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# Physics 131 

# Laboratory Manual 

Simon Fraser University Physics Department

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# Introduction 

## References:

E. M. Rogers, Physics for the Inquiring Mind. (Princeton University

Press, Princeton, NJ, 1966). Permission of citation granted by estate of Eric M. Rogers.
J. R. Taylor, An Introduction to Error Analysis, 2nd Ed.

### 0.1 What's Happening?

Physics is an experimental science. Everything we know about the world comes from observing it. That's why laboratory experience is an important aspect of learning elementary physics. Obviously you will not have time to discover and verify fundamental laws of physics in these introductory labs. Nevertheless, the laboratory serves to emphasize that contact with nature is the ultimate basis of physics.

Suppose you went to a physics lecture and you heard statements like the following:
"A motionless object will stay at rest until a force is applied. When a force is applied to an object it will move in the direction of the applied force at a velocity related to its mass. The object will remain in motion until the force is removed. Motion ceases when the applied force ceases."
"Bodies which are unsupported above a horizontal surface will fall along the line between the initial position of the body and the centre of the Earth. Heavy bodies fall very rapidly and their velocities while falling are difficult to measure. However, the nature of the fall may be determined by observing less massive bodies which fall more slowly. These considerations lead us to conclude that such objects fall at a constant rate with the velocity being larger for bigger masses but slower for smaller sized objects."

Would you believe these statements? They both seem reasonable and agree with most of our casual observations. In fact, they were accepted wisdom throughout ancient times and during the middle ages. Now, we consider these ideas to be false. Why?

The task of physics is to describe. But people describe things everyday and we don't call it physics. There are several things that distinguish the descriptions of physics from the common descriptions of everyday happenings.

First of all physical descriptions are quantitative. In the physics laboratory you won't just say the ball fell quickly. You will try to measure the speed as accurately as you can. Is the ball heavy or light? You must state the mass in grams and avoid vague, qualitative or subjective statements.

Secondly, technical words we use in physics have precise definitions and will be used to mean the same thing every time. Words like "force," "energy," "work," and "potential," are commonly used in English, but their meanings are vague and vary from time to time. These words, when used in the physics context, always have very precise meanings, and these meanings are tied to measurable quantities. Beginning physics students often confuse common meanings of words such as "energy" with their precise physical definition. (An advertisement for a candy says it is "High in Energy-Low in Calories." A
physicist thinks of calories as a measure of energy. The everyday idea of energy is much broader.)

Finally, physics strives to describe as much as possible with one law. Many things that seem completely different are actually aspects of the same principle. The law of gravitation, for example, describes how objects are attracted together.

$$
F=\frac{G M m}{r^{2}}
$$

This one law encompasses events as different as a pebble falling to the ground and the motion of the moon around the earth. The connection between these phenomena can be obtained by logical reasoning. That's why physical laws are stated in mathematical form -mathematics is a language that includes the rules of logic. If the methods of algebra, trigonometry and calculus let you use one physical law to describe a lot of different phenomena, then the law is useful. On the other hand, a physical law that only describes one particular event and doesn't relate to anything else is bad because it does not simplify our view of the world.

The two false laws quoted above are bad for all three reasons: Terms such as "force" are used vaguely. The descriptions are not quantitative. Worst of all, they describe some particular observations but do not apply in other casesthe rules have too many exceptions. For example, if things move only when a force is applied-that's fine for ox carts-then why doesn't a boat stop moving when the motor stops? What about the trajectory of a cannon ball? Contemporary physics describes all these situations in a unified way and does not require a separate explanation for each one.

Physics is a science of observation. Find out what's happening, understand qualitatively, figure out the important quantities, and measure them. Formulate a law to describe what happened accurately and quantitatively in a general way. Find the logical implications of the law for other situations. Then test to see if it works. If the descriptive law misses the mark, that is, if it doesn't predict properly, go back and try to find one that does work. This is the scientific method that has allowed us to view nature as unified and ordered rather than as a collection of independent and unrelated events. The essence here is direct experimentation and testing.

### 0.2 Your Notebook

Your lab notebook is a record of your explorations. Everything you do during an experiment should be written down here in the form of a reliable and honest record. Use notebooks with bound pages. Loose-leaf notebooks are not acceptable. Paper with squared lines is helpful for diagrams and data tables. In the notebook, explain what you were trying to do. Write down all measurements, what equipment you used and how you did them. Write all data directly into the notebook in ink. Do not use whiteout for corrections. Do not first put data on a scrap of paper and later copy it into the notebook. This risks introducing errors when you copy, and you may even lose the data. All problems you encountered or mistakes you made should also be noted. Write down any unusual observations or thoughts that occur to you even if you cannot fully explain them at the time. As the notebook is not a formal report, it doesn't need to be exquisitely neat. The guiding rule in keeping a notebook is to write down enough detail that you would be able to reconstruct, several months or years later, what you did during the experiment.

Here are some important items to include in the notebook for each experiment:

- Date
- Your name and your partner's name
- Title of the experiment
- Brief sentence listing what you hope to accomplish
- Sketch of apparatus with labels
- All data stating the instruments used to measure them and error estimates.

Put data in tables. Think carefully how to arrange the table. Try to place derived quantities beside the raw data from which they are calculated. It is usually better to put different quantities in different columns with its label and units at the top of the column as shown below.

| distance $(\mathrm{m})$ | time $(\mathrm{s})$ | velocity $(\mathrm{m} / \mathrm{s})$ |
| :--- | :--- | :--- |
| $0.25 \pm 0.002$ | $1.1 \pm 0.1$ | $0.23 \pm 0.02$ |
| 0.31 | 1.7 | $0.18 \pm .0 .01$ |
| 0.39 | 1.9 | $0.21 \pm 0.01$ |

- Sample calculations of derived quantities.

If you do the same calculation repeatedly, you don't need to display each calculation in detail.

- Graphs made on separate pieces of paper should be stapled or glued into the book.
- Conclusions and discussion.

A few brief sentences are usually enough. These introductory experiments are not extremely profound, and the graders don't like to read unnecessary verbiage.

### 0.3 Measurements

0.3.1 What is Truth?

The instructor asks you to measure a distance on the table as accurately as you can with the meter stick. You take the metre stick, place it along the length to be measured, and tilt the stick on edge so that the millimeter marks touch the table. After very carefully positioning one end of the stick at the beginning of the distance, you squint diligently at the end point. The nearest centimeter before the point is easily seen: 24 cm . Then you count the number of millimeters past the centimeter mark and get 7 mm . Now you notice that the end point is somewhere between two millimeter marks. It looks to be a little less than halfway-maybe 0.3 or 0.4 millimeters. So you split the difference and write

$$
24.735 \mathrm{~cm} .
$$

Confident of deserving praise, you give the result to the instructor who emits a barely audible moan. What went wrong?

When you measure a value you are trying to estimate something that nobody will ever know: the exact value. No instrument is perfect. For example, the marks on the meter stick might not be in exactly the right place, or the end might be a little worn making it difficult to align with the beginning point. There is always some judgment involved in reading the value like guessing between 0.3 or 0.4 mm . Even very precise instruments with digital readouts are limited. If there is a five-digit reading of 24.734 , the real value could be anywhere between 24.7335 and 24.7345.

In addition to the limitation of the measuring instrument, most things we measure are not exactly fixed and definite. On a gross scale, the length may seem constant. But as we try to refine the measurement to microscopic scale, it becomes apparent that the length is fluctuating because of vibrations or temperature changes. Irregularities in size become evident as the measurement accuracy increases.

To be honest, we must acknowledge these limitations. In lining up the beginning of a metre stick, there is the possibility of at least a 0.5 mm error one way or the other. This is written as $\pm 0.5 \mathrm{~mm}$. When we cannot really decide where between the two millimetre marks the end point lies, we should decide that it is most probably between 0.2 and 0.4 mm : another $\pm 0.2 \mathrm{~mm}$ uncertainty. In total, there is an uncertainty of at least $\pm 0.7 \mathrm{~mm}$ just in reading the meter stick. If you add in the other sources of uncertainty, that gives at least $\pm 1 \mathrm{~mm}$ or $\pm 0.1 \mathrm{~cm}$. You could write

$$
24.734 \pm 0.1 \mathrm{~cm} .
$$

The hundredth and thousandth of a cm are not significant when you are uncertain about the tenth of a cm. Therefore, you should write

$$
24.7 \pm 0.1 \mathrm{~cm}
$$

Including an uncertainty estimate and rounding off to the correct number of significant digits such as done here is simply being honest about how well you can estimate the value. Realistically, you probably should allow a little more uncertainty because of meter stick inaccuracy or irregularities at the ends of the object being measured.

Try this test: Take a length of string or wire. Give it to five people to measure and note the variation of length measurements. If you have estimated the uncertainty well, most of the values should lie within your uncertainty range. In making any measurement, always try to estimate the range of values five different people would get (assuming they are competent) and use this to guide your estimate of the uncertainty.

### 0.3.2 Scale-Reading Uncertainties

One source of uncertainty comes from reading a scale on a meter or dial. Here are some guidelines when estimating such reading inaccuracies.

If the space between marks is about the same as the width of the marks, then just estimate to the nearest mark; the uncertainty is $\pm$ one-half the smallest division. For example, if the mm marks on a meter stick are 0.5 mm wide and there is 0.5 mm space between them, then you can't get much closer than $\pm 0.5$ mm (the nearest mm ).

If the space between marks is several times the width of the marks, then try to guess whether the reading is more than half way or less than half way. For example, you could write 0.2 or 0.3 mm if it's less than halfway, 0.7 or 0.8 mm if it is more than halfway, 0.5 mm if it looks like it is exactly halfway, 0.1 or 0.9 mm if it is very near a line. Give yourself an uncertainty of $\pm 0.2$ or $\pm 0.3 \mathrm{~mm}$.

If the distance between the marks seems very wide, then try to mentally divide the interval into ten divisions. For example, if there are only cm markings on the meter stick, try to estimate the nearest millimeter and allow $\pm 1$ or $\pm 2 \mathrm{~mm}$ uncertainty.

Reading a scale is only one source of inaccuracy in the value. Add more if there is a possibility of calibration error, or if the quantity you measure is irregular or fluctuates. It is difficult to list all the things that can add to the unreliability of a measured quantity because every situation is a little different. For example, in the case of a needle on a dial or meter, you must consider parallax: the relative positions of the needle and tick marks move when you move your head from side to side.

Digital readings seem more certain. However, the accuracy is usually at least $\pm 1$ digit plus a certain percentage of the value. Meters sometimes carry an
accuracy note on them. (Look at the bottom of the digital volt meters.) If the accuracy is indicated as $\pm(2 \%+1$ digit $)$ this means that you have to add $\pm 2 \%$ of the reading to $\pm 1$ in the position of the last digit of the reading to get the total error. For example, a three-digit meter reads 25.3 mV then the accuracy is
$\pm(0.02 \times 25.3+0.1)= \pm(0.5+0.1) \mathrm{mV}=0.6 \mathrm{mV}$.
Your estimate of the uncertainty of a value may well be different from someone else's estimate. Generally, there is no exact value. Therefore, you need to keep only one significant figure for the uncertainty estimate unless its first digit is 1 . In that case, keep two digits. If your estimate is a factor of two different from the instructor's, there should be no problem. However, if your estimate differs from an instructor's by a factor of ten, it will probably be questioned.

### 0.3.3 Significant Figures and Precision

When you write values, make sure that the number of significant figures corresponds roughly with the error estimate. Even without an explicit error estimate, the number of significant figures implies an uncertainty. For example writing 24.73 cm implies $24.73 \pm 0.05 \mathrm{~cm}$. For intermediate values that will be used in calculation, it's a good idea to keep an extra significant figure to maintain accuracy: $24.734 \pm 0.05 \mathrm{~cm}$. But final results should not keep unnecessary figures.

Sometimes for large numbers, you may need to write zeros that are not significant; e.g., $24000 \pm 400 \mathrm{gm}$. In this case, it is better to write $(24.0 \pm 0.4) \mathrm{x}$ $10^{3} \mathrm{gm}$ to avoid insignificant zeros.

### 0.4 Graphical Analysis

How do we interpret experimental results given that they have uncertainties? Have you disproved conservation of linear momentum if you find that the initial momentum is $0.105 \mathrm{~kg}-\mathrm{m} / \mathrm{s}$ and final momentum is $0.114 \mathrm{~kg}-\mathrm{m} / \mathrm{s}$ ? Suppose you determine the value of $g$ in Lab 1 to be $9.93 \mathrm{~m} / \mathrm{s}^{2}$ while the accepted value is 9.8 $\mathrm{m} / \mathrm{s}^{2}$. Is the deviation significant? The important points are

1. you must consider the experimental uncertainties in order to decide these questions and
2. you can only give an argument based on probability.

This section shows some standard graphical techniques that will help you interpret experimental results. In the first part (0.4.1), we consider the case where your result is a single quantity, for instance the density of aluminum, which will be compared with its accepted value. In the next part (0.4.2), we assume that you study the relationship between two quantities by varying one of them systematically, for instance measuring the vertical velocity $v_{y}$ of a falling object at various times $t$.
0.4.1 Graphical analysis of a single quantity.

Suppose $Q_{m}$ is the value you measured for some quantity or calculated from several measured quantities. Let $\pm \Delta Q$ be the best estimate of the uncertainty of $Q_{m}$, and let $Q$ be the accepted or assumed value. Draw a (horizontal) number line marked in appropriate, easy-to-read units, choosing a scale such that $\Delta Q$ measures from 1 cm to 5 cm . Locate the three points $Q_{m}, Q_{m}-\Delta Q$, and $Q_{m}+\Delta Q$ and plot them just above the number line, as shown.


Now you have a visual comparison of $Q$ with your result that can be interpreted more easily than the set of numbers $Q, Q_{m}$, and $\Delta Q$. There is a predictable probability that a measured result $Q_{m}$ will deviate by more than $\pm \Delta Q$ from the "true" or accepted value if certain assumptions hold about the randomness of the measurement errors. At this point in your scientific career, you need no probability theory, but use the following rules of thumb:

If your graphs looks like

then you have agreement, or your disagreement is not significant.
If your graph looks like

then you were very lucky, or you overestimated your errors. The latter may seem to be a safe thing to do, but it renders your results less meaningful. So be sure to estimate errors more carefully next time.

If you get

where the deviation is as large as two times your error bar $(2 \Delta Q)$, then there is a question as to whether the disagreement is real or not. Further measurements may be useful.

But if you get

then either

1) you made a blunder, or
2) you underestimated the errors involved, or
3) you have discovered a significant departure from the accepted or assumed value. This condition (3) is the most interesting and requires the most careful scrutiny. You are not likely to discover significant departures from accepted laws in this course. In your later research, however, you may find such cases, and if you do, make sure you avoid pitfalls (1) and (2).

Sometimes, you must compare two values $Q_{1}$ and $Q_{2}$ that both have significant uncertainties. Asking the question "do they differ?" is equivalent to forming a new variable $D=Q_{1}-Q_{2}$ and asking "does $D$ differ significantly from zero?" See section 0.5 for how to calculate $\Delta \mathrm{D}$ from $\Delta Q_{1}$ and $\Delta Q_{2}$. Alternatively, you could form a ratio, $R=Q_{1} / Q_{2}$ and compare $R$ to one. There are some examples of this method in Lab 1.
0.4.2 Graphical analysis of two related quantities

In several labs, you are asked to graph one measured quantity, say $y$, against some other, say $x$, and (i) to decide whether there is a straight-line relationship
between $y$ and $x$, and, if this is the case, to determine (ii) the slope ("rise over run") and, possibly, (iii) the intercept of the straight line with the $x$ - or $y$-axis. You remember from math courses that a relationship between sets of numbers $y$ and $x$ of the form $y=y_{0}+m x$ leads to a straight-line graph, as shown in the figure. The slope is defined as

$$
\text { slope }=m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}
$$

for any pair of numbers $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$ on the line. The $y$-intercept is the value of $y$ for $x=0$, and the $x$-intercept is the value of $x$ for $y=0$. The slope, $m$, can be positive, negative or zero, and the same is true for the intercept, $y_{0}$ -

The figure shows a mathematically perfect straight line where there is no ambiguity about the relationship between $x$ and $y$. In an experiment, however, we have a small set of measured values $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{\mathrm{n}}, y_{\mathrm{n}}\right)$ with finite uncertainties $\Delta x$ and $\Delta y$ respectively. As a result, it is not always obvious that $y$ and $x$ obey a straight-line relationship. Even if they do, neither the slope nor the intercepts are accurately determined. How do we decide and how do we find the best values and their uncertainties? The following rules should help you prepare graphs and analyze your data.


1. Find the smallest and largest $x$ - and $y$-values that you want to plot, including uncertainties, and consider the size of graph paper. Choose a convenient scale for the x - and y -axes that accommodates all the relevant points, spreading them out as far as possible but is easy to read.
Note: If you are checking a relation of the form $y=m x$, then the origin is a relevant point and should be on your graph, even if you did not measure that point explicitly, as $x=0$ implies $y=0$.
In another situation, you may have to determine the $x$-intercept by extrapolation to the value $y=0$, which could be unmeasureable. In this case, too, the graph must include the point $(0,0)$.
2. Label the two axes: Mark the scale; indicate the quantity plotted and the units used.
3. Plot your data points $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right), \ldots,\left(x_{\mathrm{n}}, y_{\mathrm{n}}\right)$ with their uncertainties $\pm \Delta x_{i}$ , $\pm \Delta y_{i}$ shown as error bars as in part 0.4.1. If $\Delta x$ or $\Delta y$ is too small to be plotted, say so on the graph.
4. Now you are ready to check whether a straight line fits your data. Use a transparent ruler to find the "best line" that passes closest to all your data points. Remember that your "best line" must pass through the origin if you test a relationship of the form $y=m x$. You can claim that a straight line fits your data if it intersects at least $2 / 3$ of your error bars. If a data point is far off the line, check for errors in plotting it or recalculate it. If you find an error, correct and replot; if not, ignore it and check whether a straight line fits the rest of the points. (But comment on the presence of an outlier and on any possible explanations in your notebook.)
5. Having found the best straight line for your data, determine the slope from well-separated points on the line. (Use points on the straight line that you draw. Do not use raw data points unless they are exactly on the line.) To find the uncertainty in the slope, $\Delta m$, draw the steepest and shallowest lines that still pass through more than half of the error bars. Let these lines pass through the origin if you check a relation of the form $y=m x$ and you are sure there is no zero-point offset. Pick two well-separated points on the steepest line and two well-separated points on the shallowest line in order to calculate the maximum and minimum slopes $m_{\max }$ and $m_{\min }$. Define $\Delta m$ as $\left(m_{\max }-m_{\min }\right) / 2$. For a typical set and a reasonable choice of steepest, best and shallowest line you expect that $m \approx$ ( $m_{\text {max }}$ $\left.+m_{\min }\right) / 2$. If this is not the case, reconsider your choice of lines. The next figures show two hypothetical examples.


Graph 1: In this case both the slope and the intercept are determined by the data. The best fit line should touch at least $70 \%$ of the error bars. The maximum and minimum slope lines should touch about $50 \%$ of the error bars.


Graph 2: If the theoretical model only includes a slope the line of best fit must pass through the origin. If the best-fit line can't touch at least 70\% of the error bars, then perhaps the model is wrong. Points that look bad can sometimes be ignored, but the reasons should be discussed in your report.

### 0.5 Propagation of Errors: examples

Sum of two measurements

$$
\begin{gathered}
\text { width } w=0.24 \pm 0.03 \mathrm{~m} \\
\text { length } l=0.89 \pm 0.04 \mathrm{~m} \\
\operatorname{sum} s=w+l=0.24+0.89 \mathrm{~m}=1.13 \mathrm{~m}
\end{gathered}
$$

possible error of the sum is

$$
\Delta s=\sqrt{\Delta w^{2}+\Delta l^{2}}=\sqrt{0.03^{2}+0.04^{2}}=0.05 \mathrm{~m}
$$

The perimeter is twice the sum.

$$
p=2(w+l)=2 s=2.26 \mathrm{~m}
$$

possible error of perimeter is

$$
\Delta p=2 \Delta s=0.1 \mathrm{~m}
$$

In conclusion, write the perimeter as

$$
p=2.3 \pm 0.1 \mathrm{~m}
$$

Product of two measurements
$\begin{array}{ll}\text { width } w=0.24 \pm 0.03 \mathrm{~m} & \text { so } \quad \frac{\Delta w}{w}=\frac{0.03}{0.24}=12.5 \% \\ \text { length } l=0.89 \pm 0.04 \mathrm{~m} \text { so } & \frac{\Delta l}{l}=\frac{0.04}{0.89}=4.5 \%\end{array}$

Area is the product of width and length

$$
A=w l=(0.24 \mathrm{~m})(0.89 \mathrm{~m})=0.2136 \mathrm{~m}^{2}
$$

Possible error of area is given by

$$
\begin{gathered}
\frac{\Delta A}{A}=\sqrt{\left(\frac{\Delta w}{w}\right)^{2}+\left(\frac{\Delta l}{l}\right)^{2}} \\
\frac{\Delta A}{A}=\sqrt{12.5 \%^{2}+4.5 \%^{2}}=13 \% \\
\Delta A=(0.13)\left(0.2136 \mathrm{~m}^{2}\right)=0.028 \mathrm{~m}^{2}
\end{gathered}
$$

So one should write

$$
A=0.21 \pm 0.03 \mathrm{~m}^{2}
$$

What do you do with weird functions?
For example, what is the possible error of

$$
x=\cos (\theta)
$$

when $\theta=21^{\circ} \pm 2^{\circ}$ ?

$$
x=\cos \left(21^{\circ}\right)=0.9335
$$

The easiest way to find $\Delta \mathrm{x}$ is to substitute for the minimum and maximum values:

$$
\begin{aligned}
\Delta \cos (\theta) & =\frac{1}{2}\left|\cos \left(23^{\circ}\right)-\cos \left(19^{\circ}\right)\right| \\
& =\frac{1}{2}|0.9205-0.9455|=\frac{0.025}{2} \\
& =0.0125
\end{aligned}
$$

so write

$$
x=0.934 \pm 0.012
$$

Using Calculus to find $\Delta \cos (\theta)$ :

$$
\Delta x=\Delta \cos (\theta)=\left|\frac{\mathrm{d} \cos (\theta)}{\mathrm{d} \theta} \Delta \theta\right|
$$

## Caution:

If you use calculus, $\Delta \theta$ must be in radians.

$$
\begin{gathered}
\pm 2^{\circ}= \pm 2 \frac{\pi}{180}=0.035 \text { radians } \\
21^{\circ}=0.367 \text { radians } \\
\begin{array}{c}
\Delta x=\Delta \cos (\theta)=|\sin (0.367)(0.035)| \\
=|(0.359)(0.035)| \\
=0.0125
\end{array}
\end{gathered}
$$

write 0.012 or 0.013 as you wish.

### 0.6 Propagation of Uncertainties: General Rules

The uncertainty of calculated quantities depends directly on the uncertainties of the variables used in the calculation. For brevity, we simply state the rules for commonly encountered situations here. Later, some of these rules will be justified, but a complete understanding needs statistical methods that are too advanced for this course.

In the following, let $A, B, C, \ldots$ stand for independent quantities with uncertainties $\Delta A, \Delta B, \Delta C, \ldots$. Let $Y=f(A, B, C, \ldots)$ be the calculated quantity of interest.

