b)

The pulley is in rotation, its angular acceleration is constant because of the constant acceleration of the blocks. A particle under constant angular acceleration is the appropriate model.

c)

Newton 2^{nd} Law for block m_1 projected on the incline plane:

$$\begin{aligned} T_1 - m_1 g \sin\theta &= m_1 a \\ T_1 &= m_1(a + g\sin\theta) \\ &= 13.5(1.40 + 9.8 \times \sin 37) \\ &= 98.5 \text{ N} \end{aligned}$$

c)

Newton 2^{nd} Law for block m_1 projected on the vertical:

$$\begin{aligned} m_2 g - T_2 &= m_2 a \\ T_2 &= m_2(g - a) \\ &= 18(9.8 - 1.40) \\ &= 151 \text{ N} \end{aligned}$$

d)

Newton's Law for the rotating pulley; taking the clockwise direction as positive
:

$$\begin{aligned} T_2 r - T_1 r &= I\alpha = I\frac{a}{r} \\ I &= \frac{(T_2 - T_1)r^2}{a} \\ &= \frac{(151 - 98.5)0.22^2}{1.40} \\ &= 1.82 \text{ N.m}^2 \end{aligned}$$

0.6

A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height h.

- a)If they are released from rest and roll without slipping, which object reaches the bottom first?
- b)Verify your answer by calculating their speeds when they reach the bottom in terms of h. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)

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a)

It's the disk that will win the race.

b)

With no slipping taking place, the two objects will be in rotation and translation. Energy is conserved:

$$\begin{aligned} \Delta Ki &= -\Delta P \\ \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 &= Mgh \\ Mv^2 + I\frac{v^2}{R^2} &= 2Mgh \\ v &= \sqrt{\frac{2Mgh}{M + \frac{I}{R^2}}} \end{aligned}$$

For the disk $I = \frac{MR^2}{2}$ and for the hoop $I = MR^2$

We finally get: $v = \sqrt{\frac{4}{3}gh}$ for the disk and $v = \sqrt{gh}$ for the hoop. The disk is definitely winning the race if we compare the two values.

d)

Newton's Law for the rotating pulley; taking the clockwise direction as positive :

$$T_2 r - T_1 r = I\alpha = I\frac{a}{r}$$

$$\begin{aligned} I &= \frac{(T_2 - T_1)r^2}{a} \\ &= \frac{(151 - 98.5)0.22^2}{1.40} \\ &= 1.82 \text{ N.m}^2 \end{aligned}$$

0.7

A solid sphere is released from height h from the top of an incline making an angle θ with the horizontal.

- a)Calculate the speed of the sphere when it reaches the bottom of the incline in the case that it rolls without slipping. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)
- b)Calculate the speed of the sphere when it reaches the bottom of the incline in the case that it slides frictionlessly without rolling. (Use any variable or symbol stated above along with the following as necessary: g for the acceleration of gravity.)
- c) Compare the time intervals required to reach the bottom in cases (a) and (b).

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a)

With a moment of inertia of $\frac{2}{5}mr^2$ for a solid sphere and using energy conservation:

$$\begin{aligned} \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 &= mgh \\ \frac{1}{2}mv^2 + \frac{1}{2}.\frac{2mr^2}{5}.\frac{v^2}{r^2} &= mgh \\ v &= \sqrt{\frac{10}{7}gh} \end{aligned}$$

b)

In the case of no rolling, the rotation term $\frac{1}{2}I\omega^2 = 0$ and we simply have:

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

c)

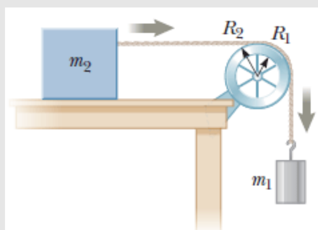
In average the solid sphere will take a time interval of $\Delta t = \frac{d}{v}$. The distance covered is the same and to compare between the times in the two cases we can evaluate $\frac{\Delta t_1}{\Delta t_2}$:

$$\begin{aligned}
\frac{\Delta t_1}{\Delta t_2} &= \frac{v_2}{v_1} \\
&= \sqrt{\frac{2gh}{\frac{10}{7}gh}} \\
&= \sqrt{\frac{14}{10}} \\
&= 1.18
\end{aligned}$$

It takes more time when rolling takes place

0.8

In the figure below, the hanging object has a mass of $m_1 = 0.415$ kg; the sliding block has a mass of $m_2 = 0.890$ kg; and the pulley is a hollow cylinder with a mass of M = 0.350 kg, an inner radius of $R_1 = 0.020\ 0$ m, and an outer radius of $R_2 = 0.030\ 0$ m. Assume the mass of the spokes is negligible. The coefficient of kinetic friction between the block and the horizontal surface is $\mu_k = 0.250$. The pulley turns without friction on its axle. The light cord does not stretch and does not slip on the pulley. The block has a velocity of $v_i = 0.820$ m/s toward the pulley when it passes a reference point on the table.



- a)Use energy methods to predict its speed after it has moved to a second point, 0.700 m away.
- b)Find the angular speed of the pulley at the same moment.

a)

The change in kinetic energy of the system {block m_1+block m_2+pulley} is equal to the net work done on the system. Only friction on the block m_2 and gravitation force on block m_1 have non-zero work. On the other hand, angular speed of the pulley is related to the sped of the objects $\omega = \frac{v}{R_2}$ and a pulley of a hollow cylinder shape has a moment of inertia of : $\frac{1}{2}M(R_1^2 + R_2^2)$

$$\begin{aligned}
\frac{1}{2}(m_1 + m_2)(v_f^2 - v_i^2) + \frac{1}{2}I(\omega_f^2 - \omega_i^2) &= m_1gh - \mu_k m_2 g \\
\frac{1}{2}(m_1 + m_2)(v_f^2 - v_i^2) + \frac{1}{2}\frac{1}{2}M(R_1^2 + R_2^2)\frac{v_f^2 - v_i^2}{R_2^2} &= m_1gh - \mu_k m_2 g
\end{aligned}$$

Rearranging should give :

$$v_f = \sqrt{v_i^2 + \frac{m_1 gh - \mu_k m_2 g}{\frac{1}{2}(m_1 + m_2) + \frac{1}{2}M(1 + \frac{R_1^2}{R_2^2})}}$$

With the given numerical values we get :

$$v_f = 1.54 \text{ m/s}$$

b)

The angular speed of the pulley :

$$\begin{aligned} \omega_f &= \frac{v_f}{R_2} \\ &= \frac{1.53}{0.030} \\ &= 51.3 \text{ rad/s} \end{aligned}$$