PHYSICS 111 HOMEWORK SOLUTION #9

April 5, 2013

0.1

A potter's wheel moves uniformly from rest to an angular speed of 0.16 rev/s in 33 s.

- a) Find its angular acceleration in radians per second per second.
- b) Would doubling the angular acceleration during the given period have doubled final angular speed?

a)

$$\omega = \omega_0 + \alpha t$$

From rest to an angular speed of 0.16 rev/s in 33s we should have:

$$\begin{aligned} \alpha &= \frac{\Delta\omega}{t} \\ &= \frac{\omega - \omega_0}{t} \\ &= \frac{0.16 \times 2\Pi}{33} \\ &= 0.030 \text{ rad/s}^2 \end{aligned}$$

b)

For double the angular acceleration we should have :

$$2\alpha = 2\frac{\Delta\omega}{t} = \frac{2\Delta\omega}{t}$$

The angular speed will be doubled as well

0.2

During a certain time interval, the angular position of a swinging door is described by $\theta = 4.95 + 9.4t + 2.05t^2$, where θ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door at the following times.

- a) t=0
- b) t=2.99 s

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a) at t=0

$$\begin{aligned}
\theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\
\theta &= 4.95 + 9.4t + 2.05t^2 \\
&= 4.95 \text{ rad}
\end{aligned}$$

$$\begin{aligned}
\omega &= \omega_0 + \alpha t \\
&= 9.4 + 4.1t \\
&= 9.4 \text{ rad/s}
\end{aligned}$$

$$\alpha = 4.1 \text{ rad/s}^2$$

b) at t=2.99 s

$$\begin{aligned}
\theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\
\theta &= 4.95 + 9.4t + 2.05t^2 \\
&= 4.95 + 9.4 \times 2.99 + 2.05 \times 2.99^2 \\
&= 51.38 \text{ rad}
\end{aligned}$$

$$\begin{aligned}
\omega &= \omega_0 + \alpha t \\
&= 9.4 + 4.1t \\
&= 9.4 + 4.1 \times 2.99 \\
&= 21.66 \text{ rad/s}
\end{aligned}$$

$$\alpha = 4.1 \text{ rad/s}^2$$

0.3

An electric motor rotating a workshop grinding wheel at 1.06×10^2 rev/min is switched off. Assume the wheel has a constant negative angular acceleration of magnitude 1.96 rad/s^2.

- a)How long does it take the grinding wheel to stop?
- b) Through how many radians has the wheel turned during the time interval found in part (a)?

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a)

The motor having an initial angular speed of $\omega_0 = 1.06 \times 10^2$ rev/min will come to a stop when its angular speed goes down to zero:

$$\begin{aligned}
\omega &= \omega_0 + \alpha t \\
0 &= 1.06 \times 10^2 \times \frac{2\pi}{60} - 1.96t \\
t &= \frac{0 - 1.06 \times 10^2 \times \frac{2\pi}{60}}{-1.96} \\
&= 5.66 \text{ s}
\end{aligned}$$

b)

The radians covered between switching off and stopping is $\Delta\theta = \theta - \theta_0$

$$\begin{aligned}
\theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\
\Delta\theta &= 1.06 \times 10^2 \times \frac{2\pi}{60} t - \frac{1.96}{2} t^2 \\
&= 1.06 \times 10^2 \times \frac{2\pi}{60} \times 5.66 - \frac{1.96}{2} \times 5.66^2 \\
&= 31.43 \text{ rad}
\end{aligned}$$

0.4

A racing car travels on a circular track of radius 275 m. Suppose the car moves with a constant linear speed of 51.5 m/s.

- a)Find its angular speed.
- b)Find the magnitude and direction of its acceleration.

a)

Angular and linear speed are always related through : $v = r\omega$

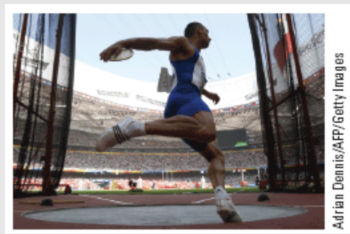
$$\begin{aligned} \omega &= \frac{v}{r} \\ &= \frac{51.5}{275} \\ &= 0.19 \text{ rad/s} \end{aligned}$$

With a constant linear speed the acceleration is radial ($a = a_r = \frac{v^2}{r}$ as $a_t = \frac{dv}{dt} = 0$) :

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{51.5^2}{275} \\ &= 9.645 \text{ m/s}^2 \end{aligned}$$

0.5

A discus thrower accelerates a discus from rest to a speed of 25.3 m/s by whirling it through 1.28 rev. Assume the discus moves on the arc of a circle 0.99 m in radius.