### 0.6.1 Rule 1: A constant multiple

$$
Y=k A
$$

where $k$ is a constant, then

$$
\Delta Y=k \Delta A
$$

### 0.6.2 Rule 2: Addition and Subtraction

If

$$
Y=A \pm B \pm C
$$

then

$$
\Delta Y=\sqrt{(\Delta A)^{2}+(\Delta B)^{2}+(\Delta C)^{2}}
$$

The generalization to the sum of four or more quantities should be obvious. The reason for taking the root-squared sum instead of just adding the uncertainties is that we are not certain whether the errors will cancel or add. If there are many terms in the sum, there will typically be some cancellation and the combined error will not likely be as large as the error given by the sum $|\Delta A|+|\Delta B|+|\Delta C|+\ldots$.

### 0.6.3 Rule 3: Multiplication and Division

If

$$
Y=A B C, Y=A B C^{-1}, Y=A B^{-1} C^{-1}, \text { or } Y=A^{-1} B^{-1} C^{-1}
$$

then

$$
\frac{\Delta Y}{Y}=\sqrt{\left(\frac{\Delta A}{A}\right)^{2}+\left(\frac{\Delta B}{B}\right)^{2}+\left(\frac{\Delta C}{C}\right)^{2}}
$$

For multiplication and division, we add the fractional (or percentage) errors.

### 0.6.4 Rule 4: Powers

If

$$
Y=A^{\alpha}, \text { where } \alpha \text { is arbitrary: integer, fraction, positive or negative }
$$ then

$$
\frac{\Delta Y}{Y}=|\alpha| \frac{\Delta A}{A}
$$

0.6.5 Examples
(1) $Y=A B^{2}$

Let $C=B^{2}$ so that $Y=A C$.
Then according to rule $3 \frac{\Delta Y}{Y}=\sqrt{\left(\frac{\Delta A}{A}\right)^{2}+\left(\frac{\Delta C}{C}\right)^{2}}$.
According to rule $4 \quad \frac{\Delta C}{C}=2 \frac{\Delta B}{B}$

Therefore

$$
\frac{\Delta Y}{Y}=\sqrt{\left(\frac{\Delta A}{A}\right)^{2}+4\left(\frac{\Delta B}{B}\right)^{2}}
$$

Example (2) $Y=\frac{1}{A}+\frac{1}{B}$
Let $C=A^{-1}$ and $D=B^{-1}$.
According to rule $4 \quad \frac{\Delta A}{A}=\frac{\Delta C}{C}$ and $\frac{\Delta B}{B}=\frac{\Delta \mathrm{D}}{\mathrm{D}}$.
However, $C=A^{-1}$ and $D=B^{-1}$ so that these latter two expressions can be written as

$$
\Delta C=\Delta A / A^{2} \text { and } \Delta D=\Delta B / B^{2}
$$

Hence,

$$
\Delta Y=\sqrt{\left(\frac{\Delta A}{A^{2}}\right)^{2}+\left(\frac{\Delta B}{B^{2}}\right)^{2}}
$$

### 0.6.6 The General Case

The four rules and the rule for the general case can be derived with the help of calculus. In this discussion, we will assume a function of the form

$$
\begin{equation*}
Y=f(A, B) \tag{1}
\end{equation*}
$$

Generalization to functions of more variables is easy.
Calculus tells us that if we change $A$ by a small amount $d A$ and $B$ by $d B$ then the change in $Y$ is given by

$$
\begin{equation*}
d Y=\frac{\square \square \square}{\square \square A} \square_{B}^{\square} d A+\frac{\square f \square}{\square B} \square_{A}^{\square} d B, \tag{2}
\end{equation*}
$$

where the subscript $A$ or $B$ on the partial derivatives has the conventional meaning that the quantity $A$ or $B$ is to be held fixed while taking the derivative. (That is the definition of partial derivative.) For convenience these subscripts are omitted from now on.

This equation tells us how fast the function $Y$ changes when we change its inputs $A$ and $B$ by some small amounts $d A$ and $d B$. We can identify these small changes with small errors in our measurements, $\pm \Delta A$ and $\pm \Delta B$. These errors can have either algebraic sign and so can the derivatives $(\partial f / \partial A)$ and $(\partial f / \partial B)$. In the worst case both terms in (2) are positive or both negative in which case you have

$$
\begin{equation*}
\Delta Y_{\text {worstcase }}=\left|\left(\frac{\partial f}{\partial A}\right) \Delta A\right|+\left|\left(\frac{\partial f}{\partial B}\right) \Delta B\right| . \tag{3}
\end{equation*}
$$

On the other hand it could turn out that you are lucky and the two terms in equation (2) tend to cancel. Then you would have

$$
\begin{equation*}
\Delta Y_{\text {bestcase }}=\left|\left(\frac{\partial f}{\partial A}\right) \Delta A\right|-\left|\left(\frac{\partial f}{\partial B}\right) \Delta B\right| \tag{4}
\end{equation*}
$$

In practice, there is no way of knowing whether the errors are going to cancel or add in the final answer. Probability theory says that in such a case, if the errors are independent and have a normal distribution, then we should add the individual errors "in quadrature"; i.e., form the root-squared sum as

$$
\begin{equation*}
\Delta Y=\sqrt{\left(\left(\frac{\partial f}{\partial A}\right) \Delta A\right)^{2}+\left(\left(\frac{\partial f}{\partial B}\right) \Delta B\right)^{2}} \tag{5}
\end{equation*}
$$

Note that $\Delta Y$ has the property

$$
\Delta Y_{\text {worst case }} \geq \Delta Y \geq \Delta Y_{\text {best case }}
$$

You can visualize this way of adding errors by means of a right triangle.


The generalization of equation (5) to an arbitrary number of variables $Y=$ $f(A, B, C \ldots)$ is

$$
\begin{equation*}
\Delta Y=\sqrt{\left(\left(\frac{\partial f}{\partial A}\right) \Delta A\right)^{2}+\left(\left(\frac{\partial f}{\partial B}\right) \Delta B\right)^{2}+\left(\left(\frac{\partial f}{\partial C}\right) \Delta C\right)^{2}+\ldots} \tag{6}
\end{equation*}
$$

With a little effort, you should be able to convince yourself that rules 1 through 4 are but special cases of Equation (5), which can also be used for more general cases such as trigonometric or exponential functions.

### 0.7 Systematic and Random Uncertainties

Broadly speaking, one encounters two types of uncertainties: systematic and random. People usually talk about systematic and random errors. The term "error" is unfortunate because there usually is no mistake involved. Rather, any measurement has limitations and returns a value that must be understood as covering a range of possible "true" values. Hopefully, this will become clearer as we proceed. In any case, we will bow to custom and use "error" rather than "uncertainty" in our discussion.

### 0.7.1 Systematic errors

"Systematic" errors are those that are repeatable from measurement to measurement. For example, if you are using a metre stick, the scale could be off. Where it reads 1 cm , for example, the true value might actually be 1.1 cm . If that were the case, every time you measured an object, you would be $10 \%$ too low. That $10 \%$ would be perfectly repeatable, since it comes from the entire scale's being incorrect.

Although this sounds like an artificial example, that is only because we have exaggerated the size of the effect. If you take a long tape measure (say 4 meters in length), then it may well read 12 meters when it is actually 12.001 m .

Another type of systematic error can arise in a calculation. For example, let's say that you are measuring the period of a pendulum's oscillation. The simplest theory of the pendulum predicts the period to be independent of the amplitude of oscillation. That calculation turns out to be correct, but only for infinitesimally small oscillation amplitudes. At finite amplitudes, there is in fact a small dependence. Because you must measure the period using a finite amplitude, there will always be a systematic difference between what you measure and the prediction based on the simple theory. If one had a more sophisticated theory that gave the dependence on amplitude, this systematic error would vanish. Absent such a theory, one could investigate how much of an error exists and try to conduct the measurements in a regime where other errors were larger.

### 0.7.2 Random errors

"Random" errors are those that give a different result every time. You measure something and record the result. You do exactly the same thing again and get a slightly different result. Of course, if you did "exactly" the same thing, you would expect to get the same result. But usually, it is impossible to do things exactly the same twice. If there are many small, uncontrollable effects, they will produce small differences in the results of repeated trials of the same measurement.

One example of a random error, which we will encounter in the first lab, arises when using a stopwatch to time events. Let us think (and try!) the following experiment: Your instructor will drop a ball from a height (say 2 meters) onto the floor. Each student has a stopwatch and times, independently, this same event. (Equivalently, one student could time the fall of many balls, but
this takes longer and is less fun!) When the instructor yells "go!" he or she drops the ball, and you start your stopwatch. When the ball hits, you hear the sound and you stop. Let's say that there are 30 people in the class. They may get the following results (in seconds):
$0.64,0.62,0.63,0.67,0.58,0.48,0.53,0.63,0.58,0.57,0.73,0.74,0.71,0.59$, $0.68,0.63,0.57,0.56,0.65,0.76,0.77,0.62,0.52,0.68,0.56,0.58,0.79,0.58$, 0.64, 0.76

The first thing to note is that the numbers differ, and by a fair amount. They range from 0.48 s to 0.79 s , a difference of almost $70 \%$ ! Does that mean one cannot say anything about the "true" value? Not at all! Using some elementary ideas from statistics, we can actually say quite a lot.

The first thing to do when confronted with such results -- called "repeated trials" in the jargon of statistics -- is to group them in a histogram. To do this, we set up bins and count how many measurements fall into each bin. For example, we can choose a bin size of 0.05 s . In Fig. 1a, we show the resulting histogram. Using this histogram, we can try to better characterize the collective results of the 30 trials.


Fig. 1: Observations grouped in bins of 0.05 sec . (a) raw histogram; (b) histogram overlaid by bell curve that describes the result of an experiment with infinitely many trials. The wide horizontal line is the standard deviation of the observations, while the narrow horizontal line is the standard deviation of the average of the observations.

In (a), we see that the distribution is starting to form a "bell-shaped curve," although 30 trials is still so small that it is not a "pretty" curve, but there are large variations from the "ideal" shape (overlaid in part b).

The next thing to do is to try to extract a "typical" value. Statistics teaches us that the "best" value we can use is the average. If $t_{i}$ is the $i$ 'th time measurement ( $i$ from 1 to 30 in this case), then the average is

$$
\langle t\rangle=\frac{1}{N} \sum_{i=1}^{N} t_{i}
$$

where $N=30$, in this case. For the measurements above, the average is 0.635 s .
The next thing that one can extract is a typical value for the trial-to-trial "fluctuation." In other words, if one takes a typical trial, how far away is it from the true value? This quantity is known as the "standard deviation." Graphically, it is the width of the best bell-shaped curve. In statistics, one can show that the best estimate from the data is the "standard deviation", $\sigma$,

$$
\sigma=\sqrt{\frac{1}{N-1} \sum_{i=1}^{N}\left(t_{i}-\langle t\rangle\right)^{2}}
$$

In words, one calculates the average and takes the difference between each point and the average. One squares this difference and adds it to a sum over all data points. This sum is normalized by $N-1$ (not $N$; it's a bit of a long story but comes about because the average is estimated from the data, rather than being known exactly). For the above data, one estimates $\sigma=0.08 \mathrm{~s}$. This is the typical deviation of each measured trial from the "exact" value. Any single measurement, for example the second one $(0.62 \mathrm{~s})$, would be reported as, say, 0.62 $\pm 0.08 \mathrm{~s}$.

Finally, the average value should intuitively be closer to the real value than any single measurement. The rules for error propagation (see 0.6.1 and 0.6.2) tell us that the uncertainty in the average should be the uncertainty in a single measurement divided by the square root of the number of measurements, i.e.,

$$
\sigma_{a v g}=\frac{\sigma}{\sqrt{N}}
$$

For the events here, this is $0.08 /(30)^{0.5}=0.015 \mathrm{~s}$. Thus, an individual measurement is likely to be off by about $\pm 0.08 \mathrm{~s}$, but the average (for 30 trials) will typically be off by only 0.015 s . The average, with the error bars for a single measurement and for the average itself are both shown in Fig. 1b (filled marker and hollow marker, respectively). For the average, we would report $0.635 \pm$ 0.015 s . (A more honest estimate of these numbers would be $0.64 \pm 0.02 \mathrm{~s}$, since the original uncertainty $(0.08 \mathrm{~s})$ was only to one significant figure).

We can also see, in Fig. 2, how increasing the number of measurements produces histograms that converge slowly to the bell-shaped solid curve. Many trials are required to get a nice-looking convergence.


Fig. 2: Effect of increasing the number of trials. (a) 30 trials; (b) 300 trials; (c) 3000 trials. Notice how the histograms converge to the bell curve, which shows what would be observed after an infinite number of trials. The $y$-axis of the histograms have all been normalized by dividing the bin height by the number of trials for each case. Bin size is 0.01 sec . (compare with 0.05 sec . in Fig. 1).

Note: By doing more and more trials, the random error can be reduced to very small values. In practice, it is not worth going below the point where other errors (particularly systematic errors) start to dominate.

## Error Analysis Problems

1. The scale shown is from a pressure gauge. The reading is in microns $(\mu \mathrm{m})$ of mercury. State the value and uncertainty limits of the reading at positions $a, b, c$, d and e.

2. Specifications on the bottom of a Fluke digital multimeter say:

ACCURACY: $\pm(\%$ OF READING + DIGITS $)$
DC VOLTAGE: $(0.1+1)$
DC CURRENT: $(0.3+1)$ EX $(0.5+1 @ 10 A)$
Decipher this inscription and rewrite the following readings with the correct number of significant figures and state the uncertainty. a) 2.125 V , b) 0.002 mV , c) $0.615 \mathrm{~A}, \mathrm{~d}) 21.21 \mathrm{~V}$, e) 9.892 . A
3. Determine the values and uncertainties of the following derived quantities from the stated values and uncertainties of the respective measured quantities.
a) voltage, $V=5.6 \pm 0.1 \mathrm{~V}$, current $I=0.125 \pm 0.02 \mathrm{~A}$, power, $P=$ ? $\pm$ ?
b) time, $t=3.67 \pm 0.05 \mathrm{~s}$, distance $x=4.52 \pm 0.01 \mathrm{~m}, \quad$ velocity, $v=$ ? $\pm$ ?
c) distances: $x=6.24 \pm 0.05 \mathrm{~m}, y=7.8 \pm 0.1 \mathrm{~m}, z=4.03 \pm 0.01 \mathrm{~m}$ area, $A=x^{2}+y z=? \pm$ ?
d) distances: $x=6.24 \pm 0.05 \mathrm{~m}, y=7.8 \pm 0.1 \mathrm{~m}, z=4.03 \pm 0.01 \mathrm{~m}$ geometric mean $\langle x\rangle=(x y z)^{1 / 3}=$ ? $\pm$ ?
e) amplitude, $A=9.000 \pm 0.001 \mathrm{~m}$, time, $t=26.0 \pm 0.5 \mathrm{~s}$ distance, $d=A \sin (t / 16)=? \pm$ ? (angle in radians)
f) amplitude, $A=9.000 \pm 0.001 \mathrm{~m}$, time, $t=51.0 \pm 0.5 \mathrm{~s}$ distance, $d=A \sin (t / 16)=? \pm ? \quad$ (angle in radians)
4. a) Five measurements of diameter are made with an estimated reading accuracy of $\pm 0.3 \mathrm{~mm}: \quad 3.20,3.19,3.21,3.22,3.20 \mathrm{~cm}$. What is the average and its uncertainty?
b) Six measurements are made on another sample with the same reading accuracy: $3.21,3.68,2.69,3.43,3.52,3.54 \mathrm{~cm}$. What is the average and its uncertainty?
5. Gold is denser than lead and more expensive. You wish to determine whether a wafer of gold has been diluted with lead by measuring its density. The wafer is supposed to be 25 g of pure gold, and you can measure its volume with an uncertainty of $\pm 0.2 \mathrm{ml}$ and its mass within $\pm 0.1 \mathrm{~g}$. What is the largest admixture of lead that can escape detection expressed as a fraction of total mass?

Phys131 Lab Manual

## Lab 1 - Pendulums

## References

E. M. Rogers, Physics for the Inquiring Mind. (Princeton University

Press, Princeton, NJ, 1966). Permission of citation granted by estate of Eric M. Rogers.
P. A. Tipler, Physics for Scientists and Engineers, 6th Ed. Ch 14, Oscillations.
Resnick, Halliday and Krane, Physics, 5th Ed. Ch 17, Oscillations.

## Objective

An "Open Lab" where you will be able to plan and do your own experiment, make your own mistakes, criticize the mistakes of others, and maybe learn from them.

## Apparatus

String, weights, ring stand, meter stick, stopwatch, electronic timer, anything else we have.

## Method

You fill in this section.


Fig. 4-2. Simple Pendulum

## EXPERIMENT: PENDULUMS

Narrow your field of investigation down to a simple pendulum, a small bob swinging to and fro on a long thread; and narrow it still more to the question, "How does the time-of-swing or period of a pendulum depend on each of the physical factors that might affect it?" Thus restricted, this is still a complex investigation unless you follow the good scientific practice of holding all other factors constant while you change one chosen factor at a time. See Fig. 4-2.

We define the PERIOD as the time of one "swing-swang," one complete cycle. What factors might affect the period? Obviously, the LENGTH of the pendulum -- most easily defined as distance from support to center of bob, but not easily, or wisely, measured directly in that form. We all know a longer pendulum takes more time to swing to-and-fro. But just how is the period $T$ related to the length $L$ ? Is there a simple mathematical rule? (There is. We can deduce it from other experimental knowledge, e.g., from force vectors and Newton's Laws of Motion; but here you should make an "empirical investigation" -- ask a straight question of nature in your own experiments.)

What other factors might affect $T$ ? Mass or weight of bob; amplitude, or size of swing; and possibly other factors?
Start by investigating how $T$ depends on
length of pendulum, $L$
amplitude of swing, $A^{\circ}$ each side of vertical
mass of bob, $M$
To avoid confusing several effects, keep two of the three, $L, A, M$, constant while you change the third, and measure $T$. Does it matter which of the three you choose to vary first? In this case, it does: there is one logically correct choice. While you are considering this, make some preliminary measurements to try out techniques.

## (0). Preliminary Measurements

A good scientist does not expect a stream of accurate measurements to flow at once from his apparatus. He experiments on his experiment, trying out techniques, gaining skill by practice. Choose a long pendulum, 2 or 3 ft long, and make accurate timings of its period with a good stopwatch (or use a magnifying glass over the seconds hand of an ordinary watch while a partner gives you signals.) Record your measurements. Compare them with your partner's. Look at the methods being used by neighbours, and get ready to criticize.

## Discussion Group

When you have practical experience of your apparatus, meet with other students and the instructor, as a "research team" to discuss difficulties and techniques. This is the time to suggest good tricks you have discovered, to criticize mistakes you saw neighbours making, to discuss the reliability of the equipment, and to decide on good techniques and plan the order of experiments. There is little use in such a discussion before you have done preliminary experimenting: that would lead to childish guessing, or else your instructor would have to step in with cookbook directions. As in a professional research group, an adult discussion needs practical experience of the apparatus.

At this time, you will find there is a good reason for investigating the effect of AMPLITUDE first, before MASS or LENGTH.

## (1). Measurements: $T$ vs. $A$

Make careful measurements of $T$ for the following amplitudes: $70^{\circ}, 60^{\circ}, 50^{\circ}, 40^{\circ}$, $30^{\circ}, 20^{\circ}, 15^{\circ}, 10^{\circ}$, and $5^{\circ}$. Plot a rough graph of $T$ vs. $A$ as you go, to guide your
further measurements. If you find the graph-points use only a narrow region of your paper, you should plot another graph with one coordinate expanded so that the blown-up graph reveals the shape better. (A blown-up graph need not have the origin on the paper--in fact its origin may be many inches off the paper.) If you run into difficulties, discuss them with your instructor--treat him or her as a source of good advice from another scientist, not as a short cut to "right answers".

When you have enough good measurements, plot a careful graph of $T$ against $A$ - with a blown-up version, too, if that seems called for.

At this stage, another conference is likely to be useful. Comparing your graph with the graphs of others, you will probably decide there is a definite relationship, but many graphs may show such large accidental errors that their form is obscured. An accurate answer to "How does $T$ depend on $A$ ?" is essential: the other parts of this investigation will be impossible without it. You will need very careful measurements of $T$ for a certain range of amplitudes. It will be obvious from your present graphs what that range is, and why measurements outside that range need not be very accurate.

## More accurate measurements

With increased skill from practice and better knowledge of techniques, make the measurements needed to settle the essential question about $T$ and $A$, and plot them in the graph of $T$ against $A$. (This sounds like a long piece of drudgery, fussing over better precision. It will take time and trouble, but the outcome is rewarding.) When you have settled the question, write your answer or conclusion clearly, and use it in part (2).

## (2). Measurements: $T$ vs. $M$

How much faster would you expect a heavy bob to pull the pendulum to and fro than a light one? Measure $T$ for a fairly long pendulum, using the conclusion of your $T$ vs. $A$ experiments to guide your arrangements. (Of course, you should not repeat your $T$ vs. $A$ investigation all over again for each bob in this investigation. Once that is settled it is settled, and its results can be used without repeating the investigation. ${ }^{1}$ Change to another bob much heavier or much lighter, and repeat your measurements. Make sure that the $L$, from support to centre of bob, is the same for both. (That is why we offered the cookbook suggestion of a long pendulum: the longer the thread the smaller the $\%$ change in $L$ if you make a small mistake in changing bobs.)

## (3). Measurements of $T$ vs. $L$

The period $T$ changes greatly as $L$ changes and a good graph (or other investigation) of the relationship between them needs many sets of measurements. Here is where cooperation among the whole laboratory group is welcome. With the answers to questions or parts (1) and (2) known, it is easy to pool comparable measurements from every member of the group.

Each student (or pair) should measure $T$ for one length, and then the group should pool their measurements for graphs. One student should choose a length between 1 metre and 0.9 m ; another student between 0.9 and 0.8 metre; and so on. Then everyone in the group will need a record of all results, to plot a graph of $T$ against $L$ and to look for the relationship.
"The Pendulum Formula" and Measuring $g$

There is a definite relationship between $T$ and $L$, a well-known one that you are likely to meet in physics books or as an example in algebra books. If you do not already know it, try guessing at it from the numbers in the communal record, or from the shape of your graph. Then plot a second graph of something else ( $1 / T$ ?, $T ?, \ldots$ ?) against $L$ that you expect to give a straight line. Or, if you prefer, ask an instructor to tell you the relationship -- as a piece of well-known hearsay -and use it to plot a straight-line graph. Draw the "best straight line" that runs "as near as possible to as many points as possible."

The pendulum formula relating $T$ and $L$ also contains " $g$," and pendulum timings offer very precise methods of measuring $g$ even today. If you do not know it, get the full formula from your instructor and calculate " $g$ " from your straight-line graph. For that you need first the slope of that line--by using the slope instead of single measurement, you are taking a weighted average of many measurements. Also calculate " $g$ " from your own single measurement that you contributed to the pool.

[^0]
# Lab 2 - DC Circuits 

## References

Tipler (6th Ed), Chapter 25: Electric Current and Direct-current Circuits Halliday, Resnick and Krane, (5th Ed.) Chapter 31, DC Circuits

## Objective

Fundamental principles of direct current electrical circuits are explored using digital and analog meters.

## THEORY

An electric current $I$ is a flow of charge $Q$ through a section of conductive material. If charge is measured in coulombs (C) and time in seconds (s), then one coulomb of charge flowing through the entire cross section of a wire per second is defined to be a current of one ampère (A) flowing through that wire.
$1 \mathrm{~A}=1 \mathrm{C} / \mathrm{s}$.
The potential difference $V$ between any two points in a circuit is the work required to move charge from one point to the other. If the potential difference is negative, energy is released during the movement of the charge, and current will move spontaneously. Potential difference is measured in volts. A potential difference of one volt $(\mathrm{V})$ between two points means that it takes one joule (J) of work to move one coulomb of charge between the two points.
$1 \mathrm{~V}=1 \mathrm{~J} / \mathrm{C}$
A useful analogy to electric current flow is the flow of liquid. Current through a wire is like flow through a pipe and a voltage difference between two points of a wire is like a pressure difference between two points of a pipe. Most of the laws of dc circuits can be visualized by reference to the fluid flow analogy.

## Kirchhoff's Laws

The two most fundamental laws of circuits are Kirchhoff's laws. These results from the conservation of energy and charge, as applied to electrical circuits. You may find it useful to think about the analogy to fluid flow.

1. Conservation of energy

The sum of voltages around any closed loop is zero.
2. Conservation of charge

The total current flowing into a junction equals the current flowing out of it.
Ohm's Law
The current flowing through an electrical wire or devices is often directly proportional to the voltage across it. Such electrical devices are said to obey Ohm's law: Their electrical resistance $R$ is defined as $V / I$. Ohm's law says that this resistance is constant. Resistance is measured in ohms ( $\Omega$ ).
$1 \Omega=1 \mathrm{~V} / \mathrm{A}$.
Many electrically conducting materials and devices do not obey Ohm's law-the ratio of voltage to current is not constant. Resistors are electrical components that are manufactured to have constant resistance over a wide range of operating conditions.

Series and Parallel Resistances
Series: When two resistors are connected in series, so that all current that passes through one must also pass through the other, the resistance of the combination is the sum of each resistance: $R=R_{1}+R_{2}$.


Parallel: When two resistors are connected in parallel so that current can flow through either one or the other, the inverse resistances add: $1 / R=1 / R_{1}+$ $1 / R_{2}$.


Meters
Three types of meters are used for electrical measurements. Voltage is measured with a voltmeter; current is measured with an ammeter; and resistance is measured with an ohmmeter. Modern meters are digital multimeters, which means that they can function as a voltmeter, an ammeter or an ohmmeter depending on which function is chosen on the front panel switches.

## Voltmeters

A voltage is measured by connecting a voltmeter to two points on the circuit. Nothing needs to be disconnected for the voltage measurement; just make electrical contact between the test leads to the points desired.


Fig. 1: Using a voltmeter to read voltage

Figure 1(a) shows a voltage source (a battery) connected to two resistors. In order to measure the voltage across $R_{2}$, connect a voltmeter as shown in Figure 1(b). The black dots are electrical connections. Figure 1(c) illustrates the meaning of the schematic in 1(b). Ideally, one wants the voltage across $R_{2}$ in circuit (a) to be the same as in (b). In reality, the voltmeter changes the voltage slightly because it conducts some current through it, diverting the current away from $R_{2}$.


Fig. 2: The internal resistance of a voltmeter
Saying that the voltmeter conducts some current is the same as saying that there is an internal resistance in the meter $R_{\mathrm{m}}$ that is less than infinite. An ideal voltmeter would not conduct current, so its internal resistance would be infinite. The figure above symbolizes this internal resistance with a grey resistor symbol. When the voltmeter is connected to both ends of $R_{2}$, the equivalent resistance between these two points becomes $\frac{R_{2} R_{\mathrm{m}}}{R_{2}+R_{\mathrm{m}}}$ which is less than $R_{2}$. Therefore, the voltage read by the real meter is less than what would be read by an ideal meter. In order to be useful, the voltmeter's internal resistance must be much larger than the resistance between the points in the circuit where it is connected: $R_{\mathrm{m}} \gg R_{2}$. The bigger the meter's resistance, the better. The digital voltmeters in the lab have resistances of $10 \mathrm{M} \Omega$. The little analog voltmeters have resistances of about $50 \mathrm{k} \Omega$.

## Ammeters



Fig. 3: How to use an ammeter
An ammeter measures the current flowing through it. Figure 3 shows that the ammeter is used to replace the wire between the two resistors. Therefore, current is forced to flow through the ammeter. Ideally the ammeter should be a perfect conductor - zero resistance - but, in fact, real ammeters have some internal resistance. In order for the current measured by the ammeter to be close to the
current in the circuit without the ammeter, its internal resistance should be small: $R_{\mathrm{m}} \ll R_{1}+R_{2}$. The digital meters have a resistance of about $1 \Omega$ on the amp scale and $1.2 \mathrm{k} \Omega$ on the mA scale.

Ammeters usually require more effort to use than voltmeters because you must take out a connection in the circuit and replace it with the meter. Voltmeters simply require touching two points in the circuit. Therefore, most people try to minimize the number of current measurements they need to make.

## Ohmmeters

Resistances are measured by an ohmmeter. Most multimeters have an ohmmeter setting. The resistance is measured by passing a small current through the resistor and measuring the voltage across it.


Fig. 4: How to use an ohmmeter
In order to measure the resistance of a component in a circuit, you must make sure that at least one of its leads is disconnected from the circuit. Otherwise, you will measure the resistance of that component in parallel with the rest of the circuit.

## Experiment

The Potentiometer
The fine nichrome wire provided has a resistance of several ohms per

${ }^{2}$ Fig. 5: The potentiometer circuit
meter. Attach a length of nichrome wire between the banana-plug terminals spanning the length of the board over the tape measure scale. Try to get the wire as straight and tight as possible. You have now made a potentiometer. Connect the DC power supply to the ends of the potentiometer. Now connect the negative terminal of the digital multimeter to the negative end of the potentiometer. The positive terminal of the voltmeter should go to a banana plug wire with an alligator clip on the other end that can be pressed against the potentiometer wire at any point along its length. The alligator clips fit on banana plugs at each end of the connecting wires. Since the banana plugs have holes into which other banana plugs can fit, you don't need to clip alligator clips onto other alligator clips.

Put the multimeter in the D.C. voltage mode, and adjust the power supply to deliver 5 V across the length of the potentiometer using the following steps:

1. Turn the voltage knob down to zero
2. Turn the current limit knob to zero.
3. Turn on the power supply
4. Turn up the current limit knob a little.
5. Adjust the voltage setting knob until you read about 5 V on the power supply's voltmeter.
If you can't get to 5 V , try increasing the current limit a little. If the current keeps increasing no matter how high the current is set, something is wrong. Make sure the current isn't being limited by the power supply-if the circuit is correct and the current limit is set high enough the current shouldn't depend on its setting.
6. Connect the digital meter across the whole potentiometer and adjust the voltage until it reads exactly 5 V .

Touch the positive terminal of the voltmeter to five equally-spaced positions along the potentiometer. Record the position in meters or centimeters and the voltage read on the digital meter. Plot a graph of the voltage vs. position along the potentiometer.

Disconnect the multimeter and reconnect it to measure the current through the potentiometer. Use the A scale not the $m A$ scale, or you'll blow the fuse! Don't change the setting on the power supply. After measuring the current for 5 V , you can calculate the resistance of the nichrome wire using Ohm's law.

Connect the multimeter to the potentiometer so as to measure the resistance directly using the ohmmeter function.

Compare the resistance measured directly with that calculated from the current and voltage. Explain why there may be a difference between the two values.

## I vs V for a Resistor and a Light Bulb

Connect the variable power supply across a $10 \mathrm{k} \Omega$ resistor. (Support the resistor on the screw terminals on the experimental board.) Use the analog Armaco milliammeter to measure current through the resistor and the digital multimeter to measure the voltage across the resistor. Draw a schematic diagram of the circuit in your notebook. Measure the current through the resistor for five different voltage settings. Plot a graph of $I$ vs. $V$ and use it to find the resistance.

Find the current-voltage characteristics of a light bulb. You will have to use another way to measure the current because it will be much larger! Connect a circuit so that you can measure the light bulb current for voltages between 0 and 40 V in steps of 5 V . Plot $I$ vs. $V$. Can you explain why this isn't a straight line? What is the lamp's resistance? Does that question make any sense? Now replot the data in the form of $\log I$ vs. $\log V$. Does this form a straight line? If so, write the equation describing the functional relation.


Fig. 6: Measuring $I$ vs. $V$ for a resistor

## Series and Parallel Resistances

You have two $10 \mathrm{k} \Omega$ resistors and one $1 \mathrm{k} \Omega$ resistor. Calculate the resistance you expect in each of the following cases and then measure it with the ohmmeter.
$1.10 \mathrm{k} \Omega$ in series with $10 \mathrm{k} \Omega$,
$2.10 \mathrm{k} \Omega$ in series with $1 \mathrm{k} \Omega$,
$3.10 \mathrm{k} \Omega$ in parallel with $10 \mathrm{k} \Omega$,
4. $10 \mathrm{k} \Omega$ in parallel with $1 \mathrm{k} \Omega$.

Answer the following questions:

1. If two nearly equal resistors are in series, what is the approximate equivalent resistance?
2. If a resistor is in series with a much smaller resistor, what is the approximate equivalent resistance?
3. If two nearly equal resistors are in parallel, what is the approximate equivalent resistance?
4. If a resistor is in parallel with a much bigger resistor, what is the approximate equivalent resistance?

## The Wheatstone Bridge

The Wheatstone bridge is useful for measuring very small changes in resistance. The figure shows the physical connections and the schematic for a Wheatstone bridge. You should be able to prove that the voltmeter will read 0 V when the ratio of the resistances on the left- and right-hand sides of the potentiometer is the same as the ratio of $R_{1}$ and $R_{2}$. If the resistance wire has a uniform resistance per unit length and the lengths on the left and right are $l_{1}$ and $l_{2}$ then:

$$
V=0 \text { when } \frac{R_{1}}{R_{2}}=\frac{l_{1}}{l_{2}}
$$



Fig 7. The Wheatstone bridge

With this arrangement it is very easy to detect small changes in one of the resistors. When the bridge is balanced, the voltage on the meter is zero and the meter may be set on its most sensitive scale. Small changes from zero are always easier to measure than small changes from a non-zero value.

Set up the apparatus and determine the ratio $R_{1} / R_{2}$ by finding the potentiometer position that gives zero voltage. Now measure the resistances with an ohmmeter. Compare the potentiometer measurement with the other determination of the ratio. Which is more accurate?

## V. Discharge of a Capacitor

The next figure shows a circuit for studying the charging and discharging of a capacitor. When the switch is in the "charge" position, the capacitor will charge up to the power supply voltage. With the capacitor fully charged to a voltage $V$, the charge on the positive plate of the capacitor is $Q=C V$ and the charge on the negative plate is $-Q$. Switching to the "discharge" position allows the charges on the two plates to recombine through the resistor $R$. The rate of recombination is given by the differential equation

$$
I(t)=\frac{d Q}{d t}=C \frac{d V_{C}}{d t} .
$$



Fig. 8: Circuit for measuring the charging and discharging of a capacitor

It can be shown that the voltage on the capacitor will decay according to the formula
$V_{C}(t)=V_{0} e^{-t / R C}$
where $V_{0}$ is the power supply voltage and $t=0$ is the time that the switch moves from charge to discharge. The numerical constant $e$ is about 2.71. The voltage will decay from $V_{0}$ to $V_{0} / e$ in the time $R C$, where $R C$ is the "time constant" of the circuit.

Connect the circuit using a $100 \mathrm{k} \Omega$ resistor and the capacitor supplied to you. The capacitor may be "electrolytic," which means that it should be connected with the polarity indicated on its case. Sometimes a black stripe indicates the negative end.

To determine the time constant, first charge the capacitor to 27.1 V with the switch in the "charge" position. Switch to "discharge" and time how long it takes for the voltage to fall to $10 \mathrm{~V} .(10 \mathrm{~V}=27.1 / e \mathrm{~V}$. $)$ Do this about three times to check the reproducibility. If the time is too fast to measure, then there is either a mistake in the wiring or you need a larger capacitor. Compare your measured time constant with what you calculate from the indicated resistor and capacitor values. (The accuracy of the indicated capacitor value is only about $\pm 20 \%$.)

Now connect the Armaco analog voltmeter to measure the voltage instead of the digital multimeter. Measure the time constant a few times and compare this with the previous measurement. If there is a difference in the two values, decide which is correct and why.

## Prelab Questions

1. The potentiometer extends between positions 1.9 cm and 54.6 cm on the scale. In part I (The Potentiometer) the voltage read by the DMM is 5.000 V and the current read the by DMM is 0.425 A . What is the resistance per unit length of the nichrome wire? (Does the DMM affect the resistance measurement?)
b) The accuracy of the multimeter is cited as $\pm 0.1 \% \pm 1$ digit on the Voltage scale and $\pm 0.3 \% \pm 1$ digit for the Amp scale. What are the uncertainties of the voltage and current readings and the resultant uncertainty of the resistance of the wire?
2. Calculate the four equivalent resistances required for the "Series and Parallel Resistances" section.
3. In the Wheatstone bridge experiment $R_{1}=10 \mathrm{k} \Omega, l_{1}=14.2 \mathrm{~cm}$ and $l_{2}=$ 83.5 cm . What is the value of $\mathrm{R}_{2}$ ?

## Resistor Colour Code

Useful Values:
brown-black 10
red-red 22
yellow-violet 47
With these three combinations of the first two bands, together with different multipliers, you can cover the whole range with steps of a factor two. For example,
$1 \mathrm{k} \Omega=10 \times 10^{2}$ brown-black-red
$2.2 \mathrm{k} \Omega=22 \times 10^{2} \quad$ red-red-red
$4.7 \mathrm{M} \Omega=47 \times 10^{5} \quad$ yellow-violet-green
It's useful to keep in mind at least two common ranges for the third band multiplier:
red $k \Omega$ range (1-9.9 k $\Omega$ )
green $\mathrm{M} \Omega$ range (1-9.9 $\mathrm{M} \Omega$ )
Except for high precision resistors, only certain values of resistors are commonly available. Standard Resistance Values for the first two digits are
$10 \%, 2 \%$ and $5 \%: \quad 10,12,15,18,22,27,33,39,47,56,68,82$
$2 \%$ and $5 \%$ only: $11,13,16,20,24,30,36,43,51,62,75,91$

## Resistor Colour Code



| digit colour | multiplier | \# zeros |  |
| :--- | :--- | :--- | :--- |
|  | silver | 0.01 | -2 |
|  | gold | 0.1 | -1 |
| 0 | black | 1 | 0 |
| 1 | brown | 10 | 1 |
| 2 | red | 100 | 2 |
| 3 | orange | 1 k | 3 |
| 4 | yellow | 10 k | 4 |
| 5 | green | 100 k | 5 |
| 6 | blue | 1 M | 6 |
| 7 | violet | 10 M | 7 |
| 8 | grey |  |  |
| 9 | white |  |  |

## EQUIPMENT NOTE: 33XR Meterman Professional Digital Multimeter

## Electrical Specifications

(Accuracy at $23{ }^{\circ} \mathrm{C} \pm 5^{\circ} \mathrm{C},<75 \%$ R.H.)

## DC VOLTS

Ranges: $400 \mathrm{mV}, 4 \mathrm{~V}, 40 \mathrm{~V}, 400 \mathrm{~V}, 1000 \mathrm{~V}$
Resolution: $100 \mu \mathrm{~V}$
Accuracy: $\pm(0.7 \%$ of reading +1 digit)
Input impedance: $10 \mathrm{M} \Omega$
Overload protection:
400 mV Range: 1000 V dc / 750 V ac rms ( 15 seconds)
Other Ranges: 1000 V dc $/ 750 \mathrm{~V}$ ac rms

## DC CURRENT

Ranges: $400 \mu \mathrm{~A}, 4 \mathrm{~mA}, 40 \mathrm{~mA}, 300 \mathrm{~mA}, 10 \mathrm{~A}$
Resolution: $0.1 \mu \mathrm{~A}$
Accuracy:
$\pm(1.0 \%$ of reading +1 digit) on $400 \mu \mathrm{~A}$ to 300 mA ranges
$\pm(2.0 \%$ of reading +3 digits) on 10A range
Burden voltage:
$400 \mu \mathrm{~A}$ Range: $1 \mathrm{mV} / 1 \mu \mathrm{~A}$
4 mA Range: $100 \mathrm{mV} / 1 \mathrm{~mA}$
40 mA Range: $12 \mathrm{mV} / 1 \mathrm{~mA}$
$300 \mathrm{~mA}: 4 \mathrm{mV} / 1 \mathrm{~mA}$
10A: $100 \mathrm{mV} / 1 \mathrm{~A}$
Input protection:
$0.315 \mathrm{~A} / 1000 \mathrm{~V}$ fast blow ceramic fuse $6.3 \times 32 \mathrm{~mm}$ on $\mu \mathrm{A} / \mathrm{mA}$ input
$10 \mathrm{~A} / 1000 \mathrm{~V}$ fast blow ceramic fuse $10 \times 38 \mathrm{~mm}$ on 10 A input 10A Input: 10 A for 4 minutes maximum followed by a 12 minute cooling period

## RESISTANCE

Ranges: $400 \Omega, 40 \mathrm{k} \Omega, 4 \mathrm{M} \Omega$
Resolution: $100 \mathrm{~m} \Omega$
Accuracy:
$\pm(1.0 \%$ of reading +4 digits) on $400 \Omega, 40 \mathrm{k} \Omega$ range
$\pm(1.2 \%$ of reading +4 digits) on $4 \mathrm{M} \Omega$ range
Open circuit volts: 0.5 V dc typical ( 3.0 V dc on $400 \Omega$ range)
Overload protection: 1000 V dc or 750 V ac rms

AC VOLTS ( $45 \mathrm{~Hz}-500 \mathrm{~Hz}$ )
Ranges: $400 \mathrm{mV}, 4 \mathrm{~V}, 40 \mathrm{~V}, 400 \mathrm{~V}, 750 \mathrm{~V}$ ac
Resolution: $100 \mu \mathrm{~V}$
Accuracy:
$\pm(1.5 \%$ of reading +4 digits)
$\pm(2.0 \%$ of reading +4 digits) 200 Hz to 500 Hz on 4 V range
Peak hold accuracy:
$\pm(3.0 \%+60$ digits) on 40 V to 750 V ranges
$400 \mathrm{mV}, 4 \mathrm{~V}$ ranges unspecified
Input impedance: $10 \mathrm{M} \Omega$
Overload protection:
400 mV Range: 1000 V dc $/ 750 \mathrm{~V}$ ac rms ( 15 seconds)
Other Ranges: $1000 \mathrm{~V} \mathrm{dc} / 750 \mathrm{~V}$ ac rms

AC CURRENT ( $45 \mathrm{~Hz}-500 \mathrm{~Hz}$ )
Ranges: $400 \mu \mathrm{~A}, 4 \mathrm{~mA}, 40 \mathrm{~mA}, 300 \mathrm{~mA}, 10 \mathrm{~A}$
Resolution: $0.1 \mu \mathrm{~A}$
Accuracy:
$\pm(1.5 \%$ of reading +4 digits) on $400 \mu \mathrm{~A}$ to 300 mA ranges
$\pm(2.5 \%$ of reading +4 digits) on10A range
Peak hold accuracy: $\pm(3.0 \%+60$ digits)
Burden voltage: See DC Current
Input protection:
$0.315 \mathrm{~A} / 1000 \mathrm{~V}$ fast blow ceramic fuse $6.3 \times 32 \mathrm{~mm}$ on $\mu \mathrm{A} / \mathrm{mA}$ input
$10 \mathrm{~A} / 1000 \mathrm{~V}$ fast blow ceramic fuse $10 \times 38 \mathrm{~mm}$ on 10A input 10A Input: 10 A for 4 minutes maximum followed by a 12 minute cooling period

## CAPACITANCE

Ranges: $4 \mu \mathrm{~F}, 40 \mu \mathrm{~F}, 400 \mu \mathrm{~F}, 4000 \mu \mathrm{~F}$
Resolution: 0.1 uF
Accuracy:
$\pm(5.0 \%$ of rdg +10 digits) on 4 uF range
$\pm(5.0 \%$ of rdg +5 digits) on 40 uF to 400 uF ranges
$\pm(5.0 \%$ of rdg +15 digits) on 4000 uF range
Test voltage: < 3.0 V
Test Frequency: 10 Hz
Input protection: $0.315 \mathrm{~A} / 1000 \mathrm{~V}$ fast blow
ceramic fuse $6.3 \times 32 \mathrm{~mm}$ on $\mu \mathrm{A} / \mathrm{mA}$ input

FREQUENCY (autoranging)
Range: 4k, 40k, 400k,4M, 40MHz
Resolution: 1 Hz
Accuracy: $\pm$ ( $0.1 \%$ of reading +3 digits)
Sensitivity: 10 Hz to $4 \mathrm{MHz}:>1.5 \mathrm{~V} \mathrm{rms}$;
4 MHz to 40 MHz : $>2 \mathrm{~V} \mathrm{rms},<5 \mathrm{~V} \mathrm{rms}$
Min pulse width: $>25 \mathrm{~ns}$
Duty cycle limits: >30 \% and $<70 \%$
Overload protection: 1000 V dc or 750 V ac rms

Phys131 Lab Manual

# Lab 3 - AC and the Oscilloscope 

## References

Tipler (6th ed.): Chapter 29: Alternating-current circuits
Halliday, Resnick and Krane, (5th Ed.) Chapter 37: Alternating-current circuits

## Objective

This experiment familiarizes the student with the operation and controls of an oscilloscope. Other objectives: to observe Lissajous figures; to use an oscilloscope to measure voltages and time intervals for alternating voltages; to compare oscilloscope measurements with those from a digital multimeter (DMM); and to use the dual channel feature to compare waveforms in a circuit with both resistance and capacitance.

## Apparatus

Oscilloscope, $1 \mathrm{X} / 10 \mathrm{X}$ probe, function generator, digital multimeter, DC power supply, 10:1 AC voltage divider, resistor, capacitor, wires and banana plugs

## Introduction

An electrical voltage is much like water pressure: the more water pressure, the greater the amount of water that flows through a pipe per second. Similarly, the greater the voltage, the more electrons flow through a resistor per second. So voltage is a kind of electrical pressure that forces electrons to flow.

You will be familiar with batteries as a source of voltage. Batteries supply a steady D.C. (direct current) voltage that remains relatively constant in time. The D.C. voltage required in this experiment will be obtained from a power supply. This power supply produces a constant DC voltage from the varying voltage that is fed into it from an ordinary bench outlet.

The power grid supplies us with a voltage that varies with time and is called alternating current (AC). Hence, many of the voltages and currents we

encounter in everyday life are alternating or varying. For example, the voltage
Fig 1: AC voltage
used in our homes alternates 60 times every second. Thus, if the two live terminals (the third terminal is grounded) of a typical house outlet were connected across a load, one terminal would be positive relative to the other for a short time, in this case $1 / 120$ of a second, and then the situation reverses. The output voltage undergoes 1 complete cycle in $1 / 60 \mathrm{sec}$, and we say that the frequency is $60 \mathrm{cycles} / \mathrm{sec}$, or 60 Hz . A graph of the voltage of one terminal relative to the other is shown in Figure 1.