- a) Calculate the final angular speed of the discus.
- b) Determine the magnitude of the angular acceleration of the discus, assuming it to be constant.
- c) Calculate the time interval required for the discus to accelerate from rest to 25.3 m/s.

a)

The final angular speed is again related to the final linear speed as :

$$\begin{aligned}\omega &= \frac{v}{r}\\ &= \frac{25.3}{0.99}\\ &= 25.56 \text{ rad/s}\end{aligned}$$

b)

From rest to 25.56 rad/s, Angular acceleration assumed to be constant can be evaluated from the time-independent equation:

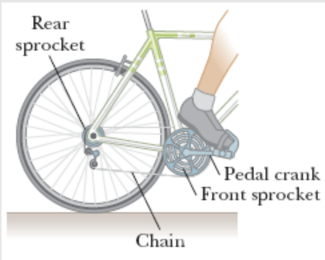
$$\begin{aligned}\omega^2 - \omega_0^2 &= 2\alpha \times \Delta\theta\\ \alpha &= \frac{\omega^2 - \omega_0^2}{2\Delta\theta}\\ &= \frac{25.56^2 - 0}{2 \times 1.28 \times 2\pi}\\ &= 40.60 \text{ rad/s}^2\end{aligned}$$

c)

$$\begin{aligned}
\theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\
\Delta\theta &= \frac{1}{2}\alpha t^2 \\
t &= \sqrt{\frac{2\Delta\theta}{\alpha}} \\
&= \sqrt{\frac{2 \times 1.28 \times 2\pi}{40.60}} \\
&= 0.63 \text{ s}
\end{aligned}$$

0.6

The figure below shows the drive train of a bicycle that has wheels 67.3 cm in diameter and pedal cranks 17.5 cm long. The cyclist pedals at a steady cadence of 79.0 rev/min. The chain engages with a front sprocket 15.2 cm in diameter and a rear sprocket 6.00 cm in diameter.



- a) Calculate the speed of a link of the chain relative to the bicycle frame.
- b) Calculate the angular speed of the bicycle wheels.
- c) Calculate the speed of the bicycle relative to the road.
- d) What piece of data, if any, are not necessary for the calculations?

a)

Links of the chain rotate around the front sprocket. The front sprocket rotates at 79.0 rev/min and has a diameter of 15.2 cm. Links on the chain will consequently have a linear speed of:

$$\begin{aligned} v &= r_{fr}\omega_{fr} \\ &= \frac{0.1502}{2} \times \frac{79 \times 2\pi}{60} \\ &= 0.621 \text{ m/s} \end{aligned}$$

b)

The bicycle wheels and the rear sprocket have the same angular speed that we can calculate from the linear speed of the chain links (links rotate around the rear sprocket also):

$$\begin{aligned} \omega_{rear} &= \frac{v}{r_{rear}} \\ &= \frac{0.621}{0.06} \\ &= 20.7 \text{ rad/s} \end{aligned}$$

c)

Again the rear sprocket and the rear wheel rotating with the same angular speed will give rise to a linear speed for both wheels and the bicycle:

$$\begin{aligned} v &= r_{wheel} \times \omega_{rear} \\ &= \frac{0.673}{2} \times 20.7 \\ &= 6.97 \text{ m/s} \end{aligned}$$

d)

The only piece of data not necessary for our calculations is the length of pedal cranks.

0.7

A wheel 1.65 m in diameter lies in a vertical plane and rotates about its central axis with a constant angular acceleration of 3.70 rad/s^2. The wheel starts at rest at t = 0, and the radius vector of a certain point P on the rim makes an angle of 57.3° with the horizontal at this time. At t = 2.00 s, find the following:

- a) the angular speed of the wheel.
- b) the tangential speed of the point P.
- c) the total acceleration of the point P.
- d) the angular position of the point P.

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a)

The wheel started at rest. Therefore, $\omega_0 = 0$:

$$\begin{aligned} \omega &= \omega_0 + \alpha t \\ &= \alpha t \\ &= 3.70 \times 2 \\ &= 7.40 \text{ rad/s} \end{aligned}$$

b)

The tangential speed of point P located on the rim:

$$\begin{aligned} v &= r\omega \\ &= \frac{1.65}{2} \times 7.40 \\ &= 6.11 \text{ m/s} \end{aligned}$$

c)

To calculate the total acceleration of the point P, we need to calculate both the radial and tangential compenents:

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha = \frac{1.65}{2} \times 3.70 = 3.05 \text{ m/s}^2$$

$$a_r = \frac{v^2}{r} = \frac{6.11^2}{\frac{1.65}{2}} = 45.18 \text{ m/s}^2$$

and finally,

$$\begin{aligned} a &= \sqrt{a_t^2 + a_n^2} \\ &= 45.28 \text{ m/s}^2 \end{aligned}$$

Its direction β with respect to the radius to P can be evaluated from

$$\tan\beta = \frac{a_t}{a_n} = \frac{3.05}{45.18}$$

i.e-

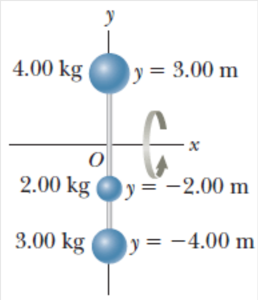
$$\beta = 3.86^\circ$$

d)

$$\begin{aligned} \theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= 57.3 \times \frac{\pi}{180} + 0 + \frac{1}{2} \times 3.70 \times 2^2 \\ &= 8.40 \text{ rad} \end{aligned}$$

0.8

Rigid rods of negligible mass lying along the y axis connect three particles. The system rotates about the x axis with an angular speed of 2.10 rad/s.



- a) Find the moment of inertia about the x axis.
- b) Find the total rotational kinetic energy evaluated from $\frac{1}{2}I\omega^2$.
- c) Find the tangential speed of each particle.
- d) Find the total kinetic energy evaluated from $\sum \frac{1}{2}m_i v_i^2$

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a)

$$\begin{aligned} I &= \sum m_i r_i^2 \\ &= 4 \times 3^2 + 2 \times 2^2 + 3 \times 4^2 \\ &= 92 \text{ kg.m}^2 \end{aligned}$$

b)

$$\begin{aligned} Ki &= \frac{1}{2} I\omega^2 \\ &= 0.5 \times 92 \times 2.10^2 \\ &= 202.86 \text{ J} \end{aligned}$$

c)

Different linear speeds for different radius.However, all particles are rotating at same angular speed: $v_i = r_i\omega$

Mass 1:

$$v_1 = r_1\omega = 3 \times 2.10 = 6.30 \text{ m/s}$$

Mass 2:

$$v_2 = r_2\omega = 2 \times 2.10 = 4.20 \text{ m/s}$$

Mass 3:

$$v_3 = r_3\omega = 4 \times 2.10 = 8.40 \text{ m/s}$$

d)

The total kinetic energy is:

$$\begin{aligned} Ki &= \sum \frac{1}{2} m_i v_i^2 \\ &= \frac{1}{2}(4 \times 6.30^2 + 2 \times 4.20^2 + 3 \times 8.40^2) \\ &= 202.86 \text{ J} \end{aligned}$$

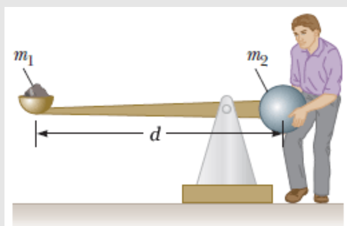
e)

Both expressions lead to the same value.
This equality can be easily proven:

$$\begin{aligned} Ki &= \sum \frac{1}{2} m_i v_i^2 \\ &= \frac{1}{2} \sum m_i r_i^2 \omega^2 \\ &= \frac{1}{2}\omega^2 \sum m_i r_i^2 \\ &= \frac{1}{2} I\omega^2 \end{aligned}$$

0.9

A war-wolf or trebuchet is a device used during the Middle Ages to throw rocks at castles and now sometimes used to fling large vegetables and pianos as a sport. A simple trebuchet is shown in the figure below. Model it as a stiff rod of negligible mass, d = 3.20 m long, joining particles of mass $m_1 = 0.130$ kg and $m_2 = 58.0$ kg at its ends. It can turn on a frictionless, horizontal axle perpendicular to the rod and 15.0 cm from the large-mass particle. The operator releases the trebuchet from rest in a horizontal orientation.