The amplitude $A$ is half the peak-to-peak value of the voltage. The effective value of an alternating voltage is equal to the steady direct voltage that would produce the same heating in a resistor as does the alternating voltage. The effective value is also called the root mean squared (rms) voltage, and it can be shown to equal $A / \sqrt{2}$ for a sinusoidal voltage. The digital multimeter has an A.C. function that reads the voltage $A / \sqrt{2}$, root mean square voltage that is indicated by the dashed line on Figure 1. An oscilloscope can also be used to measure A.C., and, in this experiment, we will compare the relative capabilities of a multimeter and an oscilloscope. Before doing this, however, we will discuss the oscilloscope in some detail.

## The Oscilloscope

## A. Types of oscilloscope

An oscilloscope produces a visual pattern of an electrical signal. The oscilloscope measures voltage and can be used to display a graph of voltage as a function of time on a screen. An oscilloscope is of great importance in technical applications (television, radar, computers) and also in research, where it is used to analyze electrical oscillations and impulses.

Older analog cathode ray oscilloscopes (CRO) use a cathode ray tube to display a voltage vs. time graph. Inside the cathode ray tube is a beam of electrons that goes from the rear to the front. At the front, a phosphor screen glows at the point where the beam hits. By deflecting the beam rapidly left-toright repeatedly while varying the horizontal position according to the voltage being measured, the oscilloscope createsa visual graph of the input.


The digital storage oscilloscope (DSO) converts the measured voltage into
Fig 2: The cathode ray oscilloscope uses an electron beam to draw a picture of the signal on a phosphor screen. (from $X Y Z$ s of the Oscilloscope by Tektronix Corp.)
numerical values with an analog-to-digital converter. The conversion is repeated at a uniform rate and the consecutive values are stored in memory while a microprocessor graphs the values on a display.


Comparing the two types reveals respective advantages and disadvantages. The
Fig 3: The serial processing architecture of a digital storage oscilloscope means that there is a significant delay between the signal and its display (from XYZs of the Oscilloscope by Tektronix Corp.)
digital storage oscilloscope records the signal into memory, so that single events are easily captured and the data can be transferred to an external computer for later study. Furthermore, the internal microprocessor of the digital storage oscilloscope allows more options for display and analysis than the analog CRT oscilloscope. On the other hand, the analog oscilloscope displays the input more rapidly than the digital scope, and the display is continuous; the digital oscilloscope leaves gaps between digitizations. Digitization gaps can miss rapid fluctuations in the input signal or introduce distortions when input frequencies exceed more than one-half of the digitization rate-the "Nyquist" limit. DSOs are the most common type of oscilloscope, on account of their economical price and small size. Nevertheless, CROs or hybrid systems are still required in special circumstances. We use a DSO in this lab.

## B. Using a Digital Storage Oscilloscope

The many controls on the oscilloscope can be grouped into three types:
-The vertical system
The vertical system controls the attenuation or amplification of the signal. Use the volts/div knobs to adjust the amplitude of the signal to the desired measurement range.
-The timebase, or horizontal system
Use the sec/div control to set the amount of time per division represented horizontally across the screen. The time base control knob is marked in units of time/div; e.g. $5 \mu \mathrm{~s} / \mathrm{div}$. This means that each horizontal division on the screen represents $5 \mu$ s.
-The triggering system
The trigger level stabilizes a repeating signal, or starts the capture of a single event.

Suppose an alternating square wave voltage is applied to the input of the oscilloscope so that it produces a pattern on the screen like that on Fig. 5. The peak-to-peak value of the applied voltage may be obtained by measuring the vertical height of the pattern seen on the screen and multiplying this number of screen divisions by the setting on the volts/div control. The period (or time for one cycle) can be determined by multiplying the horizontal length on the screen ( 6.0 divisions in Fig. 5) by the time/div setting.


Fig. 5: Measuring the period on the oscilloscope screen
Suppose the latter was set at $2 \mathrm{~ms} /$ division. Then the period for this (Fig. 5) waveform would be
$T=(6.0 \mathrm{div})(2.0 \mathrm{~ms} / \mathrm{div})=12$ milliseconds
frequency $f=\frac{1}{T}=\frac{1}{12 \times 10^{-3} \mathrm{~s}}=83$ cycles $/ \mathrm{s}=83 \mathrm{~Hz}$.

Thus, an oscilloscope allows us to visualize a rapidly varying electrical signal, and also to measure its amplitude and frequency.

The "Equipment Notes for the oscilloscope" shows the front panel of the oscilloscope used in this lab. It is a dual-channel oscilloscope so that, if required, two different electrical signals can be displayed simultaneously on the screen. One signal is connected to the input terminal of Channel 1 , and the other signal is connected to the Channel 2 input. See the "equipment notes" at the end of this lab script for further information on the controls.


Fig. 6: Oscilloscope front panel controls and connectors

## Important!

The oscilloscope (and most electronic equipment) is supplied with a 3-pin AC plug for safety. The chassis (i.e., the part you can touch) is always grounded to the round ground pin of the AC plug, which goes to earth ground. The shields of both CH 1 and CH 2 inputs are both grounded to the oscilloscope's chassis and earth ground as well. Thus the grounds of both channels are electrically connected together and you must be careful to ground both channels at the same point in your circuits;, otherwise, you will short out circuit elements. The electrical grounds of any other instruments must also be connected to the same ground point unless you know that the instrument has a "floating ground," i.e., a negative terminal not connected to the instrument's chassis and to earth ground.

## C. Probes

Oscilloscope probes
Fig 7: Connecting the probe to the internal calibration signal. are recommended for many measurements because they provide several useful features:

1. They have a little clip that can be hooked on to bare wires in a circuit. The clip can be slipped off to expose a pointed tip that can probe small electrical contacts.
2. There is another clip for attaching to a ground point in the circuit being measured.
3. Most probes are "times-ten" (10X) probes and increase the input impedance of the oscilloscope by a factor of 10 . This increase of the input impedance means that the act of making a measurement has less effect on the circuit's operation. If the scope has a $1 \mathrm{M} \Omega$ input impedance, using a 10 X probe would increase that to $10 \mathrm{M} \Omega$. If you were measuring the voltage across a $1 \mathrm{M} \Omega$ resistor, connecting the scope without a 10 X probe would decrease the voltage to half its value. Using a 10X probe would only decrease its value by one-tenth. There is also a compensation adjustment that can eliminate the effect of the small input capacitance of the oscilloscope (i.e., the imaginary part of its input impedance) which can cause the shape of the input signal to be distorted.

The 10X probe provides the oscilloscope input with only $1 / 10$ of the voltage it is measuring. One must compensate when reading the value. There is a setting under the "vertical menu" that will automatically adjust for the use of a probe. On startup, it assumes that measurements are made with a 10X probe and multiplies measurements by 10 and the vertical $\mathrm{V} / \mathrm{div}$ value byl/10 before displaying them.

## Experiment

I. Measuring the internal calibration signal (PROBE COMP) and using the Vertical Menu
(a) Attach a probe to the CH1 input and connect it to the calibration signal which is labelled "PROBE COMP". The ground clip should be clipped to the bottom tab of the signal output, and the probe hook should be hooked on the top part.
(b) Slide the $1 \mathrm{X} / 10 \mathrm{X}$ switch on the probe to 10 X .
(c) Press the Autoset button. The scope attempts to find the best settings to display the signal


You should now see a "square wave" signal on the screen: $\square$ If the signal looks like this $\checkmark \square \square$ or like this $\square$ -. compensation of the probe needs adjusting. The adjustment requires a special little screwdriver; ask an instructor or a technician to do it.
(d) Find the following information on the screen display:

Vertical V/div scale
Horizontal sec/div scale
Number of vertical divisions between bottom and top of signal
Number of horizontal divisions in one complete period
From these values, you can calculate the peak-to-peak amplitude and frequency of the signal. The readout also displays measurements of these values, thus obviating the need to actually think about what you're doing. Find these values

Peak-to-peak amplitude of the signal
Frequency of the signal
Compare with the values you got by calculating.
A little 1 on the left-hand side of the screen shows the zero voltage level for channel 1. A little arrow on the right-hand side shows the trigger voltage and a downward pointing arrow on the top shows the trigger point. Try to find these features.
(e) Turn the Volts/div know above CH1 a few click clockwise and counterclockwise. Attempt to understand the display changes that these adjustments cause.

Try adjusting the position knobs, too. Notice that the little " 1 " on the left-hand side of the screen moves as you change the position knob for CH 1 .
(f) Similarly, turn the sec/div scale clockwise and counterclockwise. Observe what happens to the signal's display. Can you understand what happens?
(g) Press the CH1 MENU button. The Vertical Menu is displayed beside the signal display. From top to bottom, the following options can be adjusted by pressing the button beside them:

Input Coupling: $\mathrm{DC} / \mathrm{AC} / \mathrm{GND}$. Change to AC and observe the change. The signal is now centred about the zero-volt level. On AC, a high-pass filter takes out the DC component and reduces the low frequency components of the signal. On GND, the signal is zero. Normally, one uses DC coupling unless it's necessary to suppress a large DC offset in the signal.

BW Limit; $20 \mathrm{MHz} /$ Off. Set this to 20 MHz to reduce noise on the signal display. Since the calibration signal has no noise, it won't make any difference this time.

Volts/div: Coarse/Fine. Fine allows fine control of the vertical scale. Set this to Fine and try adjusting the Volts/div.

Probe: $1 \mathrm{X} / 10 \mathrm{X} / 100 \mathrm{X} / 1000 \mathrm{X}$. Set this to agree with the probe you are using. If you are connecting a signal directly to the input, choose 1X. By default, it's on 10X. Set it to 1X and observe how that changes the display

Invert On/Off. Turns the signal display upside down: Positive becomes negative and vice versa. Try it. What happens?

## II. An Alternating Sine Wave Voltage

(a) Measure AC voltage with a DMM

Use the DMM to measure the AC voltage output of the voltage divider in the grey metal box that plugs into an AC outlet. This circuit cuts down the line voltage by a factor of ten so that you can study the AC line voltage without killing yourself. The voltage divider does not affect the frequency. Write down the r.m.s. voltage given by the meter.
(b) Measure AC voltage using the oscilloscope

Connect the output of the voltage divider to the CH 1 input using a direct BNC-toBNC cable and a banana-to-BNC adapter. (Make sure the ground is properly connected and watch those gender changes!) Adjust the "Probe" setting on the CH 1 vertical menu to 1 X .
The DSO allows three ways to measure amplitude and period of a signal: automatic, cursors and visual.

## Automatic measurement:

Press the "Measurement" button. Choose "Pk-Pk" option to measure the peak-topeak amplitude: the absolute difference between maximum and minimum points in the signal. The value is displayed on the screen.

## Cursors

Press "Cursors". Choose "Voltage". Use the cursor knobs to position the two dotted horizontal lines (the cursors) at the top and bottom of the signal. Press the button beside the "Delta" menu option. The voltage difference is displayed.

## Visual

Since many oscilloscopes do not have cursors or automated measurement, you should learn how to measure values by reading directly from the screen. Use the knob with the up-down arrow to position the minimum signal point on a major division line. Read the voltage of the major division line just below the signal maximum. (For example, if the top of the wave is a little over three divisions above the bottom of the wave and the vertical setting is $5 \mathrm{~V} / \mathrm{div}$, then count $5 . .10 . .15 \mathrm{~V}$.) Next, estimate the exact voltage of the maximum. (For example, if it's just over half way to the next division, estimate 18 V .)

The period of the signal can be estimated the same three ways and the frequency is just $1 /$ period in each case. Direct measurement of the frequency is also available under the "Measurement" menu.

Measure both the peak-to-peak amplitude and the period using the three ways explained above. Compare the measurements to each other and to the value from the DMM.

## Accuracy:

The claimed vertical scale calibration accuracy is $\pm 3 \%$ (for the $5 \mathrm{~V} / \mathrm{div}$ to $10 \mathrm{mV} / \mathrm{div}$ scales) or $\pm 4 \%$ ( $2 \mathrm{mV} / \mathrm{div}$ or $5 \mathrm{mV} / \mathrm{div}$ ). The probe's $10: 1$ attenuation is accurate to $\pm 2 \%$. So there is about $\pm 5 \%$ possible error on most voltage measurements. In addition, the analog-to-digital converter has eight bits, which means the full scale is divided into 256 bins. One should allow for $\mathrm{a} \pm 1$ bin conversion error so that there is a possible digitization error given by the $\mathrm{V} / \mathrm{div}$ setting multiplied by $8 / 256=1 / 32$. (Because there are eight vertical divisions over the scope range.) Example: you are reading $\mathrm{V}=10.2 \mathrm{~V}$ on the $5 \mathrm{~V} /$ div scale. From the calibration accuracy we get $\pm 5 \% \mathrm{x} 10 \mathrm{~V}= \pm 0.5 \mathrm{~V}$. From digitization accuracy using the $5 \mathrm{~V} /$ division scale we get: $\pm 5 \mathrm{~V} / 32= \pm 0.2 \mathrm{~V}$. Combining gives a generous $\pm 0.7 \mathrm{~V}$ possible error.

The time base can be considered to be calibrated accurately (digital clocks lose less than 1s per day), but the digitization rate is 250 sample bins per major time division on the oscilloscope. You should allow at least $\pm 1$ bin error on any time measurement. For example if you read a time of 1 ms on while on the $0.2 \mathrm{~ms} /$ div scale that would imply $\pm 0.2 \mathrm{~ms} / 250$ or $\pm 0.4 \mu \mathrm{~s}$ error due to digitization. Of course, if the actual points marking the time interval are uncertain, then that should be included in the error estimate as well.

## III. Examining the charge and discharge of a capacitor

Now investigate the relationship between the input and output of a circuit with both a resistor $R$ and capacitor $C$, with the output being taken across the capacitor. This circuit is similar to the $R C$ circuit of the DC circuits lab, except that the function generator switches between charging and discharging the capacitor.

Read the resistance and capacitance values and calculate the time constant, $\quad=R C$. The resistor and capacitor values are approximately $R=10.0$ $\mathrm{k} \Omega$ and $C$ is between 0.01 and $0.10 \mu \mathrm{~F}$. Set the function generator to the square wave output, and select a frequency whose period is many times longer than

Connect the above circuit. The signal from the function generator ( $V_{\mathrm{in}}$ ) should be connected to CH 1 , and the output signal across the capacitor ( $V_{\text {out }}$ ) connected to CH2. Turn on the function generator, and adjust the oscilloscope so that you can see the signals from both channels. You should see an exponential rising and falling of the output signal on channel 2 synchronized with the square wave of channel one. Trigger SOURCE should be on CH1

Note: Be careful to connect the grounded (black) terminals of the oscilloscope inputs to the same point in the circuit. Make sure you use the same output from the function generator to feed both Ch. 1 input and the filter.
(a) Measure the RC time constant. Adjust the function generator peak-topeak output voltage to a convenient voltage, $V_{0}$ and its period to be several times $R C$. The peak-to-peak output voltage across the capacitor should also be about $V_{0}$. If it is less, you probably need to decrease the frequency. Use the oscilloscope to measure the time it takes the voltage on the capacitor to fall from $V_{0}$ to $V_{0} / e$. This measured time is the time constant $\tau$. For convenience, you may choose $V_{0}=2.71 \mathrm{~V}$. Then $V_{0} / e=1 \mathrm{~V}$.
(b) Observe square wave filter output near the cutoff frequency. Increase the frequency of the function generator so that the period is about $R C$. Sketch the waveform across the capacitor, indicating the time and voltage scales.
(c) Observe square wave filter output at high frequencies. Increase the frequency further until $R C$ is many times the period. Sketch the output and note the relative magnitudes of the input and output voltages.

Use the capacitor charging and discharging model to explain in a brief sentence why the relative output voltage is nearly the same as the input voltage at frequencies much less than $1 / \tau$ but diminishes to zero as the frequency increases above $1 / \tau$.

## Equipment Note: Oscilloscope

## Manufacturer's Accuracy Specifications

Vertical scale calibration: $\pm 3 \%$, ( $\pm 4 \%$ on the 2 mV and 5 mV scales) 10X Probe attenuation: $\pm 2 \%$
Digital resolution: 8 bits ( 256 bins over the vertical range, or 32 bits / division)
Time scale: 2500 sample bins over the horizontal range, $\pm 1$ bin accuracy.

## Display and Controls



1. Icon display shows acquisition mode.
mSample mode
. mPeak detect mode
$\_$Average mode
2. Trigger status indicates the following:
$\square$ Armed. The oscilloscope is acquiring pretrigger data. All triggers are ignored in this state.
四 Ready. All pretrigger data has been acquired and the oscilloscope is ready to accept a trigger.
$\boldsymbol{m}$ Trig'd. The oscilloscope has seen a trigger and is acquiring the posttrigger data.
-Stop. The oscilloscope has stopped acquiring waveform data.
-Acq. Complete. The oscilloscope has completed a Single Sequence acquisition.
$\pm R$ Auto. The oscilloscope is in auto mode and is acquiring waveforms in the absence of triggers.
$\square$ Scan. The oscilloscope is acquiring and displaying waveform data continuously in scan mode.
3. Marker shows horizontal trigger position. Turn the HORIZONTAL POSITION knob to adjust the position of the marker.
4. Readout shows the time at the center graticule. The trigger time is zero.
5. Marker shows Edge or Pulse Width trigger level.
6. On-screen markers show the ground reference points of the displayed waveforms. If there is no marker, the channel is not displayed.
7. An arrow icon indicates that the waveform is inverted.
8. Readouts show the vertical scale factors of the channels.
9. A BW icon indicates that the channel is bandwidth limited.
10. Readout shows main time base setting.
11. Readout shows window time base setting if it is in use.
12. Readout shows trigger source used for triggering.
13. Icon shows selected trigger type as follows:
$f-$ Edge trigger for the rising edge.
l-- Edge trigger for the falling edge.
ヘ-- Video trigger for line sync.
--- Video trigger for field sync.
$ת$-- Pulse Width trigger, positive polarity.
v-- Pulse Width trigger, negative polarity.
14. Readout shows Edge or Pulse Width trigger level.
15.Display area shows helpful messages; some messages display for only three seconds. If you recall a saved waveform, readout shows information about the reference waveform, such as RefA $1.00 \mathrm{~V} 500 \mu \mathrm{~s}$.
15. Readout shows trigger frequency.

Control
AUTOSET

VOLTS/DIV

SEC/DIV

CH1 MENU

Option Buttons

Function

Automatically adjusts settings to display the input signals

Changes the vertical voltage scale
Changes the horizontal time scale
Displays a menu of settings which affect the voltage display for Channel 1.

Input coupling: AC/DC/GND
Bandwidth limit: $20 \mathrm{MHz} /$ off
V/div control fineness: Coarse/Fine
Probe attenuation: 1X, 10X, 100X, 1000X
Inversion: Off/On
Chooses the menu item directly to its left

MEASUREMENT

POSITION

CURSOR

DISPLAY

Displays the measurement menu:
Freq, Period Mean, Pk-Pk, Cyc RMS, Min, Max, Rise Time, Fall Time, Pos Width, Neg Width and None.

Three knobs which control the up-down position for CH 1 and Ch 2 and the left-right position for both channels. The CH 1 and CH 1 position function as cursor position controls when the Cursor menu is active.

Displays a menu allowing cursor readings: Type (Voltage, Time), Source (Ch1, Ch2) Delta, Cursor1, Cursor2

Gives a menu controlling how the display works:
Type (vectors, dots), Persist (off,1s,2s..infinite), Format (YT,XY), Contrast lighten/darken

## TRIGGERING

TRIGGER LEVEL

TRIG MENU

SET TO 50\%

## Input and Output Connectors

Connector
$\mathrm{CH} 1, \mathrm{CH} 2$

EXT TRIG

PROB COMP

Function

These jacks connect the input signal to the respective vertical amplifiers. Grounds go directly to scope's chassis and earth gnd.

Connects a trigger signal that is independent of CH 1 and CH 2 .

A square wave of 5 V peak-to-peak at 1 kHz is provided to allow for the checking of the calibration of the Vertical channels and for adjusting 10X probe compensation.

Another ground connection to scope chassis and earth ground.

## Equipment Note: BNC to Banana Plug Adapter

The coaxial cables in the lab have BNC type connectors. The outer shield of the coaxial cable and of the BNC connector is ground. There are several types of adapters to change from BNC connectors to unshielded wires with banana plugs. These adapters usually have a tab to indicate which banana plug goes to ground. The converters also have holes for cascading multiple banana plug connections. This diagram shows the female-BNC to male-banana version.


## Equipment Note: Function Generator

Like the oscilloscope, the function generator will be used in a number of different experiments, so you should become familiar with the controls. Most of our function generators have a control panel as shown.


The frequency generator is shown adjusted to give a 1 kHz sine wave.

| FREQUENCY | frequency = dial setting $x$ multiplier range |
| :--- | :--- |
| AMPLITUDE | Varies peak-to-peak voltage of output |
| OUTPUT | Use the output jack on the upper right |
| OFFSET | Allows DC offset to be added to the AC signal |
| DUTY | Changes the duty cycle or symmetry of output waveform <br> (fully counterclockwise on CAL gives a 50\% duty cycle) |
| FREQUENCY READOUT | Updated once every counting period as signaled by GATE light. |

## Prelab Questions

1. In Part II a voltage divider is used to decrease standard line voltage to a safe level when plugged into an electrical outlet. The output voltage of the voltage divider is $1 / 10$ th of its input voltage. Choose convenient horizontal and vertical oscilloscope scales to display the voltage divider's output. Use the grid to sketch how the oscilloscope display would appear with the scales you have chosen and indicate the scales in the blank provided. Assume that triggering level is set to near zero volts and the slope to falling edge triggering. (See equipments notes.) Scales are always multiples of 1,2 or 5 . For example: $1 \mathrm{~ms} / \mathrm{div}, 0.5 \mathrm{~s} /$ div or 20 $\mathrm{mV} /$ div.


Scales: Horizontal: $\qquad$
$\qquad$ /div; Vertical: $\qquad$ $\overline{\text { units }} /$ div
2. Here is a picture of a BNC-to-Banana plug adapter. Circle the banana plug which is connected to ground when it is properly connected.


# Lab 4 - The Magnetic Field 

## References

Tipler (6th ed.): 26-1,2; 27-2,4.
Halliday, Resnick and Krane, (5th Ed.): 32-5; 33-4.

## Objectives

I. To trace the magnetic field of a permanent magnet.
II. To create a magnetic field with an electric current.
III. To investigate the force on a moving charge in a magnetic field and to measure this force using the current balance.
IV. To create an electric field with a changing magnetic field.

## Apparatus

Permanent bar magnet
Compass
Large sheet of white paper for field tracing
Galvanometer frame
Solenoid with current balance attachment, weights
Cathode Ray Oscilloscope
Ammeter

## Introduction

## Magnetic Fields

In the first part of this experiment, you will investigate the magnetic field of a bar magnet. The direction of the magnetic field is the direction of the force on a north magnetic monopole if it was placed in a magnetic field. A north magnetic monopole is a magnet with only a north pole. Unfortunately, none have ever been found to exist, so we'll have to make do with something else: a compass. If we place a compass in a magnetic field, its north pole points in the direction of the magnetic field. Figure 1 illustrates the magnetic field of the Earth.

## Forces on Currents in Magnetic Fields

Suppose you place a loose wire in a uniform magnetic field. (See Fig. 2.) The field may be either the field of a permanent magnet or the field produced by a current that we control. If no current flows through the wire, it will droop under the force of gravity. When current passes through the wire, the wire is pushed across the magnetic field. A number of experiments can establish the rules describing the force produced by a magnetic field on a current.