- a) Find the maximum speed that the small-mass object attains.
- b) While the small-mass object is gaining speed, does it move with constant acceleration?
- c) Does it move with constant tangential acceleration?
- d) Does the trebuchet move with constant angular acceleration?
- e) Does it have constant momentum?
- f) Does the trebuchetEarth system have constant mechanical energy?

a)

Let's adopt the following notations:

- r_1 and r_2 the distance from the rotation axis to mass m_1 and m_2 respectively.
- we have $r_1 = d - r_2 = 3.20 - 0.15 = 3.05$m
- v designates the speed of m_1

The frictionless motion of the system guarantees energy conservation: at rest this energy is zero. Once the heavier mass m_2 is released to a lower position from rest, mass m_1 will rise and start gaining speed and higher gravitational potential energy. A maximum speed is reached when it is on a vertical position.
Both masses will rotate with same angular speed ω Energy conservation requires:

$$\begin{aligned} 0 &= m_1 g r_1 - m_2 g r_2 + \frac{1}{2} I \omega^2 \\ &= m_1 g r_1 - m_2 g r_2 + \frac{1}{2}(m_1 r_1^2 + m r_2^2)(\frac{v_{max}}{r_1})^2 \end{aligned}$$

Rearranging should give :

$$\begin{aligned} v_{max} &= r_1 \sqrt{\frac{2g(m_2 r_2 - m_1 r_1)}{m_1 r_1^2 + m_2 r_2^2}} \\ &= 3.05 \sqrt{\frac{2 \times 9.81(58 \times 0.15 - 0.130 \times 3.05)}{0.130 \times 3.05^2 + 58 \times 0.15^2}} \\ &= 24.55 \text{ m/s} \end{aligned}$$

b)

The overall acceleration is changing direction throughout the motion. Vector acceleration is thus not constant.

c) and d)

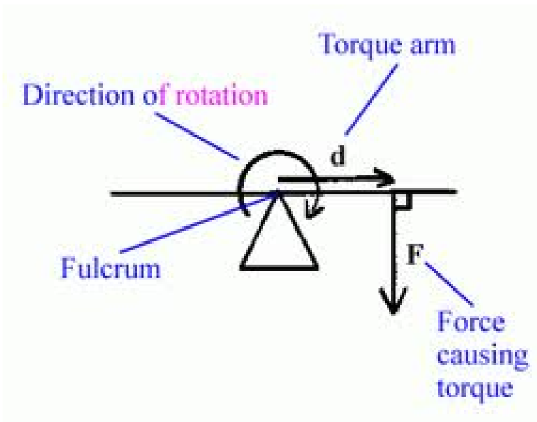
Neither tangential acceleration nor angular acceleration are constant. First of all, the two are related through $a_t = r_1 \alpha$. Let's look at α with second law :

$$\sum \tau_{net} = I\alpha$$

The net torque τ must be changing from a maximum when the arm of the trebuchet is horizontal and going to zero as it stands vertical. Remember the torque expression : $\tau = Force \times d$, where d is the perpendicular distance from the axis of rotation to the vector force line and is not to be confused with r !!. Distance d is going from r_1 to 0, changing the torque, the angular acceleration and the tangential acceleration.

e)

We started our problem with an isolated system for which $\sum \vec{F}_{ext} = \vec{0}$. The trebuchet-earth system should preserve its mechanical energy.



http://history.knoji.com/the-trebuchet/

0.10

A uniform, thin, solid door has height 2.00 m, width 0.825 m, and mass 23.5 kg.

- a) Find its moment of inertia for rotation on its hinges.
- b) Is any piece of data unnecessary?

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a)

The moment of inertia of the thin door (considered a rectangular plate with width w and height h) rotating about the hinges axis is given by :

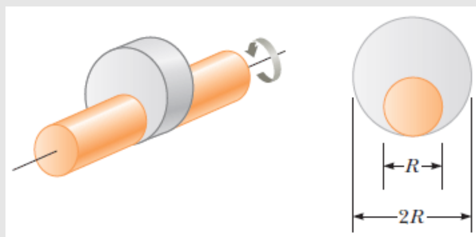
$$
\begin{aligned}
I &= \frac{1}{3}mw^2 \\
&= \frac{23.5 \times 0.825^2}{3} \\
&= 5.33 \text{ kg.m}^2
\end{aligned}
$$

b)

The height of the door is not necessary for the calculation.

0.11

Many machines employ cams for various purposes, such as opening and closing valves. In the figure below, the cam is a circular disk of radius R with a hole of diameter R cut through it. As shown in the figure, the hole does not pass through the center of the disk. The cam with the hole cut out has mass M. The cam is mounted on a uniform, solid, cylindrical shaft of diameter R and also of mass M. What is the kinetic energy of the camshaft combination when it is rotating with angular speed ω about the shaft's axis? (Use any variable or symbol stated above as necessary.)



a)

Our objects are considered uniform and the mass is distributed evenly. If the cam with the hole cut out has a mass M, we can easily convince ourselves that that the cam without the cut should have a mass of $\frac{4}{3}M$ and the mass of the cut is $\frac{1}{3}M$.

- Moment of inertia of the cam without the hole cut out (a disk of mass M and radius R)about the center is:

$$\frac{1}{2}(\frac{4}{3}MR^2) = \frac{2}{3}MR^2$$

- Using the parallel axis theorem, the moment of inertia about the rotating axis is:

$$\frac{2}{3}MR^2 + \frac{4}{3}M(\frac{R}{2})^2 = MR^2$$

- the cut-out hole (a disk of Mass $\frac{1}{3}M$ and radius $\frac{R}{2}$) moment's of inertia about the rotating axis :

$$\frac{1}{2}\frac{M}{3}(\frac{R}{2})^2 = \frac{1}{24}MR^2$$

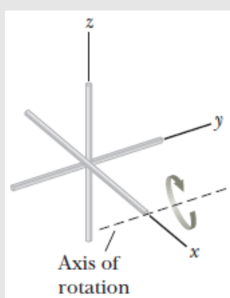
- The shaft (cylinder of mass M and radius R) moment's of inertia about the rotating axis:
$$\frac{1}{2}M(\frac{R}{2})^2 = \frac{1}{8}MR^2$$
- Finally, the overall moment of inertia of our system about the rotating axis is :
$$I = MR^2(1 - \frac{1}{24} + \frac{1}{8}) = \frac{13}{12}MR^2$$

The kinetic energy is :

$$\frac{1}{2}I\omega^2 = \frac{13}{24}MR^2\omega^2$$

0.12

Three identical thin rods, each of length L and mass m, are welded perpendicular to one another as shown in the figure below. The assembly is rotated about an axis that passes through the end of one rod and is parallel to another. Determine the moment of inertia of this structure about this axis. (Use any variable or symbol stated above as necessary.)



a)

Rotation axis is parallel to the y-axis with an offset distance of $\frac{L}{2}$. On the other hand, the y-axis is a symmetry axis for the whole system of the three rods. Therefore, a good strategy is to calculate the moment of inertia of the three-rods system about the y-axis and use the parallel axis theorem to determine the moment of inertia about the given axis.

- The x-rod contributes $\frac{mL^2}{12}$ as moment of inertia about the y-axis and so does the z-rod.
- The y-rod, a thin cylinder, contributes zero about its own y-axis
- A total of $\frac{mL^2}{6}$

The Moment of inertia of the system about the axis in consideration is therefore:

$$\begin{aligned} I &= I_0 + (3m)(\frac{L}{2})^2 \\ &= \frac{mL^2}{6} + \frac{3ml}{4} \\ &= \frac{11}{12}mL^2 \end{aligned}$$