Fig. 1: (a) The Earth seems to have a gigantic bar magnet embedded in it with a South pole in the northern hemisphere. (b) The field lines show the direction of the horizontal component of the magnetic field on the Earth's surface. (c) The complete field map reveals that there is a significant vertical component to the Earth's magnetic field at most places. (From Physics for the Inquiring Mind, E. Rogers, Princeton University Press, 1960.)
(i) The force is perpendicular to both the magnetic field and the current. If your nose points in the direction of the field and a current passes from left to right across your field of vision, the force will be upwards. Another way to describe the direction is to stretch out your right hand. Your fingers point in the direction of the field; your thumb in the direction of the current; and the force will point straight outwards from the palm of your hand.
(ii) By changing the strength of the field without changing its direction we find that the force is proportional to the magnitude of the field. By changing the direction of the field without changing its magnitude we find that only $B_{\perp}$, the field component perpendicular to the wire, contributes to the force. Therefore, the force is proportional to $B_{\perp}$, the magnitude of the perpendicular component of the magnetic field.
(iii) The force, $F$, is proportional to current, $I$. If one places two short straight wires of the same length side by side and carrying equal currents they would together experience twice the force of each wire alone. Combine the two wires and you have one wire carrying twice the current experiencing twice the force. Thus, doubling the current doubles the force.
(iv) The force, $F$, is proportional to the length of wire, $l$. Place two wires of the same length end to end carrying the same current. Again the force on both wires together doubles: Doubling the length of the wire doubles the force.


Fig. 2 (a) A thin wire hangs loose in a magnetic field because there is no current flowing through it. (Adapted from PSSC Physics, 1st ed.)

(b) When current flows through the wire in a magnetic field a force acts on it which is perpendicular to both the current flow direction and the magnetic field.

Combining these four results we have

$$
F=(\text { constant }) I_{\text {wire }} l B_{\perp}
$$

It is most convenient to choose the units of $B$ to be consistent with the units of $I$, $l$, and $F$, already defined so that the constant equals 1 . Then

$$
\begin{equation*}
F=I_{\text {wire }} l B_{\perp} \tag{4}
\end{equation*}
$$

and the unit of $B$ is $\frac{\text { newton }}{(\text { ampere })(\text { meter })}$ which is called a tesla, T.
The current balance uses a solenoid to create a magnetic field of known strength. The magnetic field inside a solenoid is constant, except near its ends, and directed along its axis. You can verify this by placing a small compass inside at various positions. It can be shown that the magnitude of the field is proportional to the number of turns per metre, $n$, and the current in the wire, $I_{\mathrm{Sol}}$.

$$
\begin{equation*}
B_{\mathrm{sol}}=\mu_{\mathrm{O}} n I_{\mathrm{sol}} \tag{5}
\end{equation*}
$$

where $\mu_{\mathrm{O}}$ is a constant. In the current balance, a wire of length $l$ is placed near the centre of the solenoid so that the current in the wire flows perpendicular to the magnetic field. The wire is on a printed circuit board, which pivots around an axis just outside of the solenoid. A balancing mass is placed on the opposite side of
the pivot point to the wire. If the arms of the balance are of equal length and the forces balance, then


Fig 3: How to "weigh" the magnetic field.
Solving for $B_{\text {Sol }}$

$$
\begin{equation*}
B_{\mathrm{sol}}=\frac{m g}{I_{\text {wire }} l} \tag{6}
\end{equation*}
$$

Therefore, we can measure the magnetic field in the solenoid by measuring the force on a wire of length $l$ in the field carrying a current $I_{\text {wire }}$ which is at right angles to the field.


Fig. 4: The current balance consists of a current-carrying conductor on a balance arm inside a solenoid. The magnetic field of the solenoid is perpendicular to the direction of current flow at the end of the arm.

## Induced EMF

Put a loop of wire in a uniform magnetic field. Let $A$ represent the area encircled by the wire and $B_{\perp}$ the component of the magnetic field perpendicular to the surface enclosed by the wire. The quantity $A B_{\perp}$ is called the magnetic flux, $\Phi$.

$$
\Phi \equiv A B_{\perp} \text { by definition. }
$$

The units of magnetic flux are Webers which are equivalent to $\mathrm{Tm}^{2}$ or $\mathrm{Nm} / \mathrm{A}$. This definition of flux will prove useful in describing the EMF induced by magnetic fields.

Please Note: In this experiment, you will measure magnetic fields. When doing so, you must keep all other sources of magnetic fields, mainly magnets but also current-carrying wires, away from the area where you are making measurements. Another magnetic field in your working area is the earth's. It is small but
obviously strong enough to affect the compass you will use for some of your measurements.

Please check your compass before you start. Sometimes, the magnetization of a compass needle can get reversed so that north points south. This is likely when the compass has been around strong magnetic fields. A reversed compass can be un-reversed. Ask for hints!

## Procedure

## I. Tracing the field of a permanent magnet

Before you start, explore the magnetic field on your work area with your compass. Try to find an area with a uniform background field and identify the direction of magnetic north. Lay a large sheet of paper out, and place the bar magnet in the middle of it with its north pole pointing towards magnetic north. Remove all other bar magnets from the area, and try to keep the magnetic environment constant throughout your measurements. Trace the position of the magnet on the paper. To trace the magnetic field, follow the procedure below:

1. Put a dot somewhere on the paper next to the north pole of the magnet.
2. Place the compass on the paper so that the south end of the needle is over the dot.
3. Make a dot where the north end of the compass needle is.
4. Move the compass so that the south end of the needle is over the new dot.
5. Repeat 3 and 4 until you think you have enough points or run out of paper.
6. Connect the dots with a smooth curve and put arrows on the line in the direction of the magnetic field.
7. Go back to 1 , choosing a different point near to the magnet. Seven lines from pole to pole starting at $45^{\circ}$ intervals from one of the bar magnet's poles gives a good field map.


Fig 5: Using a compass to map the magnetic field of a bar magnet.
Remove the magnet and label the north and south poles on the paper.
Compare your field map with those done in the Electric Field Mapping Experiment. Which electric field map of that experiment most nearly resembles the magnetic field of the bar magnet?

How would the field map change if you cut the magnet in half between the N and S poles and removed the piece labelled S ?

## II. Magnetic Fields from Currents

(a) Magnetic field of a current in a straight wire

Connect a long wire between the positive and negative outputs of the power supply. Orient the wire vertically and place a compass near the wire. Turn
on a current of more than 2 amps . Describe what happens to the compass needle as you switch the current on and switch it off. Try the compass at various positions around the wire.
(b) Magnetic field of a current loop

Take the galvanometer frame (the set of loops of wire on a plastic frame) and orient it so that the wires point in a north-south direction. Connect a power to the single loop and run a current of 1 to 2 ampères through it.

Use the "current source" mode of the power supply:
-Initially, the Voltage setting should be higher than zero and the Current set at zero (totally counterclockwise).

- Start to raise the current very slowly by gently turning the current knob clockwise.
-Look at the power supply's ammeter to measure the current.
-If the current reaches a plateau and doesn't increase as you turn the current knob clockwise, then the voltage setting is too low and needs to be increased a little.

Investigate the magnetic field around the loop using the compass. Compare the field map of the bar magnet with what you observe for the coil.

Now place the compass in the loop so that the needle is directly under the wire. At first, when there is no current, the needle should be aligned parallel to the wire. Vary the current through the loop while watching the compass needle start to turn. Increase the current until the needle points $45^{\circ}$ from north (either northeast or northwest). Record this current. The compass is under the influence of two magnetic fields: the Earth's, pointing north (more or less), and the current loop's, pointing east or west. When the compass is at $45^{\circ}$, the two fields have equal magnitude.

Next, place the compass under the 20 -turn loop, and align the needle with the wire, as before. Run current through the set of 20 loops, and find how much current needed to move the compass to $45^{\circ}$. The current is now much smaller, and the ammeter is less accurate at these levels. Nevertheless, you should be able to see the expected relationship of magnetic field, $B$, to the number of loops, $N$, and the current, $I$.


Fig 6: The magnetic fields of a coil of wire and the earth add like vectors.

## III. Force on a Moving Charge in a Magnetic Field

(a) Moving electrons in a cathode ray oscilloscope

Cathode ray oscilloscopes have an electron beam, which travels from the rear to the front, to produce an image on the fluorescent screen. There are cathode ray oscilloscopes in the lab for this experiment. Reduce the intensity knob
completely counterclockwise. Turn on the oscilloscope, and place the Horiz Mode switch on XY. Increase the intensity until you see a spot on the screen. Using the vertical and horizontal position dials, centre the dot on the screen. Keep the intensity low so the electron beam doesn't damage the screen and turn it off immediately after you finish.

Use your field map of the bar magnet to help you orient the bar magnet horizontally in front of the oscilloscope screen so that $\mathbf{B}$ is perpendicular to the electron velocity with N on the left, S on the right. Slowly move the bar magnet towards the screen in this orientation, and sketch the relative directions of $\mathbf{B}, \mathbf{v}$, and $\mathbf{F}$, as measured by the beam's deflection. Reverse the direction of $\mathbf{B}(\mathrm{N}$ on the right), and again sketch the $\mathbf{B}, \mathbf{v}$, and $\mathbf{F}$ directions.

From these observations, devise your own "rule" to help you remember the direction of the force on an electron moving in a magnetic field. Use your rule to predict the deflection of the beam when the bar magnet is held vertically near the screen with N on top, then S on top. Try it to test the prediction.
(b) The Current Balance: "Weighing" a magnetic field

Mount the current balance arm in the solenoid as shown in Fig. 7. Orient the balance so that the Earth's field runs parallel to the wire on which we are going to measure the force. The current within that section of wire will then feel no net force from the Earth's field. It may help to clean the pivoting contact with a piece of sandpaper to insure good electrical contact

Connect the power as shown. The switch and ammeter are those internal to the power supply. Note that in this case we use the same current through the solenoid and through the wire on the balance arm. (It would be better to have independent current supplies for the solenoid and the current balance arm. In order to have two independent currents, work with another group so that you can use two power supplies. The circuit must then be changed accordingly.)


Fig. 7: Connect the current from one power supply so that it goes through both the solenoid and the balance arm in series.

Clip a (non magnetic) paper clip on the arm of the balance at a position such that the arm is balanced horizontally when no current is flowing.

Fold a file card so that you can stand it next to the balance and mark the "balance" position with a tick mark.

Make sure that the balance is swinging reasonably freely by setting it swinging. If it stops within a second, there's too much friction; check to see whether the balance rubs against the sides of the solenoid. Place small masses on the outside end of the balance arm at a distance from the pivot equal to that of the current-carrying wire, which is perpendicular to $B$ on the arm in the solenoid. Increase the current until the arm comes back to its balance position. Add mass (small plastic washers), each time measuring the current needed to balance the swinging arm. (Note: The mass of a small plastic washer is comparable to the balance resolution which is 0.01 g . How can you minimize the uncertainty in this measurement?)

For each value of $m$ and $I_{\text {wire }}$, calculate $B_{\text {Sol }}$ from equation (6). The value of $n$ for your solenoid should be marked on it. Plotting $B_{\text {sol }}$ vs. $I_{\text {Sol }} n$ should give a straight line with slope $\mu_{\mathrm{O}}$. Alternatively, you can calculate $\mu_{\mathrm{O}}$ for each data pair. The second method is equivalent to forcing through the origin the straight line of the $B_{\text {Sol }}$ vs. $I_{\text {Sol }} n$ graph. This might not be justified if there is a systematic error in the data resulting in a nonzero intercept. Do your data indicate such a systematic error?

## IV. Induced EMF (qualitative) (Optional)

(a) Induction with a bar magnet and loop

Remove the current balance attachment and all wires from the solenoid. Hook the solenoid up to the oscilloscope. Set the time/div scale on the oscilloscope to $0.1 \mathrm{~ms} /$ div. Take the bar magnet and bring its north pole near to the solenoid. Quickly push the magnet into the solenoid, watching the trace on the oscilloscope screen. If you don't see anything, increase the sensitivity of the voltage setting and adjust the trigger setting. Sketch the approximate shape of the scope trace, making sure to record the V/div setting. Pull the magnet out of the solenoid, once more sketching the oscilloscope response. Repeat the above procedure with the south pole of the magnet, and on the other side of the solenoid. Sketch the apparatus, making sure to note the sense of the current in the solenoid.

Do the same for a single loop of wire and for multiple loops, which you can make yourself by wrapping wire around a suitable form. Try to keep the speed at which you move the magnet the same for both. Which creates a larger voltage: the single loop or multiple loops? How many times larger does it appear to be? Compare what happens when you move the loop instead of the magnet.


Fig. 8: Changing the magnitude of the field through a loop of wire by
moving the bar magnet back and forth and changing its direction by twisting the bar magnet.

Can you find any induced EMF in the loop for the following experiments? How does the rate of the changes affect the size of the EMF?

- Bring the north or south pole of the magnet close to the centre of the loop. Quickly twist the bar magnet so the direction that the field passes through the loop changes, but the magnitude of the field stays about the same at the centre of the coil. (Refer to your bar magnet field map.)
-Hold the magnet close to the loop, and quickly reduce the loop's area by tightening it like a noose. You may try to thread the wire through a washer or similar device to aid this attempt.

Summarize your findings about the induced EMF caused by the following factors:

1. rate of change in the magnitude of magnetic field passing through a loop;
2. rate of change of the direction of the magnetic field passing through a loop;
3. rate of change of the area of loop with magnetic field passing through it;
4. number of loops.

Show that a formula involving the flux $\Phi$ can encapsulate all these results.
(b) Inducing AC signals using a solenoid

Connect the function generator to the solenoid, and drive an AC signal of about 1000 Hz through it. Choose the sinusoidal waveform. Monitor the AC signal from the function generator on Ch1 of the oscilloscope. Connect a long plain wire between the signal input and ground input of Ch 2 . You may need to use a coaxial cable to take the signal picked up from the test loop to the oscilloscope input. This helps eliminate noise pickup when you are viewing small voltage signals.

Wrap the wire around the solenoid, and adjust the scope to see whether you can pick up the signal from the solenoid.

Try the following experiments:

1. Compare the signal strength picked up by one loop around the solenoid when the frequency generator is at $500 \mathrm{~Hz}, 1000 \mathrm{~Hz}$ and 2000 Hz . (Keep other parameters, such as loop size, the same.)
2. Compare the signal strength picked up by one loop wrapped tightly around the solenoid to that picked up by a much larger diameter loop. What happens when the loop does not encircle the solenoid?
3. Compare how the signal strength varies when the loop's orientation with respect to the magnetic field of the solenoid is changed: e.g., Surface area encircled by the loop is normal to the magnetic field vs. at a $45^{\circ}$ angle.
4. Make a much smaller loop by wrapping the insulated hookup wire around one finger. (approximately 1 cm diameter). Put the loops inside the solenoid, oriented perpendicular to the magnetic field. Compare the signal strength picked up to that picked up when the loop was encircling the solenoid and outside it.
5. Tilt the loop at a $30^{\circ}$ to $45^{\circ}$ angle with respect to field. Is there any difference in signal strength?
6. Double (or triple) the loop's area by wrapping the wire around two (or three) fingers. Insert the loop inside the solenoid, with the field perpendicular to the loop. Compare the signal strength.
7. Switch the function generator from a sine to a triangle wave. Reduce the frequency to around 250 Hz to reduce problems caused by the selfinductance of the solenoid. Observe and explain the difference in the induced waveform

## Prelab Questions

1. A horizontal magnetic field $\mathbf{B}$ is perpendicular to the horizontal component $\mathbf{B}_{\mathrm{e}}$ of the earth's magnetic field and points east.
(a) The ratio $B / B_{e}=\sqrt{3}$. In which direction will the compass needle point? (The compass needle rotates in the horizontal plane and assume $\mathbf{B}_{\mathrm{e}}$ points north.)
(b) If the compass needle points northwest, what is the magnitude of $\mathbf{B}$ ?
2. The magnetic field inside a long solenoid is uniform and parallel to the solenoid's axis. (See fig. 4.)
(a) What is the force on that portion of the current carrying wire inside the solenoid that runs parallel to the solenoid's axis?
(b) The portion of the current carrying wire inside the solenoid that is perpendicular to the axis is 2 cm long and carries 1 A current. The field inside the solenoid is $1.0 \times 10^{-2}$ tesla. What is the force on the wire? How much mass would it take to balance the force if it were placed at an equal distance from the pivot?
(c) Describe how you would use this setup as an ammeter.
3. A solenoid is 2 cm in diameter and has 3720 turns $/ \mathrm{m}$. AC current with a triangular waveform is passed through it. The current's amplitude is 1 A and frequency is 1 kHz . A circular loop of wire with a diameter of 1 cm is placed inside the solenoid with the loop's axis oriented parallel to the solenoid's. In theory, the flux through the loop is in phase with the current' and has the same waveform. On graph paper, graph the current showing the current scale on the left-hand vertical axis and, on the righthand vertical axis, show a scale of the magnetic flux through the loop. (See figure.)


Below this graph, show the graph of the induced EMF in the loop, which is proportional to the time rate of change of the flux.
On the current graph, sketch another current waveform having the same amplitude but double the frequency. (Use a dotted line or another colour.)
On the EMF graph, sketch the EMF induced by the 2 kHz current.

# Introduction to Optics Equipments 

## Introduction

Our understanding of the world is largely determined by sight. How visual sensations relate to material objects and how these sensations are transmitted is a fascinating puzzle whose pieces have been assembled slowly through the centuries. Our common understanding of the visual process has benefited from efforts of scientists and philosophers of preceding generations. Even the least scientifically educated person probably understands that the eyes passively receive light reflected from objects and that light is produced by sources such as the sun. This view is not self-evident. Ancient Greeks debated whether the eye sends out "feelers" in order to see rather than passively receiving its sensations. Observations and logic have long since refuted the "feeler" theory but other debates have persisted. The relationship of the retinal image to the mind's visual impression is still under investigation. More fundamentally, philosophers of science are trying to reconcile the basic relationship of the observer to the observed with recent findings of quantum physics.

Most naive observers intuitively understand light to be like a stream of particles or like rays. This model accounts for many of our everyday experiences. An elaborate particle theory of light flourished for many years under the patronage of Sir Isaac Newton even though it failed to explain some easily observed phenomena. The wave theory of Christian Huygens gained favour as ever-more experiments showing interference effects gave results that disagreed with expectations based on the particle theory. Later, the photoelectric effect was discovered, and the wave theory had to be changed. What resulted was a waveparticle model in which light particles, called photons, managed to retain many wave-like properties. (Or should we say that light waves managed to acquire particle properties!?)

As you experiment with light, you will try to understand what you see by making analogies to things familiar in your previous experience: light acts like a stream of pellets or like ripples in water. At first, these models help you understand; later, they fall short. Finally, you should come to accept light in its own right and realize that it is unlike anything else you know at all.

The following codes are used throughout the optics labs:
(E) Error analysis is required for this part.
(C) Less detailed instructions are given to encourage you to be creative.

## Equipment Note: The Pasco Introductory Optics System

Figure 1 shows all the equipment in the OS-8500 Introductory Optics System. The system also includes a fitted box with cutouts for each component. Please replace all components in their places after each lab period before returning the box.


Figure 1: Equipment included in the OS-8500 Introductory Optics System

## Optics Bench



Figure 2: Using the optics Bench
The optics bench is shown in Figure 2. The light source, component holders and ray table base all attach magnetically to the bench as shown. For proper optical alignment, the edge of each of these components should be mounted flush to the alignment rail which is the raised edge that runs along one side of the bench.

Note: Avoid scratching or otherwise abusing the surface of the magnetic pads. If they get dirty ask for them to be cleaned by the technician.

Incandescent Light Source


Figure 3: Using the light source
The light source is shown in Figure 3. To turn it on, connect the power cord to any grounded 105-125 Vac receptacle, and flip the switch on the rear panel to ON . If at any time the light fails to come on, check with your instructor.

The filament knob on the top of the unit moves the light bulb from side to side. The notch at the bottom indicates the position of the light bulb filament, so that accurate measurements can be made during experiments.

## Component Holders

The optics set comes with three regular component holders (Fig. 4) and one holder designed for use with the ray table. The regular component holders attach magnetically to the optics bench as in Figure 5. The ray table holder sits on the edge of the circular ray table. The notch at the top of each holder is for centring components on the holder. The notches in the base of the regular holders are for accurate distance measurements on the bench scale.


Figure 4: Component holder


Figure 5: Using The Component Holder


Figure 6: Component Alignment

## Lab 5 - RAY OPTICS, REFLECTION AND CONCAVE MIRRORS

## References

Tipler (6th ed.): 32-1: Mirrors.
Halliday, Resnick and Krane, (5th Ed.): 40-1,2,3.

Objectives (Suggested times in parentheses are approximate.)
I. Observe straight-line propagation of light. ( 15 min .)

II Locate an object by tracing rays. ( 30 min .)
III Study the law of reflection ( 45 min .)
IV Use ray tracing to locate a virtual images formed by a mirror ( 30 min .)
V Measure magnification and focal length of a cylindrical mirror ( 30 min .)
VI Form an image using a concave mirror and measure its magnification ( 30 min .)
VII Use the Virtual Image Locator to locate virtual images formed by mirrors. ( 30 min .)

## Apparatus

From the Optics Kit: Optics Bench, Light Source, Ray Table and Base Component Holders (2), Slit Plate, Ray Table Component Holder, Viewing Screen, Concave Mirror, Virtual Image Locators, Ray Optics Mirror, Slit Plate. Additional: 11x17 inch paper and a plane mirror


Fig. 1.1: Basic Ray Optics Setup

## Introduction

Many useful properties of light can be understood in terms of a simple ray theory. In these experiments, you will observe light as it travels through air of uniform density. In this case, light propagates in a straight line from its source. One can use a slit plate to select "rays" of light coming from a point source. These rays can be traced back to their origin, where they cross at their source.

If light encounters a shiny surface, it undergoes specular reflection. Rays reflect from mirrors according to the law of reflection: the angle of incidence equals the angle of reflection where the angle is measured from the normal to the reflecting surface. The rays reflected from a plane or convex mirror will not cross, but if one extends them back in a straight line to the other side of the mirror, these extensions will cross at a point called the virtual image of the source. The virtual image is at the position from where the light appears to come. In these experiments, we emphasize point sources because any object can be considered as a collection of point sources. Therefore, the virtual image of any object can be understood as a collection of virtual images of point light sources.

The law of reflection implies that the virtual image formed by a plane mirror will be at the same distance from the mirror as the object, but on the other side. If we form a virtual image using a concave mirror, then the law of reflection tells us that the object-image distance is given by the following formula:

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{r}=\frac{1}{f}
$$

where $s$ is the object-mirror distance, $s^{\prime}$ is the image-mirror distance, $r$ is the mirror's radius of curvature and $f$ is its focal length. The magnification of the mirror is given by

$$
m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}
$$

where $y^{\prime}$ and $y$ are the image and object heights respectively. (See Tipler, 3rd edition, p. 1016.)

When two objects are at different distances from an observer, the two objects appear to move with respect to each other when the observer moves. This visual phenomenon is called parallax. If two objects are at the same distance from the observer, then there is no parallax. Images exhibit the same parallax as objects. Therefore, if an object and a virtual image are at the same distance from the observer, they will not move with respect to each other as the observation point changes. Parallax can be used to locate virtual images and is the principle of the Virtual Image Locators.


Fig. 1.2: Ray Tracing

## Procedure

## Part I - Preliminary Observations on Straight-Line Propagation

Set the equipment as shown in Fig 1.1. You may leave off the viewing screen at first. Plug in the light source and turn it on. The bulb has a linear filament running up and down.

1. Does light travel in a straight line from its source? Observe the rays that emerge from the slit plate. Look at the rays on the ray table then lower your head and peer along a ray. Are you sure that the rays do not bend as they pass through the slits?
2. Find how to get sharp rays. Vary the distance between the slit plate and the light source, and note how the width, brightness and sharpness of the rays change. Observe how the distinctness and the direction of the rays change as you rotate the slit plate so that the slits are not parallel with the light bulb's filament.

Summarize your findings of steps 1 and 2 using only a few sentences.

## Part II - Locating the Filament (E)

Locate the position of the filament by tracing on a piece of paper the rays coming through the slits. ( $\mathcal{E}$ means do full error analysis.)

1. Mark the rays on the paper. Fold a piece of 11 x17 inch paper in half to make an $8.5 \times 11$ inch sheet. Tape the paper on the ray table with the fold facing the filament and directly above the ray table base. Turn on the light, and trace four rays that cross the paper.

Hint: If the rays are wide, it may be better to trace the edge of the rays.
You may mark two points of each ray before removing the paper. Choose points that are widely separated for greater accuracy.
[Do not discard the raytracing paper. One partner should attach it to his or her notebook, and the other partner should note its location.]


Fig. 1.3: Single Ray Setup
2. Extend the rays back to locate the source. Remove the paper and unfold it. Use a clear plastic ruler to trace the rays back to the source. Mark every point where two rays cross. Find the average distance between the ray table edge to the filament by means of the ray-crossing points you located. Estimate the uncertainty of this average.

Hint: How many crossing points should there be? If two points coincide, include that distance twice in the average. If you estimate the uncertainty of the average by calculating the standard deviation (SD), note that SD is the standard deviation of the distribution of individual points. The standard deviation of the average is given by $\mathrm{SD} / \sqrt{N}$ where $N$ is the number of points. As you measure more and more points, your accuracy increases even though the points may get more scattered.
3. Measure the distance between the light filament and ray table edge using the optics bench scale. Compare this distance with your ray tracing result. Do the results agree within experimental error? If not, what sources of possible error have you neglected when you estimated uncertainties?

## Part III - Measuring the Law of Reflection

Set up the equipment as shown in Figure 1.3. Adjust the components so that a single ray of light is aligned with the bold arrow labelled "NORMAL" on the ray table degree scale. Carefully align the flat reflecting surface of the ray optics mirror with the bold line labelled "COMPONENT". With the mirror properly aligned, the bold arrow on the ray table is normal (at right angles) to the plane of the reflecting surface. Turn on the light and make sure that you can see the incident ray and reflected ray on the ray table as you rotate it.

1. Measure angles of incidence and reflection. Choose about six different angles ranging from $0^{\circ}$ through $90^{\circ}$. Rotate the ray table to each of these incidence angles $\left(\theta_{\mathrm{i}}\right)$ and measure the corresponding angle of reflection $\left(\theta_{\mathrm{r} 1}\right)$. Repeat your measurements with the incident ray coming from the opposite side of the normal $\left(\theta_{\mathrm{r} 2}\right)$. Tabulate these data directly in your lab notebook.
2. Analyze your data and determine the law of reflection. What criteria must you use to determine whether your data agree with the law or not? Are the results from the two trials the same? If not, why not?

Part of the law of reflection states that the incident ray, the reflected ray and the normal to the reflecting surface lie in the same plane. Is this statement consistent with your experiments?


Fig. 1.4: Alignment of the Ray Mirror for normal incidence and for incidence at an arbitrary angle


Fig. 1.5: Finding the Virtual Image of the Filament.

## Part IV - Using Ray Tracing to Locate the Virtual Image (C)

In this part, you will use the method developed in part II to find the position of the virtual image of a filament formed by a plane mirror. The basic idea is illustrated in Figure 1.5. The (C) means that you may use your own ideas and be creative in doing this part. Take notes clearly describing the procedure in your lab notebook. Analyze the data and discuss the results. [Do not write on the ray table.]

## Part V - The Focal Length of a Concave Mirror

The image formed by a plane mirror is always virtual. A concave mirror can form either a real image or a virtual image. A real image occurs if the rays come from a distant source. If the source is closer than one focal length from the mirror, then a virtual image is formed. If the source is exactly at the mirror's focal point, the rays emerge parallel from the mirror. This is the principle used in flashlights, headlights and spotlights to produce a parallel beam of light.

Ideally, concave mirrors have a paraboloidal shape. Spherical shapes are cheaper to manufacture than paraboloids, and most inexpensive concave mirrors are spherical. For most applications, the loss in quality is insignificant, particularly where the mirror's $f$-number (focal length/diameter) is large. In the following exercise, you will use the cylindrical ray optics mirror to visualize its image-forming properties and find the mirror's focal length by producing a parallel beam of rays.

1. Mount the slit plate and ray table. Place the slit plate directly on the light source and move the ray table so its edge touches it. Use the ray table surface with a centimetre grid, and turn it so that the grid is aligned with the optical bench axis.
2. Observe the real image formed by a concave mirror. Place the ray optics mirror on the ray table; turn on the light; and observe the rays reflected from the concave surface. When the mirror is at the far end of the table, you should see the reflected rays converging at the position of the real image.
3. Rotate mirror to show aberration. If you turn the mirror so that the image position is off the optical axis of the bench, you will notice that not all rays converge at the same place. This is because the mirror's shape is circular instead of parabolic. This effect is known as spherical aberration when it occurs in spherical mirrors.
4. Measure the source-mirror and image-mirror distances and magnification. Orient the ray mirror so that the image is exactly on the optic axis. Record the object-to-mirror distance, the image-to-mirror distance. Slide the light source box a few mm away from the alignment rail,
 image.
5. Find where the mirror gives parallel reflected rays. Slide the mirror closer to the light source until the reflected rays diverge. The rays don't actually cross now, but their projections on the dark side of the mirror will cross at the position of the virtual image. Move the mirror back until the emerging rays are parallel. Measure the distance between the light source and the mirror. Remove the mirror from the table, and have your partner repeat this measurement to get two independent values for the focal length

## VI - Forming an Image with a Concave Mirror.

If you cut a strip along any diameter of a spherical mirror you get something close to a cylindrical mirror. It's not surprising then that we can understand spherical mirrors from the properties of cylindrical mirrors studied in the last part.

1. Arrange components. Set up the equipment as shown in Fig 1.7 with the concave side of the mirror facing the light source. The viewing screen should cover only half of the hole in the component holder so that light from the filament reaches the mirror.
2. Find focal point of a distant object. Estimate the focal length of the mirror by positioning the mirror on the optical bench as far from the crossed arrow target as possible. Vary the position of the viewing screen to find where the image of the target is focussed. Record this approximate value of the focal length and have your partner repeat the measurement independently.


Fig. 1.7: Setup to observe the image from a concave mirror
3. Measure object-mirror and image-mirror distances to get the Mirror's focal length. Choose five distances between 500 mm and 50 mm . These are object-mirror distances for which you will measure the image-mirror distances and the image heights. Calculate $f$ from $s$ and $s^{\prime}$ in each instance. Also, compare the magnification derived from $y^{\prime} / y$ with that calculated from $s^{\prime} / s$.

## VII- Using the Virtual Image Locators (optional)

1. Find the virtual image formed by a plane mirror.
(a) Put one of the virtual image locators (VIL\#1) on the near end of the optical bench. Mount the plane mirror on a component holder so that it extends above the holder's top with the shiny side facing VIL\#1 as in Fig. 1.8. Look for the virtual image in the mirror.
(b) After seeing the image, put the second virtual image locator (VIL\#2) near where the image appears to be. When you sight down the optical bench axis, you should see the image of the bottom half of the VIL\#1 in the mirror at the same time as you can see all of VIL\#2 through the clear plastic of VIL\#1. When you look exactly down the axis, both arrows should coincide, but you may observe parallax as you move your head left and right (or you can move the bench right and left). Move the second virtual image nearer or farther from you until the reflected image of VIL\#1 shows no parallax with VIL\#2. The far virtual image locator is now at the position of the virtual image formed by the plane mirror. Record the positions of the two virtual image locators and the mirror in your notebook. Compare object and image distances.


Fig 1.8: Virtual image locators and the plane mirror.
2.(C) Find the virtual image formed by a concave mirror. Use the same principle to find the virtual image formed by the 50 mm f.l. concave mirror. Develop your own method. Record the method and results in your notebook

Hints: Because the concave mirror's mount is opaque, you may want to use a straight edge to help align the images. You can mount the mirror sideways or
below the top of the component holder. In order to get the virtual image locator within 50 mm of the mirror, you will need to orient the component holders so that the short feet of the bases face each other. If the mirror's axis is very slightly misaligned with the optical bench axis, the virtual image will also be off axis. but much more so. Don't confuse this misalignment with parallax.

## Experiment Summary

| Part | Qualitative <br> Observations | Measurements | Calculations and <br> Graphs |
| :--- | :--- | :--- | :--- |
| I | straight line ray <br> propagation | none | none |
| II |  | ray positions | light filament position |
| III |  | $\theta_{\mathrm{r}}$ for six $\theta_{\mathrm{i}}$ angles |  |
| IV |  | incident and reflected <br> ray positions | position of filament's <br> virtual image |
| V | 2. cylindrical aberration | 3. displacement of <br> image for 10mm <br> displacement of source | 3. magnification |
|  |  | 4. source-mirror <br> distance for parallel <br> rays |  |
| VI |  | 2. apx focal length. <br> 3. image-mirror <br> distances and image <br> heights for five source- <br> mirror distances. | 3. focal lengths and <br> magnifications derived <br> both from $y^{\prime} / y$ and from <br> $s^{\prime} / s$ for each of the five <br> measurements. |
|  |  | 2. position of virtual <br> image formed by <br> concave lens. |  |
| VII | 1. lack of parallax when <br> virtual image is <br> coincident with an <br> object. |  |  |

## Prelab questions

1. When you measure the law of reflection, you are supposed to make two measurements for each angle: one clockwise from the normal, the other counterclockwise. Why do that?
2. A student stands $x=120 \mathrm{~cm}$ in front of a plane mirror, wearing a wizard hat. The distance $h$ from his toes to the top of his hat is 220 cm .
(a) Determine the location of the image of the student by ray tracing.
(b) What is the minimal vertical length of a plane mirror in which the student can see his full image (from his toes to the top of his hat)?


# Lab 6 - Refraction and Thin Lenses 

## References

Tipler (6th ed.): 31-4: Polarization. Halliday, Resnick and Krane, (5th Ed.): Ch44: Polarization.

## Objectives

I. Measure the Law of Refraction: Snell's Law ( 45 min .)
II. Use Ray Tracing to explore focussing properties of a cylindrical lens ( 20 min .)
III. Observe Total Internal Reflection and Dispersion ( 15 min .)
IV. Measure image to object relationships with a thin spherical lens ( 45 min .)
V. Explore depth of field and observe chromatic aberrations. ( 20 min .)
VI. Build a simple magnifier and measure its magnification. ( 20 min .)
VII. Build a compound microscope. ( 15 min .)
VIII. Build a telescope ( 15 min )

## Apparatus

From the Introductory optics kit: Optical bench, ray plate and base, slit plate and mask, cylindrical lens, ray table component holder, convex lenses ( 75 mm and 150 mm ), crossed-arrow target, viewing screen, virtual image locators, variable aperture, component holders.

## Introduction:

Light travels in a straight line as long as the medium is optically isotropic. (Isotropic means that there are no changes in the medium that would cause light not to travel in a straight line.) One of the most common situations where light deviates from a straight line is at the boundary of two different optically isotropic media such as air and acrylic plastic. If the angle of incidence is not normal (at right angles) to the boundary then the direction changes according to Snell's law:

$$
n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}
$$

The constants $n 1$ and $n 2$ are constants called indices of refraction and depend on the two media. The angles $\mathrm{q}_{1}$ and $\mathrm{q}_{2}$ are the angles the light ray makes with the normal to the boundary between the two media.

An important application of refraction is lenses. It is simplest to assume that the lenses are thin and spherical. The focal length of a thin spherical lens is determined by the radii of curvature of its two surfaces according to the lensmaker's formula. Let $r_{1}$ and $r_{2}$ be the radii of the spheres defining the two surfaces of the lens. The focal length of the lens is

$$
\frac{1}{f}=(n-1)\left(\frac{1}{r_{1}}-\frac{1}{r_{2}}\right)
$$

Our sign convention assumes $r$ is positive if the incident ray hits the convex side and negative if it hits the concave side. (This sign convention may be different in some textbooks.) The lens-maker's formula follows directly from Snell's law, where the angles of incidence on the spherical surfaces are small so that $\sin \theta \approx \theta$.

Illuminated objects can be thought of as a collection of point light sources. Therefore, when we study how lenses make images, we usually consider only a point source and assume that images of many point sources can be superimposed to get the image of a complex object. Light rays fan out in all directions from a point source and some of them hit the lens. If the source is very far from the lens, the rays hitting the lens are almost parallel with each other and are bent by the lens so that they all converge at the focal point a distance $f$ from the lens. If the source is not at infinity, the image position is given by

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f}
$$

where $s$ is the source-lens distance and $s^{\prime}$ the lens-image distance. (This equation is the same as for mirrors.)

The optical axis of the lens is the line joining the centres of the two spheres forming the surfaces of the lens. If a point source is on the optical axis of a lens, its image is also on the optical axis. If the source is not on the optical axis, its image will also be off the optical axis. The position of the image can be found by tracing two of the rays coming from the source. Special rays, called the principal rays, are easy to trace if we follow some simple rules:


1. Rays hitting the lens parallel to its optical axis are bent to cross the axis at the focal point.
2. Rays going through the centre of the lens are not deflected.
3. Rays from the source passing through the focal point before hitting the lens, emerge parallel to the optical axis after having passed through the lens.
One can use these rules repeatedly together with a little geometry and trigonometry to find the positions of images and the magnification of thin lens systems.

## Procedure

## Part I - The Law of Refraction: Snell's Law (E)

1. Align a ray along the table's optical axis. Mount the light source on the optical bench. Place the slit plate and slit mask on opposite sides of a component holder so that the centering notch points to the slit that is not covered by the slit mask. Put the ray table on its base, degree side up, and align all the components against the alignment rail as in Fig. 1.4. Turn on the light source and turn the filament knob and adjust the slit plate until one single ray coincides with the NORMAL line on the ray table.
2. Measure angles of incidence and refraction. Place the cylindrical lens on the COMPONENT line of the ray table so that the ray hits the flat side of the lens. Choose several angles ranging from $0^{\circ}$ to $90^{\circ}$, and measure the angle of refraction for each angle of incidence chosen. You can make two measurements for each angle: one clockwise from the normal, the other counterclockwise. This will compensate for slight misalignment of the lens on the table. Plot your points roughly as your take the data.
3. Check the reversibility of Snell's Law. Put the cylindrical lens on the other side of the COMPONENT line so that the ray hits the curved side. Measure some of the same angles you used in the last section to check the reversibility of Snell's law. What do you conclude?

Graph your results. Is your graph linear? Do the data from parts 2 and 3 agree? If not, try to modify your graphing methods so that all the data follow a linear relationship. What does the slope mean? What does the intercept mean?

## II - Focusing Properties of Cylindrical Lenses

In this part, you will use the Parallel Ray Lens to produce parallel rays, which seem to come from infinity. The Parallel Ray Lens is often called a collimating lens.

1. Measure focal length of the cylindrical lens. Remove the Slit Mask from the Slit Plate and turn over the Ray Table so that the centimeter grid is faceup. Turn on the light. Make sure you see rays diverging from the Slit Plate; then put the Parallel Ray Lens between the light source and the Slit Plate and adjust its position until the rays are parallel and the middle ray follows the centre line of the Ray Table. Place the cylindrical lens on the Ray Table so that the rays hit the flat side at normal incidence with the middle ray passing through undeflected. Ideally, all the rays should converge at a point on the table on focal length from the lens, as in Fig. 2.2. Because the lens is not thin, all the rays do not converge at one point. You can use your thumb and a finger to block all but three middle rays that cross near the ideal focal point. As you successively unblock the other rays, you can see the effects of cylindrical aberration. Measure the focal length from the curved side. [Don't mark on the Ray Table!]
Turn the lens so that the rays hit the curved side. Measure the focal length again from the curved side. Does the focal length depend on which side the rays strike first?

## III - Dispersion and Total Internal Reflection

The index of refraction in most media depends on the wavelength. For the acrylic lens, this means that different colours emerge from the lens at different angles. This effect, known as dispersion, is most obvious for large angles of refraction near $90^{\circ}$.

1. Observe dispersion. Leave the Parallel Ray Lens in place and insert the Slit Mask over the Slit Plate so that only one central ray is transmitted. Rotate the table so that the ray emerges from the Cylindrical Lens near $90^{\circ}$. You should be able to see the ray split into a rainbow of colours from blue to red on the table. You can mount the viewing screen on the Ray Table Component Holder and place this on the table to view and measure the angular dispersion of the colours. Measure the angles of the two extremes of the visible spectrum you see: blue and red. Estimate their respective refractive indices.

When light rays hit the surface between media of different refractive indices, there is always a reflected ray. There may or may not be a refracted ray going into the other medium. When the index of refraction of the medium from whence the light ray comes is larger than the index on the other side then there is an angle of incidence above which no ray is transmitted through the surface. The critical angle is the angle of incidence for which the refracted ray's angle is exactly $90^{\circ}$. Light incident above the critical angle is totally reflected. This is called total internal reflection.


Fig. 2.2
2. Measure the critical angle. Leave the Parallel Ray Lens and Slit Mask in place. With the Cylindrical Lens on the Ray Table, rotate the table until the critical angle is reached. Measure this angle. You may wish to use one of the coloured filters because the critical angle will depend on which colour you look at.

## IV - Object-Image Relationships of a Convex Lens (E)

You will now progress from the cylindrical ray-tracing lens to a real convex lens and observe the relationship between object position and image position and how these distances relate to magnification of the image.

1. Find the depth-of-field uncertainty for focussing. Set up the Crossed Arrow Target, the 75 mm lens and the Viewing Screen as shown in Fig 2.3. With the light on, move the lens until a sharp image of the target appears on the Viewing Screen. Observe the characteristics of the image: Is it magnified or reduced? Inverted or upright? About how far can you move the screen back and forth with the image still appearing sharp? This range is called Depth-of-Field and will be explored later. For now, recognize that it contributes to measurement uncertainty.


Fig. 2.3: Setup for measuring object-image relationships
2. Measure object and image distances and magnifications. Choose five or six $s$ distances between 50 and 500 mm . Measure and tabulate the respective $s^{\prime}$ values. Also, record the height of the image for each case so you can calculate the magnification.
3. Analyze the object image distances for the focal length and compare theoretical with measured magnifications. There are at least two ways to analyze the data: graphically, plotting $1 / s$ vs. $1 / s^{\prime}$ or numerically, calculating $1 / s+1 / s^{\prime}$ for each measurement. Choose one method. There are several pitfalls you should be aware of. If you graph the data, you should think about whether you need to constrain either the slope or the intercept to a certain value or not. The error analysis should indicate a more certain result if you use all five measurements than if you just used only one measurement. If you do your measurements by fixing the target-screen distance and then focusing the lens, the errors in $s$ and $s^{\prime}$ are correlated: they are not independent. In our discussion of error propagation, we have assumed that the variables are independent. You should express the
equation for $f$ in terms of statistically independent variables before applying error propagation rules.

## Part V - Depth of Field and Chromatic Aberrations.

1. Find $\boldsymbol{f}$-number and depth-of-field for the $\mathbf{7 5} \mathbf{~ m m}$ lens. Measure the diameter of the 75 mm lens and calculate its $f$-number. ( $f$-number $=$ f.1./diameter.) Position the 75 mm lens to project an enlarged image of the light bulb's filament on the screen. Record the range of viewing screen positions within which the image appears sharp. Call this the depth-of-field. Since this judgement is subjective, you and your partner should do it independently.
2. Find the diameters and depths-of-field for smaller f-numbers. Common $f$-stops (jargon for $f$-number) marked on photographic lenses are 1.4, 2, 2.8, $4,5.6,8,11,16$, and 22 . Small $f$-stops allow more light but have small depth-of-field. Conversely, large $f$-stops have large depth-of-field but cut down the light accepted. Calculate the diameters for $f / 8$ and $f / 16$ for a 75 mm lens. Set the Variable Aperture to give square apertures of these sizes. (Ideally, you want a round aperture, but most camera apertures are polygonal.) Place the aperture centred on one side of a component holder and the lens on the other. Measure the depth-of-field at each $f$-stop, and note the relative brightness of the images.
3. Observe chromatic aberration. Using a small aperture (large $f$-number), place the aperture off centre and look at the image formed. Because of the dispersion of the acrylic lens, you should see colour fringes. Note the relative positions of the red and blue fringes and explain in terms of the lens' dispersion. Why is the effect worse when the aperture is off centre?

## Part VI - The Magnifier

When an object is between a converging lens and its focal point, a virtual image is formed which appears magnified and not inverted. Because the image is virtual it cannot be projected onto a screen; however, a viewer may see it by looking through the lens. This is the secret of Sherlock Holmes.

The angular magnification of a magnifier is given by the angle subtended by the image, wherever it may be, divided by the angle subtended by the object at the eye's near point, $x_{\mathrm{np}}$. Usually, $x_{\mathrm{np}}$ is set at 25 cm , so the magnification is given by $m=$ $x_{\text {np }} / f=25 / f$, where $f$ is in cm .


Fig. 2.4: The Magnifier

1. View magnified image through $\mathbf{1 5 0} \mathbf{~ m m}$ lens. Place the viewing screen at the 250 mm mark on the optical bench. Place the 150 mm lens between the 0 and 250 mm mark. With one of your eyes at the 0 position, look through the lens to see a magnified image of the scale on the viewing screen. Move the lens back and forth until you find the largest magnification that is in focus. Where is the image? What screen-to-lens distance puts the image at infinity?
2. Measure magnification of the lens. Try to estimate the angular magnification by observing squared paper through the lens. Keep the paper about 25 cm from your eye. You should see the unmagnified squares around the lens and magnified squares through the lens. Count how many unmagnified intervals fit in a magnified interval. Compare with $x_{\mathrm{np}} / f$.
3. Repeat step 2 with the 75 mm lens.

## VII - The Compound Microscope

The magnifier is part of a compound microscope. In the microscope the eyepiece (or ocular) magnifies the enlarged real image projected by the objective lens at or near the focal point of the eyepiece. Here, we use the 150 mm lens as eyepiece. Put the crossed arrow target near the far end of the optical bench and project a magnified image of it onto the viewing screen using the 75 mm lens. You will probably have to illuminate the target with the light source.

The 150 mm lens, used as eyepiece, can be positioned to view the image. After you can see the image in the viewing screen through the eyepiece, remove the screen and turn off the light. Now you can view the magnified image directly through both lenses. To see the best image, place your eye $10-20 \mathrm{~cm}$ behind the eyepiece, not right up against it.

Try to estimate the magnification of the microscope by comparing the magnified target scale to the scale on the viewing screen when it is placed 25 cm from your eye. This is a little harder than for the magnifier or telescope. You should calculate the theoretical magnification using the formulae derived from the microscope's geometry.

In Figure 2.5, the objective forms an inverted real image of the object as shown. This first image is labelled the intermediate image in the diagram. It acts as the effective object for the eyepiece lens. The eyepiece then forms a virtual final image somewhere between infinity and the near point of the eye ( 25 cm ). The distance between the inside focal points of the objective and the eye-piece is the "optical tube length" $L$, and it is usually standardized at 18 cm in modern microscopes.

The overall magnification of the microscope is the product of two terms: the magnification of the objective and the magnification of the eyepiece. We will consider only magnitudes and not use a sign convention.
(i) The magnification of the objective is found from analysing the similar triangles in Fig. 2.5:

$$
\begin{gathered}
m_{\mathrm{O}}=\frac{\text { size of intermediate object }}{\text { size of object }} \\
m_{\mathrm{O}}=\frac{q_{\mathrm{O}}-f_{\mathrm{O}}}{f_{\mathrm{O}}} \approx-\frac{L}{f_{\mathrm{O}}}
\end{gathered}
$$



Fig. 2.5 Geometry of a Compound Microscope
(ii) The magnification of the eyepiece:

The purpose of using the microscope is to increase the size of the retinal image (in the eye). Therefore, angular magnification, which depends on the angle subtended at the eye, should be calculated. It can be shown that the angular magnification is given by

$$
M_{\mathrm{e}}=\frac{\text { angle subtended by magnified virtual image }}{\text { angle subtended by object seen by naked eye }}
$$

$$
=\frac{x_{\mathrm{np}}}{f_{\mathrm{e}}}=\frac{25 \mathrm{~cm}}{f_{\mathrm{e}}}
$$

if the virtual image is at infinity and $x_{\mathrm{np}}$ is the eye's near point. Thus the overall angular magnification of the microscope is given by

$$
M=m_{\mathrm{O}} M_{\mathrm{e}}=-\frac{L}{f_{\mathrm{o}}} \frac{x_{\mathrm{np}}}{f_{\mathrm{e}}}
$$

Manufacturers generally label objectives and eyepieces (e.g. x10, x40) using

$$
m_{\mathrm{O}}=\frac{18 \mathrm{~cm}}{f_{\mathrm{o}}} \text { and } M_{\mathrm{e}}=\frac{25 \mathrm{~cm}}{f_{\mathrm{e}}} .
$$

## VIII - Make a telescope (C)

Look up on any physics textbook how to make a telescope with the 75 mm and 150 mm lenses. Find the expression relating the lens focal lengths and their positions to the magnification of the telescope. Construct the telescope and measure its magnification by observing a distant scale.

## Other things to try

-Look at the dispersion of white light produced by a prism. Find the index of refraction as a function of the ray's angle of deviation when the rays enter and leave the prism symmetrically (the angle of minimum deviation).
-Ask for a concave lens and convex lens to build a Galilean telescope.
-Build a reflecting telescope using a convex lens and the concave mirror.

## Prelab Questions



1. The figure shows a lens focussing the image of a candle onto a white screen. Answer the following questions
a) Is the image real or virtual?
b) If the image is larger than the object, which is greater: the source-lens distance or the lens-image distance?
c) If the screen is removed, is it still possible to see the image from some position? If no, explain why not. If yes, state the position and direction the observer must look to see it.
2. The cylindrical lens of Fig. 2.2 of the lab script has a diameter of 5 cm . What would you expect its focal length to be in the thin-lens approximation? (Choose a reasonable index of refraction.)
3. Fig. 2.6 includes a ray diagram (using "standard" rays) for the formation of the intermediate image by the objective lens. Complete the following ray diagram for the eyepiece lens alone using this intermediate image as an object and show the location of the final image.


## Lab 7 - Polarization and Colour

## References

Tipler (6th ed.): 31-4: Polarization. Halliday, Resnick and Krane, (5th Ed.): Ch44: Polarization.

## Objectives

I. Produce and detect linearly polarized light ( 20 min .)
II. Change polarization angle of linearly polarized light with a half-wave plate ( 10 min .)
III. Produce and detect circularly polarized light ( 20 min .)
IV. Change the handedness of circularly polarized light ( 15 min .)
V. Analyze circularly polarized light with a linear analyzer and vice versa ( 10 min .)
VI. Produce linearly polarized light by reflection at Brewster's angle ( 20 min .)

Take a sudden-death quiz on polarization ( $0-25 \mathrm{~min}$.)
COLOUR
I. Observe colours produced by combining red, blue and green light. ( 30 min .)
II. Observe the transmitting and reflecting properties of coloured filters. ( 20 min .)

## Apparatus

From the Optics Kit: Optical Bench, Light Source, All component holders, Polarization Filters, Ray Table and Base, Viewing Screen, Slit Plate, Slit Mask, Parallel Ray Lens, red, blue and green filters.

Other Stuff: Additional linear polarizer, Left circular polarizers: 2, Right circular polarizers: 2, Quarter-wave plate, Half-wave plate

## Introduction: Polarization

In this session, you will learn how to produce and detect polarized light. The first series of short exercises will show you the properties of the filters in the polarization kit. After you know the properties of the polarizers and retardation plates, you will try to determine the polarization state of light coming from various ports of the "mystery box." Your laboratory report should list the polarization state of the light emitted from the ports of the mystery box and should describe the method used to reach your conclusions. There are four different ports, and each port has one of seven possible polarization states. The seven possibilities are

1. Unpolarized light
2. Linearly polarized light with the plane of polarization at angle $\theta$ to be determined
3. Right circularly polarized light
4. Left circularly polarized light.
5. Elliptically polarized light
6. A mixture of linear and unpolarized light
7. A mixture of circular and unpolarized light

## Theory

To describe light, one must specify its frequency, its direction of propagation and its state of polarization. In this lab, we are interested in polarization, so let us assume that we have monochromatic light propagating along the $+\hat{\mathbf{z}}$ direction of a right-handed coordinate system. Light is a transverse electromagnetic wave-the electric field is always perpendicular to the direction of propagation. Because the direction of propagation is along the $+\hat{\mathbf{z}}$ axis, the electric field vector $\mathbf{E}$ must lie in the plane formed by the $x$ and $y$ axes. This can be expressed mathematically as follows:

$$
\begin{equation*}
\mathbf{E}(x, y, z, t)=E_{x}(z, t) \hat{\mathbf{x}}+E_{y}(z, t) \hat{\boldsymbol{y}} \tag{1}
\end{equation*}
$$

The components of the electric field $E_{x}$ and $E_{y}$ do not depend on $x$ and $y$ because we assume that the wave is a plane wave propagating along the $+\hat{\boldsymbol{z}}$ direction.


Fig. 3. 1: The electric field vector of linearly polarized light
Light is linearly polarized if the electric field vector $\mathbf{E}$ is always parallel to the same line which is perpendicular to the direction of propagation. Mathematically

$$
\begin{equation*}
\mathbf{E}(z, t)=A_{x} \hat{\mathbf{x}} \cos (\omega t-k z)+A_{y} \hat{\mathbf{y}} \cos (\omega t-k z) \tag{2}
\end{equation*}
$$

The amplitudes $A_{x}$ and $A_{y}$ are real constants. The intensity of the light is proportional to the square of the amplitude. Figure 1 shows the electric field vectors at a fixed time along a line in the direction of propagation. Figure 1a illustrates light polarized along the $\hat{\mathbf{x}}$ direction $\left(A_{x} \neq 0, A_{y}=0\right)$ and 1 b shows polarization along the $+\hat{\mathbf{y}}$ direction $\left(A_{x}=0, A_{y} \neq 0\right)$. As time goes on, the entire pattern moves in the $+\hat{\mathbf{z}}$ direction. Light polarized in an arbitrary plane is a superposes these two independent possibilities $\left(A_{x} \neq 0, A_{y} \neq 0\right)$. The plane of polarization is determined by the relative magnitudes of $A_{x}$ and $A_{y}$.

Light is circularly polarized if the electric field vector moves in a circle. As there are two senses of rotation, there are again two independent polarization states, left and right:

$$
\begin{array}{ll}
\text { left } & \mathbf{E}(z, t)=A_{\mathrm{L}}[\hat{\mathbf{x}} \cos (\omega t-k z)+\hat{\mathbf{y}} \sin (\omega t-k z)] \\
\text { right } & \mathbf{E}(z, t)=A_{\mathrm{R}}[\hat{\mathbf{x}} \cos (\omega t-k z)-\hat{\mathbf{y}} \sin (\omega t-k z)]
\end{array}
$$

Figure 3.2 shows the electric field vector at a fixed time for the two circular polarization states. As time increases, the pattern moves in the $+\hat{\mathbf{z}}$ direction. If we look into the oncoming beam and track the electric field vector at any point, it moves in a circle with constant radius. By convention, clockwise rotation is


Fig. 3.2: Left and right circular polarization
called right circularly polarized light because the helical path traced by the direction of electric field at any fixed time follows the threads of a right-handed screw.

The most general polarization state is a superposition of $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ linearly polarized light with arbitrary amplitudes and phases. This general polarization state can also be considered as a superposition of left and right circularly polarized light with arbitrary amplitudes and phases of the two circular components.

Most light sources do not produce polarized light. Individual atoms in the source radiate independently. Although at any instant in time light received from a radiating atom in the source has a definite state of polarization, the state of polarization changes rapidly with time. You may think of unpolarized light as having two polarization components (left/right or $\mathbf{x} / \mathbf{y}$ ) which are radiated independently and randomly. Between polarized light which has a fixed relation in time between the amplitude and phase of the two polarization components and unpolarized light which has a random relation in time between the two polarization components, is partially polarized light. Partially polarized light is a mixture of polarized and unpolarized light.

## Polarization Filters and Retardation Plates

A linear polarizer produces polarized light by selectively absorbing light polarized perpendicular to the polarization axis and transmitting light parallel to the transmission axis. Our HN22 linear polarizers absorb more than $99.99 \%$ of the perpendicular component and transmit $44 \%$ of the parallel component. Thus the linear polarizer transmits $22 \%$ of incident unpolarized light. A perfect polarizer would transmit $50 \%$ of incident unpolarized light.

Consider unpolarized light falling on a polarizer that transmits only the components parallel to the $\hat{\mathbf{x}}$ axis. The filtered light has amplitude $A$ and intensity $I=A^{2}$. Let this filtered light fall on a second polarizer whose polarization axis $\hat{\mathbf{x}}$ ' is at an angle $\theta$ with respect to the $\hat{\mathbf{x}}$ axis. The incident light has components $A$ $\cos \theta$ and $-A \sin \theta$ along the $\hat{\mathbf{x}}^{\prime}$ and $\hat{\mathbf{y}}^{\prime}$ axes. If both polarizers are perfect, the
intensity after the second polarizer would be $|A \cos \theta|^{2}$ but for the nonideal polarizers the transmitted intensity is $0.44|A \cos \theta|^{2}$.

You will use two retardation plates in this lab: a quarter-wave plate and a half-wave plate. Both plates are made of a clear plastic film whose index of refraction depends on the polarization direction of the light passing through it. The material has two perpendicular axes called the fast axis and the slow axis. If the light beam passing through is polarized along the material's fast axis, the index of refraction is $n f$. Likewise if the light's axis of polarization is along the slow axis the index of refraction is $n_{S}$. The velocity of light in a material is $c / n$. Light polarized along the fast axis travels faster than that polarized along the slow axis implying $n_{f}<n_{S}$. Light polarized along an arbitrary direction can be resolved into components parallel to the fast axis and parallel to the slow axis. Because the faster travelling component emerges from the material first, the two polarization components, which were in phase upon entering the film, are shifted out of phase when they emerge. The phase shift, $\delta$, is given by

$$
\delta=2 \pi\left(n_{S}-n_{f}\right) \frac{d}{\lambda}
$$

where $d$ is the thickness of the film and $\lambda$ is the light's wavelength.
If $\left(n_{S^{-}} n_{f}\right) d / \lambda=1 / 4$, then the slower light component will emerge from the film $1 / 4$ wavelength behind the faster component. The half-wave plates are designed so that the two polarization components are $1 / 2$ wavelength out of phase after they have passed through the film: $\left(n_{S}-n_{f}\right) \mathrm{d} / \lambda=1 / 2$. Both the quarter- and half-wave plates work ideally for only one wavelength in the middle of the visible spectrum. The phase shift for other wavelengths deviates from exactly one-quarter or exactly one-half.

Combining linear polarizers and retardation plates
Several interesting effects occur when polarizing filters are combined with either a quarter-wave plate or a half-wave plate.

1. Linear polarizer followed by half-wave plate.

Consider a linear polarizer whose transmission axis is at an angle $\theta$ with respect to the fast axis of the half-wave plate. When the light emerges from the half-wave plate, the component polarized along the slow axis will lag the fast component by $180^{\circ}$ or $\pi$ radians. Shifting a sine wave by $180^{\circ}$ is the same as inverting it (multiplying by -1 ). Therefore, the plane of polarization is changed from $\theta$ to $-\theta$. See Fig. 3.3(a).

## 2. Linear polarizer followed by quarter-wave plate

Let the transmission axis of the linear polarizer be $\pm 45^{\circ}$ with respect to the fast axis. In this case, the slow component is shifted by $90^{\circ}$ or $\pi / 2$ radians. Shifting a sine wave by $\pi / 2$ changes it to a cosine wave. With the polarization at $45^{\circ}$ to the fast axis, both fast and slow components will have the same amplitude, but the $90^{\circ}$ phase shift will change linear polarization to circular polarization. If $\theta$ is $+45^{\circ}$, the light emerges left circularly polarized (Fig. 3.3(b)); if $\theta$ is $-45^{\circ}$, it is right circularly polarized.
3. Circular polarized light falls on a half-wave plate.

Circular polarization results when one polarization component lags the other by $90^{\circ}$. A half wave plate will add an additional $180^{\circ}$ lag to one component giving a total $270^{\circ}$ or $3 \pi / 2$ radians. A $3 \pi / 2$ lag (phase shift $=-3 \pi / 2$ ) appears the same as a $\pi / 2$ lead (phase shift $=+\pi / 2$ ). Thus, right circular polarization is transformed to left circular polarization or left to right.

Fig. 3.3(a):
A half-wave plates changes the angle of linearly polarized light to its negative.


Fig. 3.3(b):
Linearly polarized light passing through a quarterwave plate at $45^{\circ}$ becomes circularly polarized.
incident



Fig. 3.3(c):
A half-wave plate changes the handedness of circularly polarized light.

transmitted


## Experiment

## I. Production and Detection of Linearly Polarized Light

For the following experiments, mount the light source and filters on the optical bench as shown in figure 3.4.
A. Verify that light from the light source is unpolarized.

Look at the light source through a linear polarizer while rotating the polarizer through $360^{\circ}$. If the light source is unpolarized, no variation in intensity will be observed.
B. Verify that a linear polarizer can be used as a linear analyzer.

Produce linearly polarized light with a linear polarizer. Look at this light through a second linear polarizer, the analyzer. When the analyzer is rotated through $360^{\circ}$, alternate maxima and minima are seen every $90^{\circ}$. If the axis of the first polarizer is set at an angle $\theta$, then maxima are seen when the analyzer is set
at $\theta$ and $\theta+180^{\circ}$, and minima are seen when the analyzer is set at $\theta+90^{\circ}$ and $\theta+270^{\circ}$.
C. Understand the following non-intuitive experiment.

Mount two linear polarizers some distance apart, with their polarization axes oriented at $90^{\circ}$ with respect to each other. Observe that no light is transmitted through the pair. Mount a third linear polarizer between them. Upon rotation of the middle polarizer through $360^{\circ}$, maximum and minimum transmission is observed alternately every $45^{\circ}$.


Fig. 3.4: Experimental setup for exploring polarization

## Part II. The Half Wave Plate and Linearly Polarized Light

A. Verify that a half-wave plate has no effect on unpolarized light

Look at the light source through the half-wave plate while rotating the half-wave plate through $360^{\circ}$. (The half-wave plate has its fast and slow axes parallel to the edges.) Do you observe a variation in intensity?
B. Verify that a half-wave plate rotates the plane of polarization of linearly polarized light.

Use a linear polarizer to produce light polarized at an angle $-\theta$ with respect to the vertical. Next, mount the half-wave plate with its axes vertical and horizontal. Look at the light passing through the linear polarizer and the halfwave plate through a second linear polarizer. Observe minimum transmission when the polarization axis of the second polarizer is set at $\theta+90^{\circ}$ and $\theta+270^{\circ}$ and maximum transmission when its polarization axis is set at $\theta$ and $\theta+180^{\circ}$. The half-wave plate has rotated the plane of polarization from $-\theta$ to $+\theta$.
C. Verify that a half-wave plate has no effect on linearly polarized light when one of its axes is parallel to the plane of polarization.

Just as in the experiment above, mount two linear polarizers some distance apart with their polarization axes oriented at $90^{\circ}$ with respect to one another. Observe that no light is transmitted through the pair. Mount the halfwave plate between the two polaroids. Observe that no light is transmitted when the axes of the half-wave plate are parallel to the axes of the crossed linear polarizers.

Part III. Production and Detection of Circularly Polarized Light
A. Verify that a circular polarizer run backwards acts as a circular analyzer.

Produce left circularly polarized light by allowing light from the light source to exit the polarizer through the face marked "left circular polarizer." Look at this light through a left and then a right circular polarizer run backwards as a circular analyzer, i.e., through the faces marked "left circular analyzer" and "right circular analyzer." The left circular analyzer will pass the left circularly polarized light. The right circular analyzer will stop the left circularly polarized light.

Rotation of either the polarizer or the analyzer through $360^{\circ}$ has no effect. (Actually, you may find a small effect because the circular polarizers and analyzers are not perfect.)

Repeat the experiment with right circularly polarized light.
B. Verify that light from the lamp is not circularly polarized.

Look at the light source through a left and a right circular analyzer. When either analyzer is rotated through $360^{\circ}$, do you observe a variation in intensity?
C. Verify that a circular polarizer passes circularly polarized light.

Produce left circularly polarized light by allowing light from the light source to exit the polarizer through the face marked "left circular polarizer." Look at this light through a left circular polarizer through the face marked "left circular polarizer." Upon rotation of either polarizer through $360^{\circ}$, no effect is seen. Unlike the linear polarizer, the circular polarizers have a front and a back.

The same effect should be seen when right circular polarizers are used.
D. Verify that light exiting a circular polarizer run backwards as a circular analyzer is linearly polarized.

Look through a linear polarizer at light exiting a left or right circular polarizer run backwards as a circular analyzer. Observe alternate maximum and minimum transmission through the linear polarizer when the linear polarizer is rotated through $360^{\circ}$. Light exiting the analyzer is linearly polarized.
E. Make a circular polarizer with the combination of a linear polarizer and a quarter-wave plate.

Mount a linear polarizer with its polarization axis at $45^{\circ}$. Next, mount a quarter-wave plate with one of its axes vertical. For your convenience, the quarter-wave plate has been cut so that its axes are parallel to its edges. Look at the light transmitted through the combination with a right and then by a left circular polarizer run backwards as a circular analyzer. Light will be passed by one and stopped by the other. The combination produces circularly polarized light.

Rotate the quarter-wave plate by $90^{\circ}$, and again look at the light transmitted through the combination through the two circular analyzers. If the left (right) circular analyzer had passed (stopped) the light, it will now stop (pass) the light, and if the right (left) circular analyzer has stopped (passed) the light, it will now pass (stop) the light. Both left and right circularly polarized light can be produced by the combination of linear polarizer and a quarter-wave plate.

Mount a linear polarizer with its polarization axis at an arbitrary angle not $45^{\circ}$, say $30^{\circ}$, for example. Next, mount a quarter-wave plate with one of its axes vertical. Look at the light transmitted through the combination with a right and then a left circular polarizer run backwards as a circular analyzer. Light will be passed by both but with different intensities. This light is elliptically polarized.

## Part IV. Changing the Handedness of Circularly Polarized Light

A. Verify that the handedness of circularly polarized light changes upon reflection by a metal surface.

Produce left (right) circular polarized light. Observe the reflected light from a shiny metal surface at near normal incidence through a left and then through a right circular polarizer run backwards as a circular analyzer. (It helps to have the mirror far from the polarizer.) The right (left) circular analyzer will pass the reflected light, and the left (right) circular analyzer will stop the light.

## B. Understand the following non-intuitive experiment:

Place a circular polarizer on the metal mirror. Turn the polarizer over. Look at the light reflected from the mirror through both sides of the circular polarizer. The two experiments will have very different effects. Do for both circular polarizers.
C. Verify that a half-wave plate changes the handedness of circularly polarized light.

Produce left (right) circularly polarized light. Next, mount a half-wave plate. Look at the light transmitted through a right and a left circular polarizer run backwards as a circular analyzer. The right (left) analyzer will pass the light, and the left (right) analyzer will stop the light. A half-wave plate changes the handedness of circularly polarized light.

Part V. More Analysis of Polarized Light.
A. Verify that circularly polarized light is a superposition of both linear components.

Look at circularly polarized light through a linear polarizer. Upon rotation of the linear polarizer through $360^{\circ}$, why do you see very little intensity variation?
B. Verify that linearly polarized light has both circular components.

Look at linearly polarized light through a left and a right circular polarizer run backwards as circular analyzers. Both analyzers will pass light.

## Part VI - Brewster's Angle (E)

When unpolarized light reflects from a nonmetallic surface, it is at least partially polarized after reflection. Reflect a ray off the flat surface of the Cylindrical Lens, and observe the reflected ray through a linear analyzer. (You may look directly at the reflected light or project it onto a screen.) Do you observe any polarization dependence of the reflected ray? Do the same observation on the refracted ray emerging from the curved surface of the Cylindrical Lens.

Now polarize the incident ray by setting the linear polarizer at $90^{\circ}$ and placing it between the light source and the Cylindrical Lens. Project the reflected ray onto a screen. Rotate the Ray Table and Cylindrical Lens, and observe the intensity of the reflected ray as a function of angle. If you see a minimum or maximum in the reflected intensity of the polarized ray, note the angle at which this occurs. Also, record the angles of the reflected and refracted rays at this position, and calculate the angle between them. Does this angle agree, within experimental error, with what you expect from the theory of Brewster's angle in your textbook?

Set the polarizing filter at $0^{\circ}$ and try the same experiment.

Try the same experiments with the Ray Optics Mirror.


Fig. 3.5: Setup to observe polarization of reflected light and to measure Brewster's angle

## COLOUR

## Introduction

One of the most fascinating properties of light is colour. What makes light coloured? What is the difference between a coloured object and a black or white one? Consider two possible theories about colour:

1. Pure light is white. When white light reflects from an object or passes through a transparent coloured material, white light is modified to produce coloured light.
2. Pure light exists in various colours. The different colours are different species of light. White light is obtained by mixing certain colours in right proportions. Objects are coloured because they selectively transmit or reflect certain colours better than others.

You may know that theory 2 is currently accepted. But if you had never been thought about the matter, theory 1 would probably seem more reasonable. Both descriptions agree with common experience, and even the dispersion phenomenon you explored last time could be explained within either framework. Here is a case where new experiments need to be devised in order to decide between two theories. The next two exercises give you a chance to see situations you wouldn't ordinarily encounter. Do their results help distinguish between the two colour theories?

## Procedure

## Part I - Combination of Colours

Put the Slit Plate between the light source and the Ray Table, and arrange the red and blue-green filters so that the centre three rays are red, green and blue. Mount the Viewing Screen on the end of the ray table and place the Parallel Ray Lens in front of the ray table so that the three central rays converge at a point. Move the Lens or the Viewing Screen so that the red, blue and green rays all converge along the same line on the Viewing Screen. If you do this correctly, the superposition of the three coloured lines will appear white. Now cover each of the colours in sequence and observe the colour formed by the superposition of the other two.

You can probably easily understand what you see in terms of theory 2 . Try to devise an explanation for the colour effects you produced using theory 1 . Can you think up a completely convincing explanation? Is it simpler or more complex than the explanation in terms of theory 2 ?


Fig. 3.6: Combining red, blue and green rays.

## Part II - Reflection and Transmission of Coloured Filters

Set up the Ray Table and Slit Plate so that white rays pass over the table top. Hold the Green Filter by hand in the rays so that you can see both reflected and transmitted rays.
What colour are the transmitted rays? What colour are the reflected rays.

Place the Red Filter behind the Green Filter so that the light passes first through the Green Filter and then through the Red Filter. Look into the Green Filter. What colour are the reflected rays now? Which rays are reflected from the front surface of the Green Filter, and which are reflected from the front surface of the Red Filter?

Place the Blue Filter over the Slit Plate so that the rays are blue. Let these rays pass through the green filter only. What colours are reflected now? Based on your observations, what makes the Green Filter appear green?

Other things to do
Filters are used in photography to produce special effects. If the day is clear (ha, ha), go outside and look at the sky while turning the polarizing filters. Look at various angles with respect to the sun, but don't look into the sun itself. Do you know what effects the filter produces?

Red filters are used in black-and-white photography for the same purpose that the polarizing filter is used in colour photography. Try looking through the red filter to see its effect. What purpose could a blue filter serve in black-andwhite photography?...in colour photography?

## Prelab questions

1. What is Brewster's angle? Draw a diagram to explain how to measure Brewster's angle using the optics kit available in the lab.
2. Explain why it may help to wear polarizing glasses when fishing.

# Lab 8 - Interference and Diffraction 

## References

Tipler (6th ed.): 33-3,4,8; 31-8.
Halliday, Resnick and Krane, (5th Ed.): 41-1,2,4; 42-1,2,3,5.

## Objectives

I. Measure slit separation using double-slit diffraction of $\mathrm{He}-\mathrm{Ne}$ laser light. II Compare single slit diffraction patterns to double-slit patterns. III. Construct and calibrate a portable diffraction-grating spectrometer and use it to measure the spectra of several light sources. IV. Observe diffraction patterns of arrays of circular apertures and a cross.

## Apparatus

From the optics kit: Optical Bench, Light Source, Ray Table Base, Colour Filters, Slit Mask, Diffraction Plate, Diffraction Grating, Diffraction Scale.

Other Stuff: He-Ne Laser, Laser Mount, Post-it Notes.
Light Sources: Na, Hg, He, H.

## Introduction

## I. Light Waves and Double-slit Interference ( $E$ )

The preceding optics experiments could be understood using geometrical optics. The interference and diffraction phenomena you will see in this lab cannot be explained by ray optics and require a wave model to be understood. When lenses, mirrors, etc. are much larger than the wavelength of light, the wave theory gives the same results as geometrical optics. After studying interference of light, you may come away with the impression that light is a wave and that one uses geometrical optics only to simplify calculations where it is applicable. However, further experiments that are beyond the level of this course show that "light = waves" cannot be the whole story. The photoelectric effect, for example, is not comprehensible unless we let light be composed of particles. But then how do we explain interference?

In 1802, the supremacy of the particle theory of light was shattered by a simple interference experiment performed by Thomas Young. He let light from a pinhole fall on two slits in an opaque screen. If the slits were close enough to each other, then the emerging light formed alternate bright and dark fringes on a screen. Newton's particle theory could not account for these bands.

A wave model proposed by Huygens proposed that each point on a wave front be the source of a spherical wave. Thus, the superposition of all the little spherical waves produced at a wave front add up to continue propagating the wave forward. Placing a screen with two slits in the path of a wave will allow only the waves passing through the slit to continue forward. Thus, two cylindrical waves emerge from the slits that can interfere with each other at the screen. If a line on the screen is an integer number of wavelengths from both slits, then the interference is constructive and a bright line results. When the screen is


Fig. 4.1: Double slit interference forms bright fringes where waves fall on the screen in phase: $m=1$ shown here.
much farther from the slits than the slits are apart, the Fraunhofer approximation holds and bright fringes occur when

$$
\begin{equation*}
d \sin \theta=m \lambda, m=0,1,2, \ldots \tag{1}
\end{equation*}
$$

where $m$ is an integer, $d$ is the distance between slits, $\lambda$ is the wavelength and $\theta$ is the angle between the direction of incidence, which is assumed normal to the slit plane, and the fringe on the screen. Interference is darkest when

$$
d \sin \theta=(m+1 / 2) \lambda
$$

Complications disturb this ideal situation. Because the slits cannot be infinitesimally small, the brightness of the fringes varies periodically as $\theta$ increases and becomes invisibly dim at larger angles. The finite extent of the fringes arises because a single-slit diffraction pattern is superimposed on the ideal double-slit pattern. The finite width of each slit allows light passing through to interfere with itself causing complete cancellation at definite angles. If light from within each slit cancels itself out, then there is none left to produce a bright fringe. In part II of the experiment you will compare the pattern of single slits to those of double slits having the same width.

## II. The Diffraction Grating

If more than two slits are used, the angle must be very precise in order for light from all the slits to reinforce simultaneously. Therefore, the bright fringes get sharper and more clearly separated. Furthermore, if the slits are made very close to each other, the angular difference between fringes of two wavelengths increases. Combining multiple slits with small slit spacing produces a diffraction grating. Diffraction gratings can be used to display the spectrum of light. We can identify the colours produced by the various wavelengths with the aid of equation (1). Furthermore, we can explore the spectra of light emitted by vapours such as mercury, sodium, neon and hydrogen.


Fig. 4.2: The grating produces successive diffraction orders, $m=0,1,2, \ldots$.

## III. Spectroscopy and Atomic Structure

An important property of an atom is its characteristic set of energy levels. Each energy level corresponds to one of the different configurations of orbital electrons allowed by the quantum theory of atomic structure. The energy levels may be shown on an energy level diagram, such as Fig. 4.3 (drawn for a hypothetical atom).


Fig. 4.3: Energy levels in a hypothetical atom
The lowest energy state of an atom is called the ground state (level $E_{1}$ in Fig. 4.3). If an atom were totally isolated from any other interactions, it would be in its ground state. An atom can be excited to higher energy states, $E_{2}, E_{3}, \ldots E_{\infty}$, through collisions with light, electrons or other atoms. A higher energy level is the result of one or more electrons going to a higher energy orbital around the atom. Only definite, discrete levels are possible. An atom in one of the excited levels can make a spontaneous transition to one of the lower levels. When the atom passes from a higher to a lower energy level, it emits the difference in energy as light. The frequency of the light is proportional to the energy change. Thus, if an atom initially in the state $E_{2}$ makes a transition to the ground state $E_{1}$, the frequency $f$ of the light is given by

$$
\Delta E=E_{2}-E_{1}=h f
$$

where $h$ is a universal constant called Planck's constant. The observation that only certain frequencies of light are emitted by atoms agrees with the existence of discrete energy levels available to the excited atom.

In optical spectroscopy, we do not measure the frequency $f$ directly but rather the wavelength $\lambda$. For any wave motion, the frequency and the wavelength are related through

$$
f=\frac{c}{\lambda}
$$

where $c$ is the velocity of the wave in the medium. Since we are dealing with wavelengths in air, $c$ is to be interpreted as the velocity of light in air for this experiment. The energy difference $\Delta \mathrm{E}$ can now be expressed in terms of the wavelength $\lambda$ through

$$
\Delta E=\frac{h c}{\lambda}
$$

In optics, it is customary to measure the wavelength $\lambda$ in nanometers $\left(10^{-9} \mathrm{~m}=1\right.$ nanometer) and the energy difference in electron-volts ( $1 \mathrm{eV}=$ the energy gained by an electron accelerated through a potential difference of 1 volt.) The relationship between the energy difference and the wavelength can be written as

$$
\Delta E=\frac{1239.8}{\lambda(\text { in } n m)} \mathrm{eV}
$$

The set of energy levels is characteristic of the atomic species, so that each atom emits its own characteristic spectrum. The hydrogen atom with only one electron has a very simple energy level structure. The energy levels are given by

$$
E_{n}=\frac{-13.6 \mathrm{eV}}{n^{2}}, n=1,2, \ldots, \infty
$$

The integer $n$ is called the quantum number of the energy level. For a lowpressure discharge, the energy levels are sharp, and the transitions from higher energy levels to lower ones produce a line spectrum. The line spectrum of the hydrogen atom is given by the Rydberg-Ritz formula:

$$
\frac{1}{\lambda}=10.9678 \mu m^{-1}\left(\frac{1}{n_{2}^{2}}-\frac{1}{n_{1}^{2}}\right)
$$

where $n_{1}$ and $n_{2}$ are the quantum numbers of the two energy levels. The visible (Balmer series) spectrum was the first to be observed and corresponds to $n_{2}=2$.


Fig. 4.4: Setup to study diffraction using the $\mathrm{He}-\mathrm{Ne}$ laser.

## Procedure

## Part I - Double-slit Diffraction (E)

Mount the He-Ne laser on the end of the optical bench. Put the Diffraction Plate on the component holder with aperture D centred. Turn on the laser and shine the light directly onto the double slit. Don't hit the clear window above the slit. Make a screen at the far end of the Optical Bench by pasting Post-it notes just below the rulings on the Diffraction Scale and mounting it on the front of the Ray Table Holder. The scale is illuminated from the back so that you can see the diffraction pattern in the dark while measuring fringe positions. Determine the slit spacing by measuring the fringe spacing and the slit-screen distance. The wavelength of the $\mathrm{He}-\mathrm{Ne}$ laser light is 632.8 nm .

Repeat the measurements necessary to determine the spacing of slits $E$.
Hints: You may wish to use the 150 mm lens as a magnifying glass. When counting fringes you can ignore the dark bands caused by the finite slit width but you should count any double-slit fringe obscured by a dark band caused by the slit width.

## Part II - Single-slit Diffraction

Align single slit $A$ in the laser beam and observe its diffraction pattern. Repeat for slits $B$ and $C$. Refer to the Equipment Notes for slit widths and record the dark band positions as a function of slit width. What are the similarities and differences among the patterns formed by double slits and single slits?

## Part III - A Diffraction-grating Spectrometer

The principle of the transmission Diffraction Grating is similar to double slits, except more slits are used, and they are closer together. Put the Diffraction Grating in the laser beam, and make sure it is straight with the rulings vertical and the label horizontal. Move the Diffraction Scale close to the grating by hand, so that you can see the various orders of diffraction from the grating. Move the Postit note screens to the ends of the Diffraction Scale, and mount the scale at a distance that gives clear first-order spots near the $10-12 \mathrm{~cm}$ marks on the scale. From the positions of the spots, the grating-to-screen distance and laser light wavelength, you can calculate the grating line spacing.

Measure the positions of the first order spots for at least five different Grating-to-Screen distances. Plot a linear graph of these spot positions versus Grating-to-Screen distance, and determine the line spacing of the Diffraction Grating from the graph.


Fig. 4.5: A diffraction-grating spectrometer
Make a scale for your spectrometer by calculating the positions where the following wavelengths of light should fall on the Diffraction Scale: 400 nm , $450 \mathrm{~nm}, 500 \mathrm{~nm}, 550 \mathrm{~nm}, 600 \mathrm{~nm}, 650 \mathrm{~nm}, 700 \mathrm{~nm}$. Mark the wavelength scale on the Post-it notes in dark black. Then check your calibration by shining the laser beam's first order spot on your homemade scale. If the spot does not fall near 630 nm , adjust the Diffraction Scale-to-Grating distance until it is correct.


Fig. 4.6: Geometry of the diffraction-grating spectrometer

## Turn off the Laser, unplug it and remove it from the optical bench.

Put the Incandescent Light Source at the other end of the optical bench so that it shines through the slit in the Diffraction Scale. Centre the Slit Mask over the Diffraction Grating on the other side of its component holder. Turn on the Incandescent Light Source, and view its spectrum by looking through the Diffraction Grating towards the scale you made. This geometry is different from that for used to project the pattern with a laser, but the angles you see are the same. You should see a rainbow of colours, from which you can estimate the wavelengths of the spectral colours from blue to red. Put a coloured filter over the light source and look at the spectral range of each filter. You can place the blue/green filter so that you can see both spectra, one on top of the other.

The following light sources should be available in the lab: hydrogen, mercury, helium and sodium. Observe the spectral lines of their light. These spectra are caused by electron transitions between atomic energy levels in the atoms. Hydrogen has only one electron per atom and therefore has the simplest atomic structure and spectrum. You should be able to see three lines of the hydrogen spectrum, red, blue and violet. Measure their wavelengths and see whether they agree with the Rydberg-Ritz formula with $n_{2}=2$. Find the values of $n_{1}$ that give the lines you have observed. Calculate the energies of the energy levels between which the electron jumps to produce the observed lines.

## Equipment Note:

## The Laser Mount

Instructions on using the laser mount will be provided in the lab.

## The Diffraction Plate

The Diffraction Plate has ten different apertures labelled A through F. Above each line of five apertures there is a clear window, which is not a part of the aperture. This window allows you to look through the plate and see a pattern formed by the aperture as well as read the Diffraction Scale through the window. You will not need to use this window with the laser experiments. Table I lists the Diffraction Plate apertures.


Fig. 4.7: Diffraction Plate
Table I: Diffraction Plate Apertures

| Pattern | No. Slits | Slit Width <br> $(\mathbf{m m})$ | Slit Spacing <br> Centre-to-centre <br> $(\mathbf{m m})$ |
| :---: | :---: | :---: | :---: |
| A | 1 | 0.04 |  |
| B | 1 | 0.08 |  |
| C | 1 | 0.16 | 0.125 |
| D | 2 | 0.04 | 0.250 |
| E | 2 | 0.04 | 0.250 |
| F | 2 | 0.08 | 0.250 |
| G | 10 | 0.06 |  |
| H | 2 (crossed) <br> I <br> 225 Random Circular Apertures (0.06 mm dia.) |  |  |
|  | 15x15 Array of Circular Apertures $(0.06 \mathrm{~mm}$ <br> dia.) |  |  |

## Prelab Questions

1. The $\mathrm{He}-\mathrm{Ne}$ laser shines through a slit in the diffraction plate that is 50 cm from the screen. What fringe spacing would you expect for slits E and F if the stated slit spacing is correct? What difference should there be between the patterns produced by slits D and E? (See Equipment Note on the previous page.)
2. In Part III, you use the diffraction grating to make a simple spectrometer. Assume the grating is 60 cm from the diffraction scale and has a line spacing of 5276 lines $/ \mathrm{cm}$. Calculate where on the diffraction scale the visible lines of the Balmer Series of the Hydrogen atom's spectrum should appear. Make a table showing wavelengths and line positions for $n_{2}=2, n_{1}=3,4$, and 5 . Of what significance are the wavelengths you calculate when $n_{1}=1$ ? How about $n_{1}=2$ when $n_{2}=2$ ?

# Physics 131 Formal Report Writing Guidelines 

In this course you will be asked to write a formal report on one of the experiments you did. Your formal report should be no longer than 1000 words (figures, tables and short captions do not count). Your report should be based on the data you took in the lab, and include all the major results obtained from that data. Your report should be typewritten. Below is a general outline to guide you in the writing of your report.

## Writing the Report

## Make an Outline:

The outline's first level consists of the major divisions of the paper. Leave lots of space between these divisions. Under each division title, list the main things you want to say, point-by-point. On the lower levels, elaborate on these points as necessary to completely explain what you want to say. You may need to rewrite the outline a time or two as more points are added and you rearrange their order for clarity. When everything you want to say is noted in the outline, form the points into sentences and paragraphs. Your paper is complete!

## The title page and abstract:

Put the title, your name, course, section and date on the top of the title page. The abstract should be placed underneath these items. The abstract should contain a brief summary of the purpose, methods and results of the experiment.

## The sections:

1. Introduction

Say what you did and why: establish the goals and rationale. What has been done before? Put the project in a global context.

## 2. Theory

Present the simple theory necessary to understand the experiments. You may treat some or all of the theory as hypotheses tested by your experiments. Use your own words. Assume the reader has some background, but refer to elementary texts in case he or she needs to brush up. Refer to a source for a more complete explanation of the theory.

## 3. Experiment

Clearly describe what you did. Do not over-use the passive. Clear and simple diagrams showing essential features of the apparatus are necessary.

## 4. Results

Show your results in graphical form, if possible. A complete table of the raw data is usually not necessary. If calculated quantities are presented, refer to the relevant equation in the theory section, but don't show the detailed steps in the calculation. Uncertainties on both the raw data and calculated quantities should be indicated, but don't show detailed error calculations or methods-these should be known by the reader.
5. Discussion or Conclusions (Could be combined with Results)

Indicate how nearly you achieved the goals mentioned in the introduction. Discuss the meaning of the results in terms of the theory presented. Do experimental results and theory agree within experimental error? If not, suggest why not, and propose what further experiments may be needed. Make sure that your conclusions follow from your experimental results, Treat your own data and results objectively, and don't try slant the discussion to agree with your preconceived notions when the data do not support them.

## Equations:

Very simple equations may be written in-line. Complex equations with lots of subscripts, superscripts, compound fractions, integrals, or summations look clumsy in-line and should be displayed centred with a blank line above and below. Any equation that is referred to elsewhere in the text should be displayed and numbered at the right margin. Always define every symbol in all equations.

## Figures:

Figures should usually have a short caption beneath them. A caption makes it easier to understand for someone browsing through the pages. If there is more than one figure, the captions should start with a figure number. The text of the paper should refer to every figure.

## Points on style:

1. Use the active voice.

## Passive

Control of the furnace is provided by a thermostat.

Dolphins were taught by researchers in Hawaii to learn new behaviour

Active
A thermostat controls the furnace.
Researchers in Hawaii taught dolphins to learn new behaviour.

Use the passive when the doer the action is unknown or unimportant.
2. Use simple rather than complex language.

## Complex

Simple
Another very important consequence of Einstein's theory of relativity that does

According to the theory of special relativity, not follow from classical mechanics is the prediction that even when a body having mass is at rest, and hence has no kinetic energy, there still remains a fixed and constant quantity of energy within this body.
3. Delete words, sentences, and phrases that do not add to your meaning.

Wordy Concise
It is most useful to keep in mind that the Diabetes mellitus refers to a whole spectrum term diabetes mellitus refers to a whole of disorders. spectrum of disorders.

In the majority of cases, the data provided by direct examination of fresh material under the lens of the microscope are insufficient for the proper identification of bacteria.

Often, bacteria cannot be identified under the microscope.
4. Use specific, concrete terms rather than vague generalities.

## Vague

Our new scrubber means big savings for Samson Power.

The sun is hot.

Specific
Aircom's scrubbing system saves Samson $\$ 7,000$ every day.

The sun is hot-almost $10,000^{\circ} \mathrm{F}$ at its surface.

The expedition was delayed for a time because of unfavourable conditions.

The expedition was delayed one week because of snowstorms.
5. Use the past tense to describe your experimental work and results.
6. In most other writings, use the present tense.

Hypotheses, principles, theories, facts and other general truths are expressed in the present tense.
7. Avoid needlessly technical language.

Too technical
The moon was in syzygy.

Maximize the decible level.
8. Keep ideas in writing parallel.

Not parallel
The tube runs into the chest cavity, across the lungs and plunges down into the stomach.

## Clearer

The moon was aligned with the sun and the earth

Turn up the sound.

Parallel
The tube runs into the chest cavity, across the lungs and down into the stomach.

## References:

Robert W. Bly and Gary Blake, Technical Writing, (McGraw Hill, New York) 1982. Library call no. T11 B63.

William Strunk Jr. and E.B. White, The Elements of Style, 3rd ed., (Macmillan, New York) 1979
Library call no. PE1408 S772 1979 (Reference)
The Elements of Style, lst ed. is available on the world wide web at http://www.columbia.edu/acis/bartleby/strunk

## Editing your writing

The following shows how to edit your text to for conciseness and clarity. The "original abstract" is typical for a first draft and is taken from a student's report in a previous semester. Comparing the original with the edited version should help you to learn how to edit your own writing.

## Original Abstract

First, I made qualitative observations investigating the nature of images produced by concave lenses and convex lenses of various focal lengths. I found that convex lenses with small
focal lengths magnified a printed page the most. The object appeared smaller through the concave lens. Distant objects observed through both concave and convex lenses are smaller but are inverted when observed through the convex lens. These observations agree with the ray approximation methods (see figures two to four) of studying geometric optics as presented by Serway [1].

Three different methods were used to determine the focal length of various lenses: the image of a distant object method; object image methods; and the autocollimation method. Using these methods, I determined the focal length of two convex lenses to within various degrees of the rated value of the lens. The autocollimation method was the most accurate with a percentage difference of $1.3 \%$ from the rated value. Object image methods were within $2.6 \%$ of the rated
focal length. The first method was the least accurate with a percentage difference of $7.0 \%$. These uncertainties are due to systematic errors in the method and different amounts of random error.

Finally, a basic microscope was constructed and its magnification measured. The magnification was approximately 100 times. Due to the very large degree of uncertainty in the apparatus, we were interested chiefly in the order of magnitude of this value. The calculated magnification was 66 .

## Revised Abstract

In this paper, I discuss qualitative observations of images through lenses, compare three methods of measuring focal lengths and describe a compound microscope made from thin lenses.

Objects always appear upright, but smaller, when observed through a concave lens. When one observes an object through a convex lens, the image is magnified and upright if the lens-object distance is less than the focal length-the magnification being
larger for shorter focal-lengthlenses. If the convex-lens-to-object distance is greater than the focal-length, the virtual image is smaller and inverted. These observations agree with the ray model of geometric optics.

I measured the focal lengths of two calibrated convex lenses using three methods: the distant-object image method; the nearobject image method; and the autocollimation method. The deviations of the measurements from the nominal values were $7 \%$, $2.6 \%$, and $1.3 \%$ respectively for the three methods. The origins of the systematic and random errors in the three methods are discussed.

Finally, I designed a compound microscope to have an angular magnification of 66 . When built, the magnification was approximately 100 .

## A Few Basic DOs and DON'Ts

## DO

Do use relatively large font and generous line spacing, e.g., 12 pt Times and 18 pt line spacing or typewriter elite (10 pt Courier) and 1.5 or double space.

Do put your name, section, student number and all of the abstract on the title page.

Do staple your paper in the upper left-hand corner.

Do use your own words to describe the experiment.

## DON'T

Don't use very small typeface with cramped line spacing., e.g., 10 pt Times and 11 pt line spacing. (There is no room to write comments.)

Don't omit your name, start the introduction on the title page, or put all of or a part of the abstract inside.

Don't put the report in a presentation cover or folder. Don't just crimp the corner and hope the pages will not fall apart.

Don't copy large sections of the lab manual or other book.

Don't use the imperative: "Adjust the string's length to 1 m ."

Don't scan in or photocopy fancy pictures from the lab manual or other books.

Don't tear out or photocopy graphs from you lab book and stick them in your formal
report.


[^0]:    ${ }^{1}$ We assume that the factors $A$ and $M, L$, etc. affect $T$ independently, so that changing $M$ here does not affect the $T v s$. $A$ story. That is often a safe assumption in physics. In some other fields of study - such as biology, psychology, or economics - this would be a very dangerous assumption.